Query Evaluation
-- External Sorting

References:

- ❑ [RG-3ed] Chapter 13
- ❑ [SKS-6ed] Chapter 12.4

Outline

- Why sorting?
- External sorting
  - 2-way sort
  - M-way sort
- Other considerations
  - Blocked I/O
  - Double buffering
- Sorting using index
  - Clustered
  - Non-clustered

Why Sort?

- A classic problem in computer science!
- Data requested in sorted order
  - e.g., find students in increasing *GPA* order
- Sorting is the first step in bulk loading B+ tree index.
- Sorting is useful for eliminating duplicate copies in a collection of records (Why?)
- Sort-merge join algorithm involves sorting.
- Problem: sort 1Gb of data with 1Mb of RAM.

Sorting

- We may build an index on the relation, and then use the index to read the relation in sorted order.
  - May lead to one disk block access for each tuple.
- Sorting
  - For relations that fit in memory, techniques like quicksort can be used.
  - For relations that do not fit in memory, external sort-merge is a good choice.

Sorting in Commercial RDBMs

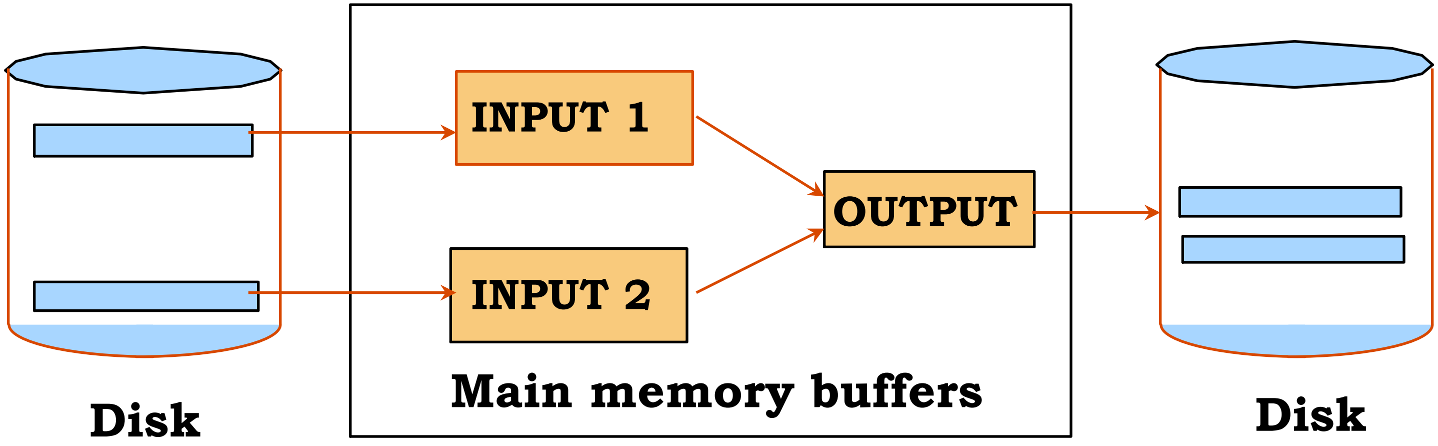
- External merge sort
  - DB2, Informix, SQL Server, Oracle 8, Sybase ASE
  - None of these systems uses the optimization that produces runs larger than available memory
  - I/O is asynchronous and prefetching
- In-memory
  - Miscrost, Sybase ASE: merge sort
  - DB2 and Sybase IQ: radix sorting
  - Oracle: insertion sort

Concepts

- Run
  - When sorting a file, several sub-files are typically generated in intermediate steps. Each sorted sub-file is referred to as a run.
- Pass
- Available buffer main memory: M
- Number of pages in the file: b_r

2-Way Sort: Requires 3 Buffers

- Pass 1: Read a page, sort it, write it.
  - only one buffer page is used
- Pass 2, 3, …, etc.:
  - three buffer pages used.



2-Way External Merge Sort

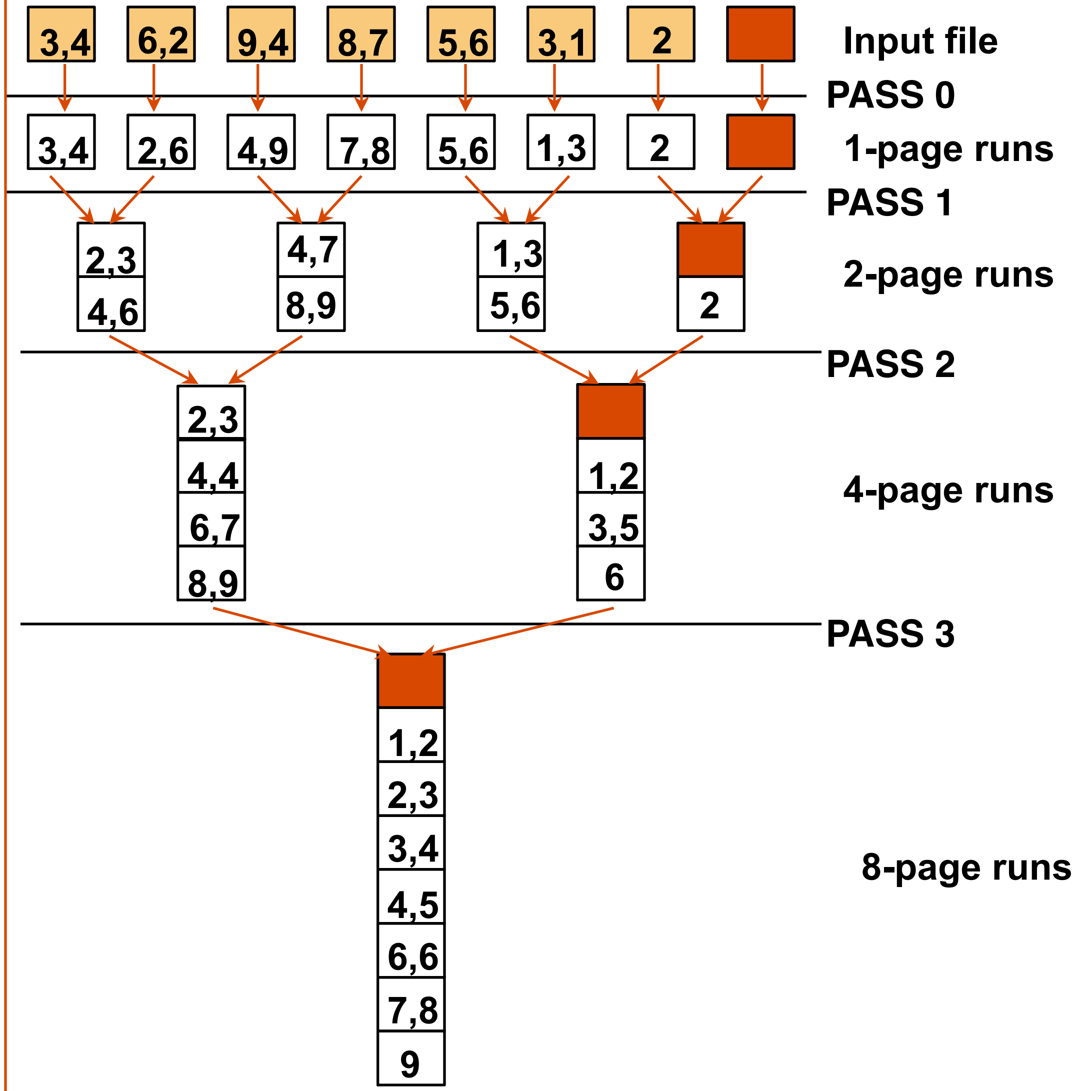
- Each pass we read + write each page in file.
- Let b_r pages in the file
  - the number of passes

$$= \lceil \log_2 b_r \rceil + 1$$

- So toal cost is:

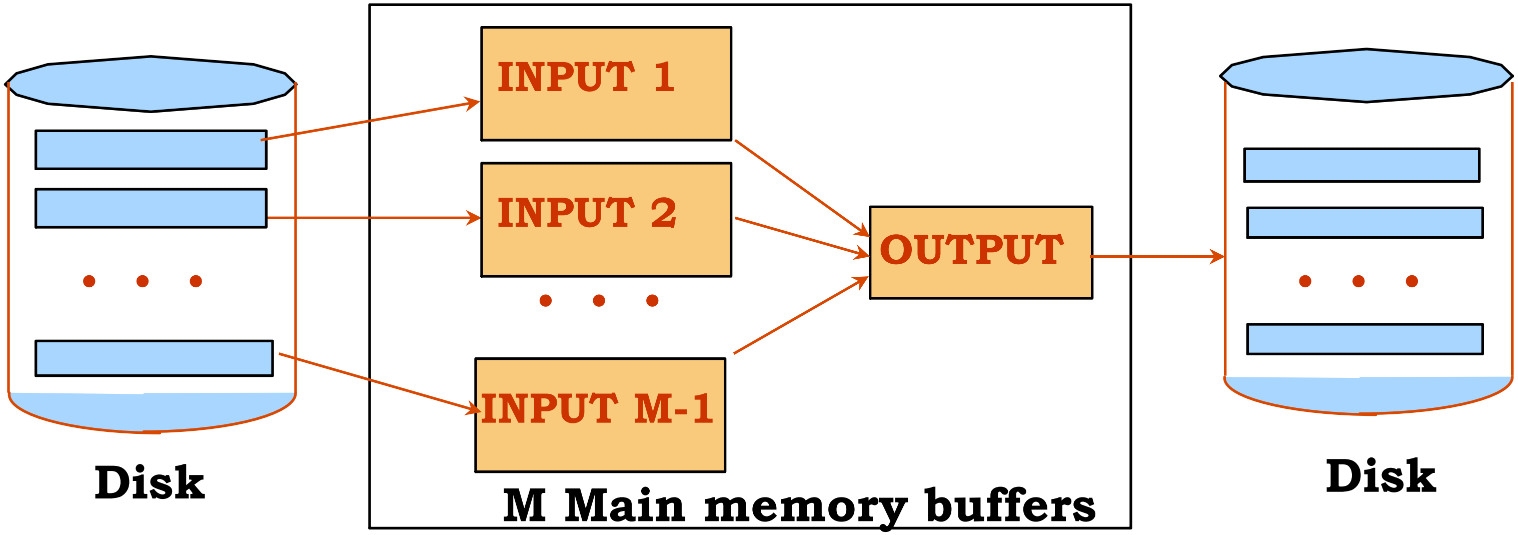
$$2b_r(\lceil \log_2 b_r \rceil + 1)$$

- Idea: **Divide and conquer:** sort sub-files and merge



General External Merge Sort

- More than 3 buffer pages. How can we utilize them?
- To sort a file with b_r pages using *M* buffer pages:
  - Pass 0: use M buffer pages. Produce $\lceil b_r / M \rceil$ sorted runs of M pages each.
  - Pass 2, …, etc.: merge M-1 runs.



External Sort-Merge

b_r: total number of pages in a file

M*:* memory buffer size (in pages).

1. **Create sorted runs.**

   Let i be 0 initially.

   Repeatedly do the following till to the end of the relation:

   (a) Read *M* blocks of relation into memory
   (b) Sort the in-memory blocks
   (c) Write sorted data to run file R_i
   (d) Increment *i*

   total number of runs N1 = $\lceil b_r/M \rceil$

2. **Merge the runs** *(next slide)…..*

External Sort-Merge (Cont.)

2. **Merge the runs ($\lceil b_r/M \rceil$ -way merge).**

   We assume (for now) that $\lceil b_r/M \rceil < M$.

   1. Use $\lceil b_r/M \rceil$ blocks of memory to buffer input runs, and 1 block to buffer output. Read the first block of each run into its buffer page
   2. **repeat**
      1. Select the first record (in sort order) among all buffer pages
      2. Write the record to the output buffer. If the output buffer is full, write it to disk.
      3. Delete the record from its input buffer page.
         If the buffer page becomes empty **then**
            read the next block (if any) of the run into the buffer.
   3. **until** all input buffer pages are empty:

External Sort-Merge (Cont.)

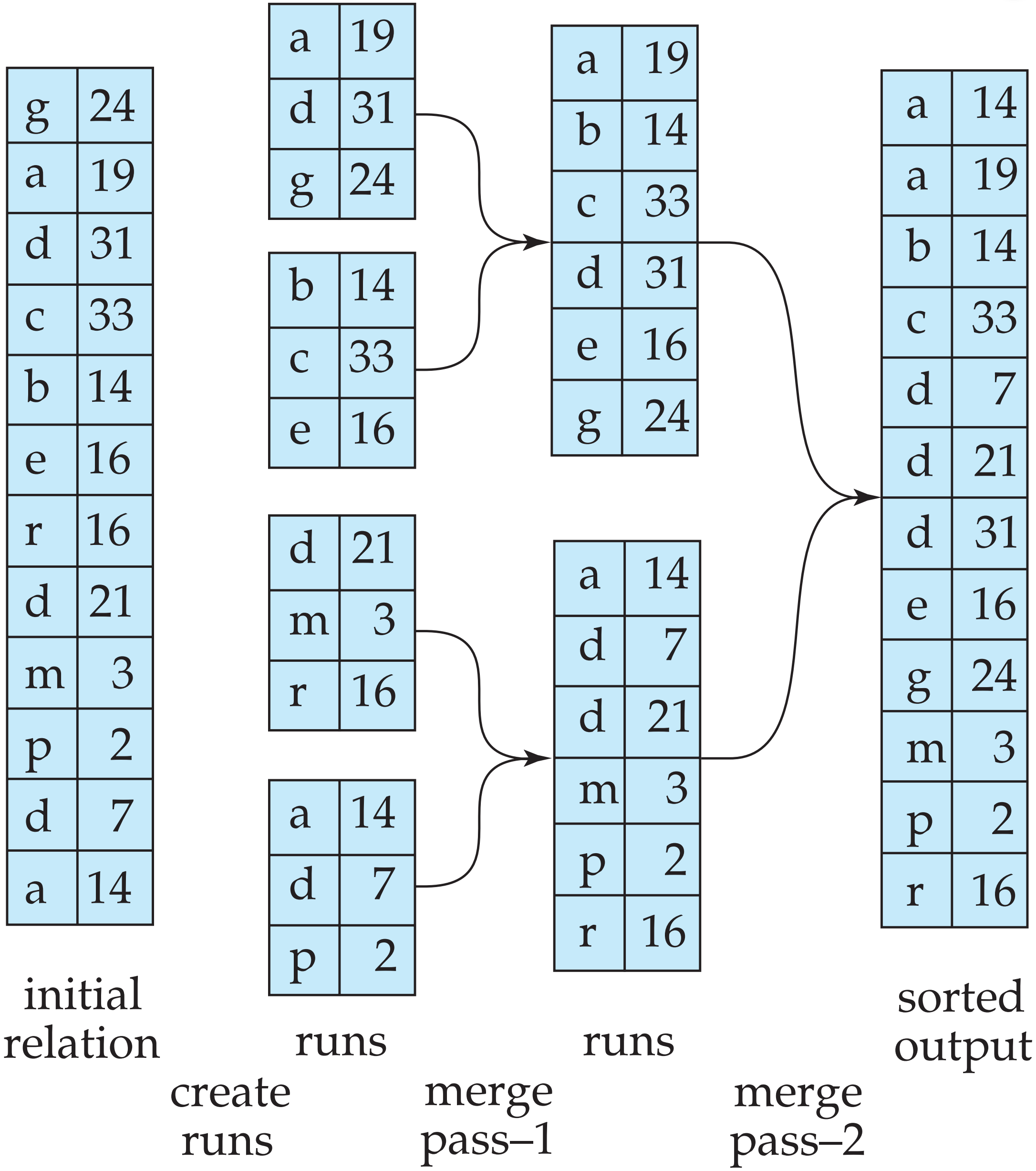
- **Merge the runs**. If $\lceil b_r/M \rceil \geq M$,

  several merge *passes* are required.

  - In each pass, contiguous groups of M-1 runs are merged.
  - Repeated passes are performed till all runs have been merged into one.

- A pass reduces the number of runs by a factor of M -1, and creates runs longer by the same factor.
  - E.g. If M=11, and there are 90 runs after the 1st pass, one pass reduces the number of runs to 9 (90/(M-1) = 90/10 = 9), each 10 times the size of the initial runs

Example: External Sorting Using Sort-Merge



g	24
a	19
d	31
c	33
b	14
e	16
r	16
d	21
m	3
p	2
d	7
a	14

a	19
d	31
g	24

b	14
c	33
e	16

d	21
m	3
r	16

a	14
d	7
p	2

a	19
b	14
c	33
d	31
e	16
g	24

a	14
d	7
d	21
m	3
p	2
r	16

a	14
a	19
b	14
c	33
d	7
d	21
d	31
e	16
g	24
m	3
p	2
r	16

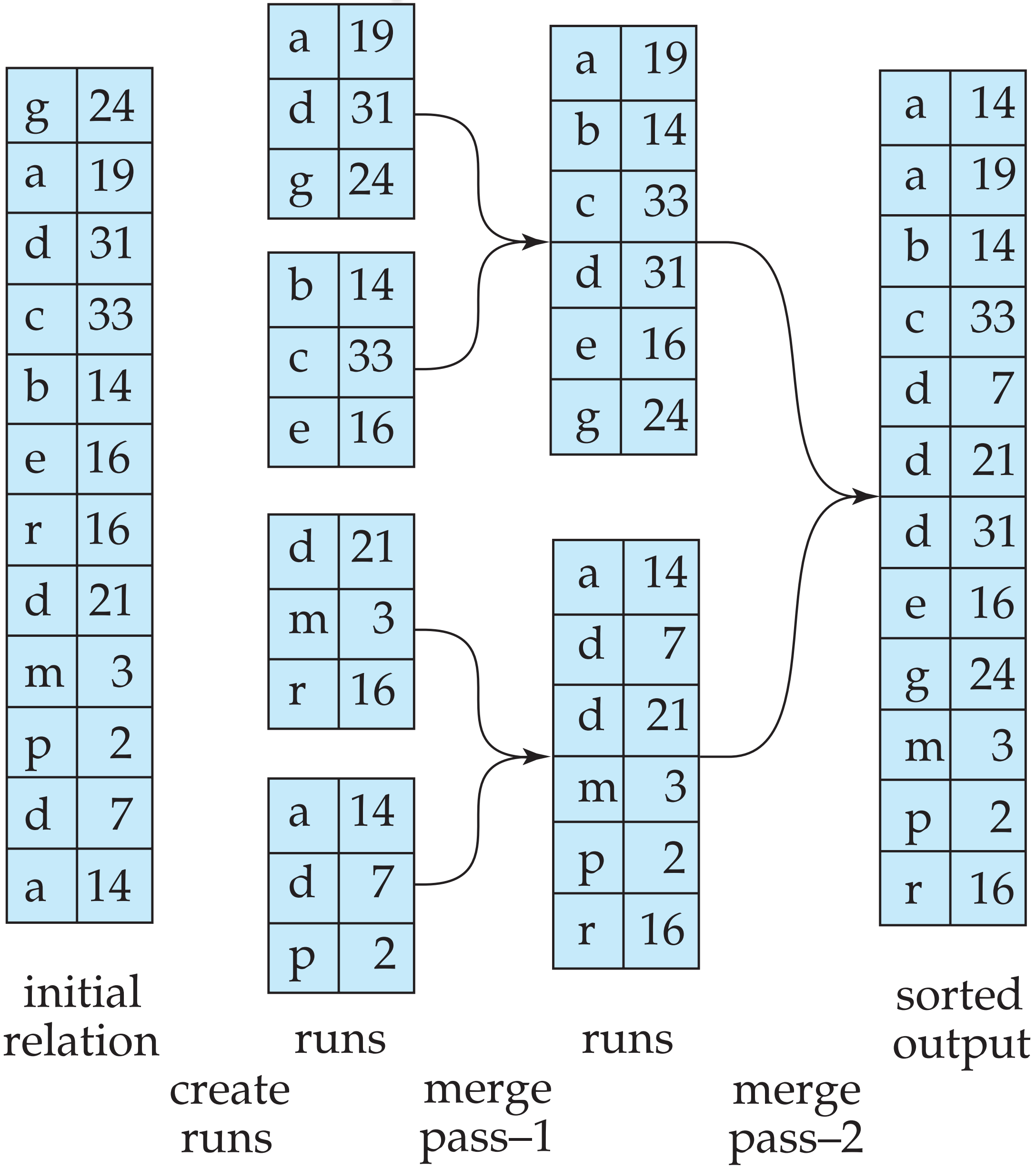
- b_r = 12 (each record uses one page.
- M = 3

External Merge Sort (Cont.)

- Cost analysis:
  - # of initial runs N1 = $\lceil b_r/M \rceil$
  - # of merge passes = $\lceil \log_{M-1} N1 \rceil$
  - # of passes (include initial pass) = $\lceil \log_{M-1} N1 \rceil + 1$
  - Block transfers for initial run creation as well as in each pass is $2b_r$
    - for final pass, we do not count write cost
      - If ignore final write cost for all operations
        - This may happen when the output of an operation is sent to the parent operation without being written to disk
        - Thus total number of block transfers for external sorting:

          b_r (2*(# of merge passes) + 1)
      - If include the final write cost: b_r (2 * # of passes)

Example: External Sorting Using Sort-Merge



initial relation

g	24
a	19
d	31
c	33
b	14
e	16
r	16
d	21
m	3
p	2
d	7
a	14

runs

a	19
d	31
g	24

b	14
c	33
e	16

d	21
m	3
r	16

a	14
d	7
p	2

runs

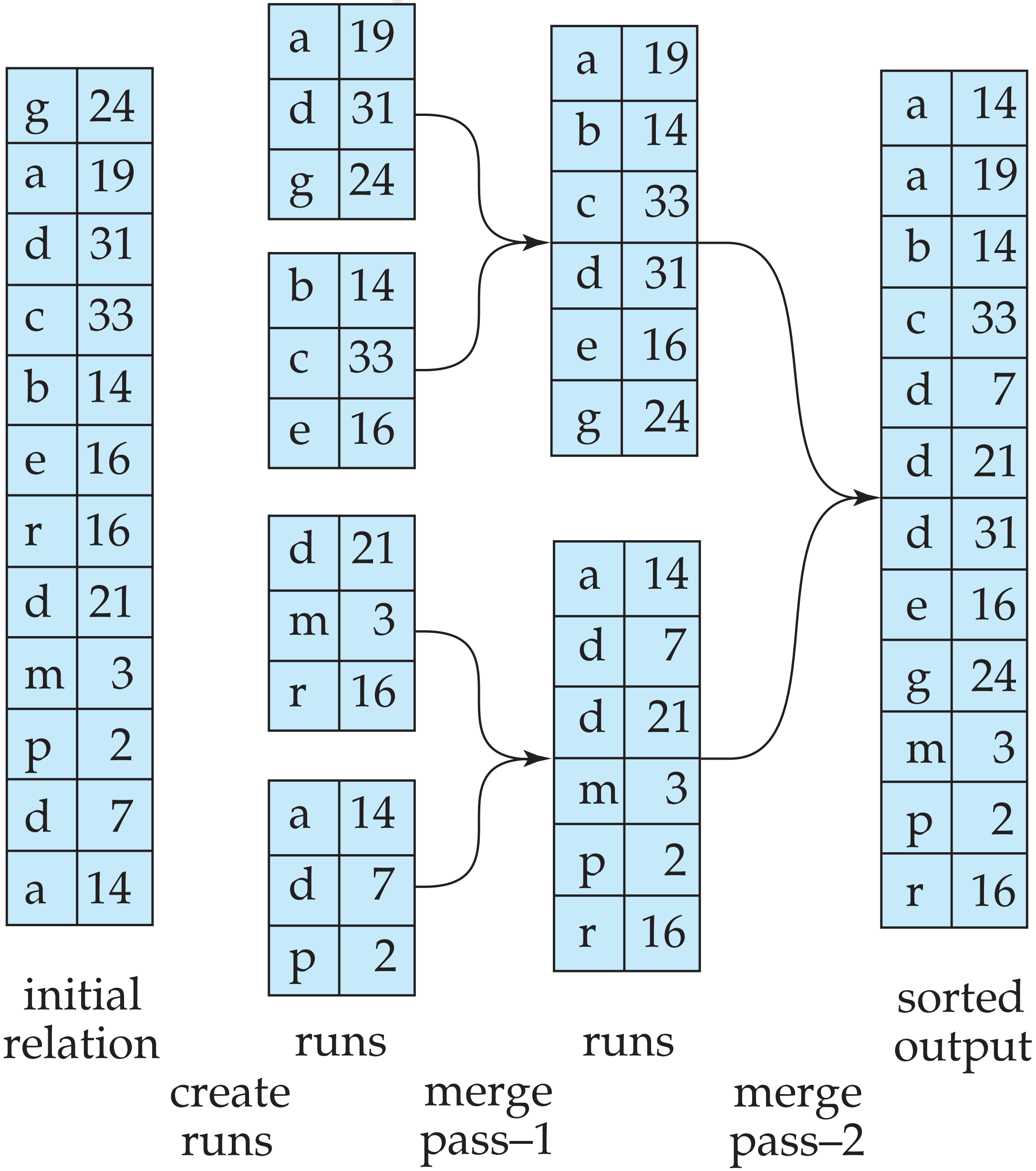
a	19
b	14
c	33
d	31
e	16
g	24

a	14
d	7
d	21
m	3
p	2
r	16

sorted output

a	14
a	19
b	14
c	33
d	7
d	21
d	31
e	16
g	24
m	3
p	2
r	16

Example: External Sorting Using Sort-Merge



- b_r = 12 (each record uses one page.
- M = 3
- b_b=1

- # of initial runs N1 = 12/3 = 4
- # of merge passes: $\log_2 4$ = 2
- # of passes: 2+1 = 3

- Transfer cost (without final writing)
  12*(2*2+1) = 12 * 5 = 60

- Transfer cost (with final writing)
  12*(2*3) = 72

Cost of External Merge Sort

- # of passes: $1+\lceil \log_{M-1} \lceil b_r/M \rceil \rceil$
- Transferring Cost = $2b_r$ * (# of passes)
- E.g., with 5 buffer pages, to sort 108 page file:
  - Pass 0: $\lceil 108/5 \rceil$ = 22 sorted runs of 5 pages each (last run contains only 3 pages)
  - Pass 1: $\lceil 22/4 \rceil$ = 6 sorted runs of 20 pages each (last run contains only 8 pages)
  - Pass 2: 2 (= $\lceil 6/4 \rceil$) sorted runs, 80 pages and 28 pages
  - Pass 3: Sorted file of 108 pages
- # of passes = $1 + \log_4 22 = 1 + 3 = 4$
- Cost = 108*(4*2) =864 (with final write)

Number of Passes of External Sort

br	M=3	M=5	M=9	M=17	M=129	M=257
100	7	4	3	2	1	1
1,000	10	5	4	3	2	2
10,000	13	7	5	4	2	2
100,000	17	9	6	5	3	3
1,000,000	20	10	7	5	3	3
10,000,000	23	12	8	6	4	3
100,000,000	26	14	9	7	4	4
1,000,000,000	30	15	10	8	5	4

- $b_r = 100$, M=3 $N1 = \lceil 100/3 \rceil = 34$, $1 + \log_2 34 = 1 + 6 = 7$
- $b_r = 100{,}000$, M=9 $N1 = \lceil 100{,}000/9 \rceil = 11112$ $1 + \log_8 11112 = 1 + 5 = 6$

Exercise

- Suppose you have a file with 10,000 pages and you have three buffer pages.
- Answer the following questions
  - Q1: How many runs will you produce in the first pass
  - Q2: How many passes will it take to sort the file completely
  - Q3: What is the total I/O cost of sorting the file (with final write)?
  - Q4: How many buffer pages do you need to sort the file completely in just two passes?

Exercise

- Suppose you have a file with 10,000 pages and you have three buffer pages.
- Answer the following questions
  - Q1: How many runs will you produce in the first pass
  - Q2: How many passes will it take to sort the file completely
  - Q3: What is the total I/O cost of sorting the file (with final write)?
  - Q4: How many buffer pages do you need to sort the file completely in just two passes?

- b_r = 10,000; M = 3
  - Q1: # of initial runs = $\lceil b_r/M \rceil$ = 3334
  - Q2: 1 + $\lceil \log_2 3334 \rceil$ = 1+12 = 13 passes
  - Q3:10000 * (2*13) = 260,000
  - Q4: Pass 0 $\lceil b_r/M \rceil$; pass 1 need to finish the merging, thus M-1 >= $\lceil b_r/M \rceil$, what is the minimum M? M = 101

Exercise

- Suppose you have a file with 100,000 pages and you have five buffer pages.
- Answer the following questions
  - Q1: How many runs will you produce in the first pass
  - Q2: How many passes will it take to sort the file completely
  - Q3: What is the total I/O cost of sorting the file?
  - Q4: How many buffer pages do you need to sort the file completely in just two passes?

External Merge Sort – blocked I/O (S.S.)

- Cost (include writing final results)
  - b_r (2 * # of passes) = $2*b_r*(\lceil \log_{M-1} N1 \rceil +1)$
- Minimize cost → minimize the number of passes → maximize the fan-in merging

- Blocked access: read a block of pages sequentially!
  - Each time: read and write a block of b pages
  - Output block pages: b
  - Input block pages: M-b
    - Number of input blocks $\lfloor (M-b)/b \rfloor$
  - Merge at most $\lfloor (M-b)/b \rfloor$ runs in each pass
    - E.g., M=10, one-page input/output block: fan-in = M-1 = 9
    - 2-page input/output block: fan-in = (10-2)/2 = 4
- # of page I/Os trade off per-page I/O cost

External Merge Sort – blocked I/O (S.S.)

- In fact, suggests we should make each buffer (input/output) be a block of pages.
  - But this will reduce fan-in during merge passes!
- In practice, most files can be sorted in just two passes, even using blocked I/O.

- First pass,
  - generate # of runs N1 = $\lceil b_r/M \rceil$
- Fan-in factor: F = $\lfloor M/b \rfloor$-1
- # of passes: 1 + $\lceil \log_F N1 \rceil$

Number of Passes of Optimized Sort (S.S.)

- Block size b = 32, initial pass produces runs of size M
- M=1000, b_r = 10,000 pages
- M=5000, b_r = 10,000,000

M	F
1000	$\lfloor 1000/32) \rfloor -1 = 30$
5000	$\lfloor 5000/32) \rfloor -1 = 155$
10000	$\lfloor 5000/32) \rfloor -1 = 311$

Number of Passes of Optimized Sort (S.S.)

- ❑ Block size b = 32, initial pass produces runs of size M
- ❑ M=1000, b_r = 10,000 pages
- ❑ M=5000, b_r = 10,000,000

M	F
1000	$\lfloor 1000/32) \rfloor - 1 = 30$
5000	$\lfloor 5000/32) \rfloor - 1 = 155$
10000	$\lfloor 5000/32) \rfloor - 1 = 311$

M=1000, b_r = 10,000 pages   N1 = $\lceil br/M) \rceil = 10$,   # of passes = $1 + \lceil \log_{30} 10 \rceil = 2$
M=5000, br = 10,000,000   N2 = $\lceil br/M) \rceil = 2000$ # of passes = $1 + \lceil \log_{155} 2000 \rceil = 3$

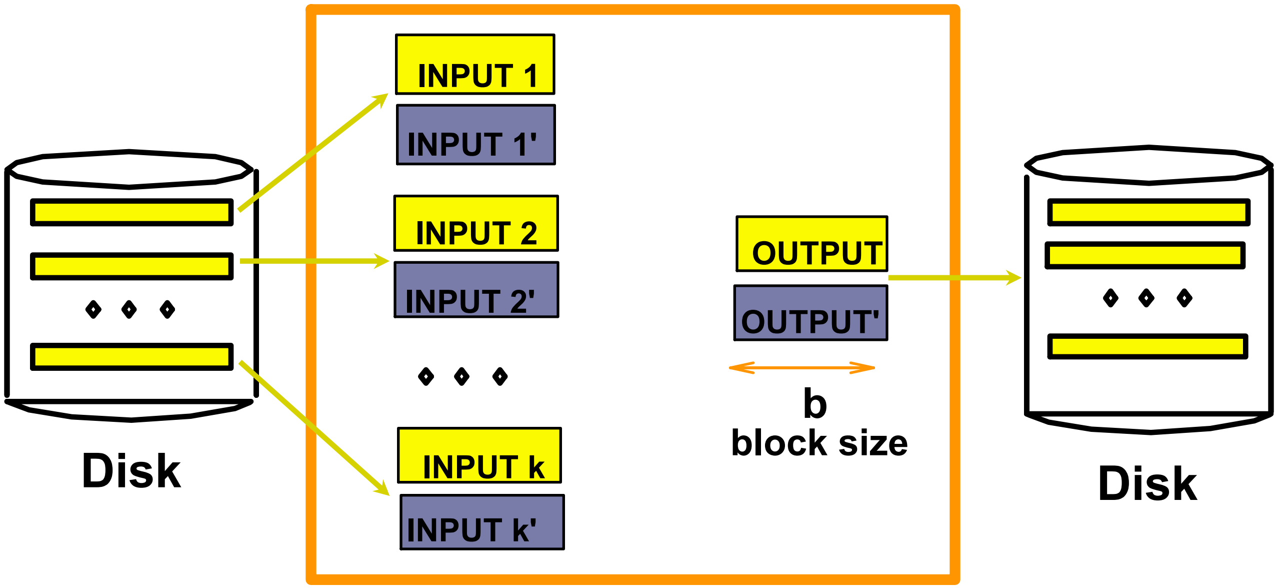
Exercise (S.S.)

- M = 5000

b	F
1	???
32	???
64	???

External sort - Double Buffering (S.S.)

- To reduce wait time for I/O request to complete, can *prefetch* into `shadow block' .
  - Potentially, more passes; in practice, most files *still* sorted in 2-3 passes.



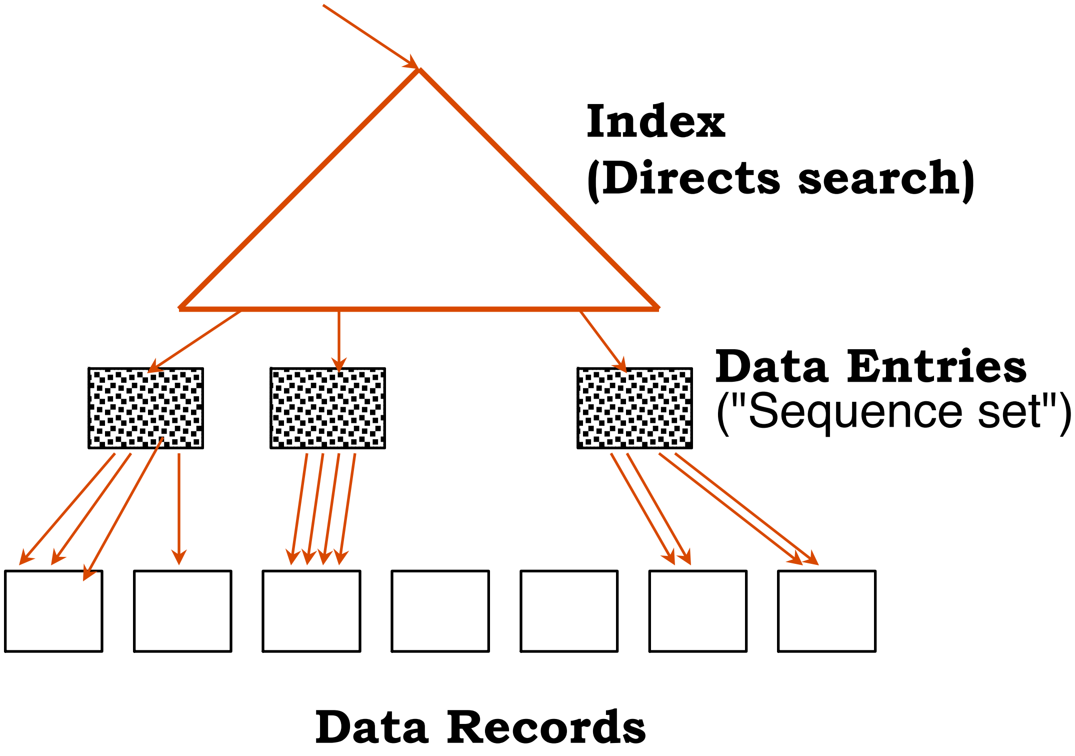
B main memory buffers, k-way merge

Using B+ Trees for Sorting

- Scenario: Table to be sorted has B+ tree index on sorting column(s).
- Idea: Can retrieve records in order by traversing leaf pages.
- Is this a good idea?
- Cases to consider:
  - B+ tree is clustered *Good idea!*
  - B+ tree is not clustered *Could be a very bad idea!*

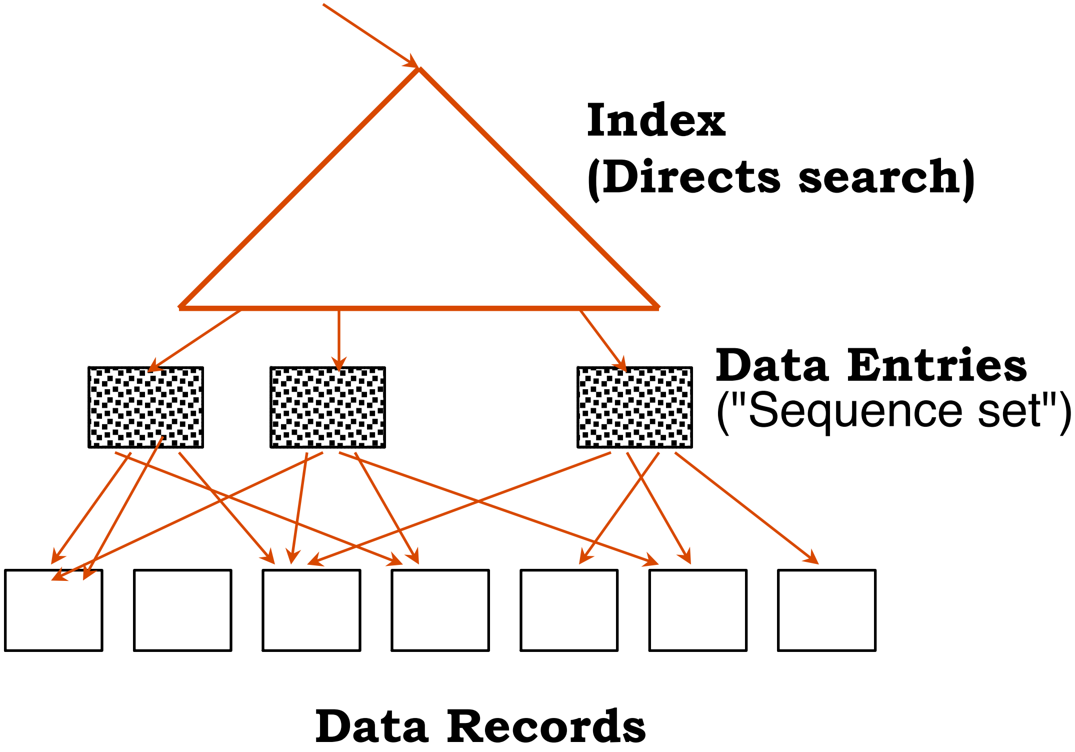
Clustered B+ Tree Used for Sorting

- Cost:
  - root to the left-most leaf (<4)
  - retrieve all leaf pages
- If <key, rid> pair is used in the index?
  - Additional cost of retrieving data records: each page fetched just once.



Unclustered B+ Tree Used for Sorting

- <key, rid> pair is used for data entries; each data entry contains *rid* of a data record. In general, one I/O per data record!



Unclustered B+ Tree Used for Sorting

- p: average number of records per data page
  - Bigger than 10
- b_r: data pages
- f: (the size of a data entry)/ (size of a data record)
  - Usually 0.1

- Cost
  - # of data records: $p*b_r$
  - Approximate number of leaf pages: $f*b_r$
  - Total cost: $(f+p)*b_r$
  - Approximation: $p*b_r$

External Sorting vs. Unclustered Index (S.S.)

br	Sorting	p=1	p=10	p=100
100	200	100	1,000	10,000
1,000	2,000	1,000	10,000	100,000
10,000	40,000	10,000	100,000	1,000,000
100,000	600,000	100,000	1,000,000	10,000,000
1,000,000	8,000,000	1,000,000	10,000,000	100,000,000
10,000,000	80,000,000	10,000,000	100,000,000	1,000,000,000

- ❑ *p*: # of records per page
- ❑ *M=1,000* and block size *b=32* for sorting
- ❑ *p=100* is the more realistic value

- ❑ Cost of unclustered index appr.: *p*br*
- ❑ Cost of sorting: calculate F, then N1 or N2, then cost

Summary

- External sorting is important; DBMS may dedicate part of buffer pool for sorting!
- External merge sort minimizes disk I/O cost:
  - Pass 0: Produces sorted **runs** of size **M** (# buffer pages).
  - Later passes: **merge** runs.
  - # of runs merged at a time depends on **M**, and **block size**.
  - In practice, # of runs is rarely more than 2 or 3.
- The best sorts are wildly fast:
  - Despite 40+ years of research, we're still improving!
- Clustered B+ tree is good for sorting;
- Unclustered tree is usually very bad.