QUESTION 3 2006

(a) The head loss in a pipe can be expressed in the form $h_f = KQ^2$. Two pipes having constants K_1 and K_2 are to be considered as a single equivalent pipe. Determine the value K_3 of this single pipe when the two are laid:

i. in series
ii. in parallel.

SOLUTION PART A

i. In series the flow is the same and total head loss is the sum of the two.

$h_{f1} = k_1Q^2 \qquad h_{f2} = k_2Q^2 \qquad h_{f1} + h_{f2} = k_3Q^2 = k_1Q^2 + k_2Q^2$

Hence $k_3 = k_1 + k_2$

ii. In parallel the friction heads are the same and the flows different.

$h_f = k_1Q_1^2 \qquad Q_1 = (h_f/k_1)^{1/2}$

$h_f = k_2Q_2^2 \qquad Q_2 = (h_f/k_2)^{1/2}$

$h_f = k_3(Q_1 + Q_2)^2$

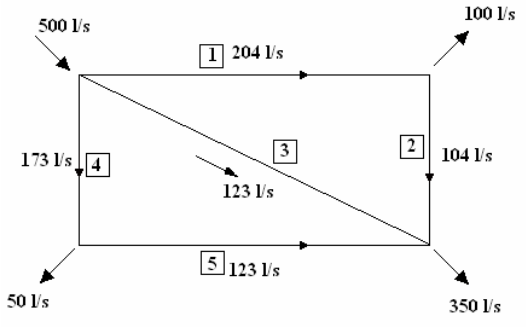
$$h_f = k_3\left\{\sqrt{\frac{h_f}{k_1}} + \sqrt{\frac{h_f}{k_2}}\right\}^2 = k_3\left\{\left(\frac{h_f}{k_1} + \frac{h_f}{k_2} + \frac{2h_f}{\sqrt{k_1k_2}}\right)\right\}$$

$$1 = k_3\left\{\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{2}{\sqrt{k_1k_2}}\right)\right\}$$

$$k_3 = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{2}{\sqrt{k_1k_2}}}$$

(b) When the flow rates are expressed in litres per second and the head losses in metres, K values for the pipe systems shown are as given in the table. Under a particular set of inputs and demands the network experienced the flow rates indicated.

The head loss in the system was considered to be excessive and a second pipe was alongside pipe 3 so that they carried flow in parallel. The equivalent single pipe for these two pipes has k = 0.000818 $ms^2/litre^2$. When the pipe had been installed the pipe flows shown changed but the inputs and demands on the system remained the same.



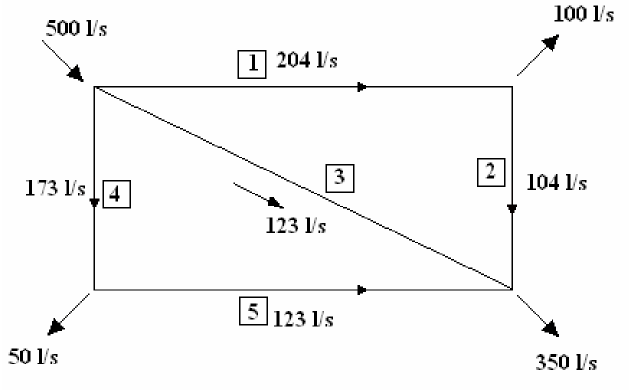
Use the flows shown as initially assumed flows and apply an iterative method of network analysis to determine the changed flows in the pipes. Make only two rounds of corrections to the initial flows.

Pipe	1	2	4	5
K ms^2/l^2	0.000570	0.012118	0.001698	0.006946

(Pipe 3 has K = 0.000818 in question)

SOLUTION PART B

The problem must be solved as two loops with a common pipe 3. Start with loop 1 with the flows shown. Data is shown for initial guess. Note clockwise flow is positive.

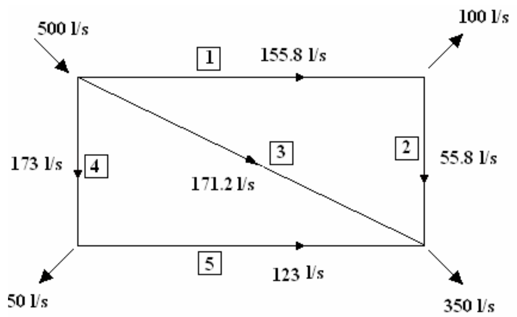


Starting data

First iteration loop 1 (pipes 1, 2 and 3)

PIPE	K	Q	h_f	h_f/Q
1	0.000570	204	23.7212	0.11628
2	0.012118	104	131.068	1.260
3	0.000818	-123	-12.376	0.1006
			142.4	1.4772

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = \frac{142.4}{2 \text{ x } 1.4772} = 48.2$$ Correct all flows in loop 1 by subtracting 48.2



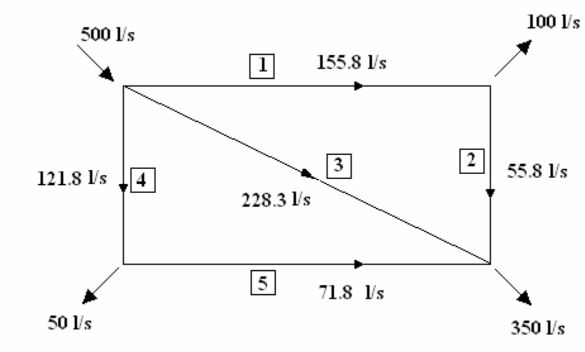
First correction shown above

First Iteration loop 2 (pipes 3,5 and 4)

PIPE	K	Q	h_f	h_f/Q
3	0.000570	171.2	23.97	0.1400
5	0.006946	-123	-105.08	-0.8544
4	0.001698	-173	-50.82	-0.2938
			-131.93	1.288

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = \frac{-131.93}{2 \text{ x } 1.288} = -51.208$$

Correct all flows in loop 2 by subtracting -51.2



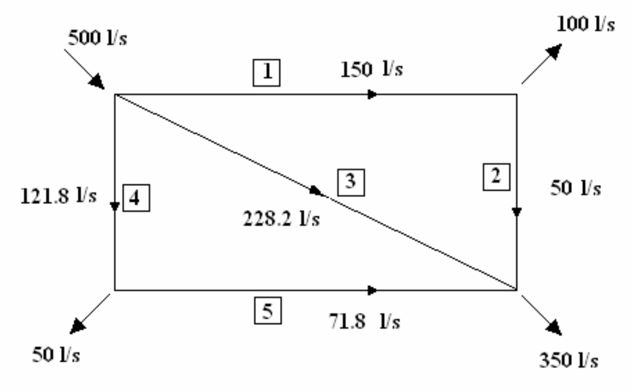
Second correction

Second iteration loop 1

PIPE	K	Q	h_f	h_f/Q
1	0.000570	155.8	13.8	0.0888
2	0.012118	55.8	37.7	0.676
3	0.003272	-222.4	-40.5	0.947
			11.1	0.947

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = \frac{11.1}{2 \text{ x } 0.947} = 5.9$$

Correct all flows in loop 1 by subtracting 5.9

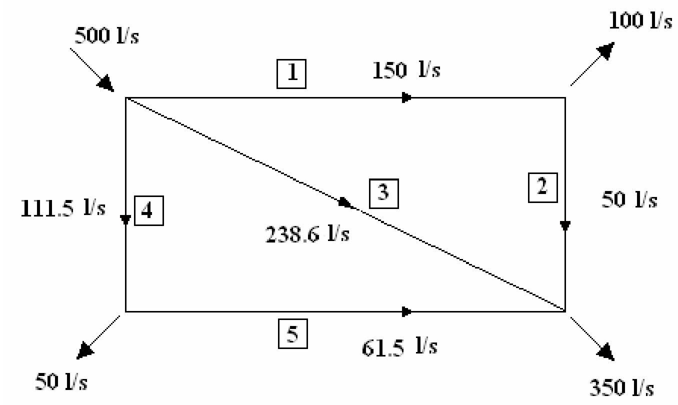


After third correction

Second iteration loop 2

PIPE	K	Q	h_f	h_f/Q
3	0.000570	228.27	42.6	0.187
5	0.006946	-71.8	-35.7	0.499
4	0.001698	-121.8	-25.2	0.207
			-18.4	0.892

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = -\frac{-18.4}{2 \text{ x } 0.892} = -10.3$$



Results after 2 iterations

D204 Q5 2004

(a) Compare and contrast the following two iterative calculation methods for complex networks of pipes.
(i) the head balance method (also known as the Hardy Cross or loop method).
(ii) the flow balance method (also known as the quantity balance or nodal method.

Explain briefly how and in what situation each of the methods may be used and state which of the correction methods shown at the end of this question is used in which method.

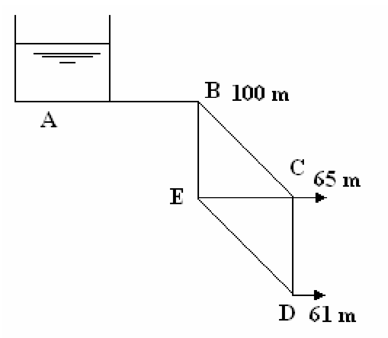
SOLUTION PART (a)
The nodal balance method is used for solving problems involving many pipes with a common junction where the total flow into the junction must be zero. The correction factor used for iteration is

$$\Delta H = \frac{2\,\Delta Q}{\sum Q/h_f}$$

The flow balance method is used for problems with multiple loops where the total head loss around a given loop is zero. The correction factor to be used is

$$\Delta Q = \frac{-\sum h_f}{2\sum h_f/Q}$$

(b) Water is supplied from a large reservoir at A to a pipe network BCDE as shown, in the diagram. The frictional resistances of the various pipes are given by the K value in the table which may be used with the formula $h_f = KQ^2$ to relate the magnitude of head loss h_f in the pipeline to the volumetric flow rate Q. Water is drawn at constant flow rates from the network at nodes C and D. The static heads (elevation + pressure head) at nodes B, C and D are 100m, 65m and 61m respectively above the local datum. Calculate the discharges at C and D and the water level in reservoir A. (The data has been added to diagram to aid the solution)



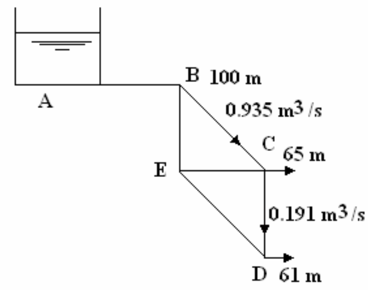
Use no more than 3 iterations and 3 significant figures

TABLE

Pipeline	AB	BC	CD	DE	CE	BE
K s^2m^5	4	40	110	25	25	35

SOLUTION PART (b)
The problem must be solved as two loops with a common pipe EC. First calculate the flows in known pipes.

BC $h_f = 35$ m $Q = (h_f/K)^{1/2} = (35/40)^{1/2} = 0.935\ m^3/s$
CD $h_f = 4$ m $Q = (h_f/K)^{1/2} = (4/110)^{1/2} = 0.191\ m^3/s$



The solution evolves around doing a flow balance at node E.

1st ITERATION Guess $h_E = 80$

PIPE	K	h_f	$Q = (h_f/K)^{1/2}$	Q/h_f
BE	35	20	0.756	0.0378 (into junction)
EC	25	-15	-0.775	0.0516 (out of junction)
ED	25	-19	-0.872	0.0349 (out of junction)
Totals			-0.89	0.135

$$\sum h_f = \frac{2\,\Delta Q}{\sum Q/h_f} = \frac{2 \text{ x } (-0.89)}{0.135} = -13.16$$

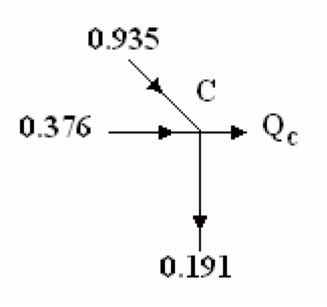
2nd ITERATION Guess h_E = 80 -13.16 = 66.84

PIPE	K	h_f	$Q = (h_f/K)^{1/2}$	Q/h_f
BE	35	20	0.0294	0.0378
EC	25	-15	-0.148	0.0516
ED	25	-19	-0.083	0.0349
Totals			0.219	0.26

$$\sum h_f = \frac{2\,\Delta Q}{\sum Q/h_f} = \frac{2 \text{ x } (0.219)}{0.26} = 1.686$$

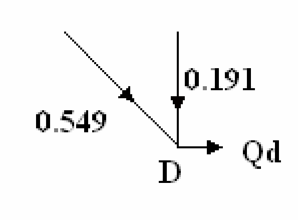
3rd ITERATION Guess h_E = 66.84 + 1.69 = = 68.53

PIPE	K	h_f	$Q = (h_f/K)^{1/2}$	Q/h_f
BE	35	20	0.948	0.0301
EC	25	-15	-0.376	0.106
ED	25	-19	-0.549	0.0729
Totals			0.0241	0.23



Further iterations will show only minor corrections giving flows of 0.945, -0.388 and -0.557. If these figures are used you get the answers given by the examiner.

We can now calculate Qc and Qd
0.935 + 0.376 – 0.191+ Qc = 0
Qc = -1.12 m^3/s



0.549 + 0.191 + Qd = 0
Qd = -0.74 m^3/s

Total flow from the reservoir is 1.12 + 0.74 = 1.86 m^3/s
Head loss pipe AB is $h_f = kQ^2 = 4 \text{ x } 1.86^2 = 13.8$ m
Head at entrance to pipe is 113.8 m

D204 Q5 2005

(a) A pipeline of diameter D = 0.5 m has a length L = 200 m, and the value of the Darcy friction λ may be assumed to have a constant value of 0.024. The pipeline contains two fully open valves, the local head loss at each of which is $0.2v^2/2g$, and three bends at each of which the head loss is $0.5v^2/2g$ where V is the velocity of water in the pipe. Calculate the value of K in the expression $h = K\,Q^2$ relating the total head loss h to the flow Q through the pipeline.

SOLUTION part (a)

Note $\lambda = 4C_f$

Straight pipe $h_f = \dfrac{4C_f L v^2}{2gD} = \dfrac{(0.024)(200)v^2}{2g(0.5)} = \dfrac{9.6v^2}{2g}$

Total for pipe line $h_f = \dfrac{9.6v^2}{2g} + \dfrac{2(0.2)v^2}{2g} + \dfrac{3(0.5)v^2}{2g} = \dfrac{11.5v^2}{2g}$

$v = \dfrac{Q}{A} = \dfrac{4Q}{\pi(.5)^2} = 5.093Q$

$h_f = \dfrac{11.5(5.093)Q^2}{2g} = 2.985Q^2$ hence $K = 2.985\ s^2/m^5$

(b) For a network of pipelines, such as that described in part (a), show that the flow correction term in an iterative head balance calculation is given by

$$\Delta Q = \frac{-\sum h_f}{2\sum (h_f/Q)}$$

SOLUTION part (b)

Starting with $h_f = K\,Q^n$ Normally n = 2 so $h_f = K\,Q^2$

Differentiate to get $dh_f = 2KQdQ = \dfrac{2KQ^2 dQ}{Q}$ and since $K\,Q^2 = h_f$

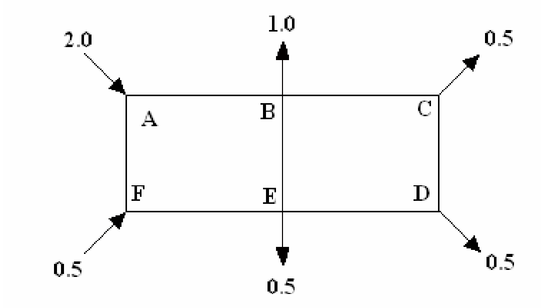
$$dh_f = \frac{2h_f dQ}{Q} \text{ or } dQ = \frac{Q dh_f}{2h_f}$$

If this relationship holds approximately true for finite changes then $\delta h_f = \dfrac{2h_f \delta Q}{Q}$ or $\delta Q = \dfrac{Q\,\delta h_f}{2h_f}$

In a balance of heads, the flow is corrected until $\Delta\theta = 0$ so the correction factor to be used for each pipe is $\delta Q = \dfrac{-Q\,\delta h_f}{2h_f} = \dfrac{-\delta h_f}{2\left(\dfrac{h_f}{Q}\right)}$ (The correction must be to reduce the flow rate).

For a network we must total all the terms to give the total correction factor of $\Delta Q = \dfrac{-\sum h_f}{2\sum (h_f/Q)}$

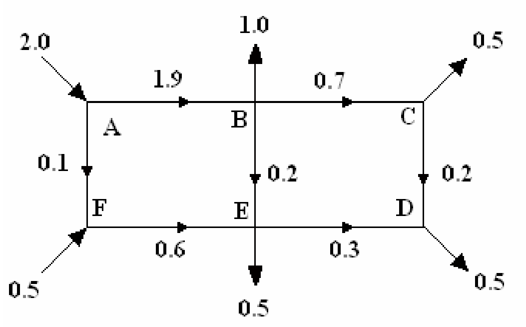
(c) The diagram shows two loops of a horizontal network with inflows and outflows in m^3/s. The K values of the seven pipes are given in the table. The pressure head at node A is 25 m. Calculate the flow rate through each pipe and the pressure head at each node. No more than two rounds of iteration are required, and final values of pressure heads may be rounded to the nearest metre.



Pipe	AB	BC	CD	DE	BE	EF	AF
K (s^2/m^5)	2	2	20	20	10	10	10

SOLUTION part (c)

The problem must be solved as two loops with a common pipe BE. First make a guess at the flow rates. Bear in mind that the net flow is zero at all nodes.



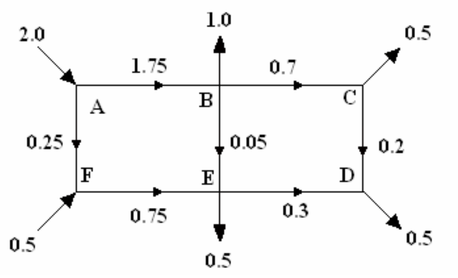
Data shown for initial guess

Start with loop ABEFA

PIPE	K	Q	h_f	h_f/Q
AB	2	1.9	7.22	3.8
BE	10	0.2	0.4	2
EF	10	-0.6	-3.6	6
FA	10	-0.1	-0.1	1
		Totals	3.92	12.8

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{3.92}{2 \times 12.8} = -0.153$$

Correct all flows in this loop by adding -0.153



First loop correction

Now do loop BCDEB

PIPE	K	Q	h_f	h_f/Q
BC	2	0.7	0.98	1.4
CD	20	0.2	0.8	4
DE	20	-0.3	-1.8	6
BE	10	0.04688	0.021973	0.46875
		Totals	0.001973	11.86875

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{0.002}{2 \times 11.869} = -0.000083$$

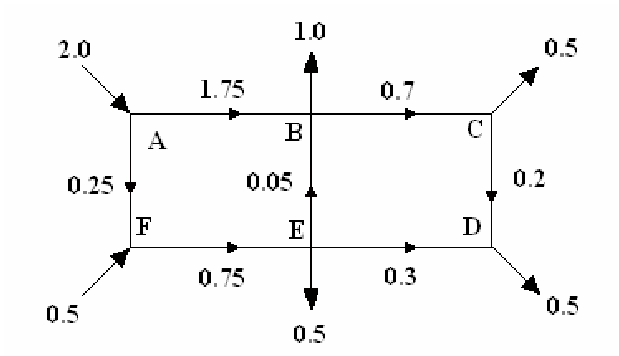
Correct loop 2 The initial guess was so good that the correction is minor

Second iteration of loop 1 is:

PIPE	K	Q	h_f	h_f/Q
AB	2	1.74688	6.10314	3.49375
BE	10	-0.0468	-0.02189	0.467919
EF	10	-0.7531	-5.67197	7.53125
FA	10	-0.2531	-0.64072	2.53125
			-0.23145	14.02417

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{0.231}{2 \times 14.02} = -0.00825$$

Final solution is



Pipe	AB	BC	CD	DE	BE	EF	AF
Q	1.75	0.7	0.2	-0.3	±0.05	-0.75	-0.25

Head at A is 25 m and rounding off the h_f values
Head B is 25 - 6 = 19 m
Head at C is 19 – 1 = 18 m
Head at D = 18 - 1 = 17 m
Head at E = 17 + 2 = 19 m or 19 - 0 = 19 m
Head at F = 19 + 6 = 25 m or 25 -1 = 24 m error due to rounding off values.

FLUID MECHANICS D209 Q1 1996

1 a) Describe with the aid of diagrams the variation of drag coefficient with Reynolds Number for the flow of a liquid past a completely immersed sphere.

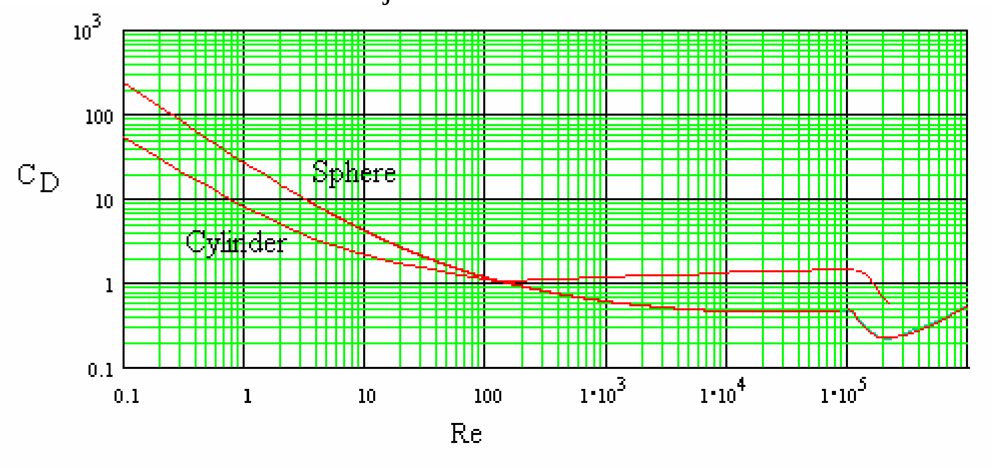
b) Spherical particles of density 2900 kg/m^3 moving horizontally enter water which is flowing upwards at a velocity of 0.27 m/s. It may be assumed that for each particle the relationship between drag coefficient C_D and Reynolds Number Re is

$$C_D = \frac{24}{R_e}\left[1 + 0.15R_e^{\ 0.687}\right]$$

Determine the diameter of the smallest particle that would move downwards.

c) Describe briefly the factors which affect the sedimentation in liquids of solid particles of different sizes.

a) Refer to tutorial 3 for notes on this subject.



b)

$$R = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} \qquad C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\pi\ d^2/4)(\rho\, u^2/2)}$$

$$C_D = \frac{4dg(\rho_s - \rho_f)}{3\rho_f\ u^2} = \frac{4d \text{ x } 9.81(2900 - 1000)}{3 \text{ x } 1000 \text{ x } 0.27^2} = 340.9d$$

$$C_D = \frac{24}{R_e}\left[1 + 0.15R_e^{\ 0.687}\right] = 340.9d$$

$$\frac{\mu}{\rho u d}\left[1 + 0.15\left(\frac{\rho u d}{\mu}\right)^{0.687}\right] = 14.2d = \frac{0.89 \text{ x } 10^{-3}}{1000 \text{ x } 0.27d}\left[1 + 0.15\left(\frac{1000 \text{ x } 0.27 \text{ x } d}{0.89 \text{ x } 10^{-3}}\right)^{0.687}\right]$$

$$4309197d^2 = \left[1 + 0.15(875.4d)^{0.687}\right]$$

$$d^2 - 203.1 \text{ x } 10^{-6} d^{0.687} - 232 \text{ x } 10^{-9} = 0$$

By plotting or any other means d = 0.0016 m or 1.6 mm

c) A moving stream can be used to separate small particles from large particles. Small particles take longer to settle out in a static pond and particles that are very tiny (colloidal) may never settle and simply colour the water.

(a) (i) Distinguish between impulse and reaction water turbines.

(ii) Describe three different types of reaction turbine and specify appropriate conditions under which each type of machine would be used.

(b) A turbine is required to work under a total head of water of 28 m and to operate at 7.14 rev/s. A one-quarter scale model of the proposed turbine is to be tested under a total head of water of 10.8 m.

(i) Determine the speed at which the model should be operated in order to predict the performance of the full scale turbine.

(ii) At the speed described in (i), the model develops 100 kW of power at a discharge of 1.085 m^3/s.
Calculate the corresponding power developed by the full-scale turbine.

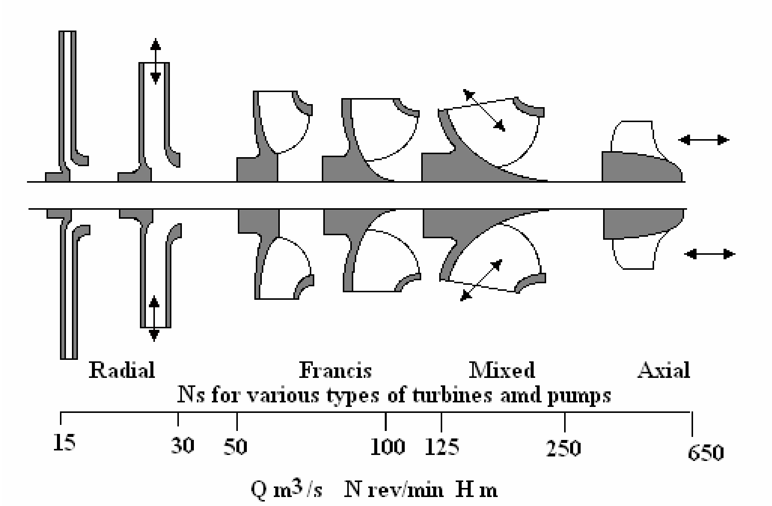
(iii) Calculate the specific speed, stating the units used, of the full-scale turbine and specify the type of machine.

IMPULSE – All the pressure is converted into Kinetic Energy in the nozzles and the KE is converted into mechanical power by the rotor.

REACTION – All the pressure is used in the rotor to accelerate the fluid over the vanes and the reaction force to this produces a torque and mechanical power.

In practice turbines like the Francis Wheel are partly impulse and partly reaction.

(b) The diagram illustrates the different types of turbines. On the left we have high pressure and on the right low pressure. The optimal efficiency for each occurs at a particular specific speed as indicated.



The Francis Wheel used Ns ≈ 75 and is used with fairly high heads and flow rates.

The mixed design uses lower heads and larger flow rates with Ns ≈ 200

The Axial flow (Kaplan) turbine uses the lowest head and is almost a free wheel propeller driven by the stream. Ns ≈ 600

The units for Head are m, speed is rev/min and flow is m^3/s

(b) H1 = 28 m N_1 = 7.14 rev/s

Model ¼ scale H_2 = 10.8 m

The dimensionless equation for turbines is $\frac{P}{\rho N^3 D^5} = \varphi\left(\frac{Q}{ND^3}\right)\left(\frac{g\,\Delta H}{N^2 D^2}\right)$

The head coefficient must be the same for both.

$$\left(\frac{g\,\Delta H}{N^2 D^2}\right)_1 = \left(\frac{g\,\Delta H}{N^2 D^2}\right)_2 \qquad \left(\frac{\Delta H}{N^2 D^2}\right)_1 = \left(\frac{\Delta H}{N^2 D^2}\right)_2$$

$$\left(\frac{28}{7.14^2 D_1^2}\right) = \left(\frac{10.8}{N^2 (D_1/4)^2}\right) \quad \left(\frac{28}{7.14^2}\right) = \left(\frac{10.8 \text{ x } 16}{N^2}\right)$$ hence N = 17.73 rev/s for the model.

The flow coefficients must also be the same

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2$$

$$Q_1 = Q_2 \frac{N_1 D_1^3}{N_2 D_2^3} = \frac{1.085 \text{ x } 7.184 \text{ x } 4^3}{17.73} = 27.82 \text{ m}^3/2$$

The Power Coefficients must be the same.

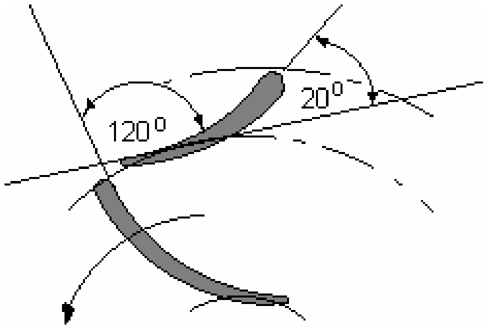
$$\left(\frac{P}{\rho N^3 D^5}\right)_1 = \left(\frac{P}{\rho N^3 D^5}\right)_2 \qquad P_1 = P_2 \frac{\rho N_1^3 D_1^5}{\rho N_2^3 D_2^5} = 100 \text{ x} \frac{7.14^3 \text{ x } 4^5}{17.73^3} = 6687.5 \text{ kW}$$

$$Ns = \left(\frac{N_1 Q_1^{1/2}}{H^{3/4}}\right) = \frac{7.14 \text{ x } 60 \text{ x } 27.82^{1/2}}{28^{3/4}} = 185.6$$

This would indicate the a mixed flow turbine (The official examiners answer is a Kaplan)

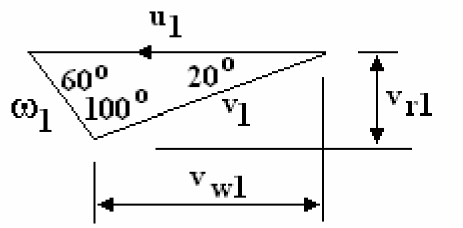
FLUID MECHANICS D203 Q10 1998

The runner (rotor) of a Francis turbine has a blade configuration as shown. The outer diameter is 0.45 m and the inner diameter is 0.3 m. The vanes are 62.5 mm high at inlet and 100 mm at outlet. The supply head is 18 m and the losses in the guide vanes and runner are equivalent to 0.36 m. The water exhausts from the middle at atmospheric pressure. Entry is shock less and there is no whirl at exit. Neglecting the blade thickness, determine:



i. The speed of rotation.
ii. The flow rate.
iii. The output power given a mechanical efficiency of 90%.
iv. The overall efficiency.
v. The outlet vane angle.

INLET
Useful head is 18 – 0.36 = 17.64 m

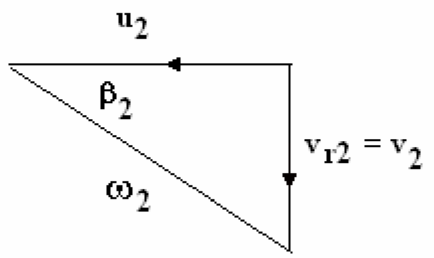


$m\ u_1\ v_{w1} = m\ u_2\ v_{w2}$
$u_1\ v_{w1} = u_2\ v_{w2}$

$(u_1\ v_{w1}/g) = \Delta H = 17.64$

sine rule $(v_1/\sin 60) = (u_1/\sin 100)$
$v_1 = 0.879\ u_1$
$(v_{r1}/\ v_1) = \sin 20$ $\qquad v_1 = 2.923\ v_{r1}$
Equate $\qquad 0.879\ u_1 = 2.923\ v_{r1} \qquad v_{r1} = 0.3\ u_1$
$v_{w1} = v_{r1}/\tan 20 = 0.824\ u_1$
$17.64 = u_1 \times 0.824\ u_1/g \qquad u_1^2 = 210 \quad u_1 = 14.5$ m/s
$v_{r1} = 0.3\ u_1 = 4.35$ m/s

EXIT



$u = \pi N D \quad N = u_1/\pi D_1 = u_2/\pi D_2$
$u_2 = u_1\ D_1/D_2 = 14.4 \times 300/450 = 9.67$ m/s
$N = u_1/\pi D_1 = 14.5 \times 60/(\pi \times 0.45) = 615$ rev/min
$v_r = Q/\pi Dh$
$v_{r1} = 4.35 = Q/\pi D_1 h_1 = Q/(\pi \times 0.45 \times 0.0625)$
$Q = 0.384\ m^3/s$
$v_{r2} = Q/\pi D_2 h_2 = Q/(\pi \times 0.3 \times 0.1) = 10.61\ Q = 4.08$ m/s
$4.08/9.67 = \tan \beta_2$
$\beta_2 = 22.8^o$

$P = m\ g\ \Delta H = 384 \times 9.81 \times 17.64 = 66.45$ kW

Output Power = 66.45 x 90% = 59.8 kW

Overall efficiency = $59800/(m\ g\ \Delta H) = 58805/(384 \times 9.81 \times 18) = 88.2\ \%$

FLUID MECHANICS D209 Q11 1996

It is required to pump water at a rate of 0.0160 m^3/s against a total head of 30.5 m. Four geometrically similar pumps, whose sizes are 100 mm, 125 mm, 225 mm and 300 mm, are available.

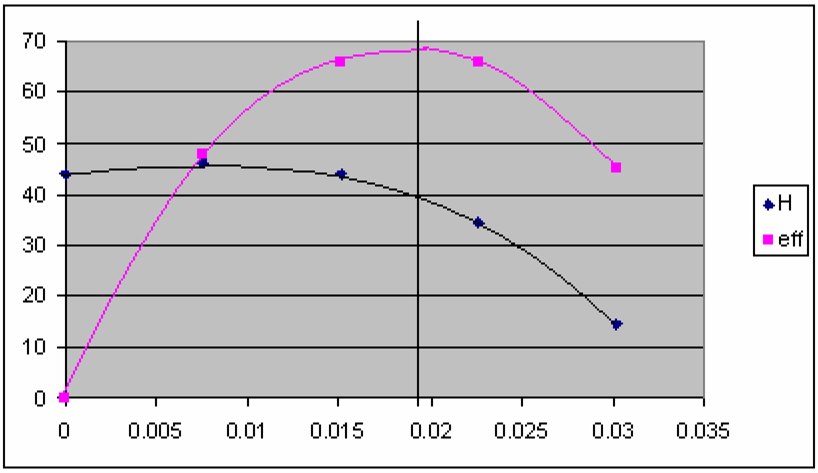
The characteristics of the 100 mm size pump, tested at 150 rad/s, are tabulated below.

Discharge	0	0.0076	0.0151	0.0226	0.0302	m^3/s
Head	43.9	46.1	43.9	34.2	14.6	m
Efficiency	0	48	66	66	45	%

(a) Determine which pump should be used, and the speed at which it should be driven, so that maximum possible efficiency is obtained.

(b) If, temporarily, only the 125 mm pump is available, determine the speed of operation and the input power from the motor, necessary to satisfy the head and discharge requirements.

By plotting the data for the 100 mm pump we can determine that the optimal point (for max efficiency) is when Q = 0.0188 m^3/s and H = 40 m. The peak efficiency is 68%



For the 100 mm pump H = 40 m Q = 0.0188 m^3/s N = 150 rad/s

$$Ns = \left(\frac{N_1 Q_1^{1/2}}{H^{3/4}}\right) = \frac{150 \text{ x } 0.0188^{1/2}}{40^{3/4}} = 1.293 \text{ rad/s} \quad (12.34 \text{ rev/min})$$

For the required condition

$$1.293 = \frac{N \text{ x } 0.016^{1/2}}{30.5^{3/4}} \text{ Hence } N = 131 \text{ rad/s (1251 rev/min)}$$

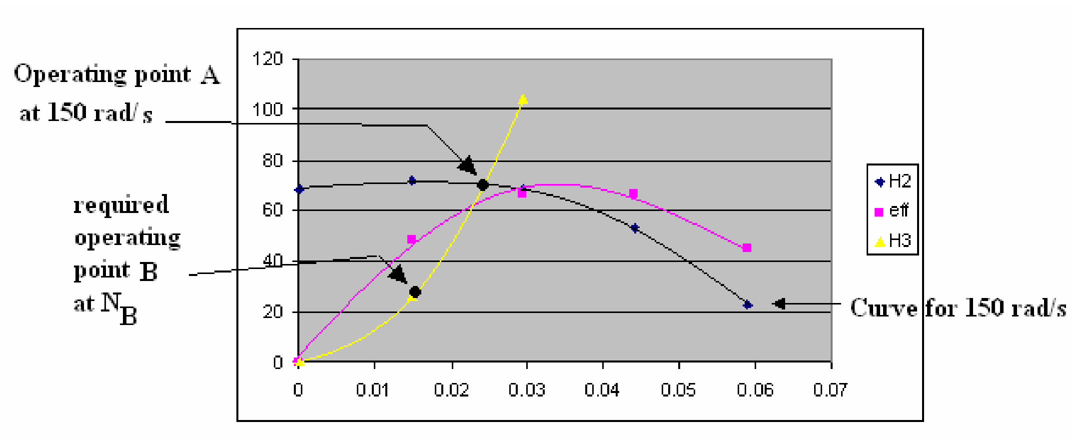
For the optimal size, remember that condition (1) is the optimal condition of the pump and condition (2) is the actual operating conditions.

Equating Flow Coefficients we get $\frac{D_2}{D_1} = \left(\frac{Q_2 N_1}{Q_1 N_2}\right)^{1/3} = \left(\frac{0.016 \text{ x } 150}{0.0188 \text{ x } 131}\right)^{1/3} = 1$

Equating head coefficients we get we get $\frac{D_2}{D_1} = \frac{N_1}{N_2}\sqrt{\frac{H_2}{H_1}} = \frac{150}{131}\sqrt{\frac{30.5}{40}} = 1$

The 100 mm seems to be the best.

(b) 125 mm pump at the same speed
The larger pump must slower to obtain the same flow. First calculate the corresponding flow and head for the 100 mm pump.



For the same Flow coefficient $Q_2 = 0.016 = Q_1\left(\frac{D_2}{D_1}\right)^3 = Q_1\left(\frac{125}{100}\right)^3 = 1.953Q_1 = 1.953 \text{ x } Q_1$

For the same Head coefficient $H_2 = 40 = H_1\left(\frac{D_2}{D_1}\right)^2 = H_1\left(\frac{125}{100}\right)^2 = 1.562H_1$

Q_1	0	0.0076	0.0151	0.0226	0.0302	m^3/s
H_1	43.9	46.1	43.9	34.2	14.6	m
Efficiency	0	48	66	66	45	%
Q_2	0	0.0148	0.0295	0.0441	0.059	
H_2	68.6	72	68.6	53.4	22.8	

Plotting H_2 and Q_2 gives the curve shown. It is assumed that the efficiency is unchanged.

As can be seen we cannot obtain the required operating point at 150 rad/s.
For the same flow coefficient between at two different speeds

$$\frac{Q_B}{N_B D_B{}^3} = \frac{Q_A}{N_A D_A{}^3} \qquad Q_B = Q_A\frac{N_B}{N_A}$$

For the same Head Coefficient at two different speeds

$$\frac{g H_A}{N_A{}^2 D_A{}^2} = \frac{g H_A}{N_A{}^2 D_A{}^2} \quad H_B = H_A\frac{N_B{}^2}{N_A{}^2} = H_A\frac{N_B{}^2}{N_A{}^2}$$

Substitute $\frac{N_B}{N_A} = \frac{Q_B}{Q_A}$ to eliminate the speed

$$H_B = H_A\left(\frac{Q_B}{Q_A}\right)^2 \quad H_A = H_B\left(\frac{Q_A}{Q_B}\right)^2$$ Where A and B correspond to different speeds.

For the case in hand let B be the values at the new speed and A the values at 150 rad/s

$$H_A = 30.5\left(\frac{Q_A}{0.016}\right)^2 = 119141Q_A^2$$

Calculate the flows at the new speed for the 125 mm pump.

Efficiency	0	48	66	66	45	%
Q_A	0	0.0148	0.0295	0.0441	0.059	
H_B	0	26	104			

Plotting H_B we get the result shown. We require the speed to produce operating point B for the same size (125 mm).

From the Flow Coefficient between points A and B.

$$\frac{Q_B}{N_B D_B^3} = \frac{Q_A}{N_A D_A^3}$$ $Q_A = 0.025 m^3/s$ and $H_A = 70$ m

$$\frac{0.016}{N_B} = \frac{0.025}{150}$$

$N_B = 96$ rad/s

Check by repeating the process with the head coefficient.

$$\frac{g H_B}{N_B^2 D_B^2} = \frac{g H_A}{N_A^2 D_A^2} \qquad N_B = N_A \sqrt{\frac{H_B}{H_A}} = 150\sqrt{\frac{30.5}{70}} = 99$$

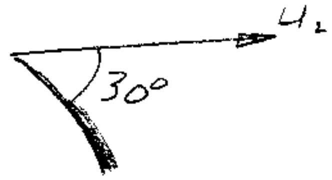
The efficiency at this point is 62%

Water Power = mgH = 16 x 9.81 x 30.5 = 4787 W

Power Input = WP/η = 4787/0.62 =7720 W

Q11 1990

ASS 8 Q2



$D = 170\,mm$

$b = 15\,mm$

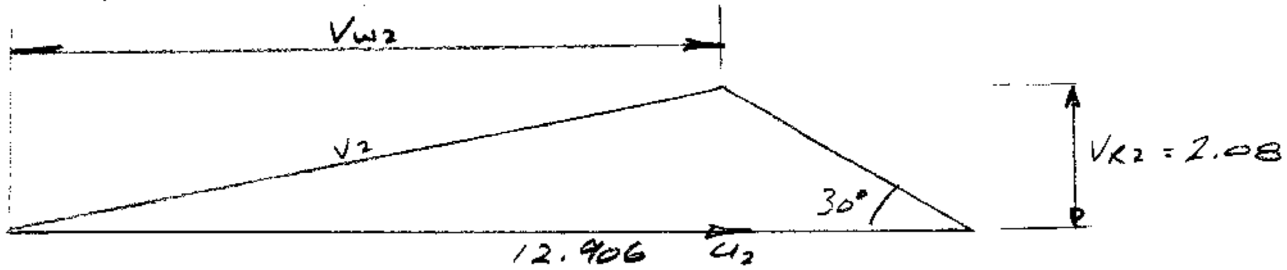
$N = 1450$ rev/min

$Q = 0.015\,m^3/s$

$K = .9$

$U_2 = \pi D N/60 = \pi \times .17 \times 1450/60 = 12.906\,m/s$

$V_{R2} = Q/A_2 = .015/(\pi \times .17 \times .015 \times .9) = 2.08\,m/s$



$V_{w2} = 12.906 - 2.08 \cot 30° = 9.3\ m/s$

$V_2^2 = 9.3^2 + 2.08^2 = 90.81$ $\quad V_2 = 9.53\,m/s$

KINETIC HEAD $= V_2^2/2g = 90.81/2g = 4.628\,m$

HEAD RECOVERED $= 35\% \times 4.628 = 1.62\,m$

HEAD LOST $= 3.00$

MANOMETRIC HEAD $= U_2 V_{w2}/g = 12.906 \times 9.3/g = 12.23\,m$

$\Delta h\,(\text{actual}) = 12.23 - 3 = 9.23\,m$

$\eta_{MAN} = 9.23/12.23 = 75.3\,\%$

$D.P. = \dot{m} U_2 V_{w2} = 15 \times 12.906 \times 9.3 = 1.8\ kW$

$W.P. = \dot{m} g \Delta h = 15 \times 9.81 \times 9.23 = 1.358\ kW$

$\eta = 1.358/1.8 = 75.4\,\%$

FLUID MECHANICS D203 Q11 1998

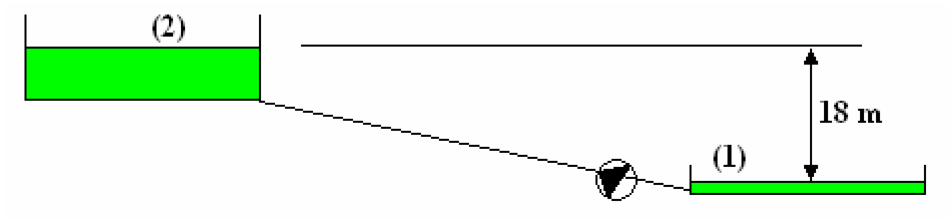
The water surface in a reservoir is 18 m above the water surface level of a river. The reservoir is to be supplied with a steady flow rate of 1100 1itres/min of water from the river using a centrifugal pump. The suction and delivery pipes will have a diameter of 100 mm and total equivalent length of 120 m. The friction factor f for the pipes may be assumed to be 0.020. Three geometrically similar pumps with impeller diameters of 165 mm, 182 mm and 214 mm respectively are available and test results for the 182 mm diameter impeller pump running at 3000 rev/min are given in the table.

(a) Determine which pump is the most appropriate to use for this application and give reasons for your choice.

(b) Calculate the pump speed which will match the supply requirements and determine the power required to drive the pump under these conditions. The water density is 1000 kg/m^3.

Table for 182 mm at 3000 rev/min

discharge q (litres/min)	0	500	1000	1500	2000	2500
head H (m)	43.8	42.5	38.8	33.0	25.2	16.3
overall efficiency η(%)	0	38	61	71	71	54



Bernoulli ΔH is the head added by the pump

$h_1 + z_1 + u_1^2/2g + \Delta H = h_2 + z_2 + u_2^2/2g + h_f +$ exit loss   velocity = 0 at free surface

$0 + 0 + 0 + \Delta H = 0 + 18 + 0 + h_f +$ exit loss

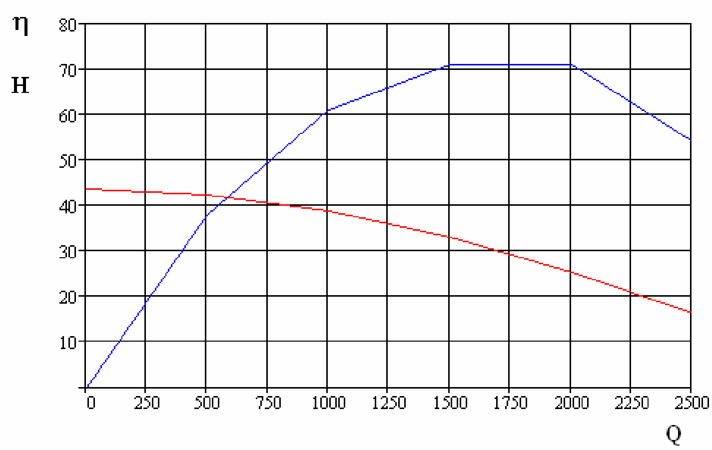
$\Delta H = 18 + h_f +$ exit loss

$$\Delta H = 18 + \frac{0.02 \times 120 \times u^2}{2g \times 0.1} + \frac{u^2}{2g} \qquad u = \frac{Q}{\pi \times 0.05^2} = 127.32Q$$

$\Delta H = 18 + 20656.7Q^2$ Given Q = 1.1/60 = 0.01833 m^3/s $\Delta H = 18 + 20656.7(0.01833)^2 = 24.94$ m

Plot the pump characteristic for 182 mm and 3000 rev/min

The optimal point is at 1750 litres/min (0.0292 m^3/s) with H = 30 m and η = 72% approx



The required Ns is

$$Ns = \frac{NQ^{1/2}}{H^{3/4}} = \frac{3000 \times 0.0292^{1/2}}{30^{3/4}} = 40$$

To achieve this, the speed must be changed to produce the required head and flow.

$$Ns = 40 = \frac{NQ^{1/2}}{H^{3/4}} = \frac{N \times 0.01833^{1/2}}{24.94^{3/4}}$$

N = 3296.6 rev/min

A higher speed means a smaller pump is required. Choose the 165 mm pump. We need to determine the operating characteristics of this pump when running at the same speed (3000 rev/min).

$$\frac{\Delta H_1}{N_1^2 D_1^2} = \frac{\Delta H_2}{N_2^2 D_2^2} \qquad \Delta H_2 = \frac{D_2^2}{D_1^2}\Delta H_1 = \left(\frac{165}{182}\right)^2 \Delta H_1 = 0.822\Delta H_1$$

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \qquad Q_2 = \frac{D_2^3}{D_1^3} Q_1 = \left(\frac{165}{182}\right)^3 Q_1 = 0.745 Q_1$$

Table for 165 mm at 3000 rev/min Efficiency assumed unchanged.

discharge q (litres/min)	0	372.5	745	1117.5	1490	1862.5
head H (m)	36	34.9	31.9	27.1	20.7	13.4
overall efficiency η(%)	0	38	61	71	71	54

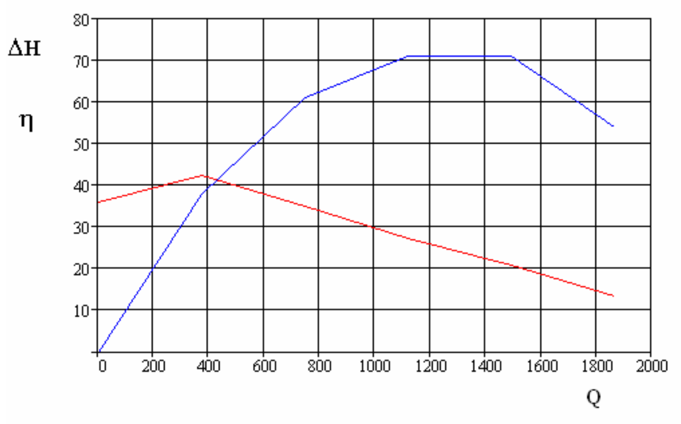
The optimal point is at 1300 l/min (0.0217 m^3/s) and 24 m head. The required Ns is

$$Ns = \frac{NQ^{1/2}}{H^{3/4}} = \frac{3000 \times 0.0217^{1/2}}{24^{3/4}} = 40.7$$

To achieve this, the speed must be changed to produce the required head and flow.

$$Ns = 40.7 = \frac{NQ^{1/2}}{H^{3/4}} = \frac{N \times 0.01833^{1/2}}{24.94^{3/4}}$$

N = 3357 rev/min



NB This work was not needed since for a geometrically similar pump we should have the same Ns (40) and hence the speed should be 3298 rev/min as calculated earlier.

The water power = mgΔH = 183.3 x 9.81 x 24.94 = 44846 W (The mass of 183 litres is 183 kg)

The power input = WP/η = 44846/0.72 = 62287 W

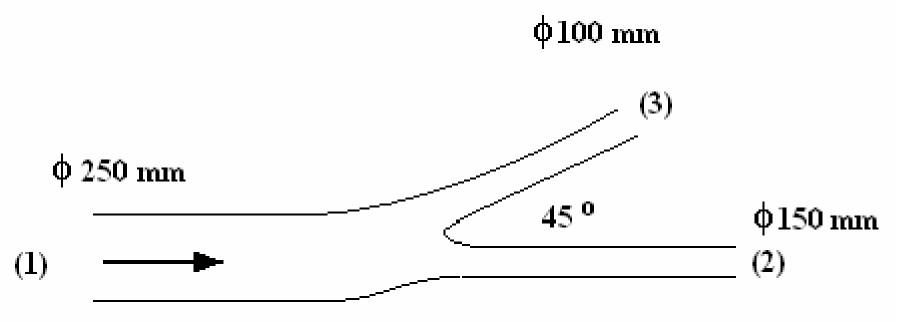
1 (a) State the conditions under which the Bernoulli equation is applicable.

(b) Water of density 1000 kg/m^3 is flowing into inlet (1) of the pipe-junction shown in the diagram. at a steady flow rate of 0.22 m^3/s. The volume of water in the junction is 0.016 m^3. The centre of the outlet (3) is situated 600 mm vertically above the main horizontal pipe running between (1) and (2). The water pressure at (1) is 230 kN/m^2 and at (2) is 200 kN/m^2; energy losses in the flow are considered negligible. Determine .

(i) the water pressure at (3).

(ii) the flows leaving the junction at (2) and (3).

(iii) the magnitude and direction of the force acting on the junction as a result of the flow.



D_1= 0.25m D_2= 0.15m D_3= 0.1m Q1= 0.22 m^3/s
p_1=230 x 10^3 N/m^2 p_2=200 x 10^3 N/m^2

$$A_1 = \frac{\pi D_1^2}{4} = 0.0491\,m^2 \quad A_2 = \frac{\pi D_2^2}{4} = 0.00177\,m^2 \quad A_3 = \frac{\pi D_3^2}{4} = 0.007854\,m^2$$

$$u_1 = \frac{Q_1}{A_1} = 4.482\ m/s$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (2)

$$p_1 + \rho\frac{u_1^2}{2} = p_2 + \rho\frac{u_2^2}{2}$$

$$230 \times 10^3 + 1000\frac{4.482^2}{2} = 200 \times 10^3 + 1000\frac{u_2^2}{2}$$

$$u_2 = 8.95\ m/s$$

FLOW RATE $Q_2 = A_2 u_2 = 0.158\,m^3/s$

CONSERVATION OF MASS $Q_3 = Q_1 - Q_2 = 0.22 - 0.158 = 0.616\,m^3/s \quad u_3 = \frac{Q_3}{A_3} = 7.847\ m/s$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (3)

$$p_1 + \rho\frac{u_1^2}{2} = p_3 + \rho\frac{u_3^2}{2} + \rho g z_3$$

$$230 \times 10^3 + 1000\frac{4.482^2}{2} = p_3 + 1000\frac{7.847^2}{2} + 1000 \times 9.81 \times 0.6$$

$$p_3 = 203.4 \times 10^3\ N/m^2$$

FORCES

Force at (1) $F_1 = m_1 u_1 + A_1 p_1 \rightarrow$

$F_1 = 11390$ N

Force at (2) $F_2 = m_2 u_2 + A_2 p_2 \leftarrow$

$F_2 = 4960$ N

Force at (3) $F_3 = m_3 u_3 + A_3 p_3$ at 45^o

F3 = 2080 N

Horizontal component = 2080 cos 45^o = 1470 N$\leftarrow$

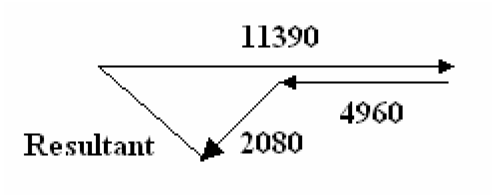
Vertical component = 2080 sin 45^o = 1470 N $\downarrow$

Total horizontal force = 11390 – 4960 – 1470 = 4960 N $\rightarrow$

Total vertical force = 1470 N $\downarrow$

RESULTANT FORCE = $\{49600^2 + 1470^2\}^{1/2}$ = 51731 N

Angle = $\tan^{-1}$ 1470/4960 = 16.5^o



(a)Describe the purpose and the operation of a surge shaft in a hydro-electric scheme.

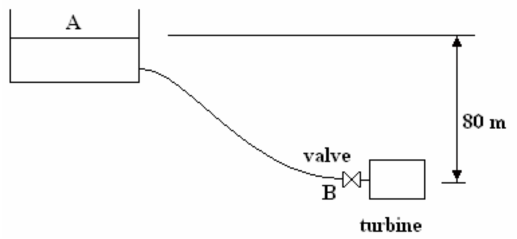
(b) A water supply dam has a hydro-electric power station installed at its foot to use the compensation flow to the river downstream of 10 m^3/s. The intake in the reservoir and the pipe-line to the turbine are equivalent to a circular pipe of diameter 2 m, length 420 m and friction factor $f = 0.01$. The head difference between the water level in the reservoir and that in the tailrace is 80 m.

Show how the flow and pressure conditions following the opening and closing, respectively, of the turbine valves determine the design requirements which have to be met if no surge shaft is installed. Assume that, in the pipeline, the velocity of sound c is 1432 m/s.

(a) The surge shaft is to protect the high pressure tunnel and penstock from pressure surges due to sudden or rapid closure of the valves. The pressure is turned into head and causes the level in the surge tank to rise and absorb the energy.

$A = \pi D^2/4 = \pi \times 2^2/4 = 3.142 \text{ m}^2$
$u = Q/A = 10/3.142 = 3.18 \text{ m/s}$

Bernoulli



$h_A + z_A + u_A^2/2g = h_B + z_B + u_B^2/2g + \text{Losses}$
$0 + 80 + 0 = h_B + 0 + 3.18^2/2g + \text{Losses}$
$80 = h_B + 0.52 + \text{Losses}$

Pipe loss $= f\,Lu^2/2gd$
$h_f = 0.01 \times 420 \times 3.18^2/2g \times 2 = 1.082 \text{ m}$

$h_B = 80 - 0.52 - 1.082 = 78.4 \text{ m}$ $p = \rho gh = 998 \times 9.81 \times 78.4 = 0.767 \times 10^6 \text{ Pa}$

The following answers may be gleaned from the examiners report.

The pipe should be designed for twice the static head. The flow should be bypassed around the valve when closed as the water still needs to be removed from the reservoir.

If the design pressure is twice the static head then the pressure rise is equivalent to 80 when the valve is closed so $\Delta p = 80 \times 9.81 \times 998 = 783 \text{ kPa}$

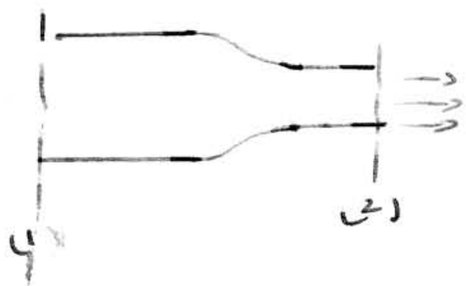
GRADUAL CLOSURE

$\Delta p = 783 \times 10^3 = \rho uL/t = 998 \times 3.142 \times 420/t$ $t = 1.87 \text{ s}$

The examiner says allow 6 s for closure and 2 s for opening.

Clearly there is more to the solution than this

Q1 2001



$\phi_1 = 40mm$
$\phi_2 = 25mm$
$\rho = 997 kg/m^3$

BERNOULLI $p_1 + \frac{\rho u_1^2}{2} = p_2 + \frac{\rho u_2^2}{2}$

$p_2 = 0$ (gauge pressure)
$p_1 = \frac{\rho}{2}(u_2^2 - u_1^2)$

$u_1 A_1 = u_2 A_2 \qquad u_1 = u_2 A_2/A_1 = u_2/r$

$r = A_1/A_2 = 40^2/25^2 = 2.56 \qquad u_1 = u_2/r$

$p_1 = \frac{\rho}{2}\left\{u_2^2 - \frac{u_2^2}{r^2}\right\} = u_2^2\left\{1 - 1/r^2\right\}$

$u_2^2 = \frac{2p_1}{\rho(1 - 1/r^2)} = \frac{2 \times 50 \times 10^3}{997(1 - 1/2.56^2)}$

$u_2^2 = 118.36 \qquad u_2 = 10.88\ m/s$
$u_1 = 10.88/2.56 = 4.25\ m/s$

$\dot{m} = \rho A_2 u_2 = 997 \times \frac{\pi \times .025^2}{4} \times 10.88$

$\dot{m} = \underline{5.284}\ kg/s$

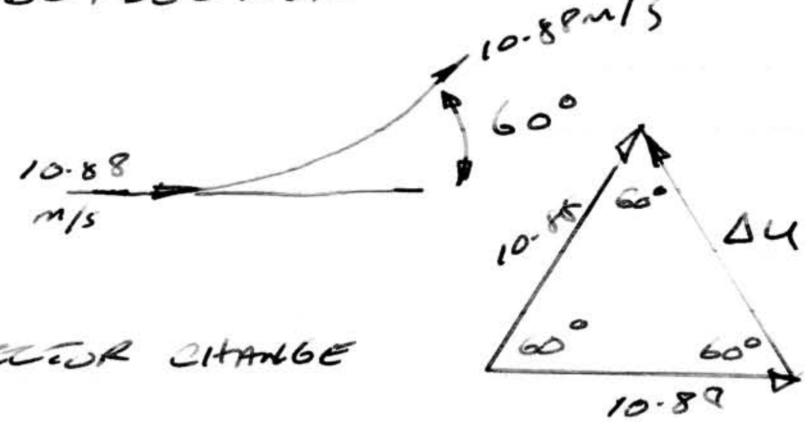
MOMENTUM FORCE $= \dot{m}\Delta u$
$= 5.284(u_2 - u_1)$
$= 5.284(10.88 - 4.249)$
$= 35N \leftarrow$

PRESSURE FORCE $= p_1 A_1 = 50 \times 10^3 \times \frac{\pi \times .04^2}{4}$

$= 62.83N \rightarrow$

NET FORCE $= \underline{\underline{27.83N}} \rightarrow$

DEFLECTION



VECTOR CHANGE

BECAUSE THE TRIANGLE IS 60°

$\Delta u = u_2 = 10.88$

$F = \dot{m}\Delta u = 5.284 \times 10.88 = 57.5\,N$

$F = \dot{m}\Delta u = \dot{m}\, u_2 = \rho A_2 u_2 \times u_2$

$F = \rho A_2 u_2^2$

$= \rho A_2 \times \dfrac{2P_1}{\rho(1 - 1/r^2)} = 200N \text{ (MAX)}$

$200 = 997 \times \dfrac{\pi \times .025^2}{4} \times \dfrac{2P_1}{997(1 - 1/2.56^2)}$

$\dfrac{200 \times 4 \times (1 - 1/2.56^2)}{\pi \times .025^2 \times 2} = P_1 \text{ (MAX)}$

$P_1 = 172.6\,kPa \text{ (MAX)}$

FLUID MECHANICS D209 Q2 1996

Planar irrotational flow past a Rankine body is produced by the combination of a uniform flow at velocity U in the positive x direction, a sink of strength Q = $2\pi m$ located on the x axis at x = +a and a source of the same strength on the x axis at x = -a.

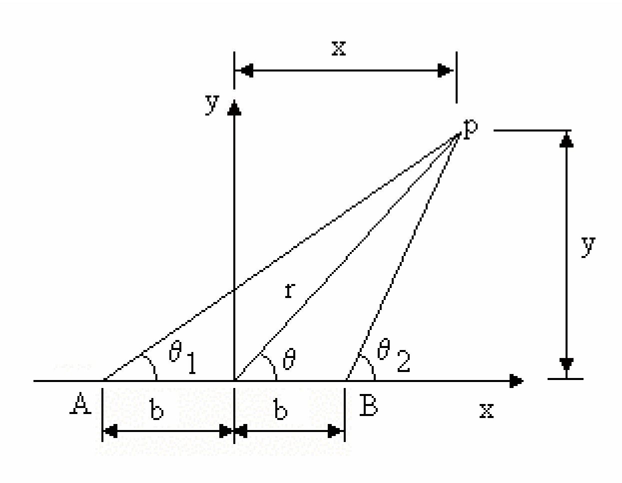
(a) Show for the above flow that the following expressions for the stream function ψ and velocity potential function ϕ apply (each term having its usual significance).
$\psi = m(\theta_1-\theta_2) + Ur\sin\theta$
$\phi = m\ln(r_1/r_2) + Ur\cos\theta$

(b) For the case where U = 4 m/s, Q = 2 m^2/s and a = 5 m, determine
(i) the length of the body
(ii) the width of the body.

(c) Sketch, without calculation, the variation of velocity and pressure, respectively, around the surface of the body.

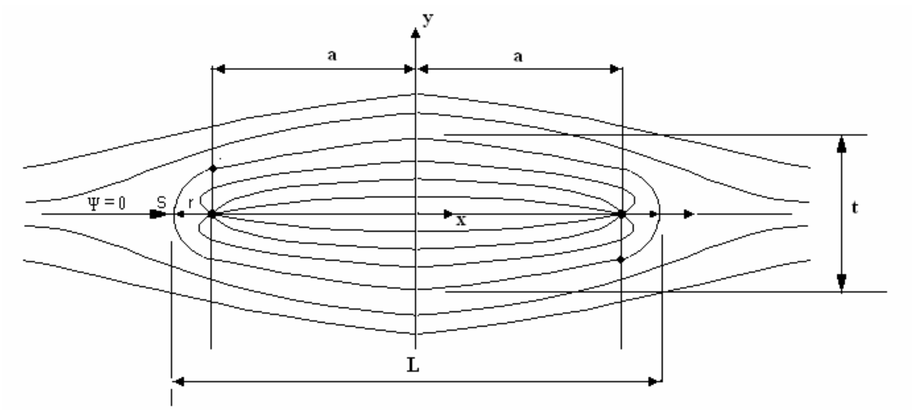
A doublet is formed when an equal source and a sink are brought close together. Consider a source and sink of equal strength placed at A and B respectively. The stream function for point P relative to A and B are respectively

$\Psi_B = m\theta_2$ for the source
$\Psi_A = -m\theta_1$ for the sink
$\Psi_C = -Uy = -Ur\sin\theta$ Uniform flow
$\Psi_P = \Psi_B + \Psi_A + \Psi_A = m(\theta_2 - \theta_1) - Ur\sin\theta$
$\Psi_P = -m(\theta_1 - \theta_2) + Ur\sin\theta$



The minus sign depends on the sign convention used.

$\phi_{AP} = m \ln r_1$
$\phi_{BP} = m \ln r_2$
Uniform flow $\phi = Ur\cos\theta$
Combined $\phi = Ur\cos\theta + m\ln(r_1/r_2)$
$Q/2\pi r = U \quad r = 1/4\pi$
$L = 2a + 2r = 10 + 1/2\pi = 10.16$ m
$t = Q/U = 2/4 = 0.5$ m



FLUID MECHANICS D203 Q2 1995

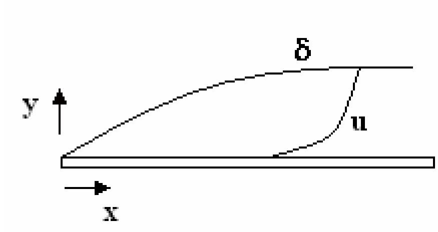
The velocity profile for flow over a flat plate with negligible pressure gradient in the flow direction may be approximated to $u/u_1=(3/2)(\eta) - (1/2)(\eta)^3$

$\eta = y/\delta$ and u is the velocity at a distance y from the plate and u_1 is the mainstream velocity. δ is the boundary layer thickness.

Discuss whether this profile satisfies appropriate boundary conditions.

Show the outline form of the derivation $C_f = 0.646 (R_{ex})^{-0.5}$ and evaluate the constant A.

SOLUTION



y = 0 u = 0 this is satisfied.
$y = \delta$ $u = u_1$ $\eta = 1$
$u/u_1=1 = (3/2)(1) - (1/2)(1)^3 = 1$ this is satisfied

$$\frac{du}{dy} = u\left\{\frac{3}{2}x\frac{1}{\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right\}$$

$$y = \delta \qquad \frac{du}{dy} = u\left\{\frac{3}{2\delta} - \frac{3}{2\delta}\right\} = 0 \text{ this is satisfied.}$$

$$\theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1 - \frac{u}{u_1}\right]dy \quad \text{leads to the solution } \theta = 39\delta/280$$

Without proof that $C_f = 2\, d\theta/dx$ leads to $C_f = (78/280)d\delta/dx = 2\tau_o/\rho u^2$

$$\tau_o = \mu\, (du/dy)_{y=0} = u\mu\left\{\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right\} = \left\{\frac{3\mu u}{2\delta}\right\}$$

$$\frac{78}{280}\frac{d\delta}{dx} = \frac{2}{\rho u^2}\left\{\frac{3\mu u}{2\delta}\right\}$$

$$\delta\, d\delta = \frac{280}{78}\left\{\frac{3\mu}{\rho u}\right\}dx$$

$$\frac{\delta^2}{2} + C = \frac{280}{78}\left\{\frac{3\mu x}{\rho u}\right\} \quad \text{but at } \delta = 0,\ x = 0 \text{ so } C = 0$$

$$\delta = \sqrt{21.538\left\{\frac{\mu x^2}{\rho u x}\right\}} = 4.64 x R_{ex}^{-1/2}$$

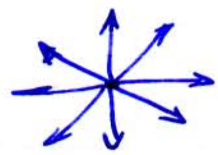
$\delta/x = 4.64\, R_{ex}^{-1/2}$

$$C_f = \frac{2\tau_o}{\rho u^2} = 2\frac{3\mu u}{2\delta\rho u^2} = \frac{3\mu}{\delta\rho u}$$

$$C_f = \frac{3\mu x}{\delta\rho u x} = \frac{3\mu}{\rho u x 4.64 R_{ex}^{-1/2}} = \frac{0.646}{R_{ex}R_{ex}^{-1/2}} = 0.646 R_{ex}^{-1/2}$$

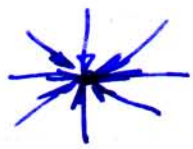
Q2 2001

a) SOURCE - FLOW EMERGING FROM A VERTICAL LINE 1M LONG AND SPREADING OUT RADIALLY IN ALL DIRECTION



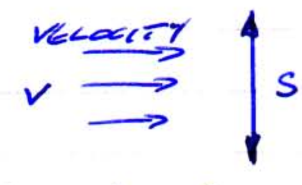
Flow emerging = $M \times 1$

SINK - OPPOSITE OF A SOURCE



DOUBLET - SOURCE AND SINK COMBINED AS THE GET VERY NEAR TO THE SAME POINT

STREAM FUNCTION ψ



THE FLUX ACROSS A LINE

$\psi = V \times S$ or $V\,dS$ if not const.

SINCE NO FLOW CROSSES A STREAM LINE CONSTANT VALUES OF ψ REPRESENT THE STREAM LINE

Q2 2001 CONT

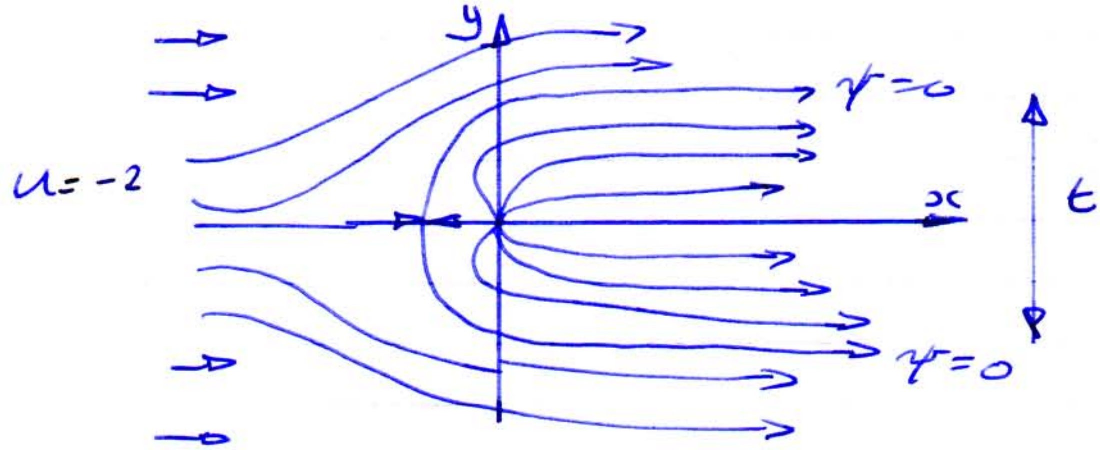
b) UNIFORM FLOW $\psi_1 = -Uy$

SOURCE $\psi_2 = \frac{m}{2\pi}\theta$

COMBINED FLOW $\psi = -Uy + \frac{m\theta}{2\pi}$

$y = r\sin\theta$ $\quad \underline{\underline{\psi = -Ur\sin\theta + \frac{m\theta}{2\pi}}}$

c)



ψ=0 IS THE DIVIDING SL

AT LARGE x FLOW IS UNIFORM

$0 = -Uy + \frac{m\theta}{2\pi}$ $\quad y = \frac{m\theta}{2\pi U}$

THE FLUX BETWEEN y=0 AND THE DIVIDING S.L. IS HALF OF THE TOTAL EMERGING FROM THE SOURCE

PUT $\theta = \pi$ (180°)

$\psi_2 = \frac{m}{2\pi} \times \pi = \frac{m}{2}$ $\quad \psi_1 = -Uy$

$\psi = 0 = -Uy + \frac{m}{2}$ $\quad y = \frac{m}{2U}$

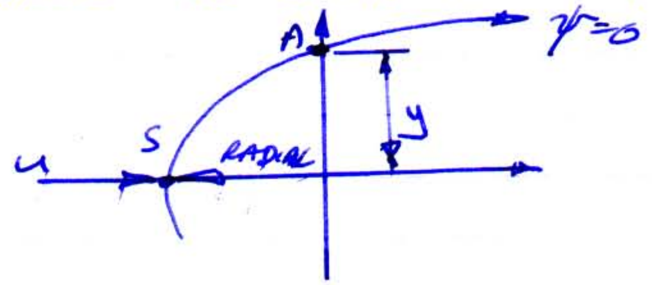
$t = 2y = \frac{2m}{2U} = \frac{m}{U} = \frac{8}{2} = 4\,m$

AT THE STAGNATION POINT u = RADIAL VELOCITY

RADIAL VELOCITY FROM SOURCE = $\frac{m}{2\pi r}$

$u = \frac{m}{2\pi r}$ $\quad r = \frac{m}{2\pi u} = \frac{8}{2\pi \times 2} = \frac{2}{\pi}$

DISTANCE TO STAGNATION POINT = $2/\pi$ m

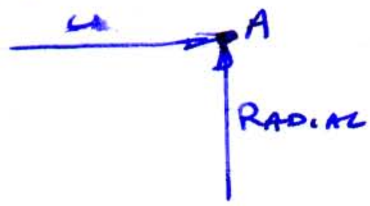


BERNOULLI BETWEEN S AND A

$\frac{\rho V_s^2}{2} + p_s = \frac{\rho V_A^2}{2} + p_A \qquad V_s = 0$ (STAGNATION)

$p_s - p_A = \frac{\rho V_A^2}{2}$

AT A WE HAVE TWO VELOCITIES



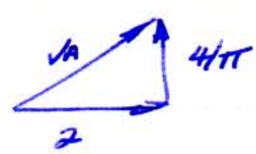
RADIAL VELOCITY $= m/2\pi r$

AT A $\psi = 0 = -uy + \frac{m\theta}{2\pi} \qquad \theta = \pi/2$

$y = \frac{m\theta}{2\pi u} = \frac{8 \times \pi/2}{2\pi \times 2 \times 2} = 1$ m

RADIAL VELOCITY $= 8/2\pi \times 1 = 4/\pi$ m/s

TRUE VELOCITY AT 'A'



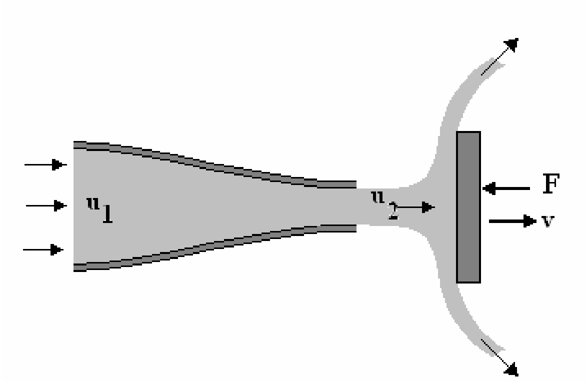
$V_A^2 = 2^2 + (4/\pi)^2 = 5.62$

$p_s - p_A = \frac{\rho V_A^2}{2} = \frac{800 \times 5.62}{2} = \underline{\underline{2248\ N/m^2}}$

FLUID MECHANICS D209 Q3 1996

A horizontal nozzle of diameter 20 mm is located at the end of a pipe 30 mm diameter. Water discharges to atmosphere at a rate of 2.5 dm^3/s as shown. There is no energy loss or contraction.

a) determine the force on the nozzle.
b) The jet strikes a flat plate as shown as flows off the plate at right angles to the jet.
Calculate the force on the plate if it is stationary.
Calculate the force on the plate if it moves at 5 m/s



Inlet to nozzle $u_1 = Q/A_1 = 2.5 \times 10^{-3} \times /\pi \times 0.03^2 = 3.537$ m/s
Exit from nozzle $u_2 = Q/A_2 = 2.5 \times 10^{-3} \times /\pi \times 0.02^2 = 7.96$ m/s
Bernoulli Use gauge pressures and $p_2 = 0$

$p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$
$p_1 = 0 + (\rho/2)(u_2^2 - u_1^2) = 500(7.96^2 - 3.537^2) = 25415$ N/m^2

Force Balance
$p_1A_1 + mu_1 = p_2A_2 + mu_2 + F$ m = 2.5 kg/s
$25415 A_1 + m(u_1 - u_2) = F$
$25415 \times \pi \times 0.03^2/4 + 2.5(3.537 - 7.96) = F$
17.96 – 11 = F = 6.96 N THIS IS THE FORCE ON THE NOZZLE.

STATIONARY VANE

$F = m \Delta U = 2.5(7.96 - 0) = 20$ N in the direction of the jet.

MOVING VANE

Mass striking the vane = $\rho A_2(u_2 - v)$

$F = m (u - v) = \rho A_2(u_2 - v) (u - v) = \rho A_2(u_2 - v)^2$

$F = 1000 \times (\pi \times 0.02^2/4) (7.96 - 5)^2 = 11.16$ N in the direction of the jet.

(a) Show by dimensional analysis that the drag D on a sphere diameter d moving constant velocity v through a stagnant fluid of density ρ and dynamic viscosity μ may be expressed as

$$D = \frac{\mu^2}{\rho}\varphi\left(\frac{\rho v d}{\mu}\right)$$

Demonstrate that Stokes' law D =3πμvd is consistent with this expression and state the circumstances that under which Stokes' law is valid.

(b) A sample of particulate material of relative density 2.65 settled 2500 mm in still water in 4.7 s. Assuming that the flow regime corresponds to flow past a sphere, for which a drag coefficient C_D = 0.44 applies, calculate the equivalent spherical diameter of the particles.

Demonstrate that, in fact, the constant drag coefficient flow regime is applicable.

(c) Water flowing at 10 m^3/s carries particulate material identical to that in (b). The material is carried in low concentration, but it is required to remove all the material from the flow. For this purpose a 10 m long settling channel of rectangular cross-section, and in which the particles must reach the bed within the length of the channel, is to be designed. The flow velocity must not exceed 0.50 m/s and it may be assumed that it does not vary with depth.
Determine the minimum appropriate width and depth for the settling channel.

D = function (d v ρ μ) = $K\ d^a\ v^b\ \rho^c\ \mu^d$

First write out the MLT dimensions.

$[D] = ML^1T^{-2}$

$[d] = L$ $\qquad ML^1T^{-2} = L^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1})^d$

$[v] = LT^{-1}$ $\qquad ML^1T^{-2} = L^{a+b-3c-d}\ M^{c+d}\ T^{-b-d}$

$[\rho] = ML^{-3}$

$[\mu] = ML^{-1}T^{-1}$

Viscosity is the quantity which causes viscous friction so the index associated with it (d) is the one to identify. We will resolve a,b and c in terms of d as before.

TIME $-2 = -b - d$ hence $b = 2 - d$ is as far as we can resolve b

MASS $1 = c + d$ hence $c = 1 - d$

LENGTH $1 = a + b - 3c - d$

$1 = a + (2 - d) - 3(1-d) - d$ hence $a = 2-d$

Next put these back into the original formula. $D = K\ d^{2-d}\ v^{2-d}\ \rho^{1-d}\ \mu^d$

Next group the quantities with same power together as follows :

$$D = K\{\rho v^2 d^2\}\{\mu\ \rho^{-1} v^{-1} d^{-1}\}^d$$

$$\frac{D}{\rho v^2 d^2} = \varphi\left(\frac{\rho v d}{\mu}\right)$$

How this becomes $D = \frac{\mu^2}{\rho}\varphi\left(\frac{\rho v d}{\mu}\right)$ is not known.

STOKES LAW

D =3πμvd if we multiply by $\frac{\rho\mu}{\rho\mu}$

$$D = 3\pi\ \mu\ v\ d\frac{\rho\mu}{\rho\mu} = 3\pi\frac{\mu^2}{\rho}\frac{\rho v\ d}{\mu} = \frac{\mu^2}{\rho}\varphi\frac{\rho v\ d}{\mu}$$ so this is consistent.

Stokes flow applies to $R_e < 0.2$

(b) $C_D = \frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2}$ u_t = 2.5/4.7 = 0.532 m/s ρ_f= 2650 kg/m^3

$$C_D = \frac{8d \ 9.81(2650 - 997)}{6 \text{ x } 997 \text{ x } 0.532^2} = 76.62d$$

0.44 = 76.62d hence d = 0.00574 m or 5.74 mm

Re =ρud/μ = 3420

Consistent with Newton flow since C_D = 0.44 Re is between 500 and 100000

(c) Q = Au A = 10/0.5 = 20 m^2

A = wD Time to cross the tank t = 10/0.5 = 20 s

Time to fall to bottom must be more.

Terminal velocity = 0.532 m/s

D = 0.532 x 20 = 10.64 m

W = 20/10.64 = 1.88 m

D203 FLUID MECHANICS

Q3 1999

$$u = u_x \frac{y}{\delta} \quad \frac{u}{u_x} = \frac{y}{\delta}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_x}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$\delta^* = \left[y - \frac{y^2}{2\delta} \right]_0^\delta$$

$$\delta^* = \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

$$\theta = \int_0^\delta \left(\frac{u}{u_x} - \left(\frac{u}{u_x}\right)^2 \right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2} \right) dy$$

$$\theta = \left[\frac{y^2}{2\delta} - \frac{y3}{3\delta^2} \right]_0^\delta = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$m = \rho A u_o = \rho\ 2h\ u_o$

At a point in the pipe $m = \rho A u_x$
$A = 2h - 2\delta^*$

$\rho\ 2h\ u_o = \rho(2h - 2\delta^*)u_x$
$2h\ u_o = (2h - 2\delta/2)u_x$
$2h\ u_o = (2h - \delta)u_x$
$2h\ u_o = 2h\ u_x - \delta\ u_x$
$2\ u_o = 2\ u_x - \delta\ u_x/h$
$(\delta/h)\ u_x = 2\ u_x - \delta\ u_o$

$$\frac{\delta}{h} u_x = 2u_x - 2u_0 = 2((u_x - u_0)$$

$$p_o + \frac{\rho u_o^2}{2} = p_f + \frac{\rho u_x^2}{2}$$

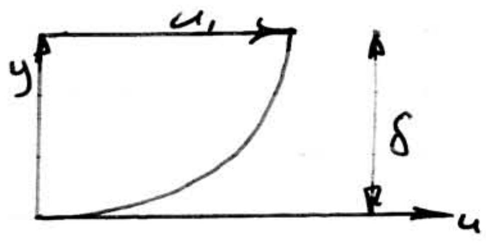
$$p_o - p_f = \frac{\rho u_x^2}{2} - \frac{\rho u_o^2}{2} = \frac{\rho}{2}\left(u_x^2 - u_o^2\right)$$

At point $\delta = h$ $\quad 2\left(1 - \frac{u_o}{u_x}\right) = 1 \quad u_x = 2\ u_o$

$$p_o - p_f = \frac{\rho}{2}\left(4u_o^2 - u_o^2\right)$$

$$\frac{p_o - p_f}{\rho u_o^2/2} = 3$$

Q3 2001



3 i) $\frac{u}{u_1} = \sin Ay$

BOUNDARY CONDITION $u = u_1$ @ $y = \delta$

$\frac{u_1}{u_1} = \sin A\delta = 1$ $A\delta = \pi/2$

$A = \pi/2\delta$

$\frac{u}{u_1} = \sin \frac{\pi y}{2\delta}$

ii) $\theta = \int_0^\delta \frac{u}{u_1}\left(1 - \frac{u}{u_1}\right)dy = \int_0^\delta \left[\frac{u}{u_1} - \left(\frac{u}{u_1}\right)^2\right]dy$

$\theta = \int_0^\delta \left[\sin\frac{\pi y}{2\delta} - \sin^2\frac{\pi y}{2\delta}\right]dy$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$ — TRIG IDENTITY

$\theta = \int_0^\delta \left[\sin\frac{\pi y}{2\delta} - \frac{1}{2} + \frac{1}{2}\cos\frac{\pi y}{\delta}\right]dy$

$\theta = \left[-\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta} - \frac{y}{2} + \frac{\delta}{2\pi}\sin\frac{\pi y}{\delta}\right]_0^\delta$

$\theta = \left[0 - \frac{\delta}{2} + 0\right] - \left[-\frac{2\delta}{\pi} - 0 + 0\right]$

$\theta = -0.5\delta + 0.6366\delta = 0.137\delta$

Q3 2001

iii WALL SHEAR STRESS $\tau_o = \left(\mu \frac{du}{dy}\right)_{y=0}$

$$\tau_o = \mu u_1 \frac{d(\sin\frac{\pi y}{2\delta})}{dy} = \mu u_1 \cos\frac{\pi y}{2\delta} \times \frac{\pi}{2\delta} \quad @ y=0$$

$$\tau_o = \frac{\mu u_1 \pi}{2\delta} \cos 0 = \frac{\mu u_1 \pi}{2\delta} \quad \text{---(1)}$$

$$C_f = \frac{2\tau_o}{\rho u_1^2} \quad \text{---(2)} \qquad \text{DEFINITION}$$

$$C_f = \frac{2 d\theta}{dx}. \quad \text{AND } \theta = 0.137\delta$$

$$C_f = \frac{2\, d(.137\delta)}{dx} = 2 \times 0.137 \frac{d\delta}{dx} \quad \text{---(3)}$$

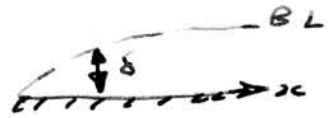
(2) → (3) $\frac{2\tau_o}{\rho u_1^2} = 0.274 \frac{d\delta}{dx}$

$$\tau_o = \rho u_1^2 \times .137 \frac{d\delta}{dx} \quad \text{---(4)}$$

(1) = (4) $\frac{\mu u_1 \pi}{2\delta} = 0.137 \rho u_1^2 \frac{d\delta}{dx}$

$$\frac{\mu \pi}{2 \times .137 \rho u_1} x = \frac{\delta^2}{2} + C \qquad C = \text{CONST of INT.}$$

@ $x = 0$ $\delta = 0$ $\therefore C = 0$



$$\delta/x = \left(\frac{2\pi}{2 \times .137}\right)^{1/2} \left(\frac{\mu}{\rho u_1 x}\right)^{1/2}$$

$$Re_x = \frac{\rho u_1 x}{\mu}$$

μ = dynamic viscosity
ρ = density
$\mu/\rho = \nu$ = kinematic visc

$$\delta/x = \frac{4.79}{Re_x^{1/2}}$$

$$C_f = \frac{2\tau_0}{\rho u_1^2} \qquad \tau_0 = \frac{\mu u_1 \Pi}{2\delta}$$

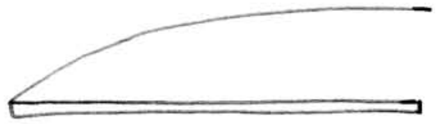
$$C_f = \frac{2\mu u_1 \Pi}{\rho u_1^2 \, 2\delta} = \frac{\mu \Pi}{\rho u_1 \delta} = \frac{\mu \Pi x}{\rho u_1 \delta x}$$

$$Re_x = \frac{\rho u_1 x}{\mu} \qquad C_f = \frac{1}{Re_x}\frac{\Pi x}{\delta}$$

SUBSTITUTE $\delta/x = 4.79\, Re_x^{-1/2}$

$$C_f = \frac{1}{Re_x} \times \Pi \times \frac{Re_x^{1/2}}{4.79}$$

$$C_f = \frac{\Pi}{4.79} Re_x^{-1/2} = \underline{\underline{0.65\, Re_x^{-1/2}}}$$

b) 

$x = 2.5\,m$

$\nu = 10^{-4}\, m^2/s$

$u_1 = 5\, m/s$

$$\frac{\delta}{x} = 4.79\, Re_x^{-1/2}$$

$$Re_x = \frac{\rho u_1 x}{\mu} = \frac{u_1 x}{\nu} = \frac{5 \times 2.5}{10^{-4}} = 12.5 \times 10^4$$

$$\delta = 2.5 \times 4.79 \times (12.5 \times 10^4)^{-1/2}$$

$$\delta = 0.03387\, m \qquad 33.87\, mm$$

(a) For laminar flow in a circular pipe, derive from first principles the following equation relating the head loss h_f, the pipe diameter d, the pipe length L, the mean velocity of flow u, the fluid density ρ, the fluid dynamic viscosity μ and the acceleration due to gravity g.

$$h_f = \frac{32\,\mu\,u\,L}{\rho g D^2}$$

(b) A 20 mm diameter, 5 m long pipe conveys oil of dynamic viscosity 1.20 N s/m^2 and density 900 kg/m^3 at a mean velocity of 0.30 m/s.
Show that the flow condition is laminar and determine

(i) The head loss
(ii) The centre line velocity
(iii) The radial location at which the velocity is equal to the mean velocity.

a) $C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$ Rearranging equation to make Δp the subject

$\Delta p = \frac{4C_f L\rho u_m^2}{2D}$ This is often expressed as a friction head h_f

$$h_f = \frac{\Delta p}{\rho g} = \frac{4C_f L u_m^2}{2gD}$$

This is the Darcy formula. In the case of laminar flow, Darcy's and Poiseuille's equations must give the same result so equating them gives

$$h_f = \frac{4C_f L u_m^2}{2gD} = \frac{32\mu 2\mu_m}{\rho g D^2}$$

b) Re = ρuD/μ = 900 x 0.3 x 0.2/1.2 = 4.5 and since this is much smaller than 2000 it must be laminar.

$$h_f = \frac{32\,\mu\,L\,u_m}{\rho g D^2} = \frac{32 \times 1.2 \times 0.3 \times 5}{900 \times 9.81 \times 0.02^2} = 16.3\,\text{m}$$

$\Delta p = \rho g h_f$ = 900x 9.81 x 16.3 = 144 kN/m^2

Centre Line velocity is twice the mean for laminar flow so u = 2 x 0.3 =0.6 m/s

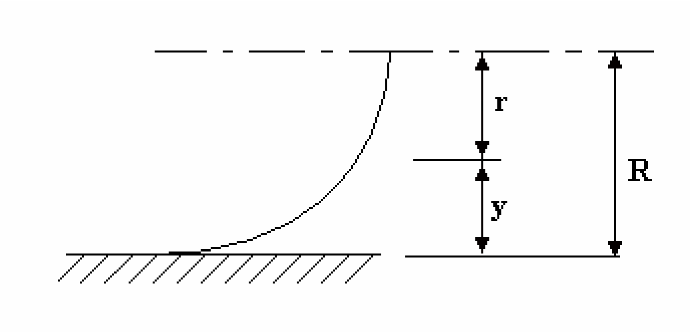
At any other radius it is given by

$$u = \frac{\Delta p\left(R^2 - r^2\right)}{4\mu L} \qquad 0.3 = \frac{144000\left(0.01^2 - r^2\right)}{4 \times 1.2 \times 5}$$

$50 \times 10^{-6} = 0.01^2 - r^2$

r = 0.00707 m or 7.07 mm

Q4 1999



For an elementary ring $dQ = 2\pi r\, dr\, u = 2\pi r\, dr\, u_1(y/R)^{1/n}$

$Y + r = R \quad r = R - y \qquad dr = -dy$

$dQ = 2\pi r\, u_1\,(R - y)(y/R)^{1/n}$

$$dQ = \frac{-2\pi u_1}{R^{1/n}} \int_R^0 \left(Ry^{1/n} - y^{1+1/n}\right) dy$$

$$Q = \frac{-2\pi u_1}{R^{1/n}} \left[\frac{Ry^{1+1/n}}{1 + 1/n} - \frac{y^{2+1/n}}{2 + 1/n}\right]_R^0$$

$$Q = \frac{2\pi u_1}{R^{1/n}} R^{2+1/n} \left[\frac{n}{n + 1} - \frac{n}{2n + 1}\right]$$

$$Q = 2\pi u_1 R^2 n \left[\frac{(2n + 1) - (n + 1)}{(n + 1)(2n + 1)}\right]$$

$u_m = Q/\pi R^2$

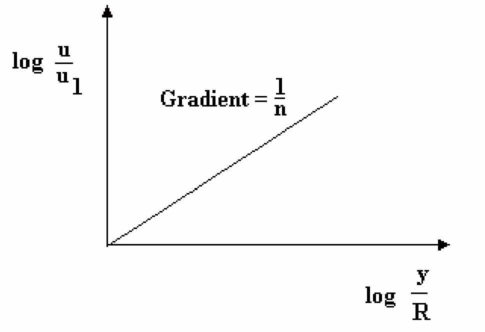
$$u_m = 2u_1 n \left[\frac{n}{(n + 1)(2n + 1)}\right]$$

$$\frac{u_m}{u_1} = \frac{2n^2}{(n + 1)(n + 2)}$$

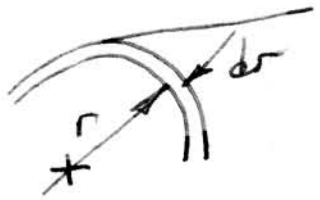
Part C

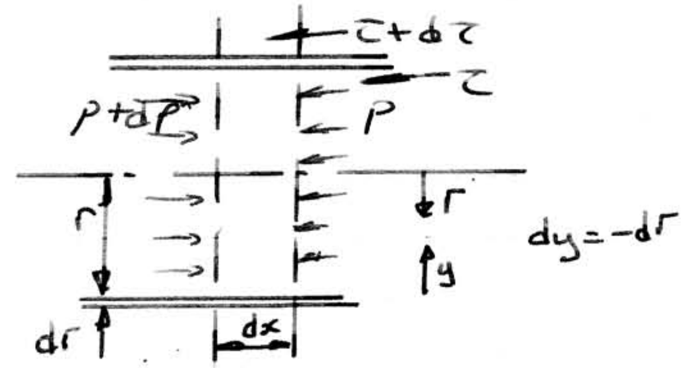
If $u/u_1 = (y/R)^{1/n}$ $\qquad \log u/u_1 = (1/n)\log(y/R)$

Plot



i) FORCE BALANCE ON AN ELEMENTAL CYLINDER





PRESSURE FORCE

$$= dp\left[\pi(r+dr)^2 - \pi r^2\right]$$

SHEARING $= (\tau + d\tau)(2\pi(r+dr))\,dx$

$- \tau \times 2\pi r\,dx$

EQUATE AND SIMPLIFY IGNORE 2nd ORDER

$$\frac{dp}{dx} = \frac{\tau}{r} + \frac{d\tau}{dr}$$

NEWTONIAN FLUID $\tau = \mu \frac{du}{dy} = -\mu \frac{du}{dr}$

$$\frac{dp}{dx} = -\frac{\mu}{r}\frac{du}{dr} + \frac{d\left(-\mu \frac{du}{dr}\right)}{dr}$$

$$\frac{r\,dp}{dx} = -\mu \frac{du}{dr} - \mu r \frac{d\left(\frac{du}{dr}\right)}{dr}$$

$$r\frac{dp}{dx} = -\mu \frac{du}{dr} - \mu r \frac{\left(\frac{du}{dr}\right)}{dr}$$

$$r\frac{dp}{dx} = -\mu \frac{du}{dr}\left\{1 + r\frac{du}{dr}\right\}$$

– SIGN SHOWS $\frac{dp}{dx}$ IS NEGATIVE

ie DERIVATION ASSUMES p decreases with x

ii) FULL DERIVATION SEE T~~1~~203-pdf page 22

$$-\frac{r}{\mu}\frac{dp}{dx} = \frac{du}{dr} + r\frac{d^2u}{dr^2} = \frac{d\left(r\frac{du}{dr}\right)}{dr}$$

INTEGRATE $$\frac{r\,du}{dr} = -\frac{r^2}{2\mu}\frac{dp}{dx} + A$$

$$\frac{du}{dr} = -\frac{r}{2\mu}\frac{dp}{dx} + \frac{A}{r}$$

INTEGRATE $$u = -\frac{r^2}{4\mu}\frac{dp}{dx} + A\ln r + B$$

BOUNDARY CONDITIONS FOR AN ANNULUS

$u = 0$ @ $r = R_1$ AND $r = R_2$

HENCE GO ON TO SHOW

$$A = \frac{1}{4\mu}\frac{dp}{dx}\frac{\{R_1^2 - R_2^2\}}{\ln R_1/R_2}$$

$$B = \frac{1}{4\mu}\frac{dp}{dx}\left[R_1^2 - \left\{\frac{R_2^2 - R_1^2}{\ln R_2/R_1}\right\}\ln R_1\right]$$

AND HENCE $$u = \frac{1}{4\mu}\frac{dp}{dx}\left[\frac{(R_2^2 - R_1^2)}{\ln R_2/R_1}\ln\frac{r}{R_1} + R_1^2 - r^2\right]$$

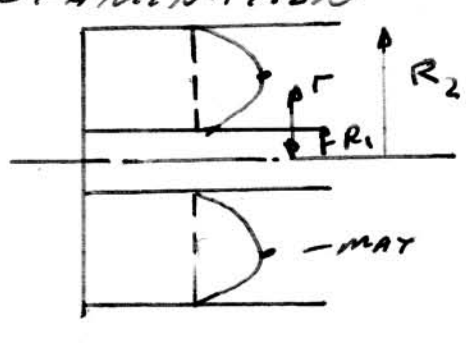
$$u = \frac{1}{4\mu}\frac{dp}{dx}\left[a\ln r/R_1 + R_1^2 - r^2\right]$$

$$= -\frac{1}{4\mu}\frac{dp}{dx}\left[r^2 - R_1^2 - a\ln R_1/r\right]$$

FULL DERIVATION WOULD TAKE FAR TO MUCH TIME IN AN EXAMINATION

FORMULAE YIELDS SAME RESULTS AS THE ONE IN THE QUESTION

VELOCITY DISTRIBUTION



Q4 2001

FOR MAX VELOCITY $\frac{du}{dr} = 0$

iii)

DIFF

$$\frac{du}{dr} = \frac{1}{4\mu}\frac{dp}{dx}\left[\frac{a}{r} - 2r\right] = 0$$

$$\frac{a}{r} = 2r \qquad \frac{a}{2} = r^2 \qquad r = \sqrt{\frac{a}{2}}$$

iv $R_1 = 0.1m \quad R_2 = 0.2m \quad \mu = 0.29\ Ns/m^2$

$$a = \frac{R_2^2 - R_1^2}{\ln R_2/R_1} = \frac{0.2^2 - 0.1^2}{\ln 0.2/0.1} = 0.04328$$

$$r = \sqrt{\frac{0.04328}{2}} = 0.147\,m$$

$dP/dx = 400\ N/m^3$

$$u = \frac{1}{4 \times .29} \times 400\left[0.04328 \ln\frac{.147}{.1} + .1^2 - .147^2\right]$$

$$= \frac{400}{4 \times .29} \times \sout{.0266674}\ 0.005065 = 1.747\,m/s$$

Q4 2008

a A circular pipe has a vertical axis. Oil spills over the open top of the pipe at a steady rate and flows down the outside of the pipe under gravity, forming a symmetrical and continuous film. A short distance down the outside of the pipe from the open top the film becomes fully developed with a constant film thickness. By choosing an axi-symmetric element of fluid in the fully developed region of the oil film, show that the following equation applies for laminar flow in the film

$$\frac{d}{dr}\left(r\frac{dv}{dr}\right) = -\frac{\rho g r}{\mu}$$

where v is the fluid velocity at radius r in the film, ρ is the density and μ the dynamic viscosity of the fluid and g the gravitational acceleration.

b. Using the result in part a above and assuming negligible drag on the oil by the surrounding air, show that the fluid velocity v at radius r in the fully developed film s given by

$$\frac{\rho g r_f^2}{4\mu}\left[2ln\left(\frac{r}{r_p}\right) - \left(\frac{r^2 - r_p^2}{r_f^2}\right)\right]$$

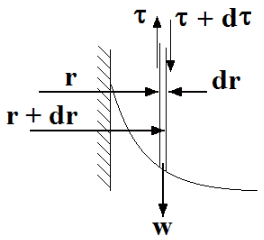
where r_f and r_p are the radii at the film surface and the pipe outside surface respectively.

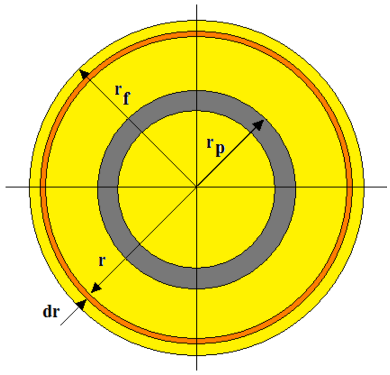
c. Calculate the volume flow rate of oil required to maintain a film thickness of 5mm when the outside diameter of the pipe is 100 mm, given that μ = 0.052 N s/m and ρ = 870 kg/m^3 for the oil.

Note: $\int x ln x dx = \frac{1}{4}x^2(2lnx - 1)$

SOLUTION

The key to this is recognising that the weight of the oil has to overcome the viscous drag. In an exam, don't spend too much time on part 1 if you can't get the correct answer. Go on to part 2 and 3 which you can do using the information provided in the question.





Consider an elementary thin cylindrical layer that makes an element of flowing down the outside of the pipe. The length is δx , the inside radius is r and the radial thickness is dr. The weight of the element is w and the shear stress on the surface increases by dτ from the inner to the outer surface. The velocity at any point is v and the dynamic viscosity is μ.

The weight is ρ g x volume $w = \rho g \delta x \{\pi(r+dr)^2 - \pi r^2\}$

The shear force opposing is $F = \{(\tau+\delta\tau)(2\pi)(r+dr) - \tau 2\pi r\}\delta x$

Force balance gives $\rho g \{\pi(r + dr)^2 - \pi r^2\} + \{(\tau+\delta\tau)(2\pi)(r+dr) - \tau 2\pi r\}\delta x = 0$

$$\rho g \delta x \pi\{(r + dr)^2 - r^2\} + 2\pi\{(\tau+d\tau)(r+dr) - \tau r\}\delta x = 0$$

$$\rho g \{(r + dr)^2 - r^2\} + 2\{(\tau+d\tau)(r+dr) - \tau r\} = 0$$

Multiply out $\rho g\{r^2 + (dr)^2 + 2rdr - r^2\} + 2(\tau r + \tau dr + d\tau\, r + d\tau dr - \tau r) = 0$

Ignore small products $\rho g\{rdr\} + (\tau dr + d\tau\, r) = 0$

$$-\rho g r = \tau + r\frac{d\tau}{dr}$$

Substitute $\tau = \mu\frac{dv}{dr}$

$$-\rho g r = \mu\frac{dv}{dr} + \mu r\frac{d\left(\frac{dv}{dr}\right)}{dr}$$

$$-\frac{\rho g r}{\mu} = \frac{dv}{dr} + r\frac{d\left(\frac{dv}{dr}\right)}{dr}$$

Partial differentiation shows that $\frac{d}{dr}\left(r\frac{dv}{dr}\right) = \frac{dv}{dr} + r\frac{d_2 v}{dr^2} = \frac{dv}{dr} + r\frac{d\left(\frac{dv}{dr}\right)}{dr}$

Hence $-\frac{\rho g r}{\mu} = \frac{d}{dr}\left(r\frac{dv}{dr}\right)$

b. $$-\frac{\rho g r}{\mu} = \frac{d}{dr}\left(r\frac{dv}{dr}\right)$$

Integrate $$-\frac{\rho g r^2}{2\mu} + A = r\frac{dv}{dr}$$

The gradient is zero when r = r_f $$-\frac{\rho g r_f^2}{2\mu} + A = 0 \qquad \frac{\rho g r_f^2}{2\mu} = A$$

$$-\frac{\rho g r^2}{2\mu} + A = r\frac{dv}{dr} \qquad -\frac{\rho g r}{2\mu} + \frac{A}{r} = \frac{dv}{dr}$$

Integrate

$$-\frac{\rho g r^2}{4\mu} + A ln r + B = v$$

$$-\frac{\rho g r^2}{4\mu} + \frac{\rho g r_f^2}{2\mu} ln r + B = v$$

v = 0 when r = r_p

$$-\frac{\rho g r_p{}^2}{4\mu} + \frac{\rho g r_f^2}{2\mu} ln r_p + B = 0$$

$$\frac{\rho g r_p{}^2}{4\mu} - \frac{\rho g r_f^2}{2\mu} ln r_p = B \qquad \frac{\rho g}{4\mu}\left(r_p{}^2 - 2r_f^2 ln r_p\right) = B$$

$$-\frac{\rho g r^2}{4\mu} + \frac{\rho g r_f^2}{2\mu} ln r + \frac{\rho g}{4\mu}\left(r_p{}^2 - 2r_f^2 ln r_p\right) = v$$

$$\frac{\rho g}{4\mu}\left(-r^2 + 2r_f^2 ln r\right) + \left(r_p{}^2 - 2r_f^2 ln r_p\right) = v$$

$$\frac{\rho g}{4\mu}\left(r_p{}^2 - r^2\right) + 2r_f^2\left(ln r - ln r_p\right) = v$$

$$\boldsymbol{\frac{\rho g r_f^2}{4\mu}\left\{2\left(ln\frac{r}{r_p}\right) - \left(\frac{r^2 - r_p{}^2}{r_f^2}\right)\right\} = v}$$

c. Volume flow rate through the elementary ring is v (2πr dr)

Total flow is $$Q = \frac{\pi\rho g r_f^2}{2\mu}\int_{r_f}^{r_p}\left\{2r\left(ln\frac{r}{r_p}\right) - r\left(\frac{r^2 - r_p{}^2}{r_f^2}\right)\right\}dr$$

$$Q = \frac{\pi\rho g r_f^2}{2\mu}\int_{r_f}^{r_p}\left\{2\left(r ln\frac{r}{r_p}\right) - \frac{r^3}{r_f^2} + \frac{r\, r_p{}^2}{r_f^2}\right\}dr$$

$$Q = \frac{\pi\rho g r_f^2}{2\mu}\left[\left(r^2 ln\left(\frac{r}{r_p}\right) - \frac{r^2}{2}\right) - \frac{r^4}{4r_f^2} + \frac{r^2\, r_p{}^2}{2r_f^2}\right]_{r_f}^{r_p}$$

$$Q = \frac{\pi(870)(9.81)(0.055)^2}{2(0.052)}\left[\left(r^2 ln\left(\frac{r}{r_p}\right) - \frac{r^2}{2}\right) - \frac{r^4}{4r_f^2} + \frac{r^2\, r_p{}^2}{2r_f^2}\right]_{r_f}^{r_p}$$

$$Q = 878.48\left[r^2 ln\left(\frac{r}{r_p}\right) - \frac{r^2}{2} - \frac{r^4}{4r_f^2} + \frac{r^2\, r_p{}^2}{2r_f^2}\right]_{0.055}^{0.05}$$

Evaluate and Q = 2.667 x 10^{-3} m^3/s

5 A fluid of density p flows at constant pressure along a flat plate. The velocity u, at a distance y from the plate, within the boundary layer is

$$\frac{u}{u_1} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$$

where u_1 is the main stream velocity and δ is the boundary layer thickness.

(a) Define the terms displacement thickness δ* and momentum thickness θ and show that

$$\theta = 2\delta/15$$

(b) Explain, in outline only, the derivation of the following equation for the shear stress τ_o on the plate

$$\tau_o = \rho u_1^2\left(\frac{d\theta}{dx}\right)$$ where x is the distance along the plate from its leading edge.

(c) From the above relationships, show that

$$\frac{\delta}{x} = \left(\frac{30\mu}{\rho u_1 x}\right)^{0.5}$$

a) The flow rate within a boundary layer is less than that for a uniform flow of velocity u_1. The reduction in flow is equal to the area under the curve in fig.2.3. If we had a uniform flow equal to that in the boundary layer, the surface would have to be displaced a distance δ* in order to produce the reduction. This distance is called the displacement thickness and it is given by :

$$\text{flow redution} = \int_0^\delta [u_1 - u]dy = u_1\delta^*$$

If this is equivalent to a flow of velocity u_1 in a layer δ* thick then :

$$\delta^* = \int_0^\delta \left[1 - \frac{u}{u_1}\right]dy$$

The momentum in a flow with a BL present is less than that in a uniform flow of the same thickness. The momentum in a uniform layer at velocity u_1 and height h is $\rho h u_1^2$. When a BL exists this is reduced by $\rho u_1^2\theta$. Where θ is the thickness of the uniform layer that contains the equivalent to the reduction. Using the same reasoning as before we get :

$$\theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1 - \frac{u}{u_1}\right]dy$$

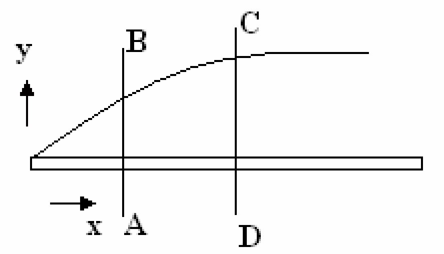
$$\frac{u}{u_1} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad \theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1 - \frac{u}{u_1}\right]dy \quad \text{substitute } \eta = y/\delta \quad \delta d\eta = dy$$

$$\frac{u}{u_1} = 2\eta - \eta^2 \qquad \theta = \delta\int_0^\delta [2\eta - \eta^2][1 - 2\eta + \eta^2]d\eta \qquad \theta = \delta\int_0^\delta [2\eta - 5\eta^2 + 4\eta^3 - \eta^4]d\eta$$

$$\theta = \delta\left[\eta^2 - \frac{5\eta^3}{3} + \eta^4 - \frac{\eta^5}{5}\right]_0^\delta = \delta\left[\left(\frac{y}{\delta}\right)^2 - \frac{5}{3}\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4 - \frac{1}{5}\left(\frac{y}{\delta}\right)^5\right]_0^\delta$$

$$\theta = \delta\left[(1)^2 - \frac{5}{3}(1)^3 + (1)^4 - \frac{1}{5}(1)^5\right] - \delta[0^5] = \frac{2}{15}\delta$$

b) By considering the momentum and mass entering the across BC it can be shown that



$$\tau_o = \rho u_1^2 \left(\frac{d\theta}{dx}\right)$$

$$C_f = \frac{2\tau_w}{\rho u^2} = 2\frac{d\theta}{dx}$$

c) $\frac{d\theta}{dx} = \frac{2}{15}\frac{d\delta}{dx}$ $\qquad \tau_o = \rho u_1^2 \frac{2}{15}\frac{d\delta}{dx}$

$$\tau = \mu \frac{du}{dy} \qquad \tau_w = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$u = u_1\left[2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right]$ $\qquad \frac{du}{dy} = u_1\left[2\eta - (\eta)^2\right]$ at y = 0

At y = 0 $\frac{du}{dy} = \frac{2u_1}{\delta}$ $\qquad \tau_o = \mu\, u_1 \frac{2}{\delta}$

$$\tau_o = \mu\, u_1^2 \frac{2}{15}\frac{d\delta}{dx} = \frac{\mu u_1^2}{\delta}$$

$$\delta d\delta = \frac{2\mu\mu_1}{\rho u_1^2}\frac{15}{2}dx$$

$\frac{\delta^2}{2} = \frac{15\mu}{\rho u_1}x + C$ but at x = 0, δ = 0 so C = 0

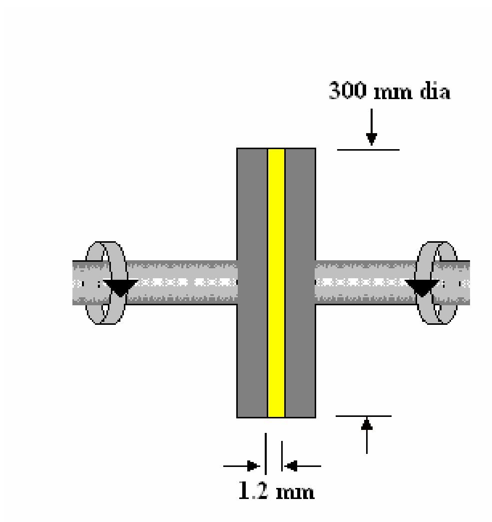
$$\delta^2 = \frac{30\mu}{\rho u_1}x$$

$$\delta^2 = \frac{30\mu}{\rho u_1}\frac{x^2}{x}$$

$$\frac{\delta^2}{x^2} = \frac{30\mu}{\rho u_1 x}$$

$$\frac{\delta}{x} = \left(\frac{30\mu}{\rho u_1 x}\right)^{0.5}$$

A simple fluid coupling consists of two parallel round discs of radius R separated by a a gap h. One disc is connected to the input shaft and rotates at ω_1 rad/s. The other disc is connected to the output shaft and rotates at ω_2 rad/s. The discs are separated by oil of dynamic viscosity μ and it may be assumed that the velocity gradient is linear at all radii.

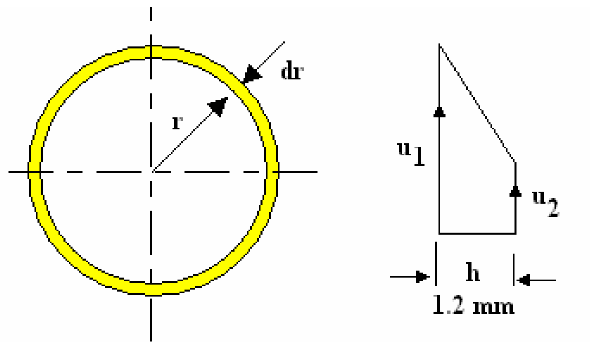


Show that the Torque at the input shaft is given by $T = \dfrac{\pi D^4 \mu(\omega_1 - \omega_2)}{32h}$

The input shaft rotates at 900 rev/min and transmits 500W of power. Calculate the output speed, torque and power. (747 rev/min, 5.3 Nm and 414 W)

Show by application of max/min theory that the output speed is half the input speed when maximum output power is obtained.

SOLUTION



Assume the velocity varies linearly from u_1 to u_2 over the gap at any radius. Gap is h = 1.2 mm

$T = \mu\, du/dy = \mu\,(u_1 - u_2)/h$

For an elementary ring radius r and width dr the shear force is

Force = τ dA = τ 2πr dr

$$dF = \mu \frac{u_1 - u_2}{h} x 2\pi r\ dr$$

Torque due to this force is $$dT = rdF = \mu \frac{u_1 - u_2}{h} x\ 2\ \pi\ r^2\ dr$$

Substitute u = ωr $$dT = rdF = \mu \frac{(\omega_1 - \omega_2)}{h} x\ 2\ \pi\ r^3\ dr$$

Integrate $$T = \mu \frac{(\omega_1 - \omega_2)}{h} x\ 2\ \pi \int_0^R r^3\ dr\ = \mu \frac{(\omega_1 - \omega_2)}{h} x\ 2\ \pi \frac{R^4}{4}$$

Rearrange and substitute R = D/2 $$T = \mu \frac{(\omega_1 - \omega_2)}{h} x\ \pi \frac{D^4}{32}$$

Put D = 0.3 m, μ = 0.5 N s/m², h = 0.012 m $$T = 0.5 \frac{(\omega_1 - \omega_2)}{0.012} x\ \pi \frac{0.3^4}{32} = 0.33(\omega_1 - \omega_2)$$

N = 900 rev/min P = 500 W Power = 2πNT/60 $T = \dfrac{60P}{2\pi\ N} = \dfrac{60\ x\ 500}{2\pi\ x\ 900} = 5.305$ Nm

The torque input and output must be the same. $\omega_1 = 2\pi N_1/60 = 94.25$ rad/s

$5.305 = 0.33(94.25_1 - \omega_2)$ hence $\omega_2 = 78.22$ rad/s and $N_2 = 747$ rev/min

$P_2 = 2\pi N_2 T/60 = \omega_2 T = 78.22 \times 5.305 = 414$ W (Power out)

For maximum power output $dp_2/d\omega_2 = 0$ $P_2 = \omega_2 T = 0.33(\omega_1\omega_2 - \omega_2^2)$

Differentiate $$\frac{dP_2}{d\omega_2} = 0.33(\omega_1 - 2\omega_2)$$

Equate to zero and it follows that for maximum power output $\omega_1 = 2\ \omega_2$

And it follows $N_1 = 2\ N_2$ so $N_2 = 450$ rev/min

D203 FLUID MECHANICS

Q5 1999

For an element of fluid the force balance is:

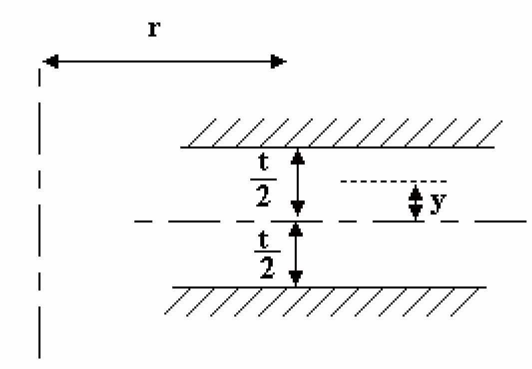
$$d\tau\, dr = dp\, dy \qquad \frac{dp}{dr} = \frac{d\tau}{dy} = \frac{d\left(\mu \frac{du}{dy}\right)}{dy} = \mu \frac{d^2u}{dy^2}$$

INTEGRATE

$$y\frac{dp}{dr} = \mu \frac{du}{dy} + A$$

INTEGRATE

$$\frac{y^2}{2}\frac{dp}{dr} = \mu u + Ay + B$$



Boundary conditions are at y = ± b/2 u = 0

Put y = t/2

$$\frac{(t/2)^2}{2}\frac{dp}{dr} = 0 + At/2 + B..........(1)$$

Put y = -t/2

$$\frac{(-t/2)^2}{2}\frac{dp}{dr} = 0 - At/2 + B..........(2)$$

Add (1) + (2)

$$(t/2)^2 \frac{dp}{dr} = 2B \qquad B = \frac{t^2}{8}\frac{dp}{dr}$$

Substitute into (1) $\frac{t^2}{8}\frac{dp}{dr} = 0 + \frac{At}{2} + \frac{t^2}{8}\frac{dp}{dr}$

It follows that A = 0

$$\frac{y^2}{2}\frac{dp}{dr} = \mu u + \frac{t^2}{8}\frac{dp}{dr}$$

$$\frac{dp}{dr}\left\{\frac{y^2}{2} - \frac{t^2}{8}\right\} = \mu\ u$$

$$u = \frac{dp}{dr}\frac{1}{8\mu}\left\{4y^2 - t^2\right\}$$

For an elementary ring radius r and height dy

dA = 2πr dy dQ = u 2πr dy

$$dQ = \frac{dp}{dr}\frac{1}{8\mu}\left\{4y^2 - t^2\right\} x\ 2\,\pi\, r\, dy$$

$$dQ = \frac{dp}{dr}\frac{2\,\pi\, r}{8\mu}\left\{4y^2dy - t^2dy\right\}$$

Integrate with respect to y

$$Q = \frac{dp}{dr}\frac{2\pi r}{8\mu}\left[\frac{4y^3}{3} - t^2 y\right]_{-\frac{t}{2}}^{\frac{t}{2}}$$

$$Q = \frac{dp}{dr}\frac{2\pi r}{8\mu}\left[\left(-\frac{4t^3}{24} + \frac{t^3}{2}\right) - \left(\frac{4t^3}{24} - \frac{t^3}{2}\right)\right]$$

$$Q = \frac{dp}{dr}\frac{\pi r}{4\mu} \times \frac{2t^3}{3}$$

$$\frac{dr}{r} = dp\,\frac{\pi t^3}{6\mu\mu}$$

$$\int_{R_i}^{R_o}\frac{dr}{r} = \frac{\pi t^3}{6\mu\mu}\int_{p}^{0} dp$$

Integrate

$$\ln\left(\frac{R_o}{R_i}\right) = \frac{\pi t^3}{6\mu\mu}\,p$$

$$Q = \frac{\pi t^3}{6\mu}\frac{p}{\ln\left(\frac{R_o}{R_i}\right)}$$

$t = 5$ mm $R_o = 0.15$ m $R_i = 0.025$ m $\rho = 800$ kg/m^3 $\mu = 0.25$ Ns/m^2 $u_m = 5$ m/s

$Q = A\,u_m = \pi \times 0.025^2 \times 5 = 9.817 \times 10^{-3}$ m^3/s

$$p = \frac{Q\ln\left(\frac{R_o}{R_i}\right) \times 6\mu}{\pi t^3} = 9.817 \times 10^{-3}\,\frac{\ln\left(\frac{0.15}{0.025}\right) \times 6 \times 0.25}{\pi \times 0.005^3} = 67.19\text{ kPa}$$

Max velocity at y = 0

$$\frac{dp}{dr} = -\frac{67190}{0.15 - 0.025} = 537 \times 10^{-3}$$

$$u = \frac{1}{8\mu}\frac{dp}{dr}\left(4y^2 - t^2\right) = \frac{1}{8 \times 0.25}(-537 \times 10^{-3})\left(-0.005^2\right) = 6.72\text{ m/s}$$

FLUID MECHANICS D203 Q6 1998

(a) Explain the terms *Stokes flow* and *terminal velocity* as applied to a particle settling in a fluid. Show that, for a spherical particle immersed in a flow for which the drag coefficient C_D *is 24/Re* (where *Re* is based on particle diameter), the terminal velocity - u is given by

$$u = \frac{d^2 g(\rho_s - \rho_f)}{18 \ \mu}$$ where p_s is the density of the particle.

(b) A gravel washing and grading plant processes gravel with a density of 2630 kg/m^3. The gravel is introduced into a stream of water at 25°C which is flowing vertically upwards with a velocity of 1.0 m/s. Treating the gravel pebbles as spherical particles, determine the diameter of the largest particle which will be carried upwards by the water flow.

(c) If the water velocity is reduced to 0.5 m/s, show that particles with a diameter greater than 5.95 mm will fall downwards through the water flow.

For spherical particles, a useful empirical correlation for the drag coefficient C_D is

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4$$

where *Re* is the Reynolds number based on particle diameter. This correlation is applicable for the range *0.2<Re<10*^{5}

....

a) For R_e<0.2 the flow is called Stokes flow and Stokes showed that R = 3πd μ u hence

R = W = volume x density difference x gravity

$$R = W = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} = 3\pi d \ \mu \ u$$

ρ_s = density of the sphere material ρ_f = density of fluid   d = sphere diameter

$$u = \frac{\pi d^3 g(\rho_s - \rho_f)}{18 \pi d \mu} = \frac{d^2 g(\rho_s - \rho_f)}{18 \mu}$$

b) C_D = R/(projected area x ρu^2/2) $C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\rho u^2/2) 6 \pi d^2/4} = \frac{4dg(\rho_s - \rho_f)}{3\rho u^2}$

$$C_D = \frac{4 \text{ x } 9.81 \text{x } (1630 - 998) d}{3 \text{x } 998 \text{ x } u^2} = 21.389 \frac{d}{u^2}$$

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4 = 21.389 \frac{d}{u^2}$$

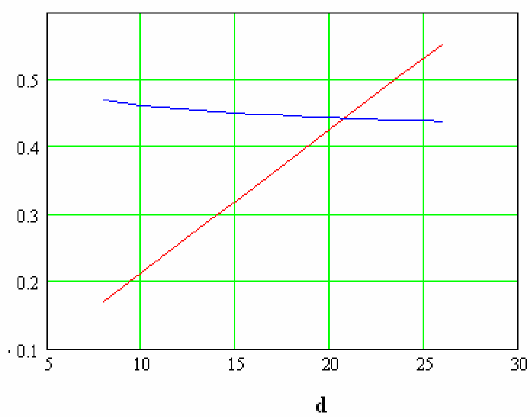
$$21.389 \frac{d}{u^2} - \frac{24}{R_e} - \frac{6}{1+\sqrt{R_e}} = 0.4 \quad \text{let}$$

$$21.389 \frac{d}{u^2} - \frac{24}{R_e} - \frac{6}{1+\sqrt{R_e}} = x$$

Re = ρud/μ = 998 x 1 x d/0.89 x 10^{-3} = 1.1213 x 10^{6}d

Make a table

D	0.001	0.003	0.01	0.02	0.03
Re	1121.3	3363.9	11213	22426	33639
x	-0.174	-0.045	0.156	0.387	0.608



Plot and find that when d = 0.0205 m (20.5 mm) x = 0.4

c) u = 0.5m/s d = 5.95mm

Re = ρud/μ = 998 x 0.5 x 0.00595/0.89 x 10^{-3} = 3336

$$C_D = 21.389\frac{d}{u^2} = 0.509$$

$$C_D = \frac{24}{3336} + \frac{6}{1+\sqrt{3336}} + 0.4 = 0.509$$

Since C_D is the same, larger ones will fall.

(a) A centrifugal pump delivers 19.4×10^{-3} m^3/s of water when operating at a speed of 1100 rev/min. The static head difference between inlet and outlet flanges is 20.8 m of water. The impeller diameter is 325 mm and outlet width is 13.5 mm. Water enters the impeller radially and the manometric efficiency is 65%.
Determine the blade angle of the impeller at exit.

(b) The static lift of the system in which the pump of (a) is installed is 12.5 m of water. Calculate

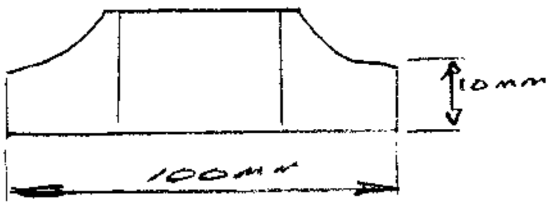
(i) the shut-off head developed by the pump when operating against a closed delivery valve at 1100 rev/min
(ii) the pump speed at which flow will commence when all valves are open.

$\Delta h_m = \Delta H/\eta_m = 20.8/0.65 = 28$ m
$\Delta h_m = u_2 v_{w2}/g \quad u_2 = \pi \times 1100/60 \times 0.325 = 18.71$ m/s
$v_{w2} = 28 \times 9.81/18.71 = 16.77$ m/s
$A = \pi Dt = \pi \times 0.325 \times 0.0135 = 0.01378$ m^2
$v_R = Q/A = 19.4 \times 10^{-3}/ 0.01378 = 1.407$ m/s
$v_{w2} = u_2 - v_R/\tan\alpha$ hence $1.407/1.94 = \tan \alpha \quad \alpha = 36^o$
Shut off $\Delta h = u^2/g = 18.71^2/9.81 = 35.68$ m

Static lift given in some text books as $N = 83.5h^{½}/D = 83.5 \times 12.5^{½}/0.325 = 908$ rev/min

Q8 1992

ASS 8 Q1

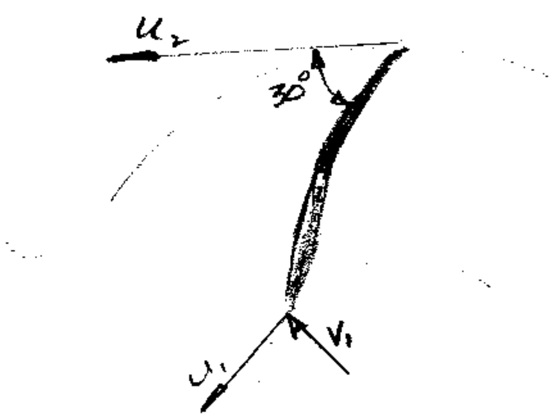


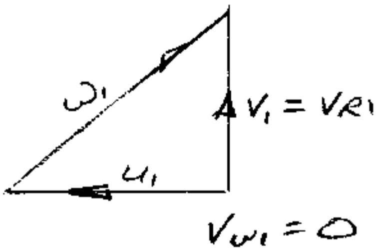
$N = 1450$ Rev/min

$K = 0.9$

$Q = 0.008\ m^3/s$

INLET



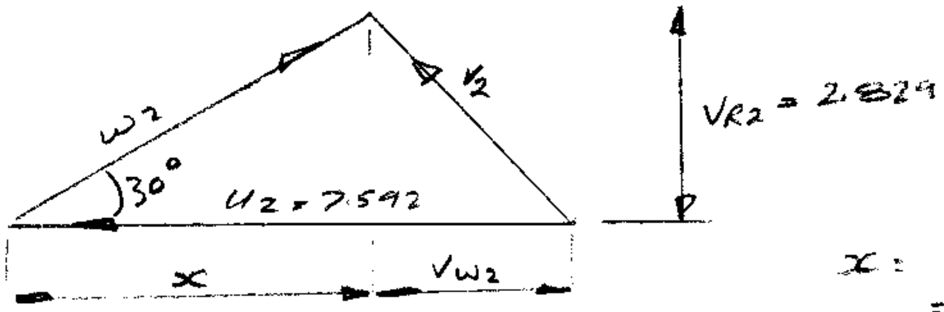


$V_{R2} = Q/A_2 = \frac{.008}{\pi \times .1 \times .01 \times .9}$

OUTLET

$V_{R2} = 2.829\ m/s$

$U_2 = \pi N D_2 = \pi \times \frac{1450}{60} \times .1 = 7.592\ m/s$



$x = 2.829/\tan 30° = 4.9\ m/s$

$V_{w2} = 7.592 - 4.9 = 2.692\ m/s$

$V_2 = \sqrt{2.692^2 + 2.829^2} = 3.905\ m/s$

KINETIC HEAD $= V_2^2/2g = 3.905^2/2g = 0.777m$

LOSS IN CHAMBER $= 25\% \times .777 = .194m$

MANOMETRIC HEAD $= U_2 V_{w2}/g = 7.592 \times 2.692/g = 2.08m$

DEVELOPED HEAD $= 2.08 - .194 = \underline{1.89m}$ ANS

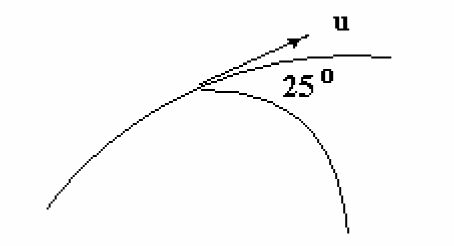
$\Delta h = U_2 V_{w2}/g = (U_2/g)(U_2 - Q/A_2 \tan\alpha_2)$

When no flow $Q = 0$

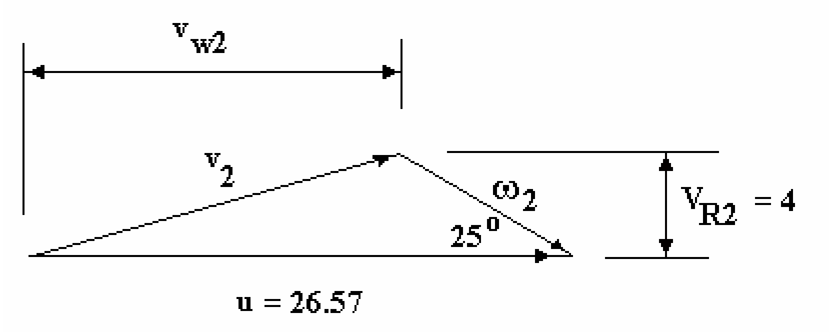
$\Delta h = \frac{7.592}{g}(7.592 - 0) = \underline{5.875m}$ Ans

Q9 1997

ρ=1000 kg/m^3 Q = 0.11 m^3/s N = 1450 rev/min D_o = 350 mm t_o = 25 mm



$u_2 = \pi$ N D/60 = π x 1450 x 0.35/60 = 26.57 m/s
V_{R2} = Q/(πDt) = 0.11/(π x 0.35 x 0.025) = 4 m/s



V_{w2} = 26.57 – 4 cot(25^o) = 18 m/s
$V_2^2 = 4^2 + 18^2$ V_2 = 18.44 m/s

Manometric head = h_m = u_2 v_{w2}/g = 48.75 m

Kinetic head = V_2^2/2g = 17.33 m

Loss = 65% x 17.33 = 11.26 m
Δh = 48.75 – 11.26 = 37.49 m
Shaft Power = m g h_m + Mechanical Losses

M = 110 kg/s

Mech Loss = 0.18 x 26.57^2/g = 12.95 m

Shaft Power = 110 x 9.81 x (48.75 + 12.95) = 66584 W

D203 FLUID MECHANICS QUESTION 1 2003

PART A

$a = f(K, \rho) = C K^a \rho^b$ dimensions are:
$[a] = [m/s] = LT^{-1}$
$[K] = [N/m^2] = [(kg\ m/s^2)(1/m^2)] = ML^{-1}T^{-2}$
$[\rho] = [kg/m^3] = ML^{-3}$ Hence

$M^0LT^{-1} = C\,(ML^{-1}T^{-2})^a(ML^{-3})^b = M^aL^{-a}T^{-2a}M^bL^{-3b}$
Equate powers

Time $-1 = -2a$ $a = ½$
Mass $0 = a + b$ $b = -½$ Substitute back $\mathbf{a = C K^{1/2} \rho^{-1/2} = C\sqrt{(K/\rho)}}$

PART B

R = function (l v ρ μ K g)

There are 7 quantities and there will be 3 basic dimensions ML and T. This means that there will be 4 dimensionless numbers Π_1, Π_2, Π_3 and Π_4. These numbers are found by choosing four prime quantities (R, μ, K and g).

Π_1 is the group formed between R and l v ρ
Π_2 is the group formed between μ and l v ρ
Π_3 is the group formed between K and l v ρ
Π_4 is the group formed between g and l v ρ

The first is formed by combining R with ρ,v and l
$R = \Pi_1\ l^a v^b \rho^c$
$MLT^{-2} = \Pi_1\,(L)^a\,(LT^{-1})^b(ML^{-3})^c$
Time **$-2 = -b$** **$b = 2$**
Mass **$c = 1$**
Length **$1 = a + b - 3c$**
$1 = a + 2 - 3$ **$a = 2$**
$R = \Pi_1\ l^2 v^2 \rho^1$ $\Pi_1 = \dfrac{R}{\rho v^2 l^2}$ and this is the Newton number.

The second is formed between μ and ρ,v and l.
$\mu = \Pi_2\ l^a v^b \rho^c$
$M^1L^{-1}T^{-1} = \Pi_2\,(L)^a\,(LT^{-1})^b\,(ML^{-3})^c$
Time $-1 = -b$ $b = 1$
Mass **$c = 1$**
Length $-1 = a + b - 3c$
$-1 = a + 1 - 3$ **$a = 1$**
$\mu = \Pi_2\ l^1 v^1 \rho^1$ $\Pi_2 = \dfrac{\mu}{l v \rho}$ and $lv\rho/\mu$ is the Reynolds number

The third group is formed between K and l v ρ

$K = \Pi_3\, l^a\, v^b\, \rho^c$
$ML^{-1}T^{-2} = \Pi_3\ L^a\,(LT^{-1})^b\,(ML^{-3})^c$
$ML^{-1}T^{-2} = \Pi_3\ L^{a+b-3c}\,M^c\,T^{-b}$

Time $-2 = -b$ $b = 2$
Mass $c = 1$
Length $-1 = a + b - 3c$
$-1 = a + 2 - 3$ $a = 0$

$K = \Pi 3 \; l^{o} \, v^{2} \; \rho^{1}$

$$\Pi_3 = \frac{K}{\rho v^2}$$

It was shown earlier that the speed of sound in an elastic medium is given by the following formula. $a = C(k/\rho)^{½}$

It follows that $(k/\rho) = a^2$ and so $\Pi_3 = C^2(a/v)^2 = C^2/ M^2$

The fourth group is formed between g and l v ρ

$g = \Pi_4 \, l^a \; v^b \, \rho^c$
$LT^{--2} = \Pi_4 \; L^a \, (LT^{-1})^b \; (ML^{-3})^c$
$M^0L^1T^{-2} = \Pi_4 \; L^{a+ b-3c} \; M^c \; T^{-b}$

Time $-2 = -b$ $b = 2$
Mass $c = 0$
Length $-1 = a + b - 3c$
$1 = a + 2$ $a = -1$

$g = \Pi_4 \; l^{-1} \, v^2$ $\Pi_4 = \frac{gl}{v^2}$

The Froude Number is defined as $Fr = v/\sqrt{(gl)}$ so $\Pi_4 = 1/Fr^2$

Putting it all together we have $\Pi_1 = \frac{R}{\rho v^2 l^2}$ **$= f(\Pi_2, \Pi_3, \Pi_4) = f(Re)(M)(Fr)$**

All powers and constants are implied in the function sign.

PART C

l = 150 m

v = 30 km/h = 8.333 m/s

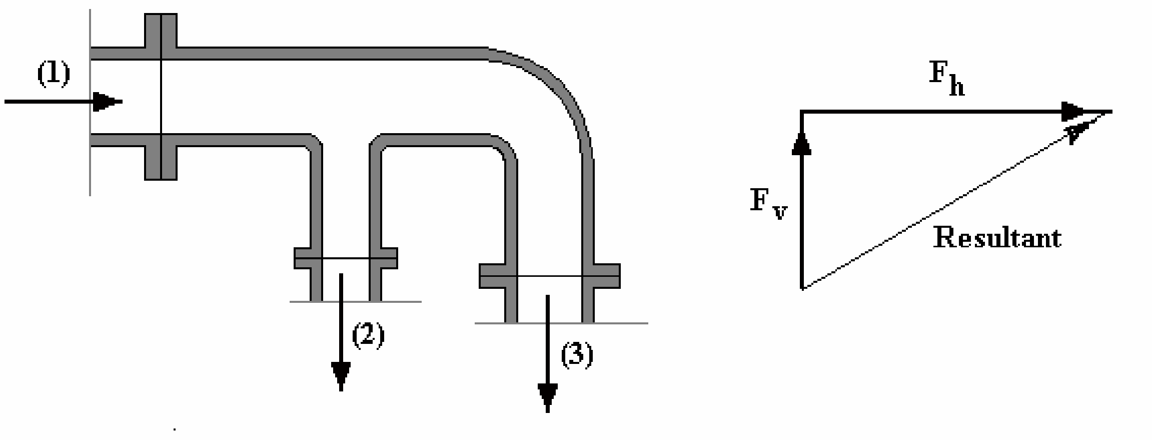
For dynamic similarity of the Froude number only

$Fr = v/\sqrt{gl} = 8.333/\sqrt{(9.81 \text{ x } 150)} = 0.217$

For the model we must have the same Froude number. Lm = 150/40 = 3.75

$Fr = 0.217 = v_m/\sqrt{gl_m} = v_m/\sqrt{(9.81 \text{ x } 3.75)}$ hence $v_m = 1.318$ m/s or 4.743 km/h

D203 FLUID MECHANICS   Q2 2003



D_1= 0.15m D_2= 0.05m D_3= 0.1m   Q1= 0.05 m^3/s
p_1=400 x 10^3 N/m^2 p_2=395 x 10^3 N/m^2

$$A_1 = \frac{p\ D_1^2}{4} = 0.018\,m^2 \quad A_2 = \frac{p\ D_2^2}{4} = 0.001963\,m^2 \quad A_3 = \frac{p\ D_3^2}{4} = 0.007854\,m^2$$

$$u_1 = \frac{Q_1}{A_1} = 2.829\ m/s$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (2)

$$p_1 + r\frac{u_1^2}{2} = p_2 + r\frac{u_2^2}{2}$$

$$400\ x\ 10^3 + 998\frac{2.829^2}{2} = 395\ x\ 10^3 + 998\frac{u_2^2}{2} = 404\ x\ 10^3$$

$$u_2 = 4.246\ m/s$$

FLOW RATE $Q_2 = A_2u_2 = 0.008336\ m^3/s$

CONSERVATI ON OF MASS $Q_3 = Q_1 - Q_2 = 0.042\ m^3/s$ $u_3 = \frac{Q_3}{A_3} = 5.305\ m/s$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (3)

$$p_1 + r\frac{u_1^2}{2} = p_3 + r\frac{u_3^2}{2}$$

$$400\ x\ 10^3 + 998\frac{2.829^2}{2} = p_3 + 998\frac{u_3^2}{2} = 404\ x\ 10^3$$

$$p_3 = 404\ x\ 10^3 - 998\frac{5.305^2}{2} = 390\ x\ 10^3\ N/m^2$$

FORCES
Horizontal force = m Δu + AΔp   in horizontal direction.
F_h = ρQ_1 (0 – 2.829) + 0.018 (0 – 400 x 10^3) = -7.21 kN   (To right on diagram)
Vertical force = m Δu + AΔp   in vertical direction. There are two calculations.
F_{v1} = ρQ_2 (u_2 – 0) + A_3p_3 = (998 x 0.008336 x 4.246) + (0.001963 x 390 x 10^3)
F_{v1} = 810.9 N Up
F_{v2} = ρQ_3 (u_3 – 0) + A_3p_3 = (998 x 0.042 x 5.305) + (0.007854 x 390 x 10^3)
F_{v2}= 3.283 x 10^3 N Up
Total vertical force = 4.094 KN Up

RESULTANT FORCE = $\{4.094^2 + 7.21^2\}^{1/2}$ = 8.291 kN

D203 FLUID MECHANICS Q3 2003

PART A

MANOMETRIC HEAD Δh_m

This is the head that would result if all the energy given to the water is converted into pressure head. It is found by equating the diagram power and water power.

$$mu_2 v_{w2} = mg\Delta h_m$$

$$\Delta h_m = \frac{u_2 v_{w2}}{g} = \frac{u_2}{g}\left\{u_2 - \frac{Q}{A_2 \tan(a_2)}\right\}$$

MANOMETRIC EFFICIENCY η_m

$$h_m = \frac{\text{Water Power}}{\text{Diagram Power}} = \frac{mg\Delta h}{mu_2 v_{w2}} = \frac{mg\Delta h}{mg\Delta h_m} = \frac{\Delta h}{\Delta h_m}$$

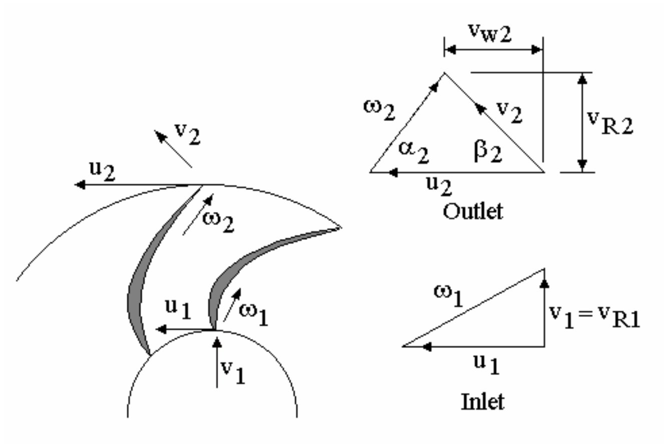
SHAFT POWER

S.P. = $2\pi NT$

OVERALL EFFICIENCY

$$h_{o/a} = \frac{\text{Water Power}}{\text{Shaft Power}}$$

PART B



D_2 = 0.2 m t_2 = 0.018 m   N = 1200/60 = 20 rev/s   Q = 0.02 m^3/s

Shaft Power = 2500 W   Outlet angle $\alpha_2 = 30^o$ recovered head = 45% of kinetic head

Tangential Velocity of blade $u_2 = \pi N D_2$ = 12.566 m/s

Radial velocity at outlet $v_{r2} = Q/(\pi D_2 t_2)$ = 1.768 m/s

Velocity of whirl at outlet $v_{w2} = u_2 - v_{r2} \cot \alpha_2$ = 9.503 m/s

Absolute outlet velocity $v_2 = \sqrt{[v_{w2}^2 + v_{r2}^2]}$ = 9.667 m/s

Manometric head $h_m = u_2 v_{w2}/g$ = 12.178 m

Kinetic Head = $v_2^2/2g$ = 4.764 m   Recovered head = 4.764 x 0.45 = 2.144 m

Manometric Efficiency $\eta_m = h_2/h_m$ = 0.176 or 17.6 %

Water Power = ρ Q g h_2 = 419.3 W

Overall Efficiency $\eta_{o/a}$ = WP/SP = 419.3/2500 = 0.168 or 16.8%

D203 FLUID MECHANICS SOLUTION Q4 2003

Comment – If anyone could do this question in the time allocated they would need to be a genius or have revised it so thoroughly ha hey could repeat it from memory. You also need to know that the stream function Ψ is taken as positive in the x direction and this is the opposite of most advanced books and that used n my tutorial. It follows that u = dΨ/dy and not - dΨ/dy

PART A

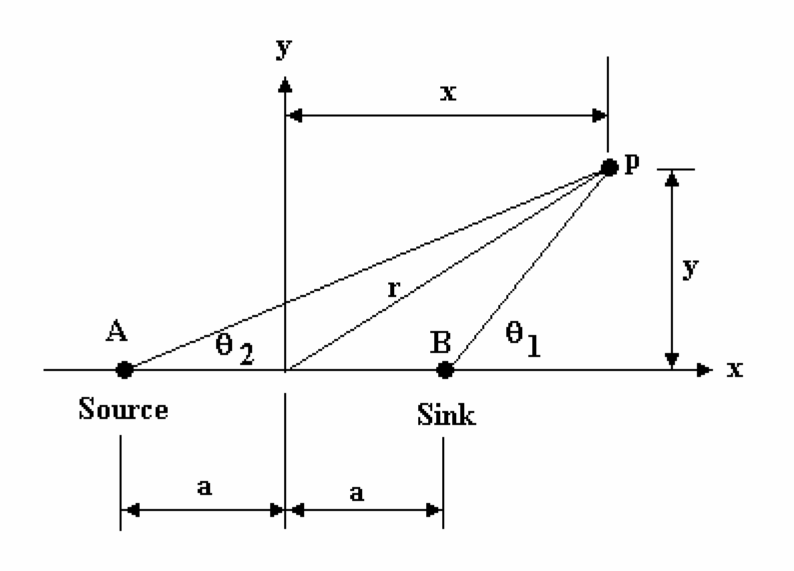
The combined stream function is found as follows.

Ψ_A= U y for a uniform flow U is the velocity of the uniform stream

$\Psi_B = \frac{m}{2p} q_2$ for the source $\Psi_C = -\frac{m}{2p} q_1$ for the sink

$$\Psi = \Psi_A + \Psi_B + \Psi_C = Uy + \frac{m}{2p}(q_2 - q_1)$$

Referring to the diagram for a source and sink placed on the x axis distance a either side :-



$$\tan\ q_1 = \frac{y}{x-a} \qquad \tan\ q_2 = \frac{y}{x+a}$$

$$\tan(q_2 - q_1) = \frac{\tan\ q_2 - \tan\ q_1}{1 + \tan\ q_2 \tan\ q_1}$$

$$\tan(q_2 - q_1) = \frac{\frac{y}{x+a} - \frac{y}{x-a}}{1 + \left(\frac{y}{x+a}\right)\left(\frac{y}{x-a}\right)}$$

$$\tan(q_2 - q_1) = \frac{y(x-a) - y(x+a)}{1 + \frac{y^2}{x^2 - a^2}}$$

$$\tan(q_2 - q_1) = \frac{y(x-a) - y(x+a)}{1 + \frac{y^2}{x^2 - a^2}}$$

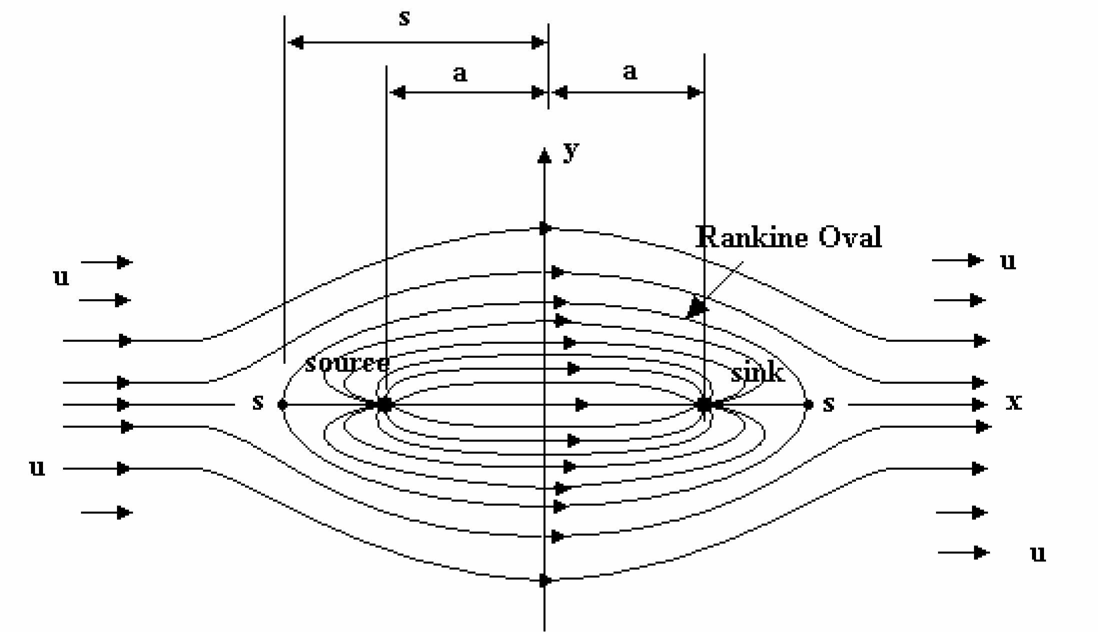
$$\tan(q_2 - q_1) = \frac{-2ay}{x^2 - a^2 + y^2}$$

$$-\tan(q_2 - q_1) = \frac{2ay}{x^2 - a^2 + y^2}$$

$$\Psi = Uy - \frac{m}{2p}\tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right)$$

PART B

$\Psi = Uy - \frac{m}{2\pi}\tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right)$ The stream pattern is like this:



The entire output of the source flows inside the Rankine Oval which is the zero stream line. There is no flux across this line. Putting $\Psi = 0$ and $x = 0$ gives:

$$0 = Uy - \frac{m}{2\pi}\tan^{-1}\left(\frac{2ay}{y^2 - a^2}\right)$$

At this point y is the half width of the oval h so change y to h and we get:

$$Uh = \frac{m}{2\pi}\tan^{-1}\left(\frac{2ay}{y^2 - a^2}\right)$$

$$\tan\left(\frac{2\pi Uh}{m}\right) = \frac{2ah}{h^2 - a^2} \qquad h = \frac{h^2 - a^2}{2a}\tan\left(\frac{2\pi Uh}{m}\right)$$

VELOCITY IN THE x DIRECTION

The velocity in the x direction is given by $u = \frac{\partial \Psi}{\partial y}$

This is easier to solve by using $\Psi = Uy + \frac{m}{2p}(\theta_2 - \theta_1) = Uy + \frac{m}{2p}\left(\tan^{-1}\frac{y}{x+a} - \tan^{-1}\frac{y}{x-a}\right)$

$$u = \frac{\partial \Psi}{\partial y} = U + \frac{m}{2\pi}\left[\left\{\frac{1}{x+a}\right\}\left\{1 + \left(\frac{y}{x+a}\right)^2\right\}^{-1} - \left\{\frac{1}{x-a}\right\}\left\{1 + \left(\frac{y}{x-a}\right)^2\right\}^{-1}\right]$$

At the stagnation point, the velocity is zero, y = 0 and x = ±s hence:

$$u = 0 = U + \frac{m}{2\pi}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right] = U + \frac{m}{2\pi}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right]$$

$$u = 0 = U + \frac{m}{2\pi}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right] = U + \frac{m}{2\pi}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right]$$

$$U = -\frac{m}{2\pi}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right]$$

$$u = 0 = U + \frac{m}{2\pi}\left[\left\{\frac{1}{s+a}\right\}\left\{1 + \left(\frac{0}{s+a}\right)^2\right\}^{-1} - \left\{\frac{1}{s-a}\right\}\left\{1 + \left(\frac{0}{s-a}\right)^2\right\}^{-1}\right]$$

$$\frac{2U\pi}{m} = -\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right] = \left[\frac{(s-a)-(s+a)}{(s+a)(s-a)}\right] = \frac{-2a}{s^2 - a^2}$$

$$s^2 - a^2 = \frac{2ma}{2U\pi} = \frac{ma}{U\pi} \qquad s^2 - a^2 = \frac{2ma}{2U\pi} = \frac{ma}{U\pi}$$

$$s^2 = \frac{ma}{U\pi} + a^2 s^2 = \frac{ma}{U\pi} + a^2$$

$$s = \sqrt{a^2 + \frac{ma}{U\pi}}$$

PART C

2a = 0.1562 a = 0.0781 u = 3 m/s h = 0.05

$$h = \frac{h^2 - a^2}{2a}\tan\left(\frac{2\pi uh}{m}\right) \text{ put } h = 0.05$$

$$0.05 = \frac{0.05^2 - 0.0781^2}{2 \text{ x } 0.0781}\tan\left(\frac{2\pi \text{ x } 3 \text{ x } 0.05}{m}\right)$$

$$-2.1696 = \tan\left(\frac{0.9425}{m}\right) \text{ remember to work in radian mode}$$

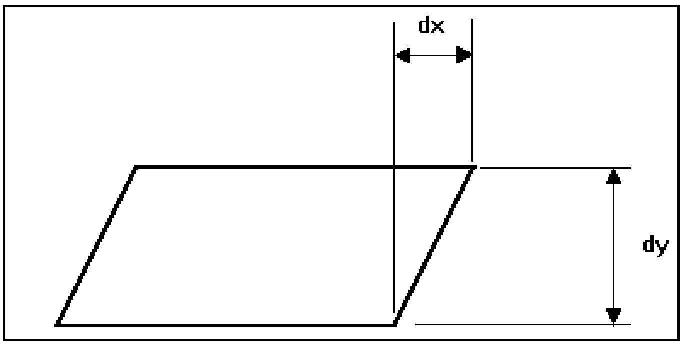
$$\pm 1.1389 = \frac{0.9425}{m} \quad m = \pm 1.208$$

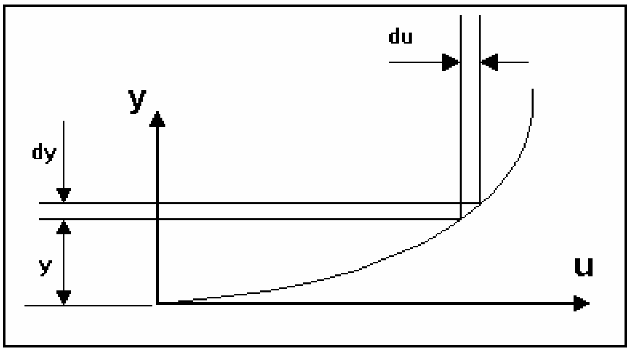
Length of Rankine Oval = 2s = 0.2 so s = 0.1 s = 0.1 = √{(ma/πu) + a^2}
0.01 = ma/πu) + a^2 (1.208 x 0.0781)/(π x 3) = 0.01 As both give 0.01 the data is correct.

VELOCITY
If anyone can show me how to solve this part I would be grateful.

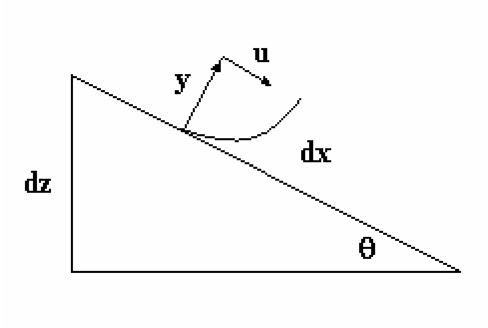
PART A

The normal equation for laminar flow is dp dy = - dτ dx so dp/dx = - dτ/dy





and since $\tau = \mu du/dy$ for a Newtonian fluid $dp/dx = - \mu d^2u/dy^2$
In this case the flow is due to gravity only and pressure is related to height z by p = ρgz
It follows that dp = ρg dz



dz/dx=sin θ hence dp = ρg dx sin θ and $dp/dx = \rho g \sin\theta = - \mu d^2u/dy^2$

$g \sin\theta = - (\mu/\rho) d^2u/dy^2 = - (\nu) d^2u/dy^2$ μ/ρ = ν the kinematic viscosity

n **d^2u/dy^2 = - g sin** q

PART B

Integrate and du/dy = - (g/ν) y sin θ + A
Integrate again and $u = - (g/\nu)(y^2/2) \sin\theta + Ay + B$ A and B are constants of integration.

Boundary conditions
Put du/dy = 0 at x = h so du/dy = 0 = - (g/ν) h sin θ + A A = (g/ν) h sin θ
Put y = 0 and u = 0 and it follows that B = 0

$u = - (g/\nu)(y^2/2) \sin\theta + \{(g/\nu) h \sin\theta\} y$
$u = (g/\nu) \sin\theta \{hy - y^2/2\}$

u = (g/2n**) sin** q**$\{2hy - y^2\}$**

PART C

$\nu = 8 \times 10^{-5}$ m^2/s h = 0.005 m Consider the flow through a small slit 1 m wide and width dy.

dQ = u dy = u = (g/2ν) sin θ{2hy - y^2} dy

Integrate between y = 0 and y = h

$$Q = \frac{g}{2\nu}\sin\theta\left[\frac{2hy^2}{2} - \frac{y^3}{3}\right]_0^h = \frac{g}{2\nu}\sin\theta\left[h^3 - \frac{h^3}{3}\right] = \frac{g}{2\nu}\sin\theta\left[\frac{2h^3}{3}\right]$$

Evaluate and

$$Q = \frac{9.81}{2 \times 8 \times 10^{-5}}\sin 40\left[\frac{2 \times 0.005^3}{3}\right] = 0.003284\ m^3/s$$

Maximum velocity is at y = h so

$$U = \frac{g}{2\nu}\sin\theta\left(2h^2 - h^2\right) = \frac{g}{2\nu}\sin\theta\left(h^2\right) = \frac{9.81}{2 \times 8 \times 10^{-5}}\sin 40\left(0.005^2\right) = 0.985\ m/s$$

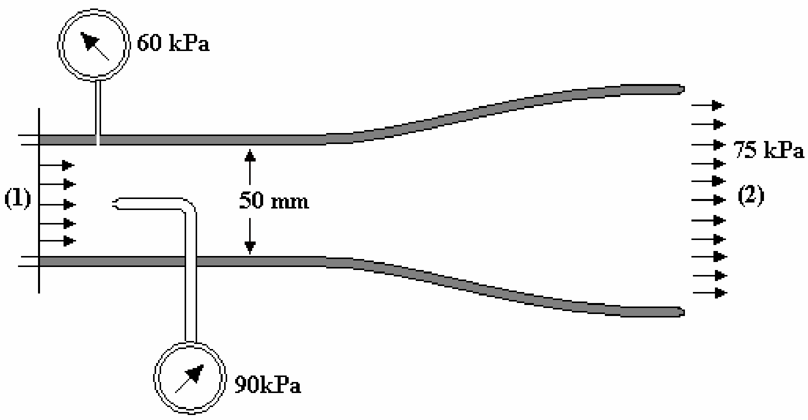
D203 Q6 2003

PART A

When the flow is supersonic shock waves occur and some of the total pressure is lost in friction so the pressure gauge does not record the true total pressure.

PART B

p_1 = 60 kPa p_2 = 75 kPa p_o = 90 kPa T_1 = 275 K D_1 = 0.05 m c_p=1005 J/kk K



$A_1=\pi D_1^2/4 = 1.963 \times 10^{-3}\ m^2$ $\rho_1 = p_1/RT_1 = 0.76\ kg/m^3$ $a_1 = (\gamma RT_1)^{1/2} = 332.4$ m/s

STAGNATION TEMP $T_o = T_1\left(\frac{p_o}{p_1}\right)^{\frac{?-1}{?}} = 275\left(\frac{90}{60}\right)^{0.286} = 308.8K$

VELOCITY $u_1=\{2c_p(T_o-T_1)\}^{1/2} = \{2 \times 1005(308.8 - 275)]^{1/2} = 260.6$ m/s

MACH NUMBER $M_1=u_1/a_1 = 260.6/332.4 = 0.784$

MASS FLOW $m = \rho_1 A_1 u_1 = 0.389$ kg/s

CHECK $T_o = T_1\left\{1+\frac{?-1}{2}M_1^2\right\} = 275\left\{1+\frac{1.4-1}{2}x0.784^2\right\} = 308.8K$

PART C

$$T_2 = \frac{T_o}{\left(\frac{p_o}{p_2}\right)^{\frac{?-1}{?}}} = \frac{308.8}{\left(\frac{90}{75}\right)^{\frac{1.4-1}{1.4}}} = 293.1\,K$$

$\rho_2 = p_2/RT_2 = 0.892\ kg/m^3$ $a_2 =(\gamma RT_2)^{1/2} = 343.2$ m/s

VELOCITY $u_2=\{2c_p(T_o-T_2)\}^{1/2} = \{2 \times 1005(308.8 - 293.1)]^{1/2} = 177.5$ m/s

MACH NUMBER $M_2=u_2/a_2 = 177.5/343.2 = 0.517$

MASS FLOW $m = 0.389\ kg/s = \rho_2 A_2 u_2$ $A_2 = m/\rho_2 u_2 = 2.458 \times 10^{-3}\ m^2$

$D_2=(4A_2/\pi)^{½}$ = 0.056 or 56 mm

Q9 D203 FLUID MECHANICS 2003

PART A

Conducting the usual force balance on a cylindrical core of radius r we have:

$\tau\ 2\pi r\, dL = \pi r^2\, dp$ but in this case $\tau = C(-du/dr)^2$ substitute for τ

$C(-du/dr)^2\ 2\pi r\, dL = \pi r^2\, dp$

$C(-du/dr)^2 = r\, dp/dL$

$$\left(-\frac{du}{dr}\right)^2 = \frac{r}{2C}\frac{dp}{dL}$$

$$\frac{du}{dr} = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} r^{\frac{1}{2}}$$

$$du = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} r^{\frac{1}{2}} dr$$

$$\int_0^u du = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} \int_R^r r^{\frac{1}{2}} dr$$

$$u = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} \frac{2}{3}\left[r^{\frac{3}{2}}\right]_R^r$$

$$u = -\frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[r^{\frac{3}{2}} - R^{\frac{3}{2}}\right] = \frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[R^{\frac{3}{2}} - r^{\frac{3}{2}}\right]$$ At the centre line r = 0 $u = U_o$

$$U_o = \frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[R^{\frac{3}{2}}\right]$$ the ratio u/U_o is hence.

$$\frac{u}{U_o} = \frac{R^{\frac{3}{2}} - r^{\frac{3}{2}}}{R^{\frac{3}{2}}} = 1 - \left(\frac{r}{R}\right)^{\frac{3}{2}}$$

Note the question omitted the C and the minus sign results from putting –dp/dL

PART B

Consider the flow rate through a thin annular ring radius r and width dr.

dQ = u x 2πr dr

$$dQ = U_o\left\{1-\left(\frac{r}{R}\right)^{\frac{3}{2}}\right\} x\ 2p\ rdr = 2p\ U_o\left\{r-\left(\frac{r^{\frac{5}{2}}}{R^{\frac{3}{2}}}\right)\right\} dr$$

$$Q = 2p\ U_o\int_0^R r-\left(\frac{r^{\frac{5}{2}}}{R^{\frac{3}{2}}}\right) dr = 2p\ U_o\left[\frac{r^2}{2}-\left(\frac{2}{7}\frac{r^{\frac{7}{2}}}{R^{\frac{3}{2}}}\right)\right]_0^R$$

$$Q = 2p\ U_o\left[\frac{R^2}{2}-\left(\frac{2}{7}\frac{R^{\frac{7}{2}}}{R^{\frac{3}{2}}}\right)\right] = 2p\ U_o\left[\frac{R^2}{2}-\left(\frac{2R^2}{7}\right)\right]$$

$$Q = 2p\ U_o\frac{3}{14}R^2 = \frac{3p}{7}U_oR^2$$

PART C

D = 0.05 mm R = 0.025 dp/dL = 20 000 N/m^3 C=0.5 Ns^2/m^2

$$U_o = \frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[R^{\frac{3}{2}}\right] = \frac{2}{3}\left(\frac{20000}{2\ x\ 0.5}\right)^{\frac{1}{2}} 0.025^{\frac{3}{2}} = 0.3726\ m/s$$

$$Q = \frac{3p}{7}U_oR^2 = \frac{3p}{7} x\ 0.3726\ x\ 0.025^2 = 0.000314\ m^3/s$$

APPLIED FLUID MECHANICS D203 SOLUTIONS 2004

1. The energy per unit mass (gH) given to a liquid by a centrifugal pump is known to be dependant on the diameter D, impeller speed N, liquid volumetric flow rate Q, liquid density ρ, liquid dynamic viscosity μ and the power supplied to the pump P.

(a) Use dimensional analysis applied to geometrically similar pumps to derive the functional expression

$$\left(\frac{g\,H}{N^2D^2}\right) = \varphi\left(\frac{Q}{ND^3}\right)\left(\frac{P}{\rho N^3D^5}\right)\left(\frac{\mu}{N\rho\,D^2}\right)$$

SOLUTION

$(gH) = f(D, N, Q, \rho, \mu, P)$ It is normal to consider $g\Delta H$ as one quantity.

There are 7 quantities and 3 dimensions so there are four dimensionless groups Π_1, Π_2 Π_3 and Π_4. First form a group with gH and ρND

$$gH = \varphi\,(\rho ND) = \Pi_1\rho^a N^b D^c$$

$$L^2T^{-2} = \left(ML^{-3}\right)^a\left(T^{-1}\right)^b\left(D^1\right)^c$$

Mass $0 = a$ Time $-2 = -b$ $b = 2$ Length $2 = -3a + c = c$ $c = 2$

$$gH = \Pi_1 N^2D^2 \qquad \Pi_1 = \frac{gH}{N^2D^2}$$

Next repeat the process between Q and ρND

$$Q = \varphi\,(\rho ND) = \Pi_2\rho^a N^b D^c$$

$$M^3T^{-1} = \left(ML^{-3}\right)^a\left(T^{-1}\right)^b\left(D^1\right)^c$$

Time $-1 = -b$ $b = 1$ Mass $0 = a$ Length $3 = -3a + c$ $c = 3$

$$Q = \Pi_2\rho^0 N^1 D^3 \qquad \Pi_2 = \frac{Q}{ND^3}$$

Next repeat the process between P and ρND

$$P = \varphi\,(\rho ND) = \Pi_3\rho^a N^b D^c$$

$$M^1L^2T^{-3} = \left(ML^{-3}\right)^a\left(T^{-1}\right)^b\left(D^1\right)^c$$

Mass $1 = a$ Time $-3 = -b$ $b = 3$ Length $2 = -3a + c = -3 + c$ $c = 5$

$$P = \Pi_3\,\rho^1 N^3 D^5 \qquad \Pi_3 = \frac{P}{\rho N^3D^5}$$

Next repeat the process between μ and ρND

$$\mu = \varphi\,(\rho ND) = \Pi_4\rho^a N^b D^c$$

$$M^1L^{-1}T^{-1} = \left(ML^{-3}\right)^a\left(T^{-1}\right)^b\left(D^1\right)^c$$

Mass $1 = a$ Time $-1 = -b$ $b = 1$ Length $-1 = -3a + c$ $c = 2$

$$\mu = \Pi_4\rho^1 N^1 D^2 \qquad \Pi_4 = \frac{\mu}{\rho\,ND^2}$$

Finally the complete equation is $\left(\frac{g\,H}{N^2D^2}\right) == \varphi\left(\frac{Q}{ND^3}\right)\left(\frac{P}{\rho N^3D^5}\right)\left(\frac{\mu}{N\rho\,D^2}\right)$

(b) Using these groups, derive the dimensionless specific speed of a pump and describe the principal applications of specific speed.

The specific speed is a parameter used for pumps and turbines to determine the best design to match a given pumped system. The formula may be derived from consideration of the pump geometry or by dimensional analysis. It is found by equating the diameter between Π_1 and Π_2 (the head and flow coefficients).

$$\Pi_2 = \frac{Q}{ND^3} \quad D = \left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} \quad \Pi_1 = \frac{gH}{N^2D^2} \quad D = \left(\frac{gH}{N^2\Pi_1}\right)^{\frac{1}{2}}$$

Equating $\left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} = \left(\frac{gH}{N^2\Pi_1}\right)^{\frac{1}{2}} \qquad \frac{1}{N}\left(\frac{gH}{\Pi_1}\right)^{\frac{1}{2}} = \frac{Q^{\frac{1}{3}}}{\Pi_2^{\frac{1}{3}}N^{\frac{1}{3}}}$

$$\frac{(H)^{\frac{1}{2}}}{Q^{\frac{1}{3}}N^{\frac{2}{3}}} = \frac{\Pi_1^{\frac{1}{2}}}{\Pi_2^{\frac{1}{3}}g^{\frac{1}{2}}} = \text{constant} \qquad \frac{(H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}} = N^{\frac{2}{3}}$$

$$\left[\frac{(H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}}\right]^{\frac{3}{2}} = N = \frac{(H)^{\frac{3}{4}}}{K^{\frac{1}{2}}Q^{\frac{1}{2}}} \qquad \frac{NQ^{\frac{1}{2}}}{(H)^{\frac{3}{4}}} = K^{-\frac{1}{2}} = \text{constant}$$

$$N_s = \frac{NQ^{\frac{1}{2}}}{(H)^{\frac{3}{4}}}$$

Traditionally the units used are rev/min for speed, m^3/s for flow rate and metres for head. It may be regarded as the speed to produce a unit flow at unit head.

(c) A centrifugal pump operating at its best efficiency point provides 0.28 m^3/s of water at a head of 2 m and consumes 6.3 kW of power. A geometrically similar pump with an impeller 40% larger is to operate at 20% higher speed. Calculate the flow, head and power required for this second pump running at its best efficiency.

We must have dynamic similarity using $D_1 = 1$ and $D_2 = 1.4$ $\quad N_1 = 1$ $\quad N_2 = 1.2$

First using the flow coefficient

$$\frac{Q_1}{N_1D_1^3} = \frac{Q_2}{N_2D_2^3} \qquad Q_2 = \frac{Q_1N_2D_2^3}{N_1D_1^3} = \frac{0.28 \times 1.2 \times 1.4^3}{1 \times 1^3} = 0.922 \text{ m}^3/\text{s}$$

Next using the head coefficient

$$\frac{gH_1}{N_1^2D_1^2} = \frac{gH_2}{N_2^2D_2^2} \qquad H_2 = \frac{H_1N_2^2D_2^2}{N_1^2D_1^2} = \frac{2 \times 1.2^2 \times 1.4^4}{1} = 5.645 \text{ m}$$

Next using the power coefficient

$$\frac{P_1}{\rho N_1^3D_1^5} = \frac{P_2}{\rho N_2^3D_2^5} \qquad P_2 = \frac{P_1N_2^3D_2^5}{N_1^3D_1^5} = \frac{6.3 \times 1.2^3 \times 1.4^5}{1} = 58.55 \text{ kW}$$

APPLIED FLUID MECHANICS D203 SOLUTIONS 2004 – QUESTION 2

(a) A viscous liquid with density 800 kg/m^3 and dynamic viscosity 0.25 Ns/m^2 flows upwards in the annular space between two concentric vertical pipes with diameters 300 mm and 200 mm respectively.

(i) Assuming that the flow is fully developed and laminar, show that the flow rate Q is given by

$$Q = \frac{-\pi}{8\mu}\left(\frac{dp}{dz} + \rho g\right)\left[\left\{R_o^4 - R_i^4\right\} - \frac{\left(R_o^2 - R_i^2\right)^2}{\ln\left\{\frac{R_o}{R_i}\right\}}\right]$$

where dp/dz is the pressure gradient in the flow direction, p and μ are the liquid density and dynamic viscosity respectively, g is gravitational acceleration and R_i and R_o are the inner and outer radii of the annular space.

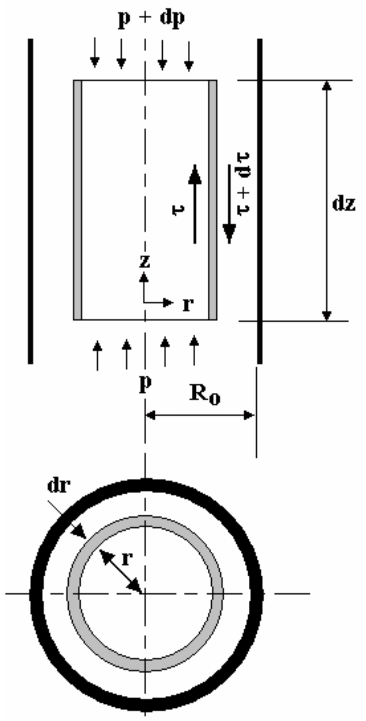
(ii) Calculate the pressure drop over one metre length of the vertical pipes when the flow rate is 0.050 m^3/s.

(iii) Verify that the flow is laminar for these conditions.

(b) For the flow described in part (a) above, calculate the maximum velocity of the liquid in the annular space and the radius at which it occurs.

COMMENT It is unrealistic to do the complete derivation in a reasonable amount of time unless there is a short cut method unknown to me. It may be acceptable to start with equation B which should be remembered and can be applied to a range of circumstances.

Consider fluid flowing vertically in a pipe. Consider a stream tube of length dz at radius r and thickness dr. The pressure vertically decreases as ρgz in the z direction.



The cross sectional area of the thin circular ring is 2πr dr
The surface area of the elementary cylinder is 2πr dz at the inside and 2π(r+dr) dz on the outside.

The shear force is 2π(r+dr) dz (τ + dτ) - 2π(r) dz (τ) which simplifies down to 2π dz (τ dr + r dτ)

Balancing forces we have

$$2\pi r\, dr\, p = 2\pi r\, dr\,(p + dp) + 2\pi r\, dr\, \rho g dz + 2\pi\, dz\,(\tau\, dr + r\, d\tau)$$

$$r\, dr\, p = pr\, dr + dp\, r\, dr + r\, dr\, \rho g dz + dz\,(\tau\, dr + r\, d\tau)$$

$$- dp\, r\, dr - r\, dr\, \rho g dz = dz\,(\tau\, dr + r\, d\tau)$$

$$-\frac{dp}{dz} - \rho g = \frac{\tau}{r} + \frac{d\tau}{dr}$$

$-\left(\frac{dp}{dz}+\rho g\right)=\frac{\tau}{r}+\frac{d\tau}{dr} \qquad \tau=-\mu\frac{du}{dy}$ for Newtonian fluids if the pressure gradient is assumed positive.

If y is measured from the inside of the pipe then r = -y and dy = - dr so $\tau=\mu\frac{du}{dr}$

$$-\left(\frac{dp}{dz}+\rho g\right)=\frac{\mu}{r}\frac{du}{dr}\mu\frac{d^2u}{dr^2}$$

$$\frac{1}{r}\frac{du}{dr}+\frac{d^2u}{dr^2}=-\frac{1}{\mu}\left(\frac{dp}{dz}+\rho g\right)$$

$$\frac{du}{dr}+\frac{rd^2u}{dr^2}=-\frac{r}{\mu}\left(\frac{dp}{dz}+\rho g\right)$$

Using partial differentiation to differentiate $\frac{d\left(r\frac{du}{dr}\right)}{dr}$ yields the result $\frac{du}{dr}+\frac{rd^2u}{dr^2}$

hence $\frac{d\left(r\frac{du}{dr}\right)}{dr}=-\frac{r}{\mu}\left(\frac{dp}{dz}+\rho g\right)$

Integrating we get $r\frac{du}{dr}=-\frac{r^2}{2\mu}\left(\frac{dp}{dz}+\rho g\right)+A$

$$\frac{du}{dr}=-\frac{r}{2\mu}\left(\frac{dp}{dz}+\rho g\right)+\frac{A}{r} \quad \text{...........(A)}$$

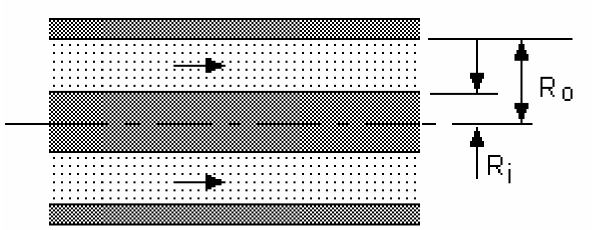
where A is a constant of integration.

Integrating again we get

$$u=-\frac{r^2}{4\mu}\left(\frac{dp}{dz}+\rho g\right)+A\ln r+B \text{................(B)}$$

where B is another constant of integration.

Equations (A) and (B) may be used to derive flow through an annular passage.



$$u=-\frac{r^2}{4\mu}\left(\left(\frac{dp}{dz}+\rho g\right)\right)+A\ln r+B$$

The boundary conditions are u = 0 at $r=R_i$ and $r=R_o$.

$$0=-\frac{R_o^2}{4\mu}\left(\frac{dp}{dz}+\rho g\right)+A\ln R_o+B \text{..........................(C)}$$

$$0=-\frac{R_i^2}{4\mu}\left(\frac{dp}{dz}+\rho g\right)+A\ln R_i+B \text{........................(D)}$$

Subtract D from C

$$0=\frac{\left(\frac{dp}{dz}+\rho g\right)}{4\mu}\left\{-R_o^2+R_i^2\right\}+A\left\{\ln R_o-\ln R_i\right\}$$

$$0=\frac{\left(\frac{dp}{dz}+\rho g\right)}{4\mu}\left\{R_i^2-R_0^2\right\}+A\ln\left\{\frac{R_o}{R_i}\right\}$$

$$A = \frac{\left(\frac{dp}{dz} + \rho g\right)}{4\mu} \frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}$$

This may be substituted back into equation D. The same result will be obtained from C.

$$0 = -\frac{R_i^2}{4\mu}\left(\frac{dp}{dz} + \rho g\right) + \frac{1}{4\mu}\frac{\delta p}{\delta x}\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\ln R_i + B$$

$$B = \frac{1}{4\mu}\left(\frac{dp}{dz} + \rho g\right)\left[R_i^2 - \left\{\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\right\}\ln R_i\right]$$ This is put into equation B

$$u = \frac{-r^2}{4\mu}\left(\frac{dp}{dz} + \rho g\right) + \frac{1}{4\mu}\frac{\delta p}{\delta x}\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\ln r + \frac{1}{4\mu}\frac{\delta p}{\delta x}\left[R_i^2 - \left\{\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\right\}\ln R_i\right]$$

$$u = \frac{\left(\frac{dp}{dz} + \rho g\right)}{4\mu}\left[-r^2 + \frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\ln r + R_i^2 - \frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\ln R_i\right]$$

$$u = \frac{\left(\frac{dp}{dz} + \rho g\right)}{4\mu}\left[\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\ln\frac{r}{R_i} + R_i^2 - r^2\right]$$

The flow through the elementary ring is $dQ = 2\pi r\, dr\, u$

$$Q = \frac{2\pi\left(\frac{dp}{dz} + \rho g\right)}{4\mu}\int_{R_i}^{R_o}\left[\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}} r\ln\frac{r}{R_i} + rR_i^2 - r^3\right]$$

$$Q = \frac{\pi\left(\frac{dp}{dz} + \rho g\right)}{2\mu}\left[\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\left(\frac{r^2}{2}\ln\frac{r}{R_i} - \frac{r^2}{4}\right) + \frac{r^2 R_i^2}{2} - \frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$Q = \frac{\pi\left(\frac{dp}{dz} + \rho g\right)}{2\mu}\left[\left(\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\left(\frac{R_o^2}{2}\ln\frac{R_o}{R_i} - \frac{R_o^2}{4}\right) + \frac{R_o^2 R_i^2}{2} - \frac{R_o^4}{4}\right) - \left(\frac{\left\{R_o^2 - R_i^2\right\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\left(\frac{R_i^2}{2}\ln\frac{R_i}{R_i} - \frac{R_i^2}{4}\right) + \frac{R_i^2 R_i^2}{2} - \frac{R_i^4}{4}\right)\right]$$

$$Q = \frac{\pi\left(\frac{dp}{dz}+\rho g\right)}{8\mu}\left[\left(\frac{\{R_o^2 - R_i^2\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\left(2R_o^2 \ln\frac{R_o}{R_i} - R_o^2\right) + 2R_o^2R_i^2 - R_o^4\right) - \left(\frac{\{R_o^2 - R_i^2\}\left(-R_i^2\right)}{\ln\left\{\frac{R_o}{R_i}\right\}} + R_i^4\right)\right]$$

$$Q = \frac{\pi\left(\frac{dp}{dz}+\rho g\right)}{8\mu}\left[\left(\{R_o^2 - R_i^2\}\{2R_o^2\} - \frac{\{R_o^2 - R_i^2\}\{R_o^2\}}{\ln\left\{\frac{R_o}{R_i}\right\}} + 2R_o^2R_i^2 - R_o^4\right) - \left(\frac{\{-R_o^2R_i^2 + R_i^4\}}{\ln\left\{\frac{R_o}{R_i}\right\}} + R_i^4\right)\right]$$

$$Q = \frac{\pi\left(\frac{dp}{dz}+\rho g\right)}{8\mu}\left[\left(\{2R_o^4 - 2R_i^2R_o^2\} - \frac{\{R_o^2 - R_i^2\}\{R_o^2\}}{\ln\left\{\frac{R_o}{R_i}\right\}} + 2R_o^2R_i^2 - R_o^4\right) + \left(\frac{\{R_o^2R_i^2 - R_i^4\}}{\ln\left\{\frac{R_o}{R_i}\right\}} - R_i^4\right)\right]$$

$$Q = \frac{\pi\left(\frac{dp}{dz}+\rho g\right)}{8\mu}\left[\left(R_o^4 - \frac{\{R_o^4 - R_i^2R_o^2\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\right) + \left(\frac{\{R_o^2R_i^2 - R_i^4\}}{\ln\left\{\frac{R_o}{R_i}\right\}} - R_i^4\right)\right]$$

$$Q = \frac{\pi\left(\frac{dp}{dz}+\rho g\right)}{8\mu}\left[R_o^4 - R_i^4 - \frac{\{R_o^4 - 2R_i^2R_o^2 + R_i^4\}}{\ln\left\{\frac{R_o}{R_i}\right\}}\right] = \frac{\pi\left(\frac{dp}{dz}+\rho g\right)}{8\mu}\left[R_o^4 - R_i^4 + \frac{\left(R_o^2 - R_i^2\right)^2}{\ln\left\{\frac{R_o}{R_i}\right\}}\right]$$

The result should have a minus in front – anyone able to point out where it has gone please let me know.

(ii) $Q = 0.05\ m^3/s \quad R_o = 0.15\ m \quad R_i = 0.1\ m \quad \mu = 0.25\ Ns/m^2$

$$0.05 = \frac{-\pi\left(\frac{dp}{dz}+\rho g\right)}{8 \times 0.25}\left[0.15^4 - 0.1^4 + \frac{\left(0.15^2 - 0.1^2\right)^2}{\ln\left\{\frac{0.15}{0.1}\right\}}\right]$$

$$0.05 = \frac{-\pi}{2}\left(\frac{dp}{dz}+\rho g\right)\left[2.089 \times 10^5\right] \qquad \left(\frac{dp}{dz}+\rho g\right) = -1524\ N/m^2 \text{per metre}$$

(iii) The mean velocity u = Q/Cross Sectional Area = $\frac{0.05}{\pi\left(0.15^2 - 0.1^2\right)} = \frac{0.05}{0.03927} = 1.273\ m/s$

$Re = \frac{\rho u D}{\mu}$ but as it is an annulus we cant use the diameter so how to check if its is laminar? My guess is it is based on radial gap. Knowing the critical value of Re is another matter.

(b) Proving where the maximum velocity occurs is another large task but a parabolic distribution reaches a maximum at mid point where r = 0.125 m

$$u = \frac{(1524)}{4 \times 0.25}\left[\frac{\{0.15^2 - 0.1^2\}}{\ln\left\{\frac{0.15}{0.1}\right\}}\ln\frac{0.125}{0.1} + 0.1^2 - 0.125^2\right] = 1.911\ m/s$$

NB this is 1.5 times the mean which is correct for a parabola.

APPLIED FLUID MECHANICS D203 SOLUTIONS 2004 – QUESTION 3

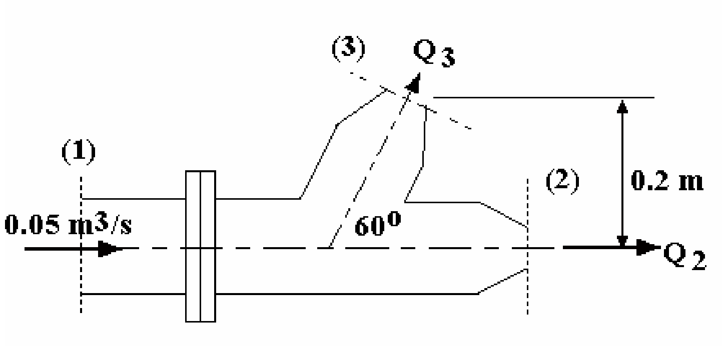
A 150 mm diameter pipe is fitted with a double nozzle with the geometry shown. At section (2) the exit diameter of the nozzle is 80 mm and at section (3) the exit diameter is 100 mm. The pipe and both nozzles lie in the same vertical plane. A steady discharge of 0.05 m^3/s of water from the pipe at section (1) emerges as jets from the nozzles into the surrounding atmosphere. Energy losses in the flow may be assumed to be negligible.

(a) Calculate

(i) the flow rate at each nozzle exit

(ii) the water pressure in the pipe at section (1).

(b) The double nozzle fitting is attached to the pipe by a bolted flange. The fitting has a material mass of 5 kg and the water volume within it is 0.005 m^3. Calculate the magnitude and direction of the resultant force applied to the group of flange bolts for the flow conditions described above.



D_1= 0.15m D_2= 0.08m D_3= 0.1m Q1= 0.05 m^3/s

p_1=400 x 10^3 N/m^2 p_2=395 x 10^3 N/m^2

$$A_1 = \frac{\pi D_1^2}{4} = 0.018\,m^2 \quad A_2 = \frac{\pi D_2^2}{4} = 0.005027\,m^2 \quad A_3 = \frac{\pi D_3^2}{4} = 0.007854\,m^2$$

$$u_1 = \frac{Q_1}{A_1} = 2.829\,m/s$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (2)

$$p_1 + \rho\frac{u_1^2}{2} = p_2 + \rho\frac{u_2^2}{2}$$

$$p_1 + 998\frac{2.829^2}{2} = 0 + 998\frac{u_2^2}{2} \quad p_1 = 499u_2^2 - 3.995 \times 10^3$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (3)

$$p_1 + \rho\frac{u_1^2}{2} = p_3 + \rho\frac{u_3^2}{2} + \rho g z$$

$$p_1 + 998\frac{2.829^2}{2} = p_3 + 998\frac{u_3^2}{2} + 0.2 \times 9.81 \times 998 \quad p_1 = 499u_3^2 - 2.037 \times 10^3$$

$$p_1 = +499u_2^2 - 3.995 \times 10^3 = 499u_3^2 - 2.037 \times 10^3$$

$$u_2^2 = u_3^2 + 3.918$$

CONSERVATION OF MASS

$$\rho Q_1 = 0.05\rho = \rho Q_2 + \rho Q_3 \qquad Q_2 = 0.05 - Q_3 \qquad u_2 A_2 = 0.05 - u_3 A_3$$

$$5.027 \times 10^{-3} u_2 = 0.05 - 7.854 \times 10^{-3} u_3$$

$$u_2 = 9.947 - 1.562u_3 \qquad u_2^2 = 98.94 + 2.441u_3^2 - 31.08u_3$$

$$u_2^2 = 98.94 + 2.441u_3^2 - 31.08u_3 = u_3^2 + 3.918$$

$$1.441u_3^2 - 31.08u_3 + 95.02$$

Solving the quadratic u_3 = 3.69 m/s the other solution 17.88 would give a flow larger than Q_1.

Q_3= A_3 u_3 = 0.029 m_3/s $Q_2 = Q_1 - Q_3$ = 0.021 m_3/s

Check $u_2 = \sqrt{u_3^2 + 3.918} = 4.185$ m/s
$Q_2 = A_2 u_2 = 0.021$ m_3/s

(ii) $p_1 = 499u_2^2 - 3.995 \times 10^3 = 4.747 \times 10^3$ N/m^2

FORCES
Momentum at (1) = $\rho A_1 u_1^2 = 998 \times 0.01767 \times 2.829^2 = 141.2$
Momentum at (2) = $\rho A_2 u_2^2 = 998 \times 0.00503 \times 4.185^2 = 87.9$

Momentum at (3) = $\rho A_3 u_3^2 = 998 \times 0.007854 \times 3.69^2 = 106.6$
Resolve vertically and horizontally
Horizontal Momentum = 106.6 cos 60^o =53.3
Vertical Momentum = 106.6 sin 60^o = 92.3

PRESSURE FORCES

Pressure force at (1) = $p_1 A_1$ = 83.9 N
Pressure force at (2) and (3) are zero since gauge pressures are being used.
Weight = 5 x 9.81 + weight of water = 5 x 9.81 + 0.005 x 998 = 54 N↓

TOTALS on FLANGE
HORIZONTAL $\Delta mv + \Delta pA$ =-141.2 + 87.9 + 53.3 - 83.9 = 83.9 N (to right)
VERTICAL Δmv - W = -92.3 -54 = -146.3 N (Down no pressure force)
Total = $\sqrt{(83.9^2 + 146.3^2)}$ = 168.6 N
Angle = $\tan^{-1}(146.3/83.9) = 60^o$ to vertical

a) Show that the combination of a uniform flow with velocity U in the x-direction, a doublet of strength Q at the origin of an x - y coordinate system and an irrotational vortex with circulation Γ also at the origin can be used to describe the inviscid flow of a fluid around a rotating cylinder. Derive expressions for the diameter of the cylinder and its rotational speed.

b) For the flow conditions corresponding to the combination given in part (a), show that
(i) the drag exerted on the cylinder by the flow is zero
(ii) the lift experienced by the cylinder is ρ U Γ per unit length of cylinder, where ρ is the fluid density.

c) A long 200 mm diameter cylinder is rotating at 1000 rev/min about a vertical axis in a steady stream of air which is flowing horizontally with a velocity of 5 m/s. The pressure and temperature of the air far upstream of the cylinder are 1.0 bar and 20°C respectively. Calculate
(i) the position of any stagnation points in the flow
(ii) the lift force on the cylinder per unit length and its direction
(iii) the minimum pressure and its location on the surface of the cylinder.

Note $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$ and $\int \sin^3 x = -\frac{1}{3}\left(2+\sin^2 x\right)\cos x$

DOING THIS QUESTION IN THE TIME ALLOTTED REQUIRES A GENIUS

(a) The stream function for UNIFORM FLOW + DOUBLET is

$\Psi = -Uy + (Q/\pi r)\sin\theta$ (depending on notation used)
$y = r\sin\theta \quad Q/\pi = B$
Q is the flow per unit depth of the source and sink making the doublet.
$\Psi = -Ur\sin\theta + B\sin\theta/r = -\sin\theta(Ur - B/r)$
In order that the cylinder be a solid surface $\Psi = 0$ hence $U r = B/r \quad B = U r^2$
And let this radius be R_o so $B = U R_o^2$

The diameter of the cylinder is hence $D = 2R_o = 2\sqrt{(B/U)} = 2\sqrt{(Q/\pi U)}$

$\Psi = -\sin\theta(Ur - U R_o^2/r^2 = -U\sin\theta\,(r - R_o^2/r)$

If we add a vortex at the origin, a circular motion is added giving the effect of a rotating cylinder. The velocity of the vortex at the same radius as the cylinder must be the tangential velocity of the cylinder.
For a vortex $ur = \omega r^2 = C = \omega R_o^2$ at the cylinder surface. u is the velocity of the stream line.
Γ = circulation $= 2\pi\omega R_o^2 = 2\pi C$
$\omega = \Gamma/2\pi R_o^2 \qquad N = \omega/2\pi = \Gamma/4\pi^2 R_o^2 \qquad R_o = \sqrt{(\Gamma/4N\pi^2)} \qquad \Gamma = 4N\pi^2 R_o^2$

For a free vortex $\Psi = C\ln(r/a) = u r \ln(r/a)$ a is the inner radius of the vortex.

The total stream function is hence $\Psi = -U\sin\theta\,(r - R_o^2/r) + C\ln(r/a)$

The tangential velocity is $v_T = d\Psi/dr = -U\sin\theta\left\{1 + \frac{R_o^2}{r^2}\right\} + \frac{C}{r}$

At the cylinder surface $r = R_o$ so $v_T = -2U\sin\theta + \frac{C}{R_o}$

The stagnation points occur where this is zero so $\sin\theta = C/2UR_o$

PRESSURE DISTRIBUTION

Apply Bernoulli between a point in the undisturbed flow and a point on the cylinder.

$$p_o - p = \frac{\rho}{2}\left(v_T^2 - U^2\right) = \frac{\rho}{2}\left\{\left(-U\sin\theta\left\{1+\frac{R_o^2}{r^2}\right\}+\frac{C}{r}\right)^2 - U^2\right\}$$

$$p_o - p = \frac{\rho}{2}\left(4U^2\sin^2\theta - \frac{4CU\sin\theta}{R_o} + \frac{C^2}{R_o^2} - U^2\right)$$

$$p_o - p = \frac{\rho U^2}{2}\left(4\sin^2\theta - \frac{4C\sin\theta}{UR_o} + \frac{C^2}{UR_o^2} - 1\right) \text{ and simplifying by putting } \frac{C^2}{U^2R_o^2} = \beta^2$$

$$p_o - p = \frac{\rho U^2}{2}\left(4\sin^2\theta - 4\,\beta\sin\theta + \beta^2 - 1\right)$$

DRAG integrate the pressure force acting on the surface horizontally.

$$D = \int_0^{2\pi}(p_o - p)R_o\cos\theta\, d\theta = \int_0^{2\pi}\frac{\rho U^2}{2}\left(4\sin^2\theta - 4\,\beta\sin\theta + \beta^2 - 1\right)R_o\cos\theta\, d\theta$$

$$D = \frac{R_o\rho U^2}{2}\int_0^{2\pi}\left(4\sin^2\theta\cos\theta - 4\,\beta\sin\theta\cos\theta + \beta^2\cos\theta - \cos\theta\right)d\theta$$

There may be a short cut but take it on advice that this comes to zero. Note that the drag in an inviscid fluid should always be zero.

LIFT integrate the pressure force acting on the surface vertically.

$$L = \int_0^{2\pi}(p_o - p)R_o\sin\theta\, d\theta = \int_0^{2\pi}\frac{\rho U^2}{2}\left(4\sin^2\theta - 4\,\beta\sin\theta + \beta^2 - 1\right)R_o\sin\theta\, d\theta$$

$$L = \frac{R_o\rho U^2}{2}\int_0^{2\pi}\left(4\sin^3\theta - 4\,\beta\sin^2\theta + \beta^2\sin\theta - \sin\theta\right)d\theta$$

Using the identities given in the question we obtain

$$L = \frac{R_o\rho U^2}{2}\left[\frac{-4}{3}\left(2+\sin^2\theta\right)\cos\theta - 4\beta\left(\frac{\theta}{2} - \frac{\sin(2\theta i}{4}\right) - \beta^2\cos\theta - \sin\theta\right]_0^{2\pi}$$

Integrating each part separately between 0 and 2π

$$\left[\frac{-4}{3}\left(2+\sin^2\theta\right)\cos\theta\right]_0^{2\pi} = \left[\frac{-4}{3}(2+0)\right] - \left[\frac{-4}{3}(2+0)\right] = 0$$

$$\left[-4\beta\left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right)\right]_0^{2\pi} = \left[-4\beta\left(\frac{2\pi}{2} - \frac{\sin(4\theta i}{4}\right)\right] - \left[-4\beta\left(\frac{0}{2} - \frac{\sin(0)}{4}\right)\right]$$

$$= \left[-4\beta(\pi - 0)\right] - [0] = -4\beta\,\pi$$

$$\left[-\beta^2\cos\theta\right]_0^{2\pi} = \left[-\beta^2\cos(2\pi)\right] - \left[-\beta^2\cos(0)\right] = 0$$

$$[-\sin\theta]_0^{2\pi} = 0$$

$$L = \frac{R_o\rho U^2}{2}\left[0 - 4\,\beta\,\pi - 0 - 0\right] = R_o\rho U^2\,\beta\,2\pi = R_o\rho U^2\;2\pi\frac{C}{UR_o} = \rho U\;2\pi\,C$$

$2\pi C$ is the circulation Γ so $L = \rho U\Gamma$

c. Dia = 200 mm $R_o = 0.1$ m N = 1000/60 rev/s $\omega = 2\pi N = 104.72$ rad/s U = 5 m/s.

$\Gamma = 2\pi\omega R_o^2 = 2\pi \times 104.72 \times 0.1^2 = 6.58$

$C = \Gamma/2\pi = 1.047$

$\sin\theta = \frac{C}{2UR_o} = \frac{1.047}{2 \times 5 \times 0.1} = 1.047$ as the maximum value can only be 1.0 there is an error or perhaps the stagnation point is off the surface.

p = 1.0 bar T = 293 K $\rho = \frac{p}{RT} = \frac{1 \times 10^5}{287 \times 293} = 1.189 \text{ kg/m}^3$

$L = \rho U \Gamma = 1.189 \times 5 \times 1.047 = 6.22$ N/m

$p = p_o - \frac{\rho}{2}\left(v_T^2 - U^2\right)$ and this will be a minimum when v_T is a maximum

$$v_T = -U\sin\theta\left\{1 + \frac{R_o^2}{r^2}\right\} + \frac{C}{r} = -2 \times 5 \times \sin\theta + \frac{1.047}{0.1} = 10.47 - 10\sin\theta$$

This will be a maximum when $\sin\theta = 0$ and $v_T = 10.47$ m/s

$p = p_o - \frac{\rho}{2}\left(v_T^2 - U^2\right) = 1 \times 10^5 - \frac{1.189}{2}\left(10.47^2 - 10^2\right)$ the change in pressure is tiny

I suspect an error somewhere but can’t find it. Anyone able to help please contact me.

APPLIED FLUID MECHANICS D203 QUESTION 5 2004

(a) Explain how momentum is transferred in the turbulent flow of a fluid and compare this with momentum transfer in laminar flow.

(b) (i) The mean shear stress τ in a turbulent flow in a pipe is given by $\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

where ρ is fluid density, l is the Prandtl mixing length and u is the time-mean velocity at distance y from the pipe wall. Assuming that the mean shear stress τ is equal to the wall shear stress τ_o and taking the mixing length l = 0.4y, show that

$$\frac{u_m - u}{u^*} = 5.75 \log_{10} \frac{R}{y}$$

u_m is the maximum velocity in the pipe. $u^* = \sqrt{\frac{\tau_o}{\rho}}$ is the friction velocity and R is the pipe radius.

(ii) Explain why this relationship does not apply close to the pipe wall.

(c) Use calculations to determine the smallest diameter of commercial galvanized steel pipe required to transport water over a horizontal distance of 200m at a flow rate of 0.10 m^3/s if the head loss is not toexceed 10m. Available commercial pipes have diameters of 80, 100, 150, 200, 250 and 300 mm. The roughness factor *k* for galvanized pipe is 0.15 mm.

The Darcy friction factor *f* for the flow may be obtained from the empirical equation

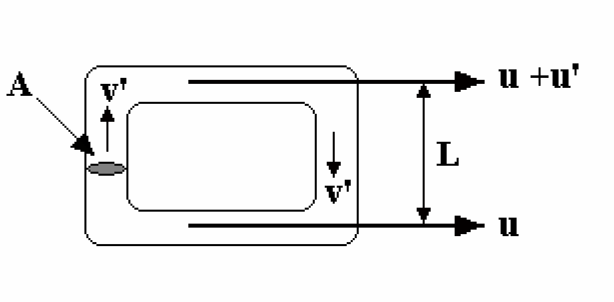
$$\frac{1}{\sqrt{f}} = -0.782 \ln\left[\frac{6.9}{Re} + \left(\frac{k/D}{3.71}\right)^{1.11}\right]$$

where Re is the Reynolds Number for the pipe based on diameter.

The Darcy friction factor *f* is used in the following formula for friction head loss in a pipe

$h_f = f\frac{Lv^2}{2gD}$ The symbols have their usual meanings.

(a) In laminar flow friction occurs between parallel layers and momentum is transferred by the drag exerted by one layer on the other. In turbulent flow eddy currents are set up to transfer momentum between layers. An idealized eddy is shown in the diagram as a closed loop of cross sectional area A with a velocity v'. Consider two layers distance L apart. The upper layer moves at velocity (u + u') and the lower layer at u.



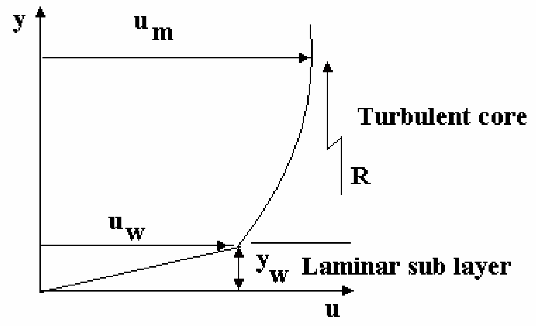
Assuming that u'/L = du/dy u' = L du/dy. The difference in velocity of the layers must be 2v' if no slippage is occurring so u' = 2v' and hence v' = ½ L du/dy.

The mass flow rate within the eddy is ρAv' and an equal mass is transferred from one layer to the other so the total interchange is 2ρAv'

The rate of change of momentum is 2ρAv'u' = $\rho A u'^2$
And this is a force F = $\rho a u'^2$

The shear stress acting on the area A is τ so $2\tau A = \rho A u'^2$ hence $\tau = \frac{1}{2}\rho u'^2 = \tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

b) The diagram shows velocity u plotted against distance y from the wall. Close to the wall is a thin laminar layer so the following only applies to the turbulent core.



$$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2 = \rho (ky)^2 \left(\frac{du}{dy}\right)^2 = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2$$

$$\tau_o = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2$$

$$u^* = \sqrt{\frac{\tau_o}{\rho}} = \frac{\sqrt{\rho}\,ky}{\sqrt{\rho}}\frac{du}{dy} = ky\frac{du}{dy} \qquad du = u^*\frac{dy}{ky}$$

Integrate $\quad u = u^*\frac{1}{k}\ln y + C$

At the centre of the pipe where y = R the velocity is u_m

$u_m = u^*\frac{1}{k}\ln R + C \quad C = u_m - u^*\frac{1}{k}\ln R$ and substitute back for C

$$u = u^*\frac{1}{k}\ln y + u_m - u^*\frac{1}{k}\ln R = u_m + u^*\frac{1}{k}\ln\frac{y}{R}$$

$$\frac{u_m - u}{u^*} = u^*\frac{1}{k}\ln\frac{R}{y}$$

put k = 0.4 and note that to convert to $\log_{10}$ we multiply by ln 10

$$\frac{u_m - u}{u^*} = u^*\frac{\ln 10}{0.4}\log_{10}\frac{R}{y} = 5.75u^*\log_{10}\frac{R}{y}$$

(c) $h_f = 10 = f\frac{Lv^2}{2gD} = f\frac{200\,v^2}{2gD}$ $\quad v = Q/A = 4Q/\pi D^2 = 4 \times 0.1/\pi D^2 = 0.127D^{-2}$

$$10 = f\frac{200\,(0.127D^{-2})^2}{2gD} = f \times 0.165\,D^{-5} \qquad f = 60.61\,D^5$$

Re $= \rho vD/\mu = 997 \times 0.127\ D^{-1}/0.89 \times 10^{-3} = 142269\ D^{-1}$

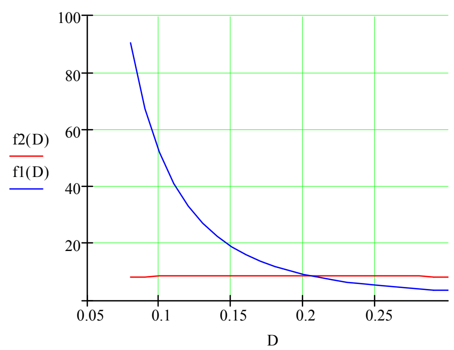
$$\frac{1}{\sqrt{f}} = -0.782\ln\left[\frac{6.9}{Re} + \left(\frac{k/D}{3.71}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{60.61D^5}} = -0.782\ln\left[\frac{6.9D}{142269} + \left(\frac{0.15 \times 10^{-3}}{3.71D}\right)^{1.11}\right]$$

$$\frac{1}{7.785D^{5/2} \times 0.782} = -\ln\left[648.5 \times 10^{-6}D + \left(40.43 \times 10^{-6}D^{-1}\right)^{1.11}\right]$$

$$0.1642D^{-5/2} = -\ln\left[648.5 \times 10^{-6}D + \left(40.43 \times 10^{-6}D^{-1}\right)^{1.11}\right]$$

Plotting both functions against D we see that a diameter that satisfies both sides is just over 200 mm.



(a) For isentropic flow in a variable area duct, derive the expression

$$\frac{dA}{A}=\frac{du}{u}\left(M^2-1\right)$$

where u is the fluid velocity, M the Mach number and A the cross-sectional area of the duct. Use the expression to explain why a convergent/divergent nozzle is required to produce a supersonic flow from a reservoir of stationary gas.

(b) Air in a reservoir has a pressure of 500 kN/m^2 and temperature of 20 °C. It is connected to a receiver by a convergent nozzle with exit diameter 35 mm. If the pressure in the receiver is maintained at 300 kN/m^2, calculate

(i) the temperature and density of the air at exit from the nozzle
(ii) the exit Mach number
(iii) the air mass flow rate through the nozzle.

(c) Repeat the calculations in part (b) when the receiver pressure is 200 kN/m^2. The following equations may be used where appropriate without proof.

$$T_o=T\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right] \qquad \frac{p}{p_o}=\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} \qquad a=\sqrt{\gamma RT} \qquad m=\rho Au$$

SOLUTION

w = specific work q = specific heat transfer u = velocity v = specific volume h = specific enthalpy
p = pressure s = specific entropy T = temperature a = sonic velocity M = u/a

CONSERVATION OF ENERGY

dq + dw = dh + d(ke) but since the flow is isentropic q = 0 and since no work is done w = 0
dh + d(u^2/2) = 0   dh = T ds + v dp but since it is isentropic ds = 0
dh + u du= 0   dh = v dp
v dp + u du = 0
v dp = - u du ……………(A)
du/u = - v dp/u^2…………(B)

CONSERVATION OF MASS

A u/v = constant take logs
log A + log u - log v = const differentiate
dA/A + du/u – dv/v = 0
dA/A = dv/v – du/u substitute (A)
dA/A = dv/v + v dp/u^2

$$\frac{dA}{A}=vdp\left(\frac{dv}{v^2dp}+\frac{1}{u^2}\right)$$ It can be shown that $a^2=-v^2\frac{dp}{dv}$

$$\frac{dA}{A}=vdp\left(-\frac{1}{a^2}+\frac{1}{u^2}\right)=\frac{vdp}{u^2}\left(-\frac{u^2}{a^2}+1\right)$$

$$\frac{dA}{A}=\frac{vdp}{u^2}\left(1-M^2\right)$$ substitute (B) $$\frac{dA}{A}=\frac{du}{u}\left(M^2-1\right)$$

If M 〉 1 the flow is supersonic so dA/A must be positive – i.e. divergent.
If M 〈 1 the flow is subsonic so dA/A must be negative – i.e. convergent.

(b) $T_o = 293$ K $p_o = 500$ kPa $p_e = 300$ kPa $A_e = \pi \times 0.035^2/4 = 962.1 \times 10^{-6}$ m^2

$$\frac{p_o}{p_e} = \left(\frac{T_o}{T_e}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{500}{300} = \left(\frac{293}{T_e}\right)^{\frac{1.4}{1.4-1}} = \left(\frac{293}{T_e}\right)^{3.5} \qquad 1.157 = \frac{293}{T_e} \quad T_e = 253.2\text{K}$$

$$\frac{T_o}{T} = 1 + \left(\frac{\gamma-1}{2}\right)M_e^2 = \frac{293}{253.2} = 1.157$$

$$\left(\frac{\gamma-1}{2}\right)M_e^2 = 0.157 = 0.2M_e^2$$

$$M_e^2 = 0.786 \quad M_e = 0.886$$

$$a_e = \sqrt{\gamma R T_e} = \sqrt{1.4 \times 287 \times 253.2} = 319 \text{ m/s}$$

$$u_e = 0.886 \times 319 = 282.7 \text{m/s}$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{300\,000}{287 \times 253.2} = 4.128 \text{ kg/m}^3$$

m = ρ A u = 4.128 x 962.1 x 10^{-6} x 282.7 =1.123 kg/s

(c) $p_e = 200$ kPa The pressure ratio is 200/500 = 0.4

$$\frac{p_o}{p_e} = \left(\frac{T_o}{T_e}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{500}{200} = \left(\frac{293}{T_e}\right)^{\frac{1.4}{1.4-1}} = \left(\frac{293}{T_e}\right)^{3.5} \qquad 1.3 = \frac{293}{T_e} \quad T_e = 225.3\text{K}$$

$$\frac{T_o}{T_e} = 1 + \left(\frac{\gamma-1}{2}\right)M_e^2 = \frac{293}{225.3} = 1.3$$

$$\left(\frac{\gamma-1}{2}\right)M_e^2 = 0.3 = 0.2M_e^2$$

$$M_e^2 = 0.786 \quad M_e = 1.225$$

The velocity is supersonic so the nozzle is choked so calculating the mass flow in the same way as part (b) is invalid.

The critical pressure ratio is $\frac{p_o}{p_t} = \left(\frac{\gamma}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$

When choked the pressure at the throat is the critical value and Mach number is 1.0

$$\frac{p_o}{p_t} = \left(\frac{\gamma}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528 \quad p_t = 0.528 \times 500 \text{ kPa} = 264.1 \text{ kPa}$$

$$\frac{T_o}{T_t} = 1 + \left(\frac{\gamma-1}{2}\right)M_e^2 = 1.2 \quad T_t = 244.2 \text{ K}$$

$$\rho_t = \frac{p_t}{RT_t} = \frac{264\,100}{287 \times 244.2} = 3.769 \text{ kg/m}^3$$

We need the throat area. This is found from

$$A_t = A_e M_e \left(\frac{p_e}{p_t}\right)^{\frac{1+\gamma}{2\gamma}} = 962.1\text{x}10^{-6} \text{ x } 1.225\left(\frac{200}{264.1}\right)^{0.857} = 928.6 \times 10^{-6} \text{m}^2$$

$$a_t = \sqrt{\gamma R T_t} = \sqrt{1.4 \times 287 \times 244.2} = 313.2 \text{ m/s}$$

$$u_e = 313.2 \text{ m/s}$$

m = ρ A u = 3.769 x 928.6 x 10^{-6} x 313.2 =1.1 kg/s This should be more than in part (b) so an error somewhere???

7 (a) Show that to avoid a shock loss at entry to the runner of an inward radial flow reaction turbine when the blade thickness at inlet to the runner is negligible, the inlet guide vane angle β_1 should have the value given by the expression

$$\beta_1 = \cot^{-1}\left[\frac{2\,\pi\, r_1^2 h_1 \omega}{Q} + \cot\alpha_1\right]$$

where r_1 is the runner blade inlet radius, h_1 is the blade width at inlet, α_1 is the runner blade inlet angle (relative to a tangent to the runner), ω is the angular velocity of the runner and Q is the water flow rate through the turbine.

(b) An inward radial flow reaction turbine is supplied with 0.700 m^3/s of water under an effective head of 16 m. The runner is rotating at 300 rev/min and its inner and outer diameters are 0.5 m and 0.75 m respectively. The runner blade width at inlet is 0.1 m and the blade inlet angle is 105° to a tangent to the runner. The flow is discharged radially from the runner to atmospheric pressure. Given that the thickness of the blades at inlet to the runner is negligible and the flow component of velocity is constant through the runner, calculate

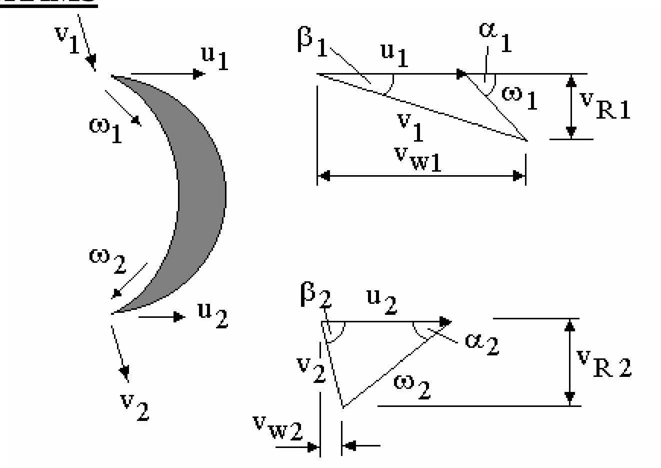
(i) the guide vane inlet angle for no shock loss in the runner
(ii) the runner blade outlet angle
(iii) the output shaft power available from the turbine if the mechanical efficiency is 93%
(iv) the overall efficiency of the turbine.

Note: $$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

SOLUTION

Comment – this seems a very reasonable question but I don't know why the trig identity is needed.

BLADE VECTOR DIAGRAMS



v_R = radial velocity and is constant

A = circumferential area = $2\pi r\, h$ $v_R = Q/(2\pi r\, h)$ h = height of the vane.

$$v_{w1} = u_1 + v_{r1}\cot\alpha_1$$

$$\cot\beta_1 = \frac{v_{w1}}{v_r} = \frac{u_1 + v_{r1}\cot\alpha_1}{v_{r1}} = \frac{u_1}{v_r 1} + \cot\alpha_1 \quad u = \omega r_1 \quad v_{r1} = \frac{Q}{2\pi\, r_1 h_1}$$ (b) (i) Q = 0.700 m^3/s N = 300

$$\cot\beta_1 = \frac{2\pi\, r_1^2\, h_1\omega}{Q} + \cot\alpha_1 \qquad \beta_1 = \cot^{-1}\left(\frac{2\pi\, r_1^2\, h_1\omega}{Q} + \cot\alpha_1\right)$$

rev/min D_1 = 0.75 m D_2 = 0.5 m Radial discharge

$R_1 = 0.375$ $h_1 = 0.1$ m $\alpha_1 = 105°$ $\omega = 2\pi N = 2\pi(300/60) = 31.416$ rad/s

$$\beta_1 = \cot^{-1}\left(\frac{2\pi\, r_1^2\, h_1\omega}{Q} + \cot\alpha_1\right) = \cot^{-1}\left(\frac{2\pi \text{ x } 0.25^2 \text{ x } 0.1 \text{ x } 31.416}{0.7} + \cot 105^o\right) = \cot^{-1}(3.965 - 0.268)$$

$$\beta_1 = \cot^{-1}(3.697) \quad \beta_1 = 15.1^o$$

(ii) From the outlet triangle with radial discharge we have

$\tan\alpha_2 = v_{R2}/u_2$ $v_{R2} = v_R = Q/(2\pi r\, h) = 2.971$ m/s

$\omega = 2\,\pi\,N = 2\,\pi$ x $300/60 = 10\pi$ rad/s

$u_2 = \omega\, D_2/2 = 7.854$ m/s

The runner blade outlet angle = $\alpha_2 = 20.7^o$

(iii) DIAGRAM POWER = D.P. = $m\,\Delta(uv_w) = m\,(u_1v_{w1} - u_2v_{w2})$ radial discharge so $u_2v_{w2} = 0$

$u_1 = \omega\, D_1/2 = 11.78$ m/s $v_{w1} = 10.985$ m/s

D.P. = $m\,(u_1v_{w1}) = \rho Q\, u_1v_{w1} = 997$ x 0.7 x 11.78 x 10.985 = 90.32 x 10^3 W

The output shaft power available from the turbine if the mechanical efficiency is 93% is

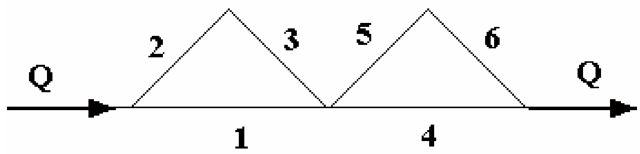
Shaft Power = 0.93 x 90.32 = 84 kW

(iv) Water Power = $mg\Delta H = \rho Qg\,\Delta H$ = 997 x 0.7 x 9.81 x 16 = 109.5 kW

The overall efficiency of the turbine = SP/WP = 76.7%

(a) The figure shows a network of pipes transporting water at a flow rate Q from the inlet node to the outlet node. The individual pipes in the network are numbered as shown and the friction head loss in the pipes is given by $(h_f)_i = k_i Q_i^2$ for I = 1,2 …..6

Derive an expression for the head loss k factor of an equivalent single pipe connected between the same inlet and outlet nodes and transporting the same total flow Q



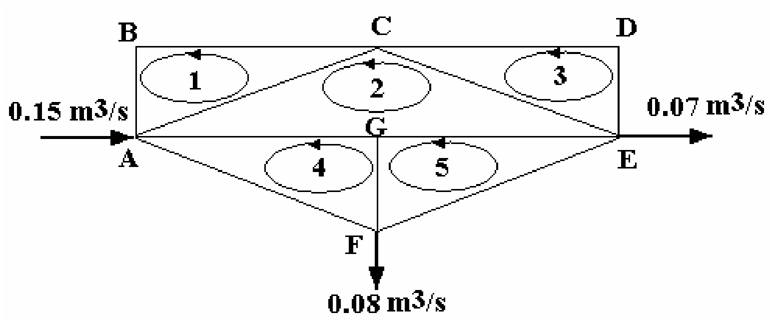
(b) A multi-loop network is shown in the second figure. The friction head loss factor k for each pipe and the elevation of each node above a common datum are Given in the table.

(i) Use the result from part (a) to simplify the pipe network.

(ii) Use two iterations of the Hardy Cross method of solution to estimate the flow distribution in the simplified pipe network.

(iii)If the pressure head at outlet E must be at least 20 m, calculate the minimum pressure head required at inlet node A.

Note. The Hardy Cross flow correction equation is $\Delta Q = \dfrac{-\sum h_f}{2\sum \dfrac{h_f}{Q}}$



The loops have been added to aid solution

Pipe	AB	AC	BC	CD	CE	DE	AG	AF	EF	GF	GE
k s^2/m^5	100	120	150	100	140	160	150	180	150	150	180

Node	A	B	C	D	E	F	G
Elevation m	12	14	10	8	10	8	10

(a) For the first half of the network (1,2 and 3)

$\Delta h_a = Q_2^2(k_2 + k_3)$ $\quad Q_2^2 = \dfrac{\Delta h_a}{k_2 + k_3}$

$\Delta h_a = Q_1^2(k_1)$ $\quad Q_1^2 = \dfrac{\Delta h_a}{k_1}$ $\quad Q = Q_1 + Q_2 = \sqrt{\dfrac{\Delta h_a}{k_2 + k_3}} + \sqrt{\dfrac{\Delta h_a}{k_1}}$

$$Q = Q_1 + Q_2 = \sqrt{\Delta h_a}\left\{\sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}}\right\} = \Delta h_a\left\{\sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}}\right\}^2 = \frac{\Delta h_a}{k_a}$$

$$k_a = \frac{1}{\left\{\sqrt{\dfrac{1}{k_2 + k_3}} + \sqrt{\dfrac{1}{k_1}}\right\}^2}$$

For the second half (4, 5 and 6) an identical result is obtained

$$Q^2 = \Delta h_b \left\{ \sqrt{\frac{1}{k_5 + k_6}} + \sqrt{\frac{1}{k_4}} \right\}^2 = \frac{\Delta h_b}{k_b} \quad \text{and } k_b = \frac{1}{\left\{ \sqrt{\frac{1}{k_5 + k_6}} + \sqrt{\frac{1}{k_4}} \right\}^2}$$

The pressure head drop over the whole network is hence

$$\Delta h = \Delta h_a + \Delta h_b = kQ^2 \text{ where } k = k_a + k_b$$

(b) The top half of the network is the same as part (a) so we may evaluate k for this part.

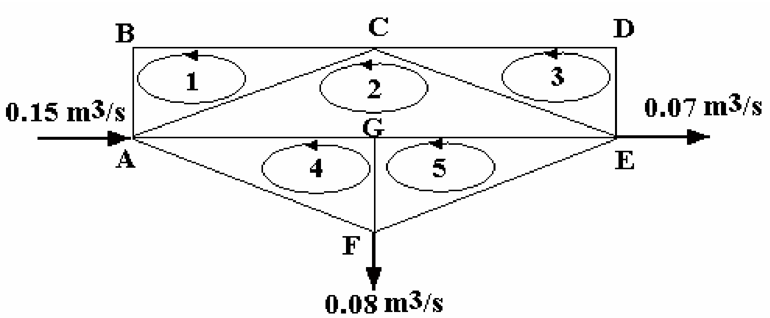
$$k = \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{k_5 + k_6}} + \sqrt{\frac{1}{k_4}} \right\}^2}} + \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}} \right\}^2}}$$

$$k = \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{100 + 160}} + \sqrt{\frac{1}{140}} \right\}^2}} + \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{100 + 150}} + \sqrt{\frac{1}{120}} \right\}^2}}$$

$$k = 88.45$$

If the flow law is applied between A and E for the top loop and assuming the friction head is equal to the change in altitude we get:

$Q_a = \sqrt{(\Delta h/k)}$ Δh = difference between A and E $\Delta h = 2$ m
$Q_a = \sqrt{(2/88.45)} = 0.150\ m^3/s$ which is the same as the total flow at A



Loop 4

Iteration 1 Note h_f is minus if Q is minus

Pipe	Flow(guess)	k	$h_f = kQ^2$	$h_f/Q = kQ$
AF	0.1	180	1.8	18
FG	-0.05	150	-0.375	7.5
GA	0.1	150	1.5	15
Totals			2.925	40.5

$\Delta Q = -(2.925)/(2 \times 40.5) = -0.036$ so Q (FG)=-0.05 - 0.036 =-0.086

LOOP 5

Iteration 1 Note h_f is minus if Q is minus

Pipe	Flow(guess)	k	$h_f = kQ^2$	$h_f/Q = kQ$
GF	-0.086	150	-1.112	12.93
FE	0.1-0.08	150		
EG		180		
Totals				

$\Delta Q=-(-0.075)/(2 \times 40.5) = 0.000926$

Guess Q AF = 0.1 Q FG = 0.05 Q GA = 0.05

h_f (AF) = 180 $(0.1)^2$ =

APPLIED MECHANICS OF FLUID D203 QUESTION 9 2004

(a) (i) Explain the importance of the Net Positive Suction Head (NPSH) for a pump installation.

(ii) Use the energy equation to derive an expression for NPSH for the case where the pump inlet is situated at a small elevation above the inlet water supply surface.

(b) A centrifugal pump produced the performance data shown in the table when running at 1500 rev/min with an atmospheric pressure of 100 kN/m^2 and water vapour pressure of 3.36 kN/m^2. The pump is required to deliver water from a sump to a reservoir whose level is 58 m above that of the sump. The suction pipe is 250 mm diameter and its effective length, after allowing for fittings is 12 m. The pump inlet is 3 m above the water level in the sump. The delivery pipe is also 250 mm diameter with an effective length of 110 m. The Darcy friction factor f for both pipes may be assumed to be 0.025.

(i) Generate a system demand curve for this application.

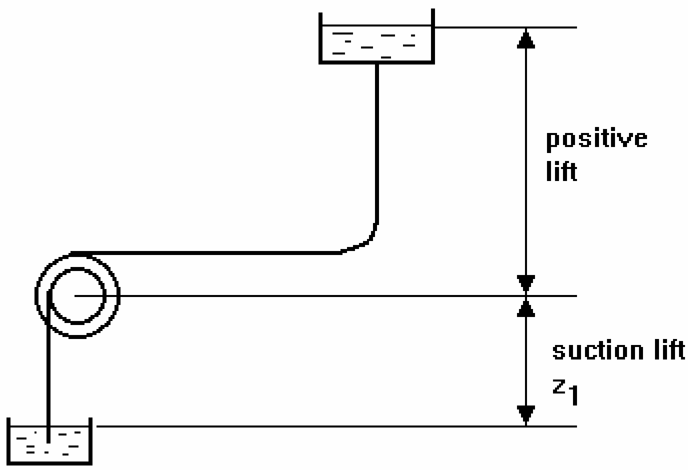
(ii) Calculate the discharge, efficiency and NPSH for the pump when running at 1500 rev/min.

(iii) Calculate the most economical speed of operation for the pump in this application and determine the discharge, efficiency and NPSH when operating at this speed. *Note.* The Darcy friction factor f is used in the following formula for friction head loss in a pipe $h_f = f\dfrac{Lv^2}{2gD}$ where the symbols have their usual meanings.

Q m^3/s	0.05	0.10	0.15	0.20	0.25	0.30
H m	70.6	69.6	67.8	64.1	57.8	49.0
P kW	80	106	128	146	163	176

(a) (i) The NPSH is important to determine the possibility of cavitation in the pump. When a liquid cavitates, it turns into a vapour and then suddenly changes back into a liquid with a load cracking sound. The bubbles of vapour cause damage to the metalwork by eroding it away. The main reason for cavitation is due to the local pressure falling below the vapour pressure of the liquid. The vapour pressure is raised with temperature and is more likely to occur in hot liquids. In pumps and turbines, the drop in pressure is often due to the wake set up behind the impeller. The system design is also important to prevent a vacuum forming due to restrictions on the suction side of the pump or negative heads on the outlet side of the turbine. An important parameter used for determining the likelihood of cavitation in pumps is the Net Positive Suction Head.

(a) (ii) Consider a pump delivering liquid from a tank on the suction side into a tank on the outlet side through a pipe.



Dynamic head = h_d = positive lift + head loss

Suction head = h_{suc} = suction lift + head loss

The head loss could include loss at entry, loss in fittings and bends as well as pipe friction.

$h_{suc} = z_1 + h_{f1} + v_1^2/2g$

The Net Positive Suction Head is the amount by which the absolute pressure on the suction side is larger than the vapour pressure (saturation pressure) of the liquid.

NPSH = absolute inlet head - vapour pressure head

Absolute inlet head = $p_a/\rho g - h_s$ where p_a= atmospheric pressure and $h_s = p_s/\rho g$

The vapour pressure varies with temperature and for water is found in thermodynamic temperatures under the heading p_s. (for saturation pressure).

Vapour pressure as a head is $p_s/\rho g$

$$NPSH = (p_a/\rho g - h_{suc}) - p_s/\rho g = (p_a - p_s)/\rho g - h_{suc}$$

(b) N =1500 rev/min p_a = 100 kPa p_s = 3.36 kPa z_1 = 3 m positive lift = 55 m f=0.025

$A = \pi D^2/4 = \pi\ (0.25)^2/4 = 0.049087\ m^2$

$v = Q/A = Q/0.049087 = 20.372Q$

Suction pipe L = 12 m D = 0.25 m f=0.025

$$h_f = f\frac{Lv^2}{2gD} = 0.025\frac{12v^2}{2g \times 0.25} = \frac{1.2v^2}{2g}0.0612v^2 = 25.38Q^2$$

Delivery pipe L = 110 m D = 0.25 m f=0.025

$$h_f = f\frac{Lv^2}{2gD} = 0.025\frac{110v^2}{2g \times 0.25} = \frac{11v^2}{2g} = 0.5607v^2 = 232.7\ Q^2$$

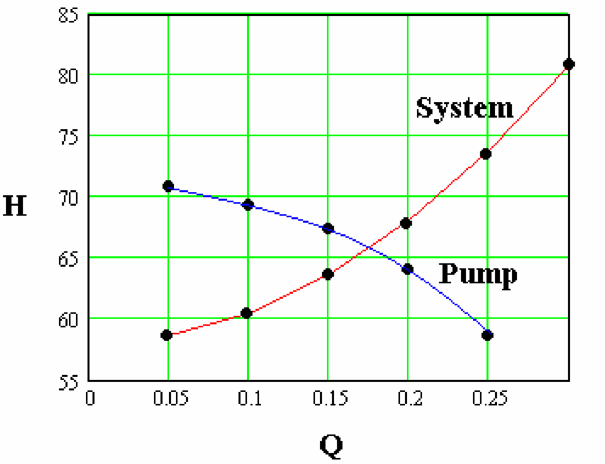
Total lift = 58 + losses and assuming only pipe friction losses

Total lift = 58 + 232.7 Q^2 + 20.372Q = 58 + 253 Q^2

Evaluate for same flow rates as in table.

$Q\ m^3/s$	0.05	0.10	0.15	0.20	0.25	0.30
H m	58.63	60.53	63.7	68.1	73.8	80.8

Plot both heads to get the demand curve.



(ii) The matching point is

$Q = 0.175\ m^3/s$ and H = 66 m

The power will be 136 kW

Water Power = ρQgH

WP = 997 x .175 x 9.81 x 66 = 113 kW

Efficiency = 113/136 = 83%

$h_{suc} = z_1 + h_{f1} = 3 + h_f = 3 + 25.38Q^2 = 3 + (25.38 \times 0.175^2) = 3.777$ m

$NPSH = (p_a - p_s)/\rho g - h_{suc} = (100 \times 10^3 - 3.36 \times 10^3)/(997 \times 9.81) - 3.777$

NPSH = 6.1 m

(iii) Calculate the efficiency for the table. WP = ρQgH

Qm^3/s	0.05	0.10	0.15	0.20	0.25	0.30
H m	70.6	69.6	67.8	64.1	57.8	49.0
P kW	80	106	128	146	163	176
WP (kW)	34.5	68.1	99.5	125.4	141.3	143.8
η %	43	64.2	77.7	85.9	86.7	81.7

Optimal efficiency occurs at Q = 0.25, H = 57.8, N = 1500

The specific speed $Ns = NQ^{1/2}/H^{3/4} = 1500 \times 0.25^{1/2}/57.8^{3/4} = 35.8$

We need the same specific speed at the matching point Q = 0.175 m^3/s and H = 66 m

$35.8 = N_2 \times 0.175^{1/2}/66^{3/4} = 0.018\ N_2$

N_2 = 1980 rev/min