QUESTION BANK

Class : 10+1 & 10+2

(Mathematics)

Question Bank for +1 and +2 students for the subject of Mathematics is hereby given for the practice. While preparing the questionnaire, emphasis is given on the concepts, students, from the examination point of view.

We hope that you might appreciate this question bank. We welcome suggestions to improve the question bank.

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SUBJECT : MATHEMATICS

CLASS 10+1

CHAPTER – SETS

(3 Marks Questions)

1. Let $A = \{\{1,2,3\}, \{4,5\}, \{6,7,8\}\}$

Determine which of the following is true or false :

(*a*) $1 \in A$ (*b*) $\{1, 2, 3\} \subset A$ (*c*) $\{6, 7, 8\} \in A$ (*d*) $\{\{4, 5\}\} \subset A$

(*e*) $\phi \in A$ (f) $\phi \subset A$

2. If $A = \{2, 3\}$, $B = \{x : x$ is a root of $x^2 + 5x + 6 = 0\}$, then find

(i) $A \cup B$

(ii) $A \cap B$

(iii) Are they equal sets?

(iv) Are they equivalent sets?

3. Let U = {1, 2, 3, 4, 5, 6, 7, 8,9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8}, Find :

(*a*) A^C (*b*) B^C (*c*) $(A^C)^C$ (*d*) $(A \cup B)^C$

4. If A = {1, 2,}, B = {4, 5, 6} and C = {7, 8, 9}, verify that :

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. How many like :

(*a*) both tea and coffee (*b*) only tea and not coffee (*d*) only coffee and not tea

6.(i) Write the following sets in set builder form :

A = {1, 3, 5, 7, 9}, E = {1, 5, 10, 15……}

(ii) Write the following sets in Roster Form :

A = {x : x is an integer and $-3 < x < 7$}, B = {x : x is a natural number less than 6}

7. Let A={1,2}, B= {1,2,3,4}, C={5,6} and D={5,6,7,8}. Verify that

(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(b) $(A \times C) \subset (B \times D)$

8. If $U = \{1,2,3,4,.....10\}$ is the universal set for the sets $A = \{2,3,4,5\}$ and $B = \{1,2,3,4,5,6\}$, then verify that $(A \cup B)' = A' \cap B'$

9. Let A and B be two finite sets such that $n(A - B) = 30$, $n(A \cup B) = 180$ $n(A \cap B) = 60$, find n(B)

CHAPTER-RELATION AND FUNCTION

(1 mark question)

Q.1 Let $R = \{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A = \{1,2,3,4\}$. The relation R is

(A) a function (B) transitive

(C) not symmetric (D) reflexive.

Q.2 Let $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A = \{3,6,9,12\}$. The relation is

(A) reflexive only (B) reflexive and transitive only

(C) reflexive and symmetric only (D) an equivalence relation

Q.3 Let R be the real number. Consider the following subsets of RxR

S = $\{(x, y): y = x+1 \text{ and } 0 < x < 2\}$

T = $\{(x, y): x - y \text{ is an integer}\}$

which one of the following is true?

(A) T is an equivalence relation on R but S is not.

(B) Neither S nor T is an equivalence relation on R.

(C) Both S and T are equivalence relations on R

(D) S is an equivalence relation on R but T is not.

Q.4 Let f(x) = [x] then f $\left(\frac{-3}{2}\right)$ is equal to :

(*a*) –3 (*b*) –2 (*c*) –1.5 (*d*) None of these

Q.5 Range of f(x) = $x^2 + 2$, where x is a real number, is :

(*a*) [2, ∞) (*b*) (2, ∞] (*c*) (2, ∞) (*d*) [2, ∞]

Q.6 The domain of $f(x) = \sqrt{1+\log_e(1-x)}$ is :

(*a*) $-\infty < x \le 0$ (*b*) $-\infty \le x \le \frac{e-1}{e}$ (*c*) $-\infty < x \le 1$ (*d*) $x \ge 1-e$

Q.7 For real x, let $f(x) = x^3 + 5x + 1$ then :

(a) f is onto R but not one-one (b) f is one-one and onto R

(c) f is neither one-one nor onto R (d) f is one-one but not onto R

Q.8 If $f(x) = x^2 - \frac{1}{x^2}$, then find the value of $f(x) + f\left(\frac{1}{x}\right)$

(a) 1 (b) 0 (c) $\frac{1}{2}x^2$ (d) $\frac{1}{2x^2}$

Q.9 Find the domain of the function

$$f(x) = \sqrt{x^2 - 7x + 10}$$

a) $(2,5)$ b) $[2,\infty)$ c) $(-\infty,2] \cup [5,\infty)$ d) $(-\infty,2) \cup [5,\infty)$

(3 marks question)

Q.1 If $G = \{7,8\}$ and $H = \{5,4,2\}$ find G x H and H x G.

Q.2 If $P\{1,2\}$ form the set $P \times P \times P$.

Q.3 Let $A = \{1,2,3,4\}$ and $B = \{5,7,9\}$

Determine :

(i) $A \times B$ and represent it graphically.

(ii) $B \times A$ and represent it graphically.

(iii) Is $n(A \times B) = n(B \times A)$

Q.4 Let $A = \{1,2,3\}$, $B = \{2,3,4\}$ and $C = \{4,5\}$ verify that

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q.5 If $A = \{4,9,16,25\}$, $B = \{1,2,3,4\}$ and R is the relation "is square of" from A to B. Write down the set corresponding to R. Also find the domain and range of R.

Q.6 If R is a relation "is divisor of" from the set $A = \{1,2,3\}$ to $B = \{4,10,15\}$, write down the set of ordered pairs corresponding to R.

Q.7 Let $A = \{1,2\}$ and $B = \{3,4\}$. Find the number of relations from A to B.

Q.8 Let $N \to N$ be defines by $f(x) = 3x$. Show that f is not an onto function.

Q.9 If 'f is a real function defined by $f(x)=\frac{x-1}{x+1}$ then prove that $f(2x)=\frac{3f(x)+1}{f(x)+3}$

Q.10 If $f(x)=\frac{1}{2x+1}, x\neq\frac{-1}{2}$, then show that $f(f(x))=\frac{2x+1}{2x+3}, x\neq\frac{-3}{2}$

Q.11 The function 't' which maps temperature in Celsius into temperature in Fahrenheit is defined by $t(c)=\frac{9c}{5}+32$ find (i) t(0) (ii) t(28) (iii) t(-10) (iv) the value of c when t(c)=212

Q.12 Find the domain of the function

$$f(x)=\frac{x^2+3x+5}{x^2-5x+4}$$

Q.13 The function f is defined by:

$$f(x)=\begin{vmatrix} 1-x, & x<0 \\ 1, & x=0 \\ x+1, & x>0 \end{vmatrix}$$

Draw the graph of $f(x)$

Q.14 Let A={9,10,11,12,13} and $f: A \to N$ be defined by

$f(x)$ = The highest prime factor of n. Find the range of f.

Q.15 Draw the graph of the function

$f(x)=|1-x|+|1+x|, -2\leq x\leq 2$

CHAPTER – TRIGONOMETRY AND FUNCTIONS

(1 Mark Questions)

1. Radian measure of $40^0 20'$ is :

(*a*) $\frac{121}{540}$ radians (*b*) $\frac{540}{121}$ radians (*c*) $\frac{121}{540}\pi$ radians (*d*) None of these

2. Radian measure of 25^0 is :

(*a*) 25π (*b*) $\frac{26}{9}$ (*c*) $\frac{26}{9\pi}$ (*d*) None of these

3. Value of sin 765^0 is :

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 765^0

4. The principal solution of tan x = $\sqrt{3}$ is :

(a) $\frac{\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

5. The most general solution of tan θ = –1, cos θ = $\frac{1}{\sqrt{2}}$ is :

(a) $\theta = n\pi + \frac{7\pi}{4}$ (b) $\theta = n\pi + (-1)^n \frac{7\pi}{4}$

(c) $\theta = 2n\pi + \frac{7\pi}{4}$ (d) None of these

6. The value of $\cos 52^0 + \cos 68^0 + \cos 172^0$ is :

(a) 1 (b) 0 (c) –1 (d) 3

7. The equation $\sqrt{3}\sin x + \cos x = 4$ has :

(a) only one solution (b) two solutions (c) infinite many solutions (d) no solution

8. The value of $\cos 15^0 \cos 7\frac{1}{2}^0 \sin 7\frac{1}{2}^0$ is :

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

9. $Sin 47^0 + \sin 61^0 - \sin 11^0 - \sin 25^0 =$

(a) $\sin 7^0$ (b) $\cos 7^0$ (c) $\sin 36^0$ (d) $\cos 36^0$

(3 Marks Questions)

1. If in two circles, arcs of the same length subted angles of 60^0 and 75^0 at the centre, find the ratio of their radii.
2. Find the angle between the minute hand and the hour hand of a clock when the time is 5:20.
3. Prove that : (a) $\sec^4 A - \sec^2 A = \tan^4 A + \tan^2 A$ (b) $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$
4. Prove that : $\frac{\text{cosec}\theta}{\text{cosec}\theta - 1} + \frac{\text{cosec}\theta}{\text{cosec}\theta + 1} = 2\sec^2\theta$

5. If $\cot\theta = \frac{-12}{5}$ and 'θ' lies in the second quadrant, find the values of other five functions.

6. Prove that : $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = \frac{-1}{2}$

7. Prove that : $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$

8. Prove that : $\tan 70^0 = \tan 20^0 + 2\tan 50^0$

9. Find the principal solutions of the following :

(a) $\tan x = \sqrt{3}$ (b) $\sec x = 2$ (c) $\operatorname{cosec} x = -6$

10. In any triangle ABC, prove that : $\sin\frac{B-C}{2} = \frac{b-c}{a}\cos\frac{A}{2}$

11. In any triangle ABC, prove that : $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{C} = \frac{a^2+b^2+c^2}{2abc}$

12. Prove that : $\tan 9^0 - \tan 27^0 - \tan 63^0 + \tan 81^0 = 4$

13. Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

CHAPTER – PRINCIPLE OF MATHEMATICAL INDUCTION

(3 Marks Questions)

1. Let P(n) be the statement "$3^{(n)} > n$"

   (a) Is P(1) true? (b) What is P(n + 1)?

   (c) If P(n) is true, prove that P(n + 1) is true.

2. By Principle of Mathematical Induction, prove that :

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

3. Prove that $10^{2n-1}+1$ is divisible by 11 for all $n \in N$

4. For every positive integer 'n', prove that $7^n - 3^n$ is divisible by 4.

5. By principle of mathematical Induction, prove that :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1} \text{ for all } n \geq 1.$$

6. Prove the rule of exponents : $(ab)^n = a^n b^n$

by using principle of mathematical Induction for every natural number.

7. By Principle of Mathematical induction. Prove that $1+3+3^2+...3^{n-1} = \left(\frac{3^n-1}{2}\right)$ for all $n \in N$

8. By using the Principle of mathematical induction 3^{2n} -1 is divisible by 8 for all $n \in N$

CHAPTER – COMPLEX NUMBERS

(1 mark Question)

1. i^{-35} is :

(*a*) i (*b*) 1 (*c*) 0 (*d*) –i

2. Solution of $x^2+2=0$ is :

(*a*) –2 (*b*) 2 (*c*) $\pm\sqrt{2}$ (*d*) $\pm\sqrt{2}\,i$

3. Complex conjugate of 3i – 4 is :

(*a*) 3i + 4 (*b*) –3i – 4 (*c*) –3i + 4 (*d*) None of these

4. Additive inverse of complex number 4 – 7i is:

(*a*) 4 + 7i (*b*) –4 + 7i (*c*) –4 – 7i (*d*) None of these

5. The imaginary part of $\frac{-1}{5}+\frac{i}{5}$ is :

(*a*) zero (*b*) $\frac{-1}{5}$ (*c*) $\frac{1}{5}$ (*d*) None of these

6. The value of $i^{13}+i^{14}+i^{15}+i^{16}$ is :

(*a*) i (*b*) –i (*c*) zero (*d*) –1

7. $i^{57}+\frac{1}{i^{125}}$ equals :

(*a*) 0 (*b*) 2i (*c*) –2i (*d*) 2

8. The complex number z = x + iy, which satisfies the equation $\left|\frac{z+5i}{z-5i}\right|=1$, lies on :

(*a*) The line y = 5 (*b*) a circle through the origin (*c*) the x – axis (*d*) None of these

9. The modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$ is :

(*a*) $\sqrt{5}$ units (*b*) $\frac{\sqrt{11}}{5}$ units (*c*) $\frac{\sqrt{5}}{5}$ units (*d*) $\frac{\sqrt{12}}{5}$ units

10. The conjugate of a complex number is $\frac{1}{1-i}$. Then that complex number is :

(a) $\frac{1}{i-1}$ (b) $\frac{-1}{i-1}$ (c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$

(5 Mark Questions)

1. Show that a real value of x will satisfy the equation $\frac{1-ix}{1+ix} = a - ib$ if $a^2 + b^2 = 1$, where a, b are real.

2. If $\frac{a+ib}{c+id} = x + iy$, show that $x^2 + y^2 = \frac{a^2+b^2}{c^2+d^2}$

3. If $(x+iy)^3 = u + iv$, then show that : $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

4. Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$

5. If z_1, z_2 are complex numbers, such that $\frac{2z_1}{3z_2}$ is purely imaginary number, find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$

6. Convert into polar form : $\frac{1+7i}{(2-i)^2}$

7. Solve : $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

8. If $|z| = 1$, prove that $\left|\frac{z-1}{z+1}\right|$ $(z \neq -1)$ is purely imaginary number. What will you conclude if $z = 1$.

9. Convert into polar form : $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

10. If $(x+iy) = \frac{a+ib}{c+id}$, then prove : $(x-iy) = \frac{a-ib}{c-id}$, and $x^2 + y^2 = \frac{a^2+b^2}{c^2+d^2}$

CHAPTER – LINEAR INEQUATIONS

(5 Marks Questions)

1. Solve the following inequations and show the graph on number line :

(a) $3x - 6 < 0$ (b) $-3x + 9 \leq 0$ (c) $7x + 5 > 33$ (d) $5x - 15 \geq 0$

2. Solve the following inequations and show that graph on number line :

(*a*) $\frac{x-3}{x-5} > 0$ (*b*) $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

3. Solve the following system of inequations :

$$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \text{ and } \frac{2x-1}{12} - \frac{x-11}{3} < \frac{3x+1}{4}$$

4. Solve the following system of inequations :

$$2(2x+3) - 10 < 6(x-2) \text{ and } \frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$$

5. Solve graphically :

(i) $|x| < 2$ (ii) $|y| \geq 3$

6. Find the region enclosed by the following inequations

$$x + y - 2 \leq 0,\ 2x + y - 3 \leq 0,\ x \geq 0,\ y \geq 0$$

7. Find the region for following inequation :

$$x + y \geq 0,\ 2x + y \leq 4,\ x \geq 0 \text{ and } y \geq 0$$

8. Solve the following system of inequalities graphically:

$4x + 3y \leq 60,\ y \geq 2x,\ x \geq 3,\ x, y \geq 0$

CHAPTER – PERMUTATION AND COMBINATION

Multiple Choice Questions : **(1 Mark Questions)**

1. 7! ÷ 5! is :

(*a*) 7! (*b*) 2! (*c*) 42 (*d*) 24

2. The value of is $\frac{12!}{10!2!}$:

(*a*) 42 (*b*) 66 (*c*) 76 (*d*) 45

3. The value of ${}^{15}C_{11} \div {}^{15}C_{10}$ is :

(*a*) $\frac{15}{11}$ (*b*) $\frac{15}{10}$ (*c*) $\frac{5}{11}$ (*d*) $\frac{5}{10}$

4. If ${}^{4}P_{n} = 5\ {}^{4}P_{3}$, then n is :

(*a*) 8 (*b*) 6 (*c*) 7 (*d*) 5

5. If n = 7 and r = 5, then the value of ${}^{n}C_{r}$ is :

(*a*) 21 (*b*) 42 (*c*) 35 (*d*) 75

6. If n = 8 and r = 3 then the value of ${}^{n}P_{r}$ is :

(*a*) 140 (*b*) 336 (*c*) 40 (*d*) 85

7. Evaluate : ${}^{10}C_{1} + {}^{10}C_{2} + {}^{10}C_{3} + \text{.........}{}^{10}C_{10}$

(*a*) 1000 (*b*) 1023 (*c*) 1050 (*d*) 1010

8. The number of ways in which 6 men and 5 women can sit at a round table if no two women are to sit together is given by :

(*a*) 30 (*b*) $5! \times 4!$ (*c*) $7! \times 5!$ (*d*) $6! \times 5!$

9. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_{n} = 3:5,$ then the value of n equal :

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1 (*e*) 5

10. If $n_{c_2} - n_{c_1} = 35$ then the value of *n* equal :

(*a*) -10 (*b*) 10 (*c*) 7 (*d*) -7

(3 Marks Questions)

1. Find n such that $\dfrac{{}^{n}P_{4}}{{}^{n-1}P_{4}} = \dfrac{5}{3},\ n > 4.$

2. In how many ways can 9 examination papers be arranged so that the best and the worst papers never come together?

3. The letters of the word 'RANDOM' are written in all possible orders and these words are written out as in dictionary. Find the rank of the word 'RANDOM'.

4. How many natural numbers less than 1000 can be formed with the digits 1, 2, 3, 4 and 5 if (a) no digit is repeated (b) repetition of digits is allowed.

5. Find out how many arrangements can be made with the letters of the word 'MATHEMATICS'. In how many ways can consonants occur together?

6. In how many ways can 5 persons – A, B, C, D and E sit around a circular table if :

(*a*) B and D sit next to each other. (*b*) A and D do not sit next to each other.

7. How many triangles can be obtained by joining 12 points, five of which are collinear?

8. If m parallel lines in a plane are intersected by a family of n parallel lines, find to number of parallelograms formed.

9. What is the number of ways of choosing, 4 cards from a pack of 52 playing cards? In how many of these :

(*a*) four cards are of the same suits (*b*) are face cards.

CHAPTER – BIONOMIAL THEOREM

(3 Marks Questions)

1. Find 'a' if the 17th and 18th terms in the expansion of $(2+a)^{50}$ are equal.

2. Using Binomial theorem, evaluate $(99)^5$.

3. (i) Find the general term in the expansion of $\left(x^2 - yx\right)^{12}, x \neq 0$.

(ii) Find the 10th term from end in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{12}, x \neq 0$.

4. (i) Write the middle term in the expansion of : $\left(x - \frac{1}{2y}\right)^{10}$

(ii) Determine the two middle terms in the expansion of : $\left(x^2 + a^2\right)^5$

5. Find the term containing x^3, if any, in $\left(3x - \frac{1}{2x}\right)^8$.

6. Find the co-efficient of $x^6 y^3$ in $(x+2y)^9$.

7. Find the term which is independent of x, in the expansion of : $\left(x^2 + \frac{1}{x}\right)^9$.

8. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$, prove that : $C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n = n.2^{n-1}$.

CHAPTER – SEQUENCE AND SERIES

(1 Mark Questions)

1. 5th term of a G.P. is 2, then the product of first 9 terms is :

(a) 256 (b) 128 (c) 512 (d) None of these

2. If a, b, c are in A.P., then : (a + 2b – c) (2b + c – a) (c + a – b) equals :

(a) $\frac{abc}{2}$ (b) abc (c) $2abc$ (d) $4abc$

3. Sum of the series is $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2$:

(a) $\frac{n}{2}\left(4n^2 - 1\right)$ (b) $\frac{n(n+1)(2n+1)}{2}$ (c) $\frac{n(n+1)(2n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

4. If a, b, c are in G.P. and x, y are with arithmetic mean of a, b and b, c respectively, then $\frac{1}{x}+\frac{1}{y}$ is equal to :

$(a)\ \frac{2}{b}$ $(b)\ \frac{3}{b}$ $(c)\ \frac{b}{3}$ $(d)\ \frac{b}{2}$ $(e)\ \frac{1}{b}$

5. If the third term of a G.P. is 3, then the product of its first 5 terms is :

(*a*) 15 (*b*) 81 (*c*) 243 (*d*) Cannot be determined.

6. 5^{th} term of a G.P. is 2, then the product of its 9 terms is :

(*a*) 256 (*b*) 512 (*c*) 1024 (*d*) None of these

7. If the p^{th}, q^{th} and r^{th} terms of G.P. are a, b and c respectively. Then

$a^{q-r}\ b^{r-p}\ c^{p-q}$ is equal to

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) -1

8. Find the number of terms between 200 and 400 which are divisible by 7.

(*a*) 28 (*b*) 23 (*c*) 29 (*d*) 27

9. Which term in the A.P. 5,2,-1,...... is -22 ?

(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 9

(3 Marks Questions)

1. Determine 2^{nd} term and r^{th} term of an A.P. whose 6^{th} term is 12 and 8^{th} term is 22.

2. Sum of the first p, q and r terms of an A.P. are a, b and c respectively. Prove that

$$\frac{q}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

3. If the 12^{th} term of an A.P. is –13 and the sum of the first four terms is 24, what is the sum of the first 10 terms?

4. Insert 3 A.M's between 3 and 19.

5. The sum of three numbers in A.P. is –3 and their product is 8. Find the numbers.

6. The digits of a positive integer having three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

7. If a^2, b^2, c^2 are in A.P., prove that : $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are also in A.P.

8. Find a G.P. for which sum of the first two terms is –4 and fifth term is 4 times the third term.

9. The value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

10. Determine the number 'n' in a geometric progression $\{a_n\}$, if $a_1=3, a_n=96$ and $s_n=189$.

11. Sum to n terms : 4 + 44 + 444 + ...

12. Find the sum of 50 terms of a sequence : 7, 7.7, 7.77, 7.777, ...

13. The arithmetic mean between two numbers is 10 and their geometric mean is 8. Find the numbers.

14. The first term of a G.P. is 2 and the sum to infinity is 6. Find the common ratio.

15. Evaluate : $.23\overline{45}$

16. Find the sum of n terms of the series : $1^2+3^2+5^2+$..........

17. Sum to n terms the series : $1^2-2^2+3^2-4^2+5^2-6^2+$..........

CHAPTER STRAIGHT LINES

(1 mark question)

Q.1 Find the distance of the point (4,1) from line $3x$-$4y$-9=0

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{-1}{5}$ (D) $\frac{-3}{5}$

Q.2 The equation of straight line passing through the point (2,3) and perpendicular to the line $4x-4y=10$ is

(A) $-3x+4y=15$ (B) $4x+3y=5$

(C) $3x+4y=18$ (D) $3x+10y=4$

Q.3. Find the value of x for which the points (x,-1) (2,1) and (4,5) are collinear.

(A) 2 (B) -1 (C) 1 (D) 0

Q.4. Find the distance between the parallel lines $3x$-$4y$+7=0 and $3x$-$4y$+5=0

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{-2}{5}$ (D) $\frac{-3}{5}$

Q.5. Find the angle between the lines $x+y+7=0$ and $x-y+1=0$

(A) 0^0 (B) 45^0 (C) 90^0 (D) 270^0

Q.6. Find the values of k for the line

$(k-3)x+(4-k^2)y+k^2-7k+6=0$ which is parallel to the x-axis

(A) ± 2 (B) 2 (C) -2 (D) 3

Q.7. Find the Equation of the line through the intersection of $3x-4y+1=0$ and $5x+y-1=0$ which cuts off equal intercept on the axes

(A) $23x+23y=11$ (B) $23x-y=11$

(C) $x+23y=11$ (D) $x+y=11$

Q.8. The co-ordinates of the foot of the perpendicular from (2,3) to the line $3x+4y-6=0$ are

(A) $\left(\frac{-14}{25},\frac{-27}{25}\right)$ (B) $\left(\frac{14}{25},\frac{-27}{25}\right)$

(C) $\left(\frac{-14}{25},\frac{27}{25}\right)$ (D) $\left(\frac{14}{25},\frac{27}{25}\right)$

Q.9. The vertices of ΔPQR are $P(2,1)$, $Q(-2,3)$ and $R(4,5)$. Find the equation of the median through the vertex R.

(A) $3x-4y+8=0$ (B) $3x+2y+9=0$

(C) $-3x+4y+1=0$ (D) $3x+4y+5=0$

Q.10. Find the equation of the line perpendicular to the line $2x-3y+7=0$ and having x-intercept is 4.

(A) $3x+2y-12=0$ (B) $3x-2y+6=0$

(C) $-3x+2y+4=0$ (D) $2x-3y-12=0$

Q.11. A line has slope m and y intercept 4, the distance between the origin and the line is equal to

(A) $\frac{4}{\sqrt{1-m^2}}$ (B) $\frac{4}{\sqrt{m^2-1}}$ (C) $\frac{4}{\sqrt{m^2+1}}$ (D) $\frac{4m}{\sqrt{1+m^2}}$

(E) $\frac{4m}{\sqrt{m^2-1}}$

(3 Marks Questions)

1. Find a point on x axis, which is equidistant from (7, 6) and (3, 4).

2. Show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-triangle.

3. Find the coordinates of the points, which divide internally and externally the line joining (1, –3) and (–3, 9) in the ratio 1 : 3.

4. Find the centroid by the triangle with vertices at (–1, 0), (5, –2) and (8, 2).

5. Find the coordinates of incentre of the triangle whose vertices are (–36, 7); (20, 7) and (0, –8).

6. A point moves so that the sum of its distances from the points (ae, o) and (–ae, o) is 2a. Prove that its locus is : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $b^2=a^2(1-e^2)$

7. State whether the two lines are parallel, perpendicular or neither parallel nor perpendicular:

(*a*) Through (5, 6) and (2, 3); through (9, –2) and (6, –5).

(*b*) Through (2, –5) and (–2, 5); through (6, 3) and (1, 1).

8. Find equation of the line bisecting the segment joining the points (5, 3), (4, 4) and making an angle 45^0 with the x-axis.

9. The perpendicular from the origin to a line meets it at the point (–2, 9), find the equation of the line.

10. Write the equation of the line for which $\tan\theta=\frac{1}{2}$, where θ is the inclination of the line and (i) y-intercept is $\frac{-3}{2}$. (ii) x– intercept is 4.

11. Find the perpendicular form of the equation of the lines from the given values of p and α : (i) p = 3 and $\alpha=45^0$, (ii) p = 5, $\alpha=135^0$

12. Find the slope and y– intercept of the straight line $5x + 6y = 7$.

13. Two lines passing through the point (2, 3) make an angle of 45^0. If the slope of one of the lines is 2, find the slope of other.

14. Determine the angle B of the triangle with vertices A(–2, 1), B(2, 3) and C(–2, –4).

15. Find the equation of the straight line through the origin making angle of 60^0 with the straight line $x+\sqrt{3}y+3\sqrt{3}=0$.

16. Find the equation of a line passing through the point (0, 1) and parallel to $3x-2y+5=0$.

17. If $3x-by+2=0$ and $9x+3y+a=0$ represent the same straight line, find the values of ‘a’ and ‘b’.

18. Find the co-ordinates of the orthocentre of the triangle whose angular points are (1,2) (2, 3) and (4, 3).

19. Prove that these lines : $2x-37=7, 3x-4y=13$ and $8x-11y=33$ meet in a point.

20. Find the equation of the line passing through the point of intersection of $x+2y=5$ and $x-3y=7$, and passing through the point : (a) (0, –1); (b) (2, –3)

CHAPTER - CONIC SECTION

(1 mark question)

Q.1 If the eq. of the circle is $x^2+y^2+8x-10y+8=0$ then its centre is

(A) (8,-10), (B) (-8,10), (C) (-4,5) (D) (4,-5)

Q.2. Find the equation of the circle whose centre is (–3, 2) and radius 4.

(A) $x^2+y^2+6x-4y+4=0$ (B) $x^2+y^2+6x-4y-3=0$

(C) $x^2+y^2-6x+4y+4=0$ (D) $x^2+y^2-6x+4y-3=0$

Q.3 The directrix of the Parabola $y^2=4ax$ is

(A) $x=-a$ (B) $x-a=0$

(C) $x=0$ (D) None of the these

Q.4 The foci of the ellipse $9x^2+4y^2=36$ are

(A) $(-5,0)$ (B) $(0,\pm\sqrt{5})$

(C) $(\pm 5,0)$ (D) $(0,-5)$

Q.5 The eccentricity of the parabola $y^2=-8x$ is

(A) -2 (B) 2 (C) -1 (D) 1

Q.6 The eccentricity of the ellipse $x^2+y^2+8y-2x+1=0$ are

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{5}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.7 If in a Hyper-bola, the distance between the foci is 10 and the transverse axis has length 8, than the length of its latus rectum is

(A) 9 (B) $\frac{9}{2}$ (C) $\frac{32}{3}$ (D) $\frac{64}{3}$

(3 Marks Questions)

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 - y^2 - 2x - 4y - 20 = 0$ externally at the point (5, 5).

2. Find the parametric representation of the circle : $x^2 + y^2 - 2x + 4y - 4 = 0$.

3. Show that the point : $x = \frac{2rt}{1+t^2}, y = \frac{r(1-t^2)}{1+t^2}$ (r constant) lies on a circle for all values of t such that $-1 \leq t \leq 1$.

4. Find the equation of the circle, the co-ordinates of the end-points of whose diameter are (3, 4) and (–3, –4).

5. For the parabola $2y^2 = 5x$, find the vertex, the axis and the focus.

6. Show that the equation $y^2 - 8y - x + 19 = 0$ represents a parabola. Find its vertex, focus and directrix.

7. Find the lengths of the major and minor axes, co-ordinates of the foci, vertices, the eccentricity and equations of the directrices for the ellipse $9x^2 + 16y^2 = 144$.

8. Find the equation of the ellipse with $e = \frac{3}{4}$, foci on y-axis, centre at the origin, and passing through the point (6, 4).

9. Find the lengths of the transverse and conjugate axes, co-ordinates of the foci, vertices and eccentricity for the hyperbola $9x^2 - 16y^2 = 144$.

10. Find the equation of the parabola satisfying the following conditions :

Vertices at $\left(\pm 0, \frac{\sqrt{11}}{2}\right)$, foci at $(0, \pm 3)$.

CHAPTER – INTRODUCTION TO 3-D GEOMETRY

(3 Marks Questions)

1. Show that the triangle with vertices (6, 10, 10) (1, 0, –5) and (6, –10, 0) is a right angled triangle.

2. Using section formula, prove that (–4, 6, 10) (2, 4, 6) and (14, 0, –2) are collinear.

3. Show that the points A (0, 1, 2), B(2, –1, 3) and C(1, –3, 1) are vertices of right angled isosceles triangle.

4. Show that the points (3, –1, –1), (5, –4, 0), (2, 3, –2) and (0, 6, –3) are vertices of parallelogram.

5. Find the third vertex of triangle whose centroid is (7, –2, 5) and whose other 2 vertices are (2, 6, –4) and (4, –2, 3).

6. Find the point in *XY*-plane which is equidistant from three points A(2,0,3), B(0,3,2) and C(0,0,1) through A.

7. Find lengths of the medians of the triangle with vertices A(0,0,6), B(0,4,0) and C(6,0,0)

8. Find the ratio in which the line joining the points (1,2,3) and (-3,4,-5) is divided by the *XY*-plane. Also, find the co-ordinates of the point of division.

CHAPTER – LIMITS AND DERIVATIVES

(3 Mark Questions)

1. Evaluate

(a) $\lim_{x\to 0}\dfrac{1-\cos x}{x}$ (b) $\lim_{x\to 0}\dfrac{a^x-1}{b^x-1}$

2. Evaluate using factor method :

(a) $\lim_{x\to 1}\dfrac{x^2-1}{x-1}$ (b) $\lim_{x\to 1/2}\dfrac{4x^2-1}{2x-1}$

3. Find the derivative of the function :

f(x)=2x^2+3x-5 at x= -1. Also prove that $f'(0)+3f'(-1)=0$

4. For each of the following functions, evaluate the derivative at the indicated value (s) :

(a) s = 4.9 t^2; t=1, t=5 (b) s = 4x^8; x = $\dfrac{-1}{2}$, x = $\dfrac{1}{2}$

5. Given : $V=\dfrac{4}{3}\pi r^3$, find $\dfrac{dV}{dr}$ and hence $\left(\dfrac{dV}{dr}\right)_{r=2}$

6. Find $\dfrac{dy}{dx}$, when $y=\sqrt{3x+2}+\dfrac{1}{\sqrt{2x^2+4}}$

7. Find $\dfrac{dy}{dx}$, when $y=\left(3x^2+2\right)^3\left(5x-1\right)^2$

8. Find $\frac{dy}{dx}$, when $y=\frac{x^2+3}{(2x+1)^2}$

9. Find $\frac{dy}{dx}$, when $y=\sin^2 x.\cos\left(x^3\right)$

(5 Mark Questions)

1. Evaluate :

(a) $\lim_{x\to 2}\frac{x^{10}-1024}{x^5-32}$ (b) $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

2. Evaluate :

$$\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)$$

3. Find $\lim_{x\to 1} f(x)$, where $f(x)=\begin{Bmatrix} x^2-1, & x\le 1 \\ -x^2-1, & x>1 \end{Bmatrix}$

4. Evaluate :

(a) $\lim_{x\to 0}\frac{1-\cos 3x}{x^2}$ (b) $\lim_{x\to 0}\left(\frac{1-\cos x}{\sin^2 x}\right)$

5. Evaluate :

$$\lim_{x\to 0}\frac{1-\cos x\cos 2x\cos 3x}{\sin^2 x}$$

6. Evaluate :

$$\underset{x\to 0}{\mathrm{Lt}}\ \frac{ax+x\cos x}{b\sin x}$$

7. Prove :

$$\lim_{x\to 0}\frac{\log\left(1+x^3\right)}{\sin^3 x}=1$$

8. Evaluate :

$$\lim_{x\to 0}\frac{e^x-1}{\sqrt{1-\cos x}}$$

9. Given $f(x)=\frac{1}{\sqrt{x}}, x>0,$ find $f(x)$ by delta method.

CHAPTER – MATHEMATICAL REASONING

(3 Marks Questions)

1. Write the negation of the following statements:

   a) Both the diagonals of the rectangle have same length.

   b) $\sqrt{7}$ is rational

   c) 27 is divisible by 3 or sky is blue.

2. Identify the quantifies in the following statement and write the negation of the statements.

   i) There exists a number, which is equal to its square.

   ii) For every real number x, x is less than x+1.

   iii) There exist a capital for every state in India.

3. Write the converse of the following statements:

   i) If a number x is odd, then x^2 is odd.

   ii) If two integers a and b are such that a > b, then (a-b) is always a +ve integer.

4. Let p : He is rich and q: He is happy be the given statements, write each of the following statements in the symbolic form, using p and q.

   i) If he is rich, then he is unhappy.

   ii) It is necessary to be poor is order to be happy.

   iii) To be poor is to be unhappy.

5. Determine the truth value of the following :

   i) 5+4=9 iff 8-2=6

   ii) Apple is a fruit iff Delhi is in Japan.

6. Show that the following statement is true by the method of contrapositive

   p : if x is an integer and x^2 is even, then x is also even.

7. Verify by the method of contraction :

   $p : \sqrt{7}$ is irrational

8. Given below the two statements :

   p : 25 is a multiple of 5

q : 25 is a multiple of 8.

Connecting, these two statements with 'And' and 'Or'. In both cases check the validity of the compound statement.

CHAPTER - STATISTICS

(5 Marks questions)

Q.1 If $\overline{x}$ is the mean and Mean Deviation from mean is MD($\overline{x}$), then find the number of observations lying between $\overline{x}$-MD($\overline{x}$) and $\overline{x}$+MD($\overline{x}$) from the following data : 22, 24, 30, 27, 29, 31, 25, 28, 41, 42.

Q.2 Calculate the mean deviation about median for the following data.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2

Q.3 Calculate the mean, variance and standard deviation for the following distribution :

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Q.4 The mean and variance of 8 observations are 9 and 9.25 respectively. If six observations are 6,7,10,12,12,13, find the remaining two observations.

Q.5 Calculate the mean and variance for the following data :

Income (in Rs.)	1000-1700	1700-2400	2400-3100	3100-3800	3800-4500	4500-5200
No. of families	12	18	20	25	35	10

7. Find the mean and variance for the data.

x_i	6	10	14	18	24	28	30
y_i	2	4	7	12	8	4	3

CHAPTER – PROBABILITY

(One mark questions)

1. In a single through of two dice, the probability of getting a total other than 9 or 11 is :

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{9}$ (*c*) $\frac{1}{18}$ (*d*) $\frac{5}{18}$

2. Two numbers are chosen from {1, 2, 3, 4, 5, 6} one after another without replacement. Find the probability that one of the smaller value of two is less than 4 :

(*a*) $\frac{4}{5}$ (*b*) $\frac{1}{15}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{14}{15}$

3. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting the other. The probability that all 3 apply for the same house is :

(*a*) $\frac{1}{9}$ (*b*) $\frac{2}{9}$ (*c*) $\frac{7}{9}$ (*d*) $\frac{8}{9}$

4. If 3 distinct numbers are chosen randomly from the first 100 natural numbers then the probability that all 3 of them are divisible by 2 and 3 is :

(*a*) $\frac{4}{25}$ (*b*) $\frac{4}{35}$ (*c*) $\frac{4}{33}$ (*d*) $\frac{4}{1155}$

5. What is the chance that a leap year, selected at random, will contain 53 Sundays?

(*a*) $\frac{1}{7}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{5}{7}$

6. Find the probability that in a random arrangement at the word 'Society' all the three vowels come together.

(*a*) $\frac{4}{7}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{1}{7}$ (*d*) $\frac{5}{7}$

7. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be a diamond :

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{1}{6}$

8. In a single throw of three dice, find the probability of getting a total of atmost 5.

(*a*) $\frac{7}{108}$ (*b*) $\frac{5}{108}$ (*c*) $\frac{1}{108}$ (*d*) $\frac{1}{216}$

9. From an urn containing 2 white and 6 green balls, a ball is drawn at random. The probability of not a green ball is :

(*a*) $\frac{1}{4}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{2}{3}$

(3 marks questions)

1. Three coins are tossed simultaneously. Find sample space.

2. A coin is tossed three times. Consider the following events :

A : No head appears.

B : Exactly one head appears.

C : At least two heads appear.

Do they form a set of mutually exclusive and exhaustive events?

3. Two dice are thrown and the sums of numbers which come up on the dice are noted. Consider the following events :

A : the sum is even.

B : the sum is a multiple of 3.

C : the sum is less than 4.

D : the sum is greater than 11.

4. A die is thrown, find the probability of the following events :

(*a*) A prime number will appear (*b*) A number less than 6 will appear

5. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be :

(*a*) a diamond (*b*) not an ace (*c*) a black card (*d*) not a black card

6. In a throw of 2 coins, find the probability of getting both heads or both tails.

7. A bag contains 8 red, 3 white and 9 blue balls. Three balls are drawn at random from the bag. Determine the probability that none of the balls is white.

8. Find the probability of 4 turning for at least once in two tosses of a fair die.

9. A and B are two mutually exclusive events, for which P(A) = 0.3, P(B) = p and P(AUB) = 0.5. Find ‘p’.

10. In a class of 25 students with roll numbers 1 to 25, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 5 or 7.

CLASS - 10+2

CONTENTS

S.No.	Chapter
1.	Relations and Functions
2.	Inverse Trigonometric Functions
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6.	Applications of Derivatives
7.	Integrals
8.	Applications of Integrals
9.	Differential Equation
10.	Vectors
11.	Three-Dimensional Geometry
12.	Linear Programming
13.	Probability

CHAPTER 1

RELATIONS AND FUNCTIONS

IMPORTANT POINTS TO REMEMBER

- Relation R from a set A to a set B is subset of $A \times B$.
- $A \times B = \{(a, b) : a \in A, b \in B\}$.
- If $n(A) = r$, $n(B) = s$ then $n(A \times B) = rs$.

  and no. of relations $= 2^{rs}$
- ϕ is also a relation defined on set A, called the void (empty) relation.
- $R = A \times A$ is called universal relation.
- Reflexive Relation : Relation R defined on set A is said to be reflexive iff $(a, a) \in R$ $\forall$ $a \in A$
- Symmetric Relation : Relation R defined on set A is said to be symmetric iff $(a, b) \in R \Rightarrow (b, a) \in R$ $\forall$ $a, b, \in A$
- Transitive Relation : Relation R defined on set A is said to be transitive if $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ $\forall$ $a, b, c \in R$
- Equivalence Relation : A relation defined on set A is said to be equivalence relation iff it is reflexive, symmetric and transitive.
- One-One Function : $f : A \to B$ is said to be one-one if distinct elements in A has distinct images in B. i.e. $\forall$ $x_1, x_2 \in A$ s.t. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

  OR

  $\forall$ $x_1, x_2 \in A$ s.t. $f(x_1) = f(x_2)$

  $\Rightarrow x_1 = x_2$

  One-one function is also called injective function.
- Onto function (surjective) : A function $f : A \to B$ is said to be onto iff $R_f = B$ i.e. $\forall$ $b \in B$, there exist $a \in A$ s.t. $f(a) = b$
- A function which is not one-one is called many-one function.
- A function which is not onto is called into.
- Bijective Function : A function which is both injective and surjective is called bijective.
- Composition of Two Function : If $f : A \to B$, $g : B \to C$ are two functions, then composition of f and g denoted by gof is a function from A to C give by, $(gof)(x) = g(f(x))$ $\forall$ $x \in A$

  Clearly gof is defined if Range of f $\subseteq$ domain of g. similarly, fog can be defined.

- Invertible Function : A function $f : X \to Y$ is invertible iff it is bijective.

  If $f : X \to Y$ is bijective function, then function $g : Y \to X$ is said to be inverse of f iff fog = I_y and gof = I_x

  when I_x, I_y are identity functions.

- g is inverse of f and is denoted by f^{-1}.
- Binary Operation : A binary operation '*' defined on set A is a function from $A \times A \to A$. * (a, b) is denoted by a * b.
- Binary operation * defined on set A is said to be commutative iff

  $$a * b = b * a \quad \forall \ a, b \in A.$$

- Binary operation * defined on set A is called associative iff $a * (b * c) = (a * b) * c \quad \forall \ a, b, c \in A$
- If * is Binary operation on A, then an element $e \in A$ is said to be the identity element iff $a * e = e * a \ \forall \ a \in A$
- Identity element is unique.
- If * is Binary operation on set A, then an element b is said to be inverse of $a \in A$ iff $a * b = b * a = e$
- Inverse of an element, if it exists, is unique.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1) Check the following functions for one-one and onto.

   (a) $f : R \to R, \ f(x) = \dfrac{2x-3}{7}$

   (b) $f : R \to R, f(x) = x^2 + 2$

   (c) $f : R \to R, f(x) = |x+1|$

2) Prove that the Greatest integer function $f : R \to R$,given by f(X)=$[x]$,is neither one-one nor onto, where $[x]$ denotes the Greatest integer less than or equal to x

3) **Consider $y : R^+ \to [4, \infty)$ given by $y = x^2 + 4$. Show that *f* is both one-one and onto, where R^+ is the set of all non-negative real numbers. Express *x* in terms of *y*.**

4) f: $R \to R$ is the function f(x) = $9x^3$

5) Find the number of all onto functions from the set {1,2,3,4,…….} to itself.

6) Let A=R-{3}and B=R-{1},consider the function f : A $\rightarrow$ B defined by

f(x) $= \frac{x-1}{x-3}$ show that f is one-one and onto and hence find f^{-1}

7) Show that the modulus function $f: R \rightarrow R$ defined by f(x) = | 2x | is neither one-one nor onto.

8) Check the function $f: R \rightarrow R$ given by $f(x) = x^3 - 6x^2 + 11x - 6$ is one-one or not.

9) Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two invertible function, then show that $(gof)^{-1} = f^{-1}og^{-1}$.

10) If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are onto functions, then show that g of $X \rightarrow Z$ is also onto.

11) If L is the set of all lines in the plane and R is the relation in L defined by R = {(l_1, l_2) : l_1 is parallel to l_2}. Show that the relation R is equivalence relation.

12) Show that the relation R, defined in a set A of all triangles as {(T_1, T_2) : T_1 is similar triangle to T_2}, is equivalence relation.

13) Show that the relation Q in R defined as Q = {(a, b) : b $\geq$ a}, is reflexive and transitive but not symmetric.

14) Show that the relation R in the set A = {a, b, c} given by R = {(b, c), (c, b)} is symmetric but neither reflexive nor transitive.

15) State the reason for the relation R in the set {1, 2, **3} given by R = {(1, 2), (2, 1)} not to be transitive**

16) Let * be a binary operation on Q defined by

$$a * b = \frac{3ab}{2}$$

Show that. is commutative as well as associative. Also find its identity element, if it exists.

17) Consider the binary operation * on N defined by a * b = LCM (a, b). for all a, b belongs to N. Write the multiplication table for binary operation *. Also find 5 * 7.

18) Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by a * b = min (a, b). Write the multiplication table for binary operation *. Also find (2 * 3) * (4 * 5).

19). If A = N $\times$ N and binary operation * is defined on A as (a, b) * (c, d) = (ac, bd).

(i) Check * for commutativity and associativity.

(ii) Find the identity element for * in A (If exists).

CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

IMPORTANT POINTS TO REMEMBER

- $\sin^{-1} x$, $\cos^{-1} x$, ... etc., are angles.
- If $\sin\theta = x$ and $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, then $\theta = \sin^{-1}x$ etc.
-

	Function	Domain Range (Principal Value Branch)
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\mathrm{cosec}^{-1}x$	$R - (1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- $\sin^{-1} (\sin x) = x \ \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

  $\cos^{-1} (\cos x) = x \ \forall \ x \in [0, \pi]$ etc.
- $\sin (\sin^{-1}x) = x \quad \forall \ x \in [-1, 1]$

  $\cos (\cos^{-1}x) = x \ \forall \ x \in [-1, 1]$ etc.
- $\sin^{-1} x = \mathrm{cosec}^{-1}\left(\frac{1}{x}\right) \forall x \in [-1,1]$

  $\tan^{-1}x = \cot^{-1} (1/x) \ \forall \ x > 0$

$\sec^{-1}x = \cos^{-1}(1/x), \ \forall \ |x| \geq 1$

- $\sin^{-1}(-x) = -\sin^{-1}x \quad \forall \ x \in [-1, 1]$

  $\tan^{-1}(-x) = -\tan^{-1}x \quad \forall \ x \in R$

  $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x \quad \forall \ |x| \geq 1$

- $\cos^{-1}(x) = \pi - \cos^{-1}x \quad \forall \ x \in [-1, 1]$

  $\cot^{-1}(-x) = \pi - \cot^{-1}x \quad \forall \ x \in R$

  $\sec^{-1}(-x) = \pi - \sec^{-1}x \quad \forall \ |x| \geq 1$

- $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, x \in [-1,1]$

  $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad \forall \ x \in R$

  $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2} \quad \forall \ |x| \geq 1$

  $\tan^{-1}x + \tan^{-1}y = \tan^{-1}(x+y/1-xy), \quad xy < 1$

  $\tan^{-1}x - \tan^{-1}y = \tan^{-1}(x-y/1+xy), \quad xy > -1$

- $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1) The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

   (A) $\left(\frac{-2\pi}{3}\right)$ (B) $\left(\frac{-\pi}{3}\right)$ (C) $\left(\frac{4\pi}{3}\right)$ (D) $\left(\frac{5\pi}{3}\right)$

2) If $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}C$, then C is

   (A) $\frac{65}{66}$ (B) $\frac{24}{65}$ (C) $\frac{16}{65}$ (D) $\frac{56}{65}$

3) If $A = \tan^{-1} x$, then the value of $\sin 2A$ is

(A) $\frac{2x}{1-x^2}$ (B) $\frac{x}{1-x^2}$ (C) $\frac{2x}{1+x^2}$ (D) None of these

4) The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ is

(A) $\pi - x$ (B) $2\pi - x$ (C) $\frac{x}{2}$ (D) $\pi - \frac{x}{2}$

5) If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1} x$, then x equals

(A) $\frac{a-b}{a+ab}$ (B) $\frac{b}{1+ab}$ (C) $\frac{b}{1-ab}$ (D) $\frac{a+b}{1-ab}$

6) The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(A) $\frac{6}{17}$ (B) $\frac{16}{7}$ (C) $\frac{7}{16}$ (D) None of these

7) $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

8) $\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(-\sqrt{3}\right)$ is equal to

(A) π (B) $\frac{-\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

9) $\tan^{-1}\sqrt{3}-\sec^{-1}\left(-2\right)$ is equal to

(A) π (B) $\frac{-\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

10. The value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

11. The number of real solution of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

(A) zero (B) one (C) both (D) infinite

12. The value of x which satisfies the equation

$\tan^{-1}x=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ is

(A) 3 (B) -3 (C) $\frac{1}{3}$ (D) $\frac{-1}{3}$

13. $\sin^{-1}(1-x)-2\sin^{-1}(x)=\frac{\pi}{2}$ then x is equal to

(A) $0,\frac{1}{2}$ (B) $1,\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

14. The value of $\tan\left[2\sin^{-1}\left(\frac{4}{5}\right)\right]$ is

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$ (C) $\frac{-24}{7}$ (D) $\frac{24}{7}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

Question 1

1).Find the principal value of $\tan^{-1}(-\sqrt{3})$

2).Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

3) Find the principal value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Question2 . **Prove the following :**

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0,1)$$

Question 3 Prove the following :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

Question 4 Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

Question 5 ***Prove that :*** $\tan^{-1} 2 + \tan^{-1} 3 = \frac{3\pi}{4}$

Question 6 ***Prove that*** $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

Question 7 ***Solve for*** x $\quad 2\tan^{-1}x = \sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) - \cos^{-1}\left(\frac{1-\beta^2}{1+\beta^2}\right)$

Question 8 ***Prove that*** $\tan^{-1}x + \cot^{-1}(x+1) = \tan^{-1}(x^2+x+1)$

Question 9 ***Prove that :*** $\sin^{-1}\frac{3}{5} \quad \sin^{-1}\frac{8}{17} - \cos^{-1}\frac{84}{85}$

Question 10 ***Prove that*** $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right), \quad |x| < a$

Question 11 **Prove that :** $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$= \frac{x+y}{1-xy}$, **if |x| < 1, y > 0 and xy > 1**

Question 12 **Prove that :** $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Question 13 **Prove that :** $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

Question 14 **Prove that** $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \quad |x| \le \frac{1}{\sqrt{2}}$

Question 14 **Prove that** $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}+\frac{x}{2}$, where, $0<x<\frac{\pi}{2}$

Question 15 **Prove that :**

$$\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$$

CHAPTER 3 & 4

MATRICES AND DETERMINANTS

POINTS TO REMEMBER

- Matrix : A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements of the matrix.
- Order of Matrix : A matrix having 'm' rows and 'n' columns is called the matrix of order mxn.
- Zero Matrix : A matrix having all the elements zero is called zero matrix or null matrix.
- Diagonal Matrix : A square matrix is called a diagonal matrix if all its non diagonal elements are zero.
- Scalar Matrix : A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.
- Identity Matrix : A scalar matrix in which each diagonal element is 1, is called an identity matrix or a unit matrix. It is denoted by I.

$\therefore \qquad I = [e_{ij}]_{n \times n}$

where, $\qquad e_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

- Transpose of a Matrix : If $A = [a_{ij}]_{m \times n}$ be an $m \times n$ matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by A' or A^T.

  Properties of the transpose of a matrix.

  (i) $(A')' = A$ (ii) $(A + B)' = A' + B'$

  (iii) $(KA)' = KA'$, K is a scalar (iv) $(AB)' = B'A'$

- Symmetrix Matrix : A square matrix $A = [a_{ij}]$ is symmetrix if $a_{ij} = a_{ji} \ \forall$ i, j. Also a square matrix A is symmetrix if $A' = A$.
- Skew Symmetrix Matrix : A square matrix $A = [a_{ij}]$ is skew-symmetrix, if $a_{ij} = -a_{ji} \ \forall$ i, j. Also a square matrix A is skew - symmetrix, if $A' = -A$.
- Determinant : To every square matrix $A = [a_{ij}]$ of order $n \times n$, we can associate a number (real or complex) called determinant of A. It is denoted by det A or $|A|$.

Properties

(i) $|AB| = |A|\ |B|$

(ii) $|KA|_{n \times n} = K^n\ |A|_{n \times n}$ where K is a scalar.

Area of triangles with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear $\Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Adjoint of a square matrix A is the transpose of the matrix whose elements have been replaced by their cofactors and is denoted as adj A.

Let $A = [a_{ij}]_{n \times n}$

$\text{adj } A = [A_{ji}]_{n \times n}$ where A_{ji} are co-factor of element a_{ji}

Properties

(i) $A(\text{adj } A) = (\text{adj } A) A = |A|\ I$

(ii) If A is a square matrix of order n then $|\text{adj } A| = |A|^{n-1}$

(iii) $\text{adj } (AB) = (\text{adj } B)(\text{adj } A)$.

Singular Matrix : A square matrix is called singular if $|A| = 0$, otherwise it will be called a non-singular matrix.

Inverse of a Matrix : A square matrix whose inverse exists, is called invertible matrix. Inverse of only a non-singular matrix exists. Inverse of a matrix A is denoted by A^{-1} and is given by

$$A^{-1} = \frac{1}{|A|} \text{adj}.A$$

Properties

(i) $AA^{-1} = A^{-1}A = I$

(ii) $(A^{-1})^{-1} = A$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A^1)^{-1} = (A^{-1})^1$

- Solution of system of equations using matrix :

1) If $AX = B$ is a matrix equation then its solution is $X = A^{-1}B$.

(i) If $|A| \neq 0$, system is consistent and has a unique solution.

(ii) If |A| = 0 and (adj A) B $\neq 0$ then system is inconsistent and has no solution.

(iii) If |A| = 0 and (adj A) B = 0 then system is consistent and has infinite solution.

2) If $AX = 0$ is a matrix equation,where is a squqre matrix

. (i) If $|A| \neq 0$,then system has trivial solution.

(ii) If $|A| = 0$ then system has infinitely many solutions.

Note- If $|A| = 0$, then $(adj\ A)\ B = 0$ as $B = 0$

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1. Let A be a square matrix of order 3×3 than $|KA|$ is equal to :

(A) $k|A|$ (B) $K^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

2. If a, b, c are in A.P. then determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \text{ is}$$

(A) 0 (B) 1 (C) x (D) $2x$

3. T_p, T_q, T_r are the p^th^, q^th^ and r^th^ terms of an A.P. then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(A) 1 (B) -1 (C) 0 (D) p+q+r

4. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, ω being a cube root of unity is

(A) 0 (B) 1 (C) ω^2 (D) ω

5. If a+b+c=0, one root of

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

(A) x=1 (B) x=2 (C) $x = a^2 + b^2 + c^2$ (D) x=0

6. The roots of the equation

$$\begin{vmatrix} a-x & b & c \\ 0 & b-x & 0 \\ 0 & b & c-x \end{vmatrix} = 0 \text{ are}$$

(A) a and b (B) b and c (C) a and c (D) a,b and c

7. Value of $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is

(A) (a-b) (b-c) (c-a) (B) $(a^2-b^2)(b^2-c^2)(c^2-a^2)$

(C) (a-b+c) (b-c+a) (c-a+b) (D) None of these

8. If A and B are any 2 x 2 matrices, then det (A+B)=0 implies

(A) det A + det B=0 (B) det A=0 or det B=0

(C) det A=0 and det B=0 (D) None of these

9. If A and B are 3 x 3 matrices then AB=0 implies

(A) A=0 and B=0 (B) |A|=0 and |B|=0

(C) Either |A|=0 or |B|=0 (D) A=0 or B=0

10. The value of λ for which the system of equations :-

$x + y + 2 = 6,\ x + 2y + 3z = 10,\ x + 4y + \lambda z = 1$ has a unique solution is

(A) $\lambda \neq -7$ (B) $\lambda \neq 7$

(C) $\lambda = 7$ (D) $\lambda = -7$

11. If the system of the equation :

$x - ky - z = 0, kx - y - z = 0, x + y - z = 0$ has a non-zero solution, then the possible values of *k* are :

(A) -1, 2 (B) 1,2

(C) 0, 1 (D) -1, 1

12. If A is a 3 x 3 non singular matrix than det [adj. (A)] is equal to

(A) $(\det A)^2$ (B) $(\det A)^3$

(C) $\det A$ (D) $(\det A)^{-1}$

13. If A is an invertible matrix of order n, then the determinant of Adj. A =

(A) $|A|^n$ (B) $|A|^{n+1}$

(C) $|A|^{n-1}$ (D) $|A|^{n+2}$

14. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(A) a+b+c (B) 1

(C) 0 (D) abc

15. If A^2-A+I=0 then the inverse of A is

(A) A (B) A+I

(C) I-A (D) A-I

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. Construct a 3 × 2 matrix A = [a_{ij}] whose elements are given by $a_{ij} = \begin{cases} i+j & \text{if } i=j \\ \dfrac{|i-2j|}{2} & \text{if } i<j \end{cases}$

2. Find A and B if 2A + 3B = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A-2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

3. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)' = B'A'$.

4. Express the matrix $\begin{bmatrix} 3 & 3 & 1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = P + Q$ where P is a symmetric and Q is a skew-symmetric matrix.

5 If A= $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, verify prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ where n is a natural number.

6. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix D such that CD – AB = O.

7 Find the value of x such that $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$

8 Prove that the product of the matrices $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$ is the null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

9. If $A=\begin{bmatrix}5 & 3\\ 12 & 7\end{bmatrix}$ show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

10 If $A=\begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$ find f(A) where $f(x) = x^2 - 5x - 2$.

11. If $A = \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

12. Find the matrix x so that $x\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

13. If $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$ then show that $(AB)^{-1} = B^{-1}A^{-1}$.

14. Test the consistency of the following system of equations by matrix method :

$3x - y = 5;\ 6x - 2y = 3$

15. Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}$, if possible.

16. By using elementary column transformation, find the inverse of $A = \begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$.

17. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $A + A' = I$, then find the general value of α

Using properties of determinants, prove the following : Q 18 to Q 24

18. $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$ if a, b, c are in A.P.

19. $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0$

20. $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

21. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(a+b+c+d)$

22. Show that :

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$

23. (i) If the points (a, b) (a´, b´) and (a – a´, b – b´) are collinear. Show that ab´ = a´b.

(ii) If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$ verify that $|AB| = |A||B|$

24 Solve the following equation for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Obtain the inverse of the following matrix using elementary row operations $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

2. Using matrix method, solve the following system of linear equations :

   x − y + 2z = 1, 2y − 3z = 1, 3x − 2y + 4z = 2.

3. Solve the following system of equations by matrix method, where x ≠ 0, y ≠ 0, z ≠ 0

   $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$,, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$, $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$. where $x \neq 0,\ y \neq 0,\ z \neq 0$

4. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations :

   x + 2y − 3z = − 4

   2x + 3y + 2z = 2

   3x − 3y − 4z = 11

5. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

6. Compute the inverse of the matrix.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix} \text{ and verify that } A^{-1} A = I_3$$

7. If the maxtric $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$

8. Using matrix method, solve the following system of linear equations :

$$2x - y = 4,\ 2y + z = 5,\ z + 2x = 7$$

9 Find the inverse of the matrix A $\begin{vmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \end{vmatrix}$ by using elementary column transformations.

10. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5

11. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A|\, I_3$.

12. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$, hence find A^{-1}.

13. By using properties of determinants prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

14. $$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^{2.}$$

15. $$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$$

16. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$ is equal to zero. Show that $xyz = -1$.

CHAPTER 5

CONTINUITY AND DIFFERENTIATION

POINTS TO REMEMBER

- A function f(x) is said to be continuous at x = c iff $\lim_{x \to c} f(x) = f(c)$

  i.e., $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$

- f(x) is continuous in (a, b) iff it is continuous at $x = c \ \forall c \in (a, b)$
- f(x) is continuous in [a, b] iff

  (i) f(x) is continuous in (a, b)

  (ii) $\lim_{x \to a^+} f(x) = f(a)$

  (iii) $\lim_{x \to b^-} f(x) = f(b)$

- Every polynomial function is continuous on R.
- If f(x) and g(x) are two continuous functions and c $\in$ R then at x = a

  (i) f(x) ± g(x) are also continuous functions at x = a.

  (ii) f(x), f(x) + c, cf(x), |f(x)| are also continuous at x = a.

  (iii) $\frac{f(x)}{g(x)}$ is continuous at x = a, provided g(a) ≠ 0.

- f(x) is derivable at x = c iff

  L.H.D. (c) = R.H.D. (c)

  $$i.e. \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

  and value of above limit is denoted by f′(c) and is called the derivative of f(x) at x = c.

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$$

- If y = f(u), x = g(u) then

  $$\frac{dy}{dx} = \frac{f'(u)}{g'(u)}.$$

- If y = f(u) and u = g(t) then $\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = f'(u).g'(t)$ (Chain Rule)
- f(x) = [x] is discontinuous at all integral points and continuous for all x $\in$ R – Z.
- Rolle's theorem : If f(x) is continuous in [a, b] and derivable in (a, b) and f(a) = f(b) then there exists atleast one real number c $\in$ (a, b) such that f´ (c) = 0.
- Mean Value Theorem : If f(x) is continuous in [a, b] and derivable in (a, b) then these exists atleast one real number c $\in$ (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.
- f(x) = $\log_e x$, (x > 0) is continuous function.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If $f(x) = |x|$ then $f(x)$ is

   (A) Continuous for all *x* (B) Continuous only at certain points

   (C) Differentiable at all points (D) None of these

2. The function $f(x) = \sin|x|$ is

   (A) Continuous for all *x* (B) Continuous only at certain points

   (C) Differentiable at all points (D) None of these

3. $\underset{h\to 0}{Lt}\dfrac{\cos^2(x+h)-\cos^2 x}{h}$ is equal to

(A) $\cos^2 x$ (B) $-\sin 2x$

(C) $\sin x\cos x$ (D) $2\sin x$

4. If $f(x)=\begin{cases}\dfrac{1-\sqrt{2}\sin x}{\pi-4x} & if \quad x\neq\dfrac{\pi}{4}\\ a & if \quad x=\dfrac{\pi}{4}\end{cases}$

is continuous at $x=\dfrac{\pi}{4}$ then, 'a' equals

(A) 4 (B) 2

(C) 1 (D) $\dfrac{1}{4}$

5. Let [ ] denote the greatest integer function and $f(x)=[\tan^2 x]$. Then

(A) $\underset{x\to 0}{Lt} f(x)$ does not exist (B) $f(x)$ is continuous at x=0

(C) $f(x)$ is not differentiable at x=0 (D) $f'(0)=1$

6. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\dfrac{dy}{dx}$ equals

(A) $-\sqrt{\dfrac{y}{x}}$ (B) $-\sqrt{\dfrac{x}{y}}$

(C) $\sqrt{\dfrac{y}{x}}$ (D) $\sqrt{\dfrac{x}{y}}$

7. If $f(x)=\begin{cases}x\sin\dfrac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$

(A) $f(x)$ is continuous and differentiable to x=0

(B) $f(x)$ is neither continuous nor differentiable at x=0

(C) $f(x)$ is continuous, but not differentiable at x=0

(D) None of these

8. If $y=\sqrt{\sin+\sqrt{\sin x+\sqrt{\sin x+..to\infty}}}$ then, $\frac{dy}{dx}$ is equal to

(A) $\frac{\sin x}{y}$ (B) $\frac{\sin x+\cos x}{2y+1}$

(C) $\frac{\cos x}{2y-1}$ (D) $\frac{\cos x}{1-2y}$

9. If $x^2+y^2=1$, then

(A) $yy''-2y^2-1=0$ (B) $yy''+y'^2+1=0$

(C) $yy''-y'^2-1=0$ (D) $yy''-2y'^2+1=0$

10. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x=0 is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 1

11. Find K, $\begin{Bmatrix} 2x+1 & x<2 \\ k & x=2 \\ 3x-1 & x>2 \end{Bmatrix}$ to be continuous at x=2 is

(A) 3 (B) -5 (C) 0 (D) 5

12. If $y=x^y$,then $\frac{dy}{dx}$ equals

(A) $\frac{x^2}{y(1-y\log x)}$ (B) $\frac{y^2}{x(1-x\log y)}$

(C) $\frac{y^2}{x(1-x\log x)}$ (D) $\frac{y^2}{x(1-y\log x)}$

13. If $x=8t^2$, $y=16t$ then $\frac{d^2y}{dx^2}$ at $t=1$ is :

(A) - 1 (B) $\frac{-1}{16}$ (C) -16 (D) 16

14. The derivative of x^6 w.r.t. x^3 is

(A) $6x^6$ (B) $3x^2$ (C) $2x^3$ (D) x^2

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

Discuss the continuity of following functions at the indicated points.

(1) $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0 & x=0 \end{cases}$ **at x=0**

(2) $g(x)=\begin{cases} \frac{\sin 2x}{3x}, & x\neq 0 \\ \frac{3}{2} & x=0 \end{cases}$ at $x=0$.

(3) $f(x)=\begin{cases} x^2\cos(1/x) & x\neq 0 \\ 0 & x=0 \end{cases}$ at $x=0$.

(4) $f(x) = |x| + |x-1|$ at $x = 1$.

(5) $f(x)=\begin{cases} x-[x], & x\neq 1 \\ 0 & x=1 \end{cases}$ at $x=1$.

6. For what value of K, $f(x)=\begin{cases} 3x^2 - kx + 5, & 0\leq x<2 \\ 1-3x & 2\leq x\leq 3 \end{cases}$ is continuous $\forall\, x\in[0,3]$.

7. If the function $f(x)$ given by

$$f(x)=\begin{cases} 2ax+b & \text{if } x>1 \\ 5 & \text{if } x=1 \\ 3ax-b & \text{if } x<1 \end{cases}$$ **is**

continuous at $x=1$, Find the value of *a* and *b*

8. Prove that f(x) = |x + 1| is continuous at x = –1, but not derivable at x = –1.

9. For what value of p,

$$f(x)=\begin{cases} x^p \sin(1/x), & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is derivable at x = 0.

10 **Discuss the continuity of the function**

$$f(x)=\begin{cases} \dfrac{1-\cos 4x}{x^2} & x \neq 0 \\ 2 & x = 0 \end{cases} \quad \textbf{at} \quad x = 0$$

11 Find $\dfrac{dy}{dx}$ **if** $y = 1+2\left(\dfrac{x}{x+1}\right)^2 + 3\left(\dfrac{x}{x+1}\right)^3$

12 Find the derivative of $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$.

13. Find the derivative of $\log_e(\sin x)$ w.r.t. $\log_e(\cos x)$.

14. **Differentiate** $y = \cot^{-1}\left(\dfrac{x}{\sqrt{1+x^2}-1}\right)$ **with respect to** x

15. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\dfrac{dy}{dx}$.

16. If $x = ae^t(\sin t - \cos t)$

$y = ae^t(\sin t + \cos t)$ then show that $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$ is 1.

17 **If** $y = \tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\dfrac{dy}{dx}$

18 **Find** $\dfrac{dy}{dx}$ **if** $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

19 **If x =** $\dfrac{\sin^3 t}{\sqrt{\cos 2t}}$**, y =** $\dfrac{\cos^3 t}{\sqrt{\cos 2t}}$**, find** $\dfrac{dy}{dx}$

20 **Find the derivative of** $\sqrt{e^{\sqrt{x}}}$

21 Give an example of the function which is continuous everywhere but not differentiable at exactly two points.

22. Discuss the applicability of Rolle's theorem for the following function on the indicated interval : f (x) = |x| on [–1, 1]

23 Discuss the applicability of Rolle's theorem for the following function on the indicated interval :
$f(x) = 3 + (x - 2)^{2/3}$ on [1, 3]

24 It is given that for the function f given by

$$f(x) = x^3 + bx^2 + ax + 1,\ x \in [1,3]$$

Rolle's theorem holds with c = $2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

25 Verify the mean value theorem for the function $f(x) = e^{x\ 1}$, in[1,2]

26 Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ for the interval [2,5]

27 Using Lagrange's mean value theorem, prove that

$\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$, where 0 < a < b.

CHAPTER 6

APPLICATIONS OF DERIVATIVES

POINTS TO REMEMBER

- Rate of Change : Let y = f(x) be a function then the rate of change of y with respect to x is given by $\frac{dy}{dx} = f'(x)$ where a quantity y varies with another quantity x.

  $\left(\frac{dy}{dx}\right)_{x=x_0}$ *or* $f'(x_0)$ represents the rate of change of *y* w.r.t. *x* at $x = x_0$

- If x=f(t) and y=g(t)

  By chain rule

  $$\frac{dy}{dx} = \frac{dy}{dt} \bigg/ \frac{dx}{dt} \quad if \quad \frac{dx}{dt} \neq 0$$

- (i) A function f(x) is said to be increasing on an interval (a, b) if $x_1 \leq x_2$ in (a, b) $\Rightarrow f(x_1) \leq f(x_2) \forall x_1, x_2 \in (a,b)$. Alternatively if $f'(x) \geq 0 \ \forall x \in (a,b)$, then f(x) is increasing function in (a, b) .

  (ii) A function f(x) is said to be decreasing on an interval (a, b). If $x_1 \leq x_2$ in (a, b) $\Rightarrow f(x_1) \geq f(x_2) \ \forall \ x_1, x_2 \in$ (a, b). Alternatively if $f'(x) \leq 0 \ \forall \ x \in (a,b)$, then f(x) is decreasing function in (a, b).

- The equation of tangent at the point (x_0, y_0) to a curve y=f(x) is given by

  $$y - y_0 = \left(\frac{dy}{dx}\right)_{(x_0, y_0)} (x - x_0)$$

  where $\left(\frac{dy}{dx}\right)_{(x_0, y_0)}$ = slope of the tangent at (x_0, y_0)

- Slope of the normal at (x_0, y_0) is given by $\dfrac{1}{\left(\frac{dy}{dx}\right)_{x=x_0}}$.

- Equation of the normal to the curve y = f(x) at a point (x_0, y_0) is given by

$$y - y_0 = \frac{1}{\left(\frac{dy}{dx}\right)_{(x_0, y_0)}}(x - x_0)$$

If $\left.\frac{dy}{dx}\right]_{(x_0, y_0)} = 0$, then the tangent is parallel to x-axis and the equation of the normal is $x = x_0$

- If $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ does not exist. Then, the tangent is parallel to y-axis and the equation of the normal is $y = y_0$.
- Let y = f(x)

  Let Δx = the small increment in x and

  Δy be the increment in y corresponding to the increment in x

  Then approximate change in y is given by

  $$dy = \left(\frac{dy}{dx}\right)\Delta x \quad \text{or} \quad dy = f'(x)\,\Delta x$$

  The approximate change in the value of f is given by

  $$f(x + \Delta x) = f(x) + f'(x)\,\Delta x$$

- Let f be a function. Let point c be in the domain of the function f at which either $f'(c) = 0$ or f is not differentials is called a critical point of f.
- First Derivative Test : Let f be a function designed on an open interval I. Let f be continuous at a critical point $c \in I$. Then

  (i) $f'(x)$ changes sign from positive to negative as x increases through c.

  i.e. if $f'(x) > 0$ at every point sufficiently close to and to the left of c and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.

  (ii) If $f'(x)$ changes sign from negative to positive as x increases through c i.e. if $f'(x) < 0$ at every point sufficiently close to and to the left of c and $f'(x) > 0$ at every point sufficiently close to and to the right of c then c is a point then c is a point of local maxima.

  (iii) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Such a point is called a point of inflexion.

- Second Derivative Test : Let f be a functions defined on an interval I and let $c \in I$.

  (i) x = c is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$.

  Then f(c) is the local maximum value of f.

(ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$. Then $f(c)$ is the local minimum value of f.

(iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.

VERY SHORT ANSWER TYPE QUESTIONS (3 marks)

1. A particle cover along the curve $6y = x^3 + 2$. Find the points on the curve at which the y co-ordinate is changing 8 times as fast as the x co-ordinate.

2. A ladder 5 metres long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall as the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 metres away from the wall?

3. A balloon which always remain spherical is being inflated by pumping in 900 cubic cm of a gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

4. A man 2 meters high walks at a uniform speed of 5 km/hr away from a lamp post 6 metres high. Find the rate at which the length of his shadow increases.

5. Water is running out of a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and attitude is 20 cm. Find the rate at which the water level is dropping when it is 5 cm from the top.

6. The length x of a rectangle is decreasing at the rate of 5 cm/sec and the width y is increasing as the rate of 4 cm/sec when $x = 8$ cm and $y = 6$ cm. Find the rate of change of

   (a) Perimeter (b) Area of the rectangle.

7. Sand is pouring from a pipe as the rate of $12cm^2$ /sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone is increasing when height is 4 cm?

8. The area of an expanding rectangle is increasing at the rate of 48 cm^2/sec. The length of the rectangle is always equal to the square of the breadth. At what rate lies the length increasing at the instant when the breadth is 4.5 cm?

9. Find a point on the curve $y = (x - 3)^2$ where the tangent is parallel to the line joining the points (4, 1) and (3, 0).

12. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

13. Show that the curves $4x = y^2$ and $4xy = k$ cut as right angles if $k^2 = 512$.

14. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - y + 5 = 0$.

16. Find the points on the curve $4y = x^3$ where slope of the tangent is $\frac{16}{3}$.

17. Show that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = be^{-x/a}$ at the point where the curve crosses the y-axis.

18. Find the equation of the tangent to the curve given by $x = a \sin^3 t$, $y = b \cos^3 t$ at a point where $t = \frac{\pi}{2}$.

19. Find the intervals in which the function $f(x) = \log(1 + x) - \frac{x}{1+x}$, $x > -1$ is increasing or decreasing.

20. Find the intervals is which the function $f(x) = x^3 - 12x^2 + 36x + 17$ is

    (a) increasing (b) decreasing.

21. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in $[0, 1]$.

22. Find the intervals on which the function $f(x)=\frac{x}{x^2+1}$ is decreasing.

23. Prove that the functions given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

24. Find the intervals on which the function $f(x)=\frac{\log x}{x}$ increasing or decreasing.

25. Find the intervals in which the function f(x) = $\sin^4 x + \cos^4 x$, $0 \leq x \leq \frac{\pi}{2}$ is increasing or decreasing.

26. Find the least value of 'a' such that the function f(x) = $x^2 + ax + 1$ is strictly increasing on (1, 2).

27. Find the interval in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$, $x > 0$ is strictly decreasing.

Using differentials, find the approximate value of (Q. No. 28 to 30).

28. $(255)^{\frac{1}{4}}$.

29. $(66)^{\frac{1}{3}}$.

30. $\sqrt{25.3}$

LONG ANSWER TYPE QUESTION (5 MARKS)

1. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.
2. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is maximum.
3. Show that of all the rectangles of given area, the square has the smallest perimeter.
4. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radium of the base.
5. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.
6. Show that the right triangle of maximum area that can be inscribed in a circle is an isosceles triangle.

7. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

8. Find the interval in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

9. Find the intervals in which the function $f(x) = (x + 1)^3 (x - 3)^3$ is strictly increasing or strictly decreasing.

10. Find the local maximum and local minimum of $f(x) = \sin 2x - x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

11. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to x-axis.

12. A solid is formed by a cylinder of radius r and height h together with two hemisphere of radius r attached at each end. It the volume of the solid is constant but radius r is increasing at the rate of $\frac{1}{2\pi}$ metre/min. How fast must h (height) be changing when r and h are 10 metres.

13. Find the equation of the normal to the curve

    $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$ at the point θ and show that its distance from the origin is a.

14. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

15. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

16. Find the equation of the tangents at the points where the curve $2y = 3x^2 - 2x - 8$ cuts the x-axis and show that they make supplementary angles with the x-axis.

18. A window is in the form of a rectangle surmounted by an equilateral triangle. Given that the perimeter is 16 metres. Find the width of the window in order that the maximum amount of light may be admitted.

19. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each cover and folding up the flaps to form the box. What should be the side of the square to be cut off so that the value of the box is the maximum points.

20. A window is in the form of a rectangle is surmounted by a semi circular opening. The total perimeter of the window is 30 metres. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.

21. An open box with square base is to be made out of a given iron sheet of area 27 sq. meter show that the maximum value of the box is 13.5 cubic metres.

22. A wire of length 28 cm is to be cut into two pieces. One of the two pieces is to be made into a square and other in to a circle. What should be the length of two pieces so that the combined area of the square and the circle is minimum?

23. Show that the height of the cylinder of maximum volume which can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed is a sphere of radius r is $\frac{4r}{3}$.

25. Prove that the surface area of solid cuboid of a square base and given volume is minimum, when it is a cube.

26. Show that the volume of the greatest cylinder which can be inscribed in a right circular cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

CHAPTER 7

INTEGRALS

POINTS TO REMEMBER

- Integration is the reverse process of Differentiation.
- Let $\frac{d}{dx}F(x) = f(x)$ then we write $\int f(x)dx = F(x) + c$.
- These integrals are called indefinite integrals and c is called constant of integrations.
- From geometrical point of view an indefinite integral is collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along with y-axis.

STANDARD FORMULAE

1. $$\int x^n dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log|x| + c & n = -1 \end{cases}$$

2. $$\int (ax + b)^n \, dx = \begin{cases} \dfrac{(ax+b)^{n+1}}{(n+1)a} + c & n \neq -1 \\ \dfrac{1}{a}\log|ax + b| + c & n = -1 \end{cases}$$

3. $\int \sin x \, dx = -\cos x + c.$

4. $\int \cos x \, dx = \sin x + c.$

5. $\int \tan x \, . dx = -\log|\cos x| + c = \log|\sec x| + c.$

6. $\int \cot x \, dx = \log|\sin x| + c.$

7. $\int \sec^2 x \, . dx = \tan x + c.$

8. $\int \operatorname{cosec}^2 x \, . dx = -\cot x + c.$

9. $\int \sec x \, . \tan x \, . dx = \sec x + c.$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c.$

11. $\int \sec x \, dx = \log|\sec x + \tan x| + c.$

12. $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c.$

13. $\int e^x dx = e^x + c.$

14. $\int a^x dx = \dfrac{a^x}{\log a} + c$

15. $\int \dfrac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c, |x < 1|$

16. $\int \dfrac{1}{1 + x^2} dx = \tan^{-1} x + c.$

17. $\int \dfrac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + c, \ |x| > 1.$

18. $\int \dfrac{1}{a^2 - x^2} dx = \dfrac{1}{2a} \log\left|\dfrac{a+x}{a-x}\right| + c$

19. $\int \dfrac{1}{x^2 - a^2} dx = \dfrac{1}{2a} \log\left|\dfrac{x-a}{x+a}\right| + c$

20. $\int \dfrac{1}{a^2 + x^2} dx = \dfrac{1}{a} \tan^{-1} \dfrac{x}{a} + c.$

21. $\int \dfrac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \dfrac{x}{a} + c.$

22. $\int \dfrac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c.$

23. $\int \dfrac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c.$

24. $\int \sqrt{a^2 + x^2} dx = \dfrac{x}{2}\sqrt{a^2 + x^2} + \dfrac{a^2}{2} \log\left|x + \sqrt{a^2 + x^2}\right| + c.$

25. $\int \sqrt{x^2 - a^2} dx = \dfrac{x}{2}\sqrt{x^2 - a^2} + \dfrac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + c.$

26. $\int \sqrt{a^2 - x^2} dx = \dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2} \sin^{-1}\left(\dfrac{x}{a}\right) + c$

RULES OF INTEGRATION

1. $\int k.f(x) dx = k \int f(x) dx.$

2. $\int k\{f(x) \pm g(x)\} dx = k \int f(x) dx \pm k \int g(x) dx.$

INTEGRATION BY SUBSTITUTION

1. $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c.$

2. $\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c.$

3. $\int \frac{f'(x)}{[f(x)]^n} dx = \frac{(f(x))^{-n+1}}{-n+1} + c.$

INTEGRATION BY PARTS

$$\int f(x).\ g(x) dx = f(x).\left[\int g(x) dx\right] - \int f'(x).\left[\int g(x) dx\right] dx.$$

DEFINITE INTEGRALS

$$\int_a^b f(x) dx = F(b) - F(a), \text{ where } F(x) = \int f(x) dx.$$

DEFINITE INTEGRAL AS A LIMIT OF SUMS.

$$\int_a^b f(x) dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + f(a+2h) + ... + f\left(a + \overline{n-1}\, h\right)\right]$$

where $h = \frac{b-a}{n}.$

PROPERTIES OF DEFINITE INTEGRAL

1. $\int_a^b f(x) dx = -\int_b^a f(x) dx.$

2. $\int_a^b f(x) dx = \int_a^b f(t) dt.$

3. $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx.$

4. $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx.$

5. $\int_{-a}^{a} f(x) = 0$; if $f(x)$ is odd function.

6. $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, if f(x) is even function.

7. $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x)=f(x) \\ 0, & if\ f(2a-x)=-f(x) \end{cases}$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Evaluate the following integrals

1. If $\frac{d}{dx}(f(x)) = 4x^3 - \frac{3}{x^4}$ such that $f(2)=0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

2. $\int \frac{1}{1+e^{-x}} dx$ is equal to :

(A) $\log(1+e^{-x})+c$ (B) $\log(1+e^{x})+c$

(C) $x - e^{-x} + c$ (D) None of these

3. $\int (\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x})dx$ is equal to

(A) $\frac{\pi}{2}x + c$ (B) $\frac{\pi}{4}x + c$

(C) $x + c$ (D) $\pi x + c$

4. $\int\left(\frac{10x^{a}+10^{x}\log_{e}10}{x^{10}+10^{x}}\right)dx$ equals

(A) $10^{x}-x^{10}+c$ (B) $10^{x}+x^{10}+c$

(C) $\left(10^{x}-x^{10}\right)^{-1}+c$ (D) $\log\left(10^{x}+10^{x}\right)+c$

5. $\int\frac{1}{\sin^{2}x\cos^{2}x}dx$ equals

(A) $\tan x+\cot x+c$ (B) $\tan x-\cot x+c$

(C) $\tan x\cot x+c$ (D) $\tan x-\cot 2x+c$

6. $\int\frac{1}{x^{2}+2x+2}dx$ equals

(A) $x\tan^{-1}(x+1)+c$ (B) $\tan^{-1}(x+1)+c$

(C) $(x+1)\tan^{-1}x+c$ (D) $\tan^{-1}x+c$

7. $\int\frac{1}{\sqrt{9x-4x^{2}}}dx$ equals

(A) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+c$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{8}\right)+c$

(C) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+c$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+c$

8. $\int\frac{x(1+x)}{\cos^{2}\left(xe^{x}\right)}dx$ equals

(A) $-\cot\left(xe^{x}\right)+c$ (B) $\tan\left(xe^{x}\right)+c$

(C) $\tan\left(e^{x}\right)+c$ (D) $\cot\left(e^{x}\right)+c$

9. $\int\frac{xdx}{(x-1)(x-2)}$ equals

(A) $\log\left|\frac{(x-1)^{2}}{x-2}\right|+c$ (B) $\log\left|\frac{(x-2)^{2}}{x-1}\right|+c$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+c$ (D) $\log|(x-1)(x-2)|+c$

10. $\int \frac{1}{x(x^2+1)}\,dx$ equals

(A) $\log|x|-\frac{1}{2}\log(x^2+1)+c$ (B) $\log|x|+\frac{1}{2}\log(x^2+1)+c$

(C) $-\log|x|+\frac{1}{2}\log(x^2+1)+c$ (D) $\frac{1}{2}\log|x|+\log(x^2+1)+c$

11. $\int x^2 e^{x^3}\,dx$ equals

(A) $\frac{1}{3}e^{x^3}+c$ (B) $\frac{1}{3}e^{x^2}+c$

(C) $\frac{1}{2}e^{x^3}+c$ (D) $\frac{1}{2}e^{x^2}+c$

12. $\int e^x \sec x(1+\tan x)dx$ equals

(A) $e^x \cos x + c$ (B) $e^x \sec x + c$

(C) $e^x \sin x + c$ (D) $e^x + \tan x + c$

13. $\int \sqrt{1+x^2}\,dx$ equals

(A) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

(B) $\frac{2}{3}(1+x^2)^{3/2}+c$

(C) $\frac{2}{3}x(1+x^2)^{3/2}+c$

(D) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+c$

14. If $\int \frac{5^{\frac{1}{x}}}{x^2} dx = k\, 5^{\frac{1}{x}}$ then the value of k is :

(A) log 5 (B) - log 5

(C) $-\frac{1}{\log 5}$ (D) $\frac{1}{\log 5}$

15. $\int_{-2}^{2} |x| dx =$

(A) 0 (B) 1 (C) 2 (D) 4

16. $\int_{0.5}^{3.5} [x]\, dx =$

(A) $2+\sqrt{2}$ (B) $2-\sqrt{2}$ (C) 4.5 (D) None of these

Where [ ] denote the greatest integer function

17. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^5 x\, dx$ has value

(A) 0 (B) -1 (C) 1 (D) None of these

18. If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$ then a is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{1}{2}$

19. $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2012}}{(\sin x)^{2012} + (\cos x)^{2012}} dx$ equals

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $2012(\sin x)^{2013}$ (D) None of these.

20. $\int_0^{10\pi} |\sin x|\, dx$ is

(A) 20 (B) 8 (C) 10 (D) 18

21. $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) 1

22. The value of $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}\,dx$ is

(A) 0 (B) 1 (C) 2 (D) 3

23. The value of integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}$ is

(A) $\frac{\pi}{2}+1$ (B) $\frac{\pi}{2}-1$ (C) -1 (D) 1

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. (i) $\int \frac{x \operatorname{cosec}\left(\tan^{-1} x^2\right)}{1+x^4}\,dx.$ (ii) $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx.$

(iii) $\int \frac{1}{\sin(x-a)\sin(x-b)}\,dx.$ (iv) $\int \frac{\cos^2 x}{1+\sin x}\,dx$

(v) $\int \cos x \cos 2x \cos 3x\,dx.$ (vi) $\int \cos^5 x\,dx$

(vii) $\int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}}\,dx.$ (viii) $\int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}}\,dx.$

(ix) $\int \frac{\mathrm{Sin}^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\,dx.$

2. Evaluate :

(i) $\int \frac{x\,dx}{x^4+x^2+1}.$ *(ii) $\int \frac{1/x \quad dx}{\left[6(\log x)^2 + 7\log x + 2\right]}$

(iii) $\int \frac{dx}{1+x-x^2}.$ (iv) $\int \frac{1}{\sqrt{9+8x-x^2}}\,dx.$

(v) $\displaystyle\int \frac{1}{\sqrt{(x-a)(x-b)}}\,dx.$

(vi) $\displaystyle\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}\,dx.$

(vii) $\displaystyle\int \frac{5x-2}{3x^2+2x+1}\,dx.$

(viii) $\displaystyle\int \frac{x^2}{x^2+6x+12}\,dx.$

(ix) $\displaystyle\int \frac{x+2}{\sqrt{4x-x^2}}\,dx.$

(x) $\displaystyle\int x\sqrt{1+x-x^2}\,dx.$

(xii) $\displaystyle\int (3x-2)\sqrt{x^2+x+1}\,dx.$

(xiii) $\displaystyle\int \sqrt{\sec x+1}\,dx.$

3. Evaluate :

(i) $\displaystyle\int \frac{dx}{x\left(x^7+1\right)}.$

(ii) $\displaystyle\int \frac{\sin x}{(1+\cos x)(2+3\cos x)}\,dx.$

(iii) $\displaystyle\int \frac{\sin\theta\cos\theta}{\cos^2\theta-\cos\theta-2}\,d\theta.$

(iv) $\displaystyle\int \frac{x-1}{(x+1)(x-2)(x+3)}\,dx.$

(v) $\displaystyle\int \frac{x^2+x+2}{(x-2)(x-1)}\,dx.$

(vi) $\displaystyle\int \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}\,dx.$

(vii) $\displaystyle\int \frac{dx}{(2x+1)\left(x^2+7\right)}$

(viii) $\displaystyle\int \frac{dx}{\sin x\,(1-2\cos x)}$

(ix) $\displaystyle\int \frac{\sin x}{\sin 4x}\,dx.$

(x) $\displaystyle\int \frac{x^2-1}{x^4+x^2+1}\,dx.$

(xi) $\displaystyle\int \sqrt{\tan x}\,dx.$

(xii) $\displaystyle\int \frac{x^2+9}{x^4+81}\,dx.$

4. Evaluate :

(i) $\int x^5 \sin x^3 dx$.

(ii) $\int \sec^3 x \, dx$.

(iii) $\int e^{ax} \cos(bx + c)\, dx$.

(iv) $\int \sin^{-1} \dfrac{6x}{1 + 9x^2}\, dx$.

(v) $\int \cos \sqrt{x}\, dx$.

(vi) $\int x^3 \tan^{-1} x \, dx$.

(vii) $\int e^{2x} \left(\dfrac{1 + \sin 2x}{1 + \cos 2x} \right) dx$.

(viii) $\int e^x \left(\dfrac{x - 1}{2x^2} \right) dx$.

(ix) $\int e^x \left(\dfrac{1 - x}{1 + x^2} \right)^2 dx$.

(x) $\int e^x \dfrac{(x^2 + 1)}{(x + 1)^2}\, dx$.

(xi) $\int e^x \dfrac{(2 + \sin 2x)}{(1 + \cos 2x)}\, dx$.

(xii) $\int \left\{ \log(\log x) + \dfrac{1}{(\log x)^2} \right\} dx$.

5. Evaluate the following definite integrals :

(i) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx.$ (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos 2x \log \sin x \; dx.$

(iii) $\displaystyle\int_0^{2} \frac{x + \sin x}{1 + \cos x} dx.$

6. Evaluate :

(i) $\displaystyle\int_1^{3} \{|x - 1| + |x - 2| + |x - 3|\} dx.$ (ii) $\displaystyle\int_0^{\pi} \frac{x}{1 + \sin x} dx.$

(iii) $\displaystyle\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx.$ (iv) $\displaystyle\int_0^{\frac{\pi}{2}} \log \sin x \; dx.$

(v) $\displaystyle\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$

(vi) $\displaystyle\int_{-2}^{2} f(x) dx$ where $f(x) = \begin{cases} 2x - x^3 & \text{when } -2 \leq x < -1 \\ x^3 - 3x + 2 & \text{when } -1 \leq x < 1 \\ 3x - 2 & \text{when } 1 \leq x < 2. \end{cases}$

(vii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ (viii) $\displaystyle\int_0^{\pi} \frac{x dx}{a^2 \cos^2 + b^2 \sin^2 x}.$

7. Evaluate the following integrals

(i) $\int_{1}^{3} \left|x^2 - 2x\right| dx.$ (ii) $\int_{0}^{1} \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx.$

(iii) $\int_{-1}^{1} \log\left(\frac{1+\sin x}{1-\sin x}\right) dx.$ (iv) $\int_{0}^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\ dx.$

8. Evaluate the following integrals :

(iii) $\int \frac{2x^3}{(x+1)(x-3)} dx$ (iv) $\int \frac{x^4}{x^4 - 16} dx$

(v) $\int_{0}^{2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx.$ (vi) $\int \frac{1}{x^4 + 1} dx.$

(vii) $\int_{0}^{\infty} \frac{x \tan^{-1} x}{\left(1+x^2\right)^2} dx.$

9. Evaluate the following integrals as limit of sums :

(i) $\int_{2}^{4} (2x + 1) dx.$ (ii) $\int_{0}^{2} \left(x^2 + 3\right) dx.$

(iii) $\int_{1}^{3} \left(3x^2 - 2x + 4\right) dx.$ (iv) $\int_{0}^{4} \left(3x^2 + e^{2x}\right) dx.$

(v) $\int_{2}^{5} \left(x^2 + 3x\right) dx.$

10. Evaluate the following integrals :

(i) $\displaystyle\int \frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x}\,dx$ (ii) $\displaystyle\int \frac{x^2}{(x\sin x+\cos x)^2}\,dx$

(iii) $\displaystyle\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ (iv) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$

(iv) $\displaystyle\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin|x|-\cos|x|)\,dx$ (vi) $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{2}} |\sin x|\,dx$

(vii) $\displaystyle\int_{0}^{1.5} \left[x^2\right] dx$ where [x] is greatest integer function

(viii) $\displaystyle\int_{-1}^{\frac{3}{2}} |x\sin \pi x|\,dx$

CHAPTER 8

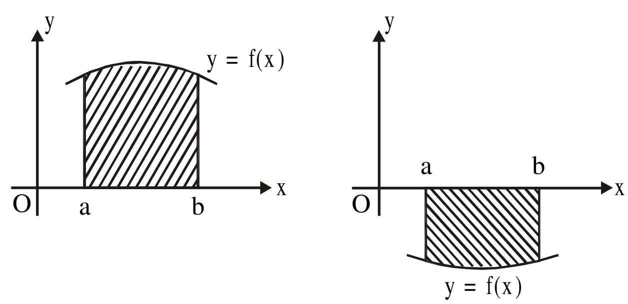
APPLICATIONS OF INTEGRALS

POINTS TO REMEMBER

AREA OF BOUNDED REGION

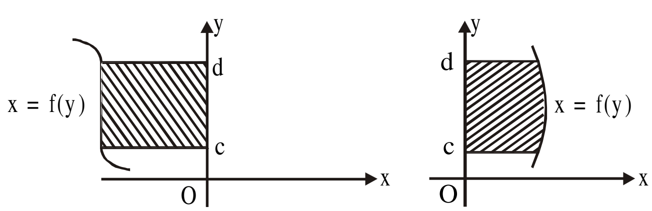
- Area bounded by the curve $y = f(x)$, the x axis and between the ordinates, $x = a$ and $x = b$ is given by

$$\text{Area} = \left| \int_a^b f(x)\, dx \right|$$

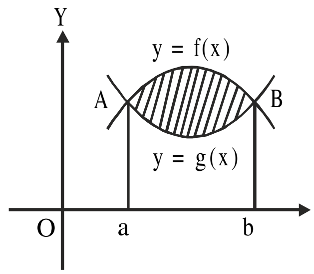


- Area bounded by the curve $x = f(y)$ the y-axis and between abscissas, $y = c$ and $y = d$ is given by

$$\text{Area} = \left| \int_c^d x\, dy \right| = \left| \int_c^d f(y)\, dy \right|$$



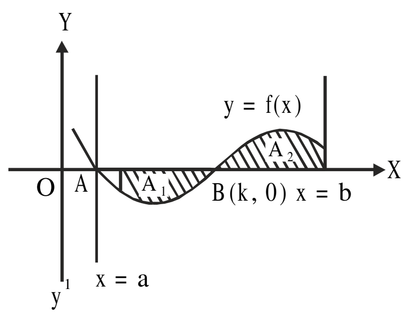
- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \leq g(x) \leq f(x)$ for all $x \in [a, b]$ and between the ordinate at $x = a$ and $x = b$ is given by



$$\text{Area} = \int_a^b \left[f(x) - g(x)\right] dx$$

- Required Area

$$= \left|\int_a^k f(x) dx\right| + \int_k^b f(x) dx.$$



LONG ANSWER TYPE QUESTION (5 MARKS)

1. F ind the area enclosed by circle $x^2 + y^2 = a^2$.
2. Find the area of region bounded by $y^2 = 4x$.
3. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
4. Find the area of region in the first quadrant enclosed by x–axis the line $y = x$ and the circle $x^2 + y^2 = 32$.
5. Find the area of region $\{(x, y) : y^2 \leq 4x,\ 4x^2 + 4y^2 \leq 9\}$
6. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.
7. Find area enclosed between the curves, $y = 4x$ and $x^2 = 6y$.

8. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

9. Using integration, find the area of the region bounded by the triangle whose vertices are

   (a) $(-1, 0), (1, 3)$ and $(3, 2)$ (b) $(-2, 2)$ $(0, 5)$ and $(3, 2)$

10. Using integration, find the area bounded by the lines.

    (i) $x + 2y = 2, \quad y - x = 1 \quad$ and $\quad 2x + y - 7 = 0$

    (ii) $y = 4x + 5, \quad y = 5 - x \quad$ and $\quad 4y - x = 5$.

11. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

12. Find the area of the region bounded by

    $y = |x - 1|$ and $y = 1$.

13. Find the area enclosed by the curve $y = \sin x$ between $x = 0$ and $x = \dfrac{3\pi}{2}$ and x-axis.

14. Find the area bounded by semi circle $y = \sqrt{25 - x^2}$ and x-axis.

15. Find area of region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.

16. Find area of smaller region bounded by ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ and straight line $2x + 3y = 6$.

17. Find the area of region bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.

18. Using integration find the area of region in first quadrant enclosed by x-axis the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

19. Find smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$.

20. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

21. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $\left(0, \dfrac{\pi}{2}\right)$.

22. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^6 |x - 5| \, dx$.

CHAPTER 9

DIFFERENTIAL EQUATION

POINTS TO REMEMBER

- **Differential Equation :** Equation containing derivatives of a dependant variable with respect to an independent variable is called differential equation.
- **Order of a Differential Equation :** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.
- **Degree of a Differential Equation :** Highest power of highest order derivative involved in the equation is called degree of differential equation where equation is a polynomial equation in differential coefficients.
- **Formation of a Differential Equation :** We differentiate the family of curves as many times as the number of arbitrary constant in the given in family of curves. Now eliminate the arbitrary constants from these equations. After elimination the equation obtained is differential equation.

- **Solution of Differential Equation**

(i) **Variable Separable Method**

$$\frac{dy}{dx}=f(x,y)$$

We separate the variables and we get

$$f(x)dx = g(y)dy$$

Then $\int f(x)\,dx = \int g(y)dy + c$ is the required solutions.

(ii) ***Homogenous Differential Equation :*** A differential equation of the

form $\dfrac{dy}{dx}=\dfrac{f(x,y)}{g(x,y)}$ where $f(x, y)$ and $g(x, y)$ are both homogeneous functions of the same

degree in x and y *i.e* of the form $\dfrac{dy}{dx}=F\left(\dfrac{y}{x}\right)$ is called a homogeneous differential equation

For solving this type of equations, we substitute $y = vx$ and then .

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

The equation into a variable separable form.

(iii) ***Linear Differential Equation :*** An equation of the from

$$\frac{dy}{dx}+Py=Q$$

where P and Q are constant or functions of x only is called a linear differential equation. For the solution of this type of equations, we find

Integrating factor (IF) $= e^{\int pdx}$

Its solution is

$$y(I.F.) = \int Q.(I.F).\,dx + c$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARKS)

(Question carrying 1 marks)

1. The degree of the differential equation.

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0 \text{ is}$$

(A) 3 (B) 2 (C) 0 (D) not defined

2. The differential equation

$$y=x\frac{dy}{dx}+\frac{1}{dy/dx} \text{ is of}$$

(A) order 2 and degree 1

(B) order 1 and degree 2

(C) order 1 and degree 1

(D) order 2 and degree 2

3. The order of the differential equation

$$\left(\frac{d^2s}{dt^2}\right)^2+3\left(\frac{ds}{dt}\right)^3+4=0 \text{ is}$$

(A) 1 (B) 2 (C) 3 (D) 4

4. The order of the differential equation.

$$\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^3} \quad \text{is}$$

(A) 2 (B) 1 (C) 3 (D) None of these

5. The solution of $\frac{d^2y}{dx^2}=0$ presents :

(A) a st. line (B) a circle

(C) a parabola (D) a point

6. The integrating factor of the differential equation.

$$x\frac{dy}{dx}-y=2x^2 \quad \text{is.}$$

(A) e^{-x} (B) e^{-y} (C) $\frac{1}{x}$ (D) x

7. The order of the differential equation :

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}=\frac{d^3y}{dx^3} \quad \text{is}$$

(A) 1 (B) 2 (C) 3 (D) 4

8. The order and degree of diff. equ.

$$\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3} \quad \text{are}$$

(A) $\left(1,\frac{2}{3}\right)$ (B) (3,1) (C) (3,3) (D) (1,2)

9. The solution of the differential equ.

$$ydx+\left(x+x^2y\right)dx=0 \text{ is}$$

(A) $\frac{1}{xy}+\log y=c$ (B) $-\frac{1}{xy}+\log y=c$

(C) $-\frac{1}{xy}=c$ (D) $\log\ y = cx$

10. A solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2-x\left(\frac{dy}{dx}\right)+y=0 \text{ is}$$

(A) $y = 2$ (B) $y = 2x$

(C) $y = 2x-4$ (D) $y=2x^2-4$

11. If $y^1=\frac{x-y}{x+y}$ then, its solution is :

(A) $y^2+2xy-x^2=c$ (B) $y^2+2xy+x^2=c$

(C) $y^2-2xy-x^2=c$ (D) $y^2-2xy+x^2=c$

12. Solution of $\frac{dy}{dx}+2xy = y$ is

(A) $y=ce^{x-x^2}$ (B) $y=ce^{x^2-x}$

(C) $y=ce^x$ (D) None of the above.

13. The general solution of $\frac{dy}{dx} + 2x = e^x$ is

(A) $y = \frac{1}{3}e^x + ce^{-2x}$ (B) $y = e^x + x^2 + c$

(C) $y = -e^x + x^2 + c$ (D) $y = e^x + c$

14. The solution of the differential equation.

$$3e^x \tan y \, dx + (1 - e^x)\sec^2 y \, dy = 0 \text{ is}$$

(A) $\cot y = c(1 - e^x)^3$ (B) $\tan y = c(1 - e^x)^3$

(C) $\tan y = c(1 + e^x)^3$ (D) None of these

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1.

i) Find the differential equation of the family of curves $y = e^x$ (A cos x + B sin x), where A and B are arbitrary constants.

(ii) Find the differential equation of an ellipse with major and minor axes 2a and 2b respectively.

(iii) Form the differential equation corresponding to the family of curves $y = c(x - c)^2$.

(iv) Form the differential equation representing the family of curves $(y - b)^2 = 4(x - a)$.

2 Solve the following diff. equations

(i) $\frac{dy}{dx} + y \cot x = \sin 2x$

(ii) $y - x\frac{dy}{dx} = \left(y^2 + \frac{dy}{dx}\right)$. (iii) $\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0$.

(iv) $x\sqrt{1 - y^2}\,dy + y\sqrt{1 - x^2}\,dx = 0$. (v) $\sqrt{(1 - x^2)(1 - y^2)}\,dy + xy\,dx = 0$.

(vi) $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$; $y(0) = 1$.

(vii) $\frac{dy}{dx} = y \sin^3 x \cos^3 x + xy e^x$.

(viii) $\tan x \tan y \, dx + \sec^2 x \sec^2 y \, dy = 0$

3. Solve the following differential equations :

(i) $x^2 y \, dx - (x^3 + y^3) \, dy = 0$.

(ii) $x^2 \frac{dy}{dx} = x^2 + xy + y^2$.

(iii) $\left(x^2 - y^2\right) dx + 2xy \, dy = 0, \quad y(1) = 1$.

(iv) $\left(y \sin \frac{x}{y}\right) dx = \left(x \sin \frac{x}{y} - y\right) dy$

(v) $(x^2+y^2) \, dy = 2xy \, dx$

(vi) $\left(x^3 + y^3\right) dx = \left(x^2 y + xy^2\right) dy$

4. (i) Form the differential equation of the family of circles touching y-axis at (0, 0).

(ii) Form the differential equation of family of parabolas having vertex at (0, 0) and axis along the (i) positive y-axis (ii) positive x-axis.

(iii) Form differential equation of family of circles passing through origin and whose centre lie on x-axis.

5. Show that the differential equation $\frac{dy}{dx} = \frac{x + 2y}{x - 2y}$ is homogeneous and solve it.

6. Show that the differential equation :

$(x^2 + 2xy - y^2) \, dx + (y^2 + 2xy - x^2) \, dy = 0$ is homogeneous and solve it.

7. Solve the following differential equations :

(i) $\frac{dy}{dx} - 2y = \cos 3x$.

(ii) $\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$ if $y\left(\frac{\pi}{2}\right) = 1$

8. Solve the following differential equations :

(i) $(x^3 + y^3) \, dx = (x^2y + xy^2) dy$.

(ii) $x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx$.

(iii) $y \left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} dx - x \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} dy = 0$.

(iv) $x^2 dy + y(x + y) \, dx = 0$ given that $y = 1$ when $x = 1$.

(v) $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$ if $y(e) = 0$

(vi) $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)dy$.

9. Solving the following differential equation

(i) $\cos^2 \frac{dy}{dx} = \tan x - y$.

(ii) $x \cos x \frac{dy}{dx} + y\,(x \sin x + \cos x) = 1$.

(iii) $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$.

(iv) $(y - \sin x)\,dx + \tan x\,dy = 0,\ y(0) = 0$.

(v) $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.

(vi) $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ given that $y(0) = 0$.

CHAPTER – 10
VECTORS

POINTS TO REMEMBER

- A quantity that has magnitude as well as direction is called a vector. It is denoted by a directed line segment.
- Two or more vectors which are parallel to same line are called collinear vectors.
- Position vector of a pt. P(a,b,c) w.r.t. origin (0,0,0) is denoted by $\overrightarrow{OP}$, where $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$ and $|\overrightarrow{OP}| = \sqrt{a^2 +^2 + c^2}$
- If $A(x_1, y_1, z_1)$ and (x_2, y_2, z_2) be any two points in space, then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ and $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- If two vectors $\vec{a}$and $\vec{b}$ are represented in magnitude and direction by the two sides of a triangle taken in order, than their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by third side of triangle taken in opposite order. This is called triangle law of addition of vectors.
- If $\vec{a}$ is any vector and λ is a scalar, than $\lambda\vec{a}$ is a vector collinear with $\vec{a}$and $|\lambda\vec{a}| = |\lambda||\vec{a}|$
- If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then $\vec{a} = \lambda\vec{b}$ where λ is some scalar.
- Any vector $\vec{a}$ can be written as $\vec{a} = |\vec{a}|\,\hat{a}$, where $\hat{a}$ is a unit vector in the direction of $\vec{a}$.
- If $\vec{a}$ and $\vec{b}$ be the position vectors of points A and B, and C is any point which divides $\overrightarrow{AB}$ in ratio m:n internally, then position vector $\vec{C}$ of point C is given as $\vec{C} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$. If C divides $\overrightarrow{AB}$ in m:n externally, then $\vec{C} = \dfrac{m\vec{b} - n\vec{a}}{m-n}$.
- The angles α, β and γ made by $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ with positive direction of *x, y* and z - axis are called direction angles and cosines of these angles are called direction cosines of $\vec{r}$ usually denoted as $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$.

  Also, $l = \dfrac{a}{|\vec{r}|}, m = \dfrac{b}{|\vec{r}|}, n = \dfrac{c}{|\vec{r}|}$ and $l^2 + m^2 + n^2 = 1$

- The numbers a, b, c proportional to l, m, n are called direction ratios.
- Scalar Product of two vector $\vec{a}$ and $\vec{b}$ is directed as $\vec{a}.\vec{b}$ and defined as $\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ ($0 \le \theta \le \pi$)
- Dot product of two vectors is commutative i.e. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$
- $\vec{a}.\vec{b} = \vec{0} \Leftrightarrow \vec{a} = \vec{0}, \vec{b} = \vec{0}$ or $\vec{a} \perp \vec{b}$
- $\vec{a}.\vec{a} = |\vec{a}|^2$ so $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ than $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + c_3b_3$
- Projection of $\vec{a}$ On $\vec{b}$ $= \left|\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right|$
- Projection vector of $\vec{a}$ along $\vec{b}$ $= \left|\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right|.\vec{b}$
- Cross product/vector product of two vector $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a}$ x $\vec{b}$ and is defined as $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where θ is the angle between $\vec{a}$ and $\vec{b}$ $(0 \le \theta \le n)$ and $\hat{n}$ is a unit vector perpendicular to both $\hat{a}$ and $\hat{b}$ such that $\vec{a}.\vec{b}$ and $\vec{n}$ form a right handed system.
- Cross product of two vector is not commutative i.e. $\vec{a}\times\vec{b} \ne \vec{b}\times\vec{a}$, but $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
- $\vec{a}\times\vec{b} = \vec{0} \Rightarrow \vec{a} = \vec{0},\ \vec{b} = \vec{0}\ or\ \vec{a} \parallel \vec{b}$
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$
- $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$ and $\hat{j}\times\hat{i} = -\hat{k}$

  $\hat{k}\times\hat{j} = -\hat{i}, \hat{i}\times\hat{k} = -\hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k},$ then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \pm\left(\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$$

- $|\vec{a}\times\vec{b}|$ is area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

- $\frac{1}{2}|\vec{a}\times\vec{b}|$ is the area of parallelogram whose adjacent diagonals are $\vec{a}$ and $\vec{b}$

- If $\vec{a},\vec{b}$ and $\vec{c}$ forms a triangle, then area of the triangle

$$=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{1}{2}|\vec{b}\times\vec{c}|=\frac{1}{2}|\vec{c}\times\vec{a}|$$

VERY SHOT ANSWER TYPE QUESTIONS (1 Mark)

1. If θ is the angle between two vectors $\vec{a},\vec{b}$ then $\vec{a},\vec{b}\geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$ (B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$ (D) $0 \leq \theta \leq \pi$

2. The vector $2\hat{i}+\hat{j}-\hat{k}$ and $4\hat{i}+2\hat{j}+10\hat{k}$ are

(A) at angle of $\frac{\pi}{3}$ (B) of equal magnitude

(C) Parallel (D) orthogonal

3. The projection of the vector $\hat{i}-2\hat{j}+\hat{k}$ on the vector $4\hat{i}-4\hat{j}+7\hat{k}$ is

(A) $\frac{5\sqrt{5}}{19}$ (B) $2\frac{1}{9}$ (C) $\frac{9}{19}$ (D) $\frac{\sqrt{6}}{19}$

4. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(A) $\vec{b} = \lambda\vec{a}$ for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) The respective components of $\vec{a}$ and $\vec{b}$ are proportional

(D) Both the vector $\vec{a}$ and $\vec{b}$ have the same direction, but different magnitude.

5. If $\hat{i}, \hat{j}, \hat{k}$ have the usual meaning in vectors, then $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i}$ is

(A) -1 (B) 0 (C) 1 (D) None of these.

6. The unit vector perpendicular to the vector $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ are

(A) $\hat{i} + \hat{j} + \hat{k}$

(B) $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

(C) $\hat{i} - \hat{j} + \hat{k}$

(D) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

7. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then $\vec{a}$ is equal to

(A) a non zero vector (B) 1

(C) - 1 (D) $|\vec{a}||\vec{b}||\vec{c}|$

8. The vector $\vec{a} = 3\hat{i} - \hat{k}, \vec{b} = \hat{i} + 2\hat{j}$ are adjacent sides of a parallelogram. Its area is

(A) $\frac{1}{2}\sqrt{17}$ (B) $\frac{1}{2}\sqrt{14}$

(C) $\sqrt{41}$ (D) $\frac{1}{2}\sqrt{17}$

9. The vector $2\hat{i}+\hat{j}-\hat{k}$ is perpendicular $\hat{i}-4\hat{j}-\lambda\hat{k}$ iff λ equals.

(A) 0 (B) -1 (C) 2 (D) -3

10. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then the vectors $\vec{a}$ and $\vec{b}$ are

(A) parallel (b) perpendicular

(c) inclined at angle $\frac{\pi}{4}$ (d) inclined at an angle $\frac{\pi}{6}$

11. The quantity $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is :

(A) not defined (b) vector

(c) scalar (d) nature depends upon $\vec{a},\vec{b},\vec{c},\vec{d}$

12. If $|\vec{a}|=|\vec{b}|$, then $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ is

(A) zero (b) negative

(c) positive (d) none of these

13. If $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(A) perpendicular (b) like parallel

(c) unlike parallel (d) coincident

14. The area of the triangle whose adjacent sides are :

$\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ is

(A) $\sqrt{42}$ (B) $\sqrt{\frac{42}{2}}$

(C) $\frac{\sqrt{42}}{2}$ (D) $\frac{2}{\sqrt{42}}$

15. The vectors $3\hat{i}+4\hat{j}-6\hat{k}$ and $-6\hat{i}+8\hat{j}+12\hat{k}$ are

(A) equal (B) of same magnitude

(C) parallel (D) mutually perpendicular

16. The work done is moving an object along a vector $\vec{d}=3\hat{i}+2\hat{j}-5\hat{k}$, if the applied force is $\vec{F}=2\hat{i}-\hat{j}-\hat{k}$,is

(A) 12 units (B) 11 units

(C) 10 units (D) 9 units

SHORT ANSWER TYPE QUESTION (3 Marks)

1. If ABCDEF is a regular hexagon, then using triangle law of addition, prove that

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=3\overrightarrow{AD}=6\overrightarrow{AO}$$

2. The scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with unit vector along the sum of the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1. Find the value of λ.

3. $\vec{a},\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude. Show that $\vec{a}+\vec{b}+\vec{c}$ makes equal angles with $\vec{a},\vec{b}$ and $\vec{c}$ with each angle as $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

4. If $\vec{\alpha}=3\vec{i}-\vec{j}$ and $\vec{\beta}=2\vec{i}+\vec{j}+3\vec{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{B}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

5. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

6. If $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the angle between $\vec{a}$ and $\vec{b}$.

7. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{c}=\hat{j}-\hat{k}$ are the given vectors then, find a vector $\vec{b}$ satisfying the equation $\vec{a}\times b=\vec{c}$ and $\vec{a}.\vec{b}=3$.

8. For any two vector, $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$

9. For any two vector, $|\vec{a}\times\vec{b}|=\sqrt{a^2b^2-\left(\vec{a}.\vec{b}\right)^2}$

10. Prove that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

11. Let $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = \vec{0}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that $\vec{a} = \pm 2\left(\vec{b}\times\vec{c}\right)$.

12. Prove that the normal vector to the plane containing three points with position vectors $\vec{a},\vec{b}$ and $\vec{c}$ lies in the direction of vector $\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}$.

13. If $\vec{a},\vec{b},\vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, than show the area of ΔABC is $\frac{1}{2}\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right|$

14. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}, \vec{b} = \hat{i} - \hat{j} - \lambda\hat{k},$ find λ such that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.

15. Let $\vec{a}$ and $\vec{b}$ be vectors such that $|\vec{a}| = |\vec{b}| = |\vec{a}-\vec{b}| = 1$, find $|\vec{a}+\vec{b}|$

16. If $|\vec{a}| = 2,\ |\vec{b}| = 5,\ \vec{a}\times\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k}$ find the value of $\vec{a}.\vec{b}$

CHAPTER 11

THREE DIMENSIONAL GEOMETRY

POINTS TO REMEMBER

- Distance between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

  $$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- (i) The coordinates of point R which divided lines segment PQ where $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio $m : n$ internally are

  $$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

  (ii) The coordinates of a point which divides join of (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio of $m : n$ externally are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$

- Direction ratios of a line (x_1, y_1, z_1) and (x_2, y_2, z_2) are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

- Direction cosines of a line whose direction ratios are a,b,c are given by

  $$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c_2}}, m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

- (i) Vector equation of a line through point $\vec{a}$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$

  (ii) Cartesian equation of a line through point (x_1, y_1, z_1) and having direction ratios proportional to a,b,c is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

- (i) Vector equation of line through two points $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

  (ii) Cartesian equation of a line through two points (x_1, y_1, z_1) and (x_2, y_1, z_2) is

  $$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

- Angle 'θ' between lines $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r}_2 = \vec{a}_2 + \mu\vec{b}_2$ is given by $\cos^{-1}\left(\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right)$

- Angle 'θ' between lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_1}{b_2} = \frac{z-z_1}{c_2}$ is given by $\cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$

- Two lines are perpendicular to each other then, $\vec{b}_1.\vec{b}_2 = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$

- Equation of plane

  (i) At a distance of p unit from origin and perpendicular to $\hat{n}$ is $\vec{r}.\hat{n} = p$ and corresponding Cartesian form is $\ln + my + nz = p$, where, l, m and n are d.c.'s of normal to plane.

  (ii) Passing through $\vec{a}$ and normal to $\vec{n}$ is $\vec{n}(\vec{r} - \vec{a}) = 0$ and corresponding Cartesian form is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where a,b,c are d.r's of normal to Plane and Pt. (x_1, y_1, z_1) lies on the plane.

  (iii) Passing through three non-collinear points is

  $$(\vec{r}-\vec{a}).\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0$$

  or $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

  (iv) Having intercept a,b and c on co-ordinate axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

  (v) Plane passing through the line of intersection of planes $\vec{r}.\vec{n}_1 = \vec{d}_1$ and $\vec{r}.\vec{n}_2 = \vec{d}_1$ is $(\vec{r}.\vec{n}_1 - d_1) + \lambda(\vec{r}.\vec{n}_2 - d_2) = 0$

- (i) Angle 'θ' between planes $\vec{r}.\vec{n}_1 = \vec{d}_1$ and $\vec{r}.\vec{n}_2 = d_2$ is given by $\left(\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right)$

(ii) Angle θ between $a_1x+b_1y+c_1z=d_1$ and $a_2x+b_2y+c_2z=d_2$ is given by

$$\cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$

(iii) The planes are perpendicular to each other iff $\vec{n}_1.\vec{n}_2 = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(iv) Two planes are parallel iff $\vec{n}_1 = \vec{n}_2$

or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- (i) Distance of a point from plane

$$\vec{r}.\vec{n} = d \ \ is \ \ \frac{|\vec{a}.\vec{n} - d|}{|\vec{n}|}$$

(ii) Distance of a point (x_1, y_1, z_1) from plane

$$ax + by + cz = d \text{ is } \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$$

- (i) Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ *and* $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar iff $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$ and equation of plane containing these lines is

$$(\vec{r} - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$$

(ii) Two lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$

or co-planar *iff* $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ and

Equ. of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

- (i) The angle θ between line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} = d$ is given as

$$\text{Sin}\,\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}$$

(ii) The angle θ between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z=d$ is given by $\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

(iii) A line is parallel to plane $\Leftrightarrow$ $\vec{b}.\vec{n}=0$ or $a_1a_2+b_1b_2+c_1c_2=0$

LESSON NO.11

THREE DIMENSIONAL GEOMETRY

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1. If the direction cosines of a line are <k,k,k> then

(A) k>0 (B) 0<k<1 (C) k=1 (D) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

2. Distance of the point (α,β,γ) from the XOY-plane is

(A) γ (B) $|\gamma|$ (C) $\sqrt{\alpha^2+\beta^2}$ (D) None of these.

3. The distance of the plane $\vec{r}.\left(\frac{2}{3}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is

(A) 1 (B) 7 (C) $\frac{1}{7}$ (D) none of these

4. The lines $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-3}{0}$ and $\frac{x-2}{0}=\frac{y-3}{0}=\frac{z-4}{1}$ are

(A) parallel (B) skew

(C) coincident (D) perpendicular

5. The distance between the planes :

$3x+2y-6z-14=0$ and $3x+2y-6z+21=0$ is

(A) 35 (B) 7

(C) 1 (D) 5

6. The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

(A) at right angles to *x*-axis.

(B) at right angles to the plane YOZ

(C) is parallel to y-axis

(D) none of these.

7. The points (0,0,0), (2,0,0), $\left(1,\sqrt{3},0\right)$ and $\left(1,\frac{1}{\sqrt{3}},\frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a

(A) square (B) rhombus

(C) rectangle (D) regular tetrahedron

8. The plane $x-2y+z-6=0$ and the $x-2y+z-6=0$ are related as

(a) parallel to the line

(b) at right angles to the plane.

(c) lines in the plane

(D) meets the plane obliquely.

9. The plane containing the point (3,2,0) and the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$ is

(A) $x-y+z=1$ (B) $x+y+z=5$

(C) $x+2y-z=1$ (D) $2x-y+z=5$

10. The line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and the plane $x-2y+z=0$ are related as the line

(A) meets the plane in a unique point

(B) lines in the plane

(C) meets the plane at right angles

(D) is parallel to the plane.

11. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+2=5$ is

(A) $\frac{10}{6\sqrt{5}}$ (B) $\frac{4}{5\sqrt{2}}$

(C) $\frac{2\sqrt{3}}{5}$ (D) $\frac{\sqrt{2}}{10}$

12. The reflection of the point (α,β,γ) in the XOY plane is

(A) (α,β,o) (B) (o,o,γ)

(C) $(-\alpha,-\beta,\gamma)$ (D) $(\alpha,\beta,-\gamma)$

13. The projection of the point (1,2,-4) in the YOZ plane is.

(A) (0,2,-4) (B) (1,0,0)

(C) (-1,2,-4) (D) (1,2,4)

LONG ANSWER TYPE QUESTIONS (5 Marks)

1) Find shortest distance between the lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

2) Find shortest distance between the lines

$$\vec{r}=(1-\lambda)\hat{i}+(\lambda-2)\hat{j}+(3-2\lambda)\hat{k}$$

and $\vec{r}=(\mu+1)\hat{i}+(2\mu-1)\hat{j}+(2\mu+1)\hat{k}$

3) A variable plane is at a constant distance 3p from the origin and meet the co-ordinate axes in A, B and C respectively. Show that the locus of centroid of ΔABC is

$$x^{-2}+y^{-2}+z^{-2}=p^{-2}$$

4) Find the foot of perpendicular from the point $2\hat{i}-\hat{j}+5\hat{k}$ on the line

$\vec{r}=\left(11\hat{i}-2\hat{j}-8\hat{k}\right)+\lambda\left(10\hat{i}-4\hat{j}-11\hat{k}\right)$. Also, find the length of perpendicular.

5) A line makes angle $\alpha,\beta,\gamma,\delta$ with a four diagonals of a cube. Prove that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$$

6) Find the point of intersection of the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

Also, find the equation of the Plane in which they lie.

7) Find the equ. of the plane passing through the intersection of planes $2x+3y-z=-1$ and $x+y-2z+3=0$ and perpendicular to the plane $3x-y-2z=4$. Also, find the inclination of this plane with *XY*–plane.

8) Prove that the image of the point (3,-2,1) in the plane $3x-y+4z=2$ lies in the plane $x+y+z+4=0$.

9) Find the equations of the two lines through the origin such that each line is intersecting the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

10) Find the equ. of plane containing the parallel lines

$$\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5} \text{ and } \frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$$

11) prove that if a plane has the intercept a,b and c and is at a distance of *p* units from the origin, then

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$$

12) Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

CHAPTER 12

LINEAR PROGRAMMING

POINTS TO REMEMBER

- Linear programming is the process used to obtain minimum or maximum value of the linear objective function under known linear constraints.
- Objective Functions : Linear function $z = ax + by$ where a and b are constants, which has to be maximized or minimized is called a linear objective function.
- Constraints : The linear inequalities or inequations or restrictions on the variables of a linear programming problem.
- Feasible Region : It is defined as a set of points which satisfy all the constraints including non-negative constraints $x \geq 0$, $y \geq 0$.
- To Find Feasible Region : Draw the graph of all the linear inequations and shade common region determined by all the constraints.
- Feasible Solutions : Points within and on the boundary of the feasible region represents feasible solutions of the constraints.
- Optimal Feasible Solution : Feasible solution which optimizes the objective function is called optimal feasible solution.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Solve the following L.P.P. graphically

   Minimise and maximise $z = 3x + 9y$

   Subject to the constraints
   $$x + 3y \leq 60$$
   $$x + y \geq 10$$
   $$x \leq y$$
   $$x \geq 0,\ y \geq 0$$

2. Determine graphically the minimum value of the objective function $z = -50x + 20y$

   Subject to the constraints
   $$2x - y \geq -5$$
   $$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0,\ y \geq 0$$

3. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stich 10 shirts and 4 pants per day. Formulate the above L.P.P. mathematically and hence solve it to minimise the labour cost to produce at least 60 shirts and 32 pants.

4. There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 61 kg and B costs Rs. 51 kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost.

5. A man has Rs. 1500 to purchase two types of shares of two different companies S_1 and S_2. Market price of one share of S_1 is Rs 180 and S_2 is Rs. 120. He wishes to purchase a maximum to ten shares only. If one share of type S_1 gives a yield of Rs. 11 and of type S_2 Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

6. A company manufacture two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finishers has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?

7. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for almost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?

8. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as L.P.P. and then solve it graphically.

9. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and one unit of capital is required. If X and Y are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

10. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows :

Machine	Area Occupied	Labour Force	Daily Output (In units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output.

11. A manufacturer makes two types of cups A and B. There machines are required to manufacture the cups and the time in minute required by each in as given below :

Types of Cup	Machine		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise and on B is 50 paise, find how many cups of each type should be manufactured to maximise the profit per day.

12. A company produces two types of belts A and B. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type A requires twice as much time as belt of type B. The company can produce almost 1000 belts of type B per day. Material for 800 belts per day is available. Almost 400 buckles for belts of type A and 700 for type B are available per day. How much belts of each type should the company produce so as to maximize the profit?

13. To Godowns X and Y have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shop A, B and C whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintals from the godowns to the shops are given in following table :

To / From	Cost of transportation (in Rs. per quintal	
	X	Y
A	6.00	4.00
B	3.00	2.00
C	2.50	3.00

How should the supplies be transported to minimize the transportation cost?

14. An Aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 in made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However atleast four times as many passengers prefer to travel by second class than by first class. Determine, how many tickets of each type must be sold to maximize profit for the airline.

15. A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food A contains 200 unit of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost but it must satisfy the requirements of the sick person.

CHAPTER 13

PROBABILITY

POINTS TO REMEMBER

- Baye's Theorem

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i) \cdot P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)}$$

  Where E_1, E_2 ...., E_n are events which constitute partitions of sample space S. i.e. E_1, E_2, ..., E_n are pairwise disjoint and $E_1 \cup E_2 \cup$ .... $\cup\ E_n$ = S to and let A be any event with non zero probability.

- Probability Distribution : Probability distribution of a random variable x is the system of numbers.

  $x : x_1, x_2$ .... x_n

  $P(x) : p_1 \ p_2$ .... p_n

Where $p_i \geq 0$ and $P_1 + P_2 + P_3 .... P_n = 1$

- Mean : The mean of x denoted by μ in given by $\mu = \sum_{i=1}^{n} x_i p_i$

  The mean of a random variable x is also called the expectation of X and is denoted by E(X). Mean of $\overline{x} = \mu = E(X)$.

- Variance : Variance of X denoted by var(X) or σ_x^2 is defined by

  $$\sigma_x^2 = \sum x_i^2 p_i - \mu^2$$

For Binomial distribution B (n, p);

$$P(x = r) = {}^{n}C_{r}\, p^{r} q^{n-r} \quad r = 0, 1, 2, \ldots\ldots\ldots\ldots. n \text{ where } q = 1 - p.$$

- Conditional Probability

  1. The conditional probability of an event E, given the occurrence of the event F is given by

  $$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}, \; P(F) \neq 0.$$

  2. $0 \leq P(E/F) \leq 1$
  3. $P(E \cap F) = P(E) \,.\, P(F/E), \; P(E) \neq 0$
  4. $P(E \cap F) = P(F) \,.\, P(E/F), \; P(F) \neq 0$
  5. $P(E'/F) = 1 - P(E/F)$
  6. $P[(E \cup F/G) = P(E/G) + P(F/G) - P((E \cap F)/G)$
  7. If E and F are independent events then $P(E \cap F) = P(E) \,.\, P(F)$.

  (I) $P(E/F) = P(E), \; P(F), \neq 0$

  (II) $P(F/E) = P(F), \; P(E) \neq 0$

- Theorem of Total Probability : Let $[E_1\ E_2, \ldots\ldots\ldots\ldots E_n]$ be a partition of a sample space and suppose that each of $E_1, E_2, \ldots\ldots\ldots\ldots\ldots E_n$ has non zero probability.

  Let A be any event associated with S then

  $P(A) = P(E_1) \,.\, P(A/E_1) + P(E_2) \,.\, P(A/E_2) \ldots\ldots\ldots\ldots + P(E_n)\ P(A/E_n)$

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. A problem is Mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved.
2. A die is rolled. If the outcome is an even number. What is the probability that it is a prime?

3. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find P (not A and not B).

4. In a class of 25 students with roll numbers 1 to 25. A student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 5 or 7.

5. A car hit a target 4 times in 5 shots B three times in 4 shots, C twice in 3 shots. They fire a volley. What is the probability that two shots at least hit.

6. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.

7. A and B throw a die alternatively till one of them throws a '6' and win the game. Find their respective probabilities of winning if A starts first.

8 Given that the events A and B are such that $P(A)=\frac{1}{2}, P(A \cup B)=\frac{3}{5}$

and $P(B) = p$, find p if

(i) they are mutually exclusive, (ii) they are independent events .

9. A drunkard man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

10. Two cards are drawn from a pack of well shuffled 52 cards. Getting an ace or a spade is considered a success. Find the probability distribution for the number of success.

11. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he win/looses.

12. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are 60% males and 40% females?

13. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds?

14. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least are defective egg.

15. Find the variance of the number obtained on a throw of an unbiased die.

16. In a hurdle race, a player has to cross 8 hurdles. The probability that he will clear each hurdle is $\frac{4}{5}$ whats the probability that he will knock down fever than 2 hurdles.

17 Bag A contain 4 red and 2 black balls. Bag B contain 3 red and 3 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red find teh probability that the transferred ball is black.

18 If a fair coin is tossed 10 times find the probability of getting.

(i) exactly six heads,

(ii) at least six heads,

.19 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of missing card to be heart.

20 A box X contain 2 white and 3 red balls and a bag Y contain 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from bag Y.

21. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ he the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, gives that he answered correctly.

22. Two urns A and B contain 6 black and 4 white and 4 black and 6 white balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white, find the probability that the balls are drawn from urn B.

23. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

24. Write the probability distribution for the number of heads obtained when there coins are tossed together. Also, find the mean and variance of the probability distribution.