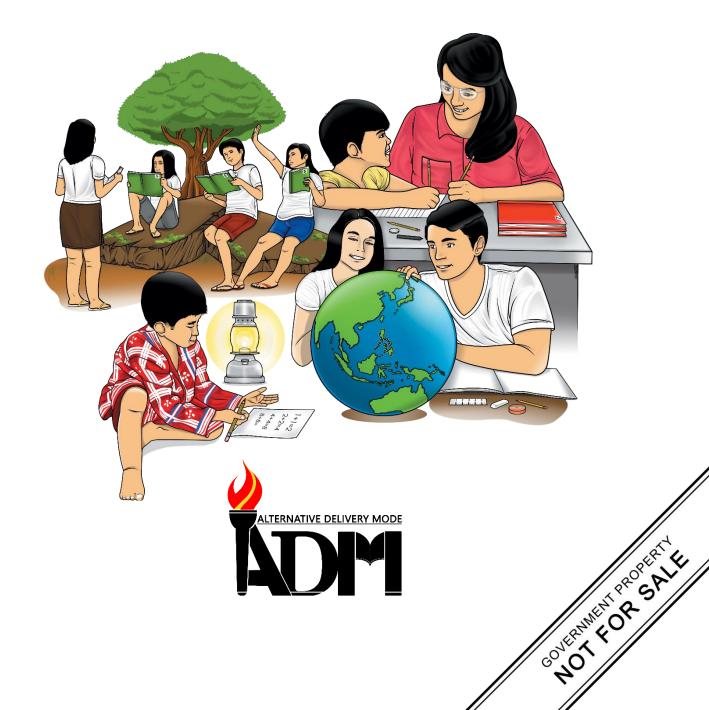




Mathematics Quarter I - Module 4

Finding the Sum of the Terms of a Given Arithmetic Sequence



Mathematics – Grade 10 Alternative Delivery Mode Quarter I – Module 4: Finding the Sum of the Terms of a Given Arithmetic Sequence

First Edition, 2020

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Mathematics Quarter 1 - Module 4

Finds the Sum of the Terms of a Given Arithmetic Sequence

M10AL – Ic - 2



Introductory Message

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This is a part of the fourth learning competency in our Mathematics 10 curriculum standard hence mastery of the skills is significant to have a smooth progress in the succeeding lessons.

For the facilitator:

Hi. As the facilitator of this module, kindly orient the learner on how to go about in reading and answering this learning material. Please be patient and encourage the learner to complete this module. Do not forget to remind the learner to use separate sheets in answering all the activities found in this module.

For the learner:

Hello learner. I hope you are ready to progress in your Grade 10 Mathematics by accomplishing this learning module. This is designed to provide you with interactive tasks to further develop the desired learning competencies on finding the sum of the terms of an arithmetic sequence. This module is especially crafted for you to be able to cope with the current lessons taken by your classmates. Please read completely the written texts and follow the instructions carefully so that you will be able to get the most of this learning material. We hope that you will enjoy learning.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
R	What I need to know	This will give you an idea of the skills or competencies you are expected to learn in the module.
	What I know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
MAR	What's In	This is a brief drill or review to help you link the current lesson with the previous one.

E	What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
	What Is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	What I have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
00	Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
A Company	Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References

This is a list of all sources used in developing this module.

The following are some reminders in using this module:

- 1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
- 2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
- 3. Read the instruction carefully before doing each task.
- 4. Observe honesty and integrity in doing the tasks and checking your answers.
- 5. Finish the task at hand before proceeding to the next.
- 6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that though this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!



WHAT I NEED TO KNOW

This module was designed and written with you in mind. It is here to help you find the sum of the terms of an arithmetic sequence. The scope of this module permits it to be used in many different learning situations. The lessons are arranged to follow the standard sequence of the course but the pacing in which you read and answer this module will depend on your ability.

After going through this module, you are expected to be able to demonstrate knowledge and skill related to sequences and apply these in solving problems. Specifically, you should be able to:

- a) define arithmetic series,
- b) find the sum of the firs n terms of a given arithmetic sequence, and
- c) solve word problems involving arithmetic series.



WHAT I KNOW

Find out how much you already know about the topics in this module. Choose the letter of the best answer. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module. Write the chosen letter on a separate sheet of paper.

1.	It is the sum of the	e terms of a sequer	nce.	
	A) mean	B) sequence	C) <i>n</i> th term	D) series
2.	Find the sum of the	ne first ten terms of	f the arithmetic sec	quence 4, 10, 16,
	 A) 310	B) 430	C) 410	D) 390
3.	Find the sum of t 27,32,	he first 25 terms	of the arithmetic s	sequence 17, 22,
	A) 1925	B) 1195	C) 1655	D) 1895

 5. The third term of an arithmetic sequence is -12 and the seventh term is 8. What is the sum of the first 10 terms? A) 5 B) 8 C) 11 D) 15 6. Find the sum of the first 50 terms of the arithmetic sequence if the first term is 21 and the twentieth term is 154. A) 9635 B) 9765 C) 9265 D) 9625 7. Find the sum of the first eighteen terms of the arithmetic sequence whose nth term is a_n = 15 + 8n. A) 1438 B) 1638 C) 1836 D) 1783 8. The first term of an arithmetic sequence is 5, the last term is 45 and the sum is 275. Find the number of terms. A) 13 B) 10 C) 12 D) 11 9. If the first n terms of the arithmetic sequence 20, 18, 16, are added, how many of these terms will be added to get a sum of -100? A) 35 B) 25 C) 15 D) 30 10. A yaya receives a starting annual salary of Php 36,000.00 with a yearly increase of Php 3,000.00. What is her total income for 8 years? A) Php 315,000.00 B) Php 372,000.00 C) Php 432,000.00 D) Php 495,000.00 11. Jane was saving for a pair of shoes. From her weekly allowance, she was able to save Php 10.00 on the first week, Php 13.00 on the second, Php 16.00 on the third week, and so on. If she continued saving in this pattern and made 52 deposits, how much did Jane save? A) Php 3,984.00 B) Php 4,568.00 C) Php 4,498.00 D) Php 5,678.00 12. Mary gets a starting monthly salary of Php 6,000.00 and an increase of Php 600.00 annually. How much income did she receive for the first three years? A) Php 276,300.00 B) Php 237, 600.00 C) Php 637, 300.00 	4.			arithmetic sequence nd the common diff C) 34		
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- 13. Mirasol saved Php10.00 on the first day of January, Php12.00 on the second day, Php14.00 on the third day, and so on, up to the last day of the month. How much did Mirasol save at the end of January?
 A) Php 2,710.00
 B) Php 2,170.00
 C) Php 1,240.00
 D) Php 1,420.00
- 14. Mrs. De la Cruz started her business with an income of Php 125,000.00 for the first year and an increase of Php 5,000.00 yearly. How much is the total income of Mrs. De la Cruz for 8 years since she started her business?

A) Php 1,104,000.00	B) Php 1,410,000.00
C) Php 1,140,000.00	D) Php 2,140,000.00

15. A hall has 30 rows. Each successive row contains one additional seat. If the first row has 25 seats, how many seats are in the hall?
A) 1,185 B) 1,815 C) 1,970 D) 1,780



Finding the sum of the first *n* terms of a given arithmetic sequence



In the previous module, it was discussed that to find the n^{th} term of a given arithmetic sequence, the formula

 $a_n = a_1 + d(n - 1)$ can be used.

This module will be discussing how to find the sum of the first n terms in an arithmetic sequence.

For example, how do we compute the sum of all the terms of each of the following sequences?

Adding manually the terms of a sequence is manageable when there are only few terms in the sequence. However, if the sequence involves numerous terms, then it is no longer practical to be adding the terms manually. It is a tedious work to do. Thus, this module will present to you a formula that will make the computation easier and faster.



To let you experience getting the sum of the terms in a sequence manually, do the following.

1. Find the sum of the first 20 natural numbers.

Solution:

a. By listing all the natural numbers from 1 to 20 and adding them, we have:

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16+ 17 + 18 + 19 + 20 = 210

- b. Thus, the sum of the first 20 natural numbers is 210.
- 2. Find the sum of all the terms of the sequence: 5, 10, 15, 20, ..., 50.

Solution:

a. By listing all the terms of the sequence and adding them, we have:

$$5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 = 275$$

b. Thus, the sum of the terms of the sequence is 275.

3. Find the sum -5, -2, 1, 4, ..., 31.

Solution:

a. By listing all the terms of the sequence and adding them, we have:

$$-5 + (-2) + 1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 = 169$$

b. Thus, the sum of the terms of the sequence is 169.

In doing this kind of solution, it is very challenging specially if you are dealing with a sequence that has many terms. For example, finding the sum of the terms of the sequence: $1, 2, 3, \ldots, 10,000$. There are 10,000 terms to be added one by one to get their sum.

To derive a formula to be used in finding the sum of the terms of an arithmetic sequence, consider the following illustration:

The terms of an arithmetic sequence with common difference, d, are

First term	<i>a</i> ₁
Second term	$a_1 + d$
Third term	$a_1 + 2d$
Fourth term	$a_1 + 3d$
:	:
$n^{ m th}$ term	$a_1 + (n-1)d$

Thus, the sum of the terms, S_n , is:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \dots + [a_1 + (n-1)d]$$

1st 2nd 3rd 4th nth

The terms of an arithmetic sequence can also be written starting from the n^{th} term and successively subtracting the common difference, *d*. Hence,

$$S_n = a_n + (a_n - d) + (a_n - 2d) + (a_n - 3d) + \dots + [a_n - (n - 1)d]$$

To find the rule for S_n , add the two equations:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \dots + [a_1 + (n - 1)d] + S_n = a_n + (a_n - d) + (a_n - 2d) + (a_n - 3d) + \dots + [a_n - (n - 1)d]$$

$$2S_n = (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \dots + (a_1 + a_n)$$

Notice that all the terms containing d added out. So,

$$2S_n = n(a_1 + a_n)$$

Divide both sides of the equation by two,

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Substituting $a_n = a_1 + (n-1)d$ to a_n , will lead to the following formula:

$$S_n = \frac{n[a_1 + a_1 + (n-1)d]}{2}$$
$$S_n = \frac{n[2a_1 + (n-1)d]}{2}$$

Thus, the sum of the terms of an arithmetic sequence is

$$S_n = \frac{n}{2} [2a_1 + d(n-1)]$$

where: S_n is the sum of the first n terms a_1 is the first term d is the common difference



In getting the sum of the terms of an arithmetic sequence. We will be using any of the following the formula:

S_n = ⁿ/₂(a₁ + a_n) if the first and last term are given
 S_n = ⁿ/₂[2a₁ + (n-1)d] if the last term is not given

Example 1. Find the sum of the first 20 natural numbers.

Given:

 $a_1 = 1$ $a_n = 20$ n = 20 $S_n = ?$

Solution:

Since the last term is given, we used the following formula: $S_n = \frac{n}{2} (a_1 + a_n)$

Substituting the given values in the formula:

$$S_{20} = \frac{20}{2} (1 + 20)$$
$$S_{20} = 10 (21)$$
$$S_{20} = 210$$

 \therefore The sum of the first 20 natural numbers is 210.

Example 2. Find the sum of the first 16 terms of the arithmetic sequence: 8, 11, 14, 17, 20, ...

Given:

 $a_1 = 8$ n = 16 d = 3 $S_{16} = ?$

Solution:

The last term is not given, so we use the formula $S_n = \frac{n}{2} [2a_1 + (n-1)d]$ By substituting the given values in the formula:

$$S_{16} = \frac{16}{2} [2(8) + (16 - 1)3]$$

= 8 [16 + (15)3]
= 8 (16 + 45)
= 8 (61)
$$S_{16} = 488$$

 \therefore The sum of the first 16 terms of the series is 488.

Example 3. If the first n terms of the sequence: 9, 12, 15, 18, ... are added, how many terms give a sum of 126?

Given: $a_1 = 9$ $S_n = 126$ d = 3n = ?

Solution:

The last term is not given so we use the following formula

$$S_n = \frac{n}{2} \left[2a_1 + (n-1)d \right]$$

Substituting the given:

$$126 = \frac{n}{2} [2(9) + (n - 1)3]$$

$$126 = \frac{n}{2} [18 + (3n - 3)]$$

$$252 = n [18 + 3n - 3]$$

$$252 = n [3n + 15]$$

$$252 = 3n^{2} + 15n$$

$$\frac{0}{3} = \frac{3n^{2} + 15n - 252}{3}$$

$$0 = n^{2} + 5n - 84$$
 by factoring
$$(n + 12)(n - 7) = 0$$

$$(n + 12) = 0 \qquad (n - 7) = 0$$

$$n = -12 \qquad n = 7$$

Since the domain of a sequence is the set of positive integers, we reject n = -12. Hence, we only accept n = 7.

 \therefore The number of terms that will add up to 126 is 7.

Example 4. Find the sum of the integers between 1 and 70, and are divisible by 3.

Given:
$$a_1 = 3$$
 $a_n = 69$ $d = 3$ $n = ?$ $S_n = ?$

Solution:

a) To solve for *n*, use the formula:

$$a_n = a_1 + (n-1)d$$

Substitute the given values:

$$69 = 3 + (n - 1)3$$

$$69 = 3 + 3n - 3$$

$$69 = 3n$$

$$n = 23$$

b) Since we already solved *n*, we can now solve for S_n .

$$S_{n} = \frac{n}{2} [2a_{1} + (n-1)d]$$

$$S_{23} = \frac{23}{2} [2(3) + (23-1)3]$$

$$S_{23} = \frac{23}{2} [6 + (22)3]$$

$$S_{23} = \frac{23}{2} (6 + 66)$$

$$S_{23} = \frac{23}{2} (72)$$

$$S_{23} = 828$$

 \therefore The sum of the integers from 1 to 70 that are divisible by 3 is 828.

Example 5. The sum of the first 15 terms of an arithmetic sequence is 765. If the first term is 23, what is the common difference?

Given:
$$a_1 = 23$$
 $n = 15$ $S_{15} = 765$ $d = ?$
Solution:
$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$
$$S_{15} = \frac{15}{2} [2(23) + (15-1)d]$$
$$765 = \frac{15}{2} [46 + (14)d]$$
$$1530 = 15(46 + 14d)$$
$$1530 = 690 + 210d$$
$$210d = 1530 - 690$$
$$210d = 840$$
$$d = 4$$

 \therefore The common difference is 4.



After knowing all the needed concept in finding the sum of an arithmetic sequence. You are now ready to answer the following exercises:

A. Find the indicated partial sum of each arithmetic series.

- 1) The first 9 terms of $5 + 8 + 11 + \cdots$
- 2) The first 30 terms of $1 + 3 + 5 + \cdots$
- 3) The first 14 terms of $6 + 9 + 12 + \cdots$
- 4) The first 25 terms of $5 + 8 + 11 + \cdots$
- 5) The first 15 terms of $-12 + (-6) + 0 + \cdots$

B. Solve for the value of *n*.

1) $S_n = -80$,	$a_1 = 10,$	$a_n = -26,$	n = ?
2) $S_n = 50$,	$a_1 = 4,$	$a_n = 16,$	n = ?
3) $S_n = -15$,	$a_1 = 12,$	d = -3,	n = ?
4) $S_n = 180$,	$a_1 = 5$,	d = 5,	n = ?

C) Answer what is asked.

- 1) Find the sum of the first 13 terms of the sequence: -3, -1, 1, 3, ...
- 2) Find the sum of the first 15 terms of the arithmetic sequence:

10, 15, 20, 25, ...?

- 3) Find the sum of the first 11 terms of the arithmetic sequence: -4, 3, 10, 17, ...?
- 4) Find the sum of the first 19 terms of the arithmetic sequence: 9,14,19,24,...?
- 5) Find the sum of the integers from 8 and 35.
- 6) Find the sum of all even integers from 10 to 70.
- 7) Find the sum of all odd integers from 1 to 50.
- 8) Find the sum of the integers from 20 to 130 and are divisible by 5.
- 9) If the sum of the first 8 terms of an arithmetic sequence is 172 and its common difference is 3, what is the first term?
- 10) If the sum of the first 9 terms of an arithmetic sequence is 216 and its first term is 4, what is the common difference?



To find the sum of the terms of an arithmetic sequence, you can use the following formulae:

A. If the first and last terms are given:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where: S_n is the sum of the first n terms a_1 is the first term a_n is the last term

B. If the last term is not given:

$$S_n = \frac{n}{2} [2a_1 + d(n-1)]$$

where: S_n is the sum of the first n terms a_1 is the first term d is the common difference



Read and understand the problems and give what is asked.

- 1. Suppose a cinema has 42 rows of seats and there are 20 seats in the first row. Each row after the first row has two more seats that the row that it precedes. How many seats are in the cinema?
- 2. A 25-layer of logs is being piled to be used on a construction. The uppermost layer is composed of 25 logs, the second upper layer contains 27 logs, and the third upper layer contains 29 logs, and so on. If the pattern continues up to the lowest layer, what is the total number of logs piled for construction?



Read and analyze each item carefully. Write the letter of the correct answer in a separate paper.

1. Which of the following is a formula for arithmetic series?

A)
$$S_n = \frac{1}{2}(a_1 + a_n)$$
 B) $S_n = \frac{n}{2}(a_1 + a_n)$ C) $S_n = \frac{1}{2}(a_1 + d)$ D) $S_n = \frac{n}{2}(a_1 + da_n)$

2. Find the sum of the first 12 terms of the arithmetic sequence 4, 11, 18, ...

A) 610 B) 530 C) 510 D) 410

3. Find the sum of the first 15 terms of the arithmetic sequence 17, 12, 7, 2, ...

A) 270 B) 287 C) -287 D) -270

4. The sum of the first 12 terms of an arithmetic sequence is 606. What is the first term if the last term is 67 and the common difference is 3.

A) 64 B) 54 C) 34 D) 44

- The second term of an arithmetic sequence is −16 and the eighth term is
 8. What is the sum of the first 10 terms?
 - A) -15 B) -20 C) 15 D) 20
- 6. Find the sum of the first 40 terms of the arithmetic sequence if the first term is 16 and the tenth term is 70.

- 7. Find the sum of the first 15th terms of the arithmetic sequence whose nth term is $a_n = 5 + 3n$.
 - A) 870 B) 860 C) 435 D) 430
- 8. The first term of an arithmetic sequence is 8, the last term is 56 and the sum is 416. Find the number of terms.
 - A) 13 B) 12 C) 11 D) 10
- 9. If the first *n* terms of the arithmetic sequence 24, 20, 16,... are added, how many of these terms will be added to get a sum of -60?
 - A) 35 B) 30 C) 25 D) 15
- 10. A yaya receives a starting annual salary of Php 60,000.00 with a yearly increase of Php 3,600.00. What is her total income for 5 years?

A) Php 672,000.00	B) Php 552,000.00
C) Php 276,000.00	D) Php 336,000.00

11. Jane was saving for a pair of shoes. From her weekly allowance, she was able to save Php 5.00 on the first week, Php 9.00 on the second, Php 13.00 on the third week, and so on. If she continued saving in this pattern and made 43 deposits, how much did Jane save?

A) Php 3,822.00	B) Php 3,827.00
C) Php 7,644.00	D) Php 6,574.00

12. Mary gets a starting monthly salary of Php 8,000.00 and an increase of Php 800.00 annually. How much income did she receive for the first four years?

A) Php 441,600.00	B) Php 388,800.00
C) Php 40,000.00	D) Php 36,800.00

13. Mirasol saved Php 8.00 on the first day of January, Php 11.00 on the second day, Php 14.00 on the third day, and so on, up to the last day of the month. How much did Mirasol save at the end of January?

A) Php 4,282.00	B) Php 4,290.00
C) Php 1,643.00	D) Php 1,590.00

14. Mrs. De la Cruz started her business with an income of Php 250,000.00 for the first year and an increase of Php 6,000.00 yearly. How much is the total income of Mrs. De la Cruz for 6 years since she started her business?

A) Php 530,000.00	B) Php 3,180,000.00
C) Php 1,590,000.00	D) Php 1,608,000.00

15. A hall has 35 rows. Each successive row contains two additional seats. If the first row has 20 seats, how many seats are in the hall?

A) 1,080 B) 1,100 C) 1,925 D) 1,890



ADDITIONAL ACTIVITY

Let us sing the song titled "Twelve Days of Christmas." Afterwards, answer the question that follows.

Verse 1:	Five golden rings	
On the first day of Christmas my true love	Four calling birds	
sent to me	Three French hens	
A partridge in a pear tree	Two turtle doves, and a partridge in a pear	
Verse 2:	tree	
On the second day of Christmas my true	Verse 7:	
love sent to me Two turtle doves, and a partridge in a pear	On the seventh day of Christmas my true love sent to me	
tree	Seven swans a - swimming	
<u>Verse 3:</u>	Six geese a - laying	
On the third day of Christmas my true love sent to me	Five golden rings	
Three French hens	Four calling birds	
Two turtle doves, and a partridge in a pear	Three French hens	
tree	Two turtle doves, and a partridge in a pear	
Verse 4:	tree	
On the fourth day of Christmas my true	Verse 8:	
love sent to me	On the eighth day of Christmas my true love sent to me	
Four calling birds	Eight maids a-milking	
Three French hens	Seven swans a - swimming	
Two turtle doves, and a partridge in a pear tree	Six geese a - laying	
Verse 5:	Five golden rings	
On the fifth day of Christmas my true love	Four calling birds	
sent to me	Three French hens	
Five golden rings	Two turtle doves, and a partridge in a pea	
Four calling birds	tree	
Three French hens		
Two turtle doves, and a partridge in a pear tree		
Verse 6:		
On the six day of Christmas my true love sent to me		
Six geese a – laying		

Verse 9:	<u>Verse 11:</u>	
On the ninth day of Christmas my true love sent to me	On the 11^{th} day of Christmas my true love sent to me	
Nine ladies dancing	11 pipers piping	
Eight maids a-milking	10 lords a-leaping	
Seven swans a - swimming	Nine ladies dancing	
Six geese a - laying	Eight maids a-milking	
Five golden rings	Seven swans a - swimming	
Four calling birds	Six geese a - laying Five golden rings	
Three French hens	Four calling birds	
Two turtle doves, and a partridge in a pear	Three French hens	
tree	Two turtle doves, and a partridge in a pear tree	
<u>Verse 10:</u>	Verse 12:	
On the tenth day of Christmas my true love sent to me	On the 12 th day of Christmas my true love sent to me	
10 lords a-leaping	12 drummers drumming	
Nine ladies dancing	11 pipers piping	
Eight maids a-milking	10 lords a-leaping	
Seven swans a - swimming	Nine ladies dancing	
Six geese a - laying	Eight maids a-milking	
Five golden rings	Seven swans a - swimming	
Four calling birds	Six geese a - laying	
Three French hens	Five golden rings	
Two turtle doves, and a partridge in a pear	Four calling birds	
tree	Three French hens	
	Two turtle doves, and a partridge in a pear tree	

Summarizing, we have the following:

12 drummers drumming	6 geese – a – laying	
11 pipers piping	5 golden rings	
10 lords-a-leaping	4 calling birds	
9 ladies dancing	3 French hens	
8 maids- a – milking	2 turtles doves, and	
7 swans-a-swimming	A partridge in a pear tree.	

Question:

How many gifts are given after the 12^{th} day of Christmas?

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8) V 8) V 12) D 13) C 14) C 13 D 13 D 14 13 C 14 13 V 14 13 V 14 14 15 14 16 17 17 17 18 17 18 17 18 18 19 19 10 19 10 10 10 10 10 10 10 10 10 10 10 10 10	IO' 2 6' II 2' 652 8' I 250 9' 1540 2' 652 3' 341 4' I 056 3' 341 J' 112 C) C)	12. B 13. C 12. B
L) C 9 V 2) B	+) 8 3) 10 5) 2	11' C 10' B 6' B
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What I Can Do 1. 2562 seats 2. 1225 logs.	Mhât's Mote A. 2) 900	What I Know 1. D 2. A 3. A



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