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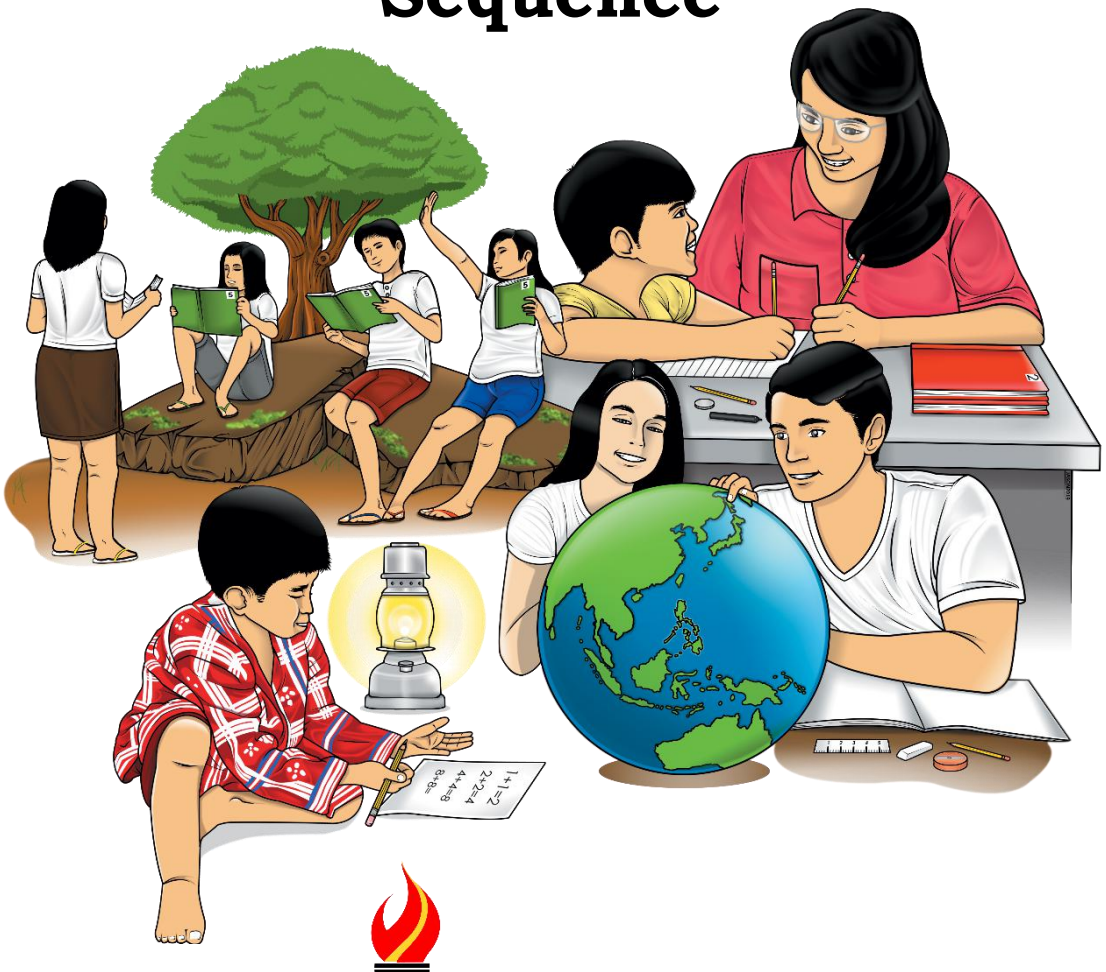
Mathematics

Quarter I - Module 2: Illustrating an Arithmetic Sequence



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Quarter I – Module 2: Arithmetic Sequence
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Mathematics

Quarter 1 - Module 2

Illustrating an Arithmetic Sequence.

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Introductory Message

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

For the facilitator:

This module deals with the second learning competency in our Mathematics 10 curriculum standards; hence mastery of the skills is significant to have a smooth progress in the succeeding lessons. This learning material also designed to equip the students with essential knowledge about defining and illustrating arithmetic sequences. With this, please be patient and encourage the learner to complete this module.

For the learner:

This module deals with the second learning competency in our Mathematics 10 curriculum standards; hence mastery of the skills is significant for you to have a smooth progress in the succeeding lessons. This learning material serves as a bridge from the previous lesson to the next lesson. By doing the prepared activities, it is expected from you to define and illustrate arithmetic sequence. Please read completely the written texts and follow the instructions carefully so that you will be able to get the most of this learning material. We hope that you will enjoy learning.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	What I need to know	This will give you an idea of the skills or competencies you are expected to learn in the module.
	What I know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this
	What's In	This is a brief drill or review to help you link the current lesson with the previous one.

	What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
	What Is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	What I have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
	Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

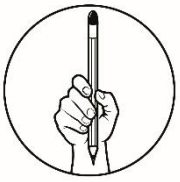
We hope that though this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you define and illustrate an arithmetic sequence. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but the pacing in which you read and answer this module is dependent on your ability.

After going through this module, you are expected to:

1. define an arithmetic sequence, and
2. illustrate an arithmetic sequence.



What I Know

Find out how much you already know about the topics in this module. Choose the letter of the best answer. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module. Write the letter of your choice on a separate sheet of paper.

1. Which of the following is a correct description of an arithmetic sequence?

   a. A term is obtained by multiplying a constant number from the preceding term.
   b. A term is obtained by squaring the preceding term.
   c. A term is obtained by adding a constant number to the preceding term.
   d. A term is obtained by extracting the square root of the preceding term.

2. Which of the following is the common difference of the arithmetic sequence 4, 7, 10, 13, …?

   a. 0 b. 1 c. 2 d. 3

3. What is the common difference in the sequence: -4, 3, 10, 17,, …?

   a. 1 b. -7 c. 7 d. -1

4. Which of the following is the missing term of the arithmetic sequence , -1, -7, _____, -19, -25, ...?
   a. -10 b. -11 c. -12 d. -13

5. Which of the following of the common difference of the arithmetic sequence -1, -8, -15, -22, ...?
   a. -5, b. -6 c. -7 d. -8

6. Which of the following is the common difference in the arithmetic sequence: $\frac{1}{2}$, 1, 1 $\frac{1}{2}$, 2, ...?
   a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\frac{5}{2}$ d. $\frac{1}{2}$

7. Which of the following is an arithmetic sequence?
   a. -7, 4, 15,26,. . . c. 21, 15, 9, 2, . . .
   b. -8, -6, -4, 0, ... d. 4, -1, -7, -13, . . .

8. Which of the following is the common difference of an arithmetic sequence whose a_2 = 1, a_5 = 7?
   a. 1 b. 2 c. 3 d. 4

9. Which of the following is the common difference of the arithmetic sequence 3a -1, 3a, 3a + 1, ... ?
   a. -a b. a c. -1 d. 1

10. Which of the following is the first positive term of the arithmetic sequence -8, -6, -4, ...?
    a. 0 b. 2 c. 4 d. 6

11. Which of the following is the value of **a** to make 3**a** +1, 4**a**, 6**a** + 1, ... an arithmetic sequence?
    a. -2 b. -1 c. 0 d. 1

12. Which of the following is not an arithmetic sequence?
    a. 2, 4, 6, 8, ... c. 3, 6, 12, 24,...
    b. 4.1, 11.1, 18.1,... d. 5, 10, 15, 20, ...

13. Which of the following is not an arithmetic sequence?
    a. 4, 7, 10, 13, ... c. 100, 98, 96, 94, ...
    b. -2, -6, -10, -14, ... d. -4,-3, 10, 17, ...

14. Which of the following is not true about the arithmetic sequence 5, 3, 1, -1, -3, ...
    a. a_2 = -3
    b. the common difference is -2
    c. a_6 = -5
    d. to obtain a term, -2 must be added to the preceding term.

15. I was advised by my physician to walk each day in the morning as my daily exercise. On my first day, I walked 40m. On the second and third day, I walked 60m and 80m, respectively, and so on. Which of the following is the distance I walked on the 10^{th} day if I continue the pattern in my daily walk?

a. 180 m b. 200 m c. 220 m d. 240 m

Lesson 1	Illustrates an arithmetic sequence.

What's In

In the previous module, you have learned that a sequence is an arrangement of objects, numbers or even figures which follows a certain pattern. Also, you have learned about the processes in finding patterns of any sequence.

Look at the sequences below. Can you see the specific pattern they follow?

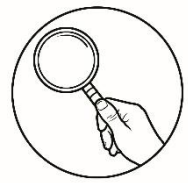
2, 4, 6, 8, …	3, 6, 12, 24, …	5, 10, 15, 20, …

Let us try to give emphasis on the differences you observed while we undergo this module.

What's New

Look at the following sequences. What is the pattern you observed in each sequence?

Sequences	Pattern
1. 2, 4, 6, 8, ...	by adding 2 to the preceding term.
2. 3, 6, 12, 24, ...	by multiplying 2 to the preceding term.
3. 5, 10, 15, 20, ...	by adding 5 to the preceding term.



What is It

Observe the following sequences:

1. 4, 7, 10, 13, ...
2. 33, 38, 43, 48, ...
3. -2, -6, -10, -14, ...
4. 100, 98, 96, 94, ...
5. $\frac{1}{2}$, 1, 1 $\frac{1}{2}$, 2, ...

Can you give the next two terms of the above sequences? How did you get those terms?

If you get 16 and 19 in item 1, then you are correct. Notice that a constant number 3 is added to the preceding term to get the next term. In item 2, 5 is added to the preceding term to get the next term. While in items 3, 4 and 5, the numbers -4, -2, and $\frac{1}{2}$ are added to the preceding term, respectively, to get the next terms.

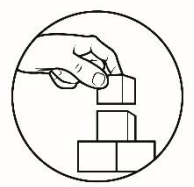
Notice that a constant or a common number is **added** to the preceding term or the number before it, to get the next term in each of the sequences above. The constant number being added is called the common difference and is represented as *d.* All these sequences are called ***arithmetic sequences.***

To find the common difference (*d),* you can simply subtract

- the second term (a_2) by the first term (a_1),
  a_2 - a_1, or
- the third term (a_3)by the second term (a_2),
  a_3 - a_2 , or
- the fourth term (a_4)by the third term (a_3),
  a_4- a_3, or
- in general, a term (a_n) by its preceding term (a_{n-1})
  $d = a_n - a_{n-1}$.

Arithmetic Sequences

A sequence in which term after the first is formed by adding a fixed number to the preceding term is called ***arithmetic sequence***. The fixed number or constant is called the common difference denoted by ***d.***



What's More

Activity 1:

From the discussion on arithmetic sequence earlier, solve the problems that follows.

1. Determine if the sequence is arithmetic or not. If it is, find the common difference and the next three terms of the sequence.

   -4, 3, 10, 17, ...

   Solution:

   a. To find out if the sequence is arithmetic, there must be a common difference between any two consecutive terms in the sequence.

      a_2 - a_1 = 3 – (-4) = 7

      a_3 - a_2 = 10 – (3) = 7

      a_4 - a_3 = 17 – 10 = 7

   Because there is a common difference between consecutive terms, the sequence is arithmetic.

b. The next three terms are obtained by adding 7 to the preceding term, so that

$a_5 = a_4 + 7 = 17 + 7 = 24$

$a_6 = a_5 + 7 = 24 + 7 = 31$

$a_7 = a_6 + 7 = 31 + 7 = 38$

So the next three terms are 24, 31, 38.

2. Write the first five terms of the arithmetic sequence with 5 as the first term and with a common difference of -2.

Solution:

First term:	$a_1 = 5$
Second term:	$a_2 = a_1 + (-2) = 5 - 2 = 3$
Third term:	$a_3 = a_2 + (-2) = 3 - 2 = 1$
Fourth term:	$a_4 = a_3 + (-2) = 1 - 2 = -1$
Fifth term:	$a_5 = a_4 + (-2) = -1 - 2 = -3$

As you notice, we add the common difference or the constant to the preceding term to get the next term. So, the first five terms of the sequence are 5, 3, 1, -1, and -3.

Remark: There is another way of finding the specified term of an arithmetic sequence but it will be discussed in the next module. The same thing is true for the general term of any arithmetic sequence.

3. Find the common difference an arithmetic sequence whose

$a_2 = 1$, $a_5 = 7$.

Solution:

a. Remember that to find a term we add the common difference d, to the preceding term, for instance,

$a_3 = a_2 + d$ Equation 1

$a_4 = a_3 + d$ Equation 2

$a_5 = a_4 + d$ Equation 3.

b. Substitute a_3 in Equation 1 to Equation 2

$$a_{4=} a_3 + d$$

$$a_{4=} (a_2 + d) + d$$

$$a_{4=} a_2 + 2d \qquad \text{Equation 4}$$

c. Substitute a_4 in Equation 4 to Equation 3

$$a_5 = a_4 + d$$

$$a_5 = (a_2 + 2d) + d$$

$$a_5 = a_2 + 3d \qquad \text{Equation 5}$$

d. Substitute the given in Equation 5

$$a_5 = a_2 + 3d$$

$$7 = 1 + 3d \qquad \text{Equation 6}$$

e. Solve for d in Equation 6.

$$7 = 1 + 3d$$

$$6 = 3d$$

$$2= d$$

So, the common difference, d is 2.

4. Find the common difference of the arithmetic sequence

3a -1, 3a, 3a + 1, …

Solution:

a. We must remember that in an arithmetic sequence, the common difference d is a term minus the preceding term. Thus

$d = a_2 - a_1$ Equation 1 or

$d = a_3 - a_2$ Equation 2

b. If we use Equation 1 and substitute the given, we have

d = 3a – (3a -1)

c. If we simplify, we obtain

d = 1.

So, the common difference is 1.

5. Find the value of **a** to make 3**a** +1, 4**a**, 6**a** + 1, … an arithmetic sequence?

Solution:

a. We must again remember that in an arithmetic sequence, the common difference d is a term minus the preceding term. Thus

$$d = a_2 - a_1 \text{ Equation 1 and}$$

$$d = a_3 - a_2 \text{ Equation 2}$$

b. If we substitution the given in Equation 1 and Equation 2,

$$\boldsymbol{d} = 4\mathbf{a} - (3\mathbf{a} + 1) \quad \text{Equation 3}$$

$$d = (6\mathbf{a} + 1) - 4\mathbf{a} \quad \text{Equation 4}$$

c. Equate Equation 3 and Equation 4

$$4\mathbf{a} - (3\mathbf{a} + 1) = (6\mathbf{a} + 1) - 4\mathbf{a}$$

d. Combine similar terms

$$\mathbf{a} - 1 = 2\mathbf{a} + 1$$

e. Further simplify

$$-2 = \mathbf{a}$$

Thus, the value of **a** is -2.

6. I was advised by my physician to walk each day in the morning as my daily exercise. On my first day, I walked 40m. On the second and third day, I walked 60m and 80m, respectively, and so on. Which of the following is the distance I walked on the 10^{th} day if I continue the pattern in my daily walk?

Solution:

a. If we analyze the problem, 40m, 60m, 80m, … follows an arithmetic sequence since there is a common difference which is equivalent to 20. Dropping the unit, the given could be written as follows

$$40 , 60, 80, \ldots$$

b. Continuing this pattern until the 10^{th} term, we have

$$40 , 60, 80, 100, 120, 140, 160, 180, 200, 220, \ldots$$

Thus, in the 10^{th} day, the distance travelled is 220 m.

Assessment 1:

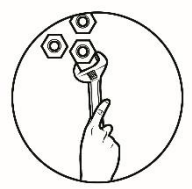
Find the common difference and the next three terms of each arithmetic sequence. Write your answer on a separate sheet of paper.

Common Difference	***Next 3 Terms***
_____	1. 24, 14, 4, ___, ___, ___
_____	2. 6, 10, 14, ___, ___, ___
_____	3. -7, 4, 15, ___, ___, ____
_____	4. 21, 15, 9, ___, ___, ___
_____	5. -8, -6, -4, ___, ___, ___
_____	6. 5, -1, -7, ___, ___, ___
_____	7. 4.1, 11.1, 18.1, ___, ___, ___
_____	8. -1, -8, -15, ___, ___, ___
_____	9. -3x, -10x, -17x ___, ___, ___
_____	10. 3a -1, 3a, 3a + 1, _____, _____, _____

What I Have Learned

Answer this question.

a. What is an arithmetic sequence?
b. How do we get the next terms of an arithmetic sequence?



What I Can Do

Activity 1: There is Math Around Us

Arithmetic sequence can be observed around us. Like the following fare rate for first 4 kms of a modernized PUJ under General Community Quarantine released by LTFRB last April 24, 2020.

Distance	Fare
First kilometer	11.00
Second kilometer	12.50
Third kilometer	14.00
Fourth kilometer	15.50

If we compute the increase of fare for every increase of kilometer distance, they are all equivalent to 1.50. With this, the fare rate is an example of an arithmetic sequence.

Aside from examples involving money, identify three situations or three things that you see or observe in your surroundings that illustrate an arithmetic sequence.

Assessment

Choose the letter that you think best answers the question. Write your answer in a separate paper.

1. Which of the following describes an arithmetic sequence?
   a. A sequence in which a term is formed by adding any number to the preceding term.
   b. A sequence in which there is an equal difference between consecutive terms.
   c. A sequence in which a term minus the preceding term is always positive.
   d. A sequence in which terms follows a pattern.

2. Which of the following is the common difference in the sequence:
   0, 4, 8, 12, ...?

   a. 1 b. 2 c. 3 d.4

3. Which of the following is the common difference in the sequence: 3, -2, -7, ...?

   a.1 b. -5 c. 5 d. -1

4. Which of the missing term in this arithmetic sequence: 23, 18, 13, 8, 3, _____, -7, -12,... ?

   a.-2 b. 2 c. -5 d. 5

5. Which of the following is the common difference in the sequence: -7, -4, -1, 2, 5,...?

   a.-3 b. 3 c. 4 d. -4

6. Which of the following is the common difference in the arithmetic sequence: 3, $\frac{13}{4}$, $\frac{7}{2}$, $\frac{15}{4}$, ... ?

   a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\frac{5}{2}$ d. 4

7. Which of the following is an arithmetic sequence?

a.1, 2, 3, 5, 7, 9...
b.1, 10, 20, 30...
c. 1, -1, -3, -5...
d. 7, -7, 7, -7 ...

8. Which of the following is the common difference of an arithmetic sequence if $a_3 = 4$ and $a_5 = 14$?

a.6 b. 5 c. 4 d. 3

9. Which of the following is the common difference of the arithmetic sequence 7p + 2, 5p +12, 3p + 22, ...?

a.2p b. -2p +10 c. 2p - 10 d. 6p

10. Which of the following is the first positive term of the arithmetic sequence: -11, -8, -5, ...?

a. -4 b. 3 c. -2 d. 1

11. Which of the following is the value of p so that the terms, 7p + 2, 5p +12, 2p – 1,... form an arithmetic sequence?

a. -8 b. -5 c. -13 d. -23

12. Which of the following is NOT an arithmetic sequence?

a. -5, -2, 1, 4
b. 11, 14, 17, 20
c. 1, 4, 7
d. 3, 7, 12, 18

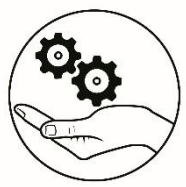
13. Which of the following is not an arithmetic sequence?

a. 1, 2, 3, 4, 5, ...
b. 3, 9, 27, 81, ...
c. 4.5, 5.0, 5.5, 6.0, ...
d. 13, 2, -9, -20, -31, ...

14. Which is NOT true about the arithmetic sequence: 25, 32, 39, 46, ...?
a. The common difference is 7.
b. The 7^{th} term is 60.
c. the 6^{th} term is 60.
d. the 8^{th} term is 74.

15. During a free-fall, a skydiver jumps 16 feet, 48 feet, and 60 feet on the first, second, and third fall, respectively. If he continues to jump at this pattern, how many feet will he have jumped during the tenth fall?

a.144 b. 156 c. 132 d. 140



Additional Activity

Determine whether the following situations illustrate an arithmetic sequence. If yes, then give what is asked.

1. I was advised by my physician to walk each day in the morning as my daily exercise. On my first day, I walked 40m. On the second and third day, I walked 60m and 80m, respectively, and so on. What is the distance I walked on the 10^{th} day if I continue the pattern in my daily walk?

2. The number of works done by a backhoe in a certain area doubles every 2 hours. If there are N number of works to start with, find the number of works done in 14 hours.

Answer Key

What's More

1.d= -10 -6,-16, -26

2.d= 4 18, 22, 26

3.d= 11 26, 37, 48

4.d=-6 3, -3, -9

5.d=2 -2, 0, 2

6.d=-6 -13, -19, -25

7.d= 7 25, 32, 39

8.d= -7 -22, -29, -36

9.d= -7x -24x,-31x,-38x

10.d= 1 3a+2, 3a+3

3a+4

What I Know

1. C
2. D
3. C
4. D
5. C
6. D
7. A
8. B
9. D
10. B
11. A
12. C
13. D
14. A
15. D

What I Have Learned

1.Arithmetic sequence - A sequence in which term after the first is formed by adding a fixed number to the preceding term.

2. By adding a fixed number or constant from the preceding term.

Assessment:

1. A
2. D
3. B
4. A
5. B
6. A
7. C
8. B
9. B
10. D
11. D
12. D
13. B
14. C
15. A
16.

Additional Activity

1. YES-
   In my 10th day of
   walking, I would have
   walked 220m.
2. No

What I Can Do

Depends on student's responses.

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