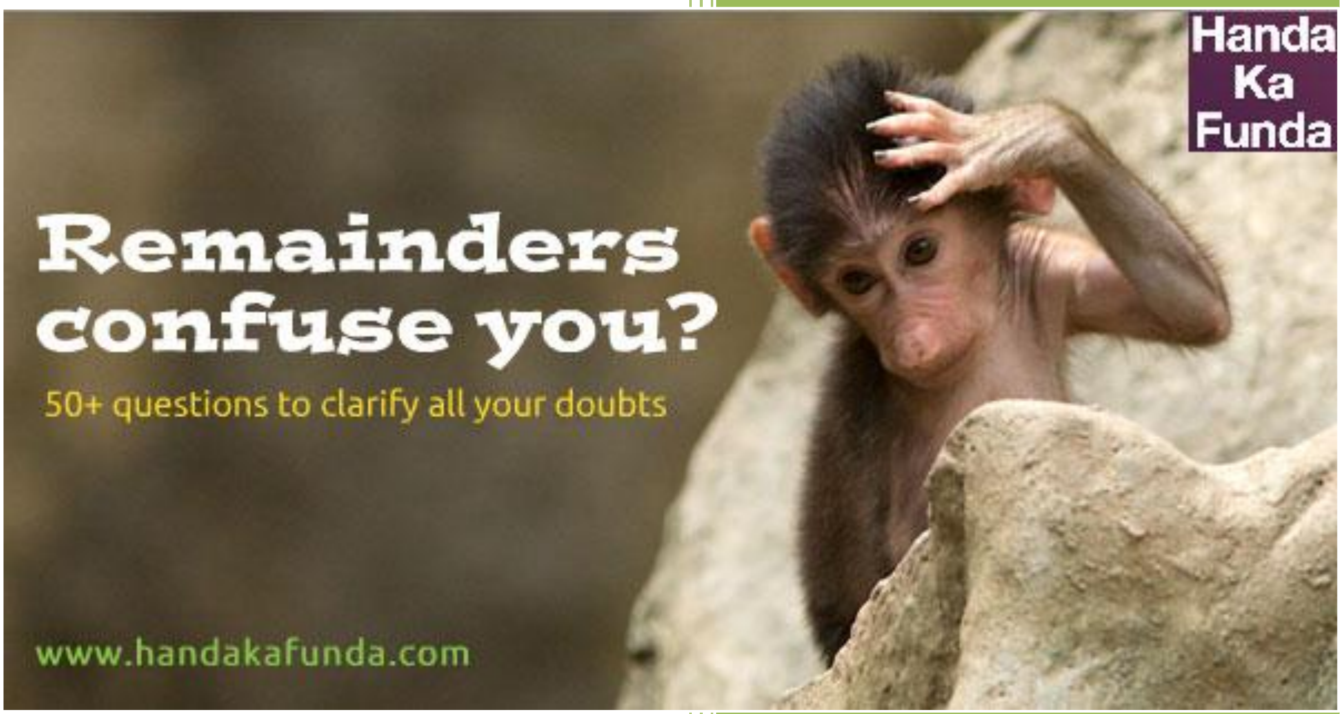
Quantitative Aptitude - Remainders



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Handa Ka Funda

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PREFACE

Hi,

I am Ravi Handa, founder of www.handakafunda.com

I have an online course for CAT Preparation that is available here: http://handakafunda.com/online-cat-coaching/ In case you are interested in my online course for CAT preparation, use coupon code **REMAINDERS** to get 101 Rs. off.

The central idea behind making this PDF is that I want to help students who spend (waste?) too much time on calculating quantitative aptitude questions based on **remainders**. Quite often, they ask the same as doubts on public forums such as Quora / Facebook / Pagalguy. I have tried to cover a variety of questions in this PDF and I hope that after you go through this ebook, you will never face a problem with remainder questions again. Let me add, remainders isn't really an important topic from CAT perspective.

Hope you enjoy this ebook and I look forward to your feedback.

Cheers,

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Basic Divisibility Rules

1) **How do I find the remainder when 12345678910...99100 is divided by 16?**
Answer: The divisibility test of 2^n is that you need to check the last 'n' digits of the number.
To find out the remainder from 16, you need to check the last 4 digits
Rem [12345....99100 / 16] = Rem [9100/16] = 12

2) **Quantitative Aptitude: What is the remainder when (111...) + (222...) + (333...) + … + (777...) is divided by 37?**
Answer: aaa = a*111 = a*3*37
aaa is divisible by 37
aaaa..... repeated 3n number of times is divisible by 37
(1111.....1) 108 times is divisible by 37
Now, 1111...111 (110 times) = 111.....1100 (108 1s and 2 0s)+ 11
= Rem [1111...111 (110 times) /37] = 11
= Rem [2222...222 (110 times) /37] = 22
.
.
= Rem [7777...777 (110 times) /37] = 77

So, we can say that
Remainder when (111...) + (222...) + (333...) + … + (777...) is divided by 37
= Rem [11 + 22 + 33 + 44 + 55 + 66 + 77 / 37]
= Rem [308/37]
= 12

3) **What is the highest possible value of 'n' for which (3^1024) – 1 is divisible by 2^n?**
Answer: We should try and split down the number to something a little more manageable by using the simple idea of
a^2 - b^2 = (a + b)(a - b)
3^1024 - 1
= (3^512 - 1)(3^512 + 1)
= (3^256 - 1)(3^256 + 1)(3^512 + 1)
= (3^128 - 1)(3^128 + 1)(3^256 + 1)(3^512 + 1)
.
.
.

= (3 - 1)(3 + 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)...(3^512 + 1)

This was fairly simple, right?
Now is where the slightly more creative part begins.
There are 11 terms given above and all of them are even
10 of these terms have an even power of 3
Rem [(3^2n + 1) / 4] = Rem [((-1)^2n + 1)/4] = Rem [(1 + 1)/4] = 2
= Terms with even powers of 3 are not divisible by 4

So in the 11 terms, 10 are not divisible by 4.
= Each of those 10 terms will give me 1 power of 2
= The 11th term, which is 3 + 1 = 4 will give me 2 powers of 2
= Total powers of 2 = n = 10*1 + 2 = 12

4) **What is the remainder when 12345678910111213141516171819202122232425262728293031323334353637383940 4142434481 is divided by 45?**

Answer: We need to find out Rem [1234.....434481/45]

This looks a little difficult if you do not any theorems for finding out remainders. Let us take a simpler approach and break down the problem into smaller parts.

These type of questions become really simple if you understand the concept of negative remainders. Always try and reduce the dividend to 1 or -1.

45 = 9*5

Let us find out the remainders separately and combine them later

Rem [1234.....434481/9]
The divisibility test of 9 is to divide the sum of the digits by 9.
The sum in this case is 1 + 2 + 3 ... 43 + 44 + 81 = 44*45 + 81
We can see that this is divisible by 9
= The number is divisible by 9
= Rem [1234.....434481/9] = 0

Rem [1234.....434481/5] = 1 (It only depends on the last digit)

So, our answer is a number which leaves a remainder of 1 when divided by 5 and is divisible by 9.
Consider multiple of 9,
9, does not leave a remainder of 1 from 5. Invalid.
18, does not leave a remainder of 1 from 5. Invalid.
27, does not leave a remainder of 1 from 5. Invalid.
36, leaves a remainder of 1 from 5. Valid. (This is our answer)

5) **What is the remainder when 123456............4647484950 is divided by 16?**
Answer: To find out the remainder from 2^n, we just need to look at the last 'n' digits.
Rem [123...484950 / 16]
= Rem [4950/16]
= 6

6) **What is the remainder when $\sum_{k=1}^{100} K!$ is divided by 18?**
Answer: We have to find out Remainder of $\sum_{k=1}^{100} K!$ when divided by 18.
= Rem [(1! + 2! + 3! ... 100!)/18]
6! is divisible by 18
7! is divisible by 18
.
.
100! is divisble by 18
= We have to find out Rem[(1! + 2! + 3! + 4! + 5!)/18]
= Rem [(1 + 2 + 6 + 24 + 120)/18]
= Rem [153/18] = 9

7) **What is the remainder when the infinite sum $(1!)^2 + (2!)^2 + (3!)^2 + \cdots$ is divided by 1152?**
Answer: We have to find out the remainder when $(1!)^2 + (2!)^2 + (3!)^2 + \cdots$ is divided by 1152
1152 = 2^7 * 3^2
= (6!)^2 is divisible by 1152
= All (n!)^2 are divisible by 1152 as long as n > 5
So, our problem is now reduced to
Rem [$((1!)^2 + (2!)^2 + (3!)^2 + (4!)^2 + (5!)^2)$/1152]
= Rem[(1 + 4 + 36 +576 + 14400) / 1152]
= Rem [15017/1152]
= 41

8) **What is the remainder when $3^{21} + 9^{21} + 27^{21} + 81^{21}$ is divided by $(3^{20}+1)$?**

Answer: $3^{21} + 9^{21} + 27^{21} + 81^{21}$

$= 3^{21} + (3^{21})^2 + (3^{21})^3 + (3^{21})^4$

$= x + x^2 + x^3 + x^4$

where $x = 3^{21}$

Now, $3^{21} = 3(3^{20}) = 3(3^{20} + 1) - 3$

$=$ Rem $[3^{21} / (3^{20}+1)] = -3$

$=$ Rem $[x / (3^{20} + 1)] = -3$

We can use this to find out out answer

Rem $[(x + x^2 + x^3 + x^4) / (3^{20} + 1)] = (-3) + (-3)^2 + (-3)^3 + (-3)^4$

$= -3 + 9 - 27 + 81 = 60$

9) **Is $2222^{7777}+7777^{2222}$ divisible by 101 and 99?**

Answer: Dividing by 101

2222 is divisible by 101 ($22*101 = 2222$)

= Any power of 2222 is divisible by 101

7777 is divisible by 101 ($77*101 = 7777$)

= Any power of 7777 is divisible by 101

If two number are individually divisible by 101, their sum is also divisible by 101.

$= 2222^{7777} + 7777^{2222}$ is divisible by 101

Dividing by 99

If a number is divisible by 11 and 9, it will be divisible by their LCM or 99.

$2222^{7777} + 7777^{2222}$ is divisible by 11 because the number in the base are individually divisible by 11.

Now, let's check divisibility from 9

Rem $[2222^{7777} / 9] =$ Rem $[(-1)^{7777} / 9] =$ Rem $[-1 / 9] = 8$

Rem $[7777^{2222} / p] =$ Rem $[1^{7777} / 9] =$ Rem $[1 / 9] = 1$

Rem $[2222^{7777} + 7777^{2222}/9] =$ Rem $[1 + 8 / 9] =$ Rem $[9/9] = 0$

$= 2222^{7777} + 7777^{2222}$ is divisble by 9

So, the number given to us is divisible by both 11 and 9
= 2222^7777 + 7777^2222 is divisible by 99

10) What is the remainder when 1212121... (up to 300 digits) is divided by 99?

Answer: To find out the divisibility for 99, we need to add the digits in blocks of two from right to left.
As a matter of fact for 999...n times, we need to add the digits in blocks of 'n' from right to left.

So, for 12121212.... 300 digits,
Sum of digits in blocks of two = 12 + 12 + 12.... 150 times = 1800

Rem [121212... 300 digits/ 99]
= Rem [1800 / 99]
= 18

11) What is the remainder when 2222.....300 times is divided by 999?

Answer: To check divisibility by 999, check the sum of the digits taken 3 at a time

Sum of the digits of 222.... 300 times (taken 3 at a time)
= 222 + 222 + 222.... 100 times
= 22200

Rem [22000/999] = 222

12) What is the remainder when 123, 123,... (up to 300 digits) is divided by 999?

Answer: To find out the divisibility for 999, we need to add the digits in blocks of three from right to left.
As a matter of fact for 999...n times, we need to add the digits in blocks of 'n' from right to left.

So, for 123123123.... 300 digits,
Sum of digits in blocks of three = 123 + 123 + 123.... 100 times = 12300

Rem [123123123... 300 digits / 999]
= Rem [12300 / 999]
= 312

Binomial Theorem

1) **Remainder when 25^10 is divided by 576?**

Answer: We need to find out the remainder of 25^10 when divided by 576.
Please note that 576 = 24^2
There are couple of methods of solving this.

Using Binomial Theorem

25^10 = (24 + 1)^10
In the expansion, there will be 11 terms where the powers of 24 will vary from 0 to 10.
If the power of 24 is greater than or equal to 2 in a term, that term will be divisible by 576
The terms that will not be divisible by 576 are the terms that have powers of 24 as 0 or 1.
Those terms are
10C1*24^1*1^9 + 10C0*24^0*1^10
= 10*24*1 + 1*1*1
= 241

So, Rem [25^10/576] = 241

Simplifying the dividend (Direct)

1) **What is the remainder when 15^2010 +16^2011 is divided by 7?**
Answer: Here we need to know that:
Rem[(a + b)/c] = Rem[a/c] + Rem[b/c]
Rem[(a*b)/c] = Rem[a/c] * Rem[b/c]

Keeping that in mind:
Rem[15^2010/7] = Rem[1^2010/7] = 1

Rem[16^2011/7]
= Rem[2^2011/7]
= Rem[2^2010/7]*Rem[2/7]
= Rem[8^670/7] * 2
= 1*2 = 2

Rem[(15^2010 + 16^2011)/7] = 1 + 2 = 3

2) **What is the remainder when 30^40 is divided by 7?**
Answer: Rem [30^40 / 7]
= Rem[2^40 / 7]
= Rem[2^39 * 2 / 7]
= Rem[8^13 * 2 / 7]
= Rem[1^13 * 2 / 7]
= Rem[1*2 / 7]
= 2

3) **How do I solve for the remainder of (19^98)/7?**
Answer: Rem [19^98/7]
= Rem [(-2)^98/7]
= Rem [2^98/7]
= Rem [2^96 * 2^2 / 7]
= Rem [8^32 * 4 / 7]
= Rem [1 * 4 / 7]
= 4

4) **What is the remainder when 7^2015 is divided by 9?**

Answer: Rem [7^2015 / 9]

= Rem [(-2)^2015 / 9]

= Rem [4*(-2)^2013 / 9]

= Rem [4*(-8)^671 / 9]

= Rem [4*1 / 9]

= 4

5) **What is the remainder when 2014^2015 is divided by 9?**

Answer: Rem [2014^2015 / 9]

= Rem [(-2)^2015 / 9]

= Rem [4*(-2)^2013 / 9]

= Rem [4*(-8)^671 / 9]

= Rem [4*1 / 9]

= 4

6) **What is the remainder when 2^2003 is divided by 17?**

Answer: Rem [2^2003 / 17]

= Rem [2^2000 * 8 /17]

= Rem [16^500 * 8 / 17]

= Rem [(-1)^500 * 8 / 17]

= Rem [1*8/17]

= 8

7) **What is the remainder when (16^27+37) is divided by 17?**

Answer: Rem [(16^27+37)/17]

= Rem [16^27/17] + Rem [37/17]

= Rem [(-1)^27/17] + 3

= -1 + 3

= 2

8) **What is the remainder when 7^121 is divided by 17?**

Answer: Rem [7^121 / 17]

= Rem [7^120 * 7 / 17]

= Rem [49^60 * 7 / 17]

= Rem [(-2)^60 * 7 / 17]

= Rem [16^15 * 7 /17]

= Rem [(-1)^15 * 7 / 17]

= Rem [(-7) / 17]

= 10

9) What is the remainder when 30^100 is divided by 17?

Answer: Rem [30^100 / 17]

= Rem[(-4)^100 / 17]

= Rem[16^50/ 17]

= Rem[(-1)^50 / 17]

= Rem[1^50 / 17]

= 1

10) What is the remainder when 54^124 divided by 17?

Answer: Rem [54^124 / 17]

= Rem[(3)^124 / 17]

= Rem[81^31 / 17]

= Rem[(-4)^31 / 17]

= Rem[(-4)^30 * (-4) / 17]

= Rem[(16)^15 * (-4) / 17]

= Rem[(-1)^15 * (-4) / 17]

= Rem[(-1) * (-4) / 17]

= 4

11) What is the remainder when 21^875 divided by 17?

Answer: Rem [21^875 / 17]

= Rem [4^875 / 17]

= Rem[4*4^874 / 17]

= Rem [4* 16^437 / 17]

= Rem [4*(-1)^437 / 17]

= Rem [4*(-1) / 17]

= Rem [-4 / 17]

= 13

12) What is the remainder when 17^200 is divided by 18?

Answer: These type of questions become really simple if you understand the concept of negative remainders. Always try and reduce the dividend to 1 or -1.

Rem [17^200 / 18]

= Rem [(-1)^200 / 18]

= Rem [1 / 18]

= 1

13) What is the remainder when 2^33 is divided by 27?

Answer: Rem [2^33/27]

= Rem [32^6 * 8 / 27]

= Rem [5^6 * 8 / 27]

= Rem [125^2 * 8 / 27]

= Rem [(-10)^2 * 8 / 27]

= Rem [800/27]

= 17

14) What's the remainder when 2^99 is divided by 33?

Answer: Rem [2^99 / 33]

= Rem [2^4 * 2^95 / 33]

= Rem [16 * 32^19 / 33]

= Rem [16 * (-1)^19 / 33]

= Rem [(-16) / 33]

= 17

15) What is the remainder when (71^71+71) is divided by 72?

Answer: Rem [(71^71 + 71)/72]

= Rem [71^71/72] + Rem [71/72]

= Rem [(-1)^72] + (-1)

= (-1) + (-1)

= -2

= 70

Simplifying the dividend (Multiple divisors)

1) **What is the remainder of (2^(90)) /91?**
Answer: We have to find out Rem [2^90/91]

This looks a little difficult if you do not any theorems for finding out remainders. Let us take a simpler approach and break down the problem into smaller parts.

These type of questions become really simple if you understand the concept of negative remainders. Always try and reduce the dividend to 1 or -1.

91 = 7*13

Let us find out Rem[2^90/7] and Rem[2^90/13]
We will combine them later.

Rem [2^90/7]
= Rem [(2^3)^30 / 7]
= Rem [8^30 / 7]
= Rem [1^30 / 7]
= 1

Rem [2^90/13]
= Rem [(2^6)^15 / 13]
= Rem [64^15 / 13]
= Rem [(-1)^15 / 13]
= -1 from 13
= 12

So, our answer is a number which leaves a remainder of 1 when divided by 7 and it should leave a remainder of 12 when divided by 13.

Let us start considering all numbers that leave a remainder of 12 when divided by 13
= 12 (leaves a remainder of 5 from 7. Invalid)
= 25 (leaves a remainder of 4 from 7. Invalid)
= 38 (leaves a remainder of 3 from 7. Invalid)

= 51 (leaves a remainder of 2 from 7. Invalid)

= 64 (leaves a remainder of 1 from 7. Valid. This is our answer)

2) **What is the remainder when 128^1000 is divided by 153?**

Answer: We have to find out Rem [128^1000/153]

This looks a little difficult if you do not any theorems for finding out remainders. Let us take a simpler approach and break down the problem into smaller parts.

These type of questions become really simple if you understand the concept of negative remainders. Always try and reduce the dividend to 1 or -1.

153 = 9*17

128^1000 = 2^7000

Let us find out Rem[2^7000/9] and Rem[2^7000/17]

We will combine them later.

Rem[2^7000/9]

= Rem [2^6999 x 2 / 9]

= Rem [8^2333 x 2 / 9]

= Rem [(-1)^2333 x 2 / 9]

= Rem [(-1) x 2 / 9]

= - 2 from 9

= 7

Rem[2^7000/17]

= Rem [16^1750 / 17]

= Rem [(-1)^1750 / 17]

= 1

So, our answer is a number which leaves a remainder of 7 when divided by 9 and it should leave a remainder of 1 when divided by 17.

Let us start considering all numbers that leave a remainder of 1 when divided by 17

= 18 (leaves a remainder of 0 from 9. Invalid)

= 35 (leaves a remainder of 8 from 9. Invalid)

= 52 (leaves a remainder of 7 from 9. Valid. This is our answer)

3) What is the remainder when 15^40 divided by 1309?

Answer: We have to find out Rem [15^40/1309]

This looks a little difficult if you do not any theorems for finding out remainders. Let us take a simpler approach and break down the problem into smaller parts.

These type of questions become really simple if you understand the concept of negative remainders. Always try and reduce the dividend to 1 or -1.

1309 = 7*11*17

Let us find out Rem[15^40/7], Rem[15^40/11] , and Rem[15^40/17]
We will combine them later.

Rem[15^40/7]
= Rem [1^40/7]
= 1

Rem[15^40/11]
= Rem [4^40/11]
= Rem [256^10/11]
= Rem [3^10/11]
= Rem [243^2/11]
= Rem [1^2/11]
= 1

Rem[15^40/17]
= Rem [(-2)^40/17]
= Rem [16^10/17]
= Rem [(-1)^10/17]
= 1

So, our answer is a number which leaves a remainder of 1 when divided by 7, 11, and 17
Such a number is 1 itself and that is our answer.

Finding Last Digit / Last Two Digits

1) **What is the last digit of 1273^122!?**

   Answer: Last digit of 1273^122!

   = Last digit of 3^122! (Because last digit of the number depends only on the last digit of the base and not the other digits)

   = Last digit of 3^4n (Last digits of powers of 3 move in cycle of 4. The cycle is 3, 7, 9, and 1. 122! is a multiple of 4, so it can be written as 4n)

   = 1

2) **K=1! +2! +3! ...+19!. What would be the last digit of K?**

   Answer: K = 1! + 2! + 3! .... 19!

   Last digit of K = Last digit (1!) + Last digit (2!) + Last digit (3!) ... Last digit (19!)

   Last digit (1!) = 1

   Last digit (2!) = 2

   Last digit (3!) = 6

   Last digit (4!) = 4

   Last digit (5!) = 0

   Last digit (6!) = 0

   .

   .

   .

   Last digit (19!) = 0

   Please note that Last digit (n!) such that n > 4 will be 0

   Last digit of K = 1 + 2 + 6 + 4 + 0 + 0 + 0 ... 0 = 3

3) **What is the remainder when N= (1! +2! +3! +4! +...1000!)^40 is divided by 10?**

   Answer: We have to find out remainder of (1! + 2! + 3! ... 1000!)^40 from 10

   = We have to find out last digit of (1! + 2! + 3! ... 1000!)^40

   Last digit of n! where n > 4 will be 0 and will have no impact on the answer.

   = We have to find out last digit of (1! + 2! + 3! + 4!)^40

Last digit of (1! + 2! + 3! + 4!)^40
= Last digit of (1 + 2 + 6 + 24)^40
= Last digit of 33^40
= Last digit of 3^40
= Last digit of 81^10
= Last digit of 1^10
= 1

4) **How do you find the remainder when 7^26 is divided by 100?**
Answer: Finding out the remainder from 100, is the same as finding out the last two digits of a number
Last two digits of 7^1 are 07
Last two digits of 7^2 are 49
Last two digits of 7^3 are 43
Last two digits of 7^4 are 01
After this, the same pattern will keep on repeating.
So, 7^(4n+1) will end in 07, 7^(4n+2) will end in 49, 7^(4n + 3) will end in 43, and 7^4n will end in 01

7^26 = 7^(4n+2) will end in 49
= Rem [7^26/100] = 49

5) **What is the remainder when 787^777 is divided by 100?**
Answer: We have to find out the remainder of 787^777 divided by 100
This is the same as finding out the last two digits of 787^777

Last two digits of the answer depend on the last two digits of the base.
= We need to find out last two digits of 87^777

The key in questions like these is to reduce the number to something ending in 1.
87^777
= 87 * 87^776
= 87 * (..69)^388 {Just looking at last two digits of 87^2}
= 87 * (...61)^194 {Just looking at last two digits of 61^2}
= 87 * (...41) {a number of the format ..a1^..b will end in (a*b)1}
= 67

6) **What are the last two digits of 2^1997?**

Answer: For finding out the last two digits of an even number raised to a power, we should first try and reduce the base to a number ending in 24.
After that, we can use the property
Last two digits of 24^Odd = 24
Last two digits of 24^Even = 76

Last two digits of 2^1997
= Last two digits of 2^7 * (2^1990)
= Last two digits of 128 * (1024^199)
= Last two digits of 28*24
= 72

7) **What are the last two digits of 2^2012?**

Answer: For finding out the last two digits of an even number raised to a power, we should first try and reduce the base to a number ending in 24.
After that, we can use the property
Last two digits of 24^Odd = 24
Last two digits of 24^Even = 76

Last two digits of 2^2012
= Last two digits of 2^2 * (2^2010)
= Last two digits of 4 * (1024^201)
= Last two digits of 4*24
= 96

8) **How do I find the last 2 digits of (123)^123!?**

Answer: For finding out the last two digits of an odd number raised to a power, we should first try and reduce the base to a number ending in 1.
After that, we can use the property
Last two digits of (...a1)^(...b) will be [Last digit of a*b]1

Let us try and apply this concept in the given question
Last two digits of 123^123!
= Last two digits of 23^123!
= Last two digits of (23^4)^(123!/4)
= Last two digits of (529^2)^(123!/4)

= Last two digits of (...41)^(a large number ending in a lot of zeroes)
= Last two digits of (...01) {Here I have used the concept mentioned above}
= 01

9) **Find the last two digits of 2025^2052+1392^1329?**
Answer: Let us break the problem into two parts.

For the first part :
Last two digits of 2025^2052
= Last two digits of 25^2052
= 25 I{Any power of 25 will have the last two digits as 25}

For the second part:
For finding out the last two digits of an odd number raised to a power, we should first try and reduce the base to a number ending in 1.
After that, we can use the property
Last two digits of (...a1)^(...b) will be [Last digit of a*b]1

For finding out the last two digits of an even number raised to a power, we should first try and reduce the base to a number ending in 24.
After that, we can use the property
Last two digits of 24^Odd = 24
Last two digits of 24^Even = 76

Let us try and apply these concepts in the given question
1392^1329
= 92^1329
= 4^1329 * 23^1329
= 2^2658 * 23 * 23^1328
= 2^8 * 2^2650 * 23 * (23^4)^332
= 256 * (1024^265) * 23 * (...41)^332
= 56 * 24 * 23 * 81
= 72

So, our overall answer will be the sum of the two parts
= 25 + 72 = 97

10) What are the last two digits of (86789)^41?

Answer: For finding out the last two digits of an odd number raised to a power, we should first try and reduce the base to a number ending in 1.
After that, we can use the property
Last two digits of (...a1)^(...b) will be [Last digit of a*b]1

Let us try and apply this concept in the given question
(86789)^41
= 89^41
= 89 * 89^40
= 89 * (..21)^40
= 89 * 01 {Here I have used the concept mentioned above}
= 89

11) What will be the last two digits of 57 ^69?

Answer: For finding out the last two digits of an odd number raised to a power, we should first try and reduce the base to a number ending in 1.
After that, we can use the property
Last two digits of (...a1)^(...b) will be [Last digit of a*b]1

Let us try and apply this concept in the given question
Last two digits of 57^69
= Last two digits of 57*(57^2)^34
= Last two digits of 57*(..49)^34
= Last two digits of 57*(..01)^17
= Last two digits of 57*(..01)
= 57

12) How do we find the last 2 digits of 19^39?

Answer: For finding out the last two digits of an odd number raised to a power, we should first try and reduce the base to a number ending in 1.
After that, we can use the property
Last two digits of (...a1)^(...b) will be [Last digit of a*b]1

Let us try and apply this concept in the given question
19^39
= 19*19^38
= 19*(361)^19
= 19*(...41) {Here I have applied the concept given above}
= (...79)

13) What is the remainder when 767^1009 is divided by 25?

Answer: Remainder of a number from 25 will be the same as the remainder of the last two digits of the number from 25.

For finding out the last two digits of an odd number raised to a power, we should first try and reduce the base to a number ending in 1.
After that, we can use the property
Last two digits of (…a1)^(…b) will be [Last digit of a*b]1

Let us try and apply this concept in the given question
Last two digits of 767^1009
= Last two digits of 67^1009
= Last two digits of 67*67^1008
= Last two digits of 67*(67^2)^504
= Last two digits of 67*(..89)^504
= Last two digits of 67*((..89)^2)^252
= Last two digits of 67*(...21)^252
= Last two digts of 67*(...41) {Here I have used the property mentioned above}
= 47

Rem [767^1009 / 25]
= Rem [47/25]
= 22

14) What are the remainders when 2^222 and 11^100 are divided by 25?

Answer: We need to solve two questions here. In both, we need to find out the remainder from 25. Solving both of them would be easier if we just find out the last two digits.

Remainder of the last two digits of a number from 25 will be the same as the remainder of the number from 25.

Rem [2^222 / 25]

Last two digits of 2^222
= Last two digits of 4*2^220
= Last two digits of 4*1024^22
= Last two digits of 4*(...76) {24^Even will always end in 76}
= Last two digits of (...04)
= 04

So, Rem [04 / 25] = 4
= Rem [2^222 / 25] = 4

Rem [11^100 / 25]

Last two digits of 11^100 = 01 {Last two digits of (…a1)^(…b) will be [Last digit of a*b]1}
= Rem [11^100 / 25] = 1

Fermat's Theorem

1) **What is the remainder of 57^67^77/17 ?**
Answer: Rem [57^67^77/ 17] = Rem [6^67^77/17]

Now, by Fermat's theorem, which states Rem [a^(p-1)/p] = 1, we know
Rem [6^16/17] = 1

The number given to us is 6^67^77
Let us find out Rem[Power / Cyclicity] t0 find out if it 6^(16k+1) or 6^(16k+2). We can just look at it and say that it is not 6^16k
Rem [67^77/16] = Rem [3^77/16] = Rem[(3^76*3^1)/16]
= Rem[((81^19)*3)/16] = Rem [1*3/16] = 3

= The number is of the format 6^(16k + 3)

= Rem [6^67^77 /17] = Rem [6^(16k + 3)/17] = Rem [6^3/17]
= Rem [216/17] = 12

2) **What is the remainder when 2^1000 is divided by 59?**
Answer: We have to find out Rem [2^1000/59]
As per Fermat's Theorem, [a^(p-1)/p] = 1 where p is a prime number and HCF(a,p) = 1

Rem [2^58 / 59] = 1
= Rem [(2^58)^17 / 59] = 1
= Rem [2^986 / 59] = 1

Rem [2^1000/59]
= Rem [2^986 x 2^14 / 59]
= Rem [1 x 128 x 128 / 59]
= Rem [1 x 10 x 10 / 59]
= Rem [100/59]
= 41

3) **What is the remainder when 2^89 is divided by 89?**
Answer: We can use Fermat's Theorem here which says
Rem [a^(p-1)/p] = 1 where p is a prime number
= Rem [2^88/89] = 1
= Rem [2^89/89] = Rem [2^88/89] * Rem [2/89] = 1*2 = 2

4) **What is the remainder when 17^432 is divided by 109?**
Answer: We have to find out Rem [17^432 / 109]

As per Fermat's Theorem, [a^(p-1)/p] = 1 where p is a prime number and HCF(a,p) = 1

Rem [17^108 / 109] = 1
= Rem [(17^108)^4 / 109] = 1
= Rem [17^432 / 109] = 1

5) **What is the remainder when 17^325 is divided by 109?**
Answer: We have to find out Rem [17^325 / 109]

As per Fermat's Theorem, [a^(p-1)/p] = 1 where p is a prime number and HCF(a,p) = 1

Rem [17^108 / 109] = 1
= Rem [(17^108)^3 / 109] = 1
= Rem [17^324 / 109] = 1
= Rem [17^324 *17 / 109] = 1*17
= Rem [17^325 / 109] = 17

6) **What is the remainder when 2^1040 is divided by 131?**
Answer: We have to find out Rem [2^1040/131]

As per Fermat's Theorem, [a^(p-1)/p] = 1 where p is a prime number and HCF(a,p) = 1

Rem [2^130 / 131] = 1
= Rem [(2^130)^8 / 131] = 1
= Rem [2^1040 / 131] = 1

Euler's Theorem

1) **What is the remainder of (121) ^(121) divided by 144?**
Answer: Euler Totient (144) = 144*(1-1/2)(1-1/3) = 48
By Euler's Theorem we can say that
Rem[a^48/144] = 1 if a and 144 are coprime to each other
= Rem [a^240/144] = 1
= Rem [11^240/144] = 1

Now, we need to find out Rem [121^121/144]
= Rem [11^242/144]
= Rem [11^240/144]*Rem [11^2/144]
= 1*121
= 121

Taking values common from Dividend and Divisor

1) **What is the remainder when 39^198 divided by 12**

   Answer: Rem [39^198 / 12]

   = Rem [3^198 / 12]

   = 3* Rem[3^197 / 4]

   = 3* Rem[(-1)^197 / 4]

   = 3* Rem[-1 / 4]

   = 3*3

   = 9

2) **What is the remainder when 3^164 is divided by 162?**

   Answer: To solve this, you need to know that Rem [ka/kb] = k Rem[a/b]

   We need to find out

   Rem [3^164/162]

   = Rem [(3^4 x 3^160) / (3^4 x 2)]

   = 3^4 Rem [3^160/2]

   = 3^4 Rem [1^160/2]

   = 3^4 x 1

   = 81

3) **What is the remainder when 21! is divided by 361?**

   Answer: Rem [21!/361]

   = Rem [(21*20*19*18!)/361]

   Using Rem [ka/kb] = k Rem[a/b]

   = 19 Rem [(21*20*18!)/19]

   Using Wilson's Theorem says For a prime number 'p' Rem [(p-1)! / p] = p-1

   = 19 Rem [(21*20*18)/19]

   = 19 Rem [(2*1*(-1))/19]

   = 19*(-2)

   = -38

   = 323

4) **What is the remainder when 2(8!)-21(6!) divides 14(7!) +14(13!)?**

Answer: We need to find out Rem [(14(7!) + 14(13!)) / (2(8!) - 21(6!))]

Let us try and simplify the divisor

2(8!) - 21(6!)

= 2*8*7! - 3*7*6!

= 16*7! - 3*7!

= 13*7!

Rem [(14(7!) + 14(13!)) / 13*7!]

= Rem [14(7!) / 13(7!)] + Rem [14(13!) / 13(7!)]

= 7!*Rem[14/13] + 0

= 7!*1

= 7!

Pattern Recognition / Cyclicity Method

1) **What is the remainder when 32^32^32 is divided by 7?**
Answer: Rem [32^32^32 / 7] = Rem [4^32^32 /7]

Now, we need to observe the pattern
4^1 when divided by 7, leaves a remainder of 4
4^2 when divided by 7, leaves a remainder of 2
4^3 when divided by 7, leaves a remainder of 1

And then the same cycle of 4, 2, and 1 will continue.
If a number is of the format of 4^(3k+1), it will leave a remainder of 4
If a number is of the format of 4^(3k+2), it will leave a remainder of 2
If a number is of the format of 4^(3k), it will leave a remainder of 1

The number given to us is 4^32^32

Let us find out Rem[Power / Cyclicity] t0 find out if it 4^(3k+1) or 4^(3k+2). We can just look at it and say that it is not 4^3k
Rem [32^32/3] = Rem [(-1)^32/3] = 1
= The number is of the format 4^(3k + 1)
= Rem [4^32^32 /7] = 4

2) **What is the remainder when 32^ (32^ (32^...infinite times)) is divided by 9?**
Answer: Rem [32^32^32... / 9] = Rem [4^32^32... /9]

Now, we need to observe the pattern
4^1 when divided by 9, leaves a remainder of 4
4^2 when divided by 9, leaves a remainder of 7
4^3 when divided by 9, leaves a remainder of 1

And then the same cycle of 4, 7, and 1 will continue.
If a number is of the format of 4^(3k+1), it will leave a remainder of 4
If a number is of the format of 4^(3k+2), it will leave a remainder of 7
If a number is of the format of 4^(3k), it will leave a remainder of 1

The number given to us is 4^32^32....

Let us find out Rem[Power / Cyclicity] t0 find out if it 4^(3k+1) or 4^(3k+2). We can just look at it and say that it is not 4^3k

Rem [32^32^32.../3] = Rem [(-1)^32^32.../3] = 1
= The number is of the format 4^(3k + 1)
= Rem [4^32^32 /9] = 4

3) What is the remainder when 34^31^301 is divided by 9?
Answer: Rem [34^31^301 / 9] = Rem [7^31^301 /9]

Now, we need to observe the pattern
7^1 when divided by 9, leaves a remainder of 7
7^2 when divided by 9, leaves a remainder of 4
7^3 when divided by 9, leaves a remainder of 1
And then the same cycle of 7, 4, and 1 will continue.
If a number is of the format of 7^(3k+1), it will leave a remainder of 7
If a number is of the format of 7^(3k+2), it will leave a remainder of 4
If a number is of the format of 7^(3k), it will leave a remainder of 1

The number given to us is 7^31^301
Let us find out Rem[Power / Cyclicity] t0 find out if it 7^(3k+1) or 7^(3k+2). We can just look at it and say that it is not 7^3k
Rem [31^301/3] = Rem [1^301/3] = 1
= The number is of the format 7^(3k + 1)
= Rem [7^31^301 /9] = 7

4) What is the remainder of 30^72^87 when divided by 11?
Answer: Rem [30^72^87 / 11] = Rem [(-3)^72^87 / 11] = Rem [3^72^87 / 11]

Now, we need to observe the pattern
3^1 when divided by 11, leaves a remainder of 3
3^2 when divided by 11, leaves a remainder of 9
3^3 when divided by 11, leaves a remainder of 5
3^4 when divided by 11, leaves a remainder of 4
3^5 when divided by 11, leaves a remainder of 1
And then the same cycle of 3, 9, 5, 4 and 1 will continue.

If a number is of the format of 3^(5k + 1), it will leave a remainder of 3
If a number is of the format of 3^(5k + 2), it will leave a remainder of 9
If a number is of the format of 3^(5k + 3), it will leave a remainder of 5
If a number is of the format of 3^(5k + 4), it will leave a remainder of 4
If a number is of the format of 3^(5k), it will leave a remainder of 1

The number given to us is 3^72^87
Let us find out Rem[Power / Cyclicity] t0 find out if it 3^(5k + what?)

Rem [72^87 / 5]
= Rem [2^87 / 5]
= Rem [2*4^43/5]
= Rem [2*(-1) / 5]
= -2
= 3
= The number is of the format 3^(5k + 3)
= Rem [3^72^87 / 11] = 5

5) **What is the remainder when 1! +2*2! +3*3! +4*4! +... +12*12! Is divided by 13?**
Answer: The trick in these type of questions is often observing the pattern

1! + 2*2! = 1 + 4 = 5 = 3! - 1
1! + 2*2! + 3*3! = 1 + 4 + 18 = 23 = 4! - 1
1! + 2*2! + 3*3! + 4*4! = 1 + 4 + 18 + 96 = 119 = 5! - 1
1! + 2*2! + 3*3! + 4*4! + 5*5! = 1 + 4 + 18 + 96 + 600 = 719 = 6! - 1
So, we can say
1! +2*2! +3*3! +4*4! +... +12*12! = 13! - 1
= Rem [(1! +2*2! +3*3! +4*4! +... +12*12!) / 13] = -1 = 12

6) **What is the remainder of 57^67^77/17 ?**
Answer: Rem [57^67^77/ 17] = Rem [6^67^77/17]

Now, by Fermat's theorem, which states Rem [a^(p-1)/p] = 1, we know
Rem [6^16/17] = 1

The number given to us is 6^67^77

Let us find out Rem[Power / Cyclicity] t0 find out if it 6^(16k+1) or 6^(16k+2). We can just look at it and say that it is not 6^16k
Rem [67^77/16] = Rem [3^77/16] = Rem[(3^76*3^1)/16]
= Rem[((81^19)*3)/16] = Rem [1*3/16] = 3

= The number is of the format 6^(16k + 3)

= Rem [6^67^77 /17] = Rem [6^(16k + 3)/17] = Rem [6^3/17]
= Rem [216/17] = 12

7) **How do you find the remainder when 7^26 is divided by 100?**
Answer: Finding out the remainder from 100, is the same as finding out the last two digits of a number
Last two digits of 7^1 are 07
Last two digits of 7^2 are 49
Last two digits of 7^3 are 43
Last two digits of 7^4 are 01
After this, the same pattern will keep on repeating.
So, 7^(4n+1) will end in 07, 7^(4n+2) will end in 49, 7^(4n + 3) will end in 43, and 7^4n will end in 01

7^26 = 7^(4n+2) will end in 49
= Rem [7^26/100] = 49

8) **What is the remainder when 7^99 is divided by 2400 and how?**
Answer: Let us try doing it by the pattern recognition / cyclicity method. It can be really long if you do not get the pattern quickly. Use other methods like Euler's Totient if you do not get a pattern quickly.

Rem[7^1 / 2400] = Rem [7 / 2400] =7
Rem[7^2 / 2400] = Rem [49 / 2400] =49
Rem[7^3 / 2400] = Rem [343/ 2400] = 343
Rem[7^4 / 2400] = Rem [2401 / 2400] = 1

After this the same pattern will keep on repeating because you got a 1.

Once we have obtained the cyclicity (number of terms in the pattern), all we need to do is to find out the Remainder of Power when divided by the Cyclicity. Whatever is this remainder, that particular value in the cycle is our answer.

In this case, power is 99 and cyclicity is 4.

Rem [Power / Cyclicity] = Rem [99/4] = 3

= Our answer will be the third value in the cycle = 343

Wilson's Theorem

1) **What is the remainder when 97! is divided by 101?**
Answer: Wilson's Theorem says for a prime number 'p'
Rem [(p-1)! / p] = p-1
This can be extended to say,
Rem [(p-2)! / p] = 1

Let us use that here. We need to find out Rem [97! / 101] = r
We know from the above theorem,
Rem [99! / 101] = 1
= Rem [99*98*97! / 101] = 1
= Rem [(-2)*(-3)*r / 101] = 1
= Rem [6r / 101] = 1
= 6r = 101k + 1
We need to think of a value of k, such that 101k + 1 is divisible by 6.
If we put, k = 1, we get 101 + 1 = 102, which is divisible by 6.
= 6r = 102
= r = 17

2) **What is the remainder when 21! is divided by 361?**
Answer: Rem [21!/361]
= Rem [(21*20*19*18!)/361]

Using Rem [ka/kb] = k Rem[a/b]
= 19 Rem [(21*20*18!)/19]

Wilson's Theorem says for a prime number 'p' Rem [(p-1)! / p] = p-1

= 19 Rem [(21*20*18)/19]
= 19 Rem [(2*1*(-1))/19]
= 19*(-2)
= -38
= 323