

# Quantum theory of vortices in *d*-wave superconductors

*Physical Review B* **71**, 144508 and 144509 (2005),  
*Annals of Physics* **321**, 1528 (2006),  
*Physical Review B* **73**, 134511 (2006),  
cond-mat/0606001.

Leon Balents (UCSB)

Lorenz Bartosch (Harvard)

Anton Burkov (Harvard)

Predrag Nikolic (Harvard)

Subir Sachdev (Harvard)

Krishnendu Sengupta (HRI, India)



Talk online at <http://sachdev.physics.harvard.edu>



BCS theory of vortices in  $d$ -wave superconductors

+

periodic potential

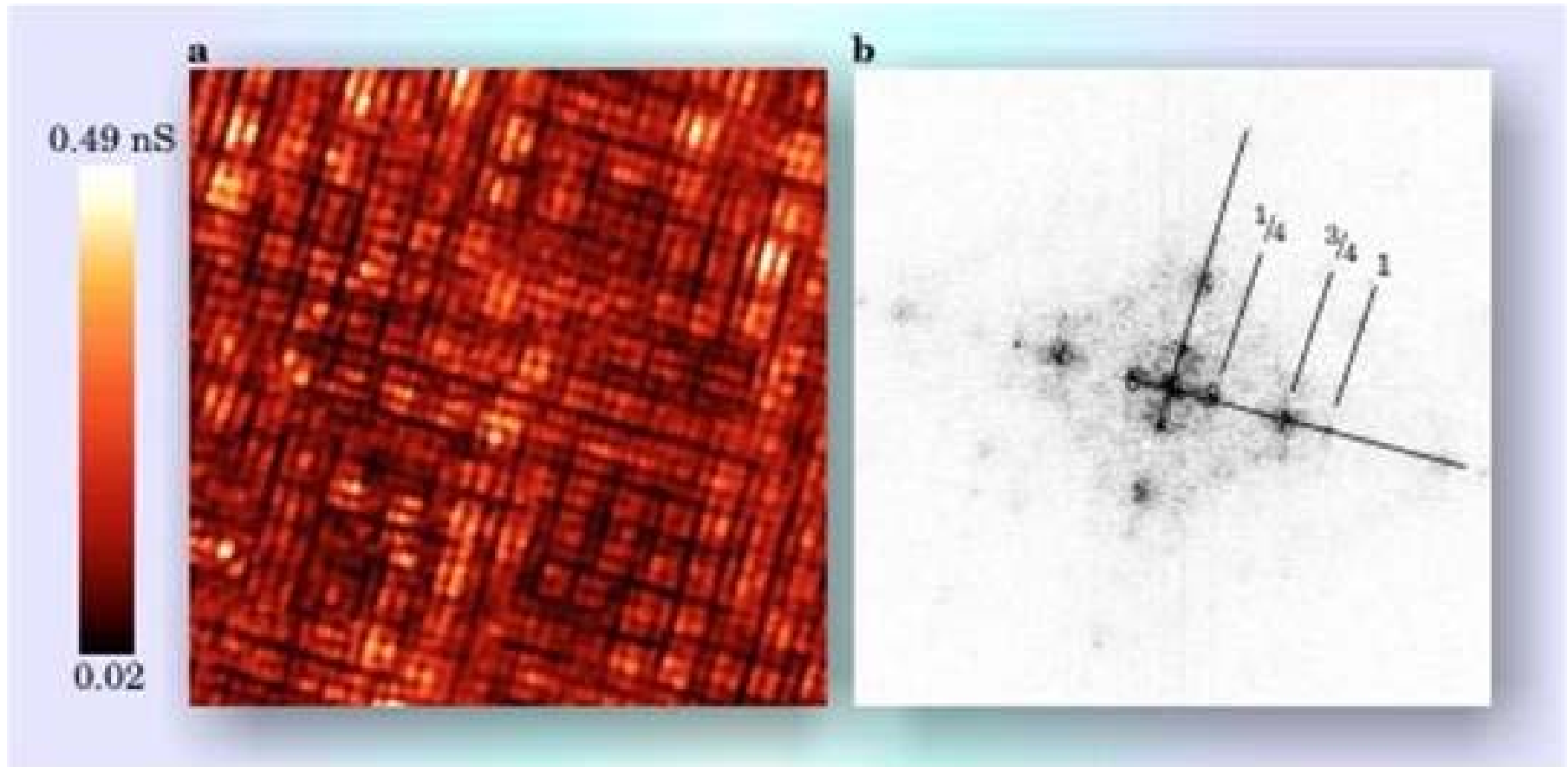
+

strong Coulomb interactions



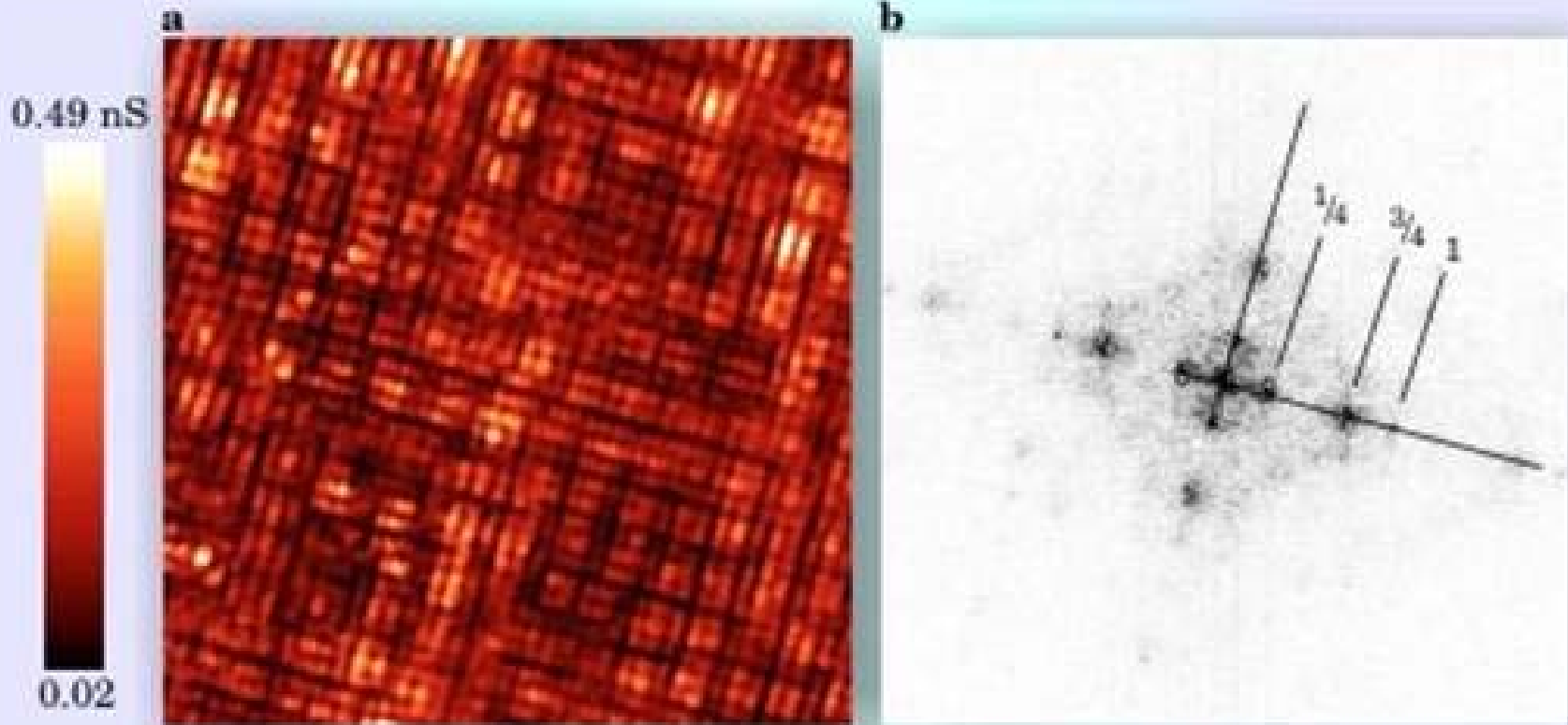
Vortices in BCS superconductors near a  
superconductor-Mott insulator transition at finite doping

# The cuprate superconductor $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$



T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004). Closely related modulations in superconducting  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  observed first by C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546 and *Physical Review B* **67**, 014533 (2003).

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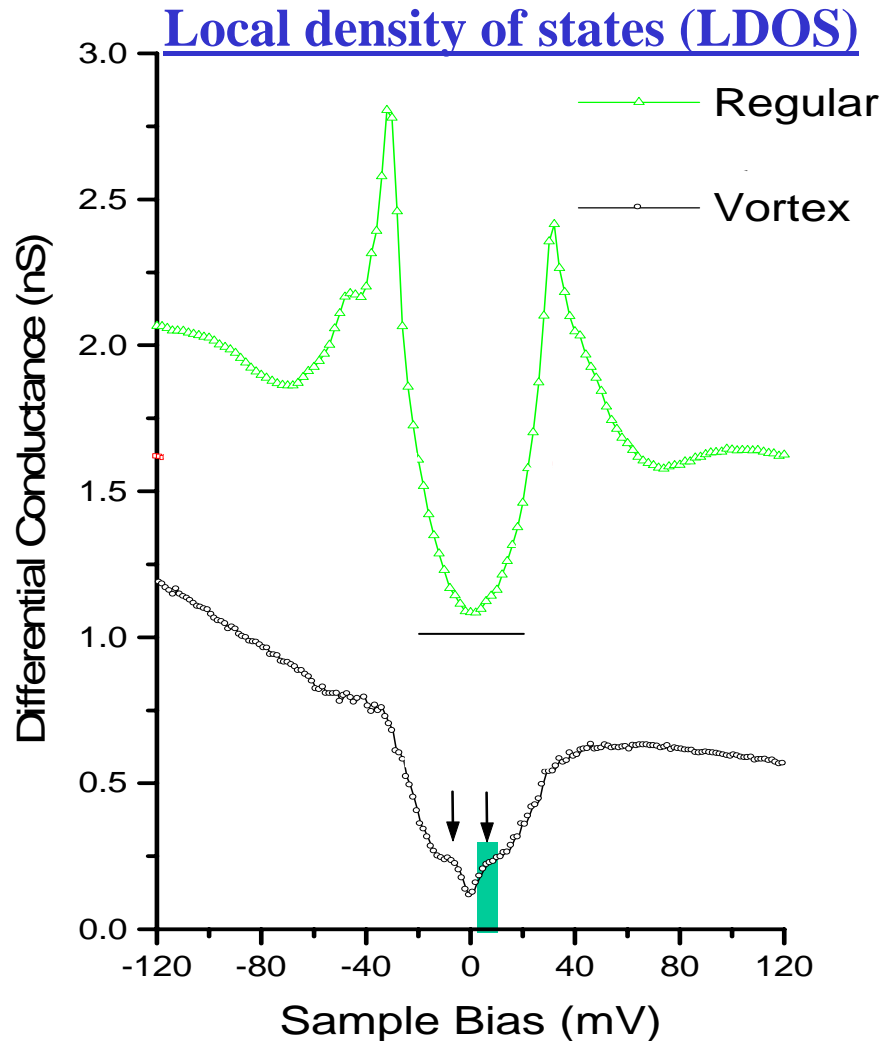


Evidence that holes can form an insulating state with period  $\approx 4$  modulation in the density

T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004). Closely related modulations in superconducting  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  observed first by C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546 and *Physical Review B* **67**, 014533 (2003).

# STM around vortices induced by a magnetic field in the superconducting state

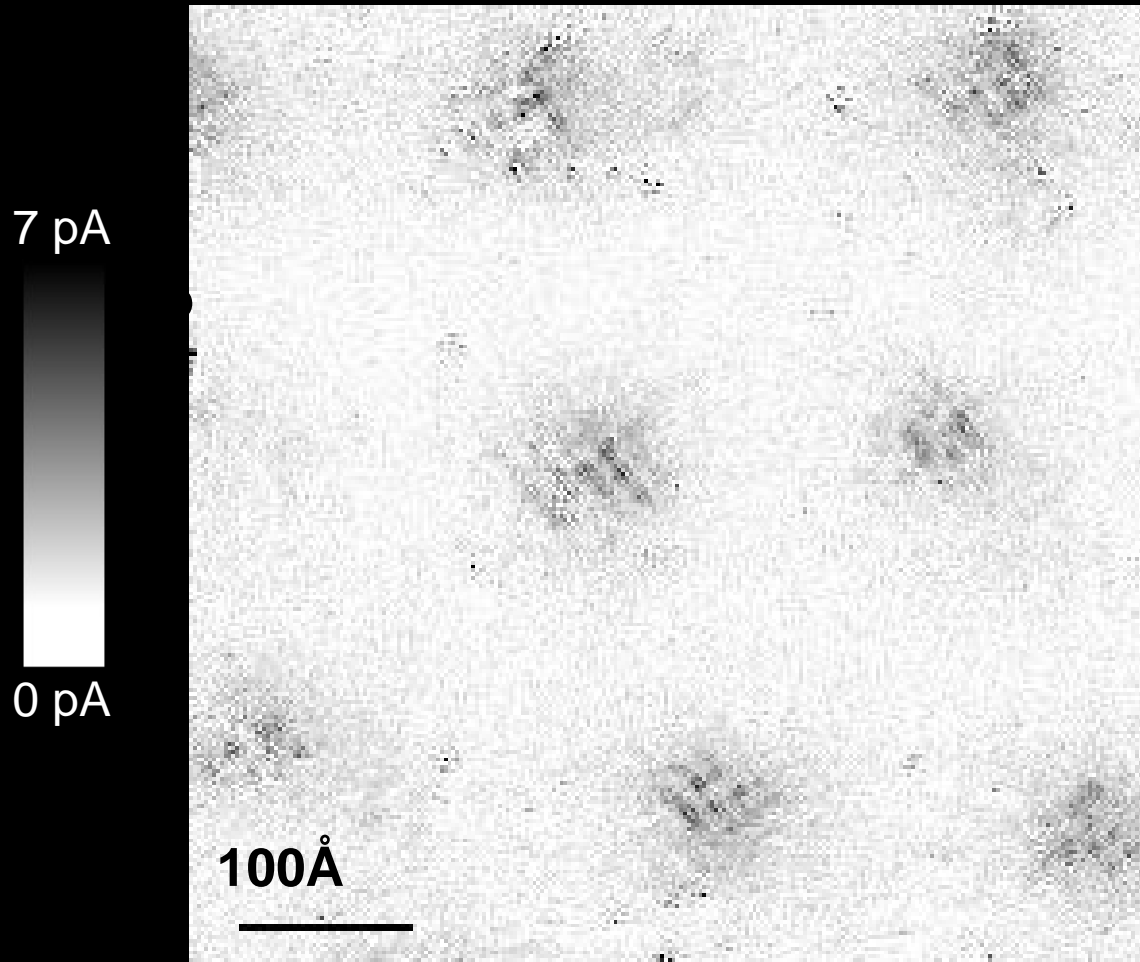
J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,  
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).



1Å spatial resolution  
image of integrated  
LDOS of  
 $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$   
( 1meV to 12 meV)  
at B=5 Tesla.

I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995).  
S.H. Pan *et al.* *Phys. Rev. Lett.* **85**, 1536 (2000).

# Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period  $\approx 4$  lattice spacings

J. Hoffman et al., *Science* 295, 466 (2002).  
G. Levy et al., *Phys. Rev. Lett.* 95, 257005 (2005).

Prediction of periodic LDOS modulations near vortices:  
K. Park and S. Sachdev, *Phys. Rev. B* 64, 184510 (2001).

## Questions on the cuprate superconductors

- What is the quantum theory of the ground state as it evolves from the superconductor to the modulated insulator ?
- What happens to the vortices near such a quantum transition ?

# Outline

- The superfluid-insulator transition of bosons
- The quantum mechanics of vortices near the superfluid-insulator transition
  - Dual theory of superfluid-insulator transition as the proliferation of vortex-anti-vortex pairs*
- Influence of nodal quasiparticles on vortex dynamics in a *d*-wave superconductor



# I. The superfluid-insulator transition of bosons

# Bosons at filling fraction $f = 1$

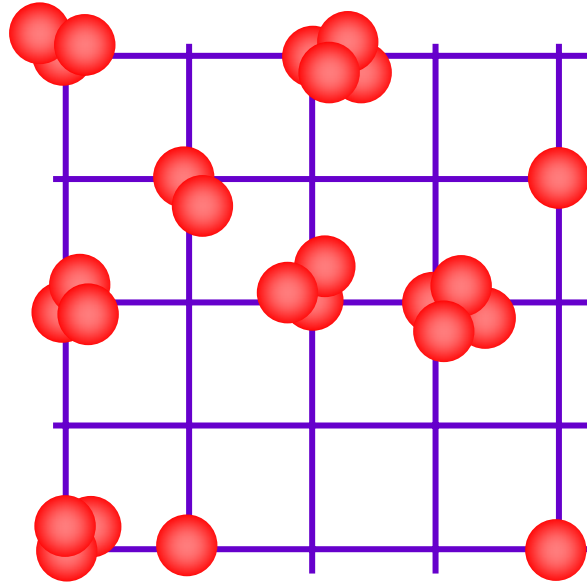
Weak interactions:  
superfluidity

**a** Superfluid state

**b** Insulating state

Strong interactions:  
Mott insulator which  
preserves all lattice  
symmetries

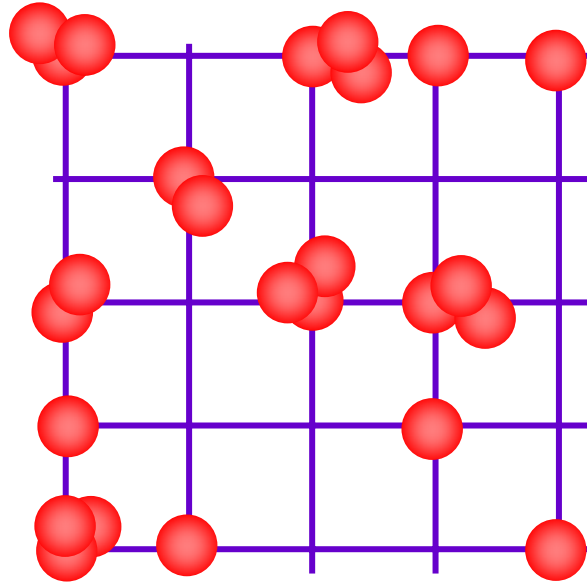
# Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

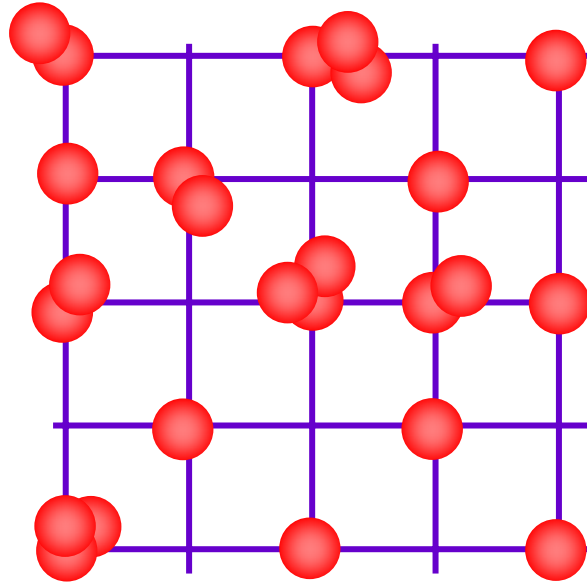
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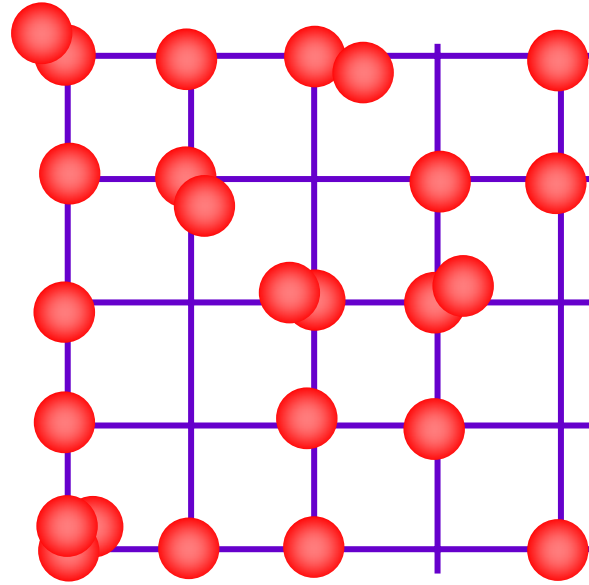
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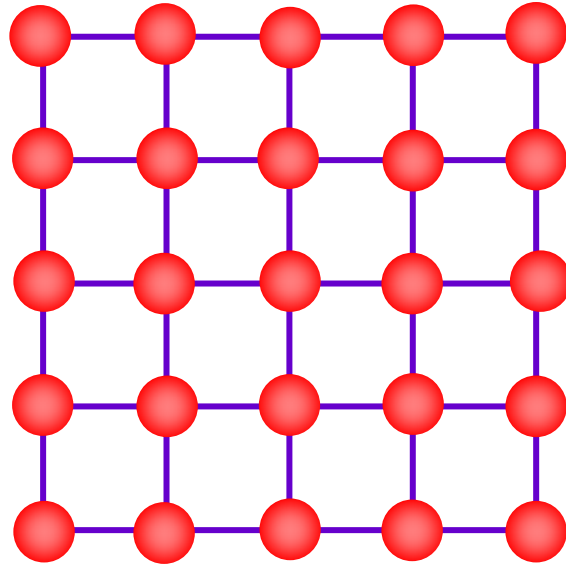
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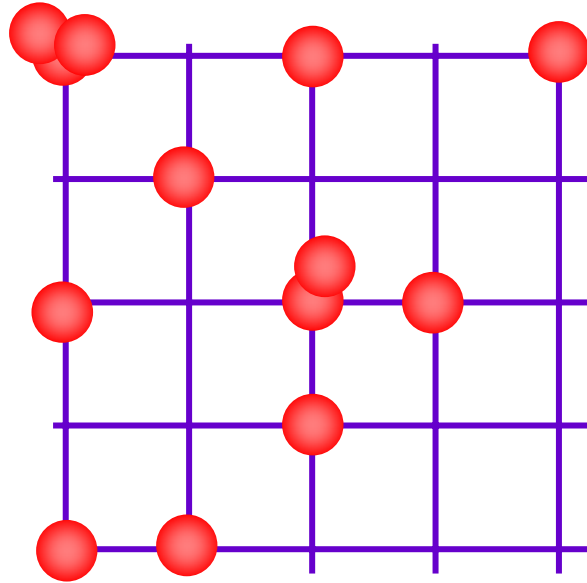
# Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Bosons at filling fraction  $f = 1/2$   
or  $S=1/2$  XXZ model

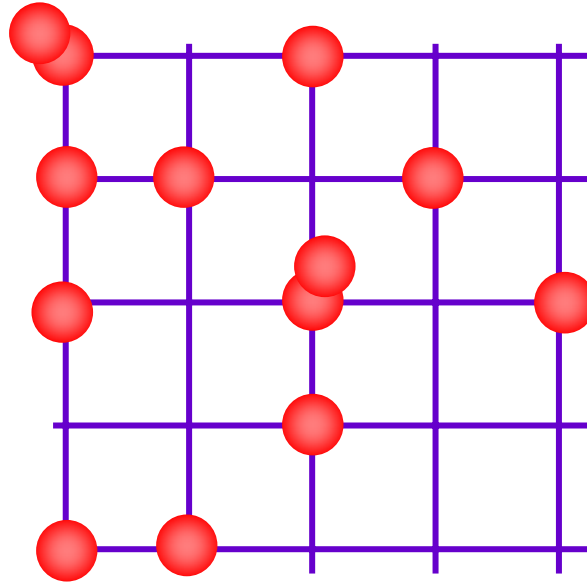


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Weak interactions: superfluidity



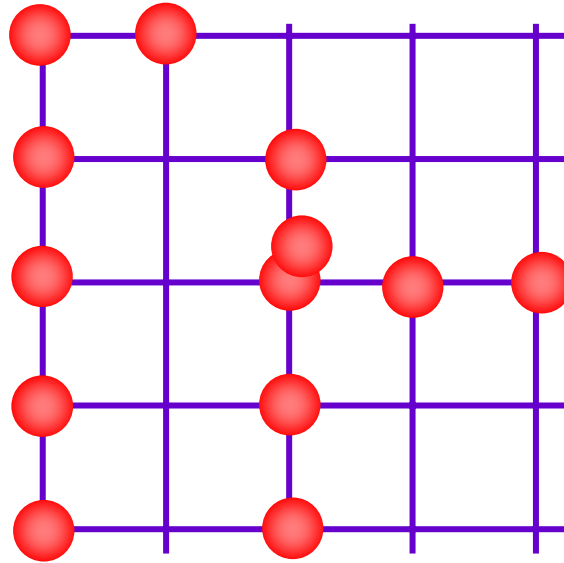
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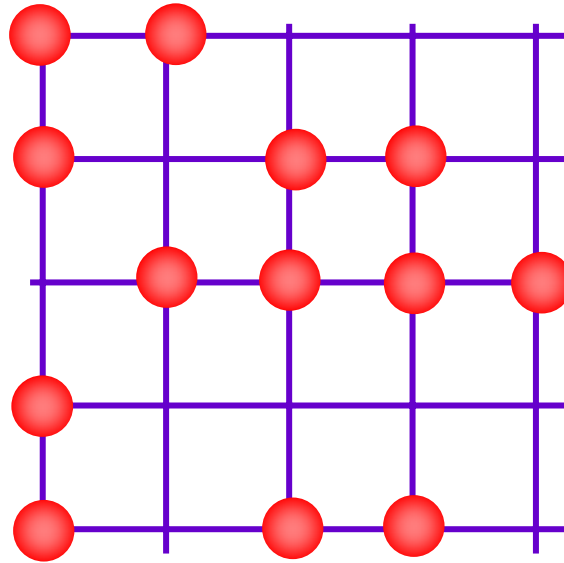
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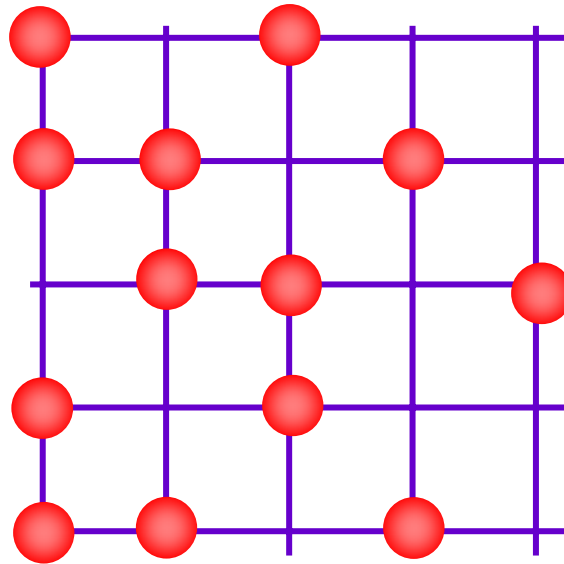
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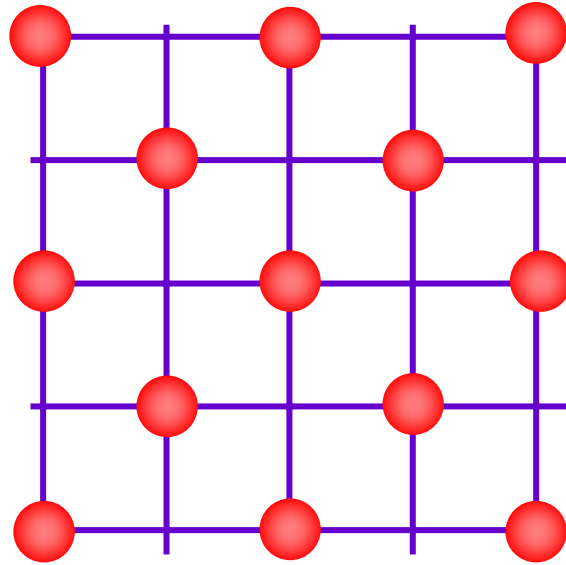
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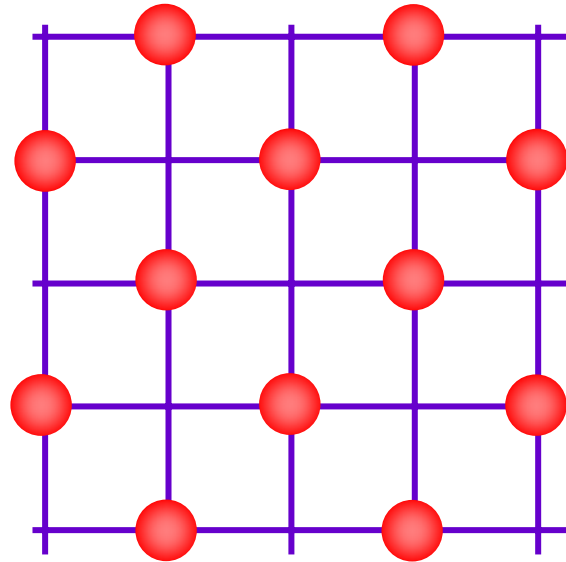
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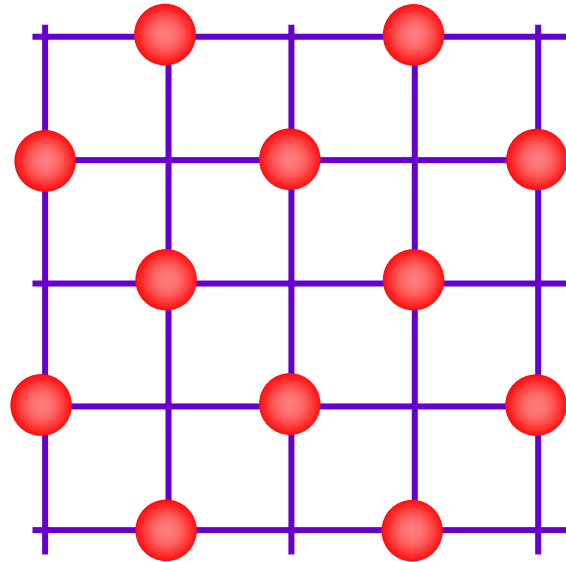
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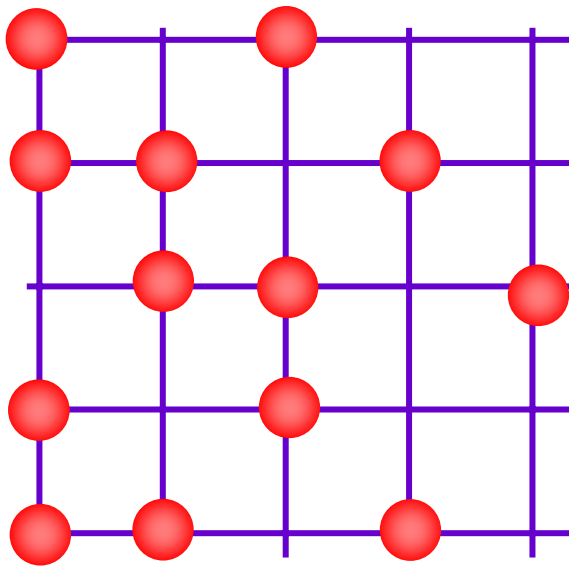
$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Insulator has “density wave” order

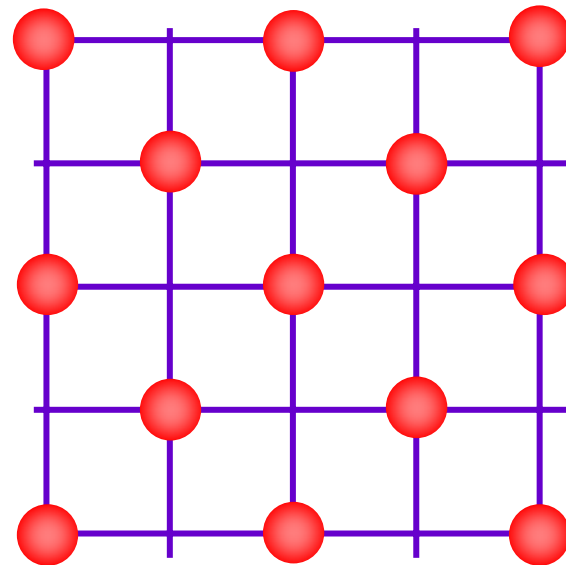
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Superfluid

?



Insulator

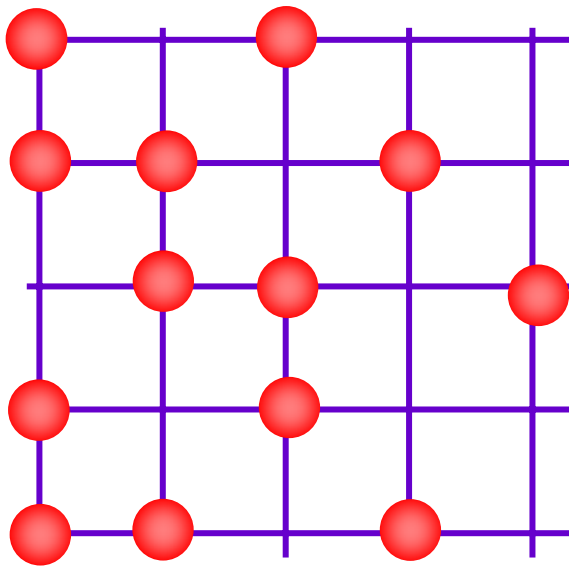
Charge density wave (CDW) order

*Interactions between bosons* →



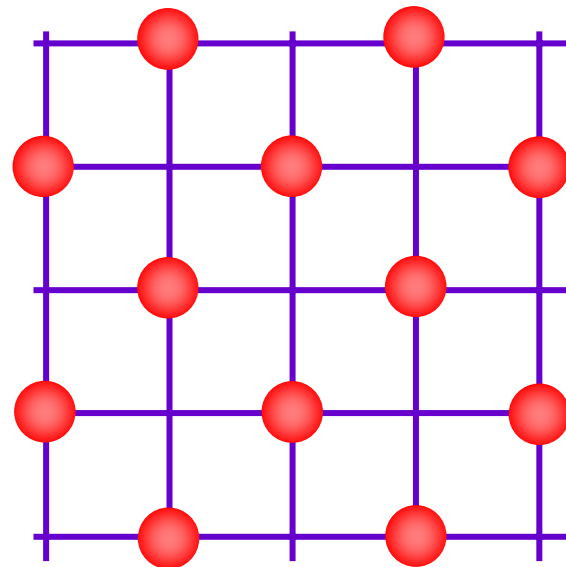
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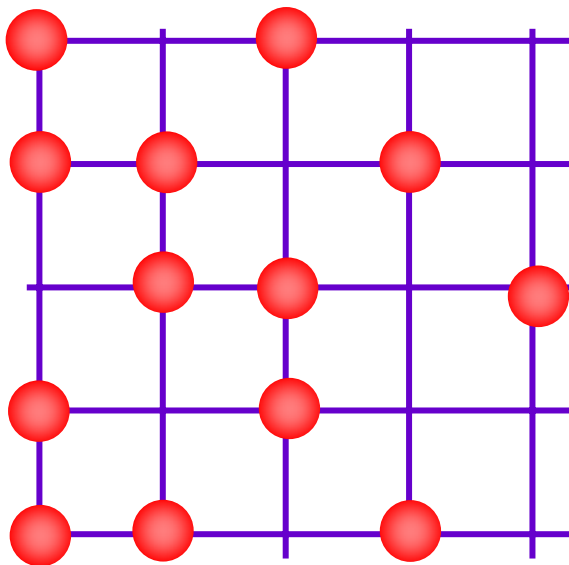
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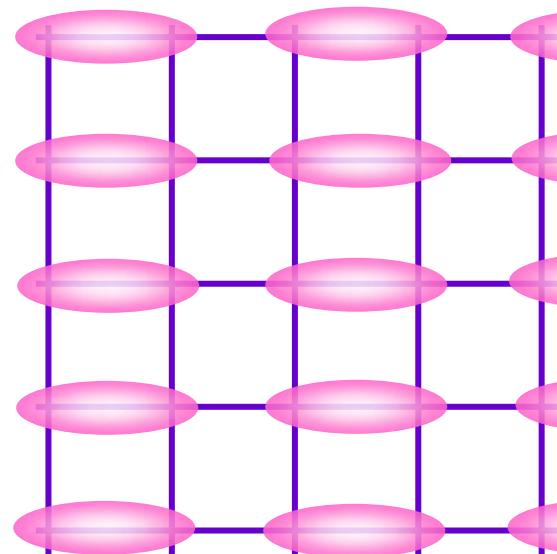
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$$\text{pink oval} = \frac{1}{\sqrt{2}} (\text{red circle} - \text{red circle})$$



Superfluid

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Insulator

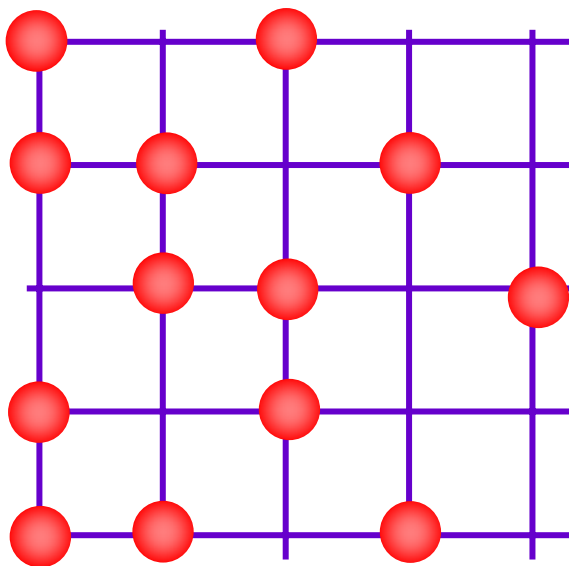
Valence bond solid (VBS) order

*Interactions between bosons* →

Bosons at filling fraction  $f = 1/2$

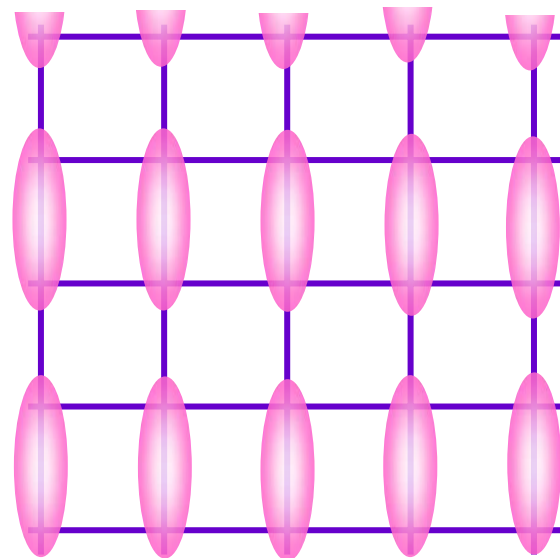
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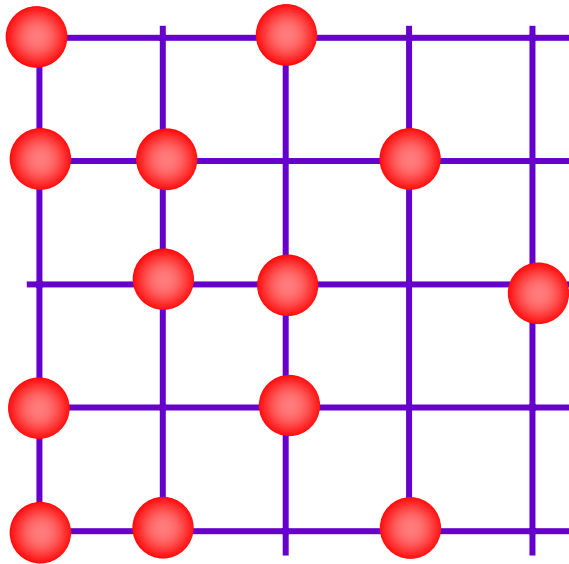
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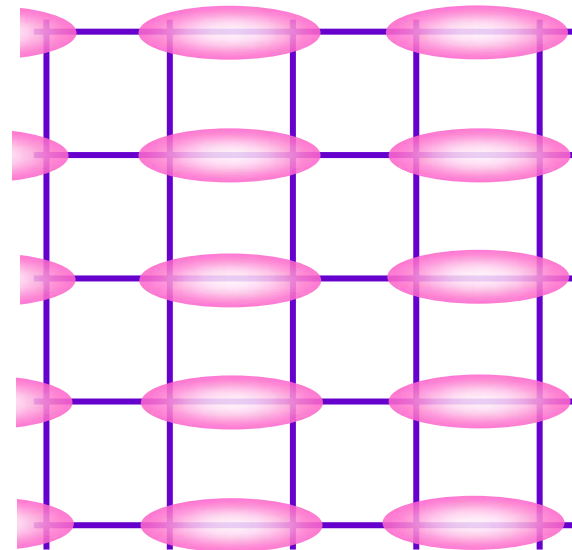
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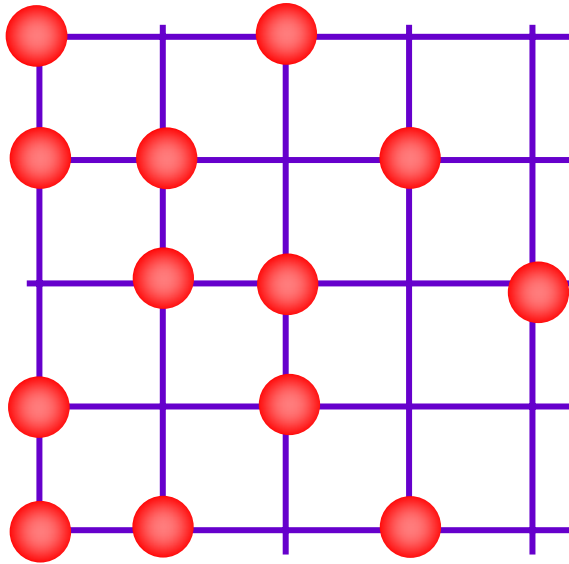
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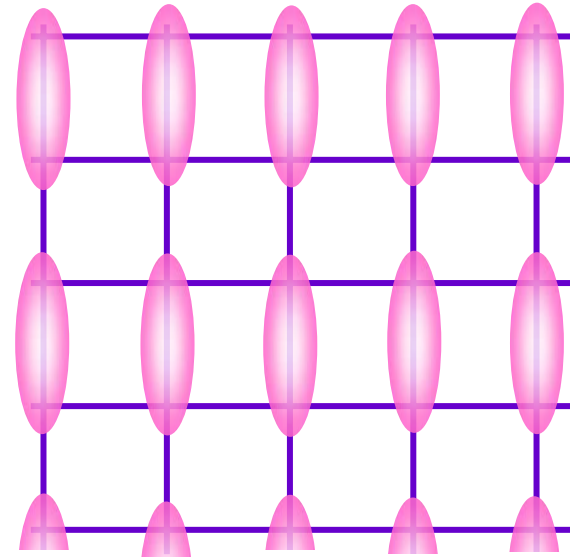
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## The superfluid-insulator quantum phase transition

**Key difficulty:** Multiple order parameters (Bose-Einstein condensate, charge density wave, valence-bond-solid order...) not related by symmetry, but clearly physically connected. Standard methods only predict strong first order transitions (for generic parameters).

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**Key theoretical tool:** *Quantum theory of vortices*

# Outline

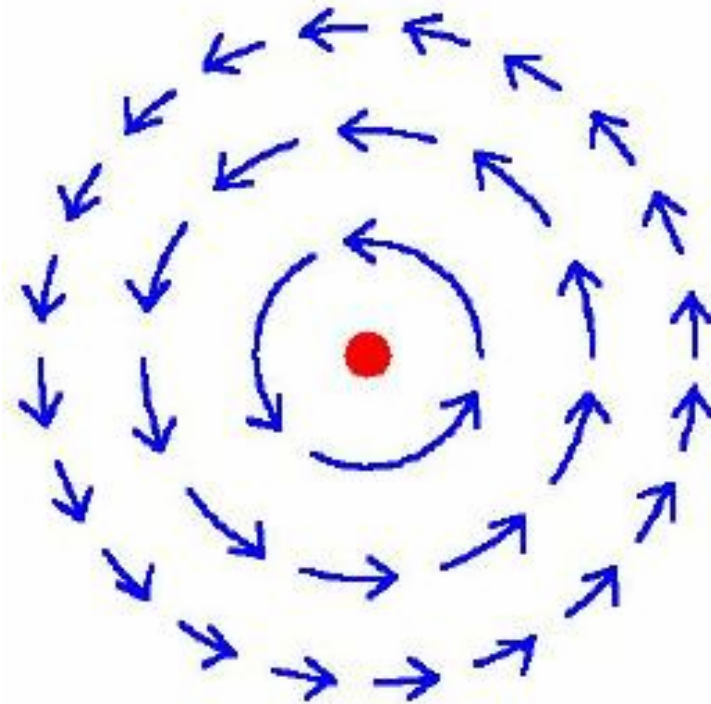
- The superfluid-insulator transition of bosons
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## II. The quantum mechanics of vortices near a superfluid-insulator transition

*Dual theory of the superfluid-insulator transition as the proliferation of vortex-anti-vortex-pairs*

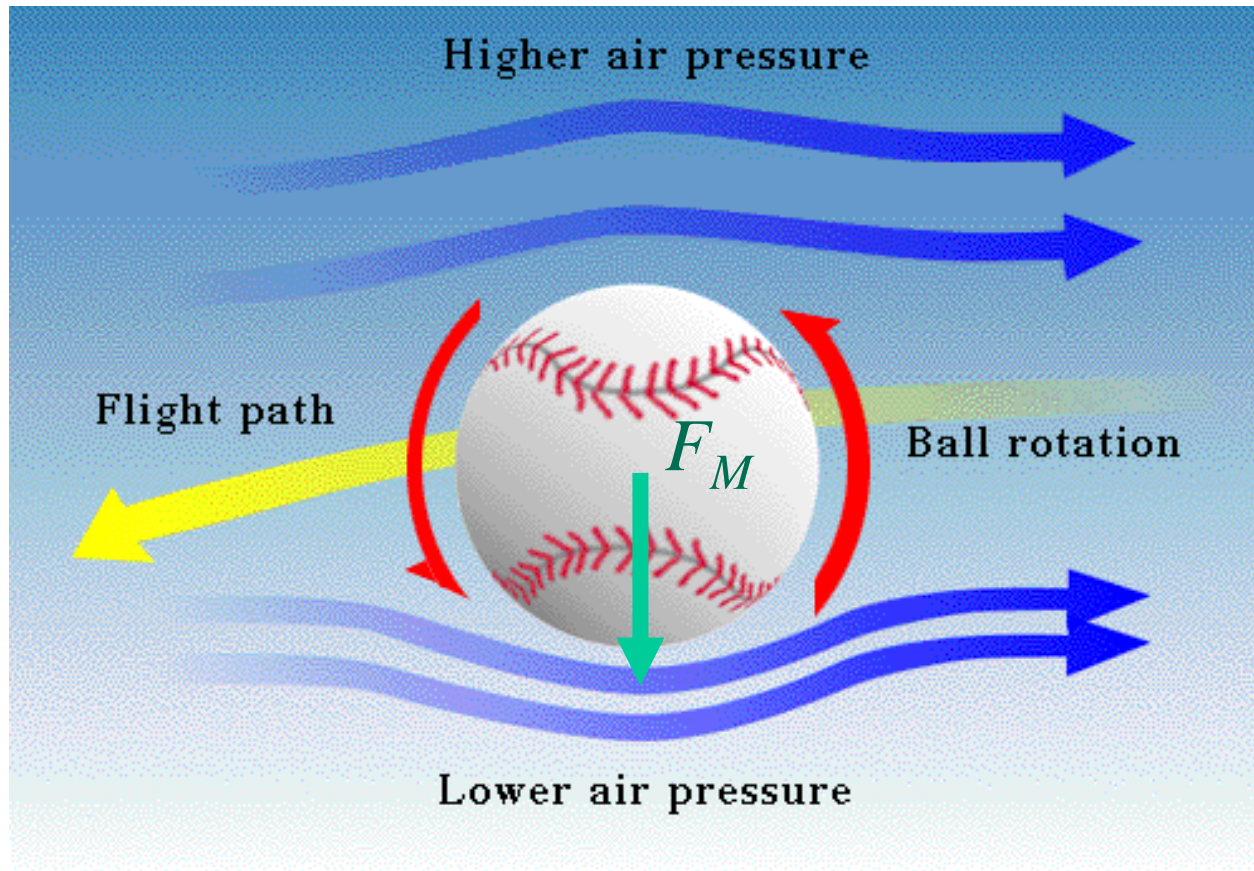
# Excitations of the superfluid: **Vortices and anti-vortices**



## Central question:

In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these “particles” ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \cdot (\text{velocity of ball}) \cdot (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho) \left( \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}} \right) \left( \oint \mathbf{v}_s \cdot d\mathbf{r} \right) \\ &= nh\rho \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}}\end{aligned}$$

where  $\rho$  = number density of bosons

$\mathbf{v}_s$  = local velocity of superfluid

$\mathbf{r}_v$  = position of vortex

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$$\begin{aligned}\mathbf{F}_M &= (m\rho) \left( \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}} \right) \left( \oint \mathbf{v}_s \cdot d\mathbf{r} \right) \\ &= nh\rho \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}} \\ &= n \left( \mathbf{E} + \frac{d\mathbf{r}_v}{dt} \times \mathbf{B} \right)\end{aligned}$$

where  $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$  and  $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

### Dual picture:

The vortex is a quantum particle with dual “electric” charge  $n$ , moving in a dual “magnetic” field of strength =  $h \times$  (number density of Bose particles)

Let the Hamiltonian of a single vortex be  $\mathcal{H}_v$ .

In general, this is a very complicated object, but we can obtain all needed information by symmetry considerations.

The Hamiltonian  $\mathcal{H}_v$  should commute with  $T_x$ , the operator which translates the square lattice by one site in the  $x$  direction (and similarly for  $T_y$ ):

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

However,  $T_x$  and  $T_y$  do not commute with each other.

Under translation along a distance  $\mathbf{s}$ , a vortex picks up a Aharanov-Bohm phase factor  $\exp\left(i \int_0^{\mathbf{s}} d\mathbf{r} \cdot \mathbf{A}\right)$ .

Consequently

$$T_x T_y = \exp(i\phi) T_y T_x$$

where  $\phi$  is the dual “flux” through a unit cell, This “flux” has the value

$$\phi = 2\pi f$$

where  $f$  is the filling fraction of bosons (Cooper pairs). We will consider the case of rational filling fraction  $f = p/q$ , where  $p, q$  are relatively prime integers.

Bosons on the square lattice at filling fraction  $f=p/q$

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp(2\pi i p/q) T_y T_x$$



## Bosons on the square lattice at filling fraction $f=p/q$

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### Theorem:

The ground state of  $\mathcal{H}_v$  is at least  $q$ -fold degenerate. We can choose a basis,  $|m\rangle$  ( $m = 0 \dots (q - 1)$ ), for the ground states such that

$$T_x |m\rangle = |m + 1\rangle$$

$$T_y |m\rangle = e^{2\pi i m p/q} |m\rangle$$

# Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any impurity breaks translational invariance, and so chooses a preferred orientation in vortex “flavor space”. This chooses some linear combination among the ground states:  $|G\rangle = \sum_{m=0}^{q-1} c_m |m\rangle$

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- The expectation value of any observable  $\mathcal{O}$ ,  $\langle G|\mathcal{O}|G\rangle$  can be related to the matrix of overlaps  $\langle m|n\rangle$  which, in turn, are linearly related to quantities  $\rho_{mn}$  which transform under  $T_x, T_y$  like the Fourier components of a density  $\rho_{\mathbf{Q}}$  at the wavevectors  $\mathbf{Q} = 2\pi f(m, n)$ :

$$T_x : \rho_{\mathbf{Q}} \rightarrow e^{i\mathbf{Q}\cdot\hat{x}} \rho_{\mathbf{Q}} \quad T_y : \rho_{\mathbf{Q}} \rightarrow e^{i\mathbf{Q}\cdot\hat{y}} \rho_{\mathbf{Q}}$$

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on,  
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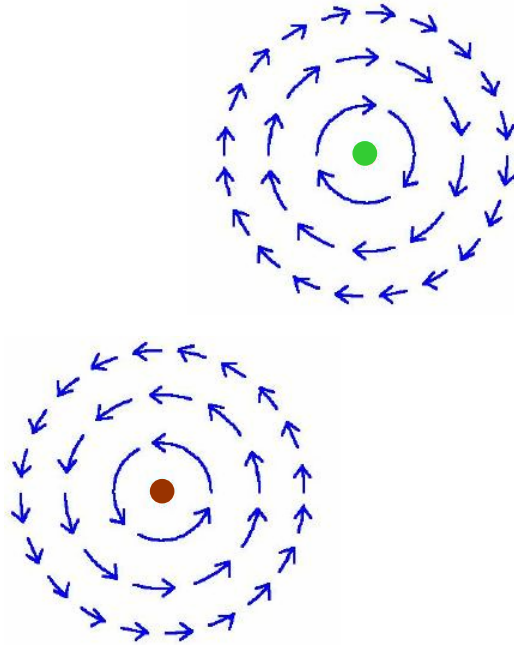
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- It can be shown that there is no linear combination  $|G\rangle$  for which all the  $\rho_{mn}$  are zero.

# Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any pinned vortex exhibits modulations in “density”-like observables at the wavevectors  $Q$  over the region in which the vortex executes its quantum zero-point motion.

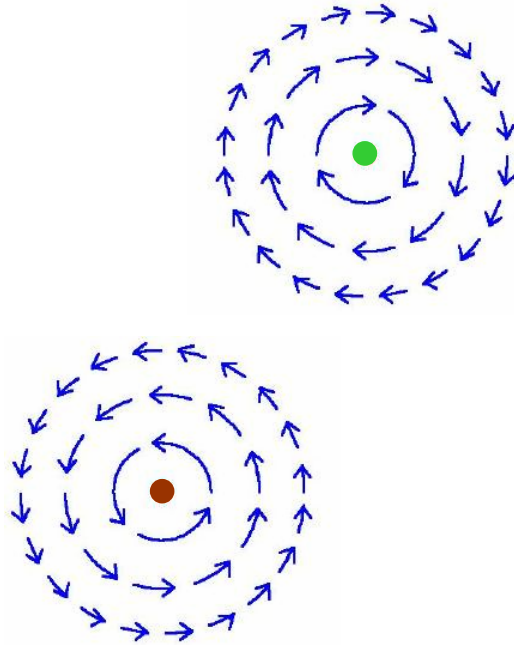
# Vortex theory of the superfluid-insulator transition



As a superfluid approaches an insulating state, the decrease in the strength of the condensate will lower the energy cost of creating vortex-anti-vortex pairs.

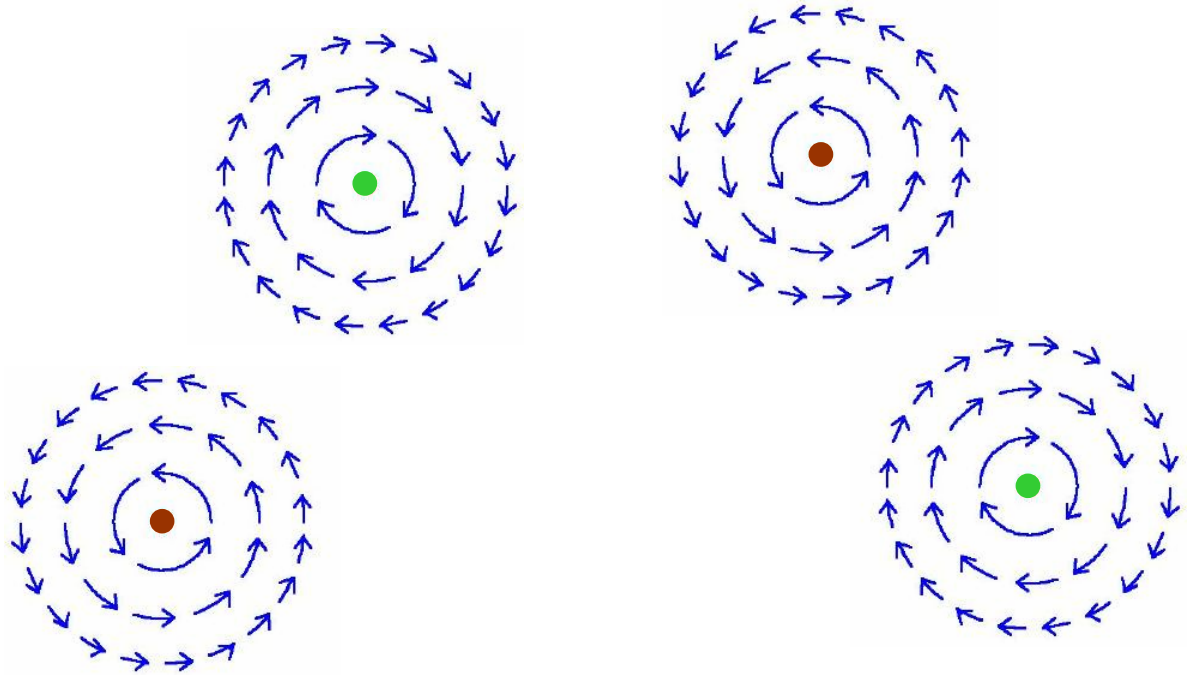


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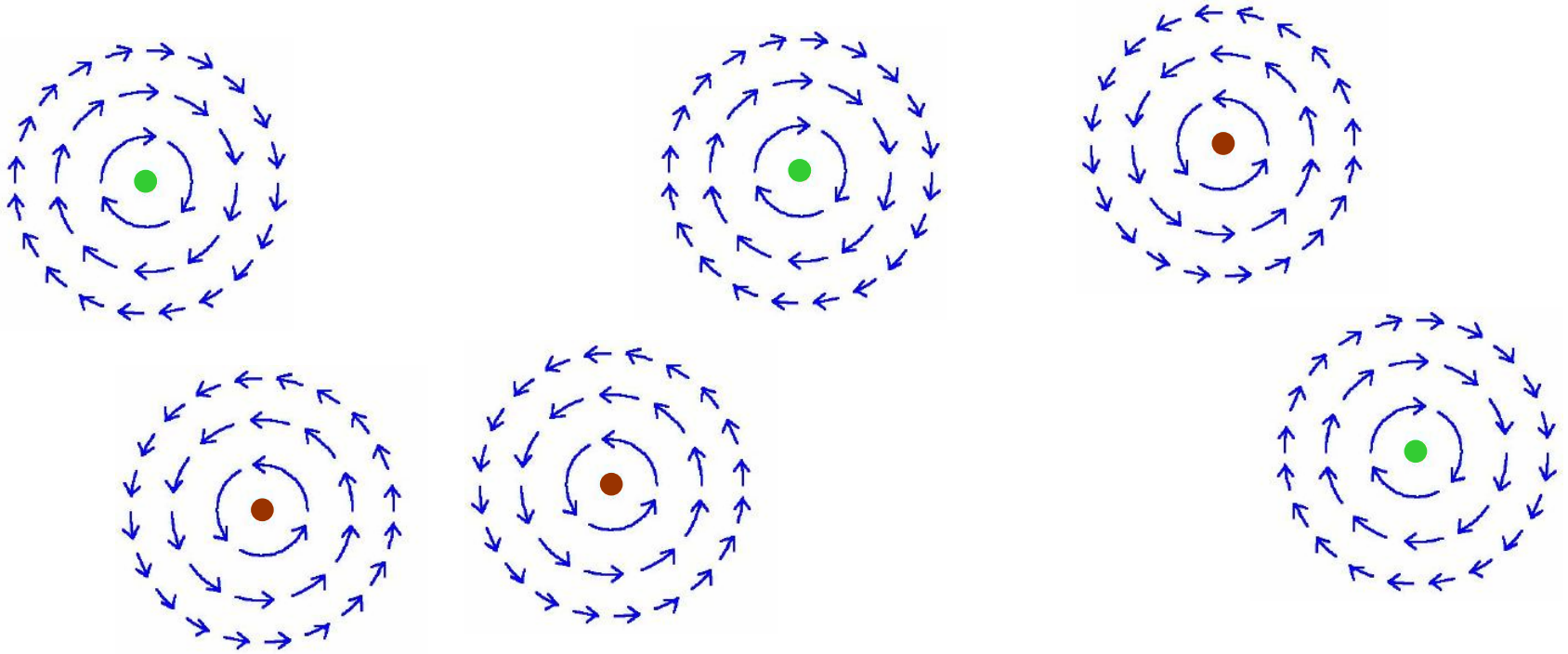
*Proliferation of vortex-anti-vortex pairs.*

# Vortex theory of the superfluid-insulator transition



*Proliferation of vortex-anti-vortex pairs.*

# Vortex theory of the superfluid-insulator transition



*Proliferation of vortex-anti-vortex pairs.*

# Implication of vortex degeneracy for superfluid-insulator transition

- Aharonov-Bohm or Berry phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev. B* **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

# Implication of vortex degeneracy for superfluid-insulator transition

- Aharanov-Bohm or Berry phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.
- Vortex zero point motion leads to a natural explanation of STM experiments.

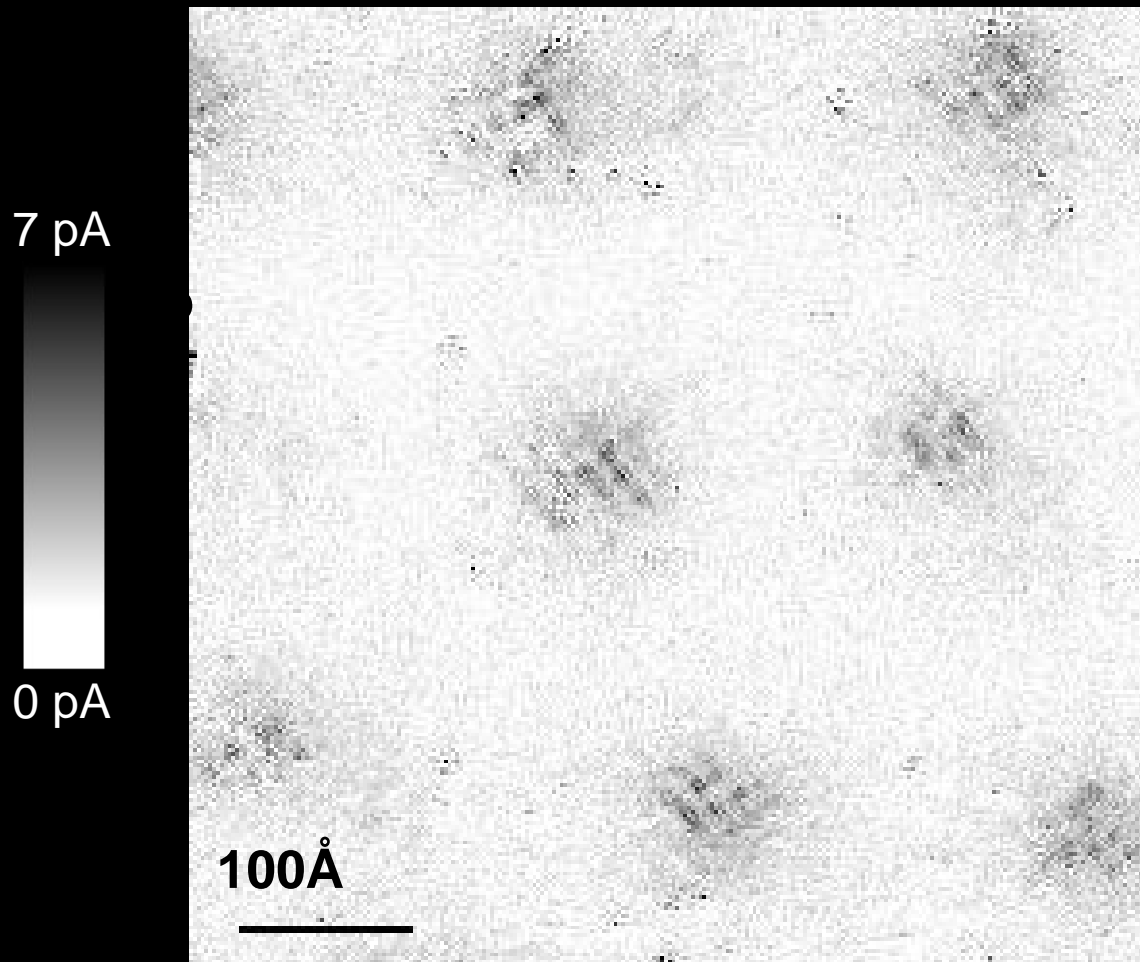
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev. B* **63**, 134510 (2001)

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T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

# Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period  $\approx 4$  lattice spacings

J. Hoffman et al., *Science* 295, 466 (2002).  
G. Levy et al., *Phys. Rev. Lett.* 95, 257005 (2005).

Prediction of periodic LDOS modulations near vortices:  
K. Park and S. Sachdev, *Phys. Rev. B* 64, 184510 (2001).

# Outline

- The superfluid-insulator transition of bosons
- The quantum mechanics of vortices near the superfluid-insulator transition
  - Dual theory of superfluid-insulator transition as the proliferation of vortex-anti-vortex pairs*
- Influence of nodal quasiparticles on vortex dynamics in a  $d$ -wave superconductor

# III. Influence of nodal quasiparticles on vortex dynamics in a *d*-wave superconductor

*P. Nikolic*



## A single vortex in a $d$ -wave superconductor

Consider a single vortex at position  $\mathbf{r}_v(\tau)$ . After the Franz-Tesanovic gauge transformation, this vortex appears as a  $\pi$  flux tube to the fermionic quasiparticles. The low energy theory for the vortex and the fermionic “Dirac” quasiparticles is then

$$\begin{aligned} \mathcal{S} &= \int d^2x d\tau \bar{\Psi} \gamma^\mu (\partial_\mu - i a_\mu) \Psi \\ &+ \text{additional terms from the “Doppler shift”} \end{aligned}$$

where

$$\vec{\nabla} \times \vec{a} = \pi \delta(\mathbf{r} - \mathbf{r}_v(\tau))$$

**First**, we integrate out the Dirac fermions to obtain an effective action for the vortex

$$\mathcal{S}[\mathbf{r}_v(\tau)] = -\text{Tr} \ln (\gamma_\mu (i\partial_\mu - a_\mu))$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$

A *finite* effective mass

$$m_v \sim \frac{\Lambda}{v_F^2}$$

where  $\Lambda \sim \Delta$  is a high energy cutoff

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$



sub-Ohmic damping with

$$C_1 = v_F^{-2} \times \left( \text{Universal function of } \frac{v_\Delta}{v_F} \right)$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$




Bardeen-Stephen viscous drag with

$$C_2 = v_F^{-2} \times \left( \text{Universal function of } \frac{v_\Delta}{v_F} \right)$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$


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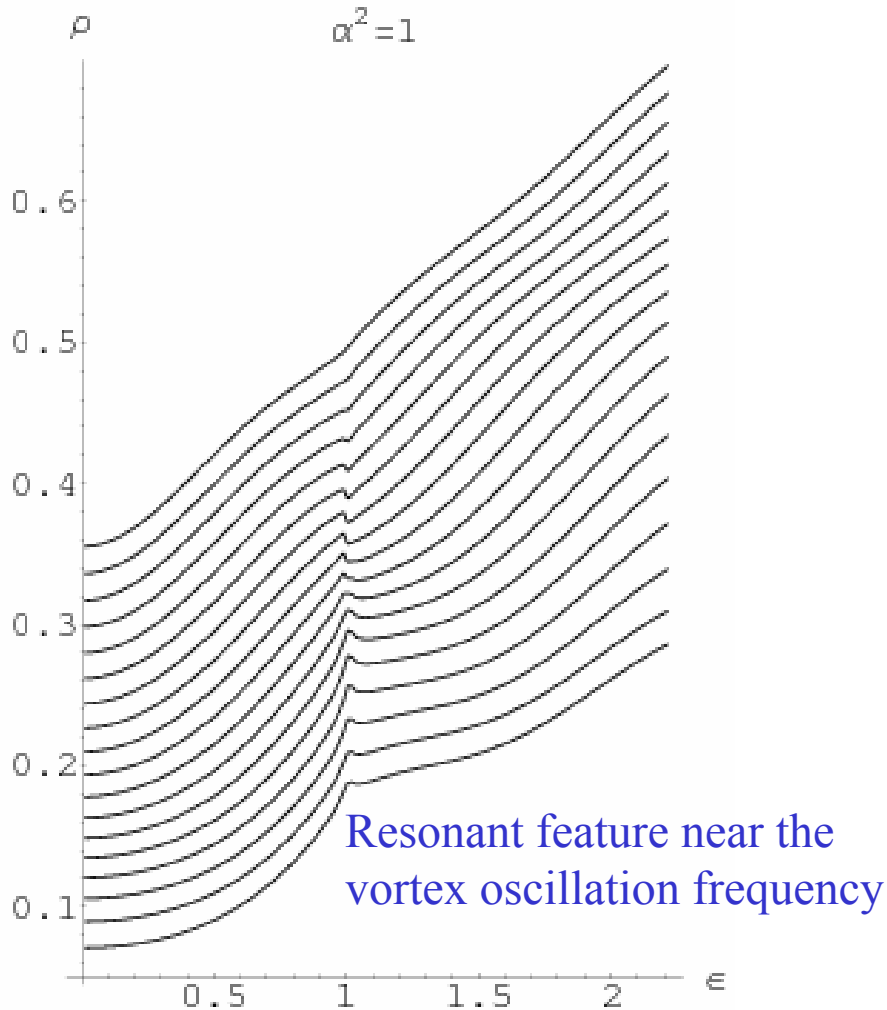
Effect of nodal quasiparticles on vortex dynamics is relatively innocuous.

Using as input (i) the size of the “checkerboard halo” in STM as a measure of the zero-point motion radius of the vortex, and (ii) the forces between the vortices as determined from an estimate of the superfluid stiffness,

we obtain as output an estimate of  $m_v \approx 2 - 9m_e$  and the vortex oscillation frequency  $\omega_v \approx 2 - 7$  meV.

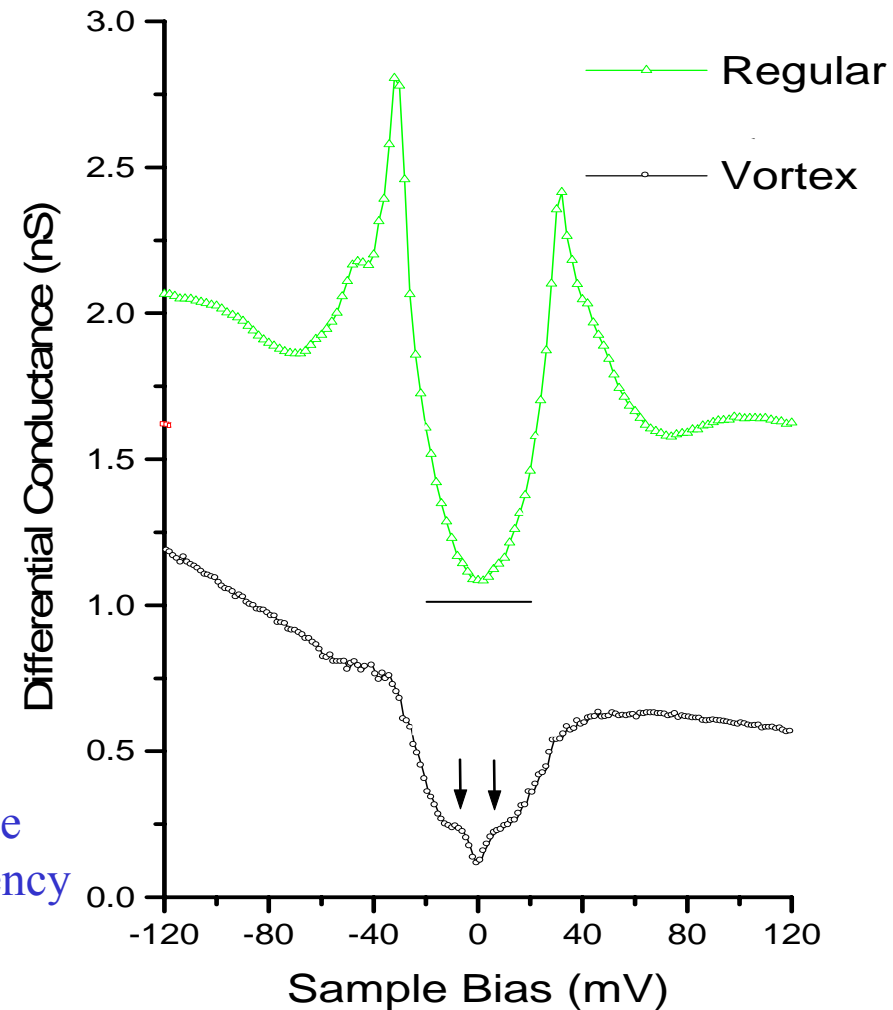
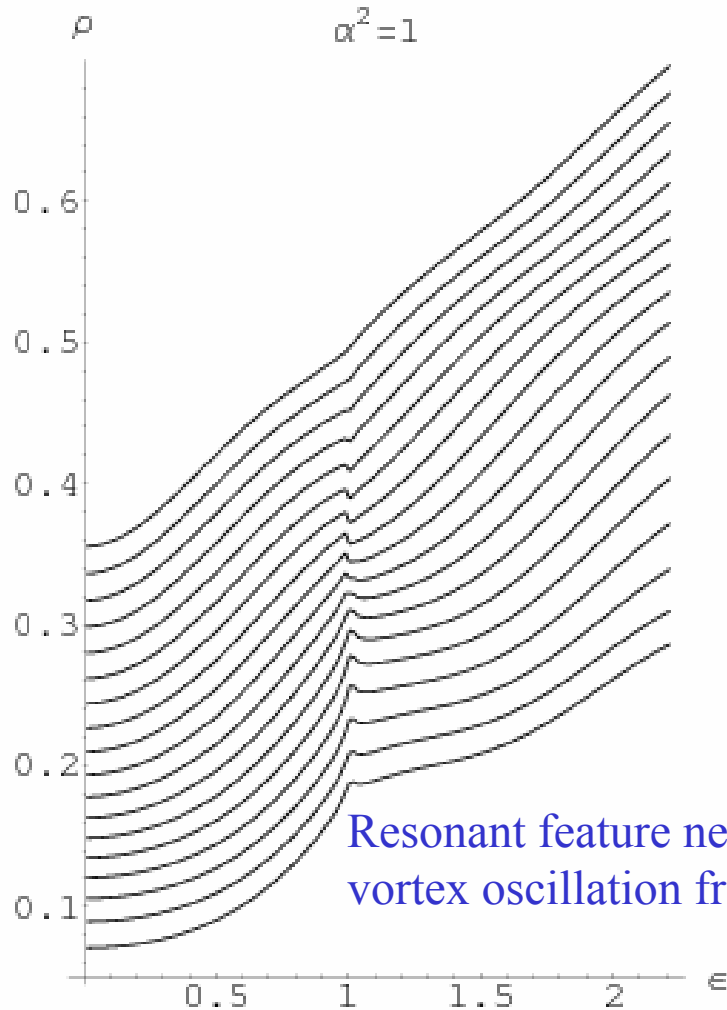
**Second**, we consider the influence of the vortex motion on the nodal quasiparticles. For this, we integrate out the vortex co-ordinates to obtain the modification of the Dirac spectrum....

# Influence of the quantum oscillating vortex on the LDOS





# Influence of the quantum oscillating vortex on the LDOS



I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995)

S.H. Pan *et al.* *Phys. Rev. Lett.* **85**, 1536 (2000).

P. Nikolic, S. Sachdev, and L. Bartosch, cond-mat/0606001

## Conclusions

- Evidence that vortices in the cuprate superconductors carry a “flavor” index which encodes the spatial modulations of a proximate insulator. Quantum zero point motion of the vortex provides a natural explanation for LDOS modulations observed in STM experiments.
- Size of modulation halo allows estimate of the inertial mass of a vortex
- Direct detection of vortex zero-point motion may be possible in inelastic neutron or light-scattering experiments
- The quantum zero-point motion of the vortices influences the spectrum of the electronic quasiparticles, in a manner consistent with LDOS spectrum
- “Aharonov-Bohm” or “Berry” phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.