Quality Variation and Quantity Aggregation in Consumer Demand for Food

Julie A. Nelson

Viewing the problem of "quality" variation in consumer demand for food as a problem of valid aggregation over goods leads to new insights. The simple sum of physical quantities, used as the measure of demand in the "quality" literature, is found to be a theoretically arbitrary and potentially misleading measure of demand when goods are heterogenous. Alternative measures of demand derived from restrictions on quality variation, consumer preferences, or relative prices, are investigated. A hypothetical example illustrates the use of a Hicksian composite commodity assumption. The empirical as well as conceptual merits of the various measures are discussed.

Key words: aggregation over goods, demand for food, Hicksian composites, quality variation.

The usual assumption made about prices in a cross-section analysis of household food expenditures is that all households are facing the same prices; estimation of price elasticities of demand is commonly left to time-series researchers who get price information from intertemporal indices. Recently, however, there has been renewed interest in the potential of cross-section analysis for estimation of price elasticities of demand for food, especially when surveys collect data on both household expenditures for food items and on the physical quantities purchased. Because we all know that "price times quantity equals expenditure," division of observed expenditure by observed quantity would seem to give the lacking price observation. Some researchers (e.g., Timmer and Alderman, Timmer) have used this simple definition. Others (e.g., Deaton 1986, 1987, 1988; Cox and Wolhgenant) have recognized that such a calculated "price" may reflect not only differences in the prices facing households (over which they presumably have no control) but also differences in quality levels of the commodity (over which households may have considerable choice). A higher price paid for beef, for example, might reflect the purchase of steak rather than hamburger. None of these works, however, have considered that quality, in addition to complicating the definition of price for demand analysis, also complicates the definition of quantity. This paper seeks to remedy this oversight by setting out the theoretical issues involved in the definition of quantity, examining potential conceptual solutions, and giving a hypothetical example of the implementation of one conceptual solution.

The following section sets out the theoretical issues and points out the problems with current formulations. The third section puts the quality issue into an aggregation theory context and describes three possibilities for defining theoretically valid quantity aggregates. Because only the Hicksian composite commodity theorem leads to interesting implications for quality choice, the following section illustrates how analysis might proceed under this assumption. A hypothetical application to demand for poultry brings in real-world measurement issues and illustrates the implication of the earlier theoretical analysis. The empirical as well as conceptual advantages and disadvantages of the various assumptions and methods are evaluated in the concluding discussion.

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The Issues and Problems

As a convention, "elementary goods" in this paper will refer to goods which are strictly homogenous, and the quantity of purchases of any such good will be denoted by x_i. The term "commodities" will refer to heterogenous goods, i.e., which vary in their characteristics. A composite commodity such as "bakery goods," for example, will cover a class of elementary goods (or distinct individual purchases) that vary in flavor, air content, fat content, freshness, convenience of packaging, etc. Because it is impossible to estimate price elasticities for every possible elementary good, some way must be found to aggregate measures of demand for these goods into measures of demand for meaningful composite commodities with corresponding meaningful price measures.

The first well-known discussions of the problems created for economic analysis by quality variation were by Houthakker and Theil in the early 1950s. The model they created, while accepted and adapted by Deaton (1986) and Cox and Wohlgenant, leads to several difficulties. Theil defines heterogenous commodity quantities as the sum of the physical quantities of elementary goods in the group (assumed to be measured in a common physical unit), and adds "quality" choice as a separate set of elements in the household utility function. That is, households are assumed to maximize

$$\text{(1)} \quad \text{Max } U(q_1, q_2, \ldots, q_G, \ldots, q_M, \nu_1, \nu_2, \ldots, \nu_G, \ldots, \nu_M) \quad \text{s.t.} \quad \sum_{G=1}^{M} p_G(\nu_G)q_G = Y,$$

where

$$\text{(2)} \qquad q_G = \sum_{i \in G} x_i$$

is the physical quantity consumed of commodity G (i.e., of elementary goods in group G), ν_G is the corresponding "quality" defined as a vector of characteristics, p_G is a composite price which depends on composite quality, Y is household income, and there are M groups. Houthakker's model is similar, except that q_G is the physical quantity of a single good chosen from the group and ν_G is a scalar indicator of quality.

First and most important among the problems created by this model is an inherent ambiguity about how the quantities, q_G, relate to the "quantity demanded" of consumer demand theory. In contrast to standard demand theory, in which quantity demanded is a function of exogenous prices and income, in this model the choice of quantity is also dependent on quality choice.

Second, the use of such physical quantities involves a selection of one dimension of physical measurement from a long list of possibilities. Physical quantity can be measured by weight, volume, protein content, caloric content, number of dietitian-identified portions, or by simple counts (as in dozens of eggs or heads of lettuce), to name just a few of the more common possibilites. The choice is trivial only for a homogenous commodity. For example, if consumers buy more dense, rich Häagen-Dazs ice cream as income rises and less of the air-filled store brand, the income elasticity of physical demand in terms of volume could be negative, even if the income elasticity of physical demand in terms of weight is positive. So is ice cream a luxury or necessity? Measurement in different physical quantity dimensions could suggest contradictory answers.

Third, it is unclear, without further assumptions, how these q_G's relate to any item of real interest. Blaylock and Smallwood state that sums of physical quantities by weight are of more interest from a nutritional standpoint than are dollar expenditures (in a constant-price framework). In addition, farmers or agricultural policy planners may be particularly interested in physical quantities by weight. Yet, these statements rest on special assumptions about the form of quality variation. If, for example, quality variation is purely in the dimension of flavor, the nutritional argument has merit; but, if it includes dimensions such as protein content, then Blaylock and Smallwood's statement concerning nutrition is clearly false. Equal weights of steak and hamburger meat, for example, may have roughly equivalent nutritional content, but equal weights of hamburger and soup bones clearly would not. Equal dollar's worth of hamburger and soup bones, on the other hand, might be roughly equivalent nutritionally.

Finally, the model outlined in problem (1) is difficult to solve in its general form. Additional assumptions are required to make the model tractable on a theoretical, much less an empirical, level. Theil makes the assumption that the prices of all qualities within a group G move proportionally in order to get theoretical results. He claims that this can be easily generalized but makes no attempt to do so. Houthakker uses a slightly less restrictive assumption on prices—the functions $p_G(\nu_G)$ are assumed to be linear in ν_G—but requires in addition that only one purchase can be made from each group. By Houthakker's definition, if a household purchases both steak and hamburger (or even two grades of hamburger), one must create distinct groups for both. This formulation clearly undoes most of the advantages of grouping in the first place.

"Quality" and Theoretically Valid Aggregation

The issue of quality can be put in terms of familiar insights from aggregation theory if we begin with a more general model of preferences defined directly over elementary goods. Suppose the consuming agent solves the problem:

$$\text{(3)} \quad \text{Max } U(x_1, x_2, \ldots, x_R) \quad \text{s.t.} \quad \sum_{i=1}^{R} p_i x_i = Y,$$

where the x_i ($i = 1, \ldots, R$), are physical quantities of the elementary goods and the p_i ($i = 1, \ldots, R$) are the corresponding exogenous prices (whose units are consistent with the measurement dimensions of the respective x_i's). Controlling for "quality variation," then, is equivalent to the problem of grouping the elementary goods, defining the composite commodity quantities, Q_G ($G = 1, \ldots, M$, each being a function of the x_i, $i \in G$) and defining corresponding composite commodity prices, P_G ($G = 1, \ldots, M$), so that solving the problem

$$\text{(4)} \quad \text{Max } U(Q_1, Q_2, \ldots, Q_G, \ldots, Q_M) \quad \text{s.t.} \quad \sum_{G=1}^{M} P_G Q_G = Y$$

is equivalent to solving the disaggregate problem. That is, one wants to create a smaller number of composite commodities ($M < R$) that can be treated as if they were single goods in all respects.

The requirements for such aggregation are stringent (Blackorby, Primont, and Russell; Deaton and Muellbauer). While either homogenous separability or strong separability with aggregators of the Gorman polar form is sufficient for treating allocation among Q_G's as dependent on only P_G's and Y, each composite commodity must be a positive linear homogenous function of its elementary goods if the product of the price and quantity indices is to equal group expenditure [as is implicitly assumed in writing the budget constraint in (4)]. Three cases are examined here: first, a degenerate case, second, a case in which homogenous separability is created entirely by restrictions on preferences, and third, a case in which homogenous separability is created by a condition on the behavior of within-group prices.

Case 1: A Degenerate Case

One possibility is to define Q_G as a simple, unweighted sum of physical quantities. That is, Q_G is defined as equivalent to $q_G = \Sigma_{i\in G} x_i$ [as in the Theil case, equation (2)], with the elementary goods within group G defined in some common unit. This approach involves an assumption that the x_i's are perfectly substitutable on a one-to-one basis. The consumer cares only about the total physical quantity and not about its composition; from the consumer's perspective, the commodity is homogenous. This, of course, would make any discussion of quality irrelevant. The consumer would choose to consume only the cheapest elementary good or would be indifferent (if within-group prices were equal). The corresponding P_G is simply the lowest p_i, $i \in G$.

To the extent that commodities are actually homogenous, or perhaps very nearly homogenous, this approach is not misleading. Timmer and Alderman use the assumption of no quality variation in their work on rice, corn, and fresh cassava in Indonesia. Only a greater institutional knowledge of the Indonesian food market would tell if their assumption of homogenous commodities is appropriate. Are multiple varieties of these commodities available, with price variations reflecting differentials in consumer valuation of the varieties? Or does everyone consume the same kind of rice?[1] Deaton (1986, 1987) and Cox and Wohlgenant clearly do not use the assumption of homogenous commodities to justify their use of the sums of physical quantities as the measure of demand because they

[1] Not only basic foodstuffs as rice may be potentially homogenous. Foods made up of combinations of other foods, such as canned fruit cocktail or canned beef stew, can be classified as homogenous (in the sense used here) if consumers perceive no difference from can to can.

explicitly allow for commodity heterogeneity elsewhere in their analyses.[2]

Case 2: Restrictions on Preference Alone

Imposing homogenous separability entirely by restrictions on preferences is possible but puts implausible restrictions on the consumer choice of quality. For exact treatment of the composite commodity like a single good without restrictions on prices, preferences must be weakly separable and within-group preferences must be homothetic. While weak separability may be a useful assumption, homotheticity of within-group preferences implies that within-group income-expansion paths are straight lines through the origin—or that group composition is independent of income. For example, the ratio of hamburger to steak must be the same for rich consumers as for poor, at constant prices. Obviously such a restriction would often be empirically rejected.

Even if one imposed this restriction, assumption of homogenous separability still does not justify the use (by Deaton 1986, 1987, 1988; Cox and Wohlgenant) of simple sums of physical quantities as measures of demand. The proper measure of demand for homothetic intragroup preferences is a quantity index, reflecting the value of a utility subfunction (see Deaton and Muellbauer, p. 130), and which, therefore, incorporates not only the specified quantity dimension of the good but all other aspects of the good which are relevant to consumer valuation.

Case 3: A Hicks' Composite Commodity Formulation

With restrictions on relative prices, on the other hand, one can get both a strict justification for use of composite commodities and a clear and nontrivial model of quality. Assume that within each group G, prices of all goods vary proportionally. That is,

$$(5) \qquad i \in G \Rightarrow p_i = P_G p_i^*,$$

where p_i^* is the "base" price of good x_i and P_G is the factor of proportionality common to all elementary goods in group G. Then, by the Hicks composite commodity theorem, a composite commodity is defined as

$$(6) \qquad Q_G = \sum_{i \in G} p_i^* x_i,$$

or a base-price weighted sum of physical quantities. The Q_G have corresponding prices P_G (which can be thought of as group-specific price-level indicators) and can be treated as if they are elementary goods. The model collapses to the one described by equation (4). The demand function for a composite commodity is

$$(7) \qquad Q_G(P, Y),$$

where P is the vector of P_G's. Because the P_G's are exogenous to the consumer, the elasticity of Q_G with respect to P_G is the desired own-price elasticity for commodity G. The elasticity of Q_G with respect to Y is the desired income elasticity.

In principle, the Hicksian composite commodity theorem does not require that goods be related in any way other than through their constant relative prices: popcorn and airplanes could be in the same grouping if their prices moved together. In thinking about broad price movements in food products, however, this criterion for grouping goods is not entirely implausible. Variations in the wholesale prices of cattle or wheat, for example (resulting from seasonal effects or transportation costs), could have downstream impacts on the prices of all varieties of beef and all varieties of bakery products. To the extent the price changes are proportional, the Hicksian grouping criterion might accord well with many of the more conventional food groupings derived from a viewpoint of common features in consumption, rather than common price movements. Because Theil also assumed proportional intragroup prices to make his model tractable, the Hicksian approach adds no additional assumptions relative to the older literature. The model, as expressed in equation (3), represents instead a return to a more general expression of preferences.

The Hicksian approach allows other expressions to be written in terms of P_G and Q_G. Expenditures on group G are

[2] The assumption of perfect substitutability is, however, made—and yet not made—in an article cited by Deaton and by Cox and Wohlgenant. Cramer writes that "If, from the consumers' point of view, the quantities of several goods can be sensibly added together, such goods belong to the same commodity," and that this "requires that the goods concerned are close substitutes . . . , but since price differentials need the support of other differences they are not identical" (pp. 351–53).

$$(8) \quad E_G \equiv \sum_{i \in G} p_i x_i = \sum_{i \in G} (P_G p_i^*) x_i = P_G \sum_{i \in G} p_i^* x_i = P_G Q_G.$$

"Unit values" can be calculated from some household surveys as

$$(9) \qquad V_G \equiv E_G / q_G = \frac{P_G Q_G}{q_G}.$$

These, in general, will not be exogenous to the consumer because they depend not only on the exogenous price level but also on the consumer's choices reflected in Q_G and q_G. They will be equal to the exogenous proportionality factors P_G only if $Q_G = q_G$, which can occur only if the base prices for all goods in group G are the same and can hence be factored out of the right-hand side of equation (6). Such identical base prices might reflect a truly homogenous good.

But what happens to quality? The aggregation view makes it clear that the "quantity/quality" distinction is a problem for the researcher, not for the consumer. The consumer is interested, not in pounds of "beef," but in purchases of pounds of particular types of beef: the x_i, not the q_G, are the arguments of the consumer's utility function. Breaking down the consumer's demand for a good into quantity and quality elements is a purely artificial exercise, perhaps interesting to the researcher for reasons other than measurements of elasticities of demand. However, if one adopts a Hicksian composite commodity model, the earlier literature on quality choice can be reinterpreted in a particularly clear way.

Because Theil's earlier approach and the Hicksian approach explained here overlap in the hypothesis of fixed "base" prices, one can follow Theil and Cramer in defining a quantity-weighted sum of elementary goods base prices as a measure of average quality within a group:

$$(10) \quad \nu_G \equiv \sum_{i \in G} \left(\frac{x_i}{q_G} \right) p_i^* = \frac{\sum_{i \in G} p_i^* x_i}{q_G},$$

where as before $q_G \equiv \Sigma_{i \in G} x_i$. The larger the proportions of higher-priced goods in the consumer's purchased bundle, the higher the measure of quality. As will be illustrated in the next section, the definition of the relative quality of different bundles depends crucially on the dimension (e.g., weight, volume, calories) in which the physical quantity (q_G) is measured. That is, by choosing one of the many dimensions in which characteristics of the good can be measured as reflecting quantity, the researcher's measure of quality defined by equation (10) is a scalar indicator of the consumer's valuation of all the omitted characteristics in the purchased bundle.

By these definitions, and for some specified dimension for the measurement of physical quantity, the following hold as identities:

$$(11) \qquad Q_G = \nu_G q_G$$

$$(12) \qquad E_G = P_G \nu_G q_G$$

$$(13) \qquad V_G = P_G \nu_G.$$

Composite quantity is a quality-adjusted quantity measure; expenditure on a composite can be broken down into exogenous price, quality, and physical quantity components; unit value has both exogenous price and endogenous quality components.

The price and income elasticities of concern can be found by taking natural logarithms of these equations and differentiating with respect to the natural logarithms of P_G or Y. Denoting the elasticity of a variable X with respect to income by β_x, and with respect to price by Θ_x, the following relationships are implied (for any group G):

$$(14) \qquad \beta_Q = \beta_\nu + \beta_q$$

$$(15) \qquad \Theta_Q = \Theta_\nu + \Theta_q;$$

and, because $\partial \ln P / \partial \ln Y = 0$ and $\partial \ln P / \partial \ln P = 1$,

$$(16) \qquad \beta_E = \beta_Q,$$

$$(17) \qquad \Theta_E = 1 + \Theta_Q,$$

$$(18) \qquad \beta_\nu = \beta_\nu, \text{ and}$$

$$(19) \qquad \Theta_V = 1 + \Theta_V.$$

Equation (14) is interpreted as the price elasticity of demand being the sum of the physical quantity elasticity and the quality elasticity. Equations (16) and (17) imply that the definition of physical quantity and quality measures is not necessarily a prerequisite for derivation of price and income elasticities of the Hicksian composite: the elasticities of demand could also be derived directly from the corresponding expenditure elasticities.

In summary, while simple sums of physical quantities are adequate measures of demand for homogenous commodities and restrictions on preferences alone give a well-defined aggregate only if shares of the individual elementary goods in heterogenous composites do not vary with income, the Hicksian composite commodity assumption permits aggregation of elementary goods consistent with freely variable choices across elementary goods with varying characteristics. Also under the Hicksian assumption, if one particular dimension is chosen to measure physical quantity, then a precise measure of commodity quality, which subsumes the consumer's evaluation of all other aspects of the goods contained in the bundle purchased, can also be defined. If goods are heterogenous, simple sums of physical quantities measure demand only for a single physical characteristic of the commodity, not demand for the commodity itself.

A comparison of the aggregation techniques used in empirically analyzing other sorts of demand data (besides the expenditure-and-physical-quantity household-level data with which this paper is primarily concerned) also illustrates that use of physical quantity as the measure of quantity demanded relies on special assumptions. In time-series work on non-food commodities, often only expenditures and price indices are observed, and an approximation to homogenous separability through restrictions on preferences is often assumed in order to justify aggregation. In this case, quantity is defined only implicitly, as the index derivable from dividing expenditure by the price index. That is, $Q \equiv E/P$ where P is an index such as the consumer price index for the commodity. A rearrangement of equation (8) shows that the Hicksian assumption yields an analogous equation for quantity: $Q_G = E_G/P_G$. On the other hand, $q_G = E_G/V_G$ [from equation (9)] and is analogous to these other quantity measures only if unit values are actually exogenous price measures (i.e., if quality effects are absent).

An Application of Hicksian Restrictions to Poultry Demand

A simple illustration demonstrates the mechanics of the Hicksian composite commodity formulation, and highlights the pitfalls that can arise from focusing only on unadjusted physical quantity measures or ignoring quality variation. Suppose we have data on a consumer's demand for the heterogenous commodity "poultry," with different combinations of price and income. Case 1 of table 1 shows that in a base situation the consumer purchases one small roasting chicken and fifteen pounds of chicken backs, with expenditures of \$10.00 and \$7.50, respectively. Total expenditure on the group, noted as E_G in the last section (from here on the "G" will be assumed to refer to "poultry," and will be dropped to simplify notation), is \$17.50.

The first problem is that quantities are reported in different dimensions. The hypothetical researcher whose results are given in columns (3) to (5) chooses to convert the "one small roasting chicken" observation to its approximate weight. As the U.S. Department of Agriculture has developed a 454-page manual for doing such conversions, the researcher finds that, on average, such a chicken weighs five pounds. In the common dimension of pounds, $x_1 = 5$ and $x_2 = 15$, in the notation of the last section, and q, the total physical quantity in pounds, is 20. Unit value (V) is total expenditure divided by total physical quantity, or \$17.50/20 = .88. Base prices (p_i^* in the notation of the theoretical exposition) are \$2.00 per pound for roasters and \$.50 per pound for the backs. In the base case, the quality measures in column (5) are identical to the unit value measures, by definition [equation (13) with $P = 1$].

Another hypothetical researcher, however, uses tables developed by dietitians which give the number of portions that can be derived from different quantities of raw poultry. If a normal-sized portion of roast chicken is one-half pound, but because of all the bones it takes a full pound of backs and necks to make a portion, this researcher's results will be those reported in columns (6) to (8). In this case, the base prices are \$1.00 per portion for roasters, and \$.50 per portion for backs.

Two (and in theory, infinitely more) quite different physical quantity and "quality" measures can describe the same purchases. The quality-adjusted quantity measure, however, does not depend on this choice of dimension. This measure, Q in the earlier notation, is appropriately deflated expenditure, which at base prices is the same as expenditure [equation (8) with $P = 1$]. This is shown in column (9).

Suppose, next, that if income rose by 10% this consumer would purchase more roasting chicken and less backs, as shown in case 2. As prices are constant, both the elasticity of expenditure and the elasticity of quality-adjusted quantity measure the income elasticity of demand at 2.7 (computed, using the ratio of log-differences formula for an elasticity, as $[\ln(23) - \ln(17.5)]/.10$). Poultry is a luxury good in this example. A use of unadjusted physical weight as the quantity measure, however, would sug-

Table 1. Hypothetical Example of Poultry Demand

			Using Weight (lbs.)			Using Standard Portions			
	(1) Quantity	(2) Expenditure (E)	(3) Quality (x_i, q)	(4) Unit Value (V)	(5) Quality (ν)	(6) Quantity (x_i, q)	(7) Unit Value (V)	(8) Quality (ν)	(9) Composite Quantity (Q)
CASE 1: "base" situation									
Roaster	1	$10.00	5			10			
Backs	15 lbs.	$ 7.50	15			15			
Total		$17.50	20	$ .88	$ .88	25	$ .70	$ .70	17.5
CASE 2: 10% higher income									
Roaster	2	$20.00	10			20			
Backs	6 lbs.	$ 3.00	6			6			
Total		$23.00	16	$1.44	$1.44	26	$ .88	$ .88	23.0
Elasticity w.r.t.									
Income		2.7	−2.2	4.9	4.9	.4	2.3	2.3	2.7
CASE 3a: 10% higher prices, no response									
Roaster	1	$11.05	5			10			
Backs	15 lbs.	$ 8.29	15			15			
Total		$19.34	20	$ .97	$ .88	25	$ .77	$ .70	17.5
Elasticity w.r.t.									
Price		1	0	1	0	0	1	0	0
CASE 3b: 10% higher prices, small response									
Roaster	9/10	$ 9.95	4.5			9			
Backs	15 lbs.	$ 8.29	15			15			
Total		$18.24	19.5	$ .94	$ .85	24	$ .76	$ .69	16.5
Elasticity w.r.t.									
Price		.4	−.3	.7	−.3	−.4	.8	−.1	−.6
Unit value			−.4			−.5			
CASE 3c: 10% higher prices, big response									
Roaster	1/2	$ 5.53	2.5			5			
Backs	15 lbs.	$ 8.29	15			15			
Total		$13.81	17.5	$ .79	$ .71	20	$ .69	$ .63	12.5
Elasticity w.r.t.									
Price		−2.4	−1.3	−1.0	−2.0	−2.2	−.14	−1.1	−3.4
Unit Value			1.3			16			

gest the contrary: its elasticity is -2.2 (computed as $[\ln(16) - \ln(20)]/.10$), implying an inferior good. Physical quantity by portion has a computed income elasticity of .4 using the same formula, implying a necessity. The solution to this dilemma of multiple proposed elasticities is to recognize that only the number 2.7 represents the income elasticity of demand for poultry. The other two numbers are elasticities for specific characteristics of the good. In each case, all attributes of the good not covered by the characteristic selected are relegated to the single index, quality. The numbers have been made up so that quality increases with income, by either index. As implied by equations (14) and (16), the sum of the income elasticities of demand for quantity and quality, by either dimension, sum to the income elasticity of expenditure of quality-adjusted-quantity.

In the income elasticity case, the equivalence of expenditure and demand elasticities should come as no surprise: even students in introductory econometrics classes are instructed to use the value of purchases rather than their units in order to adjust for quality (Studenmund and Cassidy). The extension of this intuition to the variable-price case perhaps has been overlooked. Although one cannot use simple expenditure, which itself depends on price, as the measure of demand for determination of price elasticities, the attractiveness of the Hicksian approach is that expenditure, properly deflated, is the appropriate measure of demand.

Case 3a in table 1 illustrates, as a reference, the results of a 10% increase in both prices, but with the consumption bundle unchanged relative to the base case 1. Increasing both prices by the same proportion, P, imposes the Hicksian constant relative price assumption. As percentage changes were calculated as log-differences (e.g., $\text{Ln}(p^1) - \text{Ln}(p^0)$), the P corresponding to a 10% increase is 1.1052 times the base. Expenditures and unit values rise, but everything else stays constant relative to case 1. The quality-adjusted quantity measure now differs from expenditure, and quality measures differ from unit values, as these are now being deflated by the new price level (or, equivalently, are still being measured at base prices).

In case 3b, the price increases lead to a small cutback in consumption of roasters. Comparing case 3b with case 1, the price elasticity of demand for poultry is $-.6$ [which is the price elasticity of expenditure, .4, less 1; see equation (17)]. Elasticities of the physical quantity and quality measures with respect to the true (10%) price change are similarly negative but are only half to two-thirds as large. The price increase has led to some economizing on quality (by whatever measure), but unit values still rise with price. The equivalences set out in equations (15), (17), and (19) can be confirmed (except for rounding error).

The approach that yields the price elasticities of physical quantity with respect to the true price change is analogous to the approach taken by Cox and Wolhgenant in their study of U.S. demand for vegetable commodities and by Deaton (1986, 1987, 1988) in his study of demand for several food commodities in the Ivory Coast. As in the example here, these researchers separated out the true price variation information contained in unit values from the changes in unit values caused by quality variation. However, their measures of demand are the sums of physical quantities. These measures may not be invariant to measurement in another physical dimension, as can be seen in a comparison of columns (3) and (6).

The last line of case 3b gives the elasticities of physical quantity that would be calculated if a Timmer and Alderman approach were followed, that is, one ignored possible quality variation and treated changes in unit values as changes in prices. In this particular example, the elasticities of physical quantity with respect to unit values [e.g., in column (3), $\partial\ln(q)/\partial\ln(V) = -.3/.7 = -.4$] are in the same range as the other physical quantity and quality-adjusted quantity elasticities.

Case 3c shows that the convergence of the variety of elasticities to within a relatively small range in case 3b may be only fortuitous. If roaster purchases are cut in half following the price rise, the effect on the quality composition of the bundle is dramatic enough to lead to a drop in unit values. In this case, a study of the responsiveness of physical quantities to unit values would lead to the conclusion that poultry has a positive own-price elasticity, in the case of portions, of +16 (computed as the ratio of -2.2 to $-.14$)!

Conclusion

The main theoretical result of this paper, that rigorous and nontrivial definitions of aggregate quantities and of quality variation can be maintained only under Hickson composite commodity theorem assumptions, is cold comfort to the empirical researcher. While one might simulate the assumption of constant relative prices in an experimental study or perhaps approximate it in a carefully specified study of some local market, it is unlikely that constancy of relative prices would hold either intertemporally or spatially for many goods in such much-used datasets as the Nationwide Food Consumption Survey. Results of a study applying the Hicksian assumptions to U.S. Consumer Expenditure Survey data can be found in Nelson (1987, 1990) but are somewhat unsatisfactory both in terms of the precision of the estimates and in terms of the specifications used to derive them.

Besides suggesting new research explicitly designed to incorporate the theoretical assumptions, a few conclusions can be drawn concerning current research. First, the importance of properly adjusting for quality variation depends on the importance of quality effects in the data under examination. It may very well be, for example, that rice is a fairly homogenous commodity in Indonesia, and, hence, Timmer and Alderman's treatment of demand for rice using physical quantities and unit values was theoretically appropriate (abstracting from empirical problems of measurement error). It might also be that, even when commodities are heterogenous, the composition which consumers choose might be relatively insensitive to changes in income or prices, again easing the problem of aggregation. More research put into measuring the degree of heterogeneity of a commodity, and the responsiveness of the composition of commodity aggregates to prices and income, could shed light on the appropriateness of aggregation methods used in past research. Empirical estimates of the income elasticities of physical quantity and of expenditure, which can often be easily obtained, can be compared in order to indicate the importance of quality effects. Second, methods and results devised by researchers who have sought to control for quality variation in the measurement of price, while using unadjusted physical quantities as the measure of demand, can be reinterpreted as measuring demand for one particular characteristic of the commodity, rather than for the commodity itself. For some applications, this might be sufficient, although the limitations of the approach and the possibility of getting different estimates if a different physical dimension were chosen should be noted. The old adage, "You can't add apples and oranges" is still true; the Hicksian approach adds the coda: unless the physical quantities can be weighted by unchanging base prices.

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