Measurement Error

Before we do any experiments, collect any data, or set up any process: We need to ensure we have a way to measure the results that is:

Quantitative

Accurate

Precise

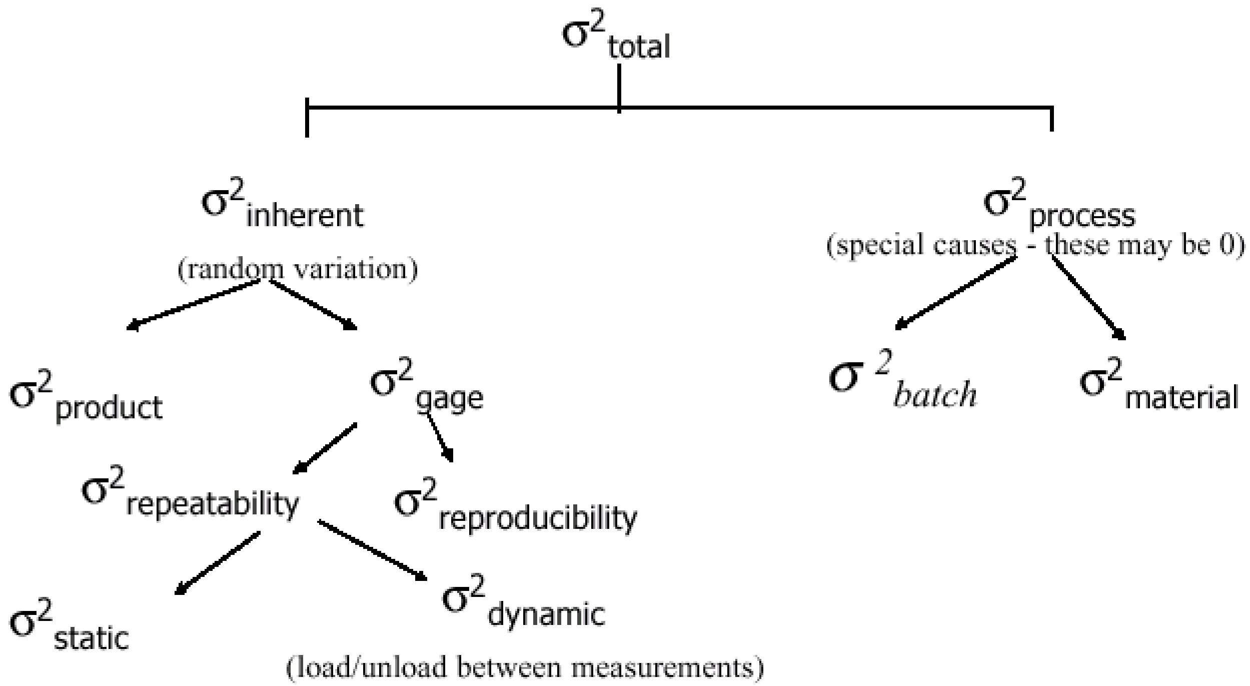
So how do we test our measurement system or gage to ensure this??

Measurement Tool (Gage) Error

We must determine the uncertainty of our measurement systems before we can compare, control or optimize our manufacturing processes.

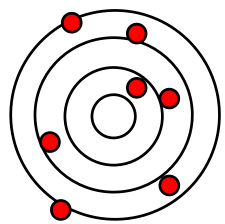
Measurement Error

Measurement Error will always exist. The idea is to measure and minimize this inherent measurement tool measuring error so that it can measure "real" differences in the "process" intended or due to some other unwanted assignable cause.

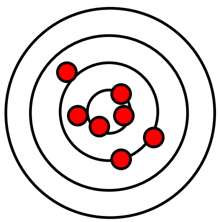


Types of Measurement Error

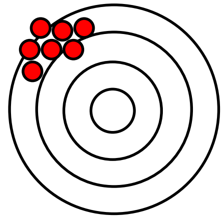
Statistical Treatment of Data



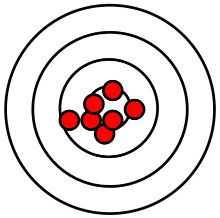
Low accuracy, low precision



High accuracy, low precision



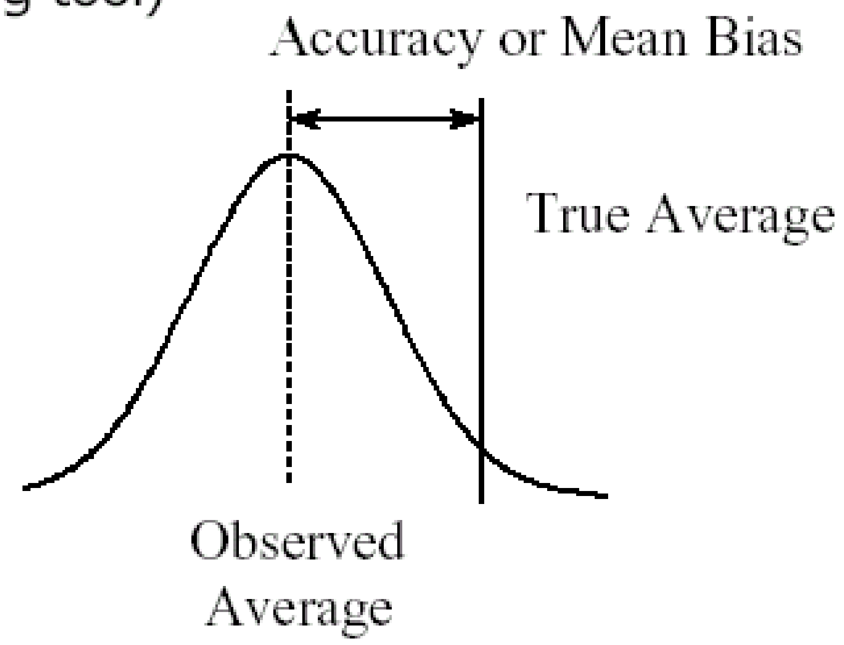
Low accuracy, high precision



High accuracy, high precision

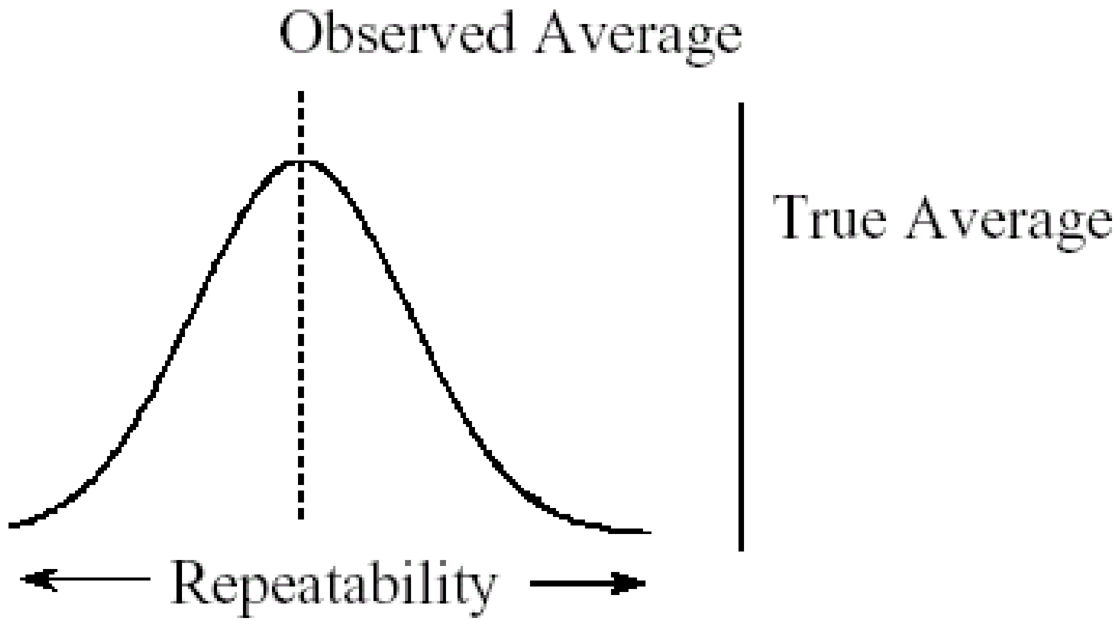
Types of Measurement Error Terms: *Accuracy*

difference between the true average and the observed average
(true average may be obtained using a more precise measuring tool)



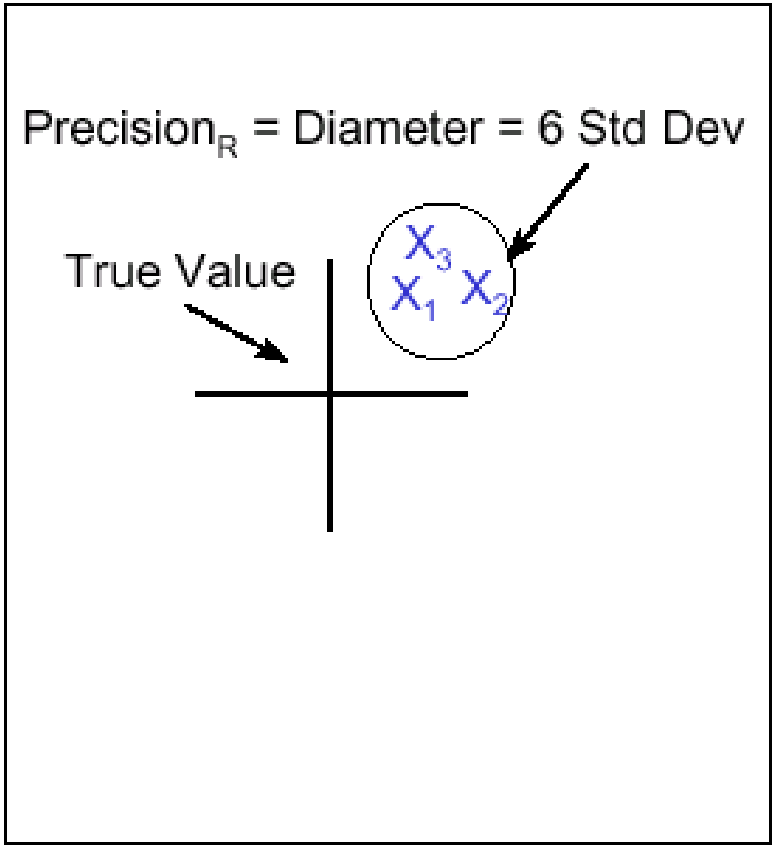
Types of Measurement Error Terms: ***Repeatability***

random variation in measurements when one operator uses the same gage to measure the same part several times.



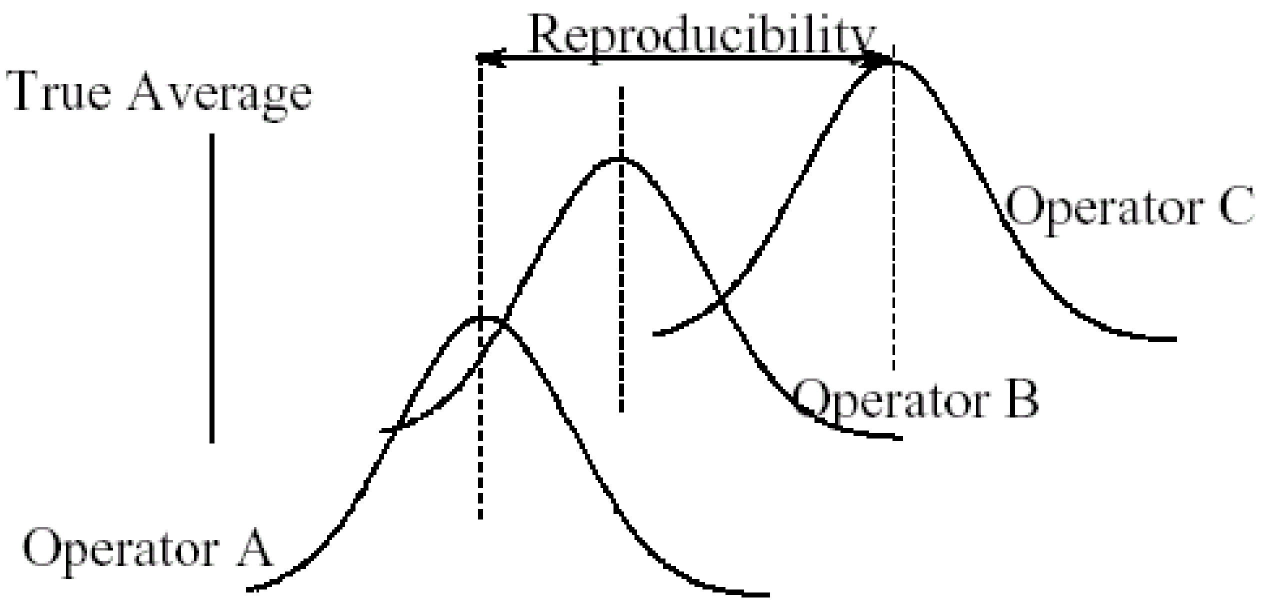
Types of Measurement Error Terms: *Repeatability*

- The variation obtained from **one gage** and **one operator** when measuring the **same part** several times.
- Machine Variation



Types of Measurement Error Terms: *Reproducibility*

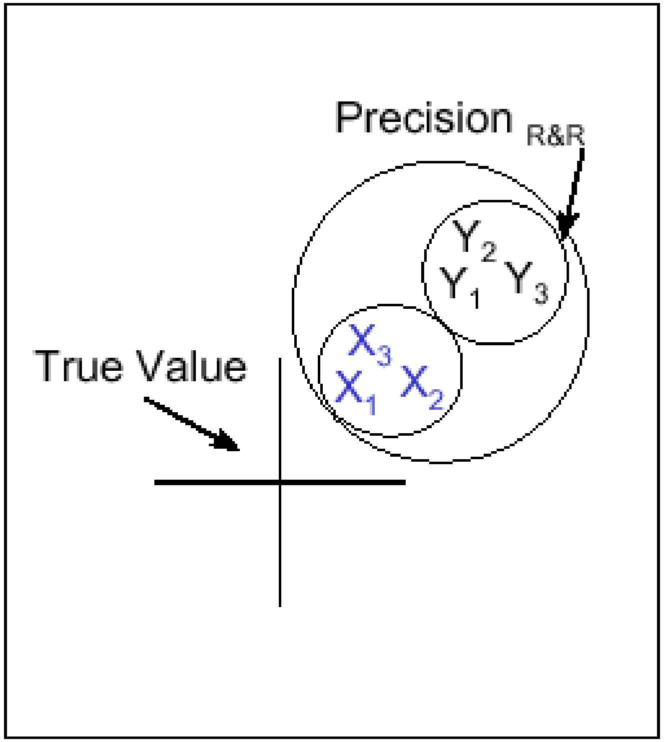
variation in average of measurements made by different **operators** using the **same** gage measuring the same part.



Types of Measurement Error Terms: *Reproducibility*

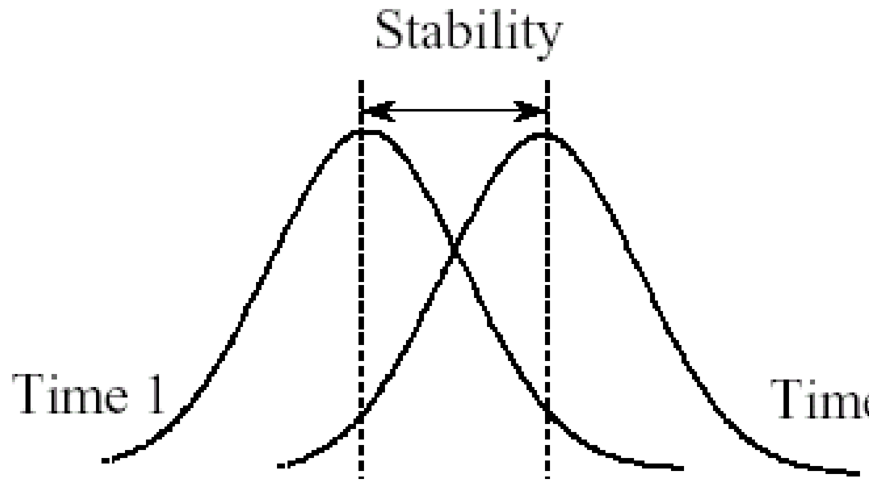
The difference in the average of the measurements made by **different** operators using the **same** gage when measuring the **same** **part**.

Operator-to-Operator Variation



Types of Measurement Error Terms: *Stability*

- difference in the average of at least 2 sets of measurements obtained with a gage as a result of time.



Measurement Error Analysis:

Accuracy and Stability Example

Measure same parts using a more precise instrument (Accuracy).

Measure same parts using the same gage at a later date (Stability).

Trial #	Part_1	Part_2	Part_3	Part_4	Part_5	Average
Current Gage	63.00	38.00	63.00	57.00	56.00	55.40
More Precise Gage	62.93	37.92	62.94	56.94	55.93	55.33
Range	0.07	0.08	0.06	0.06	0.07	0.07

Accuracy	**0.07 mm**

Trial #	Part_1	Part_2	Part_3	Part_4	Part_5	Average
Gage Now	63.00	38.00	63.00	57.00	56.00	55.40
Gage (1 month)	62.75	37.75	62.75	56.70	55.80	55.15
Range	0.25	0.25	0.25	0.30	0.20	0.25

Stability	**0.25 mm**

Method to Quantify Measurement Errors Analysis

I. Gage R&R Study : Reproducibility and Repeatability:

Methods of analysis:

Average and range.

II. ISOPLOT : Repeated Measurement Study

Measure same parts twice

T test and F test for difference between measurement systems

Measurement Tool (Gage) Error; Precision to Tolerance ideas: P/T ratio

Rules for measurement system discrimination of a process change: Typical units of measure are a ratio P/T (precision to tolerance):

At least 1/10 of 6 sigma of process variation

Other: at least 1/10 of process tolerance (spec limits).

- Precision-to-Tolerance Ratio (P/T)

P/T < 0.1 Acceptable System

0.1 < P/T < 0.3 Marginally Acceptable System

P/T > 0.3 Unacceptable System

Ref.: SEMATECH: Introduction to Measurement Capability Analysis #91090709A-ENG

Measurement Tool (Gage) Error; Precision to Tolerance ideas: P/T ratio

P/T = 0.1

* Measurement system consumes 10% of tolerance

* Effect of T on P/T

P = 1.2	P = 1.2
T = 4.0	T = 3.0
P/T = 0.3	P/T = 0.4

Measurement Tool (Gage) Error; Precision to Tolerance ideas: Repeatability and Reproducibility (R&R) Study

- Involves multiple operators and trials
- Total tear down of the setup between trials
- Provides separate estimates of repeatability and reproducibility
- $P_{R\&R}/T$

Measurement Tool (Gage) Error; Precision to Tolerance ideas: Repeatability and Reproducibility (R&R) Study

O1-T1	O1-T2	O1-T3	O2-T1	O2-T2	O2-T3
-0.0050	0.0360	0.0330	0.0040	-0.0070	-0.0420
-0.0100	-0.0460	-0.0600	-0.0170	-0.0680	-0.0700
0.0070	0.0110	0.0090	0.0230	-0.0190	-0.0040
-0.0170	-0.0820	-0.1060	-0.0890	-0.0540	-0.0580
-0.0620	-0.1000	-0.1010	-0.0270	-0.1090	-0.1000
-0.0830	-0.1480	-0.1210	-0.1200	-0.1280	-0.1870
-0.1100	-0.1460	-0.1520	-0.1200	-0.1890	-0.1900
-0.0560	-0.1490	-0.1290	-0.0900	-0.1650	-0.1780

O1, O2 - Operators 1 and 2
T1, T2, T3 - Trials 1, 2 and 3

Measurement Tool (Gage) Error; Precision to Tolerance ideas: Repeatability and Reproducibility (R&R) Study

I. To evaluate the measurement systems capability,we form a ratio comparing the gage R&R to the tolerance width (spec width) as:

%R&R as Precision to Tolerance $= 6\sigma_{R\&R}/(USL-LSL)$

II. Or a ratio comparing the gage R&R to the total process variation as:

%R&R as Precision to Total Variation $= 6\sigma_{R\&R}/ 6\sigma_{total\ variation}$

Some ratios some 5.15 x sigma instead of 6.00 as:

5.15σ = 99% of the data for normal distribution

6.00σ = 99.7% of the data for normal distribution

Measurement Tool (Gage) Error; Precision to Tolerance ideas: Repeatability and Reproducibility (R&R) Study

I. $\sigma_{R\&R}$ **= *RRS :(Sqrt(*** $\sigma^2_{repeatability}$ ***x*** $\sigma^2_{reproducibility}$***)***

II. Estimate of σ repeatability = (Average Range of measurements)/ d_2

III. Estimate of $\sigma_{reproducibility}$ ***=***

Sqrt[(average difference/ d_2***)*** ^{2} ***– (*** σ^2***repeat/(#parts x #trials)]***

IV. Total Variation TV = sqrt[(6 $\sigma_{R\&R}$***)*** ^{2} ***+ (6*** $\sigma_{process\ variation}$***)*** ^{2} ***]***

Measurement Tool (Gage) Error; Precision to Tolerance ideas: Repeatability and Reproducibility (R&R) Study

The d_2 term is used to estimate σ for small sample sizes (comes from a table:

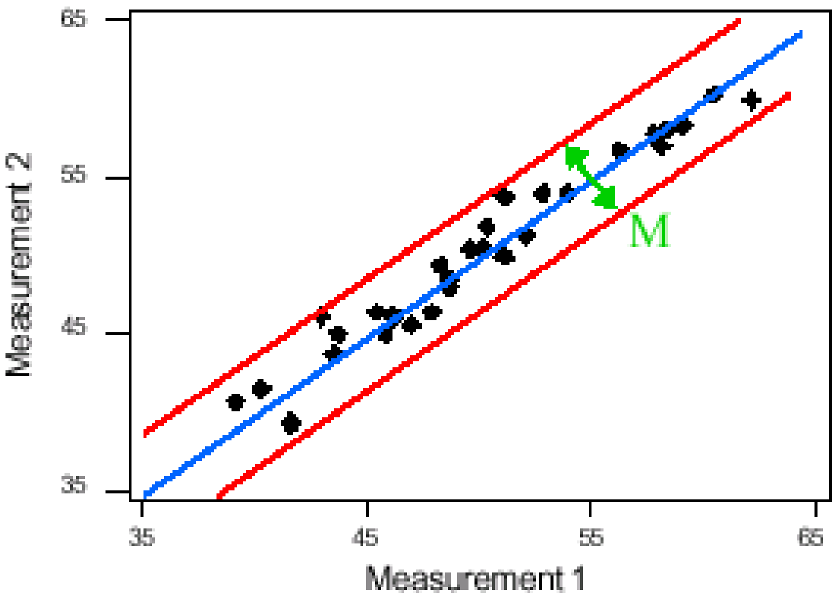
- To compute repeatability and reproducibility, the following standard values are needed.

n	d_2	d_2^*	D_3	D_4
2	1.128	1.411	0	3.267
3	1.693	1.907	0	2.574
4	2.059	2.239	0	2.282
5	2.326	2.476	0	2.114
6	2.534	2.668	0	2.004
7	2.704	2.830	0.076	1.924
8	2.847	2.960	0.136	1.864
9	2.97	3.084	0.184	1.816
10	3.078	3.179	0.223	1.777

NOTE: Repeatability: d_2 is based on number of trials
Reproducibility: d_2^* is based on number of operators
Part Variation: d_2^* is based on number of parts

Isoplot supposed a Copyrighted name by Dorian Shainin : Very similar to the R&R gage Analysis

Measurement systems require a discrimination ratio or Signal to noise ratio of > 6 to ensure one can really "detect" significant changes in the process one is measuring

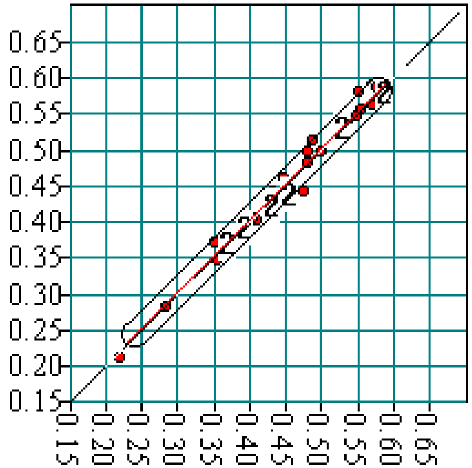


$$D \approx \text{Range}(X) \div M$$

Isoplot

Procedure: Use normal production line parts to measure

1. Measure part A and call "measurement 1"
2. At a different time (12 - 14 hours later) measure same part A call "measurement 2".
3. Keep data paired for measurement 1 and 2 on same part
4. Send part on through rest of process.
5. Obtain a second part after part A and repeat this first and second measurement
6. Continue this first and second measurement on the same part for at least 20 different parts from the normal manufacturing list over time.
7. Plot second measurement Vs first on square (equal units) graphical format



Delta P = 0.36
Delta M = 0.03
Discrimination Ratio = 10.4

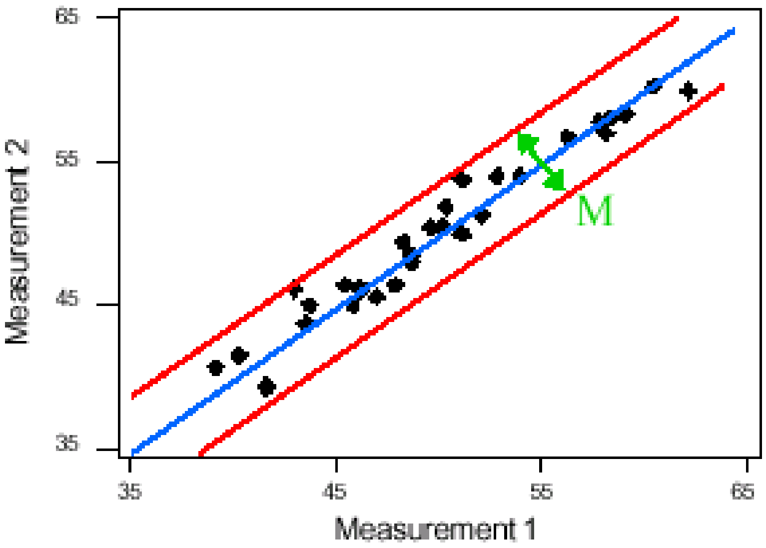
Isoplot

<u>Procedure:</u>

8. From the plot calculate a "length" or normal process variation range call this P or X as shown below.

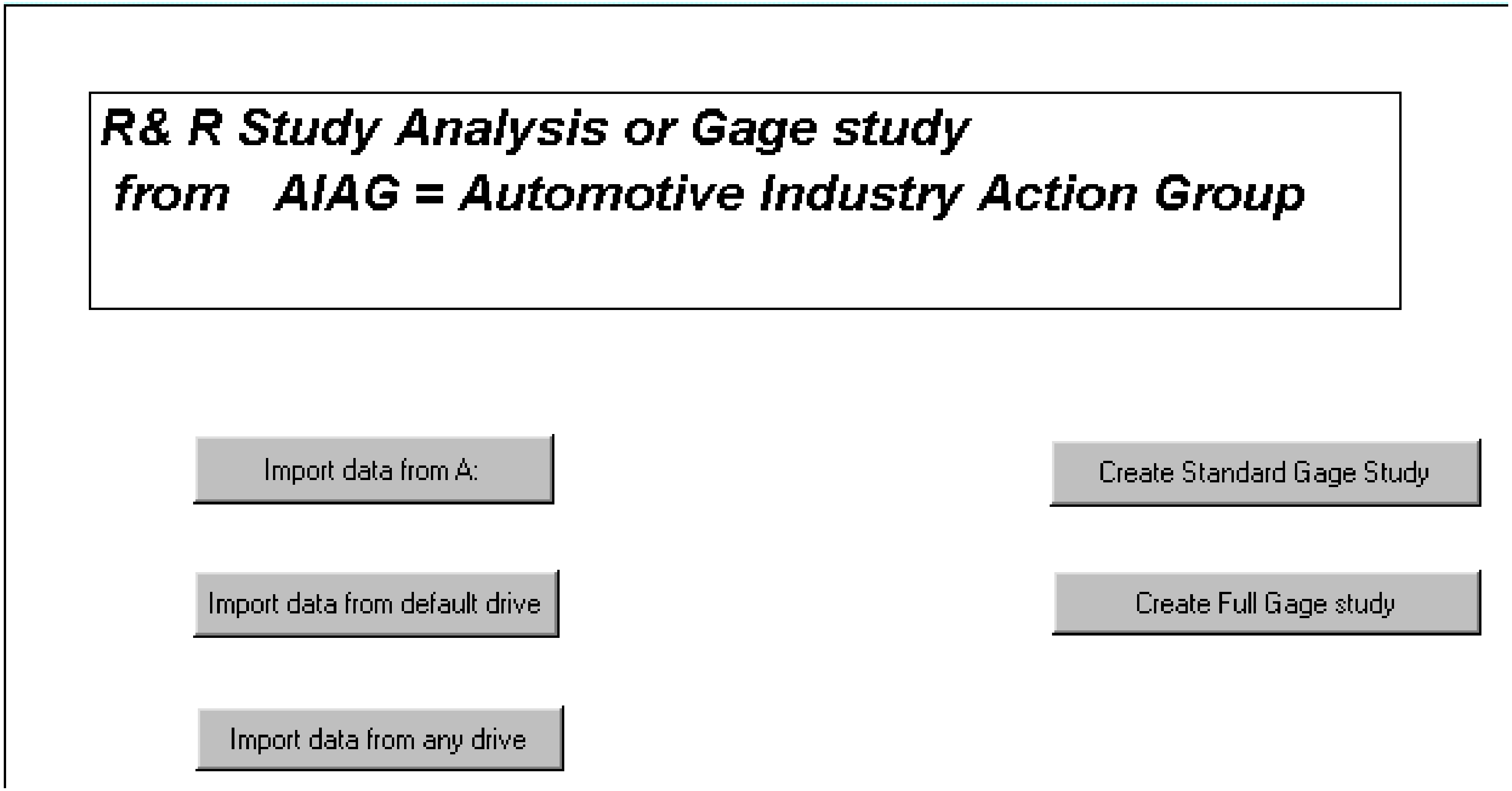
9. From the plot calculate a "width" or the measurement variation range call this M.

10. Shainin claims a 6 or greater ratio of P/M is required to ensure that the measurement system can discriminate



$$D \approx \text{Range}(X) \div M$$

Gage Study Example: Using Excel file Gage R&R Study Macro



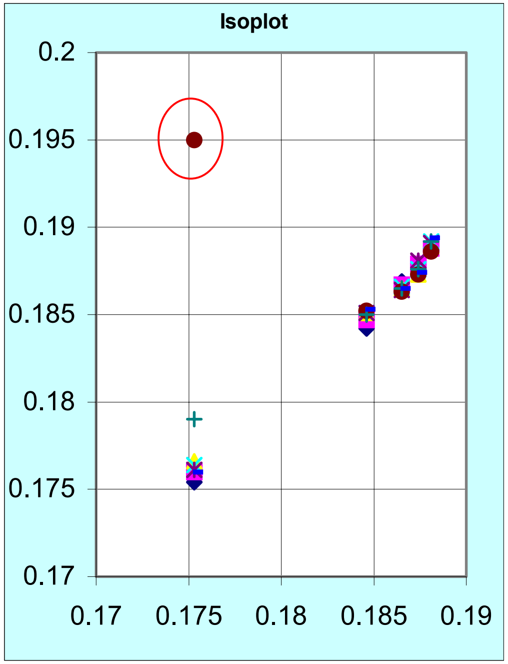
Gage Study Example: Using Excel file “Gage R&R Study” Macro Input CD data: five sites on a wafer; Measured 3 times(Trials) Repeatability and Done 3 times (Runs) Operator to Operator Reproducibility

R&R Study   STANDARD GAGE STUDY

Number of Runs =	3	Gage:	SEM 05	Date:	28-Sep-01
Number of Sites =	5	Part:	wafer 0.18um lin	Performed By:	John Doe
Number of Trials =	3	Tolerance =	0.12		

Enter data and Information in open cells
***If Range Check displays "FLAG", check data**

Run			Measurement Site on wafer 1	2	3	4	5
#1		1	0.1753	0.1865	0.1874	0.1881	0.1846
Average	Trials	2	0.1754	0.1869	0.1876	0.1888	0.1842
0.184593333		3	0.176	0.1867	0.1879	0.1888	0.1847
$(\Sigma A)^2$ 7.66680721		Range Check*	0.0007	0.0004	0.0005	0.0007	0.0005
#2		1	0.1766	0.1865	0.1873	0.1889	0.1851
Average	Trials	2	0.1764	0.1866	0.1876	0.1892	0.1851
0.18494		3	0.1761	0.1864	0.1881	0.1891	0.1851
$(\Sigma B)^2$ 7.69563081		Range Check*	0.0005	0.0002	0.0008	0.0003	0
wafer #3		1	**0.195**	0.1863	0.1873	0.1886	0.1852
Average	Trials	2	0.179	0.1865	0.1876	0.1892	0.185
0.186286667		3	0.176	0.1865	0.1874	0.1894	0.1853
$(\Sigma C)^2$ 7.80811249		Range Check*	0.019 FLAG	0.0002	0.0003	0.0008	0.0003
Part Averages			0.178422222	0.18654	0.18758	0.1889	0.18492



Gage Study Example: Using Excel file "Gage R&R Study" Macro

ANOVA	DF	SS	MS	F =MS ap/Mse	Prob>F
appraisers number of runs	2	2.40053E-05	1.20027E-05		
Parts (sites)	4	0.000604259	0.000151065		
appraisers*Parts	8	8.70102E-05	1.08763E-05	1.549229235	0.182359224
Gage(Error)	30	0.000210613	7.02044E-06	appraiser*Part Interaction is Not Significant	
Total	44	0.000925888			

Enter Process Distribution Width in Sigma's (5.15 or 6.00). For One-Sided Spec, Enter 3: **6.00**

SOURCE OF VARIATION	SIGMA	VARIATION	PERCENT OF TOTAL VARIATION	PERCENT OF TOLERANCE
Repeatability (EV - Equipment Var)	0.00280	0.01679	57.10%	13.99%
Reproducibility (AV - appraiser or Operator Var)	0.00053	0.00316	10.76%	2.64%
appraiser * Equipment Interaction (IV)				
Repeatability & Reproducibility (R&R)	0.00285	0.01709	58.10%	14.24%
Part Variation (PV)	0.00399	0.02394	81.39%	19.95%
Total Process Variation (TV)	0.00490	0.02941		24.51%

Note: Percentages will not add to 100%.

R&R = Criteria P/T = 1/6 = 16.67% The result of this analysis: PASS

For measurement systems whose purpose is to analyze a process, general rules of thumb are:

Acceptability Criteria for %R&R (% of Total Variation and % of Tolerance)

- ✓ Under 10% - acceptable system - Under 5% preferred
- ✓ 10% to 30% - may be acceptable depending on importance of the application, cost of the system, cost of repair, etc
- ✓ Over 30% - considered not acceptable - every effort should be made to improve the system

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If Prob>F is 0.05 or smaller, then appraisers*Parts interactio is significant; check plots to determine why

A significant appraisers*Parts interaction means that apprais tend to obtain different measurements from identical parts

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The AIAG method uses 5.15, other customers may use 6.00

5.15 standard deviations enclose the central 99% of a normal distribution

If %R&R of Tolerance is blank, enter a Tolerance above

14.24%

% R&R of Total Variation (RSS of EV +AV) but see the quote from AIAG below

Note: Percentages will not add to 100%.

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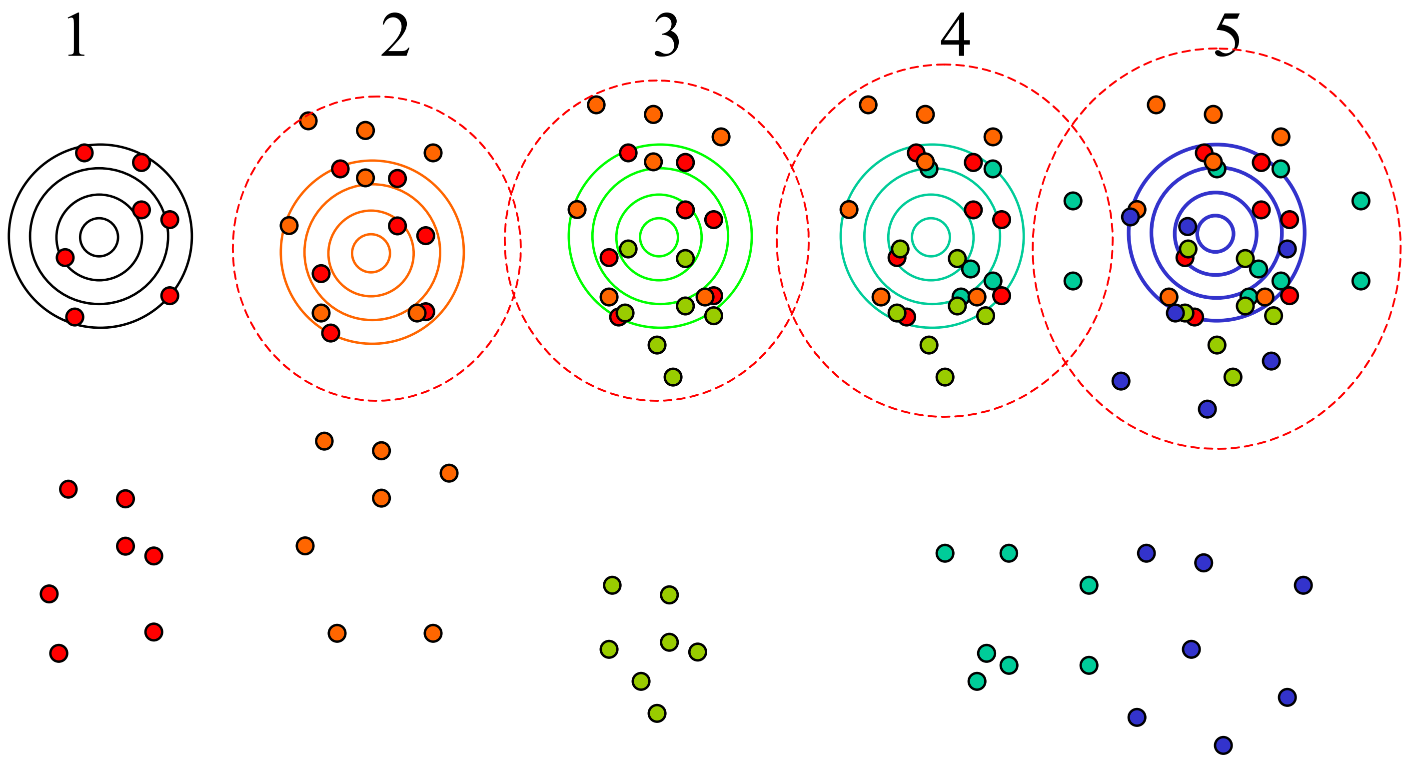
AIAG uses <10%, but within Motorola <5% is preferred (Note: For %R&R smaller is better)

AIAG = Automotive Industry Action Group

Sources of Error

<u>Variation (defined as sigma) in building a product will use a RSS approach to find total variation: Example here is fro five assembly steps</u>



Sources of Error

<u>Error propagation model: Used to find total variation after product is completely built</u>

The formula for error propagation

If f=F(x,y,z…) and you want σ_f and you have σ_x, σ_y, σ_z …, then use the following formula:

$$\sigma_f = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial F}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial F}{\partial z}\right)^2 \sigma_z^2 + \ldots}$$