Optics

Polarization
Birefringence

Quarter-wavelength and half-wavelength plate

Objects of the experiment

■ Measuring the light intensity as function of the analyzer position.

■ Using the quarter wave plate to produce circularly polarized light.

Principles

A wave plate or retarder is an optical device that alters the polarization state of a light beam travelling through it. A typical wave plate is simply a birefringent crystal or a double refracting plastic foil with a carefully chosen thickness.

If a beam of parallel light strikes perpendicularly a wave plate the light beam is splitted into two components due to its double refracting properties. The two components have planes of oscillation perpendicular to each other and slightly different phase velocities. For a quarter-wave plate the thickness of the foil is chosen in such a manner that the light component whose electric field vector oscillates in parallel to the rotation lever lags by a $\lambda/4$ behind other perpendicular oscillating light component. For a half-wave plate the thickness is chosen so that the created phase difference has the amount of $\lambda/2$.

In this experiment monochromatic light falls on a quarter-wave and half-wave plate. The polarization of the emergent light is investigated at different angles between the optic axis of the wave plates and the direction of the incident light.

Fig. 1: A half-wave plate schematically. Linearly polarized light entering a wave plate can be resolved into two waves, parallel (shown as green) and perpendicular (blue) to the optical axis of the wave plate. In the plate the parallel wave propagates slightly slower than the perpendicular one. At the far side of the plate the parallel wave is exactly half of a wavelength delayed relative to the perpendicular wave.

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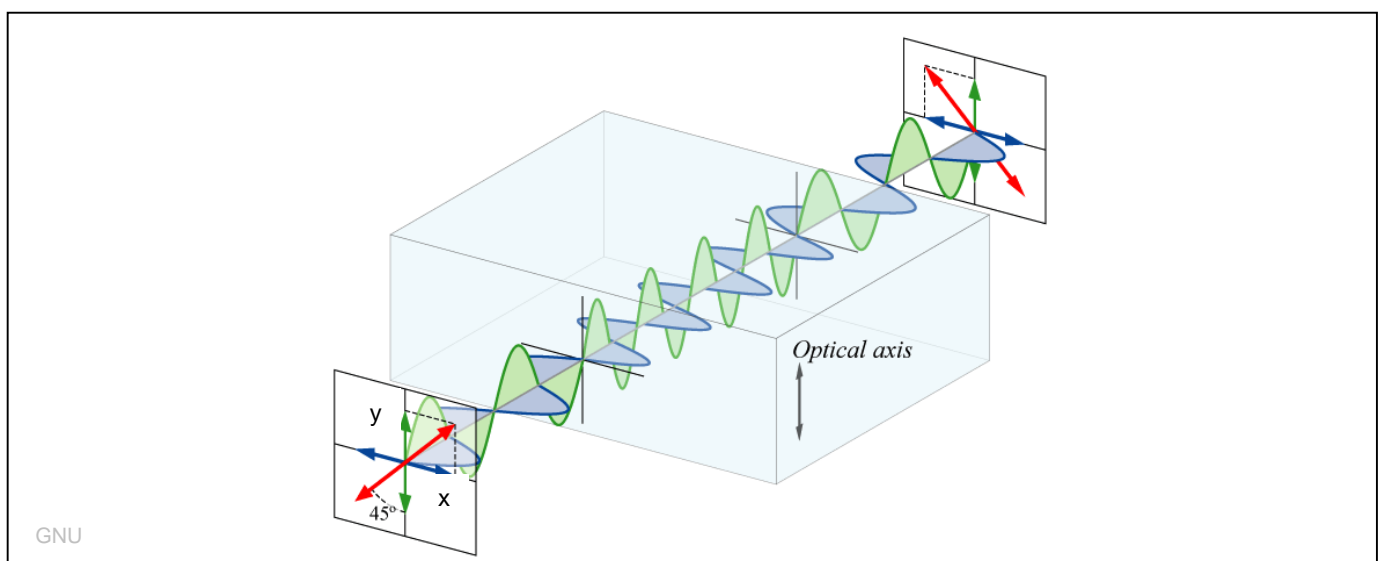
Apparatus

2 Quarter-wave plate	472 601
1 Half-wavelength plate	472 59
2 Polarization filter	472 401
1 Light filter yellow	468 30
1 Si Photocell STE 2/19	578 62
1 Holder for plug-in elements	460 21
1 Multimeter METRAHit Pro	531 282
1 Translucent screen	441 53
1 Optical bench, standard cross section 1 m	460 32
7 Optics rider 60/34	460 370
1 Halogen lamp housing 12 V, 50/90 W	450 64
1 Halogen lamp, 12 V / 90 W	450 63
1 Picture slider	450 66
1 Transformer 2 to 12 V, 120 W	521 25
2 Pair cables 100 cm, red/blue	501 46

Setup

The experimental setup is shown in Fig. 2 schematically.

Note: For the optical setup alternatively the small optical bench (460 43) or the optical bench S1 profile (460 310) can be used.

Notes on the beam path:

- The light supplied by the Halogen lamp (a) is concentrated by the condenser (b) and passes through a heat resistance filter to protect the optical components against heating up.

- Additionally, a heat protection filter filled with water might be used (in Fig. 2 indicated in dotted lines) to reduce the infrared radiation leading to a large background signal detected with the photo cell.

Optical adjustment:

- Set up the Halogen lamp (a) with the reflecting mirror and fit the condenser and picture slider in to the lamp housing.

- Insert the light filter yellow in front of the heat filter in the picture slider.

- Setup the polarizer, the λ/4-wave plate and the analyzer like shown in Fig. 2 on the optical bench. The distance between the polarizer and the halogen lamp is about 20 cm to 30 cm.

- Setup the Si-photo cell behind the analyzer and adjust the path of the light ray that the photo cell is well illuminated.

- By turning the lamp insert in the halogen lamp housing the illumination can be adjusted. Produce a sharp image of the lamp coil on a small sheet of paper positioned at the center of the Si photo cell (g).

Note: The translucent screen depicted in Fig. 2 is used to perform the experiment qualitatively.

Fig. 2: Experimental setup for investigating the type of polarization of the emergent light (schematically).

(a) halogen lamp	(d) polarizer	(g) Si photo cell
(b) picture slider with filters	(e) λ/4 or λ/2 wave plate	(h) translucent screen
(c) heat protection filter	(f) analyzer	
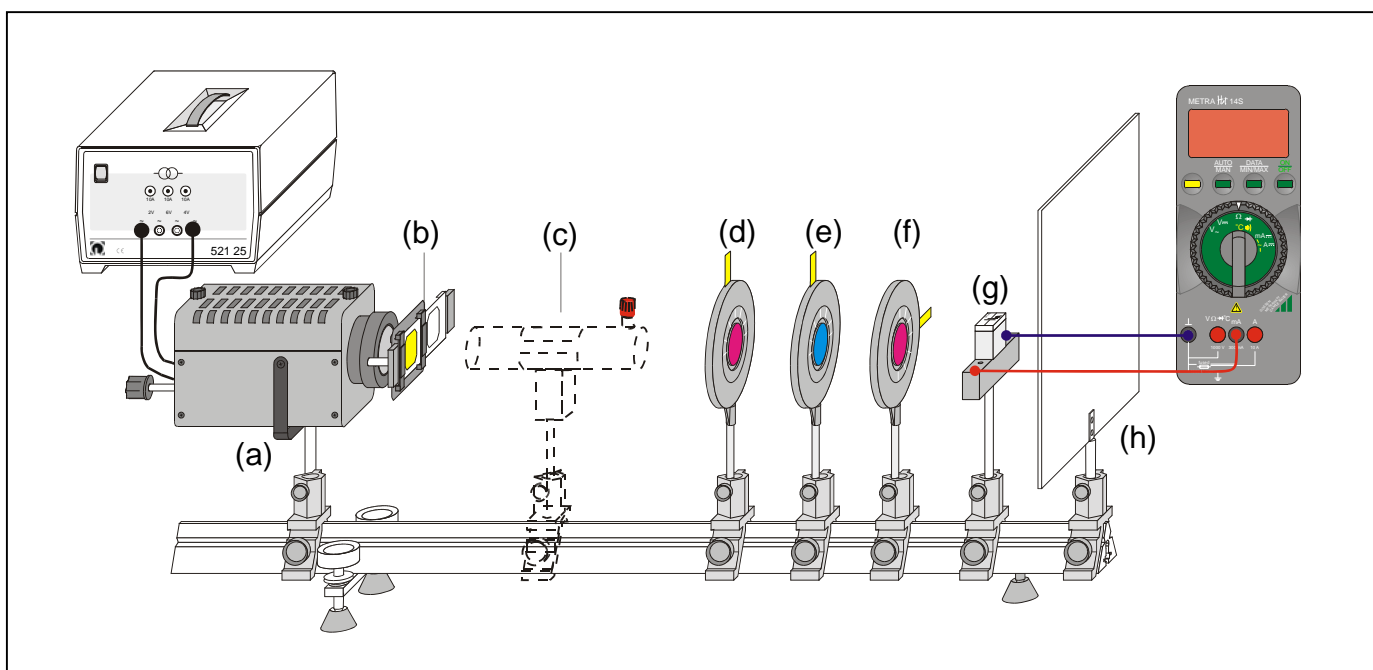
Safety notes

Care should taken that the various filters are not damaged by overheating.

■ Don't place the *polarization filter* directly in front of the light source. Use a heat protection filter to prevent damage of the diachronic plastic foil from overheating.

■ Don't *place quarter wave plate* or a *half wave plate* directly in front of a hot light source to prevent the double refracting foil from overheating.

- For measuring the photo current connect the Si-photo cell via the pair of cables red/blue to the multimeter.

Note: The photo current is proportional to the light intensity. The light intensity is proportional to the electric filed vector to the square: $I \sim E^2$

Carrying out the experiment

a) Quarter wave plate

- Remove the quarter wave plate and set the polarizer to the zero position.
- Measure the light intensity as function of the analyzer position over the range –90° to 90°.
- Clamp the quarter wave plate into the optic rider between the polarizer and analyzer.
- Measure the light intensity as function of the position of the analyzer (i.e. angles 0°, 30°, 45° and 60°) over the range –90° to 90°.

b) Half wave plate

- Set the polarizer to the zero position.
- Clamp the half wave plate into the optic rider between the polarizer and analyzer.
- Measure the light intensity as function of the position of the analyzer (i.e. angles 0°, 30°, 45°) over the range –90° to 90°.

Note: The half wave plate can be replaced also by two quarter wave plates with same orientation.

Measuring example

Table 1 and Table 2 summarizes the results. The inevitable background signal due to the infrared component have been baseline corrected.

a) Quarter wave plate

Table. 1: Measured current as a function of the analyzer position α for different quarter wave plate positions φ.
(Note: second column – measured without quarter wave plate).

position φ	–	0°	30°	45°	60°
$\frac{\alpha}{deg}$	$\frac{I}{\mu A}$	$\frac{I}{\mu A}$	$\frac{I}{\mu A}$	$\frac{I}{\mu A}$	$\frac{I}{\mu A}$
-90	0.2	0.0	29.0	39.4	31.2
-80	4.0	3.2	24.2	39.4	37.7
-70	11.9	11.2	20.8	39.7	44.2
-60	24.0	21.7	19.5	39.7	49.7
-50	37.1	35.2	20.7	39.7	54.4
-40	51.2	48.9	24.2	39.4	57.4
-30	64.5	61.2	29.2	39.2	58.0
-20	74.2	70.9	35.4	38.5	56.2
-10	80.5	76.7	41.7	38.2	52.0
0	83.2	78.2	47.8	37.7	46.4
10	81.4	75.7	53.2	37.2	39.7
20	74.0	68.5	56.2	37.2	33.3
30	62.5	57.4	57.2	37.2	27.1
40	49.2	43.9	56.2	37.7	22.4
50	34.2	33.2	52.7	38.2	19.6
60	21.2	18.4	47.4	38.2	19.2
70	9.7	8.4	41.2	38.4	20.9
80	3.0	1.7	34.7	39.0	25.1
90	0.7	0.0	28.7	39.2	30.5

b) Half wave plate

Table. 2: Measured current as a function of the analyzer position α for different half wave plate positions φ.

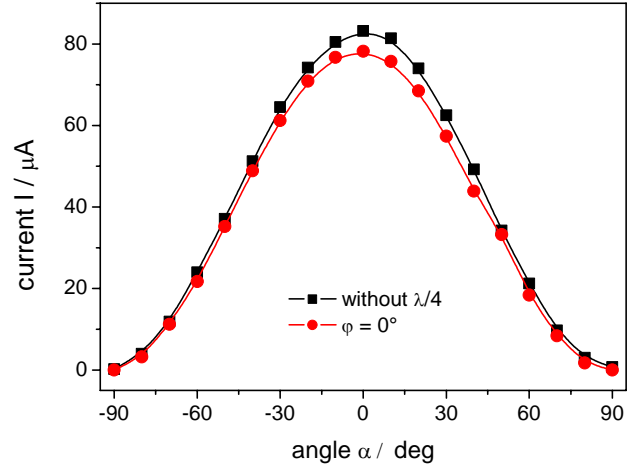| position φ	30°	45°	60°
$\dfrac{\alpha}{\text{deg}}$	$\dfrac{I}{\mu A}$	$\dfrac{I}{\mu A}$	$\dfrac{I}{\mu A}$
-90	0.0	71.9	56.1
-80	3.5	69.1	43.9
-70	11.7	63	31.7
-60	23.4	52.9	19.3
-50	35.6	42.1	9.5
-40	48.6	29.4	2.4
-30	60.3	17.7	0
-20	69.6	8.0	2.0
-10	74.5	1.9	8.4
0	75.0	0.0	18.8
10	72.0	2.8	30.8
20	64.6	9.9	43.2
30	53.5	19.4	55.3
40	40.8	31.4	64.4
50	28.6	43.9	70.9
60	16.5	54.9	72.8
70	6.8	63.8	70.6
80	1.3	69.7	64.6
90	0.2	71.3	54.8

Fig. 3:   Current I as a function of the analyzer position α: without quarter wave plate (black) and with quarter wave plate at position $\varphi = 0°$ (red). The solid lines are guides to the eyes only.
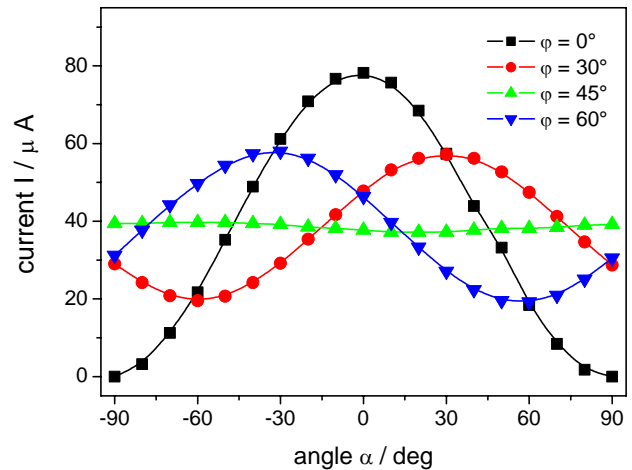

Fig. 4:   Current I as a function of the analyzer position α for various quarter wave plate positions φ. The solid lines are guides to the eyes only.

Evaluation and results

a) Quarter wave plate

E_0 is the amplitude of the electric field vector emerging from the polarizer and φ the angle between the polarizer and the quarter wave plate. At a time t the state of vibration of the two component rays is described by:

$E_1(t) = E_0(t) \cdot \sin\varphi \cdot \sin\omega \cdot t$

$E_2(t) = E_0(t) \cdot \cos\varphi \cdot \sin\omega \cdot t$

In the case of the double refracting quarter wave plates the thickness causes a path difference of $\lambda/4$ (i.e. a phase difference of $\pi/2$) between the two rays. When emerging the quarter wave plate they combine to a resultant ray which can be described by the parametric equations:

$E_1(t) = E_0 \cdot \sin\varphi \cdot \sin\omega \cdot t$

$E_2(t) = E_0 \cdot \cos\varphi \cdot \sin\omega \cdot t$

These equation describe an rotating E vector in the direction of propagation, i.e. perpendicular to the x- and y-axis about a fixed axis (Fig. 1).

For the angles $\varphi = 0°$ and $\varphi = 90°$ we obtain plane polarized light intensity after the quarter wave plate:

$I = I_0 \sim E_0^2$

This is in agreement with the experimental results shown in Fig. 3, i.e. $I \sim \cos^2 \alpha$.

For an angle $\varphi = 45°$ $\sin \varphi = \cos \varphi = \frac{1}{\sqrt{2}}$ and the amount of the rotating E vector is given by:

$$E = \sqrt{E_1^2 + E_2^2} = \frac{E_0}{\sqrt{2}}$$

The light is circularly polarized and the intensity is given by:

$$I = \frac{I_0}{2} \sim \frac{E_0^2}{2}$$

The light is transmitted without loss of intensity in all analyzer positions α. This is in agreement with the experimental results shown in Fig. 4.

At all other angles φ other that 0°, 45° and 90° the transmitted light is elliptically polarized. The tip of the E vector rotating about the axis parallel to the direction of propagation describes an ellipse with the semi axes a and b:

$$E_1(t) = E_a \cdot \sin\varphi \cdot \sin\omega \cdot t$$
$$E_2(t) = E_b \cdot \cos\varphi \cdot \sin\omega \cdot t$$

In agreement with the experimental results depicted in Fig. 4 the intensity for any angle φ between analyzer and quarter wave plate (here e.g. $\varphi = 30°$ and 60°) is given by:

$$I \sim E_0^2 \, \cos^2\varphi \, \cos^2\alpha + E_0^2 \, \sin^2\varphi \, \sin^2\alpha$$

Fig. 5: Current I as a function of the analyzer position α for various half wave plate positions φ. The solid lines are guides to the eyes only.

b) Half wave plate

The experimental result for the half wave plate (or two quarter wave plates with same orientation) is summarized in Fig. 5.

The half wave plate produces plane polarized light. For different positions φ of the half wave plate only the polarization plane changes. For example, if the position of the half wave plate is changed about 45° the polarization plane changes about 90°.

The maximum and minimum values are not changed. This is in contrast to the experimental results of the quarter wave plate.
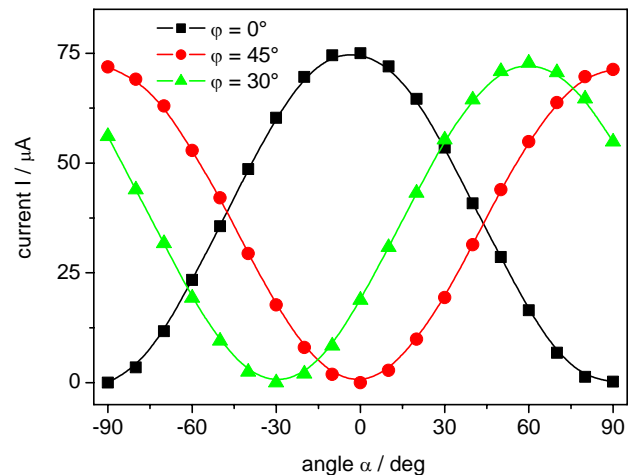
Supplementary information

Because of dispersion a wave plate will impart a phase difference that depends on the wavelength of the light. Wave plates are thus manufactured to work for a particular range of wavelengths. For the wave plates used here the phase difference produced is best for yellow light. Due to a moderate dispersion in the visible spectrum the deviations are slight.

Wave plates will give the intended effect only when the light penetrates perpendicularly. A sight convergence of the light ray does not affect the experimental results.

Further information about using the polarizer and quarter wave plates can be found in the instruction sheet 472 60.

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Related topics

Plane, circularly and elliptically polarised light, polariser, analyser, plane of polarisation, double refraction, optic axis, ordinary and extraordinary ray.

Principle

Monochromatic light falls on a mica plate perpendicular to its optic axis. At the appropriate plate thickness (λ/4, or quarter-wave plate) there is a 90° phase shift between the ordinary and the extraordinary ray when the light emerges from the crystal. The polarisation of the emergent light is investigated at different angles between the optic axis of the λ/4 plate and the direction of polarisation of the incident light.

Equipment

Photoelement f. opt. base plt.	08734.00	1
Lens holder	08012.00	3
Lens, mounted, f = +100 mm	08021.01	1
Diaphragm holder	08040.00	2
Iris diaphragm	08045.00	1
Double condenser, f = 60 mm	08137.00	1
Lamp, f. 50 W Hg high press. lamp	08144.00	1
Power supply for Hg CS/50 W lamp	13661.97	1
Interference filter, yellow, 578 nm	08461.01	1
Polarising filter, on stem	08610.00	2
Optical profile-bench, l = 1000 mm	08282.00	1
Base f. opt. profile-bench, adjust.	08284.00	2
Slide mount f. opt. pr.-bench, h = 30 mm	08286.01	8
Slide mount f. opt. pr.-bench, h = 80 mm	08286.02	1
Polarization specimen, mica	08664.00	2
Digital multimeter	07122.00	1
Universal measuring amplifier	13626.93	1
Condenser holder	08015.00	1
Connecting cord, l = 750 mm, red	07362.01	1
Connecting cord, l = 750 mm, blue	07362.04	1

Tasks

1. To measure the intensity of plane-polarised light as a function of the position of the analyser.

2. To measure the light intensity behind the analyser as a function of the angle between the optic axis of the λ/4 plate and that of the analyser.

3. To perform experiment 2. with two λ/4 plates one behind the other.

Set-up and procedure

The experiment is set up as shown in Fig. 1. The experiment lamp with the double condenser (focal length 60 mm) fitted, the lens holder with the iris diaphragm, the lens holder with the interference filter, the polariser, the holder with the λ/4 plate, the lens holder with the lens of focal length 100 mm, the analyser, and the distributor support with the silicon photo-cell are all set up on the optical bench.

First of all the path of the ray is adjusted so that the photo-cell is well illuminated (this is done without the λ/4 plate). With the polariser on zero, the analyser is then rotated until the light which it transmitted is of minimum intensity. The λ/4 plate is now clamped in the holder and rotated so that the light passing through the analyser is again at minimum intensity. The plane of polarisation of the light emerging from the polariser now makes an angle of 0° (or 90°) with the optic axis of the λ/4 plate. The light intensity is measured as a function of the position of the analyser, for angles of 0, 30, 45, 60 and 90°, over the range −90° to +90°. The resistor is plugged in parallel to the entry of the measuring amplifier.

The current intensity of the photo-cell is proportional to the intensity of the incident light.

Fig. 1: Experimental set-up for determining the type of polarisation of the emergent light.

Fig. 2: Splitting of polarised light in a double-refracting crystal (P = polariser, A = analyser).
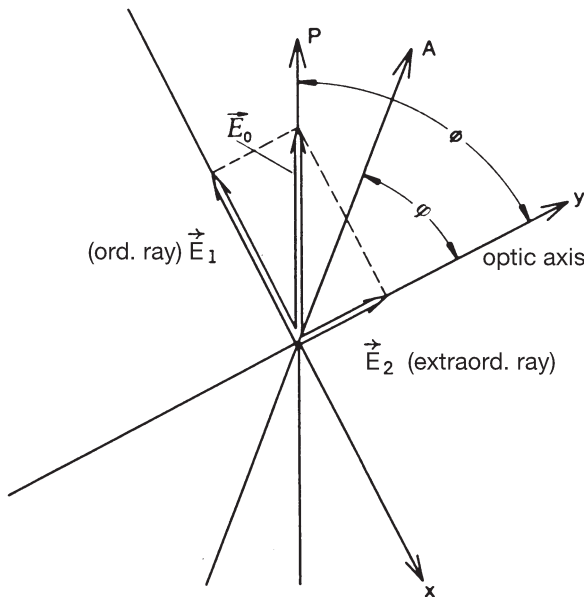
Theory and evaluation

The velocity of the light travelling in the direction of the optic axis of a double-refracting crystal has the same value, c_o, whatever the direction of its plane of polarisation. When travelling at right angles to the optic axis, polarised light has the same velocity c_o, when the electric vector is perpendicular to the optic axis (ordinary ray, see Fig. 2). If the electric vector is parallel to the optic axis the light velocity $c \neq c_o$ (extraordinary ray).

E_0 is the amplitude of an electric field vector emerging from the polariser and ϕ the angle between the direction of polarisation P and the optic axis of a double-refracting crystal.

From Fig. 2 we derive the following for the amplitudes of the ordinary and of the extraordinary ray:

$$E_1\,(t) = E_0\,(t) \cdot \sin \phi$$
$$E_2\,(t) = E_0\,(t) \cdot \cos \phi \tag{1}$$

At time t, the state of vibration in the two rays at the crystal surface is described by:

$$E_1\,(t) = E_0\,(t) \cdot \sin \phi \cdot \sin \omega t$$
$$E_2\,(t) = E_0\,(t) \cdot \cos \phi \cdot \sin \omega t \tag{2}$$

In the case of double-refracting crystals ($\lambda/4$ plates), the thickness

$$d_{\lambda 4} = \frac{\lambda}{4} \cdot \frac{1}{n_o - n_{ao}}, \tag{3}$$

where n_o is the refractive index of the ordinary ray and n_{ao} that of the extraordinary ray in the crystal, causes a path difference of $\lambda/4$ (i.e. a phase difference of $\pi/2$) between the two rays

when they combine to a resultant ray on emerging from the crystal. From (2) we obtain

$$E_x = E_1 = E_0 \cdot \sin \phi \cdot \sin \omega t$$
$$E_y = E_2 = E_0 \cdot \cos \phi \cdot \cos \omega t \tag{4}$$

(4) is the parametric representation of an E vector rotating in the direction of propagation, i.e. perpendicular to the x and y axis, about a fixed axis.

For angles of $\phi = 0°$ and $\phi = 90°$ we obtain <u>plane</u> polarised light of intensity

$$I = I_0 \sim E_0^2 \;. \tag{5}$$

For an angle of 45°, $\sin \phi = \cos \phi = \dfrac{1}{\sqrt{2}}$, and the amount of the rotating E vector is

$$E = \sqrt{E_x^2 + E_y^2} = \frac{E_0}{\sqrt{2}} \tag{6}$$

The light is <u>circularly</u> polarised and of intensity

$$I = \frac{I_0}{2} \sim \frac{E_0^2}{2} \tag{7}$$

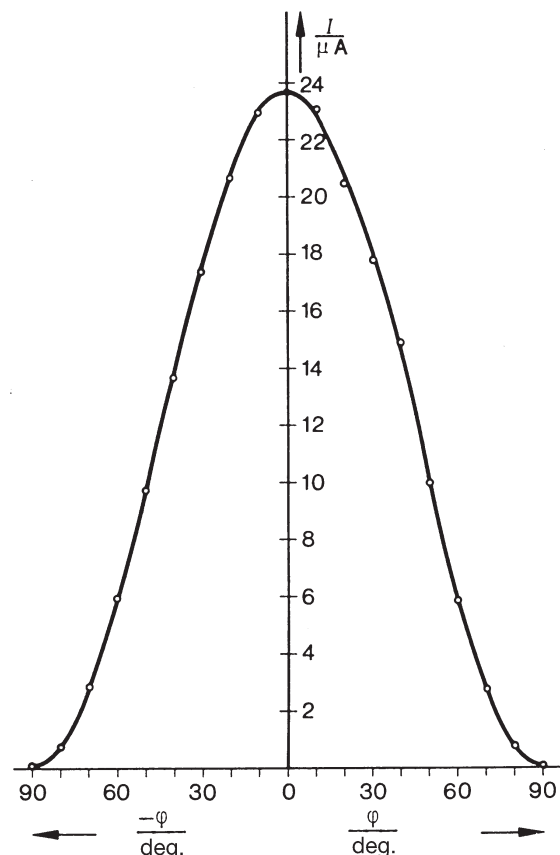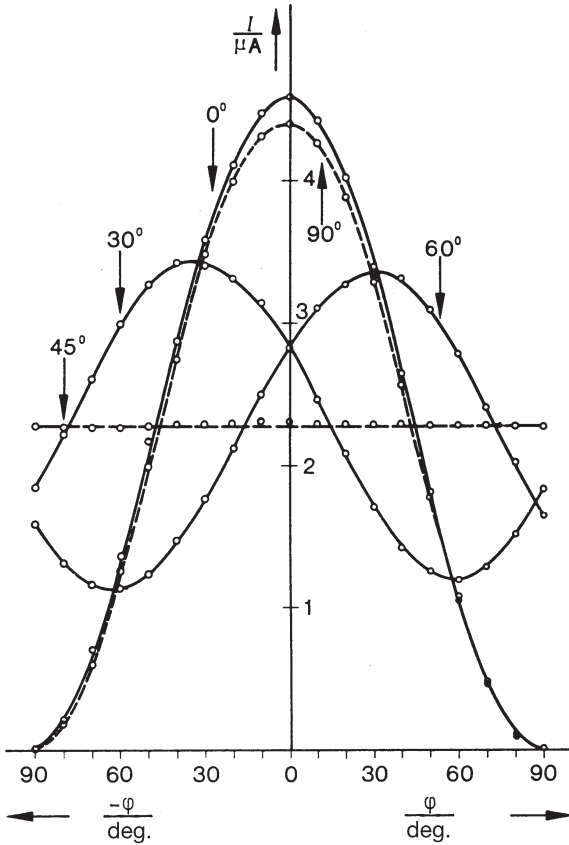and is transmitted without loss of intensity in all analyser positions.

Fig. 3: Intensity distribution of plane-polarised light as a function of the position of the analyser (without $\lambda/4$ plate).

Fig. 4: Intensity distribution of polarised light as a function of the direction of transmission of the analyser: with $\lambda/4$ plate at various angular settings.

Fig. 5: Intensity distribution of polarised light: with $\lambda/2$ plate at various angular settings.
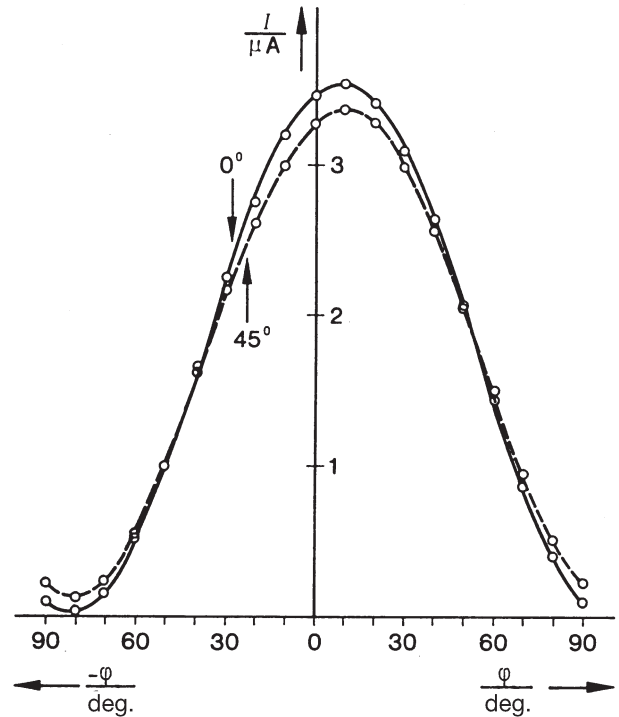
First of all the intensity distribution of plane-polarised light is measured as a function of the analyser position, without the $\lambda/4$ plate in the path of the rays (Fig. 3).

The type of polarisation of the transmitted light is determined from the corresponding intensity distribution values, for various angles between the optic axis of the $\lambda/4$ plate and the direction of transmission of the analyser (Fig. 4).

If two $\lambda/4$ plates are set one behind the other, plane-polarised light is produced whatever the direction of the optic axis of the $\lambda/2$ plate so created (Fig. 5).

At all angles ϕ other than 0°, 45° and 90°, the transmitted light is <u>elliptically polarised</u>. The tip of the E vector rotating about the axis parallel to the direction of propagation describes an ellipse with the semi-axes.

$$E_a = E_0 \sin\phi \; (x\text{-direction})$$
$$E_b = E_0 \cos\phi \; (y\text{-direction}) \tag{8}$$

For the intensity of the light transmitted by the analyser in the respective directions, we have:

$$I_a \sim E_a^2 = E_0^2 \; \sin^2\phi \tag{9}$$
$$I_b \sim E_b^2 = E_0^2 \; \cos^2\phi$$

By rotating the analyser we obtain the following for the ratio of the maximum to the minimum transmitted light intensity:

$$\frac{I_a}{I_b} = \frac{E_a^2}{E_b^2} = \frac{\sin^2\phi}{\cos^2\phi} = \tan^2\phi \tag{10}$$

For any angular setting φ between the analyser and the optic axis of the $\lambda/4$ plate, we have:

$$I \sim E_0^2 \cdot \cos^2\phi \cdot \cos^2\varphi + E_0^2 \cdot \sin^2\phi \cdot \sin^2\varphi \tag{11}$$