Quantum Mechanics

Soumitra Nandi

PH101, Lec-1

Quantum Weirdness !

- ✓ **The most fascinating aspect of quantum mechanics is that it provides a strange picture of the world.**

If you accept this picture — and given the practical successes of the theory it is difficult not to — you are left with no choice but to make fundamental changes to your idea of reality.

- ✓ The first surprise is the wave-particle duality of the building blocks of matter.

The world is not made of waves and particles, as in classical physics, but of peculiar hybrid objects with aspects of both.

- ✓ The physical state of a quantum mechanical particle-wave is described by a wave function, $\psi(x,t)$, analogous to the amplitude of a classical wave.

Does not evolve according to the classical wave equation

- ✓ Perhaps the most puzzling aspect of quantum mechanics is that it predicts probabilities only.

Its predictive power is such that quantum mechanics is considered the most successful theoretical physics construct of the human mind.

Classical point of view

- ✓ The state of the particle can be described in terms of its position x and momentum p => **take definite real values at any given moment in time.**

- ✓ We can calculate and predict to **any arbitrary accuracy** the position and momentum **(x(t), p(t))** of this particle at time t, and at a later time **t' > t .**

We can also, in principle, calculate, with unlimited accuracy, the future behaviour of any physical system by solving Newton's equations, Maxwell's equations and so on.

- ✓ In practice, there are limits to accuracy of measurement and/or calculation, but in principle there are no such limits.

- ✓ In a system with many particle (a litre of air in a bottle), we cannot hope to measure all the positions and velocities of all the particles.

We still believe that each particle does in fact possess a definite position and velocity at each instant => Difficult to get at the information.

Quantum view point

- ✓ The classical world-view works fine at the everyday (macroscopic) level – much of modern engineering relies on this !

- ✓ Non-classical behaviour is most readily observed for microscopic systems – atoms and molecules

- ✓ It is impossible to prepare any physical system in which all its physical attributes are precisely specified at the same time

  - ➢ We cannot pin down both the position and the momentum of a particle at the same time.

- ✓ Microscopic physical systems can behave as if they are doing mutually exclusive things at the same time.

  - ➢ This propensity for quantum system to behave as if they can be two places at once, or more generally in different states at the same time, is termed ‘the superposition of states’ !!

Quantum view point

- ✓ It turns out that, in Quantum Mechanics, one of the ideas that we have to abandon is the notion that we can “predict to any arbitrary accuracy" the position and momentum of any particle.

  - ➢ In fact, it is worse than this: another notion we have to abandon the idea that we can measure with arbitrary accuracy both variables at the same time t.

How do we describe the state of a particle ?

How do we describe its dynamics?

- ✓ we have to start with an entire new notion of how dynamical states of systems are described mathematically.

- ✓ We will give up the notion of absolute knowledge about the state, we can start to introduce even more abstract states which has no classical analog such as the “spin" of an electron.

An example: Two state system

The bit is a system that can only has two possible states: 1/0 or up/down or on/off or dead/live etc.

- We can endow this bit with some set of physical rules which when acted upon the system, may change it from one state to another.

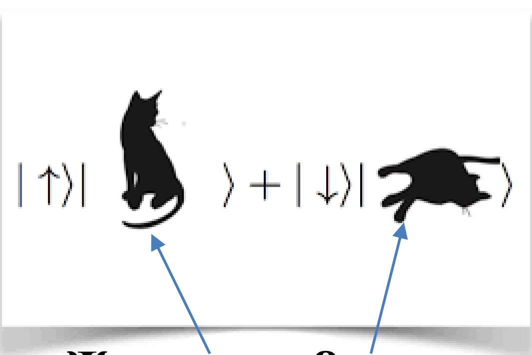
Two-state systems

✓ A bit is a classical quantity, so we can measure it with arbitrary accuracy

=> A classical cat can be either dead or alive !

What about a quantum two-state system?

What about a Quantum Cat ?



$$\Psi = \alpha\, \psi_l + \beta\, \psi_d$$

The cat is in the curious state of being both alive and dead at the same time according to an observer standing outside !

Few basic postulates

$$\Psi = \alpha\, \psi_l + \beta\, \psi_d$$

We do not have knowledge of ψ but still we can still understand few of it's properties !!

state vector

Probability amplitudes

Postulate 1 (State) : The state of aliveness/deadness of the cat is the linear superposition between two possible states !

Postulate 2 : The probability of measuring an dead/alive state is the absolute square of the inner product of the desired outcome with the state!

- ✓ Probability of measuring live state = $|\psi . \psi_l|^2 = |\psi\, \psi_l^*|^2 = |\alpha|^2$
- ✓ Probability of measuring dead state = $|\psi . \psi_d|^2 = |\psi\, \psi_d^*|^2 = |\beta|^2$

Postulate 3 (Measurement): Once the box is opened and dead/alive has been obtained, the state vector ψ collapses into the measured state !

$$\psi \rightarrow \psi_{l/d}$$

A look back in history

- Around the beginning of the 20th century, classical physics, based on Newtonian Mechanics and Maxwell's equations of Electricity and Magnetism described nature as we knew it.
- Statistical Mechanics was also a well developed discipline describing systems with a large number of degrees of freedom.
- Special Relativity was compatible with Maxwell's equations and changed our understanding of space-time and modified Mechanics.

✓ These laws are based on experimental observations and were expressed in a manner consistent with how we understand our physical system from classical point of view !!

✓ There is no uncertainty or randomness as a consequence of our ignorance of information about a system implicit in any of these laws.

The pre-quantum experimenters were unaware of the fact that the information they were gathering was not refined enough to show that there were fundamental limitations to the accuracy with which they could measure physical properties.

Unexplained observations

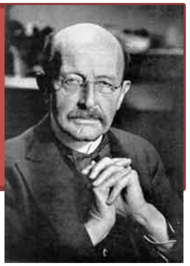
- The electron as a constituent of atoms had been found, atomic structure was rich and quite mysterious.

There were problems with classical physics !!

Many things remained unexplained

- ✓ Black Body Radiation
- ✓ The Photoelectric effect
- ✓ Basic Atomic Theory
- ✓ Compton Scattering, and eventually with the diffraction of all kinds of particles

Black Body Radiation



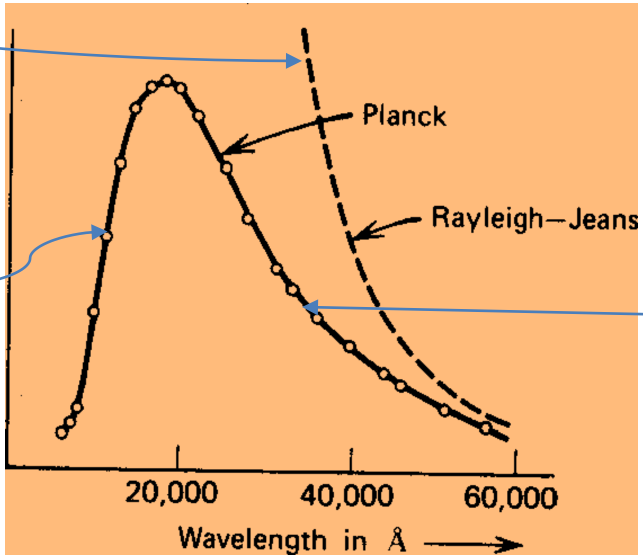
Problems leading to the development of Quantum Mechanics :

Black Body Radiation

- Classical physics predicted that hot objects would instantly radiate away all their heat into electromagnetic waves.
- The calculation, which was based on Maxwell's equations and Statistical Mechanics, showed that the radiation rate went to infinity as the EM wavelength went to zero

$$E(\nu) = \frac{8\pi\nu^2 kT}{c^3}$$

Plank solved the problem by postulating that EM energy is emitted in quanta with **E = hν**

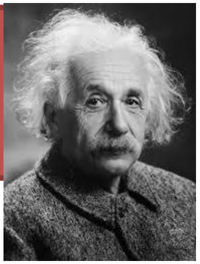


Data Points

Planck's Formula for Radiant Energy:

$$Energy(\nu,T) = \frac{8\pi\nu^3}{c^3}\frac{h}{e^{h\nu/kT}-1}$$

Photoelectric effects



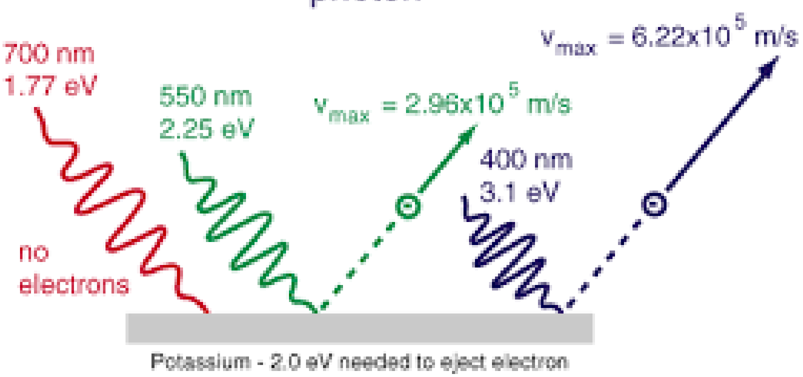
Light is wave or particle like ?? → Both

Historically, while the Young Two Slit experiment is credited with “confirming" the wave-like nature of light, it is the Photoelectric effect that shows that light also exhibits particle-like behaviour, we call light particles photons.

- When light was used to knock electrons out of solids, the results were completely different than expected from Maxwell’s equations.

✓ Electrons will be emitted or not depends only on the frequency !!

$E_{photon} = h\nu$



Photoelectric effect

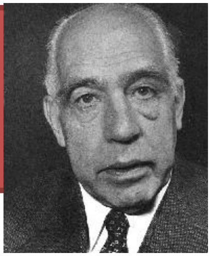
The frequency of the light required to liberate an electron is $\nu = \nu_0 > E_0/h$

Depends on the metal used

✓ The measurements were easy to explain (for Einstein) if light is made up of particles with the energies Plank postulated.

$E_k = h\nu - E_0$ → K.E of the emitted electron

Bohr's theory of Atom



Atoms

- Classical physics predicted that the atomic electrons orbiting the nucleus would radiate their energy away and spiral into the nucleus.
  - ✓ However, the energy radiated by atoms also came out in quantized amounts in contradiction to the predictions of classical physics.
  - ✓ Bohr's solution was to propose that provided the electron circulates in orbits whose radii r satisfy an ad hoc rule, now known as a quantization condition, applied to the angular momentum L of the electron, $L = n\hbar$ for $n = 1,2,3\ldots,$

Orbits would be stable !

- Therefore, atoms will have discrete energy levels !!

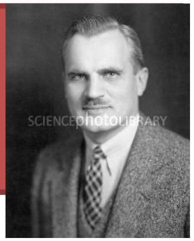
✓ Emits or absorbs light quanta by 'jumping' between the energy levels.

$$E_n = \frac{-e^2}{2(4\pi\epsilon_0)r_n} = \frac{-me^4}{2(4\pi\epsilon_0\hbar)^2 n^2} \approx \frac{-(13.6\,\mathrm{eV})}{n^2}$$

For hydrogen atom, it is consistent with the observation !

Bohr's theory was quite successful for the hydrogen atom, was an utter failure when applied to even the next most complex atom, the helium atom.

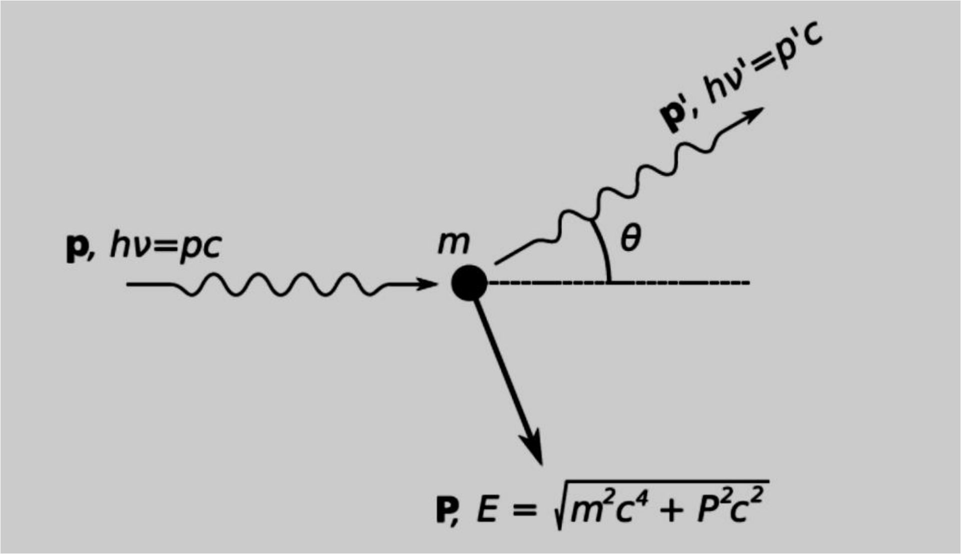
Compton Scattering



Compton scattered high energy photons from (essentially) free electrons !

- ✓ He expected to see scattered radiation with the same frequency as that of photon or few more harmonics !

Scattered photon of longer wavelength



Energy conservation

$$pc + mc^2 = p'c + \sqrt{m^2c^4 + P^2c^2}$$

Momentum conservation

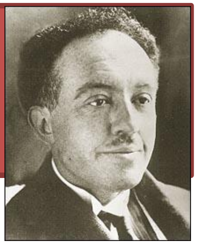
$$\mathbf{p} = \mathbf{p}' + \mathbf{P}$$

Shift in wavelength !

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

- ➢ When light scattered off the electrons, it behaved just like a particle but changes wave length in the scattering !!
  - ✓ One more evidence for the particle nature of light and Plank’s postulate.

Particles are wave



Louis de Broglie's 1924 hypothesis was much bolder than explained so far **!**

✓ **He proposed, with essentially no supporting evidence, that all particles, not just photons, have wave-like properties !**

More precisely, if light waves of frequency v can behave like a collection of particles of energy $E = hv$, then by symmetry, a massive particle of energy E, an electron say, should behave under some circumstances like a wave of frequency $v = E/h$

A wave is also characterised by its wavelength, so it is also necessary to assign a wavelength to these 'matter waves'.

✓ For a particle of light, a photon, the wavelength of the associated wave is $\lambda = c/v$!

So what is it for a massive particle?

For photon ⇨ $P = h/\lambda$

De Broglie assumed that this relationship applied to all free particles, whether they were photons or electrons or anything else.

Experimental evidences

Evidence that Particles are Waves

Electron diffraction

In 1927, Davisson and Germer showed that electrons can be diffracted by the regular array of atoms in a crystal, which acts much like a diffraction grating.

Thermal neutron diffraction

Neutrons are much heavier than electrons and uncharged, so they penetrate deep inside solids and diffract from the bulk crystal structure, not the surface.

He atom diffraction

Beams of low-energy He atoms also diffract from crystals.

Two-slit interference → Discuss in detail !

Two-slit interference patterns have been created using electrons, atoms and even C_{60} molecules !

Atomic energy levels ⇒ Direct consequence of the wave-like properties of electrons

Summary so far !

- ❑ Quantum Mechanics changes our understanding of nature in fundamental ways !
  - ✓The classical laws of physics are deterministic, QM is probabilistic !
  - ✓We can only predict the probability that a particle will be found in some region of space !
  - ✓ The amplitude squared of the waves gives the probability of observing (or detecting, or finding – a number of different terms are used) the particle in some region in space.
  - ✓Probability amplitudes are like the fields we know from electromagnetism !!

The rest of wave mechanics was built around these ideas !!