QUANTUM MODELS OF COGNITION AND DECISION

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Supported by AFOSR and NSF

1. WHY USE QUANTUM THEORY?

1. Quantum theory is a general Axiomatic theory of probability
    - Human judgments and decisions are probabilistic
    - These probabilities do not obey the Kolmogorov axioms
    - Quantum theory provides a viable alternative
2. Non Commutativity of measurements
    - Measurements change psychological states producing context effects
    - Principle of complementarity was borrowed by Niels Bohr from William James
3. Vector space representation of probabilities
    - Agrees with connectionist-neural network models of cognition

2. HOW DO WE USE QUANTUM THEORY?

Bruza, Busemeyer, Wang
(2015, *Trends in Cognitive Science*)

COMPARISON OF CLASSIC AND QUANTUM PROBABILITY THEORIES

Kolmogorov

Von Neumann

Classical

- Each unique outcome is a member of a set of points called the Sample space

Quantum

- Each unique outcome is an orthonormal vector from a set that spans a Vector space

Classical

- Each unique outcome is a member of a set of points called the Sample space
- Each event is a subset of the sample space

Quantum

- Each unique outcome is an orthonormal vector from a set that spans a Vector space
- Each event is a subspace of the vector space.

Classical

- Each unique outcome is a member of a set of points called the Sample space
- Each event is a subset of the sample space
- State is a probability function, p, defined on subsets of the sample space.

Quantum

- Each unique outcome is an orthonormal vector from a set that spans a Vector space
- Each event is a subspace of the vector space.
- State is a unit length vector, S,

$$p(A) = \left\| P_A S \right\|^2$$

Classical

- Suppose event A is observed (state reduction):

$$p(B \mid A) = \frac{p(B \cap A)}{p(A)}$$

Quantum

- Suppose event A is observed (state reduction):

$$p(B \mid A) = \frac{\left\| P_B P_A S \right\|^2}{\left\| P_A S \right\|^2}$$

Classical

- Suppose event A is observed (state reduction):

$$p(B\,|\,A)=\frac{p(B\cap A)}{p(A)}$$

- **Commutative Property**

$$p(B\cap A)=p(A\cap B)$$

Quantum

- Suppose event A is observed (state reduction):

$$p(B\,|\,A)=\frac{\|P_B P_A S\|^2}{\|P_A S\|^2}$$

- Non-Commutative

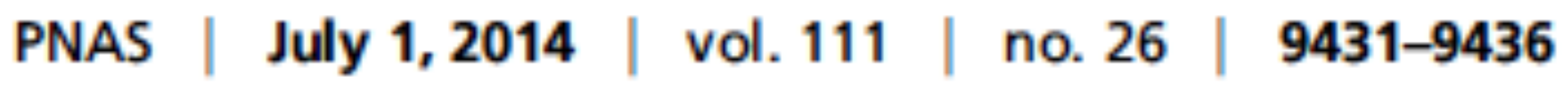
$$\|P_B P_A S\|^2 \neq \|P_A P_B S\|^2$$

SIX APPLICATIONS

1. ORDER EFFECTS

Context effects produced by question orders reveal quantum nature of human judgments

Zheng Wang[a,1], Tyler Solloway[a], Richard M. Shiffrin[b,1], and Jerome R. Busemeyer[b]

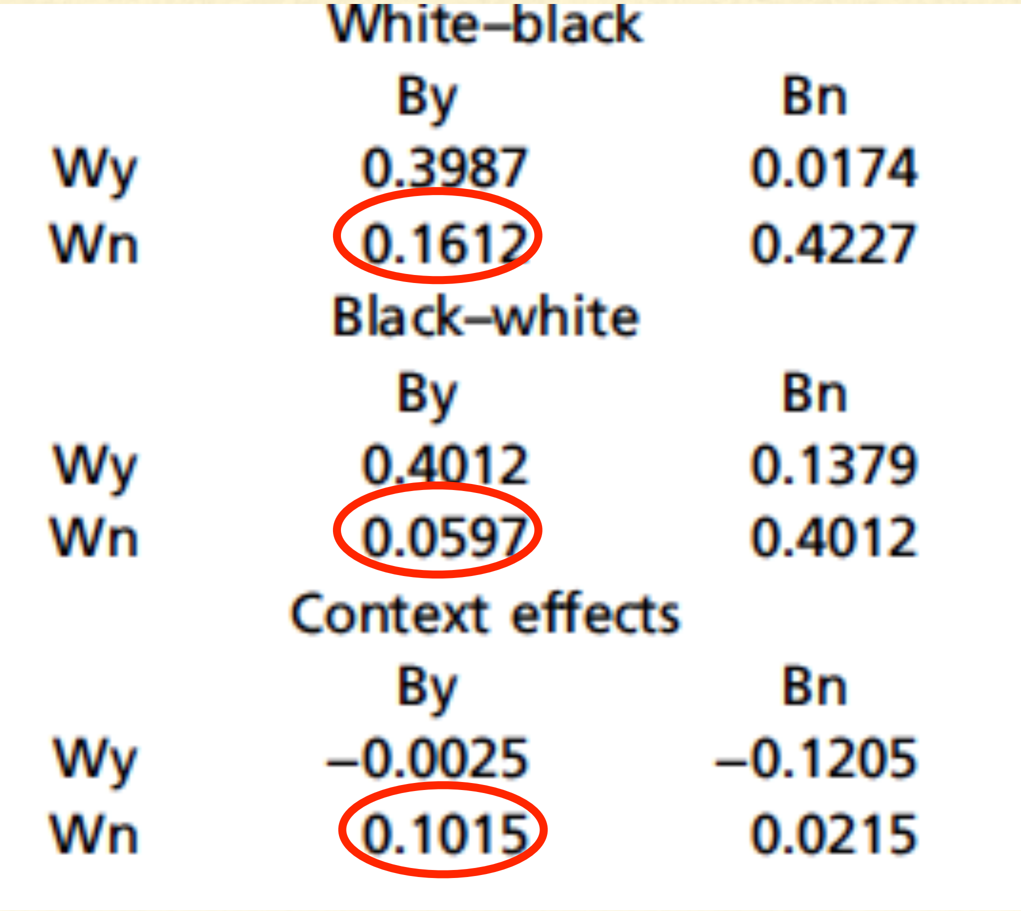


PNAS | July 1, 2014 | vol. 111 | no. 26 | 9431–9436

Hostility between white and black people:

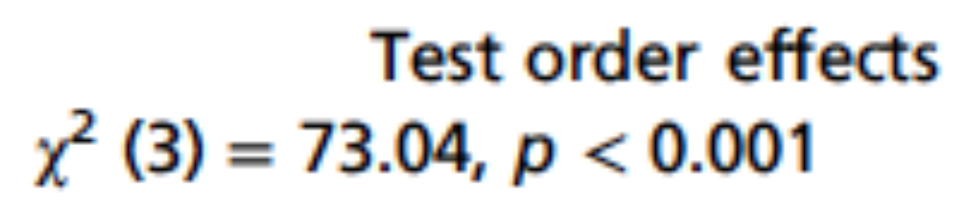
BW order (N = 500 participants)
(A) Do you think blacks dislike whites? (Y,N)
(B) Do you think whites dislike blacks? (Y,N)

WB Order (N = 500 participants)
(B) Do you think whites dislike blacks? (Y,N)
(A) Do you think blacks dislike whites? (Y,N)



	White–black	
	By	Bn
Wy	0.3987	0.0174
Wn	0.1612	0.4227
	Black–white	
	By	Bn
Wy	0.4012	0.1379
Wn	0.0597	0.4012
	Context effects	
	By	Bn
Wy	−0.0025	−0.1205
Wn	0.1015	0.0215

(Results from Gallup Pole)

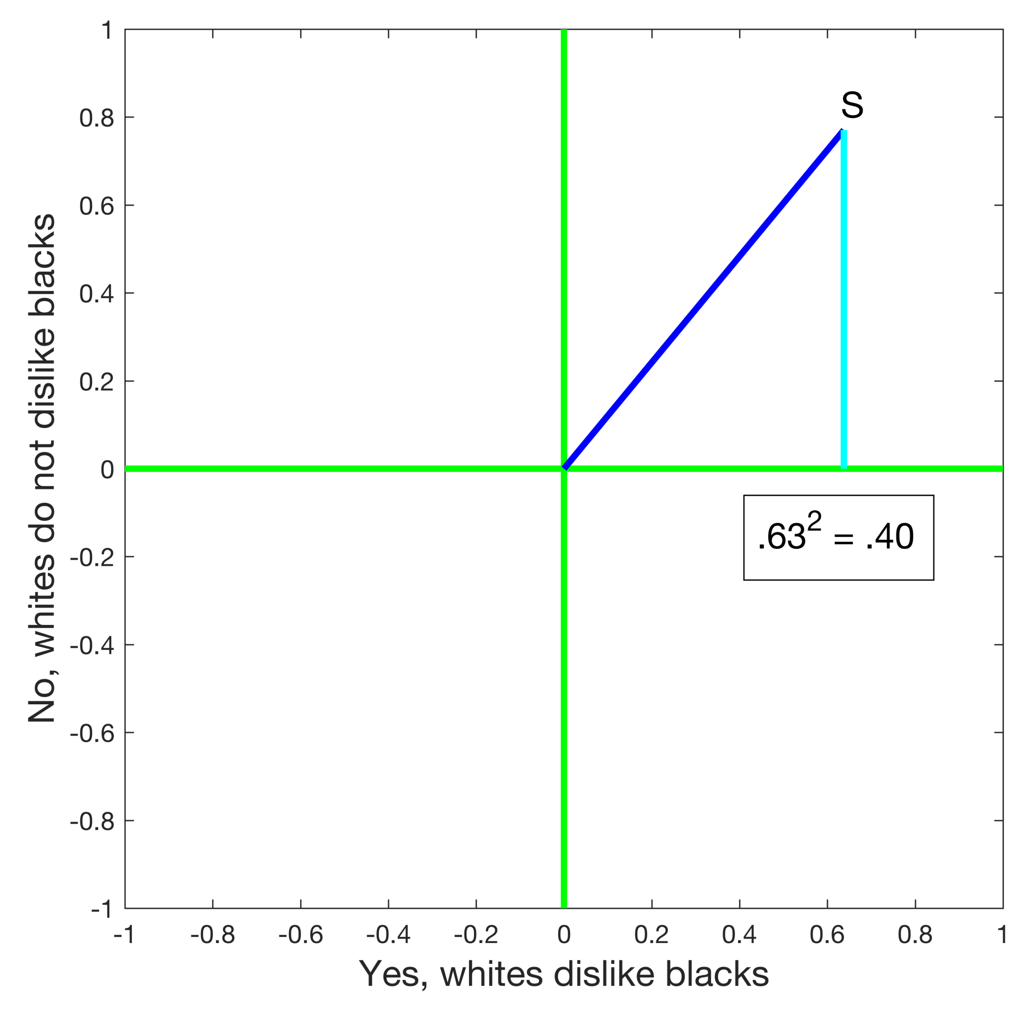


Test order effects

χ^2 (3) = 73.04, $p < 0.001$

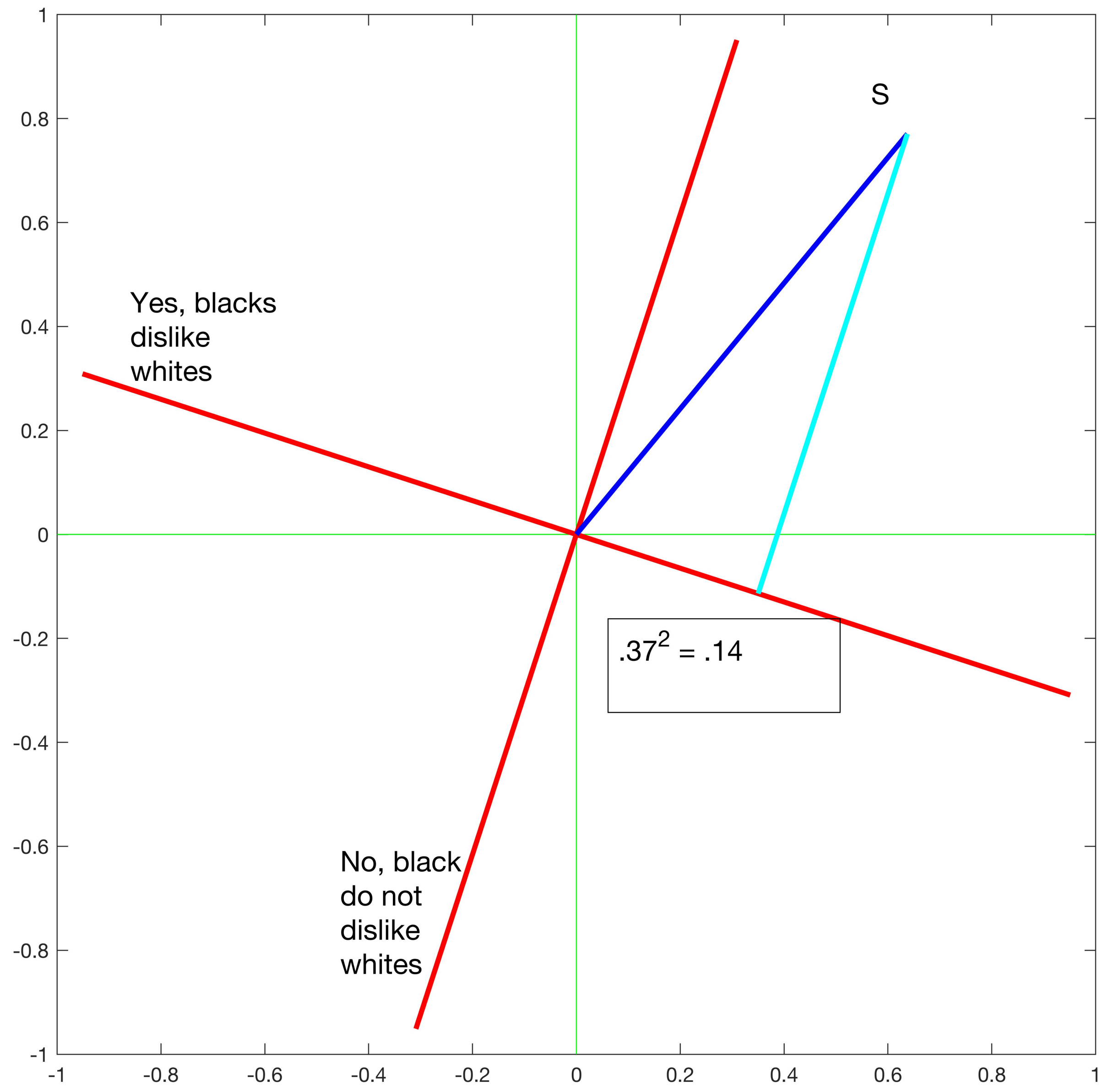
Quantum (Toy) Model

Do Whites dislike Blacks? (Question from Gallop Pole)

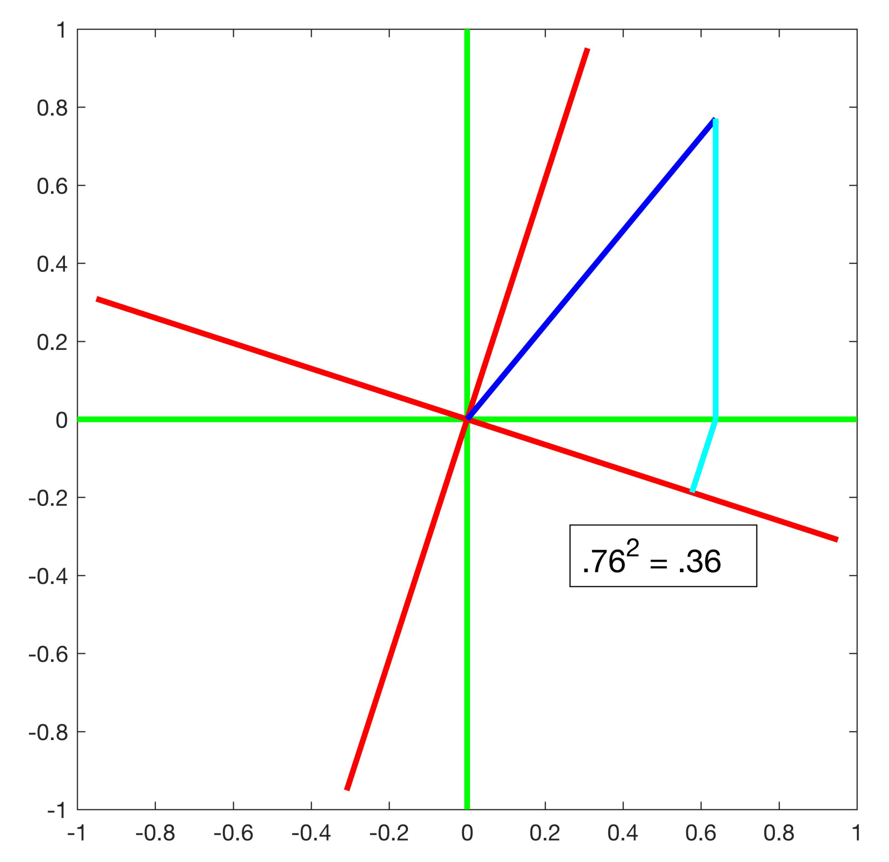


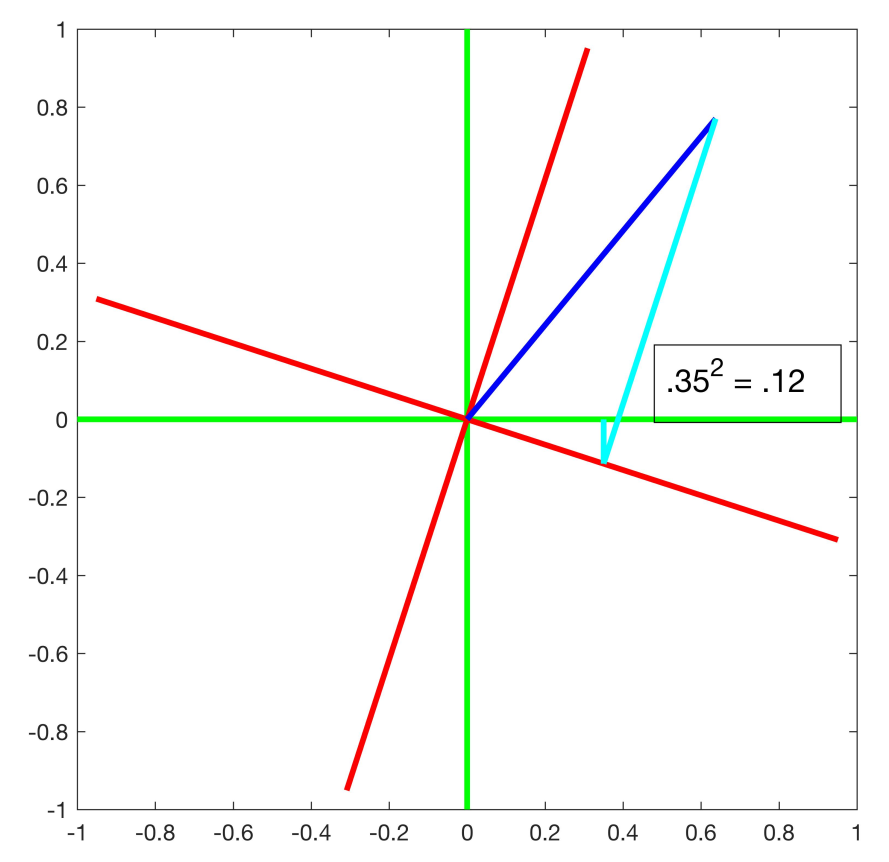
Prob(Yes| S = initial)
= .40
(Results from actual
Gallup Pole, N = 500)

Do Blacks dislike Whites?
Change in Perspective



Toy model for Order Effects





Prob(Yes to WB
and then Yes to BW| S)
= .36

Prob(Yes to BW
and then Yes to WB| S)
= .12

GENERAL N-DIMENSIONAL QUANTUM MODEL PREDICTION

Assume: One question followed immediately by another with no information in between

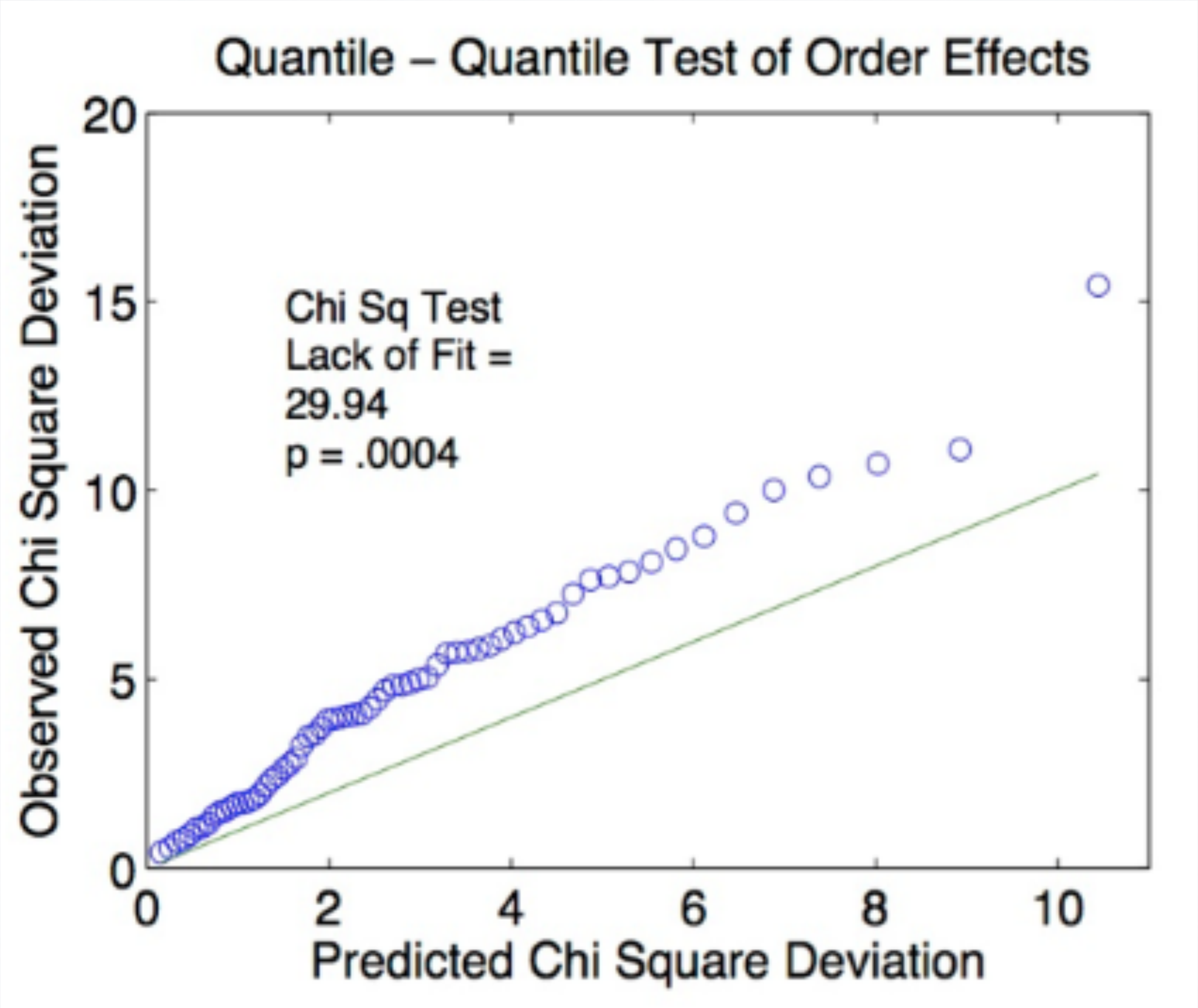
$$\Pr[A \text{ yes and then B no}] = p(A_Y B_N) = \left\| P_{\bar{B}} P_A S \right\|^2$$

$$\Pr[B \text{ no and then A yes}] = p(B_N A_Y) = \left\| P_A P_{\bar{B}} S \right\|^2$$

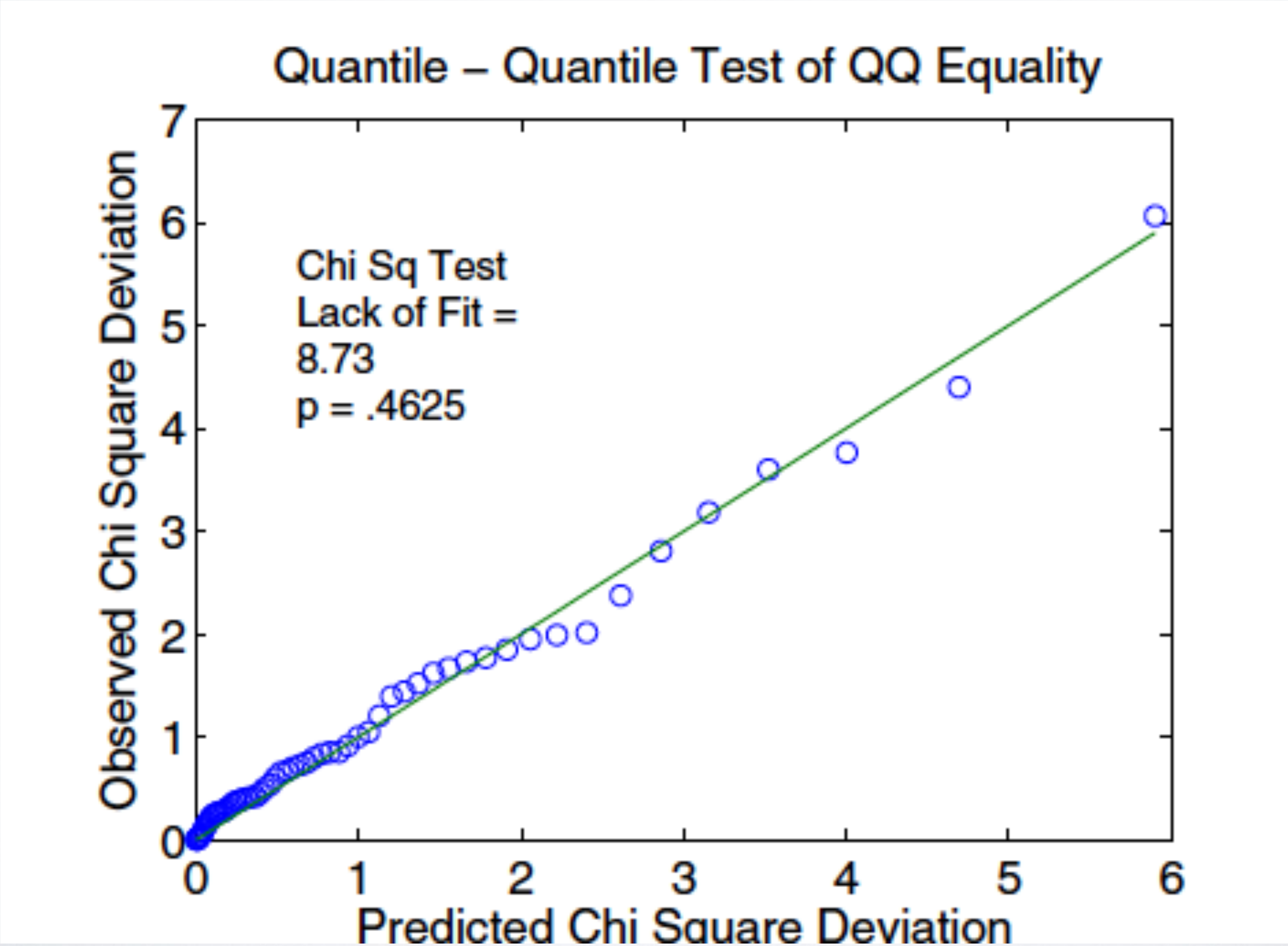
$\textit{Theorem}:$ QQ equality

$$q = \{p(A_Y B_N) + p(A_N B_Y)\} - \{p(B_Y A_N) + p(B_N A_Y)\} = 0$$

Results: 72 Pew Surveys over 10 years



Results: 72 Pew Surveys over 10 years



2. CONJUNCTION -DISJUNCTION PROBABILITY JUDGMENT ERRORS

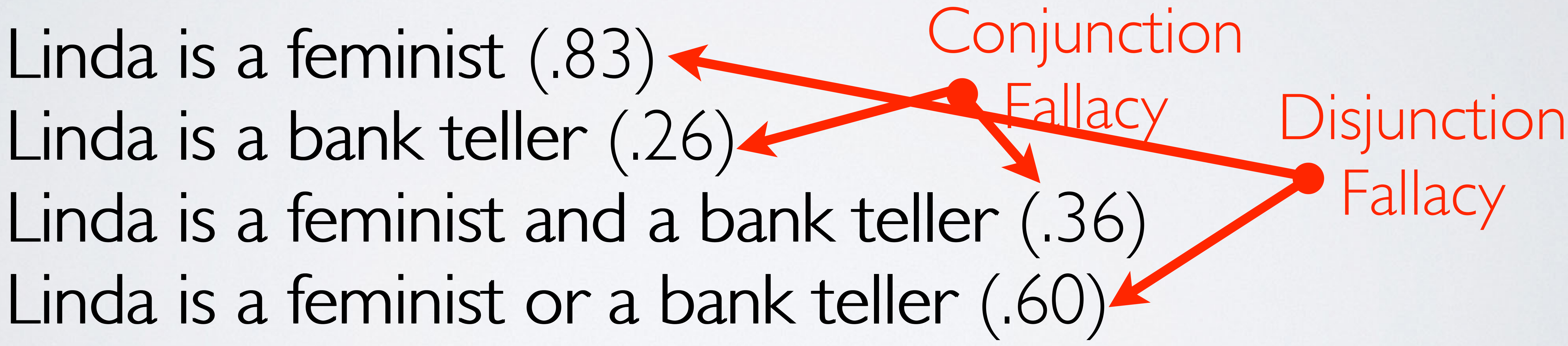
Tversky & Kahneman
(1983, *Psychological Review*)

Busemeyer, Pothos, Franco, Trueblood
(2011, *Psychological Review*)

Read the following information:

Linda was a philosophy major as a student at UC Berkeley and she was an activist in social welfare movements.

Rate the probability of the following events



Linda is a feminist (.83)
Linda is a bank teller (.26)
Linda is a feminist and a bank teller (.36)
Linda is a feminist or a bank teller (.60)

Conjunction Fallacy

Disjunction Fallacy

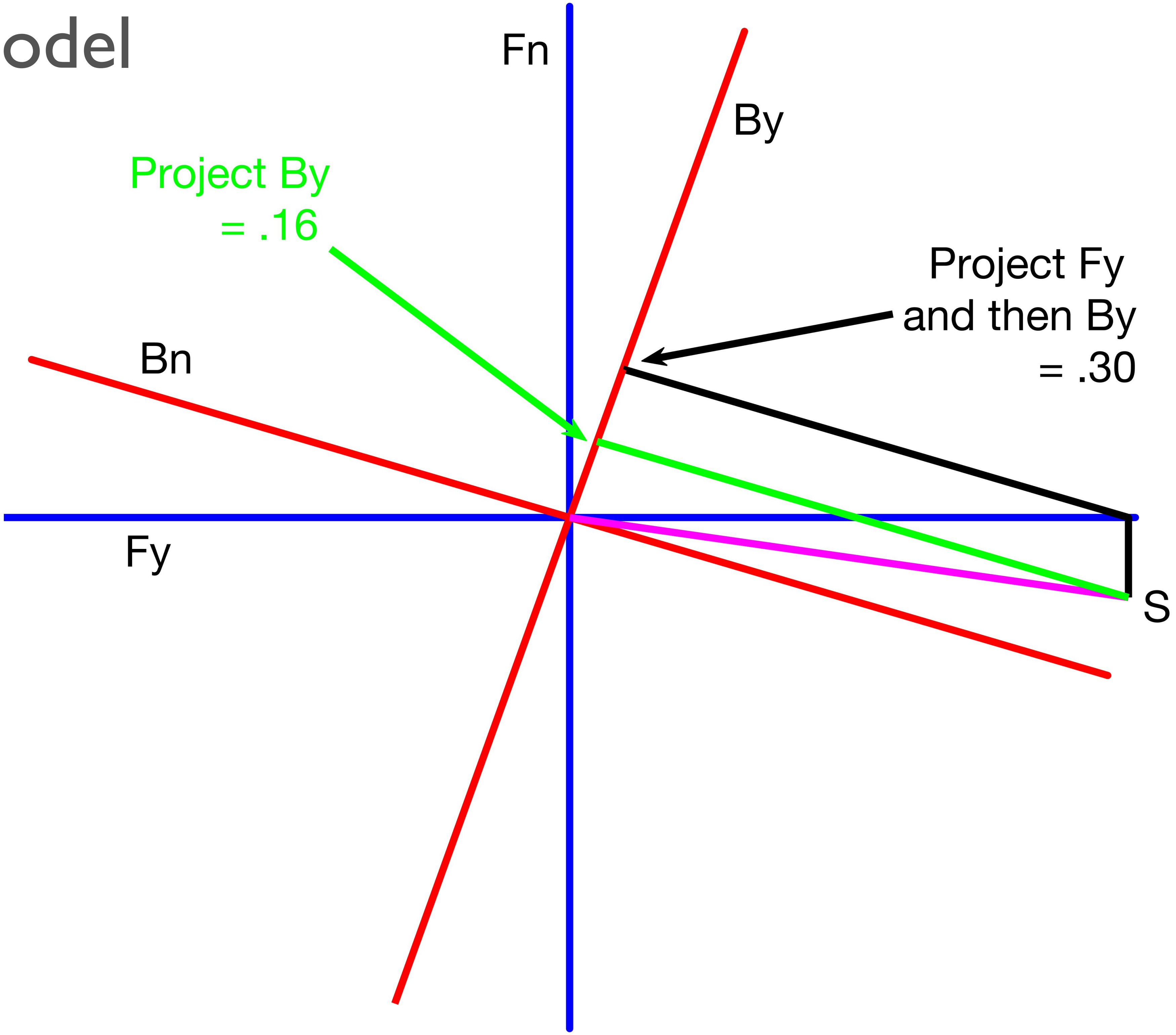
LAW OF TOTAL PROBABILITY

$$p(B) = p(F)p(B \mid F) + p(\sim F)p(B \mid \sim F)$$
$$\geq p(F)p(B \mid F)$$

CONJUNCTION - FALLACY

VIOLATES THIS LAW

Feminist Bank Teller Toy Model



Quantum Model Predictions

$$\left\|P_B S\right\|^2 = \left\|P_B I S\right\|^2 = \left\|P_B (P_F + P_{\bar{F}}) S\right\|^2$$

$$= \left\|P_B P_F S + P_B P_{\bar{F}} S\right\|^2$$

$$= \left\|P_B P_F S\right\|^2 + \left\|P_B P_{\bar{F}} S\right\|^2 + Int$$

$$Int = \left\langle S' P'_F \, P'_B \, P_{\bar{F}} S \right\rangle + \left\langle S' P_{\bar{F}} P'_B \, P_F S \right\rangle$$

$$Int < -\left\|P_B P_{\bar{F}} S\right\|^2$$

ADDITIONAL PREDICTIONS

- Order Effects: $p(F)p(B|F) =/= p(B)p(F|B)$
- Disjunction Fallacy co-occurs with Conjunction Fallacy
- Conjunction-Disjunction constraint: $p(F)p(B|F) > p(B)p(F|B)$
- Unequal Priors: $p(F)>p(B)$
- No double conjunction errors: $p(F)>p(F)p(B|F)>p(B)$
- Positive dependence: $p(B|F)>p(B)$
- Conditional Probability: $p(B|F) > p(F)p(B|F)$

3. INTERFERENCE OF CATEGORIZATION ON DECISION

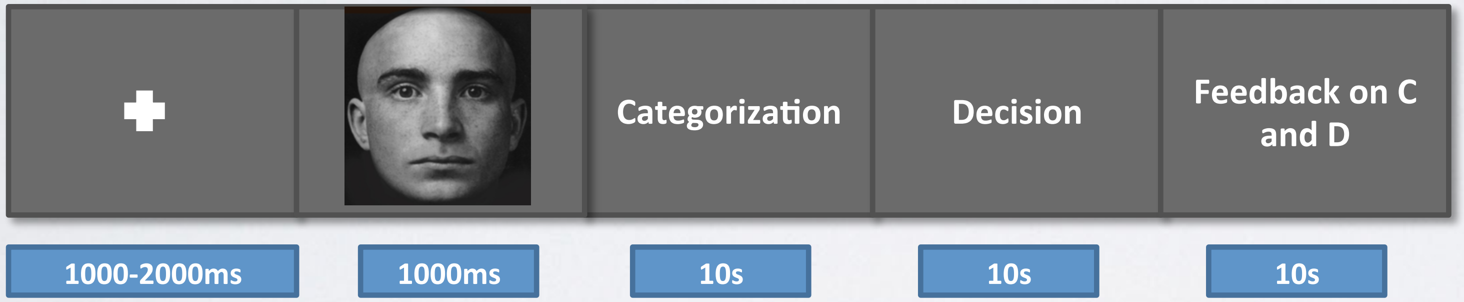
Psychological version of a double slit experiment

Busemeyer, Wang, Mogiliansky-Lambert
(2009, *J. of Mathematical Psychology*)

Wang & Busemeyer
(2016, *Cognition*)

Participants shown pictures of faces

Categorize as “good” guy or “bad” guy
Decide to act “friendly” or “aggressive”


Categorization
Decision
Feedback on C and D
1000-2000ms
1000ms
10s
10s
10s

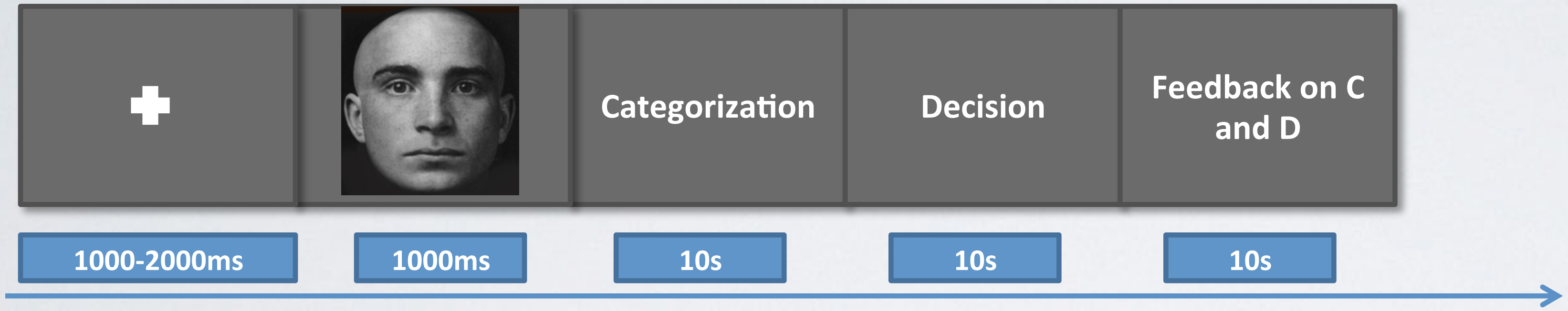
Bad Guys Good Guys

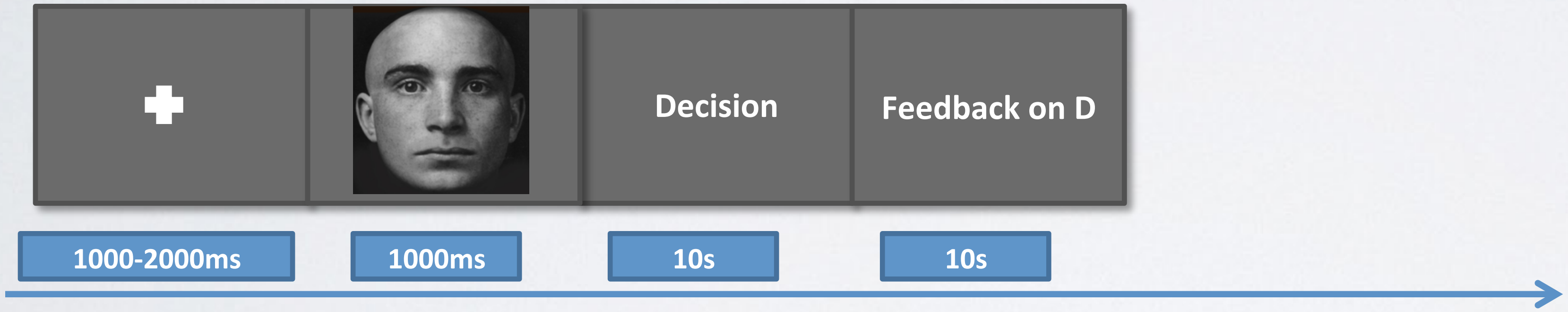
Programmed Contingencies
(Learned from Experience)

- Pr(Bad | Narrow) = .60
- Pr(Good | Wide) = .60
- Pr(Reward Attack | Bad) = .70
- Pr(Reward Withdraw | Good) = .70

Two Conditions:

C-then-D: Categorize face first and then decide





D-alone: Decide without categorization

LAW OF TOTAL PROBABILITY

G =good guy,   B=Bad guy,   A=Attack

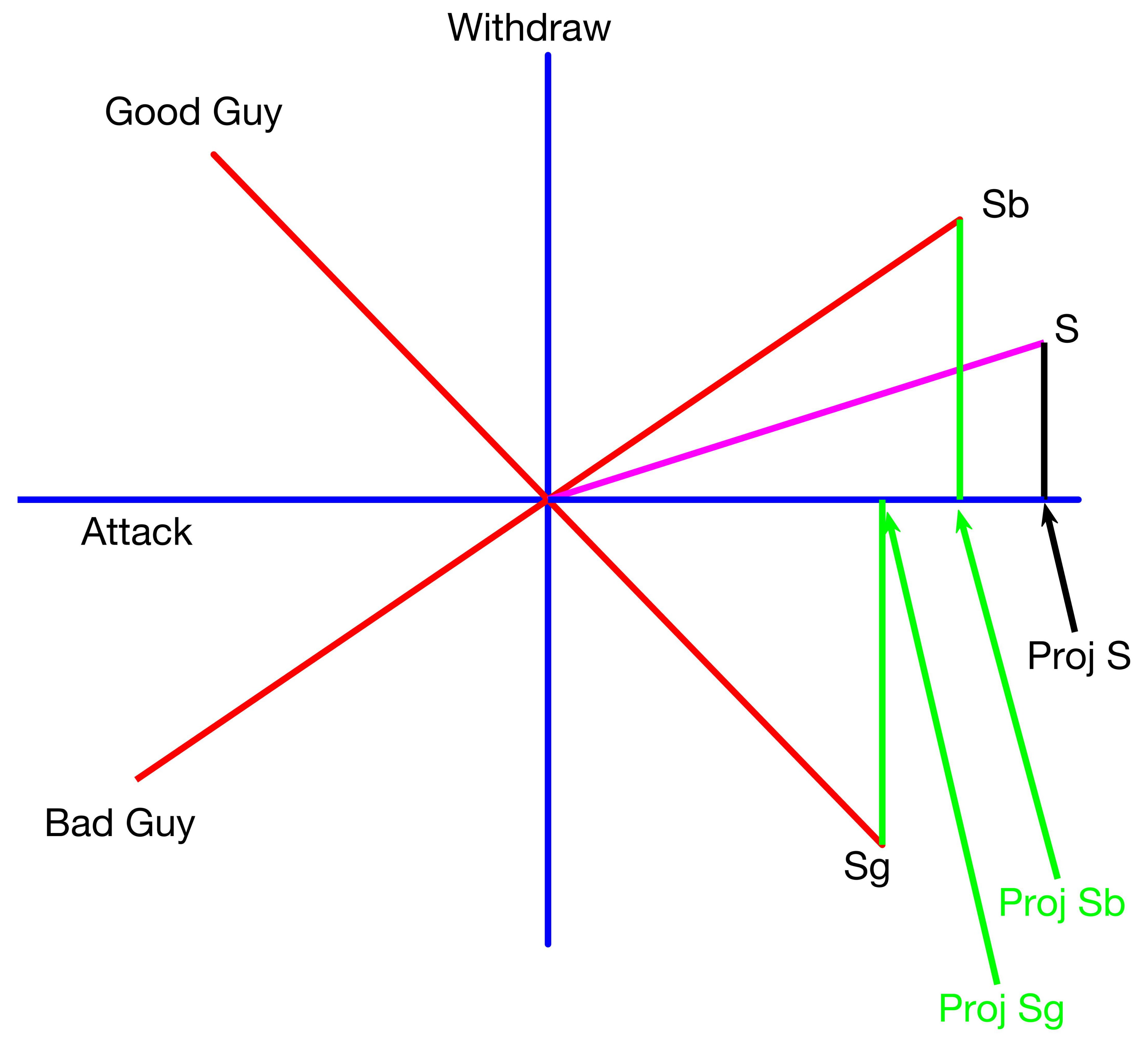
$$p(A) = p(G)p(A|G) + p(B)p(A|B)$$

D alone Condition   C-then-D Condition

RESULTS

Face	p(G)	$p(A \mid G)$	p(B)	$p(A \mid B)$	TP	P(A)
Good	0.84	0.35	0.16	0.52	0.37	0.39
Bad	0.17	0.41	0.82	0.63	0.59	**0.69**

Categorization -Decision Toy Model



QUANTUM INTERFERENCE

$$p(A \mid D\ alone) = \left\| P_A S \right\|^2 = \left\| P_A \cdot I \cdot S \right\|^2$$

$$= \left\| P_A \cdot \left(P_G + P_B \right) \cdot S \right\|^2$$

$$= \left\| P_A \cdot P_G \cdot S + P_A \cdot P_B \cdot S \right\|^2$$

$$= \left\| P_A \cdot P_G \cdot S \right\|^2 + \left\| P_A \cdot P_B \cdot S \right\|^2 + Int$$

Interference term violates of Law of Total Probability

$$Int = \langle S \mid P_G P_A P_A P_B \mid S \rangle + \langle S \mid P_B P_A P_A P_G \mid S \rangle$$

$$Finding \rightarrow Int > 0$$

4. PRISONER DILEMMA GAME

SHAFIR & TVERSKY (1992, *COGNITIVE PSYCH*)

POTHOS & BUSEMEYER, 2009,
PROCEEDINGS OF THE ROYAL SOCIETY, B.

	OD	OC
PD	O: 10 P: 10	O:5 P: 25
PC	O: 25 P: 5	O:20 P: 20

Examined three conditions in a prisoner dilemma task

Known Coop: Player is told other opponent will cooperate
Known Defect: Player is told other opponent will defect
UnKnown: Player is told nothing about the opponent

LAW OF TOTAL PROBABILITY

$p(PD) =$ probability player defects when opponent's move is unknown

$$p(PD) = p(OD)p(PD \mid OD) + p(OC)p(PD \mid OC)$$

Empirically we find : $p(PD \mid OD) \geq p(PD \mid OC)$

$$\rightarrow p(PD \mid OD) \geq p(PD) \geq p(PD \mid OC)$$

DEFECT RATE FOR TWO EXPERIMENTS

Study	Known Defect	Known Coop	Unknown
Shafir Tversky (1992)	0.97	0.84	0.63
Matthew Busemeyer (2006)	0.91	0.84	0.66

Defect rate for both known conditions exceed the unknown condition

5. TWO STAGE GAMBLING PARADIGM

VIOLATION OF "SURE THING" PRINCIPLE

Tversky & Shafir
(1992, *Psychological Science*)

Barkan & Busemeyer
(2003, *J. Behavioral Decision Making*)

Busemeyer, Wang, & Shiffrin
(2015, *Decision*)

- Participants forced to play a gamble on the first stage:
    - Equal chance to win $X or lose -$Y
- Then asked to choose whether or not to play again? on a second stage.
- Three conditions
    - Assume won first stage, play again?
    - Assume lost first stage, play again?
    - Play without knowing first stage?

Sure Thing Principle

If you prefer to play again after a win,
and
if you prefer to play again after a loss
then
you should prefer to play regardless of the first gamble outcome

Total Probability:

$$p(\text{Play}|\text{Unknown})$$
$$= p(\text{win})p(\text{Play}|\text{win}) + p(\text{lose})P(\text{Play}|\text{lose})$$

Results

Win first game: 65% chose to play again

Lose first game: 55% chose to play again

First stage Unknown: 35% chose to play again

Violation of Total Probability

QUANTUM INTERFERENCE

$$p(G) = \left\| P_G S \right\|^2 = \left\| P_G \cdot I \cdot S \right\|^2$$

$$= \left\| P_G \cdot \left(P_W + P_L \right) \cdot S \right\|^2$$

$$= \left\| P_G \cdot P_W \cdot S + P_G \cdot P_L \cdot S \right\|^2$$

$$= \left\| P_G \cdot P_W \cdot S \right\|^2 + \left\| P_G \cdot P_L \cdot S \right\|^2 + Int$$

$$Int = \left\langle S \,|\, P_W P_G P_G P_L \,|\, S \right\rangle + \left\langle S \,|\, P_L P_G P_G P_W \,|\, S \right\rangle$$

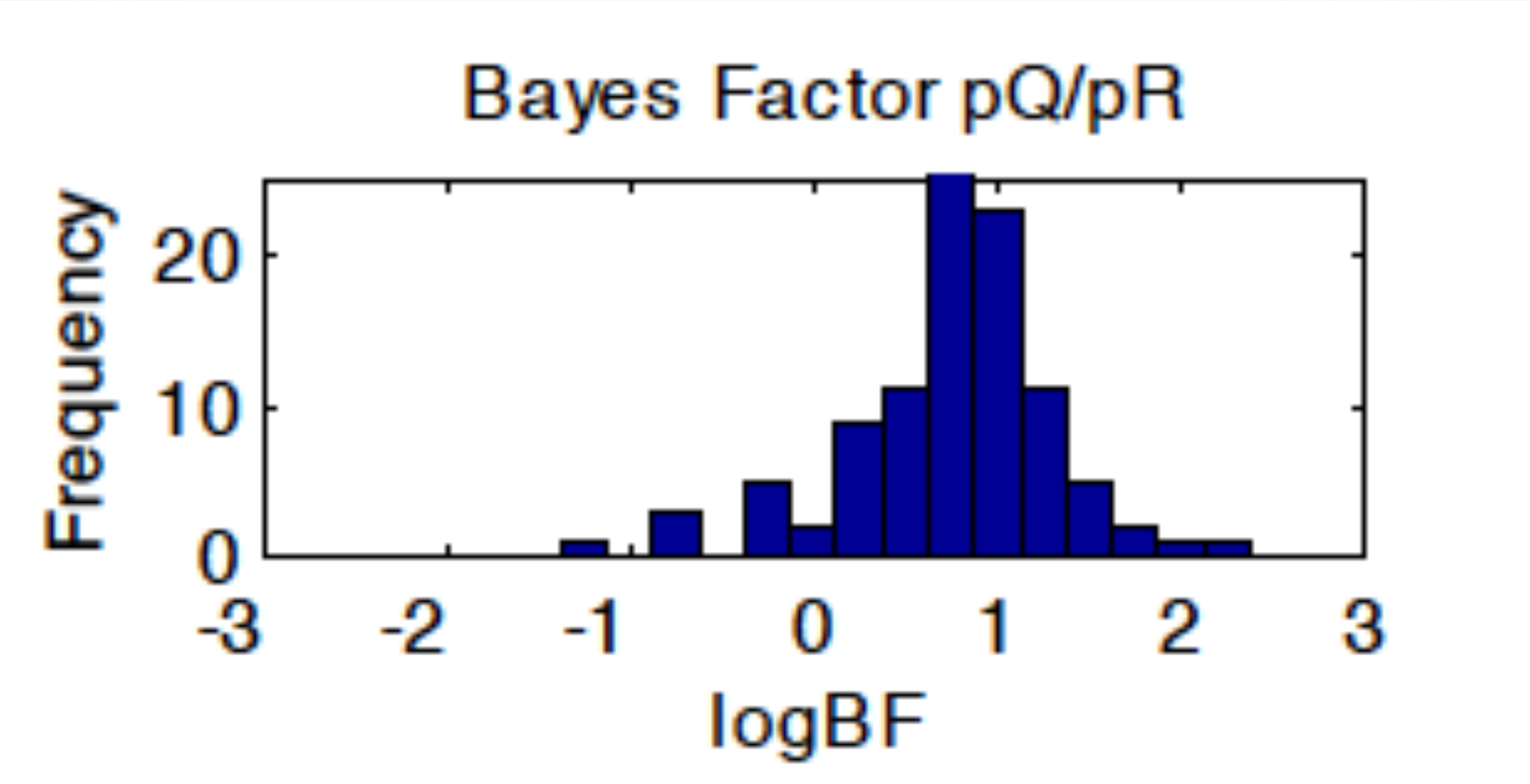
BAYESIAN MODEL COMPARISON

- Compared Quantum versus Prospect theories
- Both models used same number of parameters (four)
- Data based on Barkan & Busemeyer (2003)
    - N=100 participants,
    - 33 two-stage gambles per person,
    - obtained (plan, final) choice for each person.
- Computed Bayes' Factor separately for each person

Model Comparison of Quantum Model vs. Prospect Theory-Reference Point Model

Each model used four parameters

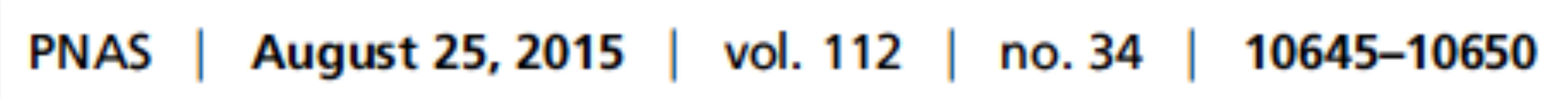
N=100 participants



6. INTERFERENCE EFFECT OF CHOICE ON LATER CONFIDENCE

Interference effects of choice on confidence: Quantum characteristics of evidence accumulation

Peter D. Kvam[a,b,1], Timothy J. Pleskac[b,1], Shuli Yu[a], and Jerome R. Busemeyer[c]



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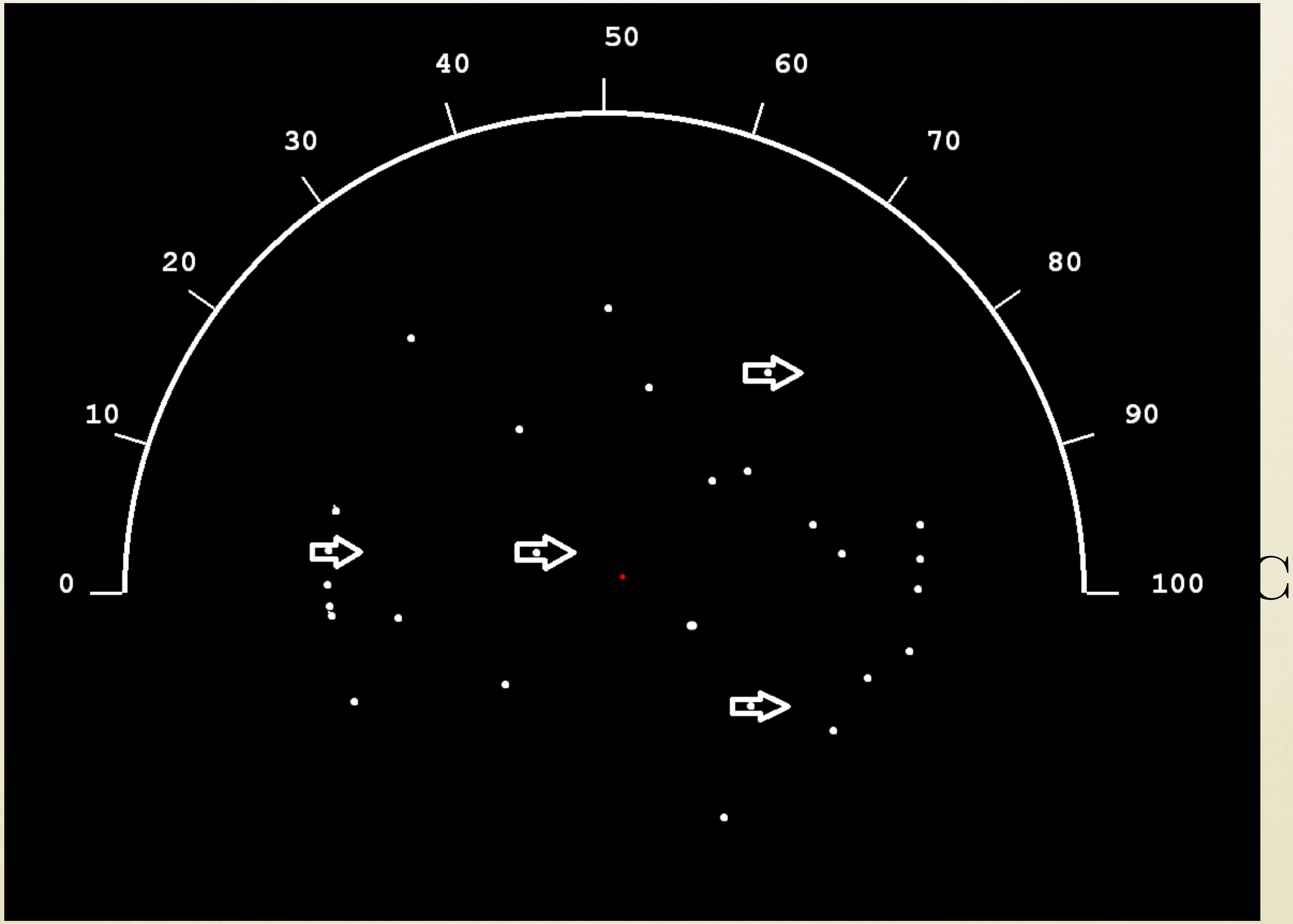
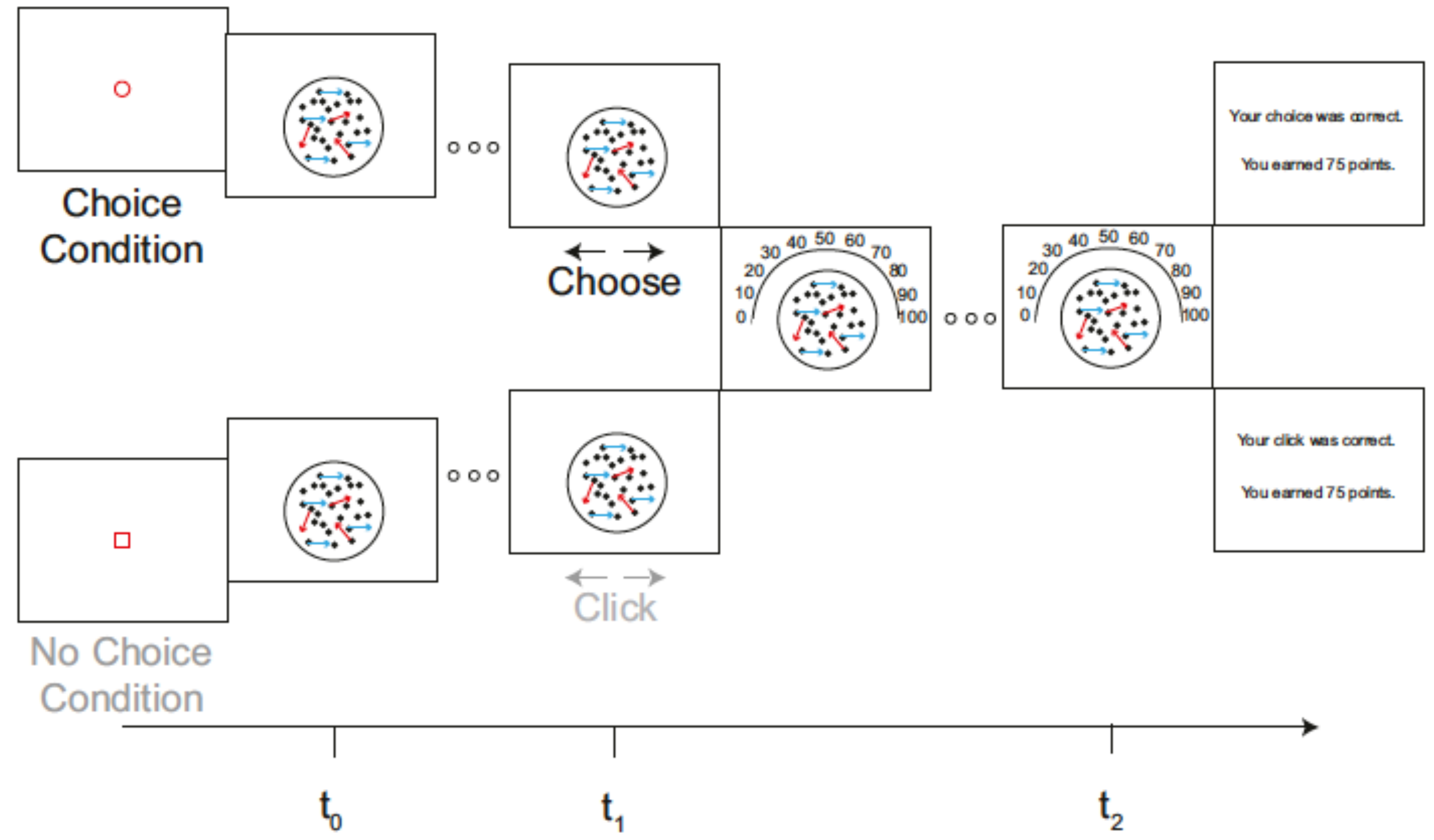
PROBABILITY RATING OF DIRECTION TO DOT MOTION TASK

Uncertain

Certain Left



Certain Right



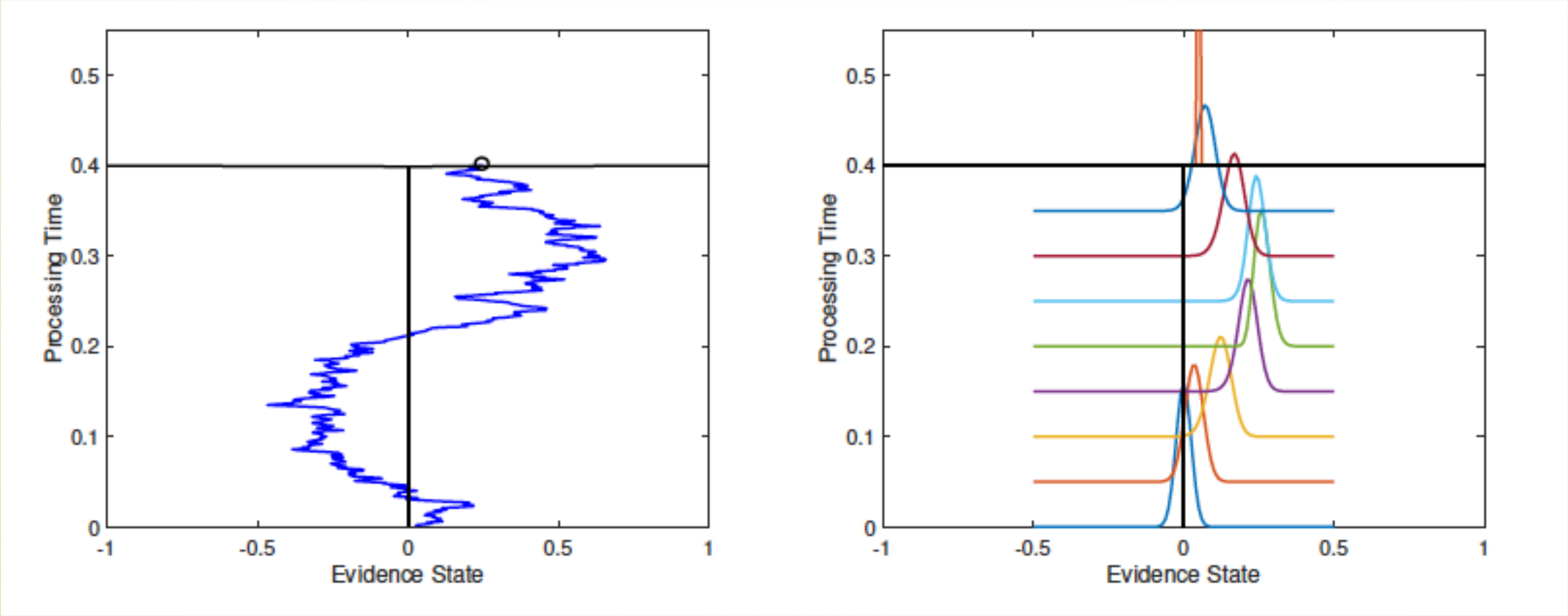
Your choice was correct.

You earned 75 points.

Your click was correct.

You earned 75 points.

Markov Quantum



INTERFERENCE PREDICTIONS

$R(t_1)$ choice at time t_1

$R(t_2)$ rating at time t_2

Markov (satisfies Total Probability)

$$\Pr[R(t_2)=r_2]=\sum_{r_1}\Pr[R(t_1)=r_1]\cdot\Pr[R(t_2)=r_2\,|\,R(t_1)=r_1]$$

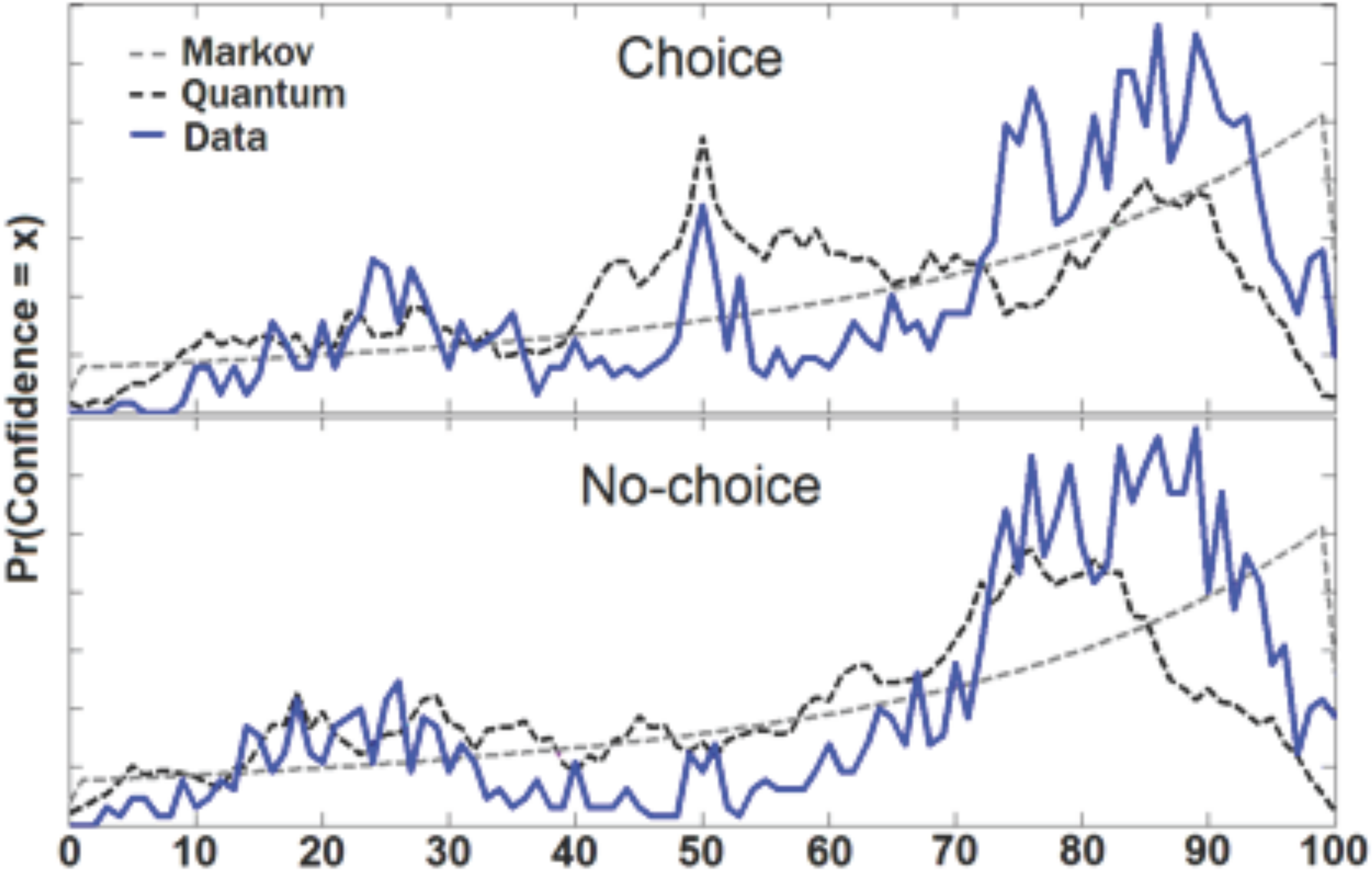
Quantum (violates Total Probability)

$$\Pr[R(t_2)=r_2]\neq\sum_{r_1}\Pr[R(t_1)=r_1]\cdot\Pr[R(t_2)=r_2\,|\,R(t_1)=r_1]$$

↑ ↑

Single **Double**

One Participant out of Nine



SUMMARY FOR ALL 9 PARTICIPANTS

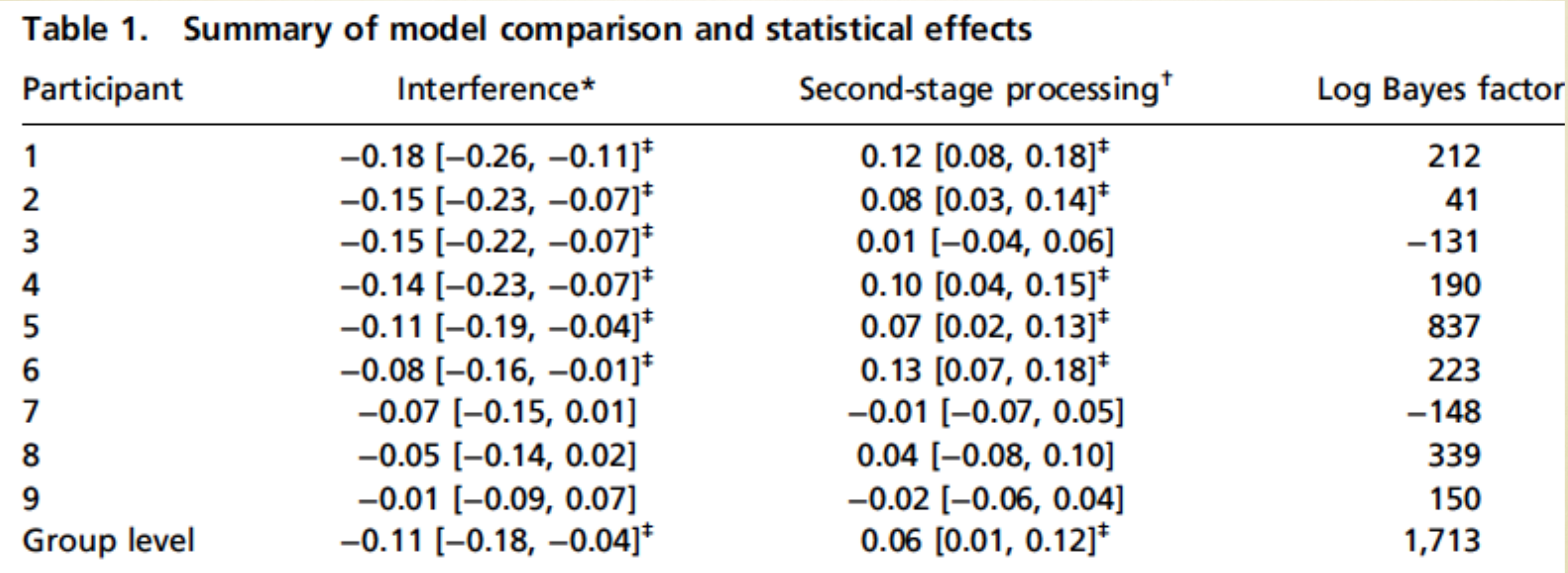


Table 1. **Summary of model comparison and statistical effects**

Participant	Interference*	Second-stage processing[†]	Log Bayes factor
1	−0.18 [−0.26, −0.11][‡]	0.12 [0.08, 0.18][‡]	212
2	−0.15 [−0.23, −0.07][‡]	0.08 [0.03, 0.14][‡]	41
3	−0.15 [−0.22, −0.07][‡]	0.01 [−0.04, 0.06]	−131
4	−0.14 [−0.23, −0.07][‡]	0.10 [0.04, 0.15][‡]	190
5	−0.11 [−0.19, −0.04][‡]	0.07 [0.02, 0.13][‡]	837
6	−0.08 [−0.16, −0.01][‡]	0.13 [0.07, 0.18][‡]	223
7	−0.07 [−0.15, 0.01]	−0.01 [−0.07, 0.05]	−148
8	−0.05 [−0.14, 0.02]	0.04 [−0.08, 0.10]	339
9	−0.01 [−0.09, 0.07]	−0.02 [−0.06, 0.04]	150
Group level	−0.11 [−0.18, −0.04][‡]	0.06 [0.01, 0.12][‡]	1,713

6 out of 9 produced credible interference effects

7 out of 9 produced BF favoring quantum over Markov

ADDITIONAL APPLICATIONS

- Order effects on inference
  - Trueblood, J. S. & Busemeyer, J. R. (2011). A quantum probability account of order effects in inference. Cognitive Science, 35, 1518-1552
- Causal Reasoning
  - Trueblood, J. S., Yearsley, J. M., & Pothos, E. M. (2017). A quantum probability framework for human probabilistic inference. Journal of Experimental Psychology: General, 146, 1307-1341
- Similarity judgments
  - Pothos, E. M., Busemeyer, J. R., & Trueblood, J. S. (2013). A quantum geometric model of similarity. Psychological Review, 120, 679-696.
- Mere Measurement Effects
  - White, L. C., Pothos, E. M., & Busemeyer, J. R. (2014). Sometimes it does hurt to ask: the constructive role of articulating impressions. Cognition, 133, 48-64

CONCLUSIONS

- Quantum theory provides an alternative framework for developing probabilistic and dynamic models of decision making

- Provides a coherent account for puzzling violations of classical probability found in a variety of judgment and decision making studies

- Forms a new foundation for understanding widely different phenomena in decision making using a common set of axiomatic principles

“Mathematical models of cognition so often seem like mere formal exercises. Quantum theory is a rare exception. Without sacrificing formal rigor, it captures deep insights about the workings of the mind with elegant simplicity. This book promises to revolutionize the way we think about thinking.”

Steven Sloman
Cognitive, Linguistic, and Psychological Sciences, Brown University

“This book is about why and how formal structures of quantum theory are essential for psychology - a breakthrough resolving long-standing problems and suggesting novel routes for future research, convincingly presented by two main experts in the field.”

Harald Atmanspacher
Department of Theory and Data Analysis, Institut fuer Grenzgebiete der Psychologie und Psychohygiene e.V.

<FURTHER ENDORSEMENT TO FOLLOW>

Much of our understanding of human thinking is based on probabilistic models. This innovative book by Jerome R. Busemeyer and Peter D. Bruza argues that, actually, the underlying mathematical structures from quantum theory provide a much better account of human thinking than traditional models. They introduce the foundations for modeling probabilistic-dynamic systems using two aspects of quantum theory. The first, “contextuality,” is a way to understand interference effects found with inferences and decisions under conditions of uncertainty. The second, “quantum entanglement,” allows cognitive phenomena to be modeled in non-reductionist way. Employing these principles drawn from quantum theory allows us to view human cognition and decision in a totally new light. Introducing the basic principles in an easy-to-follow way, this book does not assume a physics background or a quantum brain and comes complete with a tutorial and fully worked-out applications in important areas of cognition and decision.

Jerome R. Busemeyer is a Professor in the Department of Psychological and Brain Sciences at Indiana University, Bloomington, USA.

Peter D. Bruza is a Professor in the Faculty of Science and Technology at Queensland University of Technology, Brisbane, Australia.

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Busemeyer and Bruza **Quantum Models of Cognition and Decision**

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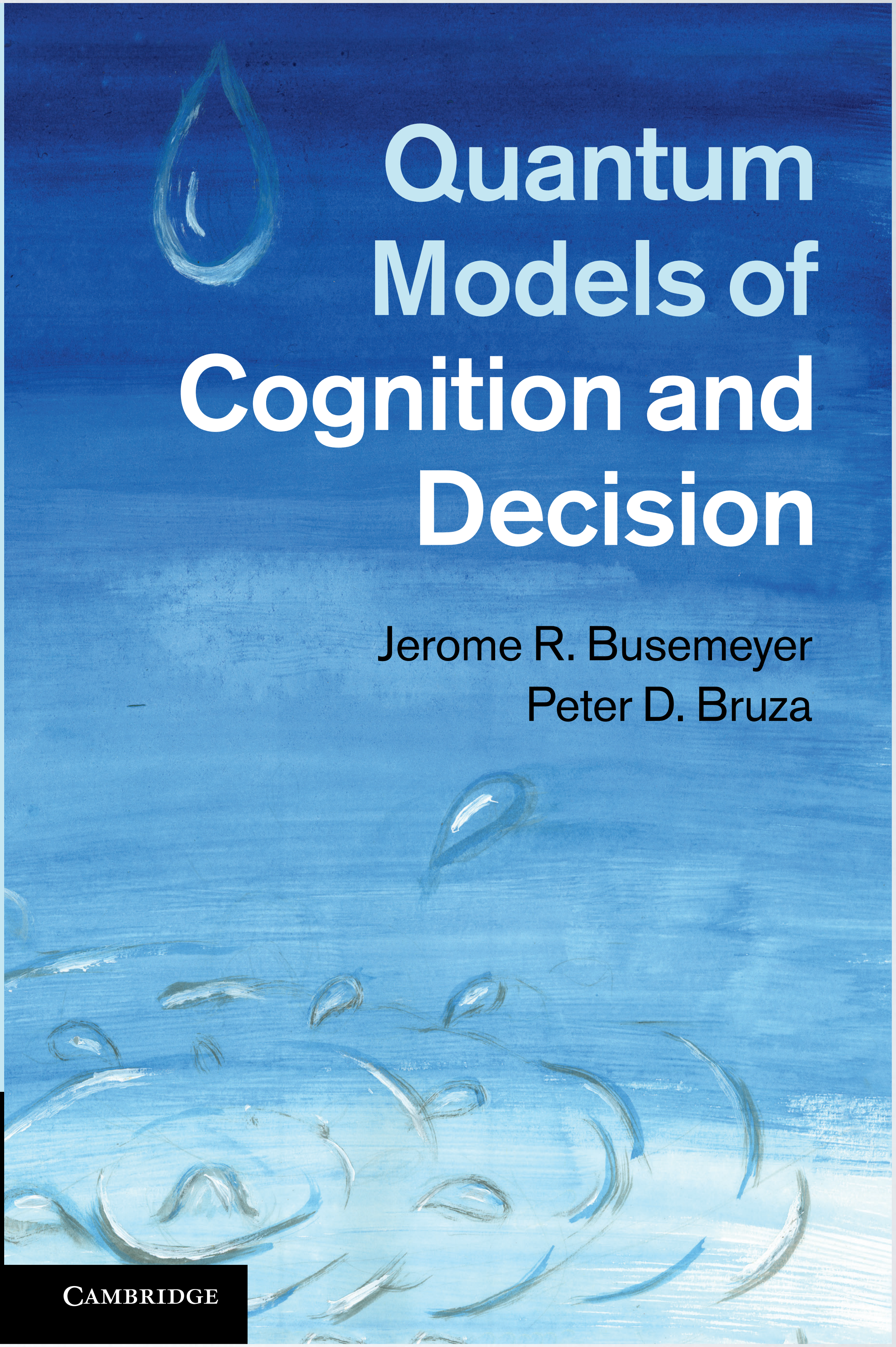

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