# Query Evaluation -- Join operation 

References:

- [SKS-6ed] Chapter 12.5
- [RG-3ed] Chapter 14.4


## Relational Operations

- We will consider how to implement:
$\square$ Selection ( $\sigma$ ): Selects a subset of rows from relation.
$\square$ Projection ( $\pi$ ): Deletes unwanted columns from relation.
$\square$ Join $(\bowtie)$ : Allows us to combine two relations.
$\square$ Set-difference (-): Tuples in relation 1, but not in relation 2.
$\square$ Union (U): Tuples in relation 1 and in relation 2.
- Aggregation (SUM, MIN, etc.) and GROUP BY
$\square$ Since each op returns a relation, ops can be composed! After we cover the operations, we will discuss how to optimize queries formed by composing them.


## Schema for Examples

Sailors (sid: integer, sname: string, rating: integer, age: real) Reserves (sid: integer, bid: integer, day: dates, rname: string)

- Similar to old schema; rname added for variations.
- Sailors:

Each tuple is 50 bytes long, 80 tuples per page, 500 pages.

- Reserves:
$\square$ Each tuple is 40 bytes long, 100 tuples per page, 1000 pages.


## Equality Joins With One Join Column

SELECT *<br>FROM Reserves R1, Sailors S1<br>WHERE R1.sid = S1.sid

- In algebra: R凶 S. Common! Must be carefully optimized.
- $R \times S$ is large. So, $R \times S$ followed by a selection is inefficient.
$\square$ We will consider more complex join conditions later.
- Cost metric: \# of I/Os. We will ignore output costs.


## Join Operation

$\square$ Several different algorithms to implement joins
$\square$ Simple nested-loop join: iteration
$\square$ Block nested-loop join: iteration

- Indexed nested-loop join
$\square$ Merge-join
$\square$ Hash-join
$\square$ Choice based on cost estimate
$\square$ Our examples use the following information
$\square$ Number of records of student: 5,000 takes: 10,000
$\square$ Number of pages of student: 100 takes: 400


## Simple nested-Loop Join

$\square$ To compute the theta join $r \bowtie_{\theta} s=\sigma_{\theta}(r \times s)$
for each tuple $t_{r}$ in $r$ do begin for each tuple $t_{s}$ in $s$ do begin
test pair $\left(t_{r}, t_{s}\right)$ to see if they satisfy the join condition $\theta$
if they do, add $t_{r} \cdot t_{s}$ to the result.
end
end
$\square r$ is called the outer relation and $s$ the inner relation of the join.
$\square$ Requires no indices and can be used with any kind of join condition.

- Expensive since it examines every pair of tuples in the two relations.


## Simple nested-Loop Join (Cont.)

- Given
$\square n_{r}, b_{r}$ : number of tuples and pages in $r$
$\square n_{s}, b_{s}$ : number of tuples and pages in $s$
$\square$ Case 1: worst case, memory hold one page of each relation
$\square b_{r}+n_{r}{ }^{*} b_{s}$


## Simple nested-Loop Join (Example)

$\square$ Number of records of student: 5,000 takes: 10,000
$\square$ Number of pages of student: 100 takes: 400

- Assuming worst case memory availability cost estimate is
$\square$ with student as outer relation:
$\square 100+5000$ * $400=2,000,100$ block transfers,
$\square$ with takes as the outer relation
- $400+10000$ * $100=1,000,400$ block transfers


## Simple nested-loop Join (Cont.)

$\square$ Case 2 (best case): enough space for both relations
$\square$ Cost for block transfer: $b_{r}+b_{s}$
If smaller relation fits entirely in memory, use that as the inner relation.
$\square$ Reduces cost to $b_{r}+b_{s}$ block transfers
$\square$ If smaller relation (student) fits entirely in memory, the cost estimate will be 500 block transfers.

## Simple nested-loop Join - analysis

$\square b_{r}$ pages in $r$, $p_{r}$ tuples per page
$\square b_{s}$ pages in $s, p_{s}$ tuples per page
$\square$ For each tuple in the outer relation R , we scan the entire inner relation S .
$\square$ Cost: $b_{r}+\left(p_{r}{ }^{*} b_{r}\right)^{*} b_{s}$
$\square$ Example
$\square$ Reserves: each tuple is 40 bytes long, 100 tuples per page, 1000 pages.
$\square$ Sailors: each tuple is 50 bytes long, 80 tuples per page, 500 pages.
$\square$ Cost: $1000+(100 * 1000)^{*} 500$ I/Os.

## Simple nested-loop join (page-oriented)

$\square$ Page-oriented Nested Loops join:
For each page of $r$,
For each page of $s$,
Write out matching pairs of tuples $\left\langle t_{r}, t_{s}\right\rangle$, where $t_{r}$ is in $r$-page and $t_{s}$ is in s-page.

- Cost: $b_{r}+b_{r}{ }^{*} b_{s}=1000+1000 * 500$
- If smaller relation (S) is outer, cost $=500+500 * 1000$


## Simple nested-loop join (page-oriented)

$\square$ Worst case: each page in the inner relation $s$ is read once for each page in the outer relation
$\square b_{r}+b_{r}{ }^{*} b_{s}$ block transfers
$\square$ Best case:
$\square b_{r}+b_{s}$ block transfers
$\square$ Example 1: 400 pages of takes, 100 pages of students
$\square$ Outer relation is student: $100+100 * 400=400,100$ transfer
$\square$ Improves 2,000,100 (simple nested-loop)

- Example 2: Reserves 1000 pages, Sailor 500 pages
- Outer relation is Reserves: $1000+1000 * 500$
$\square$ Outer relation is Sailor: $500+500 * 1000$


## Simple nested-loop join (page-oriented)

- Improvements
$\square$ If equi-join attribute forms a key on inner relation, stop inner loop on first match
$\square$ Scan inner loop forward and backward alternatively, to make use of the blocks remaining in buffer (with LRU replacement)
$\square$ Block nested-loop join
$\square$ Indexed nested-loop


## Block Nested Loops Join

$\square$ Use one page as an input buffer for scanning the inner s, one page as the output buffer, and use all remaining pages to hold '`block' ' of outer r.

For each block of M-2 pages of r do
For each page of s do
For all matching in-memory tuples $t_{r}$ in r-block, $t_{s}$ in s-page, add $<t_{r}, t_{s}>$ to result.


## Analysis of Block Nested Loops

- Cost: Scan of outer + \#outer blocks * scan of inner
- \#outer blocks = $\lceil$ \# of pages of outer relation/block size $\rceil$
- $M=$ memory size in blocks;
- Cost
- $b_{r}+\left\lceil b_{r} /(M-2)\right\rceil^{*} b_{s}$ block transfers


## Examples of Block Nested Loops

- Sailors:
$\square$ Each tuple is 50 bytes long, 80 tuples per page, 500 pages.
- Reserves:
$\square$ Each tuple is 40 bytes long, 100 tuples per page, 1000 pages.
- Example 1: With Reserves as outer, and 100-page block of Reserves:
- Block transfer cost: $1000+\lceil 1000 / 100\rceil^{*} 500=6000$
- 90-page block for Reserve, cost?
- $1000+\lceil 1000 / 90\rceil * 500=1000+12 * 500=7000$
$\square$ What is the minimum number of block pages to have this cost?
- $\lceil 1000 /(\mathrm{M}-2)\rceil=12,\lceil 1000 / 12\rceil<=\mathrm{M}<=$ floor(1000/11)


## Examples of Block Nested Loops

- Sailors:
$\square$ Each tuple is 50 bytes long, 80 tuples per page, 500 pages.
- Reserves:
$\square$ Each tuple is 40 bytes long, 100 tuples per page, 1000 pages.
- Example 2: 100-page block, Sailors as outer:
- Block transfer cost: 500+(500/100)*1000 $=5500$
- 90-page block?
$\square 500+\lceil 500 / 90\rceil * 1000=500+6 * 1000=6500$
$\square$ What is the minimum number of pages to have this cost?


## Block Nested Loops Join -- improvement

■ Hash table for outer relation $r$

- The I/O cost does not change
- The CPU cost is much lower



## Index Nested Loops Join

For each tuple $t_{r}$ in $r$ do
For each tuple $t_{s}$ in $s$ where $t_{r}==t_{s}$ do add $\left\langle t_{r}, t_{s}>\right.$ to result

- Indexed relation as the inner relation
$\square$ Does not enumerate the cross-product of $r$ and $s$


## Indexed Nested-Loop Join

- Worst case: buffer has space for only one page of $r$, and, for each tuple in $r$, we perform an index lookup on $s$.
- Cost (in I/Os): $b_{r}+\left(\left(b_{r}{ }^{*} p_{r}\right)^{*}\right.$ cost of finding matching $s$ tuples)
- For each $r$ tuple, cost of probing $s$ index is
$\square$ about 1.2 for hash index,
- 2-4 for B+ tree
$\square$ Cost of finding $s$ tuples depends on clustering.
$\square$ Clustered index: 1 I/O (typical),
$\square$ Un-clustered: up to 1 I/O per matching $s$ tuple.


## Examples of Index Nested Loops

$\square$ Example 1: Hash-index on sid of Sailors (as inner):
$\square$ Scan Reserves: 1000 page I/Os

- Reserves tuples: 100*1000 tuples.
$\square$ For each Reserves tuple: 1.2 I/Os to get data entry in index, plus 1 I/O to get (the exactly one) matching Sailors tuple.
$\square$ Find sailor entry from index: 1.2 * $(100 * 1000)=120,000$ I/Os.
$\square$ Find matching sailor tuple: $1^{*}(100 * 1000)=100,000$
- Total: 220,000
$\square$ Total: $1000+220,000=221,000$ I/Os


## Examples of Index Nested Loops

- Example 2: Hash-index on sid of Reserves (as inner):
- Scan Sailors: 500 page I/Os,
- \# of Sailors tuples: 80*500 tuples.
$\square$ For each Sailors tuple: 1.2 I/Os to find index page with data entries, plus cost of retrieving matching Reserves tuples.
$\square$ Find Reserves entry from index: 1.2 * $(80$ * 500$)=\underline{48,000}$ I/Os.
$\square$ Cost of retrieving matching Reserves tuples:
$\square$ 100,000 reservations for 40,000 sailors
$\square$ Assuming uniform distribution, 2.5 reservations per sailor (100,000/40,000).
$\square$ Cost of retrieving reserves is $2.5 \mathrm{I} / \mathrm{Os}$ per sailor tuple.
- Cost: 2.5 * $(80 * 500)=\underline{100,000}$ (un-clustered)
$\square$ Total: $500+48,000+100,000=148,500$ I/O


## Example of Nested-Loop Join Costs

- Compute student $\bowtie$ takes, with student as the outer relation.
- Let takes have a primary $\mathrm{B}^{+}$-tree index on the attribute ID, which contains 20 entries in each index node.
- students: 100 pages, 5000 tuples
- Takes: 400 pages, 10,000 tuples
- Cost of simple nested loops join (page-oriented)
- $100+100 * 400=40,100$ block transfers
- Cost of indexed nested loops join
- Since takes has 10,000 tuples, the approximate height of the tree is 4, and one/? more access is needed to find the actual data
- $100+5000$ * $5=25,100$ block transfers and seeks.
- If indices are available on join attributes of both $r$ and $s$, use the relation with fewer tuples as the outer relation.


## Exercise

- Compute student $\bowtie$ takes,.
$\square$ Let the student relation have a primary $\mathrm{B}^{+}$-tree index on the attribute $I D$, which contains 20 entries in each index node.
$\square$ students: 100 pages, 5000 tuples
$\square$ takes: 400 pages, 10,000 tuples
$\square$ with "takes" as the outer relation?


## Sort-Merge Join (r»i®s)

$\square$ Sort $r$ and $s$ on the join column (external sort)
$\square$ Merging step: and output result tuples.
$\square$ Advance scan of $r$ until current $r$-tuple $>=$ current $s$-tuple
$\square$ Current r-tuple (Tr)
$\square$ Then advance scan of $s$ until current s-tuple $>=$ current $r$-tuple; do this until current r-tuple $=$ current s-tuple.
$\square$ Current s-tuple (Gs)
$\square$ At this point, all r-tuples with same value in ri (current r partition) and all $S$ tuples with same value in $S j$ (current s partition) match;
$\square$ For each Tr, loop using another pointer (Ts) all the s-tuples with the same value as the tuple pointed by Gs
$\square$ Output $<t_{r}, t_{s}>$ for all pairs of such tuples.
$\square$ After matching one Tr with all tuples in the s partition, advance Tr
$\square$ Then resume scanning $r$ and $s$.
$\square \quad r$ is scanned once; each s group is scanned once per matching $r$ tuple. (Multiple scans of an s group are likely to find needed pages in buffer.) 25

## Example of Sort-Merge Join

| sid | sname | rating | age |
| :--- | :--- | :--- | :--- |
| 22 | dustin | 7 | 45.0 |
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |


| sid | $\underline{\text { bid }}$ | day | rname |
| :--- | :--- | :--- | :--- |
| 28 | 103 | $12 / 4 / 96$ | guppy |
| 28 | 103 | $11 / 3 / 96$ | yuppy |
| 31 | 101 | $10 / 10 / 96$ | dustin |
| 31 | 102 | $10 / 12 / 96$ | lubber |
| 31 | 101 | $10 / 11 / 96$ | lubber |
| 58 | 103 | $11 / 12 / 96$ | dustin |

## Example of Sort-Merge Join

$\square$ Cost (in I/Os): (sorting cost)+ (cost of merging)
$\square$ The cost of merging, $b_{r}+b_{s}$, could be $b_{r}{ }^{*} b_{s}$ (very unlikely!)

- With 101 buffer pages, both Reserves (1000 pages) and Sailors(500 pages) can be sorted in 2 passes;
$\square \mathrm{M}=101$, with final result write:
$\square$ Sort Reserves: $2^{*} 2^{*} 1000=4000$
$\square$ Sort Sailors: 2*2*500 = 2000
$\square$ Merge cost: $1000+500=1500$
$\square$ Total join cost: 7500.
- How about $\mathrm{M}=35$ ? $\mathrm{M}=300$ ?
- How about BNL cost?
- 2500 to 15000 I/Os


## Refinement of Sort-Merge Join

$\square$ We can combine the merging phases in the sorting of $R$ and $S$ with the merging required for the join.
$\square$ With $\mathrm{M}>\sqrt{L}$, where $L$ is the size of the larger relation,
$\square$ \# of runs of each relation is $<\sqrt{L}$
$\square$ Merging: buffer size $2 \sqrt{L}$
$\square$ Allocate 1 page per run of each relation, and "merge" while checking the join condition.
$\square$ Cost: read and write each relation in Pass $0+$ read each relation in (only) merging pass [+ writing of result tuples].
$\square$ In example, cost goes down from 7500 to 4500 I/Os.

## Sort-Merge Join (Cont.)

$\square$ Can be used only for equi-joins and natural joins
$\square$ Each block needs to be read only once (assuming all tuples for any given value of the join attributes fit in memory
$\square$ Thus the cost of merge join is:
$\square 3^{*}\left(b_{r}+b_{s}\right)$ (best)

## Example

$\square$ Compute student $\bowtie$ takes

- student: 100 pages, 5000 tuples
- Takes: 400 pages, 10,000 tuples
$\square$ Already sorted on join attribute ID.
- Merge cost $=400+100=500$ block transfers
- Not sorted, M = 3
$\square$ Sorting (write final output)
$\square$ Takes: $\left\lceil\log _{2}[400 / 3\rceil\right\rceil=8$ merge passes; $2^{*} 400^{*}(8+1)=$ 7200 block transfers; 2* [400/3 $\rceil+8^{*}(400 / 1)^{*} 2=6668$
$\square$ Students: ???
$\square$ Merging
$\square 400+100=500$ block transfers


## Hash-Join

- Partition both relations using hash function h : r tuples in partition i will only match s-tuples in partition $i$.
$\square$ Read in a partition of $r$, hash it using h2 ( $<>\boldsymbol{h}!$ ). Scan matching partition of $s$, search for matches.
$\square$ Relation $r$ is called the build input and $s$ is called the probe input.

Original Relation


Disk


## Partitions

 of R \& S

Disk


M main memory buffers Disk

## Observations on Hash-Join

\# \#partitions $\mathrm{k}<=\mathrm{M}-1$ (one input buffer), $\mathrm{M}-1$ output buffers
$\square \mathrm{M}-2>$ size of largest partition to be held in memory.
$\square$ One partition fits in the memory, good.
$\square$ Assuming uniformly sized partitions, and maximizing k, we get:
$\square \mathrm{k}=\mathrm{M}-1$ (maximum)

- If $\mathrm{b}_{\mathrm{r}} /(\mathrm{M}-1)<\mathrm{M}-2$, then $\mathrm{M}>\sqrt{b_{r}}$
$\square$ If we build an in-memory hash table to speed up the matching of tuples, a little more memory is needed. Typically $k$ is chosen as $\left\lceil b_{\text {build }} / M\right\rceil$ * $f$ where f is a "fudge factor", typically around 1.2
$\square \mathrm{M}>\sqrt{f \bullet b_{r}}$
$\square$ More specifically, $f^{*} b_{r} /(M-1)<M-2$
$\square$ The probe relation partitions need not fit in memory
$\square$ If the hash function does not partition uniformly, one or more r partitions may not fit in memory. Can apply hash-join technique recursively to do the join of this r-partition with corresponding s-partition.


## Cost of Hash-Join

$\square$ In partitioning phase, $\mathrm{R}+\mathrm{W}$ both relations ; $2\left(b_{r}+b_{s}\right)$.
$\square$ In matching phase, read both relations; $b_{r}+b_{s}$ I/Os.
$\square$ In our running example, this is a total of $4500 \mathrm{I} / \mathrm{Os}$.
$\square$ Sort-Merge Join vs. Hash Join:
$\square$ Given a minimum amount of memory (what is this, for each?) both have a cost of $3\left(b_{r}+b_{s}\right)$ I/Os.
$\square$ Hash Join is superior if relation sizes differ greatly.
$\square$ Hash Join has shown to be highly parallelizable.
$\square$ Sort-Merge is less sensitive to data skew; result is sorted.

## Cost of Hash-Join

$\square$ If recursive partitioning is not required: cost of hash join is $3\left(b_{r}+b_{s}\right)$ block transfers
$\square$ If the entire build input can be kept in main memory no partitioning is required
$\square$ Cost estimate goes down to $b_{r}+b_{s}$.

## Example of Cost of Hash-Join

$\square$ Compute student $\bowtie$ takes

- student: 100 pages, 5000 tuples
- takes: 400 pages, 10,000 tuples
- Given $\mathrm{M}=22$ pages
- student is to be used as build input. Partition it into 5 partitions, each of size 20 pages (=M-2). This partitioning can be done in one pass.
$\square$ Similarly, partition takes into 5 partitions, each of size 80. This is also done in one pass.
$\square$ Total cost:
$\square 3(100+400)=1500$ block transfers
$\square$ Always ignore cost of writing partially filled blocks
- Problem???


## Complex Joins

- Join with a conjunctive condition:

$$
r \bowtie_{\theta 1 \wedge \theta 2 \wedge \ldots \wedge \theta n} s
$$

- Either use nested loops/block nested loops, or
$\square$ Compute the result of one of the simpler joins $r \bowtie_{\theta i} s$
$\square$ final result comprises those tuples in the intermediate results that satisfy the remaining conditions

$$
\theta_{1} \wedge \ldots \wedge \theta_{i-1} \wedge \theta_{i+1} \wedge \ldots \wedge \theta_{n}
$$

$\square$ Join with a disjunctive condition

$$
r \bowtie_{\theta 1 v \theta 2 v \ldots v \ln } s
$$

$\square$ Either use nested loops/block nested loops, or
$\square$ Compute as the union of the records in individual joins $r \bowtie_{\theta i} s$ :

$$
\left(r \bowtie_{\theta 1} s\right) \cup\left(r \bowtie_{\theta 2} s\right) \cup \ldots \cup\left(r \bowtie_{\theta n} s\right)
$$

## General Join Conditions

- Equalities over several attributes (e.g., R.sid=S.sid AND R.rname=S.sname):
$\square$ For Index NL, build index on $<$ R.sid, R.sname> (if R is inner); or use existing indexes on sid or sname.
For Sort-Merge and Hash Join, sort/partition on combination of the two join columns.
- Inequality conditions (e.g., R.rname < S.sname):
$\square$ For Index NL, need (clustered!) B+ tree index.
$\square$ Range probes on inner;
$\square$ The \# of matches is likely to be much higher than that for equality joins.
- Hash Join, Sort-Merge Join is not applicable.
$\square$ Block NL is quite likely to be the best join method here.


## Summary

- No one join algorithm is uniformly superior to the others.
- The choice of a good algorithm
$\square$ Sizes of the relations being joined
$\square$ Available access methods
$\square$ Size of the buffer pool

