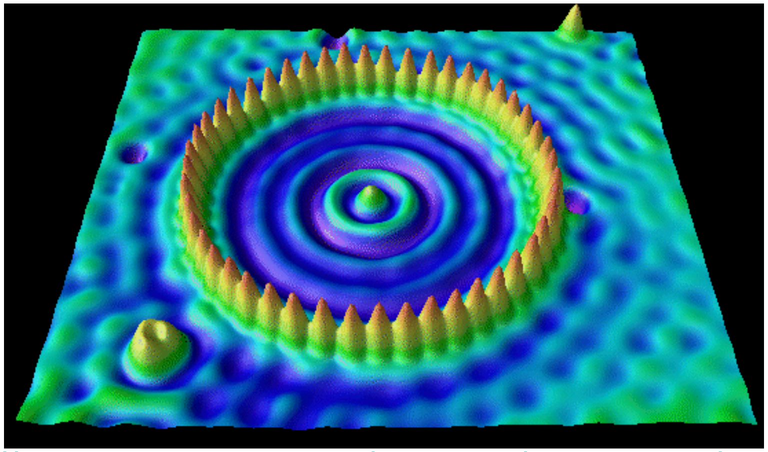
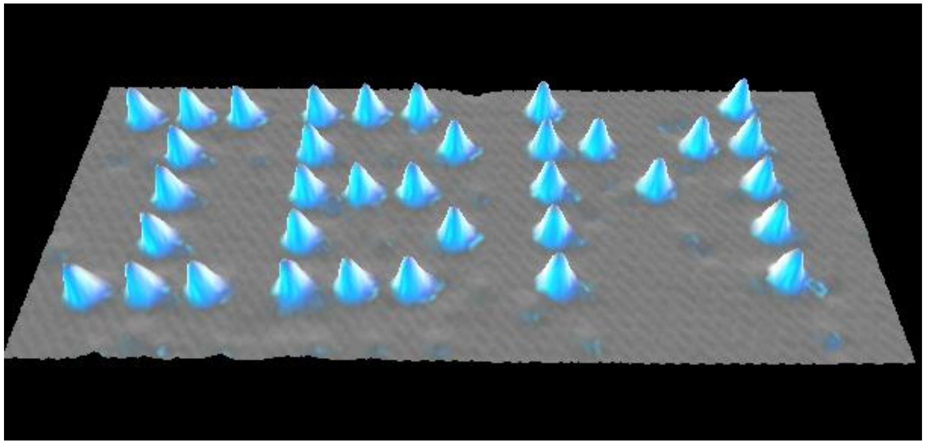
Quantum tunneling: STM & electric shock

- Homework set 10 is due on Friday.
- Homework set 9 ready to return.
- Still have some midterms to return.
- Material Covered today can be found in chapter 14, section 7 and alpha decay in Chapter 17, section 10
- Mario Livio – giving Physics Colloquium today – G1B20 4pm.

"Brilliant Blunders: From Darwin to Einstein – Colossal Mistakes by Great Scientists That Changed Our Understanding of Life and the Universe"



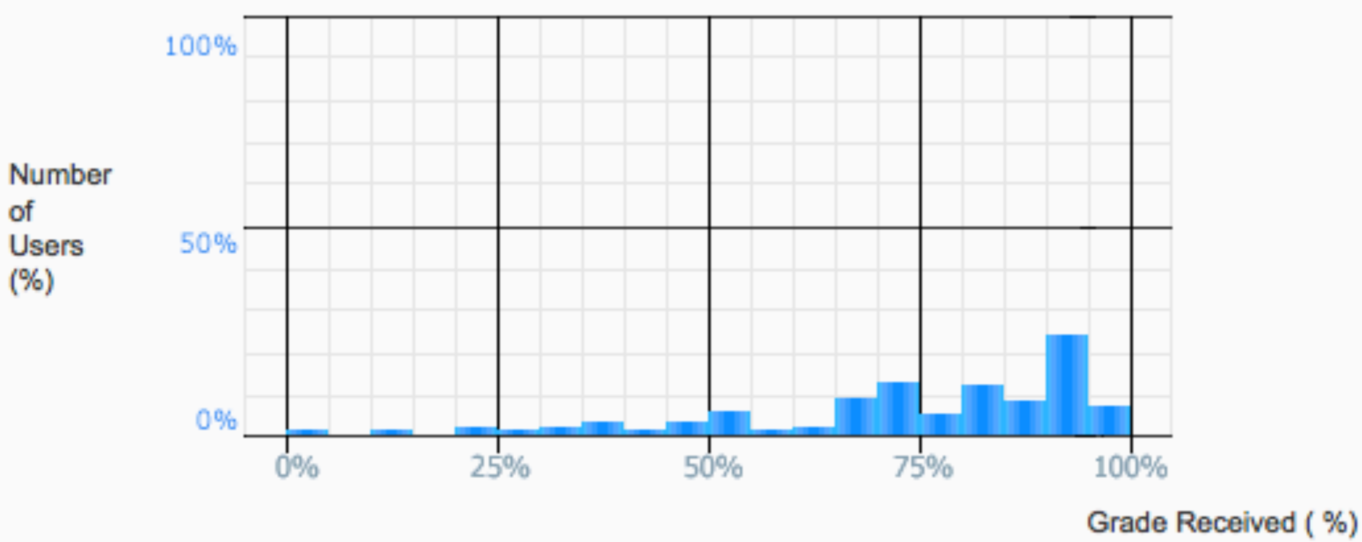


Homework #9

HW9 Class Statistics

Number of submitted grades:	101 / 122
Minimum:	0 %
Maximum:	100 %
Average:	73.56 %
Mode:	92 %
Median:	80 %
Standard Deviation:	21.24 %

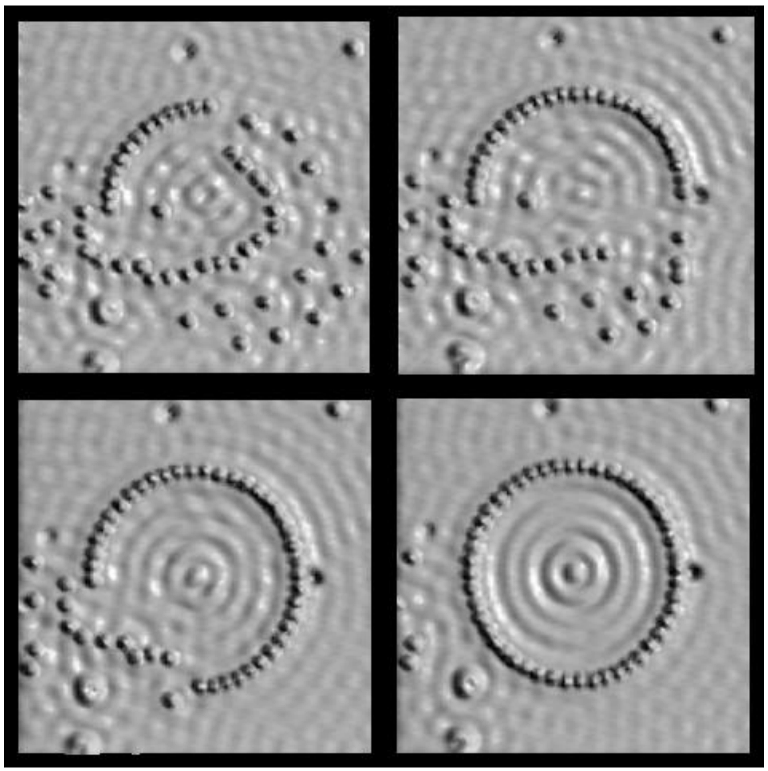
Grade Distribution



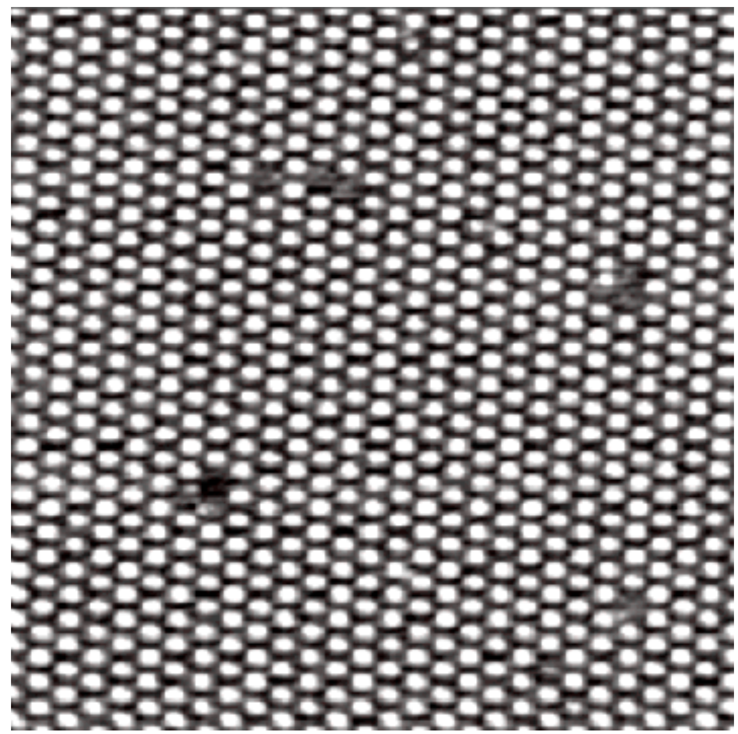
Scanning tunneling microscope

Use tunneling to measure small changes in distance. Nobel prize winning idea: invention of "scanning tunneling microscope (STM)". Measure atoms on surfaces.

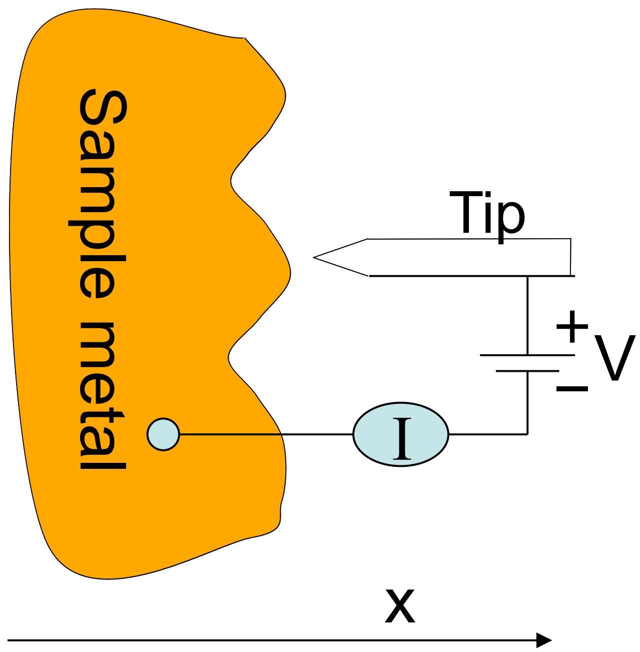
Quantum Corral- Fe atoms on a surface of single crystal Cu



Atomic Resolution on order 30nm



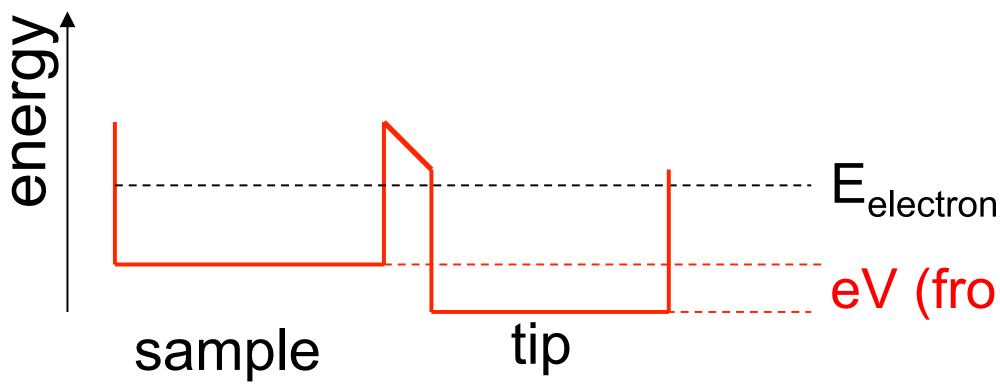
STM potential energy curve



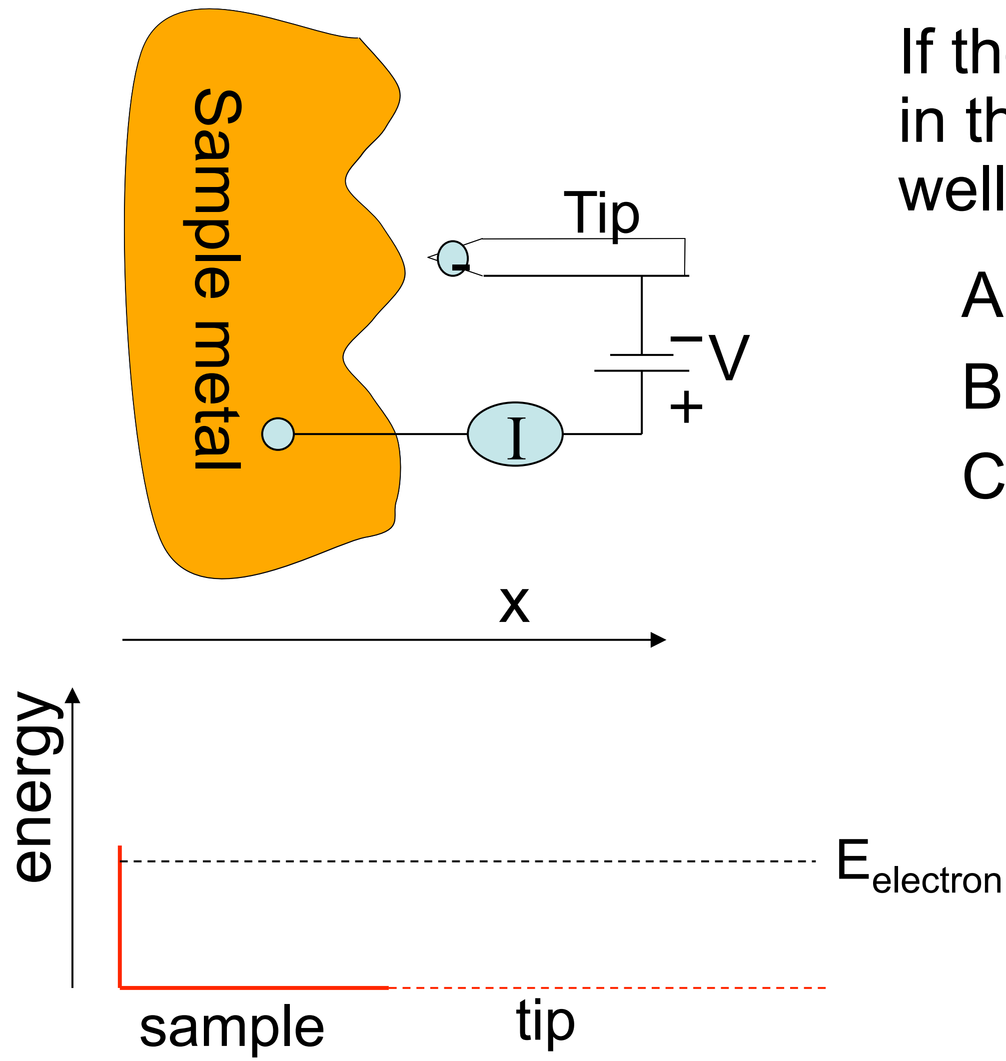
Applying a potential V has two effects

1. Allows a current to flow since electrons will be more likely to tunnel to lower potential
2. Lowers the effective potential barrier making it easier to tunnel

$$U(x) = -eV(x)$$



Clicker question 1	Set frequency to AD



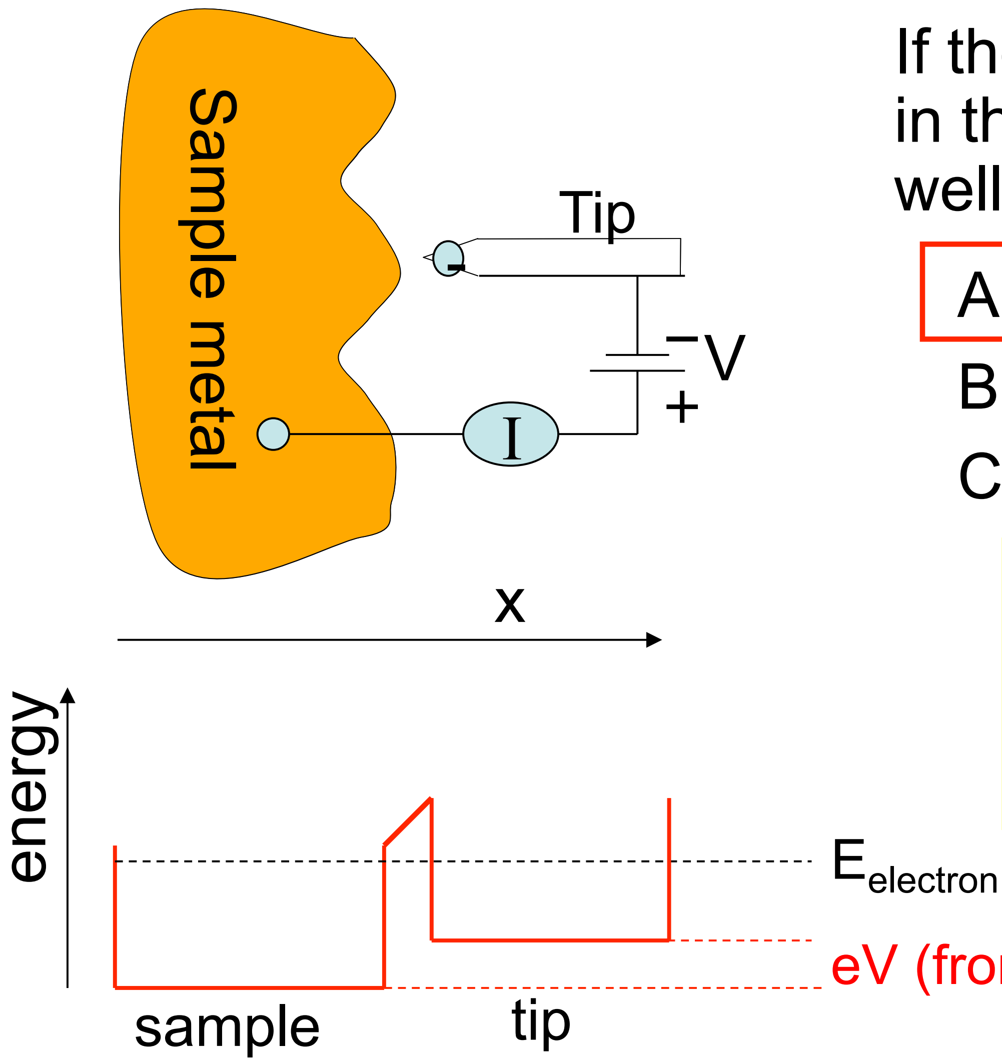
If the same voltage is applied in the *opposite* direction how well will this method work?

A. Works just as well

B. Works but not as well

C. Doesn't work at all

Clicker question 1 | Set frequency to AD



If the same voltage is applied in the *opposite* direction how well will this method work?

A. Works just as well

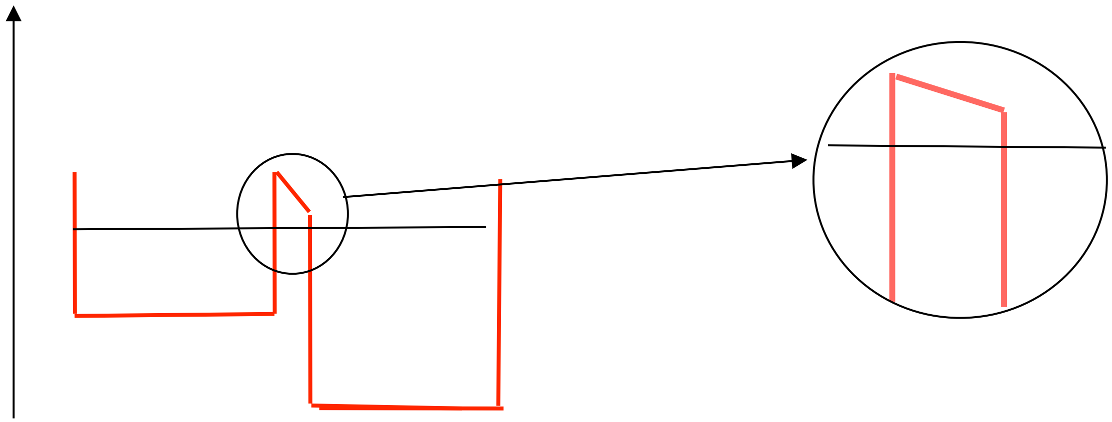
B. Works but not as well

C. Doesn't work at all

The electron will move in the opposite direction so current will be opposite but everything else is the same.

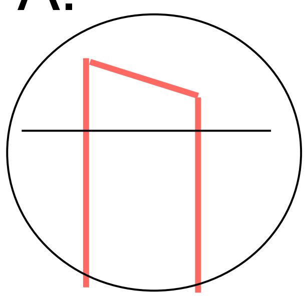
Clicker question 2

Set frequency to AD

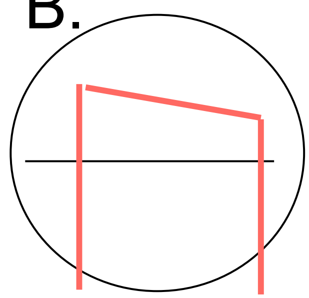


If the tip is moved closer to the sample, what will the new potential graph look like?

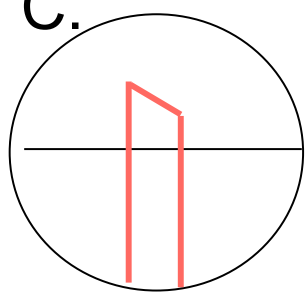
A.



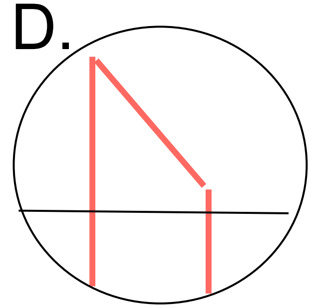
B.



C.

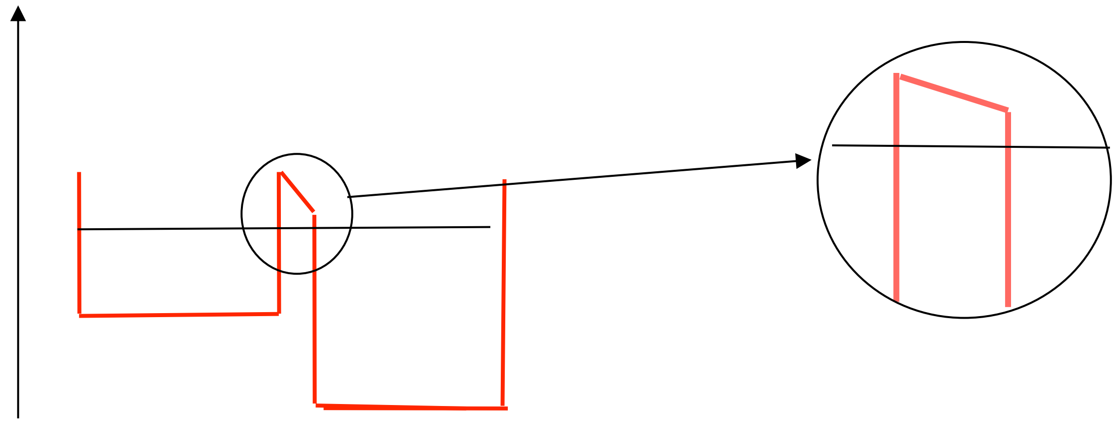


D.



Clicker question 2

Set frequency to AD



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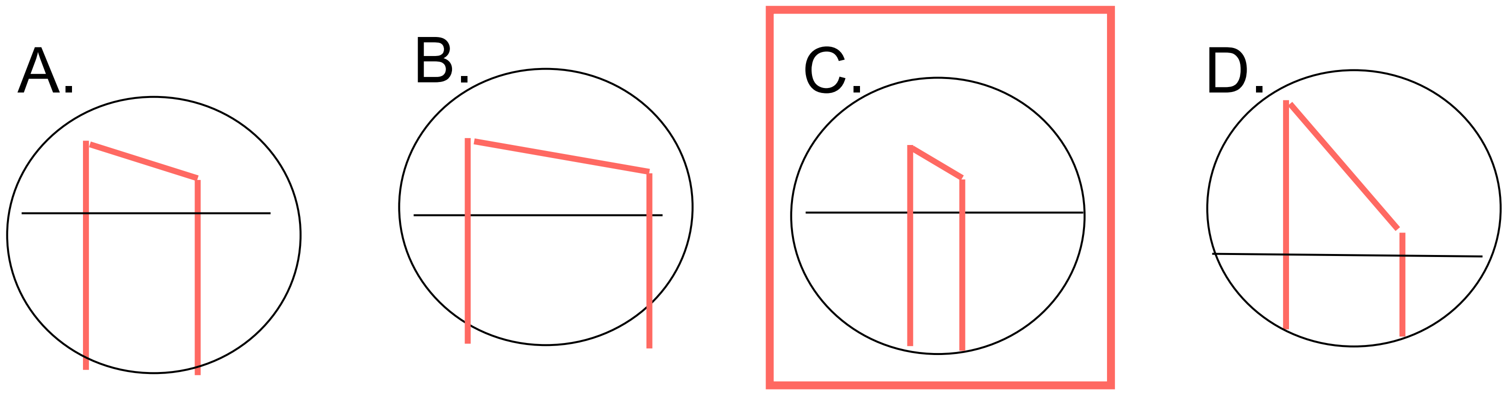


How sensitive is the STM?

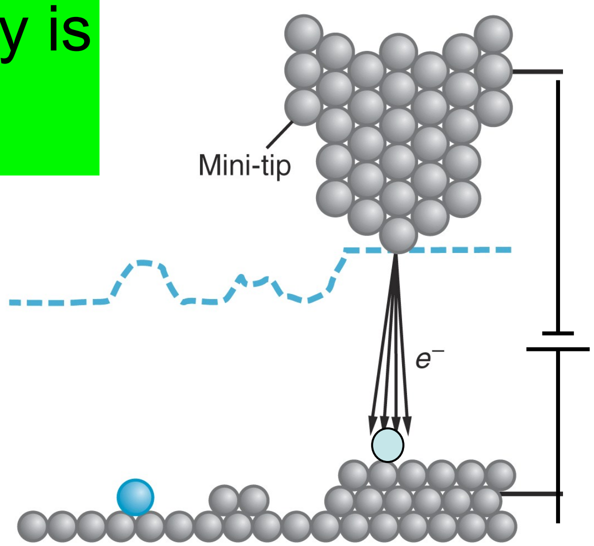
Remember tunneling probability is $P \approx e^{-2\alpha L}$ with $\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$

For work function of 4 eV $\alpha = \frac{\sqrt{2m(V-E)}}{\hbar} \approx 10\,\text{nm}^{-1}$

Note this corresponds to a penetration depth of $\lambda = 1/\alpha = 0.1\,\text{nm}$

If probe is 0.3 nm away (*L=0.3* nm), probability is

$$e^{-2\alpha L} = e^{-2(10\,\text{nm}^{-1})(0.3\,\text{nm})} = e^{-6} = 0.0025$$



An extra atom on top decreases the distance by 0.1 nm so *L* = 0.2 nm giving a tunneling probability of

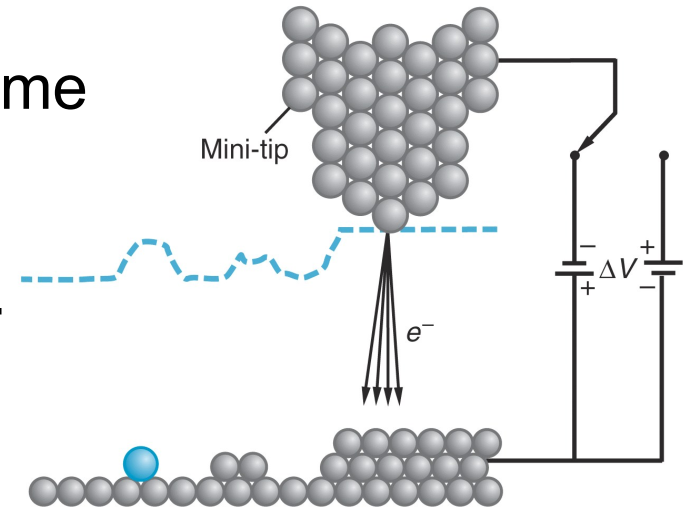
$$e^{-2\alpha L} = e^{-2(10\,\text{nm}^{-1})(0.2\,\text{nm})} = e^{-4} = 0.018$$

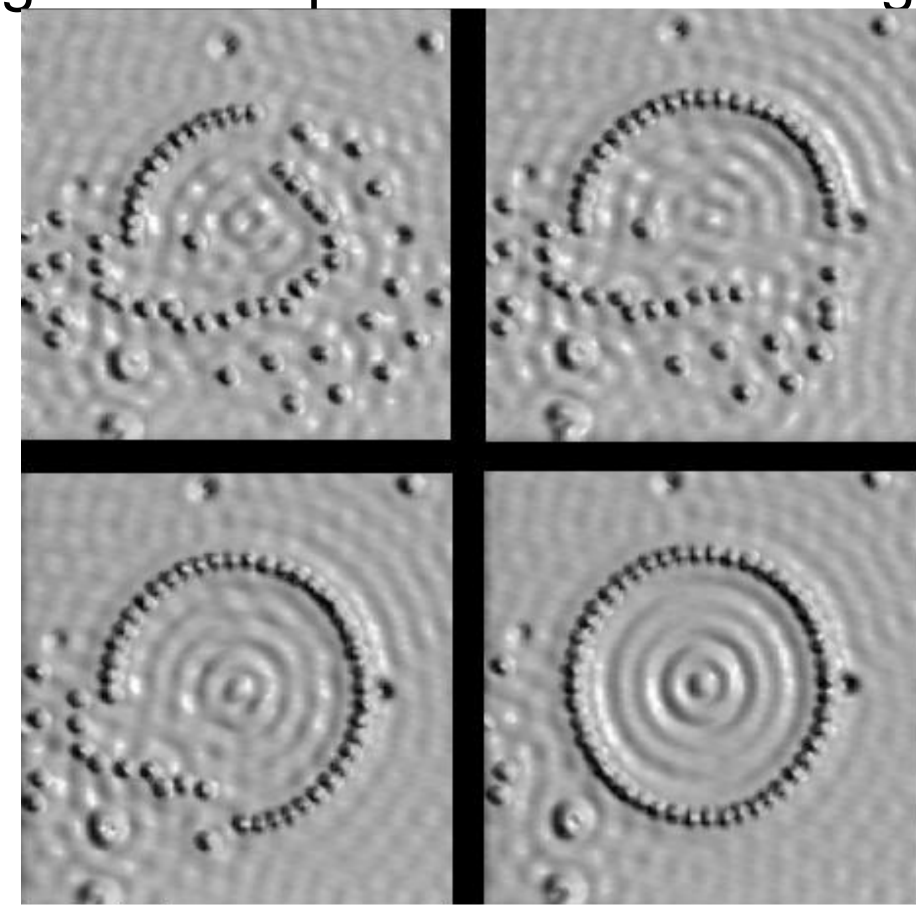
Current is proportional to the probability of an electron tunneling.

One atom increases current by 0.018/0.0025 = 7 times!

STM details

Actual STM uses feedback to keep the current (and therefore the distance) the same by moving the tip up or down and keeping track of how far it needed to move. This gives a map of the surface being scanned.





STM's can also be used to slide atoms around as shown.

Another manifestation of quantum tunneling

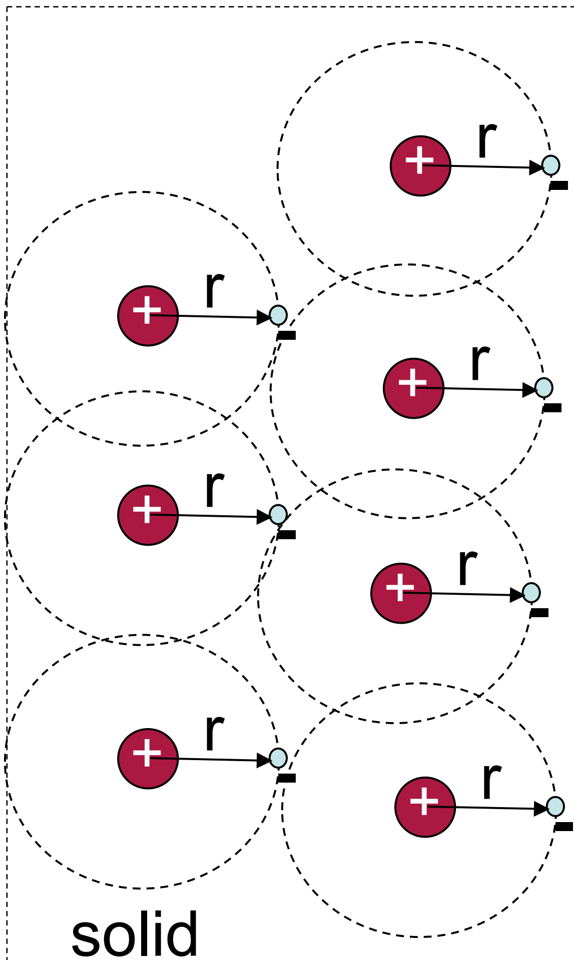
What electric field is needed to pull an electron out of a solid if we ignore quantum tunneling?

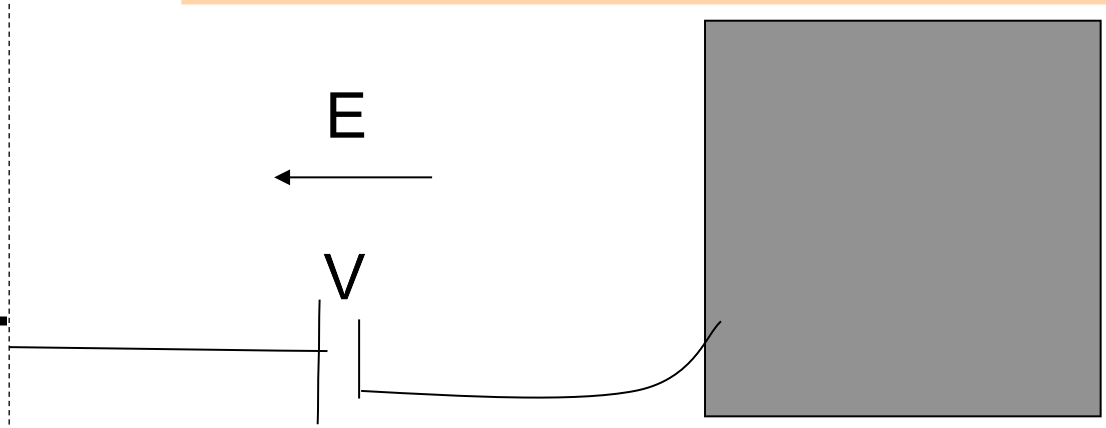
Applied force on the electron must be larger than the force by the nucleus.

Assume we are dealing with hydrogen.

Since F=qE, the applied electric field E must exceed nucleus electric field E_{nuc}.

$$E_{nuc} = \frac{kq}{r^2} = \frac{9\times10^9\ \mathrm{Nm^2/C^2}\cdot 1.6\times10^{-19}\ \mathrm{C}}{(0.053\ \mathrm{nm})^2}$$
$$= 5\times10^{11}\ \mathrm{V/m}$$



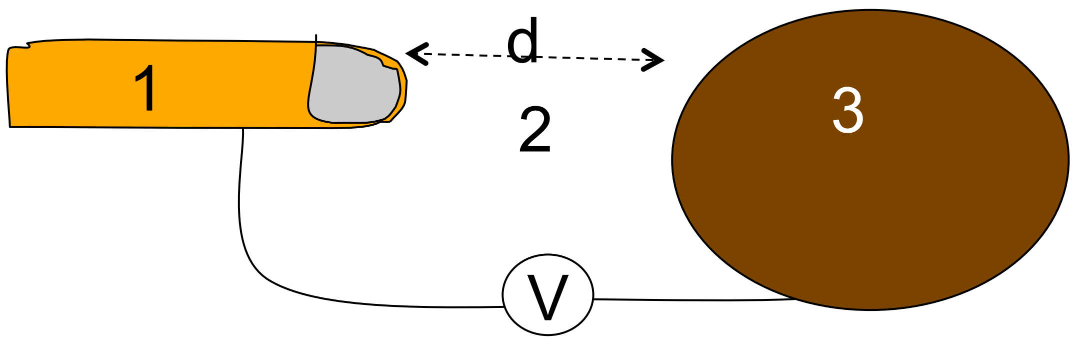


Clicker question 3	Set frequency to DA

E = 5×10^{11} V/m means need 1 billion volts for a 2 mm long spark

Do we get a billion volts by rubbing feet on rug?

NO! Electrons tunnel out at much lower voltage.



What is the minimum info needed to find the tunneling probability?

A. only d
B. only V
C. V and d
D. V, d, and work functions of finger and doorknob
E. none of the above, need additional information

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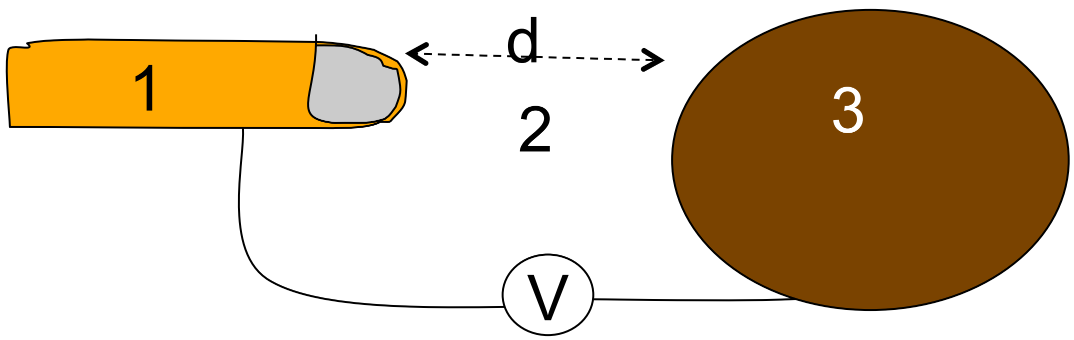

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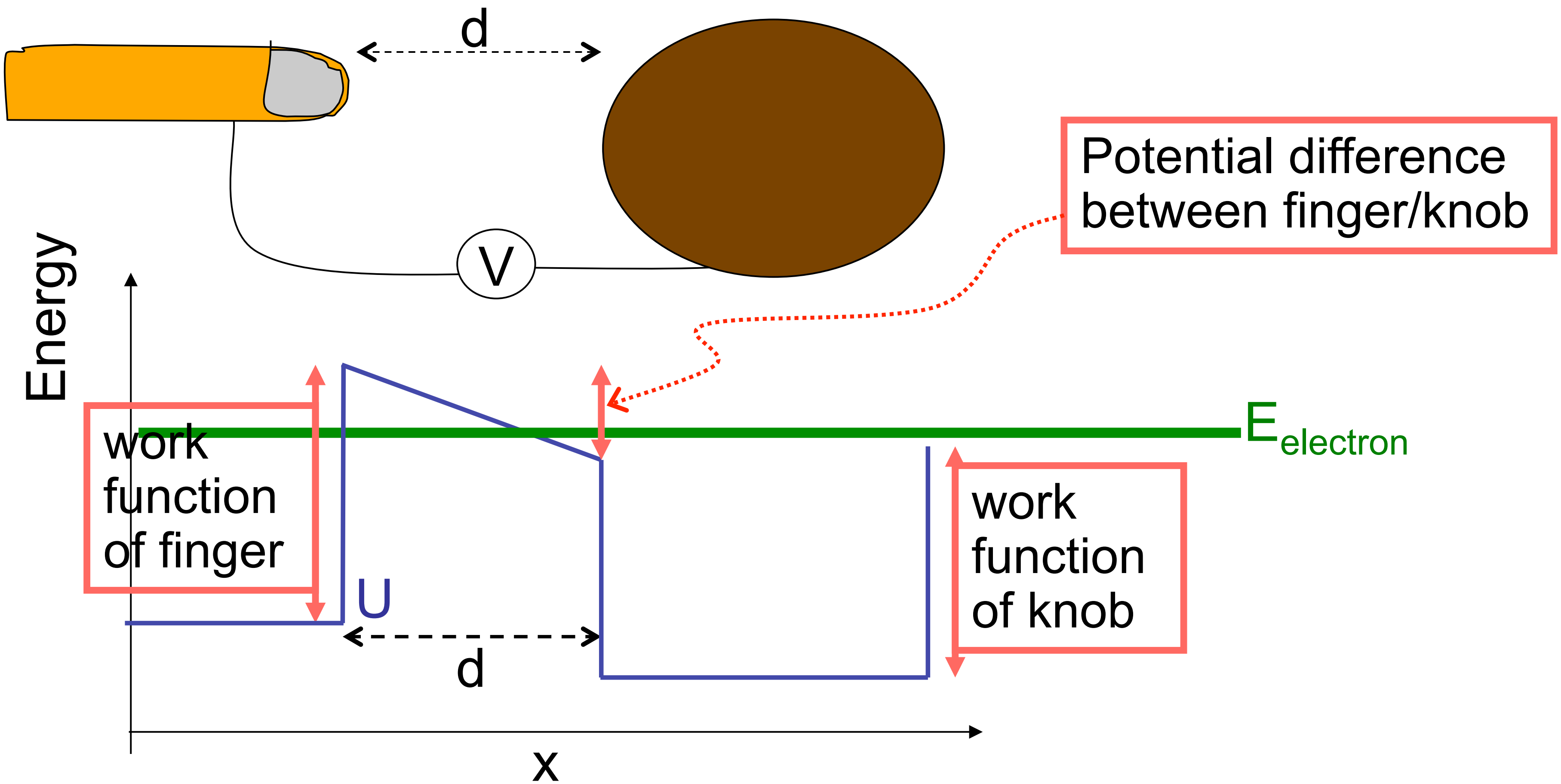
Potential energy for electric shock from door knob

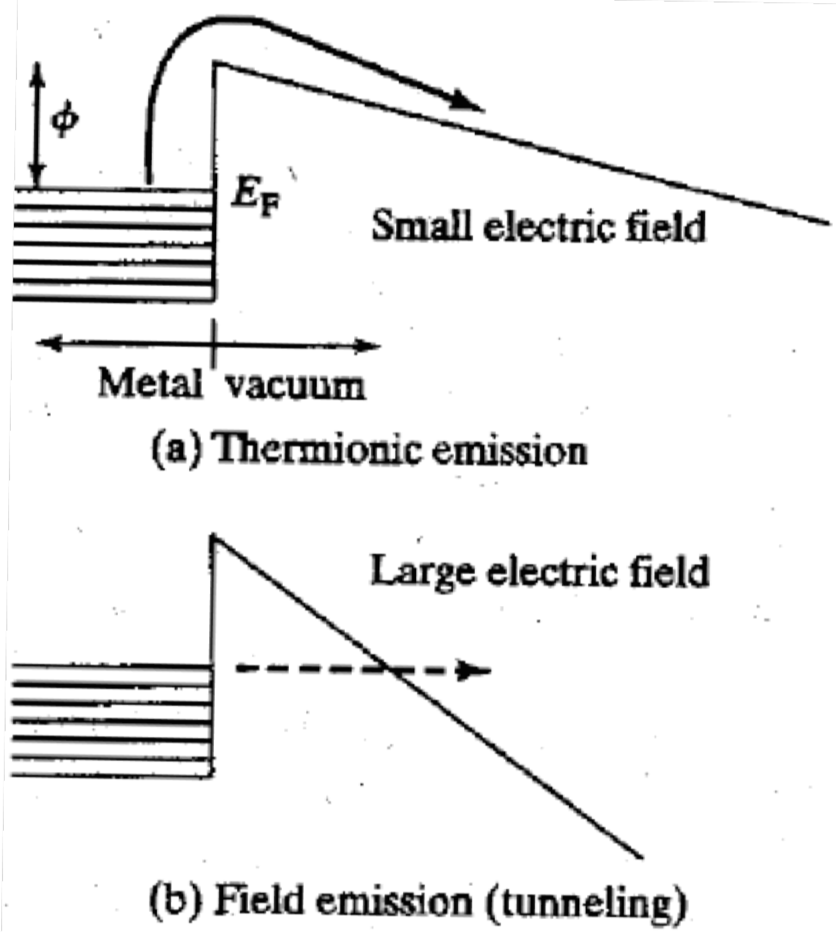


Tunneling probability:

$$P \approx e^{-2\alpha L}$$

Can effectively shorten *L* by moving finger closer **or** by increasing voltage

Let’s Talk about Freeing Electrons



$I \;\alpha\; e^{-\phi/kT}$

FIGURE 14.34

Thermionic emission versus field emission from a metal surface. **(a)** In thermionic emission, electrons at the Fermi level in the metal are thermally activated over the work function barrier. **(b)** In field emission the external electric field is so strong that the barrier becomes thin enough for electrons to tunnel through.

Limiting Case of Finite Well - Delta Potential

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}(x) + V(x)\psi(x) = E\psi(x)$$

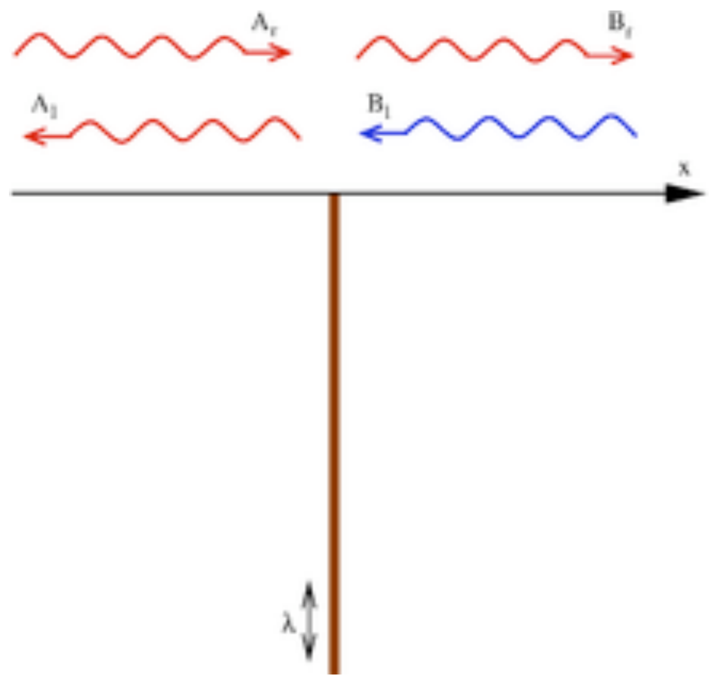
$$V(x) = \lambda\delta(x)$$

The delta potential is the potential where δ(*x*) is the Dirac Delta Function.

Called a *delta potential well* if *λ* is negative

and

a *delta potential barrier* if *λ* is positive.



$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1.$$

Let's Solve the Problem

The potential splits the space in two parts (*x* < 0 and x > 0). In each of these parts the potential energy is zero, and the Schrödinger equation reduces to

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi;$$

The solutions are e^{ikx} and e^{-ikx}, where $k = \sqrt{2mE}/\hbar$

Coefficients don't have to be the same on each side , so

$$\psi(x) = \begin{cases} \psi_L(x) = A_r e^{ikx} + A_l e^{-ikx}, & \text{if } x < 0; \\ \psi_R(x) = B_r e^{ikx} + B_l e^{-ikx}, & \text{if } x > 0, \end{cases}$$

Apply Continuity $\psi(0) = \psi_L(0) = \psi_R(0) = A_r + A_l = B_r + B_l$

Delta Potential Cont.

- Second Boundary Condition

$$-\frac{\hbar^2}{2m}\int_{-\epsilon}^{+\epsilon}\psi''(x)\,dx + \int_{-\epsilon}^{+\epsilon}V(x)\psi(x)\,dx = E\int_{-\epsilon}^{+\epsilon}\psi(x)\,dx.$$

In the limit as $\varepsilon \to 0$, the right-hand side of this equation vanishes; the left-hand side is $-\hbar^2/2m[\psi'_R(0) - \psi'_L(0)] + \lambda\psi(0)$

$$\int_{-\epsilon}^{+\epsilon}\psi''(x)\,dx = [\psi'(+\epsilon) - \psi'(-\epsilon)]$$

$$-\frac{\hbar^2}{2m}ik(-A_r + A_l + B_r - B_l) + \lambda(A_r + A_l) = 0.$$

Boundary Conditions - rewritten

$$\begin{cases} A_r + A_l - B_r - B_l &= 0; \\ -A_r + A_l + B_r - B_l &= \frac{2m\lambda}{ik\hbar^2}(A_r + A_l). \end{cases}$$

Now let $A_r = 1$, $B_l = 0$, and compute transmission coefficient

$T = vB_r^*B_r/vA_r^*A_r$

$$2A_l = \frac{2m\lambda}{ik\hbar^2}(A_r + A_l)$$

or $A_l = 1/(ik\hbar^2/m\lambda - 1)$

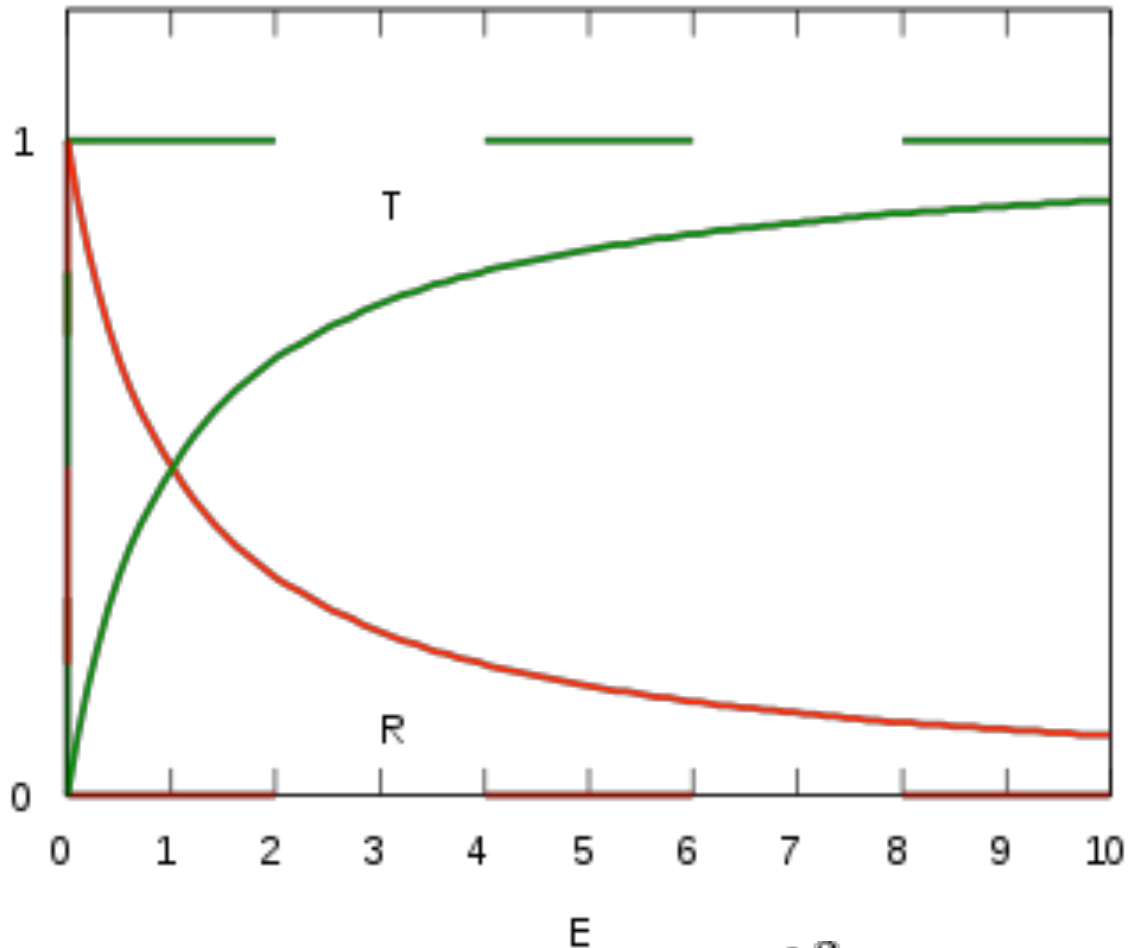
Then $B_r = 1/(1 - m\lambda/ik\hbar^2)$

$T = 1/(1+m^2\lambda^2/k^2\hbar^4)$

$$T = \frac{1}{1 + \frac{m\lambda^2}{2\hbar^2 E}}$$

Note doesn't depend on the sign of λ

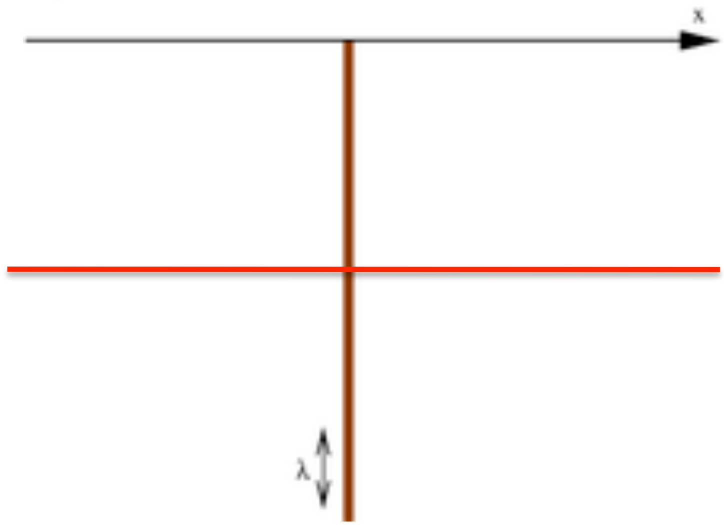
Transmission and Reflection



Doesn't matter
if there is a trough
or a barrier!

$E > 0$ in units of $\frac{\lambda^2}{2m\hbar^2}$

What happens if E < 0



E

How do we solve Schrodinger's equation?

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi;$$

Let $\kappa^2 = -\frac{2mE}{\hbar^2}$

Also note can have a bound state

$$\psi(x) = \begin{cases} \psi_L(x) = A_r e^{\kappa x}, & \text{if } x < 0; \\ \psi_R(x) = B_l e^{-\kappa x}, & \text{if } x > 0. \end{cases}$$

At x=0 $A_r = B_l$

From Normalization

$$\int_{-\infty}^{0} A_r^2 e^{2\kappa x} dx + \int_{0}^{\infty} B_l^2 e^{-2\kappa x} dx = 1$$

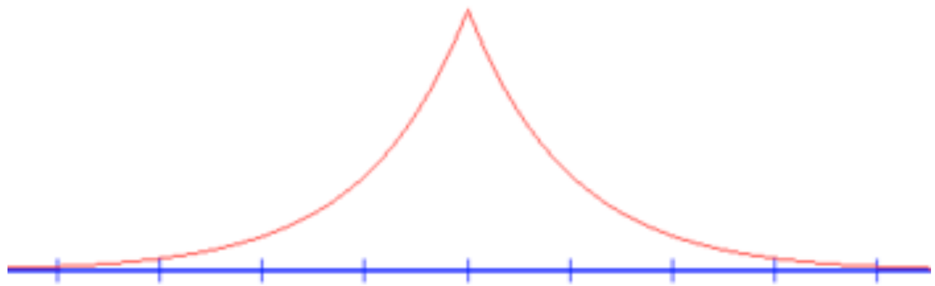
$$A_r = B_l = \sqrt{\kappa}$$

$$-\frac{\hbar^2}{2m}\int_{-\epsilon}^{+\epsilon} \psi''(x)\,dx + \int_{-\epsilon}^{+\epsilon} V(x)\psi(x)\,dx = E\int_{-\epsilon}^{+\epsilon} \psi(x)\,dx.$$

In the limit as $\varepsilon \to 0$, the right-hand side of this equation vanishes; the left-hand side is $-\hbar^2/2m[\psi'_R(0) - \psi'_L(0)] + \lambda\psi(0)=0$; so $\kappa = \frac{m\lambda}{\hbar^2}$

Also note can have a bound state

$$E = -\frac{\hbar^2\kappa^2}{2m} = -\frac{m\lambda^2}{2\hbar^2}.$$



The graph of the bound state wavefunction solution to the delta function potential is continuous everywhere, but its derivative is not at x=0.