Récoltes et Semailles

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1986

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Presentation of the Themes or PRELUDE IN FOUR PARTS

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By way of a foreword

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All that was left to write was the foreword, in order for Récoltes et Semailles to be given to the publisher. And I swear that I went into it with all the will in the world to write something suitable. Something **reasonable** this time. No more than three or four pages, carefully phrased, that would introduce this huge tome of more than one thousand pages. Something which "grabs" the attention of the jaded reader, which perhaps suggests that in these frightening "more than a thousand pages", there could be things of interest to him (or things which concern him, who knows?). It is not really my style to pander. But I was ready to make an exception for once! The "publisher crazy enough to give it a shot" (to publish this visibly unpublishable monster) had to make ends meet one way or another.

But then, it didn't come. And yet I tried my best. And not only for an afternoon, as I originally planned. Tomorrow will mark three weeks since I started, since the sheets began accumulating. What came, for sure, isn't what one could decently call a "foreword". It is yet another miss! Blame it on my old age - I have never been a salesman. Even when it comes to pleasing (oneself or friends. . .).

What came instead is a sort of long "Walk" with commentary, through my work as a mathematician. A Walk intended mostly for the "layman" - he who "never understood anything about math". And for myself as well, having never indulged in such a Walk. Step by step, I found myself unearthing and saying things that had previously remained unspoken. As if by chance, these are also things which I feel are most essential, both in my practice and its outcome. They are things which are not technical in nature. It will be up to you to decide whether or not I succeeded in my naive enterprise to "get the message through" - an exterprise which surely is also a bit mad. My satisfaction and my pleasure will come from making you feel these things. Things that many of my wise colleagues do not feel anymore. Maybe they have become too wise and too prestigious. This often leads to losing touch with the simplest and most essential things.

During this "Walk through a body of work" I also speak of my life. As well as, here and there, what Récoltes et Semailles is about. I mention this
p. A2 in more detail in the "Letter" (dated from May of last year) which follows the "Walk". This Letter was directed towards my previous students and to my "old friends" in the mathematical community. But even the Letter is not technical in nature. It can be read by any reader interested in learning through a "heartfelt" narrative, the odds and ends that led me to writing Récoltes et Semailles. Even more than the Walk, the Letter will provide a preview to a particular atmosphere in the "prestigious mathematical world". And also (just as in the Walk) of my writing style, as peculiar as it may seem, and of the spirit that is expressed through this style - a spirit which is not universally appreciated.

In the Walk and throughout Récoltes et Semailles, I speak of the activity of **doing mathematics**. It is an activity for which I have first-hand experience and know very well. Most of the things I say anbout it can surely be said of any kind of creative work, or work involving discovery. In any case it is true of all "intellectual" work, that which is done using the "brain" and in writing. All such work proceeds through the the outbreak and development of an **understanding** of the things which are being probed. But to take an example at the opposite extreme, romantic passion is also an activity of discovery. It opens us to understanding of a "physical" nature which also renews itself, develops, and deepends over time. Both of these impulses - that which, say, livens the mathematician at work and that of the lover - are much close in nature than we generally assume or we readily admit. I hope that the pages of Récoltes et Semailles will make you feel this impulse in your work and in your daily life.

Most of the Walk focuses on mathematical work itself. I remain mostly silent concerning the **context** in which this work takes place, and concerning the **motivations** at play outside of mathematical work itself. This risks giving me, or the mathematician, or the "scientist" in general a flattering but deformed image. In the style of "grand and noble passion" without any form of rectification. In accordance with the great "Myth" of Science (with a capital S, if you will!). The heroic myth, "promethean", to which writers and thinkers have succumbed (and continue to succumb). Only historians, maybe, manage to sometimes resist this tantalizing myth. The truth is, within the motivation of these "scientists", which sometimes lead them to devote themselves entirely to their work, ambition and vanity play a role just as important and universal
p. A3 as they do in any other profession. This phenomenon appears in blunt or subtle ways depending on the person - and I am no exception to this pattern. The reading of my testimony will hopefully leave no doubt about this fact.

It is true also that even the most intense ambitions are powerless at discovering or proving a novel mathematical statement - just as they are powerless (for instance) to "make one hard" (in the proper sense of the term). Whether man or woman, what "makes one hard" is not ambition, nor the desire to shine, to exhibit power, of a sexual nature in this case - quite the contrary! It is the acute perception of something strong, at once very real and very delicate. One could call it "beauty", thought this is one of a thousand faces of this thing. Being ambitious doesn't prevent one from sensing the beauty of a being or a thing. But it is **not** ambition which makes us feel it...

The person that first discovered and mastered fire was somebody just like you and me. Not at all what we refer to as “hero”, or “demi-god”, and so on. Surely, just like you and me, he has encountered the grip of anxiety as well as the time-worn remedy of vanity which alleviates the grip. At the instant at which he “knew” fire, there was no fear nor vanity. Such is the truth in the heroic myth. The myth becomes insipid when it is used to to disguise another aspect of things which is just as real and essential.

My aim in Récoltes et Semailles has been to address both aspects of the myth - that of the impulse towards understanding, and that of fear and its vain antidotes. I believe I “understand”, or at least **know** this impulse and its origin (or perhaps one day I will discover to what extent I was deluded). But concerning fear and vanity, as well as the resulting insidious creativity blocks, I know thta I have yet to thoroughly uncover this great enigma. And who knows if I will ever reach the conclusion of this mystery in the year I have left...

As I was writing Récoltes et Semailles two images emerged in order to represent the two aspects of the human journey: that of the **child** (aka the **worker**), and that of the **boss**. In the Work which we are about to undertake, we will be dealing mostly with the “child”. It is him also that is featured in the subtitle “**The Child and the Mother**”. The motivation for this name will hopefully become clear over the course of this work. p. A4

In the remainder of this reflection, however, it is the Boss who takes the lead. He is living up to his name! It would be more accurate to speak of multiple bosses of competing enterprises rather than of a **singular** boss. But it is also true that all bosses essentially resemble one another. And once we mention bosses it is implied that we will also have to deal with “villains”. In part I of the reflection (named “Fatuity and Renewal”, which follows the present introductory section). I mostly take on the role of the “villain”. In the following three parts it is mostly the “others”. Chacun son Tour! That is to say that, in addition to philosophical reflections and “confessions” (not contrite), there will be “vitriolic portraits” (to use the expression of one of my colleagues who found himself tormented). Not to mention large-scale, well-oiled “operations”. Robert Jaulin[1] assured me (half jokingly) that in Récoltes et Semailles I was making the “ethnology of the mathematical community” (or maybe the sociology I do not quite remember). It is flattering of course to learn that one has been (unknowingly) doing scholarly things! It is true that during the “investigation” segment of the reflection, I saw in passing, in the pages I was writing, a good chunk of the mathematical establishment without counting a number of my colleagues and friends of more modest status. Over the past few months, since I have been sending preliminary versions of Récoltes et Semailles this has been “brought up” again. My testimony arrived like a tome landing in a pond. There were responses of every kind (except for boredom...). Yet almost every time the response was far from what I expected. There was also a lot of silence,

[1]Rober Jaulin is an old friend of mine. From what I understand, his position with respect to the establishment of the ethnological milieu mirrors mine with respect to the “high society” of mathematics (as white wolves).

which speaks volumes. Visibly, I had (and still have) a lot to learn about what happens in people's minds, among my previous students and other colleagues - excuse me I meant about "the sociology of the mathematical milieu"! To all those that contributed to the great sociological work of my old days, I would like to express sincere recognition.

Of course, I was particularly sensitive to warm responses. There were also some rare colleagues who conveyed a sentiment (thus far unexpressed) of crisis,
p. A5 or of degradation of the inner workings of the mathematical milieu with which they identify themselves.

Outside of this milieu, among the very first to respond positively to my testimony, I would like to recognize Sylvie et Catherine Chevalley[2] Robert Jaulin, Stéphane Deligeorge, Christian Bourgois. If Récoltes et Semailles achieves a wider diffusion than that of the initial printing (addressed to a very limited social circle of people), it is mostly thanks to them. Thanks mostly to them communicating their conviction that what I strived to seize and say had to be said. And that it could have an audience outside of my colleagues (who are often sullen, sometimes even belligerent, and strictly opposed to question their position. . .). Indeed Christian Bourgois did not hesitate to risk publishing the unpublishable, and Stéphane Deligeorge did not hesitate to place my indigestible testimony alongside works of Newton, Cuivier, and Arago (I could not ask for better company). To each of them, for their repeated expressions of sympathy and trust, intervening at an especially sensitive moment, I happily extend all my gratitude.

And here we are at the beginning of a Walk through a life's work, serving as a prelude to a journey through a lifetime. A long journey, over a thousand pages long, each of which is densely packed. I spent a lifetime undergoing this journey without ever exhausting it, and it then took me more than a year to rediscover it, one page at a time. Words were sometimes hard to come by, as they were intended to convey an experience which evaded comprehension - just as ripe grapes stacked in a press occasionally seem to evade the force upon them. . . But even in those moments when words come flooding, it is not by happenstance. Each word has been carefully weighed in passing, or after the fact. Thus, this reflection/testimony/journey is not meant to be read hastily, in a day or in a
p. A6 month, by a reader rushed to reach the final word. There is **no** "final word", no "conclusion" in Récoltes et Semailles, no more than there are any such things in my life or in yours. There is only a wine, aged over the course of a lifetime, at the core of my being. The last glass which you will be drinking will be no better nor worse than the first or the hundredth. They are all "the same", and they are all different. And if the first glass is spoiled, so is the rest of the barrel; it is better to drink fresh water (if such can be found), than to drink bad wine.

But a good wine ought not to be drunk in haste, nor expeditiously.

[2] Sylvie et Catherine Chevalley are the widow and daughter of Claude Chevalley, the colleague and friend to whom the central part of Récoltes et Semailles is devoted (ReS III, the key of the Yin and the Yang). At multiple times in the reflection I speak of him and of the role he played in my journey.

Chapter 3

A walk through a life's work, or the child and the Mother

January 1986

3.1 The magic of things

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When I was little I liked going to school. The same teacher taught us reading, writing, arithmetic, singing (he accompanied us with a small violin), and he even told us about prehistoric men and the discovery of fire. I do not ever recall being bored at school during those days. There was the magic of numbers, that of words, of signs, and of sounds. That of **rhymes** as well, through songs and small poems. There seemed to be, within rhymes, a mystery which extended beyond words. I believed this until the day somebody told me that there was a simple "trick" to it; that rhyme was simply when one ends two consecutive spoken movements by the same syllable, so that, as if by magic, these phrases became **verses**. It was a revelation! At home, where I found a good audience, for weeks or months on end, I amused myself by making verses. At one point, I even started exclusively speaking in rhymes. That period has passed, fortunately. Yet even to this day, I still sometimes write poems - but without trying to force the rhyme, if it doesn't seem to come by itself.

On another occasion an older friend, who was already in high school taught me about negative numbers. It was also a fun game, although I rapidly exhausted it. And then there were crosswords - I spent days and weeks constructing them, evermore interwoven. Within that game the magic of form, that of signs, and that of words found themselves combined. But even that passion subsided without leaving a trace.

During my high school years, the first of which took place in Germany and the others in France, I was a good student without quite being the "star student". I devoted myself without restraint to the courses which I cared most about, and tended to neglect the others, without really caring for the impression I made on my "prof". During my first year of high school in France, in 1940, I was interned at a concentration camp with my mother at Rieucros, near Mende. It was wartime, and we were foreigners - "undesirables", as they said. But the administration of the camp turned a blind eye towards the kids, however undesirable they may be. We came and went as we pleased. I was the oldest, and the only one to go to high school. It was a four or five kilometers long walk, p. P2 often in rainy and windy weather, wearing makeshift shoes that always got wet.

I still remember my first "math examination", in which the teacher gave me a bad grade, for my proof of one of the "three cases of equality of triangles". My proof wasn't exactly that of the book, which he followed religiously. Yet, I knew very well that my proof was no less convincing than that of the book which I followed in spirit, through repeated invocation of the traditional "we slide this figure in such and such a way onto that figure". Visibly, this teacher did not feel capable of judging things (namely, the validity of my reasoning) on his own. He had to report to a higher authority, that of a book in this case. Since I still remember this incident, I must have been stricken by such dispositions. In the years that followed, I have been presented with more than enough evidence to realize that such dispositions are far from exceptional - rather, they are the quasi-universal norm. There is a lot to be said on this subject - one which I approach more than once from various angles in Récoltes et Semailles. Somehow, to this day, I find myself invariably taken aback whenever I am confronted with such behavior…

During the last few years of the war, while my mother was still interned at the camp, I lived in a youth refugee house called "Secours Suisse", in Chambon sur Lignon. Most of us were jewish, and when we were told (by the local police) that there would be raids by the Gestapo, we went to hide in the woods for a night or two, in small groups of no less than 3, without quite realizing that our life was on the line. The region was filled with jews hiding in Cévenol country, and many of us survived thanks to the solidarity of the lcoal population.

What struck me most about "Collège Cévenol" (where I was raised), was the extent to which my peers were disinterested in learning. As for myself, I devoured our textbooks at the beginning of the school year, thinking that this time around, we would finally learn **truly** interesting things; and for the rest of the year I utlized my time as best as I could while classes dragged along inexorably one trimester at a time. Yet we had some wonderful professors. The natural history professor teacher, mister Friedel, was a person with remarkable intellectual and social qualities. However, as he lacked authority, the class was acting out of control, to the extent that it became impossible to hear what he had to say, as his voice was lost in the hurly-burly. This may be the reason I p. P3 haven't become a biologist!

I spent a fair amount of time, including class time (shh…), solving math problems. The ones in the book soon became insufficient. Perhaps it was because they tended to resemble one another after a while; but mostly, I believe, because they seemed to come out of the blue à la queue-leue-leue, with no indication as to where they came from or where they're going. These were the books problems, not mine. And yet, natural questions were plentiful. For instance, once the three side lengths a, b, and c of a triangle are known, so that the triangle itself is known (up to its position), there has to be an explicit "formula" that expresses the area of the triangle as a function of a, b, and c. Likewise, for a tetrahedron of which the six side lengths are known - what is the volume? I struggled through that one for a bit, but I must have gotten there eventually. In any case, when a problem "grabbed me", I did not count the hours or days that I spent working on it, even if it meant losing track of everything else! (And such remains the case to this day...)

What I found least satisfying, in our math textbooks, was the absence of a serious definition of the notion of length (of a curve), of area (of a surface), or of volume (of a solid). I promised myself to make up for this omission as soon as I could. This is what I devoted most of energy to between the years of 1945-1948, while I was a student at the University of Montpellier. University lectures weren't for me. Without ever quite realizing it, I must have been under the impression that all my professor did was recite the contents of the textbooks, just like my first math teacher at the lycée de Mende. I barely ever set foot on university grounds, just enough to keep up to date with the perennial "program". Books sufficed to cover said program, but it was clear that they offered no answers to the questions I was asking myself. Truly, they did not even **see** them, no more than my high-school textbooks did. As long as we were provided with recipes for all sorts of calculations, such as lengths, areas, volumes, through single, double, triple integrals (dimensions higher than 3 were carefully avoided...), the problem of providing an intrinsic definition was omitted by both my professors and textbook authors.

From my then limited experience, it seemed that I was the only person in the world to be gifted with a curiosity for mathematical questions. Such was, in any case, my unexpressed conviction, during those years spent in complete intellectual solitude (which did not bother me).[1] To be fair, it never occurred to me at that time to investigate whether or not I was the only person in the p. P4
world to take interest in what I was doing. My energy was sufficiently absorbed by the task I set for myself: to develop a fully satisfactory theory.

I never doubted that I would succeed in reaching the end of the story, as long as I was committed to scrutinizing these structures, spelling out on paper

[1]Between 1945-1948, I lived with my mother in a small hamlet about 10 kilometers away from Montpellier, Mairargues (near Vendargues), lost in the middle of vineyards. (My father disappeared in Auschwitz in 1942.) We scraped by on my meager student funding. To make ends meet, I took part in the harvest every year, and after the harvest season I would sell wine under the table (in contravention of the legislation, or so I hear...). On top of that, there was a self-regulating garden which supplied us with an abundance of figs, spinach, and even (towards the end) tomatoes planted by a complacent neighbor, amidst a sea of splendid poppies. It was the good life - although occasionally a bit rough along the edges, when we had to replace a pair of glasses, or a pair of worn-out shoes. Luckily, because my mother was weak and sick due to her long stay in the camps, we received free medical assistance. We would never have been able to afford a doctor otherwise...

what they were telling me. The intuition behind **volume**, say, was irrecusable. It could only be the reflection of a **reality**, momentarily elusive, but perfectly reliable. What had to be done was simply to seize this reality - a bit, perhaps, the way the magic reality of the "rhyme" had been seized, "understood" one day.

When I began this pursuit, at age 17, freshly out of high-school, I thought it would only take a few weeks. I spent three years on the project. It even caused me to fail an exam at the end of my second year of university - that of spherical trigonometry (in the "further astronomy" module), because of a stupid computational mistake. (I was never very good at computations, I must say, ever since I left high school. . .). That is why I had to spend a third year in Montpellier to complete my bachelor's instead of going to Paris right away - the only place, I was told, where I would be able to find people aware of what was considered important in Mathematics. My informant, Mister Soula, assured me that the last problem left in mathematics had been resolved 20 or 30 years ago by a so-called Lebesgue. He had apparently developed (funny coincidence!) a theory of measure and integration which brought *point final* to mathematics.

Mister Soula, my "diff calc" teacher, was a benevolent man who took a liking p. P5 to me. I was still not convinced by his claim. There must have already been, within me, the prescience that mathematics is a thing which is infinite in scope and depth. Does the sea have a "*point final*"? Yet I never thought of looking for that book by Lebesgue which Mister Soula had told me about, and he probably never held it either. In my mind there was nothing in common between anything a book contained and the work that **I** had been doing, in my own way, in order to answer questions which intrigued me.

3.2 The importance of being alone

When I finally made contact with the mathematical world in Paris, one or two years later, I ended up learning, among many other things, that the work which I had been doing independently, and with the means at hand, was (essentially) what "everybody" knew as the "Lebesgue theory of measure and integration". According to the two or three experts to whom I mentioned my work (or even showed a manuscript), I had just wasted my time redoing something "already known". I actually do not recall being disappointed. At that moment, the idea of receiving "credit", or even simply receiving approbation for the work that I was doing, must have still been foreign to my mind. Furthermore, my energy was completely taken by the process of familiarizing myself with an entirely different milieu and mostly learning what was considered in Paris to be the basic toolkit of the mathematician.[2]

Yet, thinking back to those three years, I realized that they were not in any way wasted. Unknowingly, I learned in solitude what is essential to the work of a mathematician - something no master could truly teach. Without ever having

[2] I briefly narrate this rough transition period in the first part of Récoltes et Semailles (ReS I), in the section "The Welcome Stranger" (nb. 9).

been told, without ever having to encounter someone with whom I could share my quest for understanding, I knew "in my gut" that I was a mathematician: somebody who "does" math, in its fullest sense - the way one makes "love". Mathematics had become, for me, a mistress always accommodating my desires. These years of solitude laid the foundation for a trust that has never been shaken - not by the discovery (upon arrival in Paris at age 20) of the scope of my ignorance and the vastness of what I had to learn; nor (more than 20 years later) by the eventful episode of my permanent departure from the mathematical world; nor, in these last few years, by the often crazy episodes of a "Funeral" (anticipated and cleanly executed) of my person and life's work, orchestrated p. P6 by those who used to be my closest companions...

To phrase it differently: I learned in those crucial years to "be alone".[3] That is, I learned to approach the things which I want to know with my own eyes, rather than rely on the expressed or implicit ideas that eminate from the group with which I identify, or a group to which I attribute authority. An unspoken consensus told me, both in high school and in university, that there was no need to question the notion of "volume", which was presented as "well-known", "self-evident", "unproblematic". Naturally I turned a blind eye to this consensus - just as Lebesgue, a few decades earlier, had to **turn a blind eye**. It is in this act of "turning a blind eye", of being oneself rather than the mere expression of the reigning consensus, of not to remain inscribed within the imperative circle to which they assign us - it is within this solitary act, above all else, that "**creation**" lies. Everything else comes after.

In the following years, within the mathematical world which welcomed me, I had the opportunity to meet multiple people, both older and younger, which were clearly more brilliant, "gifted" than I was. I admired the facility with which they learned new notions, as if at play, juggling them as if they had known them their whole life - while I felt heavy-handed and clumsy, laboriously making my way, akin to a mole, through an amorphous mountain of important things (or so I was told) which I had to learn, despite having no sense of their ins and out. Actually, I was far from the brilliant student who aced every prestigious *concours* and assimilating at once the most prohibitive courses.

Many of my more brilliant peers went on to become competent famous mathematicians. In hindsight, after 30-35 years, it does not seem to me that they left a deep imprint upon the mathematics of today. They did things, often times p. P7 beautiful things, in a pre-existing context which they would never have considered altering. They unknowingly remained prisoners in their imperious circles, which delimitate the Universe of a given time and milieu. In order to overcome them, they would have had to rediscover within them the ability which they

[3]This formulation is somewhat clumsy. I never had to "learn to be alone", for the simple reason that I never **unlearned** during the course my childhood, this innate skill which I had since birth, just as we all do. Yet these three years of solitary work, during which I could walk to my own beat, following my own exigence criteria, confirmed within me a degree of trust and tranquil confidence in my relationship with mathematics which owed nothing to the reining trends and consensus. I make allusion to these again in the note "Roots and Solitude" (Res IV, n°171_3) notably page .

had since birth, just as I did: the capacity to be alone.

The small child has no difficulty being alone. He is solitary by nature, even though he enjoys the occasional company, and knows when to ask for mom's permission teat. And he knows, without having ever been told, that the teat is his, and that he **knows** how to drink. Yet often times we lose touch with out inner child. And thus we constantly miss out on the best without even seeing it...

If in Récoltes et Semailles I address somebody other than myself, it is not a "public". I address myself to you, reader, as I would a **person**, and a person **alone**. It is to the person inside of you that knows how to be alone, the child, with whom I would like to speak, and nobody else. I am aware that the child is often far away. He has gone through all sorts of things for quite some time. He went hiding god knows where, and it can be hard, often times, to get to him. One could swear that he has been dead forever, or rather that he has never existed - and yet I am sure that he is there somewhere, well alive.

I know too what the **sign** is that I am being heard. It is when, beyond all cultural and experiential differences, what I share about my personal experiences echos within you and finds resonance; when you find within it **your own life**, your own self-experience, in a new light which you may never have considered before that. It is not about an "identification" with something or someone far from you. Rather, perhaps, you will rediscover a bit of your own life, or of what is **closest** to you, as you follow my own rediscovery of myself throughout Récoltes et Semailles, including within these very pages which I am currently writing.

3.3 The inner journey - or myth and testimony

Before all else, Récoltes et Semailles is a **reflection** upon myself and my life. Because of this, it is also a **testimony**, in two distinct ways. It is a testimony about my **past**, upon which the principle component of the reflection is concerned with. But it is also, at the same time, a testimony about the immediate present, about the very moment at which I am writing, and during which
p. P8 Récoltes et Semailles is born, in the course of hours, nights, and days. These pages serve as faithful witnesses to a long meditation upon my life, such as it was really carried out (and continues to be carried out at this very moment...).

These pages have no literary pretense They only constitute a document about myself. I only allowed myself to modify them within very narrow bounds[4] (notably for occasional stylistic edits). If there is pretense, it is only that of faithfulness. And that is saying a lot.

This document is also far from an "autobiography". You will learn neither my date of birth (which would be of little relevance unless one is making astrological predictions), nor the names of my mother and father or what they did in life, nor the names of the person who was my spouse and other women

[4]Thus, the occasional rectification of mistakes (material and of viewpoint) does not appear in the first pass but rather in footnotes or in later reconsideration.

who have been very important in my life, nor those of the children that were born from these unions, nor what these children have made of their lives. This does not mean that these things were not important in my life. Rather, the way this reflection on myself was engaged and developed never incited me to give a description of those things, which I lightly touch on here and there, but never take the time to consciously flesh them out with names and numbers. It never seemed to me that doing so would add anything whatsoever to the point which I was making at any given time. (Whereas in the few pages above I was brought, almost inadvertently, to include perhaps more material details on my life than you will find in the thousand pages to come. . .)

And if you were to ask me what "point" I have attempted to make over the course of these thousand pages, I would answer: it is to narrate, and thereby **discover**, the **inner journey** that my life has been and still is. This narrative/testimony of a journey is happening simultaneously at the two level which I have mentioned above. There is the exploration of a journey past, of its roots and of its origin, tracing all the way back to my childhood. And there is the continuation and renewal of this "very" journey, over the course of the days during which I am writing Récoltes et Semailles in spontaneous response to a violent stimulus coming from the outside world.[5] p. P9

External facts come to nourish the reflection, only to the extent they they induce and provoke new developments in my inner journey, or help clarify it. The burial and the plunder of my mathematical work, of which I will speak at length, has been such a provocation. It awoke in me a host of powerful reactions, and at the same time revealed to me the profound, and hitherto unknown links that continue to tie me to the work I have created.

It is true that my being "good at math" is not necessarily a reason (and even less so a good reason) for you to be interested in my particular journey - nor is the fact that I have had trouble with my colleagues, after shifting milieu and lifestyle. Colleagues, or even friends abound, who find it ridiculous to publicly spread one's "inner moods" - as they say. To them, what matters are "results". The "soul", meaning that within us which **witnesses** the production of these result, as well as apprehends them in various ways (as much in the life of the "producer" as in those of his peers), is looked down upon, sometimes even targeted with open derision. This attitude aims to display some form of modesty, but what I see is a sign of withdrawal or asynchrony promoted by the very air which we breath. I do not write for he who is stricken by this latent self disgust, which makes him reject the best I have to offer. A disdain for what truly makes his **own life**, and for what makes mine: the superficial or profound, course or subtle motions which animate the psyche, that "soul" which lives the experience and reacts to it, which freezes or blossoms, which retreats or learns. . .

The narrative of an inner journey can only be told by the person living it and no one else. Even though the narrative is only aimed towards oneself, it often

[5] For more details about this "violent stimulus", see "The Letter", notably sections 3 through 8.

times inserts itself within the construction of a **myth**, of which the narrator is the hero. Such a myth is born, not in the creative imagination of a people and a culture, but rather from vanity of he who dared not assume a humble reality, but instead substitutes a construction for it. But a **true** narrative (if such a thing exists), of a journey such as it was truly lived, is to be prized. And this is not because of renown which is (rightly or wrongly) attributed to the narrator, but simply by virtue of its **existence**, and of its truthfulness. Such a testimony is precious, whether it comes from an illustrious person, a small clerk with no future and with family responsibilities, or from a common criminal.

If there is value for one in such a narrative, it is first and foremost that of self confrontation, through this unvarnished testimony of the experience of an other. But also (to phrase it differently) to erase within oneself (be it only for the span of a reading) this disdain by which one holds one's **own journey**, and that "soul" of which one is both the passenger and the captain. . .

p. P10

3.4 The painting of mores

In speaking of my past as a mathematician, and later in discovering (almost against my will) the twists and turns of the intricacies of the gigantic Burial of my work, I was brought, inadvertently, to paint the picture of a particular milieu and era - an era affected by the decay of certain values which provided meaning to the work of individuals. That is what I mean by "painting of mores", centered around a "fait divers" which is thoughtlessly unique in the annals of Science as I have said rather clearly earlier, I believe, you will not find in Récoltes et Semailles a "folder" concerning a certain unordinary "case", quickly bringing you up to date. And yet a friend of mine, looking for such a folder, blindly passed by nearly everything constituting the substance and flesh of Récoltes et Semailles.

As I explain in much more detail in the Letter, the "investigation" (or the "painting of mores") carries on in parts II and "The Funeral (1) - or the robe of the Emperor of China" and "The Funeral (3) - or the Four Operation". Page after page I persistently extract one after another, a number of juicy facts (to say the least) which I attempt to "classify" bit by bit. Slowly, these fact assemble into a global painting which progressively emerges from the fog, taking on brighter colors and sharper contours. In these daily notes, the raw facts "which just appeared" are inextricably mixed with personal reminiscing, as well as with commentaries and reflections of a psychological, philosophical, or even (occasionally) mathematical nature. That's how it is, and there is nothing I can do about it!

Starting with work I had already done, which occupied me for over a year, producing a sort of "investigation proceedings" folder should not have taken longer than a few hours or days worth of work, depending on the curiosity or demands of the interested reader. I tried at one point to produce such a folder. That is how I started writing a note which was to be called "the four

operations".[6] But in the end I could not bring myself to do it! That is decidedly p. P11
not my style of expression, and in my old age less so than ever. I now consider, having written Récoltes et Semailles, that I have done enough for the benefit of the "mathematical community", and therefore can leave, without remorse, the task of producing the necessary "folder" to others (in particular to any of my colleagues who would feel concerned).

3.5 The heirs and the builders

It is now time for me to say a few words about my mathematical work, which has played an important role in my life, and continues to do so (to my own surprise). I come back to this work more than once in Récoltes et Semailles - sometimes in a way that should be understandable by all, and other times in slightly more technical terms.[7] The latter will mostly go "above the heads", not only of the "profane", but also of the mathematical colleague who may not be completely "in" the field in question. One can of course feel free to skip the sections which seem too "involved". Just as one can try to go through them, glimpsing as one goes, a shadow of the "mysterious beauty" (in the words of a non-mathematician friend of mine) of the universe of mathematical things, appearing as a multitude of "strange inaccessible islands" in the vast moving waters of reflection...

Most mathematicians, as I mentioned earlier, are inclined to constrain themselves to a conceptual framework, a "**universe**" fixed once and for all - the one, essentially, which they have found "ready made" at the time of their studies. They are like the heirs of a large and beautiful fully-furnished house, with its lounges, kitchens, workshops, and its kitchenware and tools left and right, with which there is, I trust, plenty to cook and tinker. How this house was built, progressively over the course of multiple generations, and how and why these tools (and not others...) were conceived and built, why the pieces are disposed and organized in such a way - these are all questions the heirs would never think of asking themselves. This is the "universe", the "given", in which we must live, and that is that! Something which appears massive (and which most of the time we have only been able to partially explore), yet at the same time **familiar**, and mostly: **immutable**. They mostly busy themselves with maintaining or embellishing a patrimony: fixing a faulty piece of furniture, restoring a facade, sharpening a tool, or even sometimes, for the most enterprising, building an
entire workshop, or a whole new piece of furniture. It even happens, when they p. P12
fully commit to the task, that the piece of furniture is truly beautiful, so that the whole house appears embellished by its addition.

[6] The note eventually exploded into part (also named "The four operations") of Récoltes et Semailles, comprising about seventy notes running over more than four-hundred pages.

[7] One will also find here and there, in addition to mathematical notes concerning my previous work, sections containing new mathematical developments. The longest of these is "the five pictures (crystals and $\mathcal{D}$-modules)" in ReS IV, note n°171 (ix).

Even more rarely, one of them will consider modifying one of the main tools, or even, under repeated and insistent pressure or need, to imagine and build a whole new tool. And in so doing, he often feels on the brink of profusely apologizing for what he feels is infringing on the piety owed to the familial tradition, which he has disturbed through his brazen innovation.

In most of the rooms of the house, the windows and shutters are carefully closed, probably on account of a fear that a foreign wind would blow in. And when the pretty new furnishings, here and there, together with their progeny, begin to clutter the rooms and invade the corridors, none of these heirs will agree to face the fact that his familiar and cozy Universe is beginning to feel cramped. Rather than come to terms with such a fact, most will prefer to awkwardly slither, and try not to get trapped, some between a Louis XV buffet and a rocking chair in rattan, others between a boisterous toddler and an Egyptian sarcophagus, while others, as a last resort, will try to climb over a heteroclite and crumbling pile of chairs and benches. . .

The picture I have just sketched is not unique to the world of mathematicians. It illustrates the deeply engrained and immemorial conditioning which one encounters in every milieu and sphere of human activity, regardless, as far as I can tell, of the society or era in question. I have mentioned such a phenomenon already, and I do not in any way pretend to fall outside of its influence. As will be clear from my testimony, the contrary is true. It only happens to be the case that at the relatively limited level of the act of intellectual creation, I was barely affected[8] by this conditioning which may be called "cultural blindness" - the incapacity to see (and to evolve) outside of the "Universe" fixed by the surrounding culture.

As for myself, I feel that I belong to the lineage of mathematicians whose spontaneous vocation and joy was to continuously construct new houses[9] In
p. P13 so doing, they cannot help but invent all of the required tools, utensils, and furnishings for both the construction of the house from its foundation, and to fill the kitchens and workshops of the house in abundance, so that one may live in it comfortably. Yet, once everything down to the last sapling and stool has been taken care of, the builder rarely lingers on the premises, of which every stone and every piece of wood carries a trace of the hand which shaped and placed it. The builder's place lies not in the quietude of fully finished universes, however welcoming and harmonious they may be, whether they are a product of his own hands or those of his predecessors. His place is in the open air. He is friends with the wind, and does not fear solitude at work for weeks, years or, if need be, for an entire lifetime if no welcome succession presents itself. Just like everybody else, the builder only has two hands - but two hands which at each moment know what they need to do, which refuse neither the largest nor the most delicate tasks, and which never tire of comprehending, again and again, the multitude of things which become them. Two hands might be few, given

[8]I believe the main reason for such immunity is a certain favorable climate which surrounded me until age 5, the note "The innocence" (ReS III,n°107).

[9]This archetypal picture of a "house" to be built, surfaces and is formulated for the first time in "Yin, the Servant, and the new masters" (ReS III , n°135).

that the World is infinite. They will never exhaust it! And yet, two hands can be a lot...

History is not my strong suit, but if I had to give a list of mathematicians inscribed in this lineage, names that spontaneously come to my mind are those of Galois, Riemann (from the past century), and Hilbert (at the beginning of the current century). If I were to name a candidate among the elders which welcomed me into the mathematical world,[10] the name of Jean Leray comes to my mind before any other, even though my contact with him has always been episodic.[11] p. P14

I have just roughly sketched two pictures: that of the "homebody" mathematician, who is content with maintaining and embellishing a heritage, and that of the builder-pioneer,[12] who is drawn to repeatedly crossing these "invisible and imperious circles" which delimitate a given Universe.[13] These two groups may also be called, somewhat bluntly but also suggestively, "conservatives" and "innovators". Both have their raison d'être, in one collective adventure that is carried out through the generations, through centuries and millennia. During the fruitful periods of a science or art there is neither opposition, nor is there antagonism among these two temperament.[14] They are different and mutually complementary, just as dough and yeast.

Between these two extremes (not at all opposed by nature), one can find a plethora of intermediary temperament. There is the "homebody" that would never think of leaving a familiar dwelling, and would be even less willing to take on the task of building another, god knows where, yet will not hesitate, when the house gets cramped to build a basement, raise the ceiling, or even, if need be, to build a dependency of modest proportions.[15] Without being a

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[10] I speak of these beginnings in the section "The welcome stranger" (ReS I , n°9).

[11] I was nonetheless (following H. Cartan and G. Serre) one of the first users and promoters of one of the great innovative notions introduced by Leray, that of a sheaf, which has been an essential tool throughout my work as a geometer. It is also the notion which has provided me with the key to enlarge the notion of a topological space into that of topos, about which I will be speaking later.

Leray differs from the portrayal I have given of the "builder", I believe, in that he does not seem drawn to "construct houses from their foundations to their completion". Rather be was compelled to lay out vast foundations, in places where nobody would have thought to look while leaving to others the care of carrying the construction to its completion, and once the house is built, to settle into the premises (be it only for a short time)...

[12] I have just surreptitiously attached herein two qualifiers with male connotation (that of "builder" and that of "pioneer"), which express very different aspects of the impulse of discovery, one which is of a nature more delicate than what these qualifiers might evoke. Such a discussion will be carried out later in this walk-reflection, in the step "The discovery of the Mother - or the two versants" (n°17).

[13] At the same time, and without really meaning to do so, the builder-pioneer assigns to the old Universe (if not for himself, at the very least for his more sedentary colleagues) new boundaries, thereby inscribing circled which may be larger, but are just as invisible and imperious as those which they have come to replace.

[14] Such has been the case, notably in the mathematical world, during the period (1948-1969) of which I was a direct witness, as I myself belonged to that world. Following my departure in 1970, there seems to have been a large scale reaction, a sort of "consensus of disdain" for the "ideas" in general, and more specifically the great innovative ideas that I have introduced.

[15] Most of my "elders" (about whom I speak for instance in "a welcome debt" (Introduc-

builder at heart, he will often view with a sympathetic eye, or at the very least without concern nor secret reprobation towards another who had shared the p. P15 same dwelling, and who is already out and about assembling beams and stones in some impossible boonies, with the confidence of somebody who already sees a castle...

3.6 Viewpoint and vision

Allow me to return to myself and my work.

If I excelled in the art of mathematics, it was not through the ability and perseverance to solve problems left by my predecessors, but rather through a natural tendency within me to discover **questions**, evidently crucial, yet that nobody had yet seen, or to excavate the **"right notions"** that were missing (often without anyone realizing until the new notion appeared), as well as the **"right statements"** of which nobody had thought. Often, notions and statements mesh in such a perfect way, that there can be no doubt in my mind as to their validity (give or take small adjustments at most) - so that often, when it boils down to "travail sur pièces" destined for publication, I refrain from going further, and from taking the time to flesh out a proof that often, once the statement and its context are well-understood, consists of no more than a matter of "trade", not to say routine. Things which solicit ones attention are countless, and it is impossible to follow them all to their end! The fact remains that carefully proved propositions and theorems in my written and published work appear in the thousands, and almost all of them have entered the patrimony of things commonly accepted as "known" and frequently used all over mathematics.

I am led more towards the discovery of fertile **viewpoints** than towards the discovery of questions, notions, and statements, by my particular type of genius, which is constantly leading me to introduce, and more or less develop, entirely novel **themes**. It is this, I reckon, which is my most essential contribution to the mathematics of my time. In fact, these innumerable questions, notions, and statements which I just mentioned, only truly make sense for me once they are subjected to such a "viewpoint" - or more precisely they **arise** spontaneously from it; in the same way that a light (even a dim one) appearing in a pitch black night seems to invoke from the shadows contours which it suddenly reveals to us. Without this light uniting them in a common sheaf, the ten, or one-hundred, or one thousand questions, notions, statements would appear as a heterogeneous p. P16 amorphous pile of "mental widgets" all isolated from one another - rather than as the many parts of a **Whole** which, while perhaps remaining invisible, escaping within the folds of the night, is nonetheless clearly felt.

The fertile viewpoint is that which reveals to us, organized as the many

tion n°10)) conform to this intermediary temperament. I was thinking notably about Henri Cartan, Claude Chevalley, André Weil, Jean-Pierre Serre, Laurent Schwartz. With the exception maybe of Weil, they have all turned a "sympathetic eye", without "concern nor secret reprobation" towards the solitary adventures into which they saw me embark.

living parts of a common Whole, enveloping them and giving them meaning, these pressing questions which no one had asked, and (as if in response, perhaps, to these questions) these extremely natural notions which nobody had thought of expressing, and these statements finally which seem to immediately follow, and which nobody had dared to conjecture, for as long as the questions which brought them about, and the notions that allowed us to formulate them had remained hidden. Even more so than what we call "key theorems" in mathematics, it is the fertile viewpoints which, in our art,[16] constitute the most powerful tools of discovery - or rather, they are not tools, but they are the very eyes of the researcher who passionately strives to understand the nature of mathematical things.

Thus, the fertile viewpoint provides us with an "eye" which at once helps us **discover** and **recognize the unity** within the multiplicity of what is discovered. And such unity is truly the very life and breath which connects and animates these discoveries.

But just as the word itself suggests, a "viewpoint" by itself remains fragmentary. It reveals to us **one of the aspects** of a scenery or panorama, among a multiplicity of others which are equally valuable, equally "real". It is when complementary viewpoints of a common reality are conjugated, that is, when our "eyes" are multiplied, that the gaze is able to penetrate further ahead in the reckoning of things. The richer and more complex the reality which we desire to know, the more important it is to be equipped with several "eyes"[17] in order to apprehend it in all its ampleness and subtlety.

By virtue of our innate ability to grasp the "multiple" as the **One**, it also happens, sometimes, that a sheaf of viewpoints converging to a unique and vast scenery, gives rise to a novel thing; a thing which transcends each of the partial perspectives, in the same way that a living being transcends each of its limbs and p. P17
organs. This new thing could be called a **vision**. The vision unites the known viewpoints which constitute it, while also revealing to us other viewpoints that were previously ignored, just as the fertile viewpoint makes us discover and apprehend, as part of the same Whole, a multiplicity of new questions, notions, statements.

To say this in another way: the vision is to the viewpoints, from which it seems to arise and which it unites, as the clear and warm daylight is to the various components of the solar spectrum. A vast and profound vision is like an inexhaustible source, made to inspire and guide the work, not only of the one within whom the vision was once conceived and who made himself its servant, but that of generations, fascinated perhaps (as he first was) by these distant horizons which it lets us glimpse...

[16] Such a phenomenon is not exclusive to "our art", but (it seems to me) it appears in every act of discovery, at the very least when such an act happens at the level of intellectual reckoning.

[17] Every viewpoint leads to the development of a language which is best suited to expressing it. Having several "eyes" or several "viewpoints" to apprehend a situation, also means (at least in mathematics) having several different languages to tackle the situation.

3.7 The "great idea" - or the trees and the forest

The so-called "productive" period of my mathematical life, meaning it was marked by proper publications, ran between 1950 and 1969, 20 years that is. And for 25 years, between 1945 (when I was seventeen) and 1969 (when I was in my forty-second year), I invested nearly the totality of my energy in mathematical research. An excessive investment certainly. This cost me a long period of spiritual stagnation, an incremental "thickening", to which I will be coming back multiple times in the pages of Récoltes et Semailles. Yet, within the limited scope of a purely intellectual activity, and through the burgeoning and maturation of a vision restricted to the world of mathematical things, these were years of intense creativity.

During this long period of my life, the near totality of my time and energy were devoted to what might be called "**travail sur pièces**": a minute process of shaping, assembling, and honing, required for the construction from start to finish of houses that an inner voice (or an inner demon...) called for me to build, following a blueprint that it whispered to me at every step of the way. Occupied by the tasks of the "trade": such as those of a sculptor, bricklayer, carpenter, even plumber, woodworker, cabinet-maker - I rarely stopped to write down, even in rough sketches, the master-plan, which was invisible to all (as it only appeared later...) except to me, who over the course of the days, months,
p. P18 and years, guided my hand with the certainty of a sleepwalker. [18] I have to say

[18]The metaphor of the "sleepwalker" was inspired by the title of the wonderful book "the sleepwalkers" by Koestler (Calman Lévy), presenting an "Essay on the history of the conceptions of the universe", starting from the origins of scientific thought, all the way to Newton. One of the facets of this history which struck Koestler, and which he highlights is the extent to which, often, the path from a given state of our understanding of the world, to some other state which (logically and with hindsight) seems very close, sometimes takes the most astounding detours, which appear to defy reason; and to think that yet, despite those thousand detours that could conceivably have lost them forever, and with the "certainty of sleepwalkers", men who have gone on the quest for the "keys" of the Universe find, as if unintentionally and without even realizing it, other "keys" that they would never have thought of, and which nonetheless appear to be "the right ones".

From what I have observed all around me, at the level of mathematical discovery, these extraordinary detours in the path towards discovery are the actuality of some high-caliber researchers, but not of all. This could be due to the fact that for the past two or three centuries, research in the natural sciences, and even more so in mathematics, has freed itself from the religious presupposition or metaphysical imperatives, pertaining to a given culture or era, which have been strong barriers to the deployment (for better or for worse) of a "scientific" understanding of the Universe. It is nonetheless true that some of the most fundamental and evident ideas and notions in mathematics (such as the notions of displacement, group, the number 0, symbolic arithmetic, the coordinates of a point in space, the notion of a set, or that of topological "shape" without even mentioning negative numbers and complex numbers) took millennia without making an appearance. These oversights are signs of this ingrained "block", deeply embedded in the psyche, against the conceptualization of entirely novel ideas, even in the cases where these are of a childlike simplicity and seem to impose themselves with the strength of evidence, over the course of generations or even millennia...

Returning to my own work, I have the impression that within it, the "mess-ups" (perhaps more frequent than in the work of most of my colleagues) pertain exclusively to matters of detail, generally spotted quickly by my own hand. These are simple "potholes", of purely "local" nature, and with no serious implications concerning the validity of the examined sitthat the "travail sur pièces" to which I like to devote a loving care, was not at all displeasing to me. Moreover, the mode of mathematical expression which was professed and practiced by my elders, gave preeminence (to say the least) to the technical aspect of one's work, and in no way encouraged the "digressions" that would have idled on the "motivations"; or even those which appeared to bring out of the mist some vision which perhaps was inspiring, but which, because it failed to be presented in the form of tangible constructions in wood, stone, or hard cement, was likened more to dream fragments, than to the craft of a p. P19 conscientious and diligent artisan.

At the quantitative level, my work during these intense years of productivity manifested itself mostly in the form of about twelve-thousand pages of publications, in the form of articles, monographs, or seminaries,[19] as well as hundreds, if not thousands, of new notions which have entered into the mathematical patrimony, with the very names which I had given them upon first discovering them.[20] In the history of mathematics, I think I may have been the person who introduced the largest number of new notions into our science, and at the same time, the person who was brought, as a consequence, to invent the largest number of new names, with the intention of expressing these notions with delicacy, and in as suggestive a way as I could.

These indications give no more than a very rough feel for my work, ignoring what truly lies at its heart, its life and vigor. As I wrote about earlier, what I have brought best to mathematics are the new "**viewpoints**" that I have been able to first **glimpse**, and later patiently **excavate** and to some extent develop. These new viewpoints, like the notions I just mentioned, inserting themselves in a vast multiplicity of different situations, are also nearly enumerable.

The fact remains that certain viewpoints are more vast than others, those which by themselves encompass a multitude of partial viewpoints, within a multitude of particularly different situations. Such a viewpoint can be called, rightly, a "**great idea**". Through its internal fecundity, such an idea gives rise to a vast progenitor of ideas which themselves inherit its fecundity, although most (if not all) are of a lesser scope than the mother idea.

The task of **expressing** a great idea, of "saying" it, can be as delicate as its conception and slow gestation within the person who conceived of it. To better put it, the laborious process of patiently excavating the idea, day after day,

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uation. At the same time, at the level of ideas and larger guiding intuitions, my work is free of any "mistakes" as incredible as that might seem. It is this always reliable certainty in apprehending each moment, if not the eventual conclusions of an argument (which often remain hidden from sight), at the very least the most fertile directions which present themselves to take me straight to what is essential - it is that certainty which has brought to my mind Koestler's metaphor of the "sleepwalker".

[19]Starting from the 1960's, part of these publications were written in collaboration with colleagues (mostly J. Dieudonné) and students.

[20]The most important of these notions are reviewed in the Thematic Sketch, as well as in the Historical Commentary which accompanies it - both of these are included in Volume IV of the Reflections. Some of these names were suggested by friends or students, such as the term "smooth morphism" (J. Dieudonné) or the panoply "site, stack, gerbe, and band" in the thesis of Jean Giraud.

p. P20 from the veils of mist which surround it at birth, to slowly succeed in giving it a tangible form, as a painting which grows richer, firmer, and finer over the course of weeks, months, years. To simply **name** the idea by some striking formula, or by more or less technical keywords, can fit in the span of a few lines, or a few pages - but rare are those who, without knowing it beforehand, will know how to listen to this "name" and recognize its face. And when the idea reaches full maturity, a hundred pages might suffice to express it to the full satisfaction of the worker within whom it was born - just as it can happen that ten-thousand pages, crafted and weighted at length, may not suffice.[21]

In this case, as in others, among those who gained awareness of the project, which presented the idea in full bloom, many people could see the vigorous trees, and used them (some to climb, others used them as lumber, some as firewood, ...) yet few could see the forest from the trees...

3.8 The vision - or twelve themes for a harmony

Perhaps one might say that the "great idea" is the viewpoint which, not only itself reveals new and fertile ideas, but that which introduces a novel and vast **theme** which embodies it. Every science, when understood not as a tool to gain power or domination, but rather as a journey towards the understanding of our species through the ages, is nothing but the harmony, more or less vast and more or less rich from one era to the next, which deploys itself through generations and centuries, through the delicate counterbalancing of all the themes which appeared one by one, as if summoned from the void, to integrate into the science
p. P21 and be interlaced within it.

Among the multiple viewpoints, which I have unearthed in mathematics, there are **twelve**, in hindsight, which I would call "great ideas".[22] To see my

[21] Upon leaving the mathematical world in 1970, the totality of my publications (including several works in collaboration) on the central theme of schemes amounted to some $10,000$ pages. Yet this only represented a modest fragment of the vast program which I glimpsed ahead, concerning schemes. This program was abandoned sine die dès my departure despite the fact that almost everything which had already been published for the use of everyone had already entered the common patrimony of results and notions considered "well known".

The part of my program concerned with schemes together with its extensions and ramifications, which I had completed at the time of my departure, alone represented the most vast foundational work ever accomplished in the history of mathematics, and surely one of the most vast in the history of Science as well.

[22] Here are, for the mathematically inclined reader, the twelve key ideas, or "maître thèmes" (in chronological order of appearance):

1. Topological tensor products and nuclear spaces.
2. "Continuous" and "discrete" duality (derived categories, "six operations").
3. Riemann-Roch-Grothendieck yoga (K-theory, relationship with intersection theory).
4. Schemes.
5. Topos.
6. Étale and ℓ-adic cohomology.
7. Motives, Motivic Galois Groups (Grothendieck $\otimes$-categories).

mathematical work, to "feel" it, is to see and "feel" at least some of these ideas, and the great themes that they introduce and which lie at the heart of my work.

By the nature of things, some of these ideas are "greater" than others (which in turn are "smaller"!). In other words, among these novel themes, some are vaster than others, and some plunge deeper into the heart of the mystery of mathematical things.[23] There are three (and not the least of them) which, p. P22 having appeared only after I had left the mathematical world, remain at the embryonic stage; they "officially" do not even exist, as no proper publication can be pointed to as a birth certificate.[24] Among the nine themes that had appeared prior to my departure, the latest three, which I had left growing in full swing, remain today at a stage of infancy, due to a lack (following my departure) of caring hands that would provide for these "orphans" left behind in a hostile

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8. Crystals, crystalline cohomology, yoga of "de Rham coefficients", "Hodge coefficients", ...
9. "Topological algebra": ∞-stacks, derivators; cohomological formalism of topoi, serving as inspiration for a new conception of homotopical algebra.
10. Tame topology.
11. The yoga of nonabelian algebraic geometry, Galois-Teichmüller Theory.
12. "Schematic" or "arithmetic" point of view for regular polyhedra and regular configurations of all kinds.

Apart from the first theme, a large portion of which appeared in my thesis of 1953 and was further developed in the period in which I worked in functional analysis between 1950 and 1955, these themes were discovered and developed during my time working as a geometer, starting in 1955.

[23] Among these themes, the most **vast** in its **reach** seems to me to be that of **topoi**, in that it provides the idea of a synthesis of algebraic geometry, topology, and arithmetic. The most vast in terms of the **extent of the developments** to which it has given birth thus far, is the theme of schemes. (See the note ref.) It is that theme which provides the framework "par excellence" in which eight of the other listed themes are developed (namely all but themes 1,5,10), and it also provides the central notion for a fundamental renewal of algebraic geometry, and of the algebraic geometric language.

On the other extreme, the first and last of these twelve themes seem to me to be of more modest dimensions than the others. Yet, concerning the last, introducing a new viewpoint on the ancient theme of regular polyhedra and regular configurations, I doubt that the life of a mathematician who would devote themselves exclusively to this would suffice to exhaust it. As for the first of all of these themes, topological tensor products, it played a role of a new tool ready for use, rather than a source of inspiration for later developments. The fact remains that I still, to this day, receive sporadic echoes of more or less recent solutions (20 or 30 years later) to some of the questions which I had left open.

The deepest (to my eyes) of these twelve themes, are the notion of **motives**, and the closely related **yoga of nonabelian algebraic geometry, and Galois-Teichmüller theory**.

From the viewpoint of **powerful tools**, perfectly polished under my care, and commonly used in various "cutting edge fields" in research during the past two decades, the themes of "**schemes**" and "**étale and ℓ-adic cohomology"** have been the most noteworthy. For a well-informed mathematician, I believe there is no doubt that the schematic tools, as well as the theory of ℓ-adic cohomology which arises from it, are among some of the most important contemporary acquisitions, having come to nourish and renew our science over the course of the previous generations.

[24] The only "semi-official" text where these themes are briefly sketched, is the Esquisse d'un Programme written in January 1984, in the context of a detachment request to the CNRS. This text (which is also mentioned in the Introduction 3, "compass and luggage") will be included in principle in volume 4 of the Reflexions.

world[25]. As for the remaining six themes, which reached full maturity during the two decades preceding my departure, one could say (modulo one or two caveats[26]) that they had by then already entered the mathematical patrimony: among geometers in particular, "everybody" nowadays invokes them routinely and without even noticing (the way Monsieur Jourdan wrote prose). They are part of the air we breathe when "doing geometry", or when doing arithmetic,
p. P23 algebra, or analysis of a "geometric flavor".

The twelve main themes of my work are far from isolated from one another. Rather, they together constitute in my eyes a **unity** of spirit and aim, manifesting itself in the form of a common and persistent backdrop throughout both my "written" and "unwritten" work. And as I wrote these lines, I seemed to once again encounter the same key - a calling of sorts! - which had permeated the three years I spent working "free of charge", intensely and alone, at a time when I did not even bother to inquire as to whether there existed mathematicians other than myself in the outside world, being so completely entranced by the fascination I felt for that which was calling me...

This unity is not only that of a common style among works produced by the same hands. The aforementioned themes are interlinked in countless ways, both subtle and obvious, akin to how the different themes of a singular and vast counterpoint are interlinked, intertwined in their deployment, while each remains clearly recognizable. They come together in a harmony that carries them forward and breathes meaning into each in turn, in the form of a movement and plenitude to which all others contribute. Each of the partial themes seems to be issued from this vaster harmony and to be reborn within it moment after moment, rather than the harmony appearing as the "sum" or "result" of pre-existing constituent themes. And to tell the truth, I cannot shake away the feeling (surely unreasonable...) that it is in a way this harmony - hitherto invisible, but which surely already "existed" in the hidden bosom of things yet to be born - which engendered one by one the themes which were only to take their full meaning through it, and that its muted and pressing voice was what was already calling me during these years of intense solitude, at the outset of my teenage years...

The fact remains that the twelve "maître-thèmes" of my work all contribute to a common symphony, as if by some secret predestination - or, to re-invoke a different picture, they incarnate as many different "viewpoints" that come together to constitue a single and vast **vision**.

This vision only began to emerge from the mist, unveiling some of its recognizable contours, around the years 1957-58 - which were years of intense gesta-
p P24 tion[27]. As strange as it may seem, this vision felt so close, so "obvious", that up

[25]Right after my departure, following the unceremonious burial of these three orphans, two of them were exhumed with great fanfare and with no mention of the original craftsman, one in 1981 and the other (given the fussless success of the operation) starting the following year.

[26]The "one or two caveats" concerns mostly the yoga of grothendieckian duality (derived categories and six functor formalism), and that of topoi. These (among other things) will be discussed in detail in parts II and IV of Récoltes et Semailles (The Funeral (1) and (3)).)

[27]The year 1957 was that during which I was led to unearth the **Riemann-Roch** theme (Grothendieck version) - which turned me almost overnight into a "superstar". This was also

until just a year ago[28] I had not thought of giving it a name (even though one of my passions has always been to constantly **name** the things that I discovered, as a first method of apprehension...). It is true that I cannot think of a particular moment during which I experienced the appearance of this vision, or one which I could recognize as such in hindsight. A novel vision is something so vast that its appearance really cannot be attributed to a particular moment. Rather, the vision has to penetrate and take possession - over the course of many years, if not generations - of the individual or group whose activity it is to contemplate and scrutinize; as if new eyes had to laboriously come into being, behind the familiar eyes which they have come to replace. The vision is also too vast for there to be any chance to "capture it", the way we may capture the first notion to come our way. There is therefore nothing surprising, after all, about the fact that the very thought of naming something this vaste, yet so close and diffuse, only came with hindsight, once the vision had reached full maturity.

To tell the truth, until two years ago, my relationship with mathematics was limited (with the exception of teaching) to the act of **doing** it - following an impulse that ceaselessly moved me **forward**, into an "unknown" that continu- p P25
ally attracted me. The idea never occurred to me to stop in my stride and to interrogate myself, to turn around even for an instant and perhaps to see the outline of a path taken, or to situate past work (be it to situate it **in my life**, as something to which profound links long ignored continue binding me; or to situate it in the collective adventure that is **"mathematics"**.)

Stranger thing even, in order to bring myself to "lay out" and to reckon with this half-forgotten work, or to even consider giving a **name** to the vision that lay at its heart, I would have had to suddenly confront the reality of a Funeral

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the year of my mother's passing, and through it that of an important break in my life. It was one of the most intensely creative years of my life, mathematically and otherwise. That year was the first time I felt that I had more or less "gone round" what constitutes mathematical work, and that it might be time for me to begin devoting my time to something else. A need for self-actualization had visibly surfaced in me, for the first time in my life. I considered becoming a writer, and stopped all mathematical activity for several months. I eventually decided that I should at least produce a write-up of the mathematical projects which I was already working on, something that I thought would only take a few months, or up to a year at the longest...

The time was not yet ripe, it seems, for the "grand saut". The fact is that my mathematical work, once taken back up, completely re-absorbed me, and did not let me go for another twelve years!

The year that followed this interlude (1958) was perhaps the most prolific in my life as a mathematician. It is during that year that the burgeoning of the two central themes of the novel geometry took place, starting with the launch "en force" of the **theory of schemes** (which was the topic of my talk at the international congress of mathematicians in Edinburgh the summer of that same year), and followed by the formulation of the notion of "**site**", a provisory technical prototype for the crucial notion of **topos**. With nearly 30 years of hindsight, I can now say that this was the year during which the novel geometry was born, in the wake of the two key tools of this geometry: schemes (which are a metamorphosis of the prior notion of "algebraic variety"), and topoi (which are an even deeper metamorphosis of the notion of space).

[28] I contemplate naming this vision for the first time in the reflection of December 4^{th} 1984, in the sub-note (n$^o 136_1$) to the note "Yin the servant (2) - or the generosity" (ReS III, page 637).

of gigantic proportions: the funeral, through silence and derision, both of the vision and of the craftsman in whom it was borne...

3.9 Form and structure - or the way of things

Without planning to, I ended up writing this "foreword" as a sort of presentation "en règle" of my work, intended (mostly) to the non-mathematician reader. Having come too far to turn back, it is time to finish "the introductions"! I would like to attempt to say a few words about the **substance** of these marvelous "great ideas" (or "maître-theèmes") which I have mentioned in the above pages, and about the nature of this so-called "vision" within which these key ideas supposedly come together. In keeping with the non-technical nature of this foreword, I would undoubtedly only be able to convey an extremely vague picture (provided indeed that anything at all can even "go through"...)[29].

Traditionally, we draw a distinction between three types or "quality", or "aspects" of things in the Universe which lend themselves to mathematical thought:
p. P26 they are **number**[30], **size**, and **shape**. They could also be called respectively the **"arithmetic"** aspect, **"metric"** (or analytic) aspect, and the **"geometric"** aspect of things. In most situations under study in mathematics, these three aspects are present simultaneously and are tightly interacting. However, very often, there is a clear prevalence of one of the three. It seems to me that for most mathematicians, it is rather clear (to those who know them, or who are aware of their work) which is their base temperament, whether they are "arithmeticians", "analysts" or "geometers" - and this is so even for those who have many strings to their bow and who have done work in all registers and diapasons imaginable.

My first and solitary reflections, concerning the theories of measure and integration, are situated without any ambiguity in the "size", or "analysis" category. The same goes for the first of the new themes which I have introduced in mathematics (which seems to me to be of lesser proportions than the eleven others). The fact that I have entered the world of mathematics "through" analysis seems due, not to my particular temperament, but rather to what may be called "fortuitous circumstances": it was because the most blatant gap in my secondary and university education, to my mind enamored with generality and rigor, happened to concern the "metric" or "analytic" aspect of things.

[29] A picture may remain "vague", but it nonetheless can have the potential of being faithful, and of successfully restoring the essence of what is being considered (in this case, my work). Inversely, an image can be sharp yet distorted, and it can moreover include the accessory while entirely omitting the essential. Thus, if you "connect" to what I have to say about my work (in which case surely some part of the image will "go through" successfully), you can flatter yourself to have better understood what is the essence of my work than perhaps any of my wise colleagues!

[30] By "number" here I mean the "natural numbers" 0,1,2,3, etc or (at most) numbers (such as the rational numbers) which can be expressed from them using elementary operations. These numbers do not lend themselves, unlike the "real numbers", to the measure of quantities subject to continuous variations, such as the distance between two points varying along a line, a plane or in space.

The year 1955 marked a crucial turn in my mathematical work: that of the transition from "analysis" to "geometry". I still remember the associated striking impression (entirely subjective of course), as if leaving arid and surly steppes to suddenly find myself in a "promised land" of sorts, full of lush riches, multiplying themselves ad infinitum wherever I cared to look, to reap or to investigate... And this impression of overwhelming wealth, beyond all measure[31], only confirmed itself and deepened over the course of the years, up to this very day. p. P27

This is to say that if there is one thing in mathematics which fascinates me more than any other (and undoubtedly always has), it is neither "number" nor "size", but invariably **shape**. And among the thousand and one faces under which shape chooses to reveal itself to us, that which has fascinated me more than any other and continues to do so is the **structure** hidden in mathematical things.

The structure of a thing is not something which it is possible for us to "invent". We can only patiently unravel it, humbly get to know it and **"discover"** it. If there is any ingenuity involved in this line of work, and if we sometimes take up the role of a blacksmith or that of a tireless builder, it is never to "model" or to "construct" "structures" - they didn't have to wait for us to exist, and to be precisely what they are! Rather, it is to **express**, as faithfully as we can, those things which we are in the process of scanning and discovering, the structure that is reluctant to surrender and which we attempt to grasp, fumblingly, and through a perhaps fledgling language. Thus are we constantly led to **"invent" the language** best suited to ever more finely express the intimate structure of mathematical things, and to "construct" by means of this language, slowly and from the ground up, the "theories" that are supposed to report what has been apprehended and seen. Underlying this process is a continual, uninterrupted back-and-forth motion between the **apprehension** of things and the **expression** of that which has been apprehended, through a language that grows finer and is created anew over time, under the constant pressure of immediate needs.

As the reader will have no doubt guessed, these "theories", "constructed from the ground up", are but the **"beautiful houses"** which were discussed earlier: the houses which we inherit from our predecessors and those which we are led to build with our own hands, as we listen and follow the calling of things. Having mentioned earlier the "ingenuity" (or imagination) of the builder or the blacksmith, I should add that what lies at its heart is not the arrogance of he who asserts "I want this, and not that!" and who decides according to his whim; it is a pitiful architect who sets off with all of his plans fixed in his mind, before having even seen and felt the terrain, surveying its requirements and

[31] I chose to use the words "accablant, au delà de toute mesure" (in the French version) to express as well as I could the German expression "überwältigend", and its English counterpart "overwhelming". In the preceding sentence, the (inadequate) expression "striking impression" is also to be understood in this sense, namely: when the impressions and sentiments evoked in us after confronting a splendor, a grandeur, or a beauty out of the ordinary suddenly submerge us to such an extent that any attempt at expressing that which we feel seems doomed from the start.

possibilities. What characterizes the value of the ingenuity and imagination of a researcher is the **quality of his attention** as he listens to the voice of things - for the things of the Universe never tire of talking about themselves and to reveal themselves to he who cares to listen. Thus, the most beautiful house, p. P28 that in which the love of the builder is most evident, is not that which is larger or higher than the others. Rather, a house is beautiful if it faithfully reflects the structure and beauty hidden in things.

3.10 The novel geometry - or the marriage of number and size

But I digress once more - my intention was to speak of maître-thèmes, united under a common vision-mère, akin to rivers returning to the Sea from which they were born...

This vast and unifying vision can be described as a **novel geometry**. It appears to have been what Kronecker had dreamed of, in the last century[32]. Yet reality (which a bold dream can sometime lead us to anticipate or glimpse at, thereby encouraging us to pursue its discovery...) always surpasses in wealth and resonance even the deepest and most daring of dreams. Surely, concerning more than one arc of this novel geometry (if not all of them), nobody, even up to the very day preceding its appearance, could have seen it coming - the builder no more than anybody else.

One may say that "number" is capable of describing the structure of "discontinuous", or **"discrete"** aggregates: systems - often finite - composed of "elements" or "objects" isolated from one another, so to speak, without a notion of "continuous motion" from one object to the next. "Size", on the other hand, is the quality par excellence when it comes to **"continuous variation"**; as such, it is capable of describing structures and phenomena of a continuous nature: motion, spaces, "varieties" of all kinds, force fields, etc... Thus, arithmetic appears (roughly) to be the **science of discrete structures**, and analysis the p. P29 **science of continuous structures**.

As for geometry, for the more than two thousand years that it has existed as a science (in the modern sense of the term), it sits "halfway" between these

[32] I only know about "Kronecker's dream" from hearsay, from when somebody (it might have been John Tate) told me that I was realizing that very dream. In the education that I received from my elders, historical references were extremely rare. I mostly learned through direct communication with other mathematicians, both orally and in the form of letter correspondences, rather than by reading authored texts - be they ancient or contemporary. The main, and perhaps the only external inspiration that preceded the sudden and energetic beginnings of the theory of schemes in 1958, was Serre's famous article FAC ("Faisceaux Algébriques Cohérents"), which appeared a few years earlier. Apart from that article, much of my inspiration in further developing the theory came on its own, and renewed itself over the course of the years, through mere considerations of internal simplicity and coherence, in an effort to recover in this new context what was "well known" in classical algebraic geometry (material which I absorbed as it took a new form in my hands), and to follow what this "well known" material led me to anticipate.

two types of structures, the "discrete" and the "continuous"[33]. Indeed, for a long time, there wasn't really a **"divorce"** between **two** geometries of a fundamentally different kind, the one discrete and the other continuous. Rather, there were two different viewpoints involved in the investigation of the **same** geometric figures: the first focused on the study of "discrete" properties (notably, properties of a numerical or combinatoric nature), and the second treating of "continuous" properties (such as that of position in an ambient space, or that of "size" measured in terms of the pairwise distance between points, etc...).

It is only at the turn of the last century that a divorce emerged, marked by the apparition and development of what may be called **"(abstract) algebraic geometry"**. Roughly speaking, this consisted in introducing, for every prime number p, a theory of (algebraic) geometry "in characteristic p", modeled after the (continuous) analog of (algebraic) geometry developed over the course of the preceding centuries, yet in a context which appeared to be fundamentally "discontinuous", "discrete". These new geometric objects grew in importance since the beginning of the century, notably due to their close connections to arithmetic, the science par excellence of discrete structures. This seems to have been one of the leading ideas in the work of **André Weil**[34], perhaps even his principal idée-force (although it remained relatively tacit in his written work, as it should) - namely, that "the" theory of (algebraic) geometry, and particularly the "discrete" geometries associated to the various prime numbers, p. P30 were to provide the key for a vast renewal of the theory of arithmetic. It is in this spirit that the celebrated **"Weil conjectures"** were formulated in 1949. These were utterly spectacular conjectures which gave a glimpse, for these novel "varieties" (or "spaces") of a discrete nature, of the possibility of certain kinds of constructions and arguments[35] which up until then seemed only conceivable in the context of usual "spaces" in the sense understood by analysts - namely, so-called "topological" spaces (where the notion of continuous variation takes place).

Before all else, the novel geometry can be viewed as a synthesis between these two worlds, hitherto adjacent and tightly connected, but nonetheless sep-

[33] Admittedly, it was traditionally the "continuous" aspects which were the focus of the geometer, while properties of a "dicrete" nature, and notably arithmetic and combinatorial properties, were glossed over or hastily treated. It is with awe that I discovered, about a decade ago, the wealth of the combinatorial theory of the icosahedron, a theme which is not even scratched (and which was probably not even seen) in Klein's classical treatise on the icosahedron. I perceive another striking sign of this negligence (twice millennial) of geometers regarding discrete structures which spontaneously appear in geometry: it it the fact that the notion of group (notably group of symmetries) has only appeared in the last century, and that it furthermore was first introduced (by Evariste Galois) in a context which was not yet considered as pertaining to "geometry". It must be said that even nowadays, many algebraists have yet to understand that Galois theory is really, in essence, a **"geometric" vision**, coming to renew our understanding of so-called "arithmetic" phenomena...

[34] André Weil, French mathematician who emigrated to the United States, is one of the "founding members" of the "Bourbaki group" - which will be discussed further in the first part of Récoltes et Semailles (as will Weil himself, occasionally).

[35] (Intended for the mathematical reader) The "construction and arguments" mentioned here are those linked to the cohomology theories of smooth or complex manifolds, notably those leading to the Lefschetz fixed point formula and to Hodge theory.

arated: the **"arithmetic" world**, in which (so-called) "spaces" with no notion of continuity live, and the **world of continuous size**, where "spaces" in the proper sense of the term live and are accessible using the tools of the analyst, who (for this very reason) considers them worthy to dwell in the mathematical city. **Under the novel vision, these two hitherto separated worlds are merged into one.**

The first embryo of this vision of "arithmetic geometry" (the term I hereby suggest for this novel geometry) can be found in Weil's conjectures. In the development of some of my principal themes[36], these conjectures served as my main source of inspiration, throughout the years 1958-1969. Before me, **Oscar Zariski** on the one hand, and later **Jean-Pierre Serre** on the other, had developed some "topological" methods tailored for the unruly spaces of "abstract" algebraic geometry, inspired by the standard methods previously used for the
p. P31
"nice spaces" everybody knows[37].

Of course, their ideas played an important role during my first steps towards the development of the theory of arithmetic geometry; this was the case more so as starting points and as **tools** (which I had to more or less entirely remodel to cater to the needs of a much more general context), than as a source of inspiration that would have nourished my dreams and projects over the course of months and years. In any case, it was clear from the get-go that these tools, even remodeled, were far from sufficient in light of what was required to take even the very first steps in the direction of Weil's fantastic conjectures.

3.11 The magical fan - or the innocence

The two crucial "idées-forces" in starting up and developing the novel geometry were those of **schemes** and **topoi**. They appeared at roughly the same time and in tight symbiosis[38], and they acted as one and the same **motor nerve** in the spectacular expansion of the novel geometry, beginning the very year of the

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[36] I am here referring to the four "median" themes (n^{o}5 through 8), namely those of **topos**, **étale and** l**-adic cohomology**, **motives**, and (to a lesser extent) **crystals**. I unearthed these themes one by one between 1958 and 1966.

[37] (Intended for the mathematical reader) Zariski's main contributions in this sense seems to me to have been his introduction of the "Zariski topology" (which was later an essential tool for Serre in FAC), as well as his "connectedness principle", what he called his "theory of holomorphic functions" - which became in his hands the theory of formal schemes, and his "comparison theorems" between the formal and the algebraic (with, as a second source of inspiration, Serre's fundamental GAGA article). As for Serre's contribution alluded to in the above text, it refers of course, before all else, to his insertion of the viewpoint of sheaf theory into abstract algebraic geometry (a viewpoint which was introduced by **Jean Leray** about a dozen years earlier in an entirely different context), which takes place in his other fundamental article cited earlier: FAC ("Faisceaux algébriques cohérents").

[38] I speak of these beginnings, which took place in 1958, in the footnote on page 23. The notion of site, or **"Grothendieck topology"** (provisory prototype of the notion of topos) appeared in the immediate succession of the notion of scheme. It is that notion which in turn led to the new language of "localization" and "descent", used at every step of the development of the schematic theme and the schematic tools. The more intrinsic and geometric notion of topos, which remained implicit at first during the succeeding years, was mostly uncovered starting in 1963 with the development of étale cohomology, and later slowly imposed itself in

apparition. To complete this tour of my work, all that remains is for me to say a few words about these two ideas.

The notion of scheme is the most natural, the most "obvious", when it comes to englobe into a unique notion the infinite series of notions of (algebraic) "variety" which we manipulated previously (one such notion for **each** p. P32
prime number[39]...). In fact, a unique "scheme" (or "variety" nouveau style) gives rise to a well-defined "(algebraic) variety in characteristic p" for every prime number p. The collection of these various varieties in different characteristics can then be viewed as a sort of "(infinite) fan of varieties" (one for each characteristic). The "scheme" is precisely this magical fan, which links to one another several different "branches", its "avatars" or "incarnations" in every possible characteristic. Through this very fact, it provides an efficient "crossing point" between different "varieties", which had hitherto appeared as more or less isolated, separated from one another. They thus became englobed in a common "geometry" through which they were linked - what may be called **schematic geometry**, first sketch of the "arithmetic geometry" into which it was to grow in the following years.

The very idea of a scheme is of a childlike simplicity - so simple, so humble, that no one before me had even thought to look so low. So "silly", in fact, that for many years and despite the evidence pointing to the contrary, many of my erudite colleagues found this whole affair "not serious"! It actually took me months of intense and solitary work, to convince myself that it indeed "worked" just fine - that this new language, so silly, that I had the incorrigible naivety to persist in testing, was after all adequate in capturing in a new light and with greater finesse, and in a common framework, some of the very first geometric intuitions attached to the prior "geometries in characteristic p". It was the kind of exercise, considered mindless and doomed in advance by any "well informed" person, which I was without a doubt the only one, among all of my colleagues and friends, to ever dare attempt, and even (under the impulse of some secret demon...) carry to a successful end despite everybody's expectation!

Rather than letting myself be distracted by the consensus which prevailed around me, regarding what is considered "serious" and what isn't, I simply **trusted**, as I had before, the humble voice of things, and followed that within me which knew how to listen. The reward was immediate, and beyond all expectations. Within the span of a few months, without even "meaning to", I had discovered powerful and unexpected new tools. They allowed me to not p. P33
only recover old results, reputedly difficult, in a more telling light, but also to surpass them, as well as to finally tackle and solve problems in "geometry in characteristic p" which to that day seemed out of reach using the methods known at the time[40].

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my eyes as the more fundamental notion.

[39]This series should also include the case $p = \infty$ corresponding to algebraic varieties "in zero characteristic".

[40]The summary of these "energetic beginnings" to the theory of schemes was the subject of my talk at the International Congress of Mathematicians in Edinburgh, in 1958. The text of this exposé appears to me to be one of the best introductions to the schematic viewpoint, as

In our probing of things in the Universe (mathematical or otherwise), we dispose of a crucial rehabilitating power: **innocence**. By this I mean the original innocence which we have all received at birth and which rests within us, often the target of our scorn and of our deepest fears. It alone unites the humility and audacity which allow us to penetrate into the heart of things, while also allowing these things to penetrate into us and impregnate us with their meaning.

This power does not only come as a privilege for the extraordinarily "gifted" - (say) with an exceptional intellectual power allowing them to absorb and manipulate, with ease and dexterity, an impressing quantity of known facts, ideas, and techniques. Such gifts are admittedly precious, and susceptible to generate the envy of those (like myself) who were not so gifted at birth, "beyond all measure".

Yet, it is not those gifts, nor even the most burning of ambitions, accompanied with a relentless will, which allow us to cross the "invisible and imperious circles" which enclose our Universe. Only innocence can cross them, without noticing or even caring to, during the times where we find ourselves alone and listening to the voice of things, intensely absorbed in child's play...

3.12 Topology - or surveying through the mist

The innovative idea of "scheme" is, as we just discussed, that which allows us to bring together the various "geometries" associated to different prime numbers (i.e. to different "characteristics"). These geometries nonetheless each remained essentially "discrete" or "discontinuous" in nature, in contrast with the traditional geometry inherited from centuries past (going back all the way to Euclid). The new ideas introduced by Zariski and Serre restored to some
p. P34 extent a "dimension" of continuity for these geometries, which was instantly inherited by the "schematic geometry" which had just appeared, with the aim of uniting them. But we were still a long way from approaching the "fantastic conjectures" (of Weil). These "Zariski topologies" were, from this viewpoint, so coarse that it was almost as if we had remained at the stage of "discrete aggregates". What was visibly missing was some novel principle, one which would allow us to connect these geometric objects ("varieties", or "schemes") to the usual "nice" (topological) "spaces"; those spaces, say, in which "points" are clearly **separated** from one another, something which failed in Zariski's unruly spaces, where points had the poor tendency to cluster around one another...

Decidedly, it was the appearance of such a "novel principle", and nothing else, which would allow for this "marriage of number and size", of the "geometry of the discontinuous" with that of the "continuous", of which Weil's conjectures were a first premonition.

The notion of **"space"** is undoubtedly one of the oldest in mathematics. It is so fundamental to our "geometric" apprehension of the world around us, that

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a means to (perhaps) motivate the geometer reading it to gain familiarity with the imposing (later) treatise "Eléments de Géométrie Algébrique", which exposes in depth (and without omitting any technical detail) the new foundations and new techniques of algebraic geometry.

is has remained more or less tacit for over two millennia. Only in the last century did this notion finally begin to progressively detach itself from the tyrannical stranglehold of immediate perception (that of a unique "space" surrounding us), and from its traditional ("euclidian") theorization, in an effort to acquire an autonomy and dynamic of its own. Nowadays, it is one of the most universal and most commonly used notions in mathematics, one with which every mathematician is almost surely familiar. It is moreover a remarkably multifaceted notion, one of a thousand and one faces, depending on the structure with which we choose to equip these spaces, from the richest of all (such as the venerable "euclidian" structures, or the "affine" or "projective" structures, or the "algebraic" structures of the eponymous "varieties"", which generalize and relax the latter) to the most bare: those where all forms of "quantitative' information seems to have been loss forever, and where alone subsist the quintessence of the qualitative notion of **"proximity"** or that of **"limit"**[41], as well as the most elusive version of the notion of (so called "topological") **shape**. The most bare of all such notions, which to this day had served as a sort of vast and common p. P35
conceptual bosom englobing all the others, was that of **topological space**. The study of such spaces constitutes one of the most fascinating and vigorous branches of geometry: that of **topology**.

As elusive as this "purely qualitative" structure incarnated by a (so-called "topological") "space" might seem at first sight, in the complete absence of data of a quantitative nature allowing us to hold on to some familiar intuition of "largeness" or "smallness" (such as the distance between two points), we have nonetheless succeeded, over the course of the previous century, to finely capture these spaces in the tight and supple meshes of a carefully "tailor-made" language. Better yet, we have invented and created from the ground up all sorts of "meters" and "metersticks" that let us, against all odds, attached "measurements" of sorts (called "topological invariants") to these tentacular "spaces" that previously seemed to escape all attempts to measure them - akin to an elusive mist. It is true that most of these invariants, including the most essential, are of a subtler nature than the simple notions of "number" or "size" - they are themselves more or less delicate mathematical structures, attached to the space under consideration (by means of more or less sophisticated constructions). One of the oldest and most crucial of these invariants, introduced already in the last century (by the mathematician **Betti**), consists of various so-called "cohomology groups" (or "spaces") associated to a space[42]. These cohomology groups

[41]When speaking of the notion of "limit", I mean mostly that of "passing to the limit", rather than the notion (more familiar to the non-mathematical reader) of "boundary".

[42]What Betti actually introduced were so-called **homological** invariants. **Cohomology** constitutes a more or less equivalent version of those, a "dual" notion that was only introduced much later. The latter notion gained prevalence over the initial one of "homology" due (doubtlessly) to Jean Leray's introduction of the viewpoint of sheaves, about which I write in more detail later. At the technical level, one could say that a large portion of my work as a geometer consisted in unearthing, and in developing to various extents, the cohomology theories that were missing for all kinds of spaces and varieties, particularly for "algebraic varieties" and for schemes. Along the way, I was brought to reinterpret the traditional homological invariants in cohomological terms, and thereby to reintroduce them in an entirely new light.

p. P36 are involved (mostly "between the lines", admittedly) in Weil's conjectures, and they are what constitutes their deeper "raison d'être" (in my eyes at least, after having been "brought to speed" by Serre's explanations), what gives them their full meaning. But the very possibility of associating such invariants to the "abstract" algebraic varieties occurring in these conjectures, in a way that would respond to the very specific desirata imposed by the needs of the situation, was no more than a mere hope. I doubt that anybody other than Serre and myself (including, and especially, André Weil himself![43]) really believed this could work...

A little earlier, our understanding of cohomological invariants found a deep enrichment and renewal through the work of **Jean Leary** (produced while he
p. P37 was held in captivity in wartime Germany, during the first half of the 1940s). The essential innovation was the notion of (abelian) **sheaf** over a space, to which Leray associated a sequence of "cohomology groups" (said to "take values in that sheaf"). It was as if the good old classical "cohomological meterstick" of which we hitherto disposed to "survey" a space was suddenly multiplied into

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Many other "topological invariants" were introduced by topologists, in their attempts to capture various kinds of properties of topological spaces. Apart from the "dimension" of a space and (co)homological invariants, the first invariants to have been introduced were the "homotopy groups". I introduced another one in 1957, the (so-called "Grothendieck") group $K(X)$, which immediately took off successfully, and whose importance (in topology as well as in arithmetic) continued to grow over time.

The introduction of a host of new invariants, of a nature more subtle than the currently known and used invariants, yet which I sense are fundamental, is part of my program of "tame topology" (*"topologie modérée" in French*) (of which a brief sketch can be found in the "Sketch of a Program" ("Esquisse d'un Programme" in French) to appear in volume 4 of the Reflections). This program is based on the notion of "tame theory", or "tame space", which constitutes, in the same way as the topos, a (second) "metamorphosis of the notion of space". This one is much more obvious (in my eyes) and much less deep than the latter. I expect that its immediate implications in "classical" topology will nonetheless be felt much more acutely, and that it will fundamentally transform the "profession" of the topological geometer, via a profound transformation of the contextual framework in which his work takes place (as was the case in algebraic geometry with the introduction of the viewpoint of scheme theory). I have already send my "Esquisse" to many of my old friends and reputed topologists, but none of them seems to have found it particularly interesting...

[43]Paradoxically, Weil had a persistent "block", seemingly visceral, against the cohomological formalism - even though his famous conjectures were in large part what inspired the development of cohomological theories in algebraic geometry, starting from the year 1955 (with Serre marking the starting point, with his fundamental article FAC, already mentioned in an earlier footnote). It seems that, for Weil, this "block" fit into a general aversion against "big machinery", i.e. that which fits into a formalism (which cannot be summed up into a few pages) or takes the form of a relatively nested "construction". Weil was not a "builder", admittedly, and it was almost against his own will that he was forced, during the 1930s, to develop the initial foundations of "abstract" algebraic geometry, foundations which (in view of his propensity) turned out to be a "Procrustean bed" for the user.

I do not know whether he held a grudge against me for going beyond, and for engaging in the construction of vast dwellings which later allowed for Kronecker's dream as well as his own to find an incarnation as a language and as a collection of efficient and delicate tools. In any case, he has never communicated a single word to me regarding the work in which he saw me involved, or regarding the work that had already been done. I received no echo concerning Récoltes et Semailles either, of which I had sent him a copy more than three months earlier, accompanied by a hearty handwritten dedication.

an unimaginably large multiplicity of new "metersticks" of all sizes, shapes, and substance imaginable, each of them intimately linked to the space at hand, and conveying to us information about it with perfect precision, information which it alone was in a position to reveal. The notion of a sheaf was the "idèe maîtresse" of a profound transformation in our probing of spaces of all kinds, and surely one of the most crucial ideas to have appeared in the course of this century. Thanks mostly to the later work of Jean-Pierre Serre, Leray's ideas first bore fruit during the decade following their appearance, in the form of an impressive restart of the theory of topological spaces (notably the theory of so-called "homotopical" invariants, intimately linked to cohomology), as well as in the form of another restart, no less capital, in so-called "abstract" algebraic geometry (through Serre's fundamental FAC article, which appeared in 1955). My own work in geometry, beginning in 1955, fits in the continuity of Serre's work, and thus also falls in line with Leray's innovative ideas.

3.13 Topoi - or the double bed

The viewpoint and language of sheaf theory introduced by Leray brought us to view "spaces" and "varieties" of all kinds in a new light. They did not touch the notion of space itself; rather, they allowed us to probe more finely, with new eyes, the traditional "spaces" which were already familiar to all. However, it turned out that this notion of space was inadequate to capture the most essential "topological invariants" expressing the "shape" of "abstract" algebraic varieties (including those which applied to Weil's conjectures), or the shape of general "schemes" (which generalized the old varieties). For the anticipated "marriage of number and size", the existing notion of space was a rather cramped bed, where only one of the future spouses (namely, size) could with some effort manage to fit, but certainly not both at once! The "novel principle" which remained to be found, to arrive at the promised wedding, was nothing but the spacious "bed" which the future spouses were missing, escaping everybody's notice up to this point...

This "double bed" appeared (as if by magic) with the idea of the **topos**. This idea englobes in a common topological both the traditional (topological) p. P38
spaces, incarnating the world of continuous size, and that (so-called) "spaces" (or "varieties") of the impertinent abstract algebraic geometers, as well as countless other types of structures which seemed hitherto to be irremediably locked in the "arithmetic world" of "discontinuous" or "discrete" aggregates.

It is the viewpoint of sheaf theory which was the silent and sure guide, the efficient (and not in any way secret) key which led me without delay nor detours towards the bridal chamber and its ample marital bed. A bed so vast indeed (akin to a vast and tranquil deep river), that

"all of the king's horses
could drink from it at once..."

[44] - as goes an old air which you surely have sung in the past, or at least heard. And the person who first sang it has better felt the secret beauty and serene strength of the topos than any of my wise students and friends of yesteryear...

The key was the same, both in the initial and provisory approach (via the convenient but non-intrinsic notion of "site") and in that of topos. It is the idea of the topos which I would now like to try describing.

Consider the set of **all** sheaves on a given (topological) space, which is, in a sense, the prodigious arsenal formed by **all** the "metersticks" used to probe this space[44]. We consider this "set" or "arsenal" equipped with its most evident structure, which appears - so to speak - "in front of our nose"; namely, the structure of a "category". (The non-mathematical reader needs not worry if the term is unfamiliar. Its meaning will not be needed in what follows.) It is this "probing superstructure" of sorts, called "category of sheaves" (on the space considered), which will henceforth be considered as the "incarnation" of what is most essential in the space. Such a perspective is legitimate (in terms of mathematical "common"-sense), in that we can entirely "recover" a topological
p. P39 space[45] from its associated "category of sheaves" (or probing arsenal). (To verify this fact is a simple exercise - admittedly, the question first needs to be asked...) This suffices to assure us that (if it fits our purposes for one reason or another) we can henceforth "forget" the initial spaces, and only retain and use the associated "category" (or arsenal), thought of as the most adequate incarnation of the "topological (or spatial) structure" which we are trying to express.

As often happens in mathematics, we have succeeded (through the crucial idea of a "sheaf", or "cohomological meterstick") to express one notion (that of "space" in this case) in terms of another (that of "category"). Each time, the discovery of such a **translation** of one notion (expressing a certain type of situation) in terms of another (corresponding to another type of situations) enriches our understanding of both notions, through the unexpected confluence of specific forms of intuition pertaining to one notion or the other. Thus, a situation of a "topological" nature (incarnated by a given space) is hereby translated into a situation of an "algebraic" nature (incarnated by a "category"); alternatively, the "continuity" incarnated by the space is "translated" or "expressed" by the structure of the associated category, which is of "algebraic" nature (something which had hitherto been perceived as being of a fundamentally "discontinuous"

[44]Translator's note: In French, the poem reads:

"tous les chevaux du roi
y pourraient boire ensemble..."

[44](Intended for the mathematical reader) To be precise, I am here speaking of sheaves of **sets**, not of the **abelian** sheaves which were introduced by Leray as the most general type of coefficients that could be used to construct "cohomology groups". I believe I was the first to have systematically worked with sheaves of sets (starting from 1955, in my article "A general theory of fibre spaces with structure sheaf" at the University of Kansas).

[45](Intended for the mathematical reader) Strictly speaking, this only holds for so-called "sober" spaces. These nonetheless accounts for the near-totality of the spaces which one commonly encounters, including all of the "separated" spaces that are dear to the analysts.

or "discrete" nature).

But on the other side, there is more. The first of these notions, that of a space, seemed to be a "maximal" notion - so general already, that we could hardly imagine how one could go about finding any "reasonable" further extension. On the other hand, these "categories" (or "arsenals") which one encounters on the other side of the looking glass[46], starting from topological spaces, are of a very particular form. They enjoy a collection of very specific properties[47], which liken them to a "pastiche" of the simplest such category - that obtained as the mirror of a one point space. Thus, a "space nouveau style" (or topos), generalizing the traditional notion of topological space, can be described simply as a "category" which, without necessarily coming from a specific topological space, possesses all of the good properties (singled out once and for all) pertaining to such a "category of sheaves".

p. P40

This, then, was the novel idea. Its appearance can be viewed as a consequence of the (nearly childlike) observation that what really matters in a topological space isn't its "points" or its subsets of points[48] (and the proximity relations between them, etc...); rather, it is the **sheaves** on that space that matter, and the category which they form. I have only, in effect, carried Leray's initial idea to its ultimate consequence - and, having done this, **taken the step**.

As with the very idea of sheaves (due to Leray), or that of schemes, indeed as for any "great idea" which comes to disrupt an ingrained vision of things, the notion of topoi is unsettling due to its natural character, its "evidence"; due to a simplicity (which is almost, one could say, naive or simplistic, even "silly") of the particular flavor which makes us so often exclaims: "Oh, that's all there was to it!", in a half disappointed, half envious tone; with perhaps an undercurrent pertaining to the "zany", the "unreasonable", which is often reserved to that which is shocking through an excess of unexpected simplicity. A simplicity which reminds us, perhaps, of the long buried and repressed days of our childhood.

3.14 Mutation of the notion of space - or breath and faith

The notion of scheme constitutes a vast enlargement of the notion of "algebraic variety", and as such it has led to a profound renewal of the algebraic geometry

[46]The "looking glass" in question, as in Alice Through the Looking Glass, is that which returns as "image" of a given space the associated "category", considered as a sort of "double" of the space, "on the other side of the looking glass".

[47](Intended for the mathematical reader) I am here referring mostly to the properties which I have introduced in category theory under the name of "exactness properties" (at the same time as the modern categorical notion of general inductive and projective "limits"). See "Sur quelques points d'algèbre homologique", Tohoku math. journal, 1957 (p. 119-221).

[48]Indeed, it is possible to construct very "large" topoi which nonetheless have only one "point", or even no "points" at all!

inherited from my predecessors. As for the notion of topos, it constitutes an unexpected enlargement, or rather, **a metamorphosis of the notion of space**. It thereby bears promise for a similar renewal of topology, and beyond it, of geometry. Already, it has played a crucial role in the expansion of the novel geometry (especially through the themes of l-adic and crystalline cohomology issued from it, and the resulting proof of the Weil conjectures). Like its older sister (and quasi-twin), it possess the two complementary characteristics that are essential to any fruitful generalization, which are described in what follows.

First, the new notion is not **too vast**, in that inside the new "spaces" (called
p. P41 "topoi" so as not to bother delicate ears[49]), the most essential "geometric" intuitions and constructions[50], familiar within the traditional spaces of yesteryear, admit a more or less obvious transposition. In other words, when working with the new objects, we have at our disposal the entirety of the rich array of the images, mental associations, notions, and to some extent the techniques which hitherto were restricted to the old fashioned objects.

Second, the new notion is at the same time **sufficiently vast** to englobe a host of situations which were hitherto not considered to be suitable for intuition of a "topologico-geometric" nature - an intuition which we had in the past precisely reserved to the study of ordinary topological spaces (and for good reasons...).

Crucially, from the perspective of the Weil conjectures, this new notion is vast enough to allow us to associate to any "scheme" such a "generalized space" or "topos" (called the "étale topos" associated to the scheme under consideration). Some of the "cohomological invariants" of this topos ("silly" as they may be!) then seemed to stand a good chance to provide us with "all that was needed" to make sense of these conjectures, and (who knows) to give us the means of proving them.

It is in these pages which I am currently writing that, for the first time in my life as a mathematician, I take the time to depict (even if only to myself) the collection of "maître-thèmes" and great governing principles underlying my mathematical work. This leads me to better appreciate the place and scope of each of these themes and of the "viewpoints" which they incarnate, within the grand geometric vision which unites them and from which they are issued. It is through this process that : the notion of **schemes**, and that of **topoi**.

p. P42 It is the second of these ideas, that of topoi, which presently appears to me as the deeper of the two. If, by adventure, I had not taken it upon myself in

[49]The name "topos" was chosen (in association with "topology", or "topological") to suggest that this is the "objet par excellence" to which topological intuition applies. In line with the rich clod of mental images that this name evokes, it is to be considered as essentially the equivalent of the term (topological) "space", only with a stricter emphasis on the "topological" specificity of the notion. (Thus, one can speak of "vector spaces", but not, as far as I am aware, of "vector topoi"!) It is necessary to keep the two expressions conjointly, each with its particular specificity.

[50]Among these "constructions" is comprised notably that all of the familiar "topological invariants", including cohomological invariants. Concerning the latter, I had already laid all of the necessary groundwork in the previously mentioned article ("Tohoku", 1955) necessary to express them in an arbitrary "topos".

the late 1950s to diligently develop, day after day, over the course of twelve long years, a "schematic tool" of perfect accuracy and power, it seems almost unthinkable that within the following ten to twenty years, others would not have eventually introduced (be it even against their will) the notion which was visibly required, putting together at the very least a handful of "prefabricated" shabby barracks, instead of the spacious and comfortable dwellings which I took to heart to assemble stone by stone and to erect with my own hands. On the other hand, I cannot think of anyone else on the mathematical scene who, in the course of the past three decades, could have had the naiveté, or innocence, required to take (in my stead) the most crucial step of all in introducing the childlike idea of the topos (or even that of "sites"). In fact, even presupposing that the idea had been generously provided, and with it the shy promise it seemed to enclose - I see nobody else, be it among my old friends or among my students, who would have had the breath, and most importantly the faith, to carry this humble idea (so derisory in appearance, while the goal seemed infinitely distant...) to completion[51]: from its fledgling beginnings to full maturity and the "mastery of étale cohomology" into which it ended up transforming between my hands, during the years that followed.

3.15 All of the king's horses...

Indeed, deep is the river, and vast and tranquil are the waters of my childhood, in a kingdom I thought I had long since left. All of the king's horses could drink from these waters at once, at ease and to their full satisfaction, without exhausting them! The water comes from glaciers, ardent as the distant snowfalls, and it is soft as the loam of the plains. I just told you about one of these horses, which a child brought to drink at length and to its fill. I also saw another horse p. P43
come to drink for a moment, possibly following the footsteps of the same kid - but not for long. Someone must have chased it away. And that is all, I must say. And yet I see countless herds of parched horses roaming the plains - in fact, just this morning, their neighs got me out of bed, at an undue hour, even though I am in my sixties and cherish tranquility. There was no choice, I had to wake up. It hurt to see them in this state, weak and beaten up, even though there was no shortage of fresh water or green pastures. It was as if a malicious

[51](Intended for the mathematical reader) When I speak of "carrying this humble idea to terms", I refer to the idea of étale cohomology as an approach to the Weil conjectures. It is under this inspiration that I discovered the notion of site in 1958, and that this notion (or the closely tied notion of topos), as well as the formalism of étale cohomology, were developed between the years 1962 and 1966 following my impulsion (and assisted by certain collaborators who will be mentioned at a later point).

When I speak of "breath" and of "faith", I am referring to qualities of a "non-technical" nature, yet which here really appear to be the essential required qualities. At another level, I could also add what could be called "cohomological flair", meaning the kind of flair which had developed within me while working on the edification of cohomological theories. I was under the impression that I had communicated that flair to my cohomology students. In hindsight, seventeen years after my departure from the mathematical worlds, I realize that it did not persist in any of them.

spell had been cast on a region which I once knew to be welcoming, blocking off all access to these generous waters. Or perhaps was it a plot orchestrated by the country's horse dealers to drop prices, who knows? Or was this a country where no children were left to lead the horses to the water, and where the horses suffered from thirst, for lack of a child to show them the way back to the river...

3.16 Motives - or the heart within the heart

The theme of topoi was issued from that of schemes, in the same year that schemes appears - yet it far surpasses the mother theme in its scope. It is the theme of topoi, not that of schemes, which constitutes this "bed", or "deep river", in which geometry and algebra, topology and arithmetic, mathematical logic and category theory, the world of the continuous and that of "discontinuous" or "discrete" structures come to be wed. If the theme of schemes is to be seen as the **heart** of the novel geometry, then the theme of topoi is its envelop, or its **dwelling**. It is the vastest of my conceptions, in its ability to subtly grasp, with a unique language rich in geometric resonance, the common "essence" of situations far removed from one another, coming from one region or another of the vast universe of mathematical things.

Yet, the theme of topoi is far from having encountered the success of that of schemes. Now is not the time to speak of the strange vicissitudes that have affected this notion - I touch on this topic on various occasions in Récoltes et Semailles. Two of the "maître-thèmes" of the novel geometry are nonetheless issued from that of topoi, namely two complementary "cohomology theories", both conceived with the intention of formulating an approach towards the Weil conjectures: the **étale** (or **"l-adic"**) **theme**, and the **crystalline theme**. The first concretized in my hands into the l-adic cohomology tool, which has already imposed itself as one of the most powerful mathematical tools of the century. As for the crystalline theme, reduced to a quasi-occult status following my departure, it was finally exhumed (under pressure of needs) in June 1981, in the limelight and under a borrowed name, in circumstances stranger even than
p. P44 those surrounding topoi...

The l-adic cohomology tool was, as predicted, the key tool used to establish the Weil conjectures. I proved several of them myself, and the last step was taken masterfully three years later by Pierre Deligne, the brightest of my "cohomologist" students.

Around the year 1968, I had also formulated a stronger and more "geometric" version of the Weil conjectures. The latter remained "tied" (so to speak!) to an apparently irreducible "arithmetic" aspect, even though the very spirit of these conjectures is to express and capture the "arithmetic" (or the "discrete") by means of the "geometric" (or the "continuous")[52]. In this sense, the ver-

[52](Intended for the mathematical reader) The Weil conjectures are subject to hypotheses of an "arithmetic" nature, stemming from the fact notably that the varieties under consideration are defined over a finite field. From the perspective of cohomological formalism, this leads to the **Frobenius endomorphism** playing a special role in the situation. In my approach,

sion of the conjectures which I had formulated seem to me more "faithful" to "Weil's philosophy" than Weil's own conjectures - a philosophy left unwritten and rarely told, yet which was perhaps **the** main tacit motivation underlying the extraordinary expansion of geometry over the course of the past four decades[53]. My reformulation essentially consisted in extracting a "quintessence" of sorts of what had to remain true in the context of so-called "abstract" algebraic varieties, of classical "Hodge theory", valid for "ordinary" algebraic varieties[54]. I called this new, entirely geometric version of Weil's famous conjectures the **"standard conjectures"** (for algebraic cycles).

In my mind, this constituted a new step towards these conjectures, following the development of the l-adic cohomology tool. But it was also and most importantly a principle through which one could approach what I consider to this day to be the deepest theme which I have introduced in mathematics[55]: that of p. P45 **motives** (which itself was born out of the "l-adic cohomology theme"). This theme is akin to the **heart** or soul, the part most concealed and hidden from view, of the schematic theme, which itself lies at the heart of the novel vision. And the handful of key phenomena unearthed by the standard conjectures[56] can in turn be conceived of as forming a sort of ultimate quintessence of the motivic theme, akin to the vital **'breath"** of this most subtle of themes, constituting the **"heart within the heart"** of the novel geometry.

Here is a rough overview of what these motives are about. We saw earlier, for a given prime number p, the importance (notably from the perspective of the Weil conjectures) of knowing how to construct "cohomology theories" for "(algebraic) varieties in characteristic p". In fact, the "l-adic cohomology tool" provides just such a theory; it even provides an **infinity of different cohomology theories**, one for each prime number different from the underlying characteristic p. There remains visibly a "missing theory", corresponding to the case where l equals p. To provide such a theory, I conceived of yet another cohomology theory (which I already alluded to earlier) called "crystalline cohomology". Furthermore, in the important case where p is infinite, we have at our disposal three additional cohomology theories[57] - and there is no reason why

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the crucial properties (of the kind "generalized index theorem") concern **arbitrary** algebraic correspondences, and make no hypothesis of an arithmetic nature on the given base field.

[53]However, there was following my departure in 1970 a clear reactionary movement, concretized by a situation of relative stagnation, which I evoke more than once in Récoltes et Semailles.

[54]Here, "ordinary" means "defined over the field of complex numbers". Hodge theory (said to be the "theory of harmonic integrals") was then the most powerful known cohomology theory in the context of complex algebraic varieties.

[55]This is at least the deepest theme which I introduced during the "public" period of my mathematical activity, between the years 1950 and 1969, up to my departure from the mathematical scene. I consider the themes of nonabelian algebraic geometry and Galois-Teichmüller theory, developed starting from the year 1977, to be of comparable depth.

[56](Intended for the algebraic geometers) There eventually came a time when these conjectures had to be reformulated. For a more detailed commentary on this, see "A tour of the construction sites" (ReS IV note n^o 178, p. 1215-1216) and the footnote on p. 769 of "Conviction and knowledge" (ReS III, n^o 162).

[57](Intended for the mathematical reader) These theories correspond, respectively, to Betti cohomology (defined by transcendental methods, via an embedding of the base field into the

we wouldn't be led in the future to introduce other new cohomology theories, sharing analogous formal properties. Contrarily to what happens in ordinary topology, we are thus faced with a disconcerting abundance of different cohomology theories. One had the net impression that, in a sense which had yet to be clarified, all of these theories had to be "essentially the same", that they "gave
p. P46 the same results"[58]. It is in order to express this intuition of "kinship" between different cohomology theories that I have formulated the notion of **"motive"** associated to an algebraic variety. In choosing this term, I intended to suggest the interpretation of "common motive" (or "common **reason**") underlying this multitude of different cohomological invariants associated to the given variety, by means of the multitude of a priori available cohomology theories. Under this framework, these different cohomology theories would appear as different thematic developments, each in the most appropriate "tempo", "key" or "mode" ("major" or "minor"), of a single "base motive" (called "**motivic** cohomology theory"), while the latter would at the same time be the most fundamental, or "finest", of all of the different thematic "incarnations" (meaning, of all of the possible cohomology theories). Thus, the motive associated to an algebraic variety would constitute the "ultimate" cohomological invariant, the invariant "par excellence", from which all others (associated to the various possible cohomology theories) could be deduced, as various musical "incarnations", or as different "realizations". All the essential properties of "**the** cohomology" of the variety could be "read" (or "heard") on the corresponding motive, so that all of the familiar properties and structures pertaining to particular cohomological invariants (such as l-adic or crystalline invariants) would simply appear as the
p. P47 faithful reflections of properties and structures **internal to the motive**[59].

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field of complex numbers), Hodge cohomology (defined by Serre), and De Rham cohomology (defined by myself), the latter two tracing their origins to the 1950s (and Betti cohomology to the last century).

[58](Intended for the mathematical reader) For instance, if f is an endomorphism of the algebraic variety X inducing an endomorphism of the cohomology group $H^i(X)$, the "characteristic polynomial" of the latter should have **integral** coefficients, independently of the specific cohomology theory we have chosen (e.g. l-adic cohomology for varying l). Ditto for general algebraic correspondences when X is assumed to be proper and smooth. The sad truth (which hints at the state of deplorable abandon of the cohomology theory of algebraic varieties in characteristic $p > 0$ since my departure) is that this has not yet been proven as the time of writing, even in the special case where X is a smooth projective **surface** and $i = 2$. In fact, as far as I am aware, no one after my departure cared to look into this crucial question, typical of the kind of questions which appear as subordinate to the standard conjectures. The decree in vogue is that the only endomorphism worth our attention is the Frobenius endomorphism (which was treated separately by Deligne with the means at hand)

[59](Intended for the mathematical reader) Another way to think about the category of motives on a field k is to visualize it as a sort of "enveloping abelian category" of the category of separated schemes of finite type over k. The motif associated to such a scheme X (also called the "motivic cohomology of X", which I denote by $H^*_{\mathrm{mot}}(X)$) thereby appears as an "abelianized" avatar of sorts of X. Crucial here is the fact that, just as an algebraic variety X can be subject to "continuous variation" (so that its isomorphism class depends on continuous "parameters", or "modules"), the motive associated to X, or more generally a "variable" motive, should also be subject to continuous variation. The latter aspect of motivic cohomology stands in stark contrast to the behavior of all classical cohomological invariants, including l-adic invariants, with the only exception of Hodge cohomology for complex algebraic varieties.

Such is, expressed in the non-technical language of a musical metaphor, the quintessence of an idea once again of childish simplicity, at once delicate and bold. I developed this idea on the margins of the foundational tasks which I considered more urgent, under the name of "theory of motives" or "philosophy (or yoga) of motives" throughout the years 1963-69. It is a theory of fascinating structural richness, about which a great deal remains conjectural to this day[60].

I express myself about this "yoga of motives", which I hold particularly dear to my heart, at various points in Récoltes et Semailles. Now is not the time to repeat what I have written about it elsewhere. I shall only mention that the "standard conjectures" ensue in the most natural way from this yoga of motives. At the same time, they also provide an approach to one of the possible constructions of the notion of motive.

These conjectures seemed to me (and they still do to this day) to be one of the two most fundamental standing questions in algebraic geometry. Neither this question, nor the equally crucial other question (concerning the "resolution of singularities") has been resolved as of today. But while the second question appears today, as it did a hundred years ago, to be prestigious and formidable, the question which I had the honor of formulating was relegated by the peremp- p. P48
tory decrees of fashion (beginning in the years that followed my departure from the mathematical scene, just as it happened to the motivic theme itself[61]) to the status of benign grothendieckian farce. But once again I am getting ahead of myself...

This should give an idea of the extent to which "motivic cohomology" is a finer invariant, capturing much more tightly the "arithmetic shape" (if I dare use this expression) of X than traditional invariants of a purely topological nature can. In my vision, motives constitute a subtly hidden and delicate "thread" linking together the algebro-geometric properties of an algebraic variety and the properties of an "arithmetic" nature incarnated by its associated motive. The latter can be considered as an object of a "geometric" nature in its very spirit, albeit one where "arithmetic" properties subordinate to the geometry are, so to speak, "laid bare".

Thus, the motive appears to me to be the deepest "shape invariant" which we have been able to associate to an algebraic variety to this day, with the exception of its "motivic fundamental group". Both invariants represent in my eyes the "shadows" of a "motivic homotopy type" which is yet to be described (and about which I write a few words in passing in the note "A tour of the construction sites - or tools and vision" (ReS IV, n^{o}178, see construction site 5 (Motives), notably page 1214)). It is the latter object which in my eyes ought to be the most adequate incarnation of the elusive intuition of the "arithmetic (or motivic) shape" of an arbitrary algebraic variety.

[60] I have explained my vision of motives to anyone who cared to listen, throughout these years, without going through the trouble of publishing anything about this subject in print (having to cater to several other tasks at the service of all). This later allowed certain of my students to plunder more at ease, under the tender eye of all of my old friends, well informed about the situation. (See the footnote on the following page.)

[61] In fact, the motivic theme was exhumed in 1982 (a year after the crystalline theme), in its original name this time (and in a narrowed form, accounting only for the characteristic zero case), with no mention of the name of the original craftsman in sight. This is an example among others of a notion or theme buried following my departure as grothendieckian phantasmagories, only to be exhumed one after the other by certain of my students during the ten to fifteen years that followed, with a modest pride and (need I mention it again) without mentioning the original craftsman...

3.17 Toward the discovery of the Mother - or the two sides

To tell the truth, my reflections upon the Weil conjectures themselves, in the process of establishing them, remained sporadic. The panorama which had begun to open up in front of me, and which captured my attention in my attempt to scrutinize it and render it, far surpassed in amplitude and in depth the hypothetical needs of a proof; in fact, it even surpassed all that these famous conjectures had allowed us to glimpse in the first place. With the sudden appearance of the schematic theme and that of topoi, a new and unsuspected world had suddenly opened up. "The conjectures" played a central role, to be sure, akin somewhat to the role which the capital city of a vast empire or continent would play in relation to the countless surrounding provinces, with most of the latter nonetheless maintaining only the most distant rapports with the brilliant and prestigious capital. Without ever having to spell it out for myself, I knew that I had from that point onwards become the servant to a great task: that of exploring this immense and unknown world, of apprehending its contours all the way to the farthest frontiers; and at the same time, of roaming in every direction and establishing with tenacious and methodical care the inventory of neighboring and distant provinces alike, drawing maps with scrupulous fidelity and precision, in which the smallest of hamlets and cottages would find their place...

It is mostly the above work which ended up absorbing the greatest part of my energy - a patient and vast foundational work which I alone perceived with clarity and, most importantly, "felt in my guts". It is that work which by far occupied the most of my time, between 1958 (the year where the schematic theme and that of topoi appeared, one after the other) and 1970 (the year of my departure from the mathematical scene).

I often had to step on the breaks, feeling restrained as if by a persistent and clinging weight by the endless tasks which (once what was essential had been established) felt closer to an act of "stewardship" than to a thrust into
p. P49 the unknown. I had to constantly resist the urge to plow ahead - the urge of the pioneer or explorer, gone at the discovery and exploration of unknown and unnamed worlds, ceaselessly calling him to come to know them and give them a name. That urge, and the energy which I devoted to it (almost covertly!) were constantly present in the background.

Yet, I knew deep down that it was precisely that energy, snatched (so to speak) from what I owed to my "tasks", which was of the rarest and most unbound essence - that it was **there** that lay the "creation" in my work as a mathematician: in this intense attention for apprehending, within the obscure, amorphous and moist folds of a warm and inexhaustible nourishing womb, the first traces of shapes and contours yet to be born and which seemed to be calling me in an effort to take form, to become incarnate and come to life... In the process of discovery, this intense attention and ardent solicitude are an essential force, akin to the sun's heat contributing to the underground gestation of sowings buried in the nourishing earth, carrying them toward their humble and miraculous hatching in broad daylight.

In my work as a mathematician, I mostly see two forces or urges at play, both equally profound, yet of a different nature (or so it seems to me). To evoke one and the other, I have used in turn the image of the **builder** and that of the **pioneer** or explorer. Put next to each other, they now both strike me suddenly as very “yang”, very “masculine”, even “macho”! They bear the haughty resonance of myths, that of “grand occasions.” Surely they were inspired within me by my old “heroic” vision of creative work, the super-yang vision. As such, they offer a strongly tinted perspective, not to say frozen, “ten-hut”, of a much more fluid reality, humbler and “simpler” - a **living** reality.

In this male urge of the “builder”, pushing me ceaselessly toward new construction sites, I can nonetheless discern equally the urge of the **homebody**: that of a person profoundly attached to **“the”** home. Before anything else, it is **“his”** house, that of “close ones” - the location of an intimate living entity to which he feels a sense of belonging. Only later, as the circle of what is considered “close” progressively grows, does it become a “home for all”. And in this impulse to “make houses” (the way we would “make” love...), there is first and foremost a **tenderness**. There is the pulse of a **contact** with the materials we come to shape one by one, with loving care, and which we only really come to know through this affectionate contact. Then, following the building of the walls and the laying of the beams and ceiling, comes the profound satisfaction of p. P50
installing one room after another, and to progressively witness the emergence, amidst these halls, rooms, and nooks, of the harmonious order of a living house - beautiful, welcoming, good for living. Indeed, **home** is also, first and foremost and secretly for each of us, **the mother** who surrounds us and shelters us, at once refuge and solace; and perhaps (on an even deeper level, and despite the fact that we are constructing it from scratch) it is also that from which we ourselves are issued, that which hosted and nourished us during the times forever forgotten that preceded our birth... It is also the **Bosom**.

And the picture that thus spontaneously appeared, in an effort to go beyond the prestigious title of “pioneer” and to grasp the hidden reality which it covered, was also free from any “heroic” emphasis. Once again, it was the archetypal image of the maternal that appeared to me - that of the nourishing “womb” and of its amorphous and obscure workings...

These two urges which at first seemed to be “of a different nature” are actually closer to one another than I had thought. They both share the nature of an **“urge of contact”**, carrying us toward meeting **“the Mother”**: toward That which **both** incarnates what is near, “known”, **and** what is “unknown”. To abandon myself to one urge or to the other, is to “return to the Mother”. It is to renew contact at once with what is near, “more or less known”, and with what is “unknown” yet at the same time felt, at the brink of making itself known.

The difference herein is one of tonality, of dosage, rather than one of nature. When I am “building houses”, it is the “known” that dominates, and when I am “exploring”, it is the unknown. These two “modes” of discovery, or rather,

these two aspects of the same process, the same work, are inextricably linked. They are essential and complementary to one another. In my mathematical work, I discern a constant back-and-forth motion between these two modes of approach, or rather, between moments (or periods) during one mode or the
p. P51 other dominates[62]. However, it is also clear that in each moment, both modes are nonetheless present. When I am building, adjusting, or when I am clearing, cleaning, organizing, it is the "yang", or "masculine" "mode" or "side" of work which sets the pace. When I am fumbling through the elusive, the amorphous, the nameless, I am closer to the "yin", or "feminine" side of my being.

My point is not that either side of my nature should be minimized or renounced, as they are both essential - the "masculine" builds and engenders, and the "feminine" conceives and hosts the slow and obscure gestations. I "am" one and the other - "yang" and "yin", "man" and "woman". Yet I also know that the most delicate essence, the one most unrestrained when engaging in creative processes lies on the "yin" , the "feminine" side - a side which is humble, obscure, and often of feeble appearance.

It is that side of work which, for as long as I can remember, has exerted upon me the most powerful fascination. And this was so despite the fact that the ruling consensus was encouraging me to invest the largest part of my energy to the other side, through which tangible, not to say finished and completed "products" are incarnated and established - products whose contours are well defined, a testament to their reality with the evidence of carved stone...

I clearly perceive, in hindsight, how these consensuses have weighted upon me, but also how I managed to "bear the weight" - seamlessly! Admittedly, the "conception" or "exploration" part of my work was kept at a minimum. Yet, looking back at what constituted my work as a mathematician, it is manifest with striking evidence that what constitutes the essence and power of this work is the side which is nowadays overlooked, one which is free from derision and from condescending disdain: that of **"ideas"**, even of **"dreams"**, and not in any capacity that of "results". In trying to capture what is most essential in what I have contributed to the mathematics of today, by means of a sweeping glance that embraces the forest rather than linger on the trees, I have seen, not a track record of "great theorems", but rather a living fan of fertile ideas[63],

[62]What I say here about mathematical work is equally true for "meditation" work (which is frequently discussed in Récoltes et Semailles). I have no doubt that this is in fact a phenomenon that is featured in any work of discovery, including that of the artist (writer of poet, for instance). The two "sides" which I describe above can both be seen alternatively as that of **expression** and the accompanying "technical" requirements, and that of **reception** (of all sorts of perceptions and impressions), which become **inspiration** under the effect of intense attention. Both sides are present at every moment of the process, and there is a constant motion of "back-and-forth" between the "times" where one mode predominates and times where the other mode does.

[63]This is not to say that so-called "great theorems" are missing from my work, including theorems resolving questions which were asked by others and which no one had known how to solve before I did. (I review some of them in the footnote (***) on page 554, in the note "The rising sea..." (ReS III, n^o 122).) Rather, as I already mentioned at the early stages of this "walk" (at the step "Viewpoint and vision", n^{o}6), these theorems only gain their full meaning in the nourishing context of a greater theme, one initiated by one of these "fertile

coming together under one common and vast vision.

3.18 The child and the Mother

When this "foreword" started turning into a walk through my work as a mathematician, with my little synopsis on the "heirs" (dyed-in-the-wool) and the "builders" (incorrigible), there came to my mind a name for this failed attempt at a foreword: "The child and the builder". Over the course of the days that followed, it became clear that the "child" and the "builder" were actually the same character. Thus, this name turned into "The builder child". A name, I must say, that has a certain charm, and completely pleases me! p. P52

The reflection thus led to the realization that this haughty "builder", or (more modestly) the child-who-plays-at-building-houses, was but once of the **two** faces of the famous child-at-play. There is also the child-who-likes-exploring-things, who enjoys rummaging and burrowing away in the sand or the nameless muddy waters, in the most improbable and bizarre places... Probably in an attempt to put up a front (even if only for myself...), I started introducing him under the flamboyant title of "pioneer", followed that of "explorer", a more down to earth title that nonetheless remained charged with prestige. It begged the question of which title between "builder" and "pioneer-explorer" sounded more manly, more appealing! Heads or tails?

And yet, in looking closer, we find that our intrepid "pioneer" was actually a **girl** all along (who I had fancied dressing up as a boy) - a sister of the ponds, the rain, the drizzle and the night, silent and almost invisible as a result of her tendency to fade into the shadows, and always forgotten (when we aren't mocking her...). I myself have found a way to forget her, day after day - to doubly forget her I should say: at first, I only cared to see the boy (who plays at building houses...) - and, even when I finally couldn't help but to see the **other**, I still mistook her for a boy...

As for the nice name for my walk, it can no longer do. It is a strictly yang name, entirely "macho", a limping-name. To become acceptable in earnest, the name would need to feature **the other** as well. Yet, strangely, **"the other" doesn't really have a name**. The only one that somewhat does it is "ex- p. P53
plorer", but it is decidedly a boy's name, there is nothing to be done about it. In this matter, language is being difficult, it manages to trick us while leaving us unaware, in a way which is visibly influenced by ancestral prejudice.

We could perhaps get away with the title of "The builder-child and the explorer-child", leaving unspoken the fact that one is a "boy" and the other is a "girl", and that it is in fact a single boy-girl child who, at once, builds while exploring and explores while building... However, yet another aspect of things

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ideas". Their proof then follows, naturally and effortlessly, from the very nature, the "depth" of the theme which carries them - the way the river's waves seem to gently be born from the depths of its waters, smoothly and effortlessly. I speak in analogous terms, but with a different imagery, in the aforementioned note "The rising sea...".

appeared to me yesterday, in addition to the double-sided yin-yang associated to that which contemplates and explores, and that which names and builds.

The Universe, the World, or even the Cosmos, are strange and highly remote things at the core. They don't really concern us. It is not towards **them** that our deeply seated urge for knowledge carries us. Instead, what attracts us is their most tangible and immediate **Incarnation**, that which is closest to us, most "carnal", loaded with profound echoes and rich in mystery - That which blends with the origins of our being of flesh and with those of our species; and That also which at all times awaits us "at the other end of the tunnel", silently and with arms open. It is **from her**, the Mother, the One who gave us birth as she gave birth to the World, that the urge emanates, and that the roads of desire soar - and it is towards **Her** encounter that they lead us, towards **Her** that they flow, to continually return and surrender to Her.

Thus, on this detour off the path of this unexpected "walk", I find myself faced with a once familiar parable which I had started to forget - the parable of **the child and the Mother**. It could also be interpreted as a parable for **"Life, in the quest for itself**. Or alternatively, at the most humble level of individual existence, as a parable for **"the being in the quest of things"**.

It is both a parable and the expression of an ancestral experience, deeply embedded in the psyche - that of the most powerful aboriginal symbol nourishing deep-seated creative layers. I seem to hereby recognize, expressed in the immemorial language of archetypal imagery, the very breath of the creative power which animates man's flesh and mind, as much in in its most humble and ephemeral manifestations as in its most dazzling and durable ones.

This "breath", as well as the carnal imagery which embodies it, is the most humble thing in the world. It is also the most fragile, the most ignored and the most scorned upon... p. P54

The history of the vicissitudes of this breath over the course of your existence is nothing but **your** adventure, the "adventure towards knowledge" in **your** life. And the wordless parable that expresses this adventure is that of the child and the Mother.

You are the child, issued from the Mother, hosted within Her, nourished by Her power. And the child sets off from the Mother, the Near-by, the Well-known - and evolves towards the Mother, the Infinite, forever Unknown and full of mystery...

End of the "Walk through a life's work".

Epilogue: the invisible Circles

3.19 Death is my cradle (or three toddlers for one moribund)

Up until the appearance of the viewpoint of topoi, towards the end of the 1950s, the evolution of the notion of space seems to me to have been an essentially

"continuous" one. It appears to have happened smoothly and without jumps, starting from the euclidian theorization of the space which surrounds us and from the geometry we inherited from the greeks, which focused on the study of certain "figures" (lines, planes, circles, triangles, etc...) existing in that space. Admittedly, profound changes have occurred in the ways in which the mathematician of "philosopher of nature" conceives of "space"[64]. But all of these changes seem to me to be embedded within an essential "continuity" - they never led the mathematician, attached (as we all are) to familiar mental images, to a sudden **change of scenery**. It was something akin to the changes, profound perhaps but nonetheless gradual, which happen over the course of the years in a person which we have known as a child, and whom we would have seen grow up from the time they took their first steps to their adult age, and finally to maturity. Such changes are imperceptible for long periods of calm, and at other times they can be tumultuous. But even during the most intense periods of growth and maturing, and even though we may have lost sight of that person for months or years, there never comes a time where there can be any doubt, any hesitation, as to the fact that it is still him, a familiar person p. P55 who we know well, and who we are re-encountering, be it with altered traits.

I believe I can also say that, towards the middle of this century, this familiar person had already aged quite a bit - like a man who at last had become worn and exhausted, overwhelmed by a flux of new tasks requiring aptitudes he never had. Perhaps was he even already dead, without anybody caring to take note of it or acknowledging it. "Everybody" was still pretending to be busy working in the house of a living man, to the extent that he may have well have still been alive.

As such, in the eyes of the regulars of the house, accustomed to the venerable old man sitting stiff and completely still on his couch, you can imagine the importunate effect that would have the sudden appearance of a tiny and vigorous toddler, pretending in passing, with a straight face and with self-evident conviction, that **he** is actually Mister Space! (and you can feel free to drop the "Mister" from now on...) If he at least shared family features, he could be seen as a natural child, who knows... but not at all! At first sight, nothing in him is reminiscent of the old Father Space whom we knew so well (or thought we knew...), and who we believed with certainty (and that was the least one could ask...) that he was eternal...

This was the famous "mutation of the notion of space" mentioned above. It is **this** which I came to "see", as something evident, starting at least from the beginning of the 1960s, without ever really realizing it up to the time of writing. And I now suddenly see with renewed clarity, by mere virtue of this pictorial evocation and of the cloud of associations which simultaneously generates: the

[64]My initial plan in writing this Epilogue was to include a brief sketch of these "profound changes", and to highlight this "essential continuity" which I perceive. I have since renounced to this idea, so as not to prolong this Walk beyond measure - a Walk which is already much longer than I had expected! I think I will come back to this topic in the Historical Commentaries planned in volume 4 of the "Reflections", intended this time to the mathematical reader (something which completely changes the task of exposition).

traditional notion of "space", as well as the closely related notion of "variety" (of all kinds, including that of "algebraic variety"), had become so old, by the time I arrived on the scene, that they may as well have been dead...[65] And I believe I can say that it is with the successive appearance of the viewpoint of schemes (and of its outgrowth[66], leading to more than ten thousand pages of foundations) followed by that of topoi, that a crisis-whose-name-shall-not-be-said was finally defused.

p. P56

In the above imagery, we actually shouldn't be speaking of one but rather of two toddlers from elsewhere, products of a sudden mutation. Two toddles who share between them an undeniable "family resemblance", even though neither resembles the deceased old man. Furthermore, in taking a closer look, we could almost say that the Scheme toddler establishes a "kinship link" between deceased Father Space (aka Varieties-of-all-kinds) and the Topos toddler[67].

3.20 A look at the next-door neighbors

The aforementioned situation seems very close to that which occurred at the beginning of this century, with the appearance of Einstein's theory of relativity. There had been an even more striking conceptual cul-de-sac, in the form of a sudden **contradiction** which seemed insoluble. As is appropriate in these circumstances, the novel idea that was to restore order within chaos was an idea of childlike simplicity. What is remarkable (and in line with an often repeated scenario...) is that among all of these brilliant, eminent, prestigious people who were suddenly on their knees, trying all their might to "save what could still be saved", nobody thought about that idea. It had to come from a young and unknown man, freshly minted (possibly) in the ranks of university students, who explained to his illustrious elders (perhaps while embarrassed by his own audacity) what was needed to "save the phenomena": one needed only

[65] This assumption (which some may find peremptory) is to be taken with a "grain of salt". It is no more nor less valid than the assumption that the "newtonian model" (which I invoke again below) of (celestial or terrestrial) mechanics was "moribund" at the beginning of this century, when Einstein came to the rescue. It is a fact that to this day, in most "common" situations in physics, the newtonian model is perfectly appropriate, and it would be folly (given the margin of error inherent to the measurements involved) to go looking for relativistic models. In the same way, in several situations in mathematics, the old and familiar notions of "space" and "variety" remain perfectly adequate, without trying to speak of nilpotent elements, or topoi, or of "tame structures" ("structure modérées"). In either of these two scenarios, for an increasing number of contexts intervening in state-of-the-art research, the old conceptual frameworks have become inept at expressing even the most "common" of situations.

[66] (Intended for the mathematical reader) Among this "outgrowth", I notably include formal schemes, "multiplicities" of all kinds (notably scheme-theoretic and formal multiplicities), as well as so-called "rigid analytic" spaces (introduced by Tate, following a "blueprint" I issued, and inspired by the notion of topos as well as by that of formal schemes). This list is also in no way exhaustive...

[67] One could also add to these two toddlers a third and younger one, who appeared during less lenient times: I am speaking of the Tame space ("espace modéré") toddler. As I mention elsewhere, he was attributed no birth certificate, and it is in complete illegality that I have nonetheless included him among the twelve "maître-thèmes" which I had the honor of introducing in mathematics.

separated space and time[68]! Technically, everything was in place for this idea p. P57 to burgeon and be welcomed by the scientific community. And it is a credit to Einstein's elders that they accepted to welcome this novel idea without too much resistance. There lies the sign that it was still a great era...

From the mathematical viewpoint, Einstein's novel idea was commonplace. However, from the viewpoint of our conception of **physical space**, it was a profound mutation, a sudden "change of scenery". It was the first mutation of its kind, since the mathematical model of physical space formulated by Euclid some 2400 years ago, and adopted identically for the needs of mechanics by all physicists and astronomers since antiquity (including Newton), in their attempts to describe terrestrial and celestial mechanical phenomena.

Einstein's initial idea was subject to further refinements, taking the form of a subtler, richer and more flexible mathematical model, drawing from the rich arsenal of already existing mathematical notions[69]. With the "theory of general relativity", this idea was enlarged into a vast **vision** of the physical world, encompassing in one sweep the subatomic world of the infinitesimally small, the solar system, the milky way and distant galaxies, and the trajectory of electromagnetic waves in a space-time curved at each point by the matter present at that point[70]. That was the second and last time in the history of cosmology and physics (following Newton's initial great synthesis three centuries earlier) that a vast and unifying vision had appeared, capturing the entirety of physical phenomena in the Universe in the language of a mathematical model.

This einsteinian vision of the physical Universe was in turn overwhelmed by later developments. "The entirety of physical phenomena" which one is to take into account has had time to expand in scope, since the beginnings of this century! There appeared a multitude of physical theories, each meant to account (with varying levels of success) for a select and limited number of p. P58 phenomena, amidst the immense limbo of all "observed phenomena". And we are still awaiting the audacious toddler who will find, while playing with the new key (if one exists...), the "all-in-one-model" that was long dreamt about, and which would kindly "work" to capture all phenomena at once...[71] p. P59

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ing a "satisfactory" model (or, if needs be, a family of such models, "linked" to one another in as satisfactory a way as possible...), be it of a "continuous", "discrete", or "mixed" nature, will surely require a great deal of conceptual imagination, as well as a consummate flair to grasp and to bring to light mathematical structures of a novel kind. This type of imagination or "flair" appears to me to be s rare commodity, not only among physicists (where Einstein and Schrödinger seem to have been among the rare exceptions), but even among mathematicians (and here I am speaking in full knowledge of the facts).

In summary, I expect that the awaited renewal (if one is yet to come...) will need to come from someone who is a mathematician at heart, and well-versed in physics' great challenges, rather than from a physicist. And most importantly, this person will need to have the "philosophical openness" to grasp the heart of the problem - for this problem is not a technical one, but rather a fundamental problem in the "philosophy of nature".

[68] This is admittedly a relatively brief description of Einstein's idea. At the technical level, there was a need to designate the structure which was to be put on the new space-time (something which was already "in the air" with Maxwell's theory and Lorenz's ideas). The essential leap here is not of a technical nature, but rather it is of a **"philosophical"** nature: one needed to realize that the notion of simultaneity of distant events admitted no experimental evidence. This was the "childlike observation", that "the Emperor is naked!", which allowed

The comparison between my contribution to the mathematics of my time,

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us to cross the famous "imperious and invisible circle which delimitates a Universe"...

[69]Here I am alluding mostly to the notion of "Riemannian manifold", and to tensorial calculus on such manifolds.

[70]One of the most striking features which distinguishes the euclidian (or newtonian) model of space-time from Einstein's first model (known as "special relativity") is that the **global topological shape** of space-time remains indeterminate, rather than being imperatively prescribed by the very nature of the model. The question of determining which global shape describes our physical reality appears to me (as a mathematician) as one of cosmology's most fascinating questions.

[71]We refer to this hypothetical theory as a "unified theory", one which would "unify" and reconcile the multitude of aforementioned partial theories. I have the feeling that the fundamental reflection which has yet to be undertaken will have to take place at two distinct levels:

1^o) A reflection of a "philosophical" nature, on the very notion of "mathematical model" meant to represent a slice of reality. Since the success of Newton's theory, physicists have taken as a tacit axiom the fact that there **exists** a mathematical model (even a unique model, **"the"** model) which would perfectly capture physical reality, without the least "departure" nor smudge. This consensus, which has been the golden standard for over two centuries, is a sort of fossil relic of Pythagoras' vision that "All is number". Perhaps there lies a new "invisible circle", which came to replace the old metaphysical circles to delimitate the physicist's Universe (while the genus of "philosophers of nature" seems to have gone definitively extinct, briskly taken over by that of computers...). If we take the time to stop and ponder this phenomenon for even just an instant, it will clearly appear to us that the validity of this consensus is not in any way evident. There are even very serious philosophical reasons as to why it should a priori be put into question, or at the very least for us to expect its validity to be subject to very strict limitations. Now is the time to submit this axiom to close scrutiny, and perhaps even to "demonstrate" beyond all doubt that it it **not** well-founded: that there cannot exist a unique rigorous mathematical model that would account for the entirety of so-called "physical" phenomena documented to this day.

Once the notion of "mathematical model", and that of "validity" of such a model (within the limitations of explicit "margins of error" associated to the measurements involved) have both been satisfactorily formulated, the question of finding a "unified theory" or at least an "optimal model" (in a sense to be made precise) will finally become well-posed. At the same time, we will then surely have a clearer idea of the degree of arbitrariness attached (perhaps by necessity) to the choice of such a model.

2^o) In my opinion, it is only **after** such a reflection has been carried through that the "technical" question of formulating an explicit model, one that is more satisfactory than its predecessors, can take on its full meaning. This would then be the right time, perhaps, to break free from another one of the physicist's tacit axioms, one tracing back to antiquity and profoundly embedded in our very mode of perception of space: namely the assumption that space and time (or space-time), the "place" where "physical phenomena" occur, are of a **continuous nature**.

Some fifteen or twenty years ago, while browsing through the modest volume constituting Riemann's complete works, I was stricken by a remark he made "in passing". He observed that the ultimate structure of space may well be "discrete", and that the "continuous" representations which we employ may be a simplification (perhaps excessive, in the long run) of a reality that is more complex; that the "continuous" is easier to grasp for the human mind that the "discontinuous", and that the continuous serves, as such, as an "approximation" to gain insight into the discontinuous. I find this to be a remark of surprising acuity coming from a mathematician, at a time when the euclidian model of physical space had never been put into question; in a strictly logical sense, it was rather the discontinuous which had traditionally been used as a technical mode of approach towards the continuous.

The mathematical developments of the past few decades have indicated an even more intimate symbiosis between continuous and discontinuous structures than we may have imagined existed as recently as during the first half of the century. The fact remains that the act of find-and that of Einstein to physics, came to light for two reasons: both works led to a **mutation of our conception of "space"** (in the mathematical sense for one, and in the physical sense for the other); next, they both take the form of a **unifying vision**, encompassing a vast multitude of phenomena and situations which had hitherto appeared as separate from one another. This in my view indicates a clear **kinship of spirit** between his work[72] and mine.

I do not consider this kinship to be affected by the evident difference in **"substance"**. As I have implied earlier, the einsteinian mutation concerned the notion of physical space, and in formulating it Einstein needed only draw from the arsenal of already known mathematical notions, without ever needing to expand it or to alter it. His contribution consisted in extracting, among the known mathematical structures of his time, those that were most apt to[73] serve as "models" to the world of physical phenomena, in place of the moribund model bequeathed by his predecessors. In this sense, his work was very much that of a **physicist**, and beyond it, that of a **"philosopher of nature"**, in the connotation used by Newton and his contemporaries. This "philosophical" p. P60 dimension is absent from my mathematical work, wherein I was never led to ask myself questions concerning the eventual relations between the conceptual construction of "ideals", taking place in the Universe of mathematical things, and the phenomena which take place in the physical universe (or even, the lived experiences happening in the psyche). My work was that of a **mathematician**, deliberately facing away from questions of "applications" (to other sciences) or of the "motivations" or psychic roots of my work. More precisely, it was the work of a mathematician who was carried by his very particular genius for ceaselessly expanding the arsenal of notions at the very core of his art. This is how I was led, without even realizing and as if at play, to overturn the most fundamental notion of all for the geometer: that of **space** (and that of "variety"), namely the conception of the **"place"** where geometrical things live.

The novel notion of space (as a sort of "generalized space" where the points which are supposed to constitute the "space" have more or less disappeared) doesn't in any way resemble, in substance, the notion which Einstein brought to bear in physics (a notion which is not at all puzzling for the mathematician). The comparison which does hold in this sense is that with **quantum mechanics**, as discovered by **Schrödinger**[74]. In this new framework for mechanics, the traditional "material point" disappears and is replaced by a sort of "probabilistic cloud", more or less dense from one region of ambient space to another, depending on the "probability" that the point happens to occupy this region.

[72]I do not claim to be familiar with Einstein's work. In fact, I have read none of his papers, and I have only learned about his ideas very approximatively on the basis of hearsay. I nonetheless have the impression of being able to discern "the forest", even though I have never gone through the effort of scrutinizing any of its trees...

[73]For more comments on the use of the adjective "moribund", see a preceding footnote (footnote 55).

[74]From what I understand (based on echoes which came from various directions), we generally consider that there were three "revolutions" or great upheavals in physics: Einstein's theory, the discovery of radioactivity by Pierre and Marie Curie, and the introduction of quantum mechanics by Schrödinger.

We can clearly feel, through this novel viewpoint, an even deeper "mutation" in the way in which we conceive of mechanical phenomena than in the viewpoint incarnated by Einstein's model. It is a mutation which does not simply consist in replacing one mathematical model, a bit tight around the corners, by a similar model, shaped a bit larger or better adjusted. Rather, this new model is so unlike the good old traditional models that even the mathematician specializing in mechanics must have felt suddenly out of his depth, even lost (or outraged...). To go from Newton's mechanics to Einstein's must feel, for the mathematician, a bit like going from the good old provincial dialect to newfangled Parisian slang. On the other hand, to pass to quantum mechanics must have felt, I imagine, like going from French to Chinese...

The aforementioned "probabilistic clouds", coming to replace the reassuring material particles of yesteryear, strangely remind me of the elusive "open p. P61 neighborhoods" that populate topoi, comparable to evanescent ghosts, serving to surround imaginary "points", to which a recalcitrant imagination continues to hold against all odds...

3.21 "The unique" - or the gift of solitude

This brief visit to the "next-door neighbors", the physicists, can hopefully serve as a reference point for the reader who (like most people) knows nothing about the world of mathematicians, but who has surely heard about Einstein and his famous "fourth dimension", or even about quantum mechanics. After all, even though the inventors never planned for their discoveries to concretize into Hiroshimas, and later into both military and (so-called) "pacific" atomic bidding wars, the fact remains that physics discoveries have a tangible and near-immediate impact on the world of men in general. The impact of a mathematical discovery, on the other hand, especially in so-called "pure" mathematics (meaning with no motivation towards "applications") is less direct, and surely more complicated to delineate. I am not aware, for instance, of any "use" that my contributions to mathematics may have had, say in constructing any engine. If my contributions as such have no merit, so be it; I find this fact reassuring. As soon as there are applications in sight, one can be sure that the military (and, after that, the police) will be the first to put their hands on them - as for industry (even so-called "pacific" industry), I can't say it is always that much better...

For my own sake, or for that of the mathematical reader, it would be in order for me to attempt to situate my work, by means of "reference points", within the history of mathematics itself, rather than looking for analogies elsewhere. I have been thinking about this for the past few days, within the bounds of my rather vague knowledge of the history in question[75]. Already during the

[75] Ever since childhood, I was never a big fan of history (nor geography in fact). (In the fifth part of Récoltes et Semailles (only partially written), I have the occasion to detect "in passing" what I think might be the deeper reason underlying this partial "blockage" regarding history - a blockage which has been subsiding, I believe, over the course of recent years) The

"Walk", I had the occasion to mention a "lineage" of mathematicians, with whose temperaments I identify my own: Galois, Riemann, Hilbert. If I was better informed about the history of my art, I would possibly be able to prolong p. P62 this lineage farther into the past, or to perhaps insert a few additional names which I only know about on the basis of hearsay. What I find striking is that I cannot recall, even in allusions made by friends or colleagues who were better versed in mathematical history than I was, ever coming across a mathematician other than myself who brought to bear a multitude of innovative ideas that were all part of one vast unifying vision, rather than more or less disjoint from one another (something which Newton and Einstein both accomplished in physics and cosmology, respectively, as did Darwin and Pasteur in biology). I am only aware of two "moments" in the history of mathematics, during which a far-reaching novel vision was born. One of these moments was that of the birth of mathematics, as a science in the sense which we presently understand, some 2500 years ago in ancient Greece. The other moment is that of the birth of infinitesimal and integral calculus during the seventeenth century, a period associated with the names of Newton, Leibniz, Descartes and others. As far as I understand, the vision that was born in either of these moments was not due to the work of a single individual, but rather to the collective work of an era.

Of course, between the era of Pythagoras and Euclid and the beginning of the seventeenth century, mathematics had plenty of time to change appearance, and ditto for the mathematics of the "calculus of the infinitesimally small", created by seventeenth century mathematicians, in contrast with the mathematics of the middle of the present century. Yet, to the best of my understanding, the profound changes that have intervened during these two intermediary periods, the first lasting for more than two thousand years and the second for three centuries, were never concretized or condensed within a novel vision expressed in a specific work[76], in an analogous way to Newton's then Einstein's great

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mathematical education I received from my elders, in the "bourbachique circle", did little to improve this predisposition - the occasional historical references were all the more rare.

[76]Hours after writing these lines, I realized that I had omitted the vast synthesis of contemporary mathematics undertaken in the (collective) treaty of Mr. Bourbaki. (The Bourbaki group will be amply treated in the first part of Récoltes et Semailles). I think this happened for two reasons.

On the one hand, this synthesis is limited in scope to "putting in order" a vast collection of well-known ideas and results, without bringing forth novel ideas of its own. If there was any such novel idea, it was that of a precise mathematical definition for the notion of "structure", which turned out to form a precious guiding thread throughout the treaty. But this idea appears to me to be closer to that of an intelligent and imaginative lexicographer, rather than serving towards the renewal of a language which would provide a renewed understanding of (mathematical) reality.

On the other hand, since the 1950s, the idea of structure was overridden by contemporary events, notably with the sudden influx of "categorical" methods in some of the most dynamic branches of mathematics, such as topology and algebraic geometry (thus, the notion of "topos" does not fit into the "bourbachique bag" of structures, a bag which is decidedly narrow around the edges!). Upon deciding, in full knowledge of the facts, not to engage down this "mess", Bourbaki renounced to their initial ambition, which was to provide the foundations and the linguistic basis from which the totality of contemporary mathematics could be expressed.

Nonetheless, Bourbaki did fix a language, as well as a certain style of writing about and

p. P63 syntheses in physics and cosmology, at two crucial moments in their history.

It seems that in my quality of servant to one vast unifying vision born within me, I am "one of a kind" in the history of mathematics from its origins to the present day. I apologize for singling myself out more than seems admissible! Much to my relief, I nonetheless believe to have discerned a potential (and providential!) **brother**. I had the occasion to mention him earlier, as first in my lineage of "brothers in temperament": I am referring to **Evariste Galois**. Within his brief and dazzling life[77], I seem to discern the beginning stages of a grand vision - precisely that of the "marriage of number and size", within a novel geometric vision. I mention elsewhere in Récoltes et Semailles[78] how a sudden intuition appeared to me two years ago: that the mathematical work which at the time exerted upon me the most powerful fascination was in a way a "revival of Galois' heritage". This intuition, which was rarely evoked since, nonetheless had time to mature in silence. The retrospective reflection upon my life's work which I have been pursuing for the past three weeks has doubtlessly contributed to this maturing. The most direct kinship which I presently recognize with a mathematician from the past, is that which links me to Evariste Galois. Rightly or wrongly, it seems to me that the vision which I have been developing for fifteen years of my life, and which has continued to mature and grow within me during the sixteen years that have passed since my departure from the mathematical scene - that this vision is also that which
p. P64 Galois would inevitably have developed[79], had he been around in my stead, and had a premature death not come to brutally stop him in his magnificent stride.

There is yet another reason which contributes to this feeling of "essential kinship" - a kinship which doesn't stop merely at the level of "mathematical temperament", nor to the defining aspects of one's work. I also sense a kinship of destiny between his life and mine. Yes, Galois died stupidly, at the age of twenty one, while I am in my sixties and determined to live to old age. The fact remains that Evariste Galois was throughout his life a **"marginal"** in the official mathematical world, just as I have been a century and a half later. In Galois' case, a superficial judgement could lead to the conclusion that his marginality was "accidental", and that he simply had not had enough time to "impose himself" through his innovative ideas and through his work. In my case, my marginality, during the first three years of my life as a mathematician,

approaching mathematics. This style was originally meant to be the (very partial) shadow of a certain spirit, Hilbert's living and direct inheritance. Over the course of the 1950s and 1960s, this style eventually prevailed - for better and (mostly) for worse. For about twenty years, it ended up becoming a rigid "canon", or a purely superficial "rigor", while the spirit that once animated it seems to have disappeared, never to return.

[77]Evariste Galois (1811-1832) died in a duel, at the age of twenty-one. There have been, I believe, several biographies written about him. As I young man, I read a romanticized biography written by Infeld, a physicist, and it left a lasting mark on me.

[78]See "Galois' heritage" (ReS I, section 7)

[79]I am also persuaded that someone like Galois would have gone significantly farther than I did myself: in part because of his entirely exceptional gifts (which I do not share); but also because, unlike me, he would not have given in to devoting the major part of his energy to endless and painstaking formatting tasks, touching on what is more or less understood at any given time...

was due to my (perhaps deliberate...) ignorance of the very existence of a world of mathematicians to which I could have confronted myself; and since my departure from the mathematical scene sixteen years ago, it was the consequence of a deliberate choice. It is this choice, surely, which led to retaliation in the form of a "flawless collective will" to remove from mathematics any trace of my name, and with it of the vision to which I had made myself a servant.

But beyond these accidental discrepancies, I believe that I can discern a common cause to this "marginality", one which I deem to be essential. I do not interpret this cause as being embedded in historical circumstances, nor is it related to peculiarities of "temperament" or "character" (in fact, the latter doubtlessly differ between Galois and myself as much as they could possibly differ from one person to the next), and even less so to levels of "giftedness" (visibly prodigious for Galois, and comparatively modest for myself). If there is an "essential kinship", I conceive of it at a much humbler and more elementary level.

I have sensed such a kinship at a few rare occasions during my life. It is through it that I still feel "close" to another mathematician, who was my elder: **Claude Chevalley**[80]. The connection to which I am referring is founded on a certain "naïveté", or an "innocence", about which I have had the occasion p. P65
to speak earlier. This connection is expressed by means of a propensity (often under-appreciated by one's surroundings) to look at things with one's own eyes, rather than through certified glasses graciously handed down by some more or less vast group of people, invested with authority for one reason or another.

This "propensity", or inner attitude, is not a privilege that comes with maturity, it is rather an attribute of childhood. It is a gift we all received at birth, at the same time as we received life - and it is a humble and formidable gift. It is often buried deep within, but some have learned to preserve it to some extent, or perhaps to retrieve it...

It could also be called **the gift of solitude**.

[80]I write about Claude Chevalley at various places in Récoltes et Semailles, particularly in the section "Encounter with Claude Chevalley - or liberty and good sentiments" (ReS I section 11), and in this note "A farewell to Claude Chevalley" (ReS III, n^o 100)

Chapter 4

A letter

May 1985

4.1 The one-thousand page letter

p. L1

The text which I am hereby sending you, of which a limited number of copies were typed and printed by my university, is neither an off-print, nor a preprint. Its title, Récoltes et Semailles, makes this much clear. I am sending it to you the way I would send a long letter - including the personal dimension, indeed. If I have decided to send it to you, rather than await for you to learn about it some day (if your curiosity leads you to it) in the form of some publicly available volume in a library (if there even exists an editor crazy enough to engage in such an adventure...), it is because I am addressing this letter to you more than to others. I have thought of you more than once in the course of writing it - I must say that I have now been writing this letter for more than a year, and devoting all my energy to the task. It is a gift I am making you, and I took great care in the process to give out what I had best to offer (at any given moment). I do not know whether or not you will welcome this gift - until your response (or absence thereof) brings me the answer.

At the same time as I am sending you Récoltes et Semailles, I am also sending it to all of the colleagues, friends, and (ex-)students of the mathematical world with whom I was close at one time or another, as well as to those who appear in my reflection in some form, both named and unnamed. There is a chance that you yourself appear in what follows, and if you make the effort to read it not only with your eyes and head but also with your heart, you will surely recognize yourself even in places where you are not explicitly named. I am also sending Récoltes et Semailles to a handful of other friends, both inside and outside the scientific community.

This "letter of introduction" which you are currently reading, which announces and introduces a "one-thousand page letter" (to begin with...), will also serve as a Foreword. The latter has not yet been written at the time of writing these words. Additionally, Récoltes et Semailles consists of five parts (including an introduction "with drawers"). I am hereby sending you parts I (Fatuity and Renewal), II (The Burial (1) - or the Robe of The Emperor of China), and IV (The Burial (3) - or the Four Operations)[1]. These are the parts which seemed to concern you in particular. Part III (The Burial (2) - or the Key to the Yin and Yang) is the most personal segment of my testimony, and p. L2 at the same time it is the part which, more so than the others, appears to me to hold "universal" value, beyond the particular circumstances that surrounded its creation. I refer to that part in various places in part IV (The Four Operations), which can nonetheless be read independently, and even (to a large extent) independently of the three parts which precede it[2](*) If reading what I have sent you prompts you to respond (as is my wish), and if it makes you want to read the missing part as well, do let me know. It will be my pleasure to send it to you, as long as your response makes it clear that your interest goes beyond superficial curiosity.

4.2 Birth of Récoltes et Semailles (a lightning fast retrospective)

In this pre-letter, I would like to tell you in the span of a few pages (if at all possible) what Récoltes et Semailles is about - to do so in more details than can manage the subtitle: "Reflections and testimony about the past of a mathematician" (my past, as you will have guessed...). Récoltes et Semailles contains many things, and many will see them as several different things: a **voyage** of discovery through the past; a **meditation** on existence; a **painting of mores** of a milieu and an era (or a painting of the insidious and unstoppable transition from one era to the next...); an **investigation** (almost police-like at times, and elsewhere approaching the style of a cloak-and-dagger novel taking place in the underbelly of the mathematical megapolis...); a vast **mathematical divagation** (which will lose more than one reader...); a practical treaty in applied psychoanalysis (or, alternatively, a book of **"psychoanalytic fiction"**); a eulogy of **self-knowledge**; **"My confessions"**; a private **diary**; a psychological study of the processes of **discovery** and **creation**; an **indictment** (unforgiving, as it must...), or even a **settling of accounts** in the mathematical "beau monde" (in which no punches are pulled...). If there is one thing I can guaran-

[1] I am singling out colleagues who appear in my reflection in some way, but who I do not know personally. To those I am only sending "The Four Operations" (which particularly concerns them), as well as "booklet O" which consists of the present letter together with the Introduction to Récoltes et Semailles (as well as the detailed table of contents for the first four parts).

[2] (*) More generally, you will notice that each "section" (in Fatuity and Renewal) and each "note" (in any of the following three parts of Récoltes et Semailles) has its own unity and autonomy. Each can be read independently of the rest, the way one can find interest and stimulation in simply observing a hand, a foot, a finger, a toe, or any part, large or small, of the human body, without forgetting that it is part of a Whole, and that it is only with respect to that Whole (which remains unspoken) that the part takes on its full meaning.

tee, it is that I never once was bored in the process of writing it, and that from it I have learned and seen a great deal. If you find the time to read it among your other important duties, I doubt that you will be bored. Unless you force p. L3
yourself to read it, who knows...

As such, this work is not only meant to be read by mathematicians. It is true that certain parts of it will cater to mathematicians more than to others. In this pre-letter to Récoltes et Semailles, I would like to summarize and highlight what it is, then, which concerns you in particular as a mathematician. The most natural way to go about this is to simply tell you how, one thing leading to another, I progressively got around to writing the four or five "tomes" mentioned above.

As you know, I left the "grand monde" of mathematics in the year 1970, following an issue of military funding at my home institution (the IHES). After a few years devoted to anti-military and ecological activism, in the style of a "cultural revolution" - about which you probably received echoes here and there - I practically disappeared from the public sphere, and settled at a remote provincial university. Rumor has it that I am spending my days herding sheep and digging wells. The truth is that, in addition to several other occupations, I am bravely continuing to give university lecture, like everybody else (teaching was my first source of income, and it remains so to this day). It even just so happened that I would sometimes spend a few days, or even a few weeks or a few months doing mathematics again - I have filled several boxes with my doodles, which I am probably the only person in a position to decrypt. However - at least at first sight - these projects revolved around very different topics than the ones I used to work on. Between the years 1955 and 1970, my theme of predilection had been cohomology, more specifically the cohomology of varieties of all kinds (and of algebraic varieties in particular). I considered that I had done enough work in that direction for others to carry on without my help, and decided that if I were to continue doing mathematics, I might as well change things up...

In 1976, a new passion appeared in my life, one about which I felt as strongly as I once felt about mathematics, and which is closely related to the latter. It was a passion for what I call "meditation" (things need to be named after all). This name, like any other, risks leading to countless misunderstandings. As for mathematics, the process of meditation is one of discovery. I express myself on this subject at various points in Récoltes et Semailles. It visibly held enough in store to keep me occupied for the rest of my life. In fact, I have more than once gotten to thinking that mathematics was now a matter of the past, and that it was time for me to orient myself towards more serious matters - time to
"meditate". p. L4

I nonetheless ended up facing the evidence (four years ago) that my passion for mathematics was still very much alive. In fact, to my own surprise, and despite my long-standing conviction (for almost fifteen years) that I would never publish a single new line of mathematics in my lifetime, I found myself suddenly engaged in the writing of a mathematical project which seemed never-ending and would require producing volume after volume; and while I was at it, I might as well just write out all that I had to say about mathematics in an (infinite?)

series of books which would be called “Mathematical Reflections”, and that would be that.

This project began two years ago, during the spring of 1983. I was then already too busy writing (volume 1 of) “Pursuing Stacks” (“À la Poursuite des Champs” in French), which also was to constitute volume 1 of the (mathematical) “Reflections”, to stop and reflect on what was happening to me. Nine months later, as is fit, this first volume was virtually complete, and all that was left for me to do was to write the introduction, re-read everything, add annotations - and off it could be sent to be printed...

The volume in question is still not finished to this day - it hasn’t moved by an inch for the past year and a half. The introduction that I had left to write grew to take over twelve hundred pages (typewritten), and when all will be said and done I estimate it will be some fourteen hundred pages long. You will have guessed that this “introduction” is nothing but Récoltes et Semailles. Last I checked, it was supposed to constitute volumes 1 and 2, as well as part of volume 3 of the much-feted “series” in the works. The latter had to undergo a name change and is now called “Reflections” (i.e. not necessarily mathematical). The remainder of volume 3 will consist mostly of mathematical writings, ones which I deem more pressing than Pursuing Stacks. The latter can wait until next year for me to come around to adding annotations, an index, and, of course, an introduction...

End of the first Act!

4.3 The death of the boss - or abandoned construction sites

I sense that it is time for me to provide some explanations as to why I so abruptly left a world in which I had apparently felt at ease for more than twenty years of my life; why I had the strange idea of “coming back” (like a ghost...) when everybody seemed to have been doing just fine without me for the past fifteen years; finally, as to why the introduction to a mathematical work of six or seven hundred pages grew in turn to reach the length of twelve (or fourteen) hundred pages. As I cut to the chase, I will doubtlessly sadden you (sorry!), perhaps even upset you. For you, as I once did, surely prefer to see through “rose-colored glasses” the milieu to which you belong, the one in which you have found your place, your name and so on. I know what this is like... And what
p. L5 follows might cause some teeth grinding...

I mention the episode of my departure at various places in Récoltes et Semailles, without lingering on it. This “departure” serves rather as an important rupture in my life as a mathematician - it is in relation to that “reference point” that the events of my life as a mathematician arrange themselves, taking place on either side of a “before” and an “after”. A very strong **shock** was needed to uproot me from a milieu in which I was firmly entrenched, and from a clearly delineated “trajectory”. This shock came in the form of a confrontation, within a milieu with which I strongly identified myself, with a certain kind of corruption[3] which I had chosen to ignore up to that point (by simply abstaining from participating in it). In hindsight, I eventually realized that beyond that singular event, there was a deeper force at work within me, signaling an intense **need for an inner renewal**. Such a renewal could not be accomplished nor pursued in the lukewarm atmosphere that is the scientific vacuum of an institution of high-standing. Behind me lay twenty years of intense mathematical creativity and of mathematical devotion beyond measure - and, at the same time, twenty years of spiritual stagnation, "in a silo"... Without realizing it, I was suffocating - what I needed was some fresh air! My providential "departure" marked the sudden end of a long stagnation period, and it also marked the first step taken towards an equalization of the deep forces at play within my being, which were folded and screwed in a state of intense disequilibrium, frozen in place... This departure was, truly, a **new start** - the first step in a new journey...

As I said earlier, my passion for mathematics nonetheless remains alive. In recent times, it has found expression in the form of reflections that have remained sporadic, and going in directions which are different from those that I had been working on "before". As to the **body of work** which I was leaving behind, what I had produced "before", including both published texts and, perhaps even more importantly, material which hadn't yet reached the stage of writing and publication - it could almost appear as if it had effectively become detached from my person - and it seemed so to me. Up until last year, with the beginning of Récoltes et Semailles, I never thought of ever "weighing in" on the scattered echoes that reached me here and there. I knew that all that I had done in mathematics, and in particular what I had produced during my "geometric" p. L6
period between 1955 and 1970, were things that **had** to be done - and that the things which I had seen or glimpsed were things that **had** to appear, that **must** be brought to light. Additionally, I knew that the work which I had done, as well as the work which was done under my direction, was work well done, and that I had applied myself to it entirely. I had devoted all of my strength and love to it, and (or so it seemed to me) it could henceforth carry on autonomously - as a living and vigorous body which no longer needed to rely on my parental care. On this front, I left with a perfectly clear conscience. There was no doubt in my mind that the written and unwritten things which I was leaving behind were in good hands, put under the care of others who would make sure that they would deploy themselves, grow and multiply following the intrinsic nature of living and vigorous things.

During these fifteen years of intense mathematical work, a vast **unifying vision** had hatched, matured, and grown within me, taking the form of a handful of very simple **idées-force**. It was the vision of an "arithmetic geometry", a synthesis of topology, geometry (both algebraic and analytic), and arithmetic,

[3] I am referring to the open collaboration, "establishments" at the head, of scientists from all of the world's countries with military institutions, as a convenient source of funding, prestige, and power. This question is barely scratched in passing, once or twice, in Récoltes et Semailles, such as for instance in the note "Respect" from April 2^{nd} of last year (n^o 179, pages 1221-1223).

a first embryo of which I had found in the Weil conjectures. This vision was my principal source of inspiration during those years, a period which I mostly remember as the one during which I managed to formulate the key ideas of this novel geometry, and to develop some of its main tools. Over time, this vision and these "idées-force" became second nature to me. (And this feeling of "second nature" persists in me to this day, despite having ceased all contact with these ideas for nearly fifteen years!) I found them to be so simple, so obvious, that it was natural for "everyone else" to internalize them and to make them their own over time, at the same time as I went through these motions myself. It is only recently, during the past few months, that I realized that neither this vision, nor the "idées-force" which had been my constant guides, could be found in writing in any existing publication, save perhaps for tacit appearances between the lines. Most importantly, I also realized that this vision which I thought I had imparted to others, and the "idées forces" which carry it, remained ignored by all to this day, twenty years after having reached their full maturity. I alone, the worker and servant to these things which I had the privilege of discovering, remain the sole vessel in which they have remained alive.

Some tool or other which I have crafted will find itself used in various places to "break open" a problem reputed for being difficult, the way one would break open a safe. The tool is visibly solid. Yet, I am aware of a "force" it has other
p. L7 than that of a crowbar. The tool is part of a Whole, just as a limb is part of a body - it is part of a Whole from which it is issued, which gives it its full meaning and infuses it with strength and life energy. Granted, you can use a bone (if it is big enough) to break open a skull. But such is not its true function, its "raison d'être". Yet, I am witnessing these tools being dispersed, grabbed by one person or another, like bones being carefully butchered and cleaned, after being torn from a body - a living body that they are pretending to ignore...

What I am hereby spelling out in carefully chosen terms, at the term of a long reflection, I probably first noticed progressively and vaguely, over the course of successive years. It first occurred to me at the level of the unformulated which does not yet seek to take the form of a thought or conscious image, nor that of clearly articulated speech. I had decided that the past, after all, no longer concerned me. The echoes that occasionally reached me, although filtered, were nonetheless eloquent. I had considered myself a worker among others, busying myself on five or six "construction sites"[4] in full swing - a more experienced worker perhaps, the senior who for many years was the only person working on these sites, waiting for a welcome succession; senior, perhaps, but not fundamentally different from the others. And yet, upon his departure, it was as if a masonry enterprise had gone bankrupt, following the unexpected passing of the boss: from one day to the next, so to speak, the construction sites were deserted. The "workers" were gone, each of them carrying some small gewgaw

[4] I speak about these deserted "construction sites", and I eventually list them, in the series of notes "The deserted construction sites" (n^{o}176 through 178) written three months ago. A year prior, before the discovery of the Burial, I had already touched on this, in the first note in which I resume contact with my previous work and its recent course, titled "My orphans" (n^{o}46).

which they thought may be of use at a later time. The cash register was gone, and as such there was no longer any reason for them to tire themselves out at work...

The above is once again a formulation that is the result of a slow process of decantation, the end product of a reflection and an investigation that took place over the course of more than a year. Yet, this was surely something I already felt "on some level" in the first few years following my departure. Putting aside Deligne's work on the absolute values of Frobenius eigenvalues (the "million dollar question", from what I have recently gathered...) - whenever I happened to come across a close acquaintance from yesteryear, someone with whom I had worked on the same construction sites, and asked them "so... ?", I was always met with the same eloquent gesture, arms in the air as if asking for grace... Visibly, they were all too busy working on other things which were more important than the projects which were close to my heart - and, just as visibly, while they were all chugging along with an occupied and important air, nothing much was actually being done. The essential feature of mathematical work had disappeared - the presence of a **unity** which gave meaning to each partial task, and also, I believe, the presence of a **warmth**. What remained was a scattered collection of tasks unattached to a whole, with each worker hiding their little bounty away in a corner, or scrambling for a way to bring it to fruition. p. L8

I couldn't help but to feel sorrow over the fact that everything seemed to have stopped in its tracks; I no longer heard news about motives, topoi, the six functor formalism, De Rham and Hodge coefficients, nor about the "mysterious functor" which was supposed to unite under one umbrella De Rham and l-adic coefficients for all prime numbers, nor about crystals (except to learn that they remained at a standstill), nor about the "standard conjectures" and other conjectures which I had formulated and which, evidently, represented crucial questions. Even the vast foundational work begun in the Eléments de Géométrie Algébrique (with the unflagging help of Dieudonné), which one needed only push along a set track, was left behind: everybody was content to simply settle between the four walls and amidst the furniture that someone else had patiently assembled, built, and polished. With the worker gone, nobody had the inspiration to rise up in turn and to get their hands dirty, in order to construct the many buildings that were yet to be erected, **houses** which would be good to live in, for oneself and for others...

I once again couldn't resist rendering on the page these fully conscious images which emerged and came to light as a result of a work of reflection. There is no doubt in my mind that these images were already present in some form in the deeper layers of my being. I must have already felt the insidious reality of a **Burial** of my life's work and of my person on April 19^{th} of the previous year - it suddenly appeared to me on that day, with undeniable strength and under that very name, "The Burial". Yet, at the conscious level, I never felt offended nor even afflicted. Whatever a person, close to me or not, chose to do with their time was entirely their concern. If what had once motivated them or inspired them no longer did, that was their business, not mine. If the same shift seemed to happen, without fault, to every single one of my ex-students, p. L9 it was yet again each of their personal business, and I had other things to do than to start looking for an explanation, and that was that! As for the things which I had left behind, and to which a profound and ignored link continued to tie me - despite their visible state of abandon, left on deserted construction sites - I knew that they needn't fear "the assault of time" nor the fluctuations of fashion. Even if they had not yet entered the common patrimony (something which I mistakenly believed had already happened), they would inevitably take root eventually, be it in ten years or in a hundred years, no matter...

4.4 A wind of burial...

Even though I chose to ignore the diffuse perception of a large scale Burial for years on end, this burial nonetheless obstinately returned to haunt me in different guises, less innocuous than that of a mere disaffection with my work. I slowly came to realize, in ways I could not quite explain, that many of the constituent notions of the forgotten vision had not only fallen into disuse, they had also become, within a certain "beau monde", the objects of condescending disdain. Such was notably the case for the crucial and unifying notion of topos, which lays at the very heart of the novel geometry - and which provides a common geometric intuition for topology, algebraic geometry, and arithmetic. The notion of topos was also pivotal to my formulation of the étale and l-adic cohomology tools, as well as to the key ideas (since then more or less forgotten, admittedly...) underlying crystalline cohomology. In fact, it was my very name which, insidiously, mysteriously and over the course of the years, had become an object of derision - becoming a synonym for rambling discourses ad infinitum (such as those I produced on the famous "topoi", or on these "motives" that I kept dwelling on and which nobody had ever seen...), for counting angels dancing on the head of a pin for thousands of pages on ends, and for plethoric and gigantic discussions about things which everyone already knew anyway without having to read about them somewhere... Such was the tone being held, albeit in muted voices, by means of innuendos, and with all of the delicacy that is in order among "high-minded people of esteemed company".

During the reflection pursued in Récoltes et Semailles, I believe I was able to point towards the deep forces at play in various characters, forces which are responsible for the airs of derision and condescension they tend to display when confronted with a work whose scope, life and breath are beyond them. I have also discovered (apart from the particular traits of my person which have p. L10 influenced my work and my fate) the secret **"catalyst"** who incited these forces to take the form of brazen contempt in the face of eloquent signs of an intact creativity; the Chief Funeral Officer, in sum, of this Burial muffled by derision and contempt. Strangely, this catalyst was also the person to whom I was the closest - the only one who eventually assimilated and made his own a certain vision, full of life and of intense power. But I am getting ahead of myself...

Truth be told, these "whiffs of subtle derision" which reached me here and there did not affect me that much. They remained in a way anonymous, up until three or four years ago. I saw in them a sign of somewhat bleak times, but they did not appear to be directly targeting me, eliciting neither anxiety nor concern. What did affect me more directly were signs of a distancing away from my person which I received from several of my old friends in the mathematical world, friends to whom I continued (my departure from a mutual world notwithstanding) to feel connected through sympathy, in addition to the links created by a shared passion and a common past. Yet, here again, even though such signs pained me, I never stopped to look further into them, and the thought never occurred to me (as far as I remember) to connect the dots between these three series of signs: the abandoned construction sites (and the forgotten vision), the "wind of derision", and the distancing of many old friends from my person. I wrote to each of these friends, and none responded. It is actually no longer a rare occurence, nowadays, for letters that I write to old friends of students about things which I hold close to my heart to remain unanswered. New times, new ways - what was there to be done? I simply stopped writing them. Yet (if you are one of them) this letter will be the exception, a word which is once again directed your way, and it will be up to you to decide whether to welcome it this time around, or to shut it off once more...

If I remember correctly, the first signs that certain old friends were distancing themselves away from me trace back to 1976. This was the year during which another "series" of signs started to appear, which I would like to presently mention, before going back to Récoltes et Semailles. To be more precise, these two series of signs appeared jointly. At the time of writing, it seems to me that they are in fact inextricably linked, that they are in essence two aspects or "faces" of a single reality which came into being that year in the field of my own life. The aspect which I am about to address concerns a systematic "stonewalling", muted and with no reply, directed under a "flawless consensus"[5] p. L11
towards some students and ex-students **post**-1970 who, through their work, their style, and their inspiration clearly bore the mark of my influence. It was perhaps on this occasion that I first perceived the "whiff of subtle derision" which, through them, targeted a certain style and **approach** to mathematics - a style and a vision which (according to a consensus which had apparently already become universal in the mathematical establishment) **had no place in mathematics**.

This was again something which was clearly perceived at an unconscious level. During the same year, it ended up becoming visible to my conscious attention, in the wake of an aberrant scenario (regarding the impossibility of publishing a thesis which was visibly brilliant) that had happened five times over, with the burlesque obstination of a circus gag. Thinking back to these

[5]This "flawless consensus" is mentioned sporadically in Fatuity and Renewal, and it eventually becomes the object of a circumstantial testimony and a reflection in the following part, The Burial (1), with the "Procession X" or "The Funeral Service", consisting of the "coffin-notes" (n^o93-96) and the note "The Gravedigger - or the entire Congregation". The latter concludes this part of Récoltes et Semailles, and at the same time constitutes the first culmination of the "second breath" of the reflection.

events, I realize that a certain reality was "giving me a sign" with benevolent insistence, while I continued to play deaf: "Hey, look this way goofy, pay some attention to what's happening right here under your nose, it concerns you I promise...!!". I slightly arose from my torpor, look over (for an instant), half-bewildered half-distracted: "oh yes, well, it's a little strange, it sure looks like somebody's after somebody else, something must have gone wrong, and with such a perfect set I must say that's quite hard to believe!".

It was so hard to believe that I scurried to forget about both the gag and the circus. I must say that I had several other interesting occupations at hand. This didn't prevent the circus from returning to my attention in the following years - no longer in the form of gags this time around, but rather showing a certain penchant for humiliation, or for full-on punches in the face; with the caveat that we are amongst distinguished people and that the punch in the face therefore also comes in most distinguished forms, necessarily, albeit just as effective - the choice of a form was left to the discretion of the distinguished
p. L12 people in question...

The episode which felt like a "full-on punch in the face" (of someone else) took place in October 1981[6]. This time around, and for the first time since frequent signs of a new spirit had started reaching me, I was affected - certainly more so than I would have been had I been the one who was punched, rather than someone else whom I held in affection. This someone else was a student of mine, as well as a remarkably gifted mathematician, who had just accomplished beautiful things - but that was just a detail. What wasn't a detail, on the other hand, was that three of my students from "before' showed direct solidarity in favor of an act which the person in question received (rightly) as a humiliation and an affront. Another two of my old students had already treated him with condescension, taking the air of well-off people chasing away a good-for-nothing[7]. Yet another student was to follow suit three years later (again in the style "punch in the face") - but of course that was something I did not yet know. What occupied me at the time was amply sufficient. It was as if my past as a mathematician, never once examined, was suddenly taunting me in the form of hideous rictus on behalf of five of my ex-students, who had gone on to become important, powerful and disdainful characters...

There had never been a better time to probe in the direction of what was suddenly calling my attention with such violence. But something inside me had decided (without ever saying it out loud...) that this past from "before" did not really concern me after all, that there was no reason to look further into it; that if I was under the impression that it was calling me with a voice that I knew well - that of the time of contempt - there must have been a mistake. Yet, I was overcome with anxiety for days on end, maybe even weeks, without deciding to act upon it. (It was only last year, in the course of writing Récoltes et Semailles and thereby returning to this episode, that I became aware of this anxiety which had been put under control as soon as it had appeared.) Instead of noticing it

[6] This episode is recounted in the note "Coffin 3 - or the slightly-too-relative jacobians" (n^{o}95), notably on pages 404-406 .

[7] This occurrence is mentioned in passing in the note mentioned in the previous footnote.

and probing it, I became agitated and I wrote left and right the "letters that were in order". The parties involved even took the time to respond, naturally sending me evasive responses which did not go past the surface of anything. The waves eventually grew calmer, and everything returned to order. I never had to think back on any of it, until last year. When I did, I nonetheless noticed that there had remained a wound of sorts, or rather a painful splinter which one p. L13 avoids touching; a splinter which **maintains** a wound which is trying to heal itself...

This was surely the most painful and difficult of my experiences in my life as a mathematician - when I was shown (without consenting to really **become aware** of what my eyes were seeing) "such a cherished student or companion of yesteryear taking pleasure in discreetly crushing another cherished individual in whom he recognizes me". This impacted me even more strongly than the rather crazy discoveries which I made last year, and which (from an outsider's perspective) might seem just as incredible... It is true that this experience had brought into play several others, in the same tonalities although slightly less violent, and which as a result had been "over-seeded".

I am reminded that this very year 1981 had marked a drastic shift in my relationship with the only one of my ex-students which whom I had maintained regular contact following my departure, as well as the one who for the past fifteen years had taken the role of a "preferred interlocutor" on the mathematical side of things. This was indeed the year during which the "signs of an affectation of disdain" which had already appeared in previous years[8] "suddenly became so brutal" that I stopped all mathematical communication with him. That was a few months before the aforementioned punch-in-the-face episode. In hindsight the coincidence seems striking, but I had never tried to bring the two events together. They were filed in separate "cabinets"; cabinets which someone had declared, in addition, to be of no consequence - the matter was settled!

Thus also reminds me that a certain **Symposium** took place during the month of June of the same year 1981, which was memorable on a number of counts - a symposium which would have deserved to enter into History (or what remains of it...) under the indelible name of "Perverse Symposium". I became aware of it (or rather, it dropped on me!) on May 2^{nd} of last year, two weeks after the discovery (on April 19^{th}) of the Burial in the flesh - and I suddenly understood that I had come across **"the Apotheosis"**. The apotheosis of a burial, but also the **apotheosis** of the **scorn** targeted at what, for the more than two thousand years during which our science has existed, was the tacit and unshakeable foundation of the mathematician's code of ethic - namely, the elementary rule that one must not present as his own ideas and results which were taken from somebody else's work. In taking note presently of of p. L14 the remarkable temporal coincidence between two events which at first sight can seem to be of very different nature and scope, I am taken aback by the revelation of the profound and evident link there is between the **respect** of the **person**, and the respect of the elementary ethical rules of an art or a science,

[8] This episode is treated in the note "Two turning points" (n^{o}66).

preventing its practice from turning into a "free-for-all", and preventing its practitioners, known for their excellence and responsible for setting the tone, from turning into a "maffia" without scruples. But once again I am getting ahead of myself...

4.5 The journey

I believe I have by now covered most of the context in which my "return to mathematics", as well as, one thing leading to another, my writing of Récoltes et Semailles took place. It was only in late March of last year, in the last section of 'Fatuity and Renewal ("The weight of a past" (n^o 50)), that I started pondering the reasons behind and the meaning of this unexpected return. Regarding the "reasons", the most influential of those was surely the impression, at once vague and imperious , that powerful and vigorous things which I thought I had left in caring hands were actually "in a tomb, in which they had been left in decay, away from benefits of the wind, the rain, and the sun, for the fifteen years during which they had remained out of sight".[9] I must have understood, over time and without daring to admit it to myself before today, that I was the one that would have to finally break apart the old timbers imprisoning living things that were not made to rot away in locked coffins, but to flourish in the open air. Moreover, the false airs of self-importance and the insidious derision surrounding these upholstered and unwieldy coffins (so as to resemble the bemoaned deceased, surely...) played a role in "eventually awakening a fighting spirit within me which had fallen into slumber over the past ten years", and in giving me the desire to throw myself into the brawl...[10]

These are the circumstances in which, some two years ago, what I thought at first would be a brief oversight of one of these "construction sites" which I had left behind, a matter of a few days or a couple weeks at most, turned into a great mathematical feuilleton in N volumes, inserting itself into the famous new series of "Reflections" ("mathematical" reflections, awaiting the removal of this unnecessary qualifier). From the very moment I realized that I was in the process of writing a mathematical work destined for publication, I knew that I would also be including, in addition to a more or less standard "mathematical"
p. L15 introduction, a second "introduction" of a more personal nature. I sensed that it was important for me to explain the motivations behind my "return", which wasn't a return to a **milieu**, but only a "return" to an intense mathematical activity and to the publication of my own mathematical works, to last for an indeterminate period. I also intended to describe the spirit with which I now approached mathematical writing, one which is in many ways different from the spirit of my earlier writing - the present spirit is closer to that of the "travel log" accompanying a journey of discovery. And then there were naturally other things weighing on my mind, linked to the above, yet which I felt an even more pressing need to communicate. I went without saying that I was to take my time

[9] Quote copied from the note "The melody by the tomb - or sufficiency" (n^{o}167), page 826.
[10] See "The weight of a past" (section n^{o}50), notably p.137. (**).

in expressing what I had to share. I viewed these things, although still diffuse, as crucial in making sense of the volumes which I was about to write, as well as of the "Reflections" in which they would fit. I wasn't about to surreptitiously slip them in, apologizing to the busy reader in passing for wasting their precious time. If there were things in "Pursuing Stacks" which they and others had to be aware of, they were precisely the ones which I set out to express in this introduction. If twenty or thirty pages didn't suffice in saying these things, I would write forty, or even fifty - and I didn't intend to force anyone to read me...

Thus was born Récoltes et Semailles. I wrote the first pages of the introduction, meant to be completed by June 1983, during a slow period in the writing of the first volume of Pursuing Stacks. I resumed writing in February of last year, at a time when the volume had been essentially completed for several months[11]. I intended to use this introduction as an opportunity to clarify a few things that remained somewhat blurred in my mind. But I also had no doubt that it was about to be, just like the volume which I had just written, a **journey** or **discovery**; a journey into an even richer and vaster world than the one that I was getting ready to oversee, in the volume just written and in those to follow. Over the course of days, weeks, months, without really understanding what was happening, I continued this new journey at the discovery of a certain past (which had obstinately eluded me for the past three decades...), as well as of myself and the threads linking me to that past; it was alsoa journey at the p. L16
discovery of some people to whom I had been close in the mathematical world, and whom I knew so little; and lastly, in the same stride, a journey of mathematical discovery wherein, for the first time in fifteen to twenty years[12], I took the time to return to some burning questions which I had left at the time of my departure. As such, I would say that I am actually pursuing **three** intimately interlinked journeys of discoveries in the pages of Récoltes et Semailles. And none of the three has reached completion by page twelve hundred and counting. The echoes with which this testimony will be met (including echoes of silence...) will be part of the "continuation" of the journey. As to a conclusion - this is the kind of journey that never really reaches its end; not even, perhaps, on the day of our death...

And so I have come full circle, back to where I started: telling you in advance, inasmuch as it can be done, "what Récoltes et Semailles is about". But it is also true that, whether or not you had this question in mind, the previous few pages gave some sort of an answer. Perhaps it would be more interesting for me to continue in my stride and to begin **telling**, rather than "announcing".

[11] In the meantime, I spent a fair amount of time thinking about the "structural surface" for a system of pseudo-lines, obtained in terms of the set of all possible "relative positions" of a pseudo-line in relation to such a system. I also wrote "Sketch of a Program" ("Esquisse d'un Programme" in French, which will be included in volume 3 of the Reflections.

[12] In the 1950s and 1960s, I often repressed a desire to pursue such burning and fruitful questions, as I was entirely at the mercy of countless foundational tasks which no one else would have or could have taken up in my stead, and which no one had the stamina to pursue further following my departure...

June 1985

4.6 The obscure side - or creation and disdain

The previous pages were written during a "low activity point" last month. Since then, I have finally added the finishing touch to the "Four Operations" (the fourth part of Récoltes et Semailles) - all that remains is for me to complete this letter or "pre-letter" (which itself seems to be assuming prohibitive proportions...) and it will then be ready for typesetting and printing. I was starting to lose faith, after the almost year and a half during which I have been "on the verge of finishing" these famous notes! When I first sat down to write this unusual "introduction" to a work of mathematics, in February of last year (and already in June of the year before that), there were (I reckon) three things which I intended to speak about.

First, I wanted to describe the intentions behind my decision to resume a mathematical activity, to write about the spirit in which I had written the first volume "Pursuing Stacks" (which I had then just completed), and about
p. L17 the spirit in which I intended to pursue an even vaster journey of exploration and mathematical discovery in the "Reflections". From there, I would then only have to present the meticulous and well-pressed foundations of some new mathematical universe in gestation. It would read rather like a "travel log", in which the work is carried day after day, in plain sight and as it is **actually** happening, including all the mistakes and mess-ups, the frequent look-backs as well as the sudden leaps forward - a work irresistibly being pulled forward day after day (in spite of the countless incidents and unforeseen circumstances), as if by an invisible thread - by some elusive yet nonetheless tenacious and sound vision. A work that is often fumbling, especially during the "delicate times" in which a spring of intuition arises, barely perceptible, nameless and faceless as of yet; or during the early steps of some new journey, while still on the lookout for and at the pursuit of initial ideas and intuitions - the latter of which often prove to be elusive and escaping the meshes of language, indicating that what is actually missing is the formulation of an adequate language in which they could be captured with finesse. The task then becomes that of creating such a language, to condense it out of the intangible mist with which we are initially confronted. Thus, what was at first only intuited, before being glimpsed without being really seen or "touched", slowly trickles out of the imponderable, extracting itself from its shadowy and hazy mantel to assume an existence in flesh and bone...

It is this part of the work which, albeit puny looking - not to say (often) harebrained - is often the most delicate and essential part of the process: it is truly there that something new becomes manifest, through intense attention, solicitude, and respect towards the fragile and infinitely delicate thing about to be born. It is the most creative part of all - that of conception and slow gestation within the warm shadows of the maternal womb, inside which the initial double gamete becomes amorphous embryo and continuously transforms itself over the course of days and months, by means of an obscure and intense process, seamless and invisible, into a new being made out of flesh and bone.

This is also the “obscure”, “yin”, or “**feminine**” part of the work of discovery. The complementary “clear”, “yang”, or “**masculine**” part would be closer to a work of hammer and chisel upon a piece of hardened steel (using ready-made tools whose efficacy has already been established...). Both aspects have their individual raison d'être and function, and they are inextricably tied p. L18
to one another by symbiosis - or rather, they are the **wife** and **husband** forming the indissoluble couple of the two original cosmic forces, whose ever-renewing embrace gives rise to the obscure creative work of conception, gestation, and birth - the birth of the **child**, of the novel thing.

The second topic which I felt the need to write about, in this famous personal and “philosophical” introduction to a work of mathematics, was the nature of creative work. It had been years since I first realized that the nature underlying this process was largely ignored, overshadowed by a host of clichés, repressions and ancestral fears. I only started discovering the extent to which this was a reality over the course of the days and months which I devoted to the reflection and the “investigation” undertaken in Récoltes et Semailles. Already from the “get go” of this reflection - while writing some pages back in June 1983 - I was taken aback by a fact which, while inconspicuous at first sight, is nonetheless stupefying once one stops to think about it: namely, that this “most creative part of all” within a work of discovery which I mentioned above **is reflected almost nowhere** in the texts and monologues which are supposed to present work of this kind (or at least present its most tangible outgrowths); such is the case in textbooks and other texts of a didactic nature, in articles and original memoirs, as well as in oral lectures, seminary presentations, etc. It is as if there existed, for what seems like millennia, tracing back to the very origins of mathematics and of other arts and sciences, a sort of “conspiracy of silence” surrounding these **“unspeakable labors”** which precede the birth of each new idea, both big and small, and which thereby lead to a renewal of our understanding of a portion of this world in which we live, a world engaged in perpetual creation.

To tell the truth, it seems that this most crucial aspect, or stage, of the work of discovery (as well as creative work in general) is subjected to a repression so efficient, so interiorized by the very people who come to know firsthand such a process that we could almost swear that these same people have eradicated every memory of this practice from their consciousness - akin to how a woman living in a highly restrictive puritan society might have eradicated from her living memory, in relation to each of the children who fall under her daily care, the moment of the embrace (grudgingly endured) tied to their conception, the long months of pregnancy (suffered as an impropriety), and the long hours of childbirth (experienced as a distasteful ordeal, followed by deliverance at long last). p. L19

This comparison may seem outrageous, and it may indeed be so, if I were to apply it to the spirit of the mathematical milieu to which I belonged myself, as I remember it from some twenty years ago. But in the course of my reflec-

tion in Récoltes et Semailles I was led to realize, especially during the past few months (while writing "The Four Operations"), that there had been since my departure from the mathematical world a stupefying **degradation** in the spirit which nowadays rules over the milieux which I once knew, and (to what seems to be a large extent) in the mathematical world at large[13]. It is even possible, in view of my very particular mathematical personality and of the circumstances surrounding my departure, that the latter may have played a catalyzing role in an evolution which was already underway[14] - an evolution to which I was then blind (as much so as any other friend or colleague of mine, with the possible exception of Claude Chevalley). The aspect of this degradation which I am mostly thinking about in writing these lines (and which is just **one** aspect among many others[15]) is the **tacit disdain**, if not outright derision, directed towards work (mathematical, in this case) which does not resemble pure hammer-and-chisel
p. L20 labor - disdain for the more delicate (and often lesser looking) creative processes; for all that pertains to **inspiration, dream, vision** (however powerful and fruitful it might be); almost (in the extreme) for any idea, however clearly it may have been conceived and formulated: in sum, disdain for anything that hasn't been written and **published** in black on white, in the form of plain statements, classifiable and classified, ready to be incorporated into the "databases" inscribed in the inexhaustible memories of our supercomputers.

There has been (to borrow an expression from C.L. Siegel[16]) an extraordinary **"flattening"** and a **"narrowing"** of mathematical thought, as a result of the fact that an essential dimension has been stripped away: the totality of its "obscure side", its "feminine" side. It is true that, in accordance with an ancestral tradition, this side of the work of discovery was to remain largely hidden, and nobody (virtually) ever **spoke** about it - but, until now (to the best of my knowledge), the ability to establish a raw contact with the profound sources of dream, and to in turn nourish great ideas and great designs, had never yet been

[13]This degradation is not in any way limited to the "mathematical world". It can be detected in the entirety of the scientific sphere and, beyond it, in the contemporary world at a global scale. I begin an assessment and reflection upon this situation in the note "The muscle and the bowel", which opens the reflection on the yin and the yang (note n^{o}106).

[14]I further examine this evolution in the note cited in the previous footnote. Links between this evolution and the Burial (of my person and of my work) appear and are examined in the notes "The Funerals of the Yin (yang buries yin (4))", "Providential circumstance - or Apotheosis", "The disavowal (1) - or the reminder", "The disavowal - or the metamorphosis" (n^o 124, 151, 152, 153). See also the more recent notes (in ReS IV) "The unnecessary details" (n^{o}171 (v), part (c) "Things that don't resemble anything - or desiccation") and "The family book" (n^o'73, part (c) "The one among all - or the acquiescence").

[15]The aspect which is most often the focus of Récoltes et Semailles, particularly in the two "investigative" parts (ReS II or "The robe of the Emperor of China", and ReS IV or "The Four Operations"), is also, perhaps, the aspect that I found most "flabbergasting", namely the degradation of the ethical code of the profession, expressed in the form of sacking, discrediting, and shameless scheming operated by some of the most prestigious and brilliant mathematicians of the time, and (to a large extent) in full view of all. For some of the other, more delicate aspects, which are in fact directly tied to this one, I refer the reader to the aforementioned note (n^{o}173 part (c)) "Things that don't resemble anything - or desiccation".

[16]This expression is cited and commented upon in the note that was just cited in the preceding footnote

lost. It seems as if we have recently entered an **era of desiccation**, in which access to these dream sources, which are admittedly not yet dried-up, has been condemned by the unappealable verdict of general disdain and by the reprisals of derision.

We therefore seem to be approaching the risk of eradication, within each individual, of not only the **memory** of work carried out close to the source, of a "feminine" nature (often derided as "muddy", "sluggish", "inconsistent" - or at the other extreme as "trivial", "child's play", "long-winded"...), but also the loss of this very work and its outgrowths - when this work is where novel notions and visions are conceived, grow, and come to life. Such an event would lead us to an era during which the practice of our art will be reduced to arid and vain demonstrations of cerebral "weightlifting", and to intellectual bidding wars towards the "breaking open" of competition problems ("of proverbial difficulty") - an era of sterile, feverish, "super-macho" hypertrophy following more than three centuries of continuous creative renewal.

4.7 Respect and fortitude

But I digress once again, looking ahead regarding what my reflection has taught me. My initial goal, prior even to the start of this reflection, was twofold: to present a "declaration of intent", and (something which we just saw is closely related to the latter point) to share my thoughts on the nature of creative work. However, there was also a third goal which, although less perceptible at the conscious level, fulfilled a more profound and essential need of mine. This third p. L21
point was sparked by sometimes troubling "interpellations" issued by many of my old students or colleagues, and regarding my past as a mathematician. On the surface, this need manifested itself as a desire to "empty my bag", to say out lout some "uncomfortable truths". Yet, at a deeper level, there was surely also the need to **become acquainted** at last with a past which I had hitherto chosen to ignore. It is as a result of this need more than anything else that Récoltes et Semailles came into being. This long reflection was my "response", day in and day out, to the inner impulse I felt towards understanding, and to the recurring interpellations that reached me from the outside world, the "mathematical world" which I had left with no intention of return. With the exception of the first pages of "Fatuity and Renewal", namely the first two chapters ("Work and discovery" and "The dream and the dreamer"), and beginning with the following chapter "The birth of fear" (p. 18) which features an unplanned "testimony", I believe that this need to confront my past and to fully accept it was the principal force at work in the writing of Récoltes et Semailles.

The interpellation that had reached me from the mathematical milieu, and which returned with renewed strength throughout Récoltes et Semailles (especially during the "investigation" undertaken in parts II and IV), had taken from the get-go a self-important character, if not one of ("delicately calibrated") disdain, derision, or scorn - directed both towards myself (sometimes), and (most often) towards those who had dared take inspiration from me (without being

aware of the consequences awaiting them), and who were thereby "categorized" as being linked to me, by some tacit and implacable decree. Once again I am detecting herein the "obvious and profound" connection between **respect** (or the absence thereof) for others; respect for the creative act and for some of its most delicate and essential fruits; and, finally, respect for the most evident rules of scientific ethic, namely those that are rooted in an elementary respect of oneself and of others, and which I am tempted to call "**rules of decency**" in the practice of our art. All of these are but aspects of an elementary and essential "**respect of oneself**". If I had to summarize, in a single lapidary sentence, what Récoltes et Semailles has taught me about the certain world which I once called my own, and with which I identified myself for more than twenty years,
p. L22 I would say: it is a world that has **lost the value of respect**[17].

The above was something I had already strongly felt, if not outright formulated, during the preceding years. It was only further confirmed and clarified, in unexpected and sometimes stupefying ways, over the course of writing Récoltes et Semailles. It is notably already apparent starting from the point at which the general reflection of a "philosophical" nature suddenly turns into a personal testimony (in the section "The welcome stranger" (n^{o}9, p.18) at the start of the aforementioned chapter "The birth of fear").

Yet, this observation is not to be interpreted as an acerbic or bitter recrimination, but rather (through the logic internal to the writing and through the attitude which it induces in the reader) as an **interrogation**. That is, I invite mathematicians to ask themselves: what was the nature of my own involvement in this degradation, this loss of respect which I am nowadays witnessing? Herein lies the principal interrogation which pervades and carries forth the first part of Récoltes et Semailles, up until the point where it is finally resolved by a clear and unequivocal observation[18]. Previously, this degradation had seemed to me as having suddenly "materialized itself", as something that was equally unexplainable, outrageous, and unacceptable. During the reflection, I discovered that it had actually been taking its course insidiously, without anybody taking notice of its evolution either around them or within them, throughout the 1950s and 1960s, **including in my own case**.

This humbling realization, although evident and without show, marks the first crucial turning point in the testimony, and immediately brings with it a qualitative change[19]. This was the first essential realization which I was to acquire about my mathematical past and about myself. This newly gained aware-

[17] Once again, this formulation preserves its relevance outside of the limited milieu in which I had ample opportunities to come to this conclusion, where it seems to also summarize a certain degradation of the entirety of contemporary society. (Compare with the footnote on page L19.) Within the more self-contained context of summarizing the "investigation" pursued in Récoltes et Semailles, this formulation appears in note 2 from last April, titled "Respect" (n^{o}179).

[18] See the sections "Athletic mathematics" and "Enough with this merry-go-round (n^{o}40, 41).

[19] Beginning the following day, the testimony deepens into a meditation on myself, a particular quality which it preserves in the following weeks, all the way to the end of this "first breath" of Récoltes et Semailles (ending with the section "The weight of a past", n^{o}50).

ness of my **shared responsibility** in the general degradation (an awareness which makes itself felt more or less sharply at various points of the reflection) remains a kind of background note and a reminder throughout Récoltes et Semailles. Such is the case, mostly, at the times when the reflection takes the form p. L23 of an investigation on the disgraces and inequities of an era. Together with the desire to understand, this curiosity which carries forward any authentic work of discovery, I believe that it is this humbling awareness (forgotten at various points along the way but always reappearing, in the places where it is least expected...) which prevented my testimony from ever turning into a compendium of sterile recriminations on the world's shortcomings, or even into a "settlement of scores" with some of my old students or friends (or both). This absence of complacency towards myself also provided me with a sense of inner calm, or fortitude, which in turn safeguarded me from the risk of complacency towards others, including that of "false discretion". I said all that I felt the need to say at every point of the reflection, be it about myself, about one or another of my colleagues, students, or friends, or about a milieu or an era, without ever having to jostle against my reluctance: every time such inner opposition surfaced, I needed only examine it carefully for it to disappear without a trace.

4.8 "My close acquaintances" - or connivance

The purpose of this letter isn't to list all of the "key moments" (or "sensitive moments") in segments of or all of Récoltes et Semailles[20]. I would only like to mention that there were four great steps, or "breaths", that can be clearly distinguished in this work - akin to respiratory motions, or to the successive waves of a rising tide, vast and mute masses, at once immobile and in motion, limitless and nameless, issued from the unknown and bottomless sea that is "me", or rather, from a sea infinitely vaster and deeper than "I" am, and which carries and nourishes me. These "breaths" or "waves" materialized into the four presently written parts of Récoltes et Semailles. Each wave came of its own accord and unexpectedly, and at no point could I tell where it was taking me nor when it would end. And whenever one wave ended and a new one took its place, there was a period of time during which I still believed myself to be at the end of a cycle (leading, at the very end, to the end of Récoltes et Semailles!), whereas I was already being lifted and carried forward by another breath of the same vast movement. It is only in hindsight that this movement becomes clearly p. L24 apparent and that a **structure** unequivocally reveals itself within what has been lived as an act in motion.

Naturally, this movement did not come to an end upon the (provisory!) completion of Récoltes et Semailles, and it not end with this letter either, the latter being but a "measure" of this movement. In the same way, it wasn't born on a given day in June 1983, or February 1984, when I sat down in front

[20]A short retrospective-recap of the first three parts of Récoltes et Semailles can be found in the two groups of notes "The evening fruits" (n^{o}179-182) and "Discovering a past" (n^{o}183-186).

of my typewriter to begin (or resume) writing the introduction to a certain mathematical work. Instead, it was born (or rather re-born) on the day that meditation appeared in my life...

But I digress once more, letting myself be carrier (and taken away...) by images and associations born in the moment, rather than diligently sticking to the thread of a planned "message". Today's intention was to continue the narration, however briefly, of the "discovery of the Funeral" written last April, at a time when I thought I was already done with Récoltes et Semailles two weeks earlier - to review the way in which a series of major and incredible discoveries cascaded upon me in the span of just three or four weeks, discoveries so massive and wild that it took me months to even begin "believing the testimony of my sound perceptive capacities", and to free myself from an insidious **incredulity** in the face of evidence[21]. This secret and tenacious incredulity was only dissipated last October (six months after the discovery of the "Funeral in all its glory"), following the visit at my home of my friend and ex-student (albeit secretly) Pierre Deligne[22]. For the first time, I was confronted to the Funeral not through the intermediary of **texts** going over (in nonetheless eloquent prose!) the discrediting, sacking, and massacre of a life's work, as well as the burial of a certain style and approach to doing mathematics - only this time the confrontation was direct and tangible, assuming familiar traits and a known voice, whose intonations were affable and ingenuous. The Funeral was in front of me at last, "in flesh and bone", with the occupied and anodyne traits which I recognized, but which I saw for the first time with new eyes and a renewed attention. Here
p. L25 was before me the person who, over the course of my reflection in the preceding months, had revealed himself to be the Chief Officer at my solemn funeral rites, at once the "Priest in a chasuble" and the principal architect and "beneficiary" or an unprecedented "operation", secret inheritor of a work abandoned to derision and sacking...

This encounter takes place at the beginning of the "third wave" of Récoltes et Semailles, just as I had embarked upon a long meditation on the yin and the yang, at the pursuit of an elusive and tenacious association of ideas. At the time, this brief episode was only mentioned in passing, in the form of an echo of a few lines. It nonetheless marks an important moment, whose fruits will only make themselves known months later.

There was a second similar confrontation to the "Funeral in flesh and bone", which happened just ten days ago and came to relaunch, "at the last minute" once more, an investigation that kept finding new impetuses. This time, it was just a phone call with Jean-Pierre Serre[23] . This "jumbled" conversation served to confirm in a striking and unexpected way what I had just (a few days earlier)

[21]I attempt to express this difficulty in the tale "The robe of the Emperor of China" in the note of the same name (n^{o}77), and again in the note "Duty fulfilled - or the moment of truth" (n^{o}163).

[22]I narrate this visit in the note cited in the previous footnote.

[23]This is essentially a paraphrase of the note "The Gravedigger - or the entire Congregation" (n^{o}97, page 417).

admitted to myself[24], almost against my will, concerning the role played by Serre in my Funeral and his "secret compliance" to what was happening "right under his nose" while he pretended not to see or feel anything.

Here too, of course, the conversation itself was perfectly "cool" and amicable, and it indeed seems that Serre's friendly disposition towards me is entirely sincere and genuine. The fact remains that I could clearly discern, or should I say "touch", this compliance of his that I had just admitted to myself; undoubtedly "secret" (as I wrote above), but most of all hastened, as I was able to observe in person beyond any doubt. A hastened and unreserved compliance, to bury what has to be buried, and to replace, wherever deemed fit and by any means, a real and undesirable paternity (which Serre knows about firsthand) by a bogus and welcome one...[25] I received a striking confirmation of an intuition which had already appeared a year earlier, when I wrote[26]: p. L26

"Seen in this light[27], the principal officer Deligne no longer appears as the one who created a trend reflecting the underlying forces determining his own life and actions, but rather as the **instrument** designated (because of his role as "legitimate heir"[28]) by a highly cohesive **collective will**, and given the impossible task of erasing both my name and my personal style from contemporary mathematics."

If Deligne thus appeared as the "instrument" designated by a "highly cohesive collective will" (while also being its first and principal "beneficiary"), Serre now appears to me as being the very **incarnation** of this collective will, as well as the **guarantor** of the resulting unreserved compliance; an acquiescence to all kinds of trickeries and frauds including the "vast" operations of shameless collective mystification and appropriation, as long as these practices contributed to the "impossible task" targeted towards my modest and departed self, or towards any other person[29] who dared join forces with me and to appear, against

[24] In part (c) ("The one among all - or the compliance") of the same note (n^o173).

[25] This is essentially a paraphrase of the note "The Gravedigger - or the entire Congregation" (n^o97, page 417).

[26] This quote is taken from the same note (see previous footnote), also page 417.

[27] "In light" of the deliberate comment mentioned above, regarding the need to eliminate at all cost "undesirable paternity ties" (or even "intolerable" ties, to use the original expression from the note in question).

[28] This role of "heir" that Deligne takes on is both occult (in that not a single line published by Deligne could suggest that he has learned anything from me) and felt and admitted by all. There lies a typical aspect of Deligne's double-play and of his particular style, in that he was able to masterfully play with this ambiguity, cashing in the advantages of his tacit role as heir while simultaneously disavowing the deceased master and taking the direction of the large scale burial operation.

[29] I am here thinking about **Zoghman Mebkhout**, about whom I write for the first time in the Introduction 6 ("The Burial"), then later in my note "My orphans" (n^o46), as well as in the notes (written at a later time, after the discovery of the Burial) "Failure of a teaching (2) - or creation and fatuity" and "A sensation of injustice and powerlessness" (n^os 44', 44"). I explore the iniquitous operation of concealment and appropriation of Mebkhout's pioneering work over the course of the eleven notes forming Procession VII of the Burial, "The Colloquium - or Mebkhout's sheaves and Perversity" (n^os 75-80). An investigation and a more extensive narrative about this (fourth and last) "operation" constitutes the most substantial part of the investigation "The four operations", under the fitting name **"The Apotheosis"** (notes n^os 171 (i) through 171 ?).

p. L27 all odds, as a "continuator of Grothendieck".

One of the paradoxical and disconcerting aspects of this Burial, among others, is that it was carried out chiefly, not to say exclusively, by people who had once been my friends or my students within a community in which I had never counted any enemies. It is mostly because of this reason, I believe, that Récoltes et Semailles concerns you more than others, and that this letter is meant to serve as an **inquiry**. For if you are one of the mathematicians among my ex-students and friends, your are certainly no stranger to the Burial, be it through your acts or complicity, or even through your silence towards me regarding something which has been taking place at your doorstep. And if (by some miracle) you choose to welcome these humble words and the testimony they bring you, rather than to remain locked behind closed doors and to dismiss the unwelcome messengers, you might then learn that the burial undertaken by all and with your participation (be it active or through your tacit acquiescing) did not only affect someone else's work, the fruits and testimony of his frenetic love affair with mathematics; rather, at a deeper and more hidden level than this (unspoken) burial, there is a living and essential part of your own being, and of your original power to know, to love, and to create, which you have elected to bury with your own hands in the guise of another person.

Among all of my students, Deligne had occupied a special place, upon which I elaborate at length during the reflection[30](*). He was, by far, the "closest" to the vast vision which had been born and had grown within me long before we met, and he was in fact the only one (among my students as well as others) to have intimately absorbed this vision and made it his own[31]. And among the friends sharing with me a common passion for mathematics, it was Serre who, having taken on the role of an elder, had been the closest (by far once again) in the unique role of "catalyst" for some of my greatest undertakings for a decade
p. L28 - and I owe him most of the great key-ideas which inspired my mathematical thought during the 1950s and 1960s, up until my departure from the mathematical world. This special relationship that they both had with me had to do in parts with their exceptional abilities, the latter of which guaranteed their equally exceptional rise in the ranks of the mathematicians of their generation and of the ones to follow. Other than these similarities, the temperaments of Serre and Deligne appear to me to be as dissimilar as could be, and they stand at antipodal points from one another in many ways.

In any event, if I had to name mathematicians who were in some way or another "close" to me and to my work (and who are furthermore known), I would have to name Serre and Deligne: the first as an elder and as a source of

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[30](*)See on this topic the group of seventeen notes "My friend Pierre" (n^os 60-71) in ReS II.

[31]This "vast vision" which Deligne successfully "assimilated and made his own" had generated a powerful fascination within him, and it continues to fascinate him despite his own will, when an imperious force is simultaneously pushing him to destroy it, to blow up its fundamental unity and to appropriate the scattered pieces. Thus, his occult antagonism towards a renounced and "late" master appears as the expression of a division within his being, which profoundly marked his work following my departure - a work which remained well below the rather prodigious abilities which I had known him to possess.

inspiration for my work during the crucial gestative period of my vision; and the second as the most talented of my students, for whom I was in turn (and remained, Burial or not...) his main (and secret...) source of inspiration[32]. The fact that a Burial could have been set in motion right after my departure (turning the latter into a proper "death") and furthermore concretized into an endless procession of "operations" both large and small serving the same end is only conceivable with the joint and solidary contribution of both of these individuals, the ex-elder and the ex-student (or even ex-"disciple"): one of them, feeling the need to destroy the **Father** (under the grotesque and derisory effigy of a plethoric and bombastic **powerpuff girl**) discretely and efficiently took charge of the operations, rallying some of my students to his cause along the way[33]; and the other granted an unconditional and unlimited "green light" to the carrying of the (four) operations (namely, the discrediting, massacre, cutting up, and partitioning of inexhaustible remains...).

4.9 The plunder

As I alluded to earlier, I have had to surmount a considerable amount of inner resistance, or rather to resolve them through a patient, meticulous and tenacious process, so as to manage to distance myself from certain familiar images that had taken hold in my mind with great inertia and which had weakened my ability to perceive reality in a direct and nuanced way over the course of decades (as is the case for everybody else, including yourself most likely) - namely, I am referring to the picture I had of a certain mathematical world to which I continue to be linked through my past and through my work. One of the most solidly anchored p. L29
such images, or ready-made ideas, is that it seemed downright unconceivable that a world-renowned intellectual, or even a man recognized to be a great mathematician, could engage in fraudulent behavior of any scale (and in any capacity, let alone to let that become a matter of habit...); and it seemed just as unconceivable that, abstaining (out of habit once again) from engaging in such behavior himself, he could nonetheless welcome such operations ("at times defying any sentiment of decency") organized by someone else when, for one reason or another, they are beneficial to him.

The inertia I faced was so great that it was only two months ago, at the end of a long reflection that had taken place over the course of a whole year, that I started timidly perceiving that Serre could have had something to do with this Burial - something which now appears to me as evident, independently from the eloquent conversation which I recently had with him. There was in my mind a certain tacit "taboo" surrounding his person, as well as all other members of the "Bourbaki milieu" which had warmly welcomed me at the start of my career to a lesser degree. He represented the very incarnation of a certain kind of "elegance" - an elegance that includes not only mere form but also a rigor

[32] See on this topic the preceding footnote.

[33] I am here referring precisely to the five other students who had (like Deligne) chosen the cohomology of varieties as their principal theme.

and a scrupulous probity.

Before my discovery of the Burial on April 19 of last year, I could never have even dreamed that one of my ex-students could have been capable of dishonesty in the exercise of their profession, whether towards me or anyone else; and such a supposition would have appeared most aberrant if applied to the most brilliant of my ex-students, the same one who was closest to me! Yet, from the moment of my departure and through the following years up to this very day, I have had ample occasion to realize the extent to which his relationship with me was divided. More than once, I had also seen him use of his power to discourage and to humiliate (as if for sports) when the occasion was fitting. Each of these events profoundly affected me (more so, surely, than I would have liked to admit at the time...). These were clear enough signs of a profound imbalance, which (as I have had ample occasion to observe) does not solely affect him, even in the very small circle of my ex-students. This imbalance, based on one's loss of respect for others, is no less flagrant nor less profound than the imbalance caused by
L. 30 so-called "professional dishonesty". The fact remains that the discovery of his dishonesty came to me as an utter surprise and as a shock.

In the weeks that followed this breathtaking discovery, followed by a "cascade" of others of a similar nature, I started to realize that a certain chicanery had already begun among certain of my ex-students[34] during the years preceding my departure. That this was the case was clearest for the most brilliant of them - the one who, after my departure, set the pace and (as I wrote earlier) "discretely and efficiently took charge of the operations". With almost twenty years of hindsight, this swindling appears as an evidence, "blindingly obvious".

If I had then chosen to ignore what was happening around me, busy that I was chasing the "white whale" in a world where "all is for the best" (or so I thought), I own realize that I failed to assume the responsibility that was mine regarding the students who learned under my supervision about the trade that I love; a trade which goes beyond a simple savoir-faire and the development of a certain "flair". Through my complaisance towards brilliant students, who I fancied (through a tacit decree) to treat as being "one of a kind", absolving them
check name of section
from all suspicion, I have contributed my own share[35] to the the hatching of (what seems like) unprecedented corruption, which I now witness to be diffusing into a world and into people whom I once held dear.

Admittedly, due to their immense inertia, it took me intense and sustained inner work to manage to separate myself from what are customarily called "illusions" (not without a hint of regret...), and which I would rather refer to as ready-made ideas; ideas about myself, about I milieu with which I had once identified, and about people whom I had loved and who I perhaps still love. I

[34] See the preceding footnote.

[35] This "contribution" appears notably in the note "One of a kind" (n^o 67') as well as in the two notes "The ascension" and "The ambiguity" (n^{o}s 63', 63"), then once again (under a slightly different light) at the end of the note "The eviction" (n^o 169). Another type of "contribution" appears in "Fatuity and Renewal", in the form of attitudes of fatuity towards young mathematicians who were less visibly brilliant. This coming into awareness of my share of responsibility in the general trend of degradation culminates in the section "Mathematics for sports" (n^o 40)

have managed to “separate myself” from these ideas, or rather, **to allow them to separate themselves from my person**. This process took work, but it was never a struggle; it brought me occasional bouts of sadness, among many other things, but never a feeling of regret or bitterness. Bitterness is a mechanism which allows one to elude awareness of a given fact, or of the message of p. L31
a lived experience; it keeps one locked in a tenacious illusion about themselves, at the cost of another “illusion” (its negative so to speak) about the world and about others.

It is therefore without bitterness or regret that I now part with these ready-made ideas which I had once “held dear”, through force of habit and because they had “always been there”. They had become almost like a second nature to me; nonetheless, this “second nature” was not “me”; as such, to separate myself from it bit by bit is neither tearing nor frustrating, the way it would be for someone letting go of things which are of value to them. The “plunder” about which I am speaking comes as a reward, as the fruit of a **labor**. It is immediately followed by a positive feeling of relief, a welcome **liberation**.

4.10 Four waves in one movement

Naturally, this letter doesn’t look anything like I had expected upon starting it. I thought I would mainly be giving a little “recap” on the Burial: here’s roughly what happened; whether or not you believe it (even I struggled to believe it at first...) and whether you like it or not, this is indubitably what it is - you need only open the right periodical or book at the right page, it’s all written in black and white. In fact, everything is unearthed at length in Récoltes et Semailles; see “The Four Operations”, note such and such - take it or leave it! And in case you would rather abstain from reading me, others will do it in your place...

In reality, none of the above was included - even though this letter is already nearing thirty pages in length, and I only expect to go on for five or six more pages. Without even meaning to, one page leading to the next, I was led to telling you the essential things, while this “bag” of items which I was impatient to unravel (and which itself was made clearly visible in the first few pages!) is still full! I don’t even feel the urge to write about those things anymore, the need dissipated along the way. I understood that now was not the time...

Truth be told, part IV of Récoltes et Semailles (the longest of all), titled “Burial (3)” or “The Four Operations”, grew out of a “note” initially expected to be “a little recap” once more, in which I would sum up at a high-level the things I had learned during the surprise-investigation () of the preceding year, which translate “et en coup de vent”
had been pursued in part II (“The Burial (1)”, or “The robe of the Emperor of China”). I thought I was setting up to write a five or ten pages note, tops. Eventually, one thing leading to the next, the investigation was taken up again, and I was off to write nearly four hundred more pages - nearly twice the length p. L32
of the part which I was suppose to summarize and synthesize! As a result, the recap in question is still missing, while six hundred pages of Récoltes et Semailles are devoted to the investigation around the Burial. It’s a little silly, for sure.

But there is always time to add a third part to the Introduction (which is no longer at the stage where ten or twenty pages will make a difference anyways), before handing my notes to a printer.

The five parts of Récoltes et Semailles (the first of which is not yet finished, and probably won't be for another few months) represent an alternation between (three) "meditation" waves and (two) "investigation" waves. This in some sense reflects, in shortened format, my life during the past nine years, which also consisted of an alternation between "waves" issued from the two passions that sustain me today, namely my passion for meditation and my passion for mathematics. In fact, the two parts (or "waves") of Récoltes et Semailles which I have given the cookie-cutter name "investigation" are precisely those which have directly emerged from my ties to my past as a mathematician - they were engendered by the mathematical passion within me and by the ego-attachments that took roots within it.

The first wave, "Fatuity and Renewal", consists of a first encounter with my past as a mathematician, leading to a meditation on my present, whose rooting in my past I freshly unraveled. Without the slightest amount of advance planning, this part lays out the "base tone" to persist throughout the rest of Récoltes et Semailles: it serves as a providential and indispensable inner preparation for me to undertake the discovery of the "Burial in all its glory" in what closely follows, during the second wave "The Burial (1) - or the robe of the Emperor of China". More than a simple "investigation", this part really retraces the process of discovery day after day, its impact on my being, and the efforts I have made to face what suddenly befell me without warning and to situate the unbelievable relative to my lived experience, so as to eventually be able to articulate what had happened in the context of the familiar. This movement leads to a first provisory conclusion, in the note "The Gravedigger - or the whole Congregation" (n^{o}97), the first essay in which I attempt to find an explanation and a meaning to something which, for years and now more acutely than ever, presented a formidable challenge to all common sense!

p. L33 This same second movement leads into a "sickness episode"[36] which forced me into absolute rest, putting an end to all intellectual activity for over three months. This happened at a time when I again thought I had almost brought Récoltes et Semailles to an end (modulo some last "housekeeping" tasks...). Upon resuming a normal activity around the end of September of last year, as I was getting ready to bring the last touch to the notes which I had left unattended, I thought that I would only need to add two or three final notes, including one regarding the "health-incident" which I had just gone through. Instead, one week after the other and one month after the other, there came a thousand more pages - more than twice what had already been written - and this time around, it was clear to me that I was still not done[37]! In fact, this long interruption, during which I nearly lost contact with a warm (nearly burning!)

[36]This episode is covered in the two notes "The incident - or mind and body" and "The trap - or ease and exhaustion" (n^{o}98, 99), opening "Procession XI" titled "The (living) dead".

[37]"Still not done", if only because a part V is yet to come, which is not complete at the time of writing.

substance, practically forced me to come back to this substance with fresh eyes, in order not to stupidly "wrap-up" the last part of a "program" with which I had lost living contact.

This is how the third wave in the vast movement that is Récoltes et Semailles was born - a long "meditation wave" on the theme of the yin and the yang, the "shadow" and "light" sides present in the dynamic of things and in human existence. This meditation grew out of a desire to reach a more in-depth understanding of the profound forces at work in the Burial, yet it acquires from the get-go its own autonomy and unity, orienting itself towards the universal as well as the intimately personal. It is during this meditation that I discover the (admittedly obvious, once the question has been posed) fact that, in my spontaneous approach to discovery, be it in mathematics or elsewhere, the "base tone" is "yin", "feminine"; I also come to realize that, unlike what happens in most cases, I have remained truthful to my original nature[38], never bending it or correcting it in an attempt to conform to the dominating values put forward by the environing milieux. At first, this discovery appears as a mere curiosity. Later, p. L34 it is revealed to be an essential key to understanding the Burial. Furthermore - and this is something which appears to be even farther reaching - I can now see the following thing clearly and without any doubt: that if, with my entirely un-extraordinary intellectual abilities, I was able to birth vast, powerful, and fertile work and vision, it is chiefly thanks to this very fidelity to my nature, this absence of any concern within me for conforming to the norms, which allows me to abandon myself with complete trust to the quest for original knowledge, without in any way cutting away from or amputating what gives this quest its strength, finesse, and indivisible nature,

Nonetheless, creativity and its sources are not at the center of attention in the meditation "The Burial (2) - or the Key to the Yin and the Yang"; rather, the dominating theme is the "conflict", the state of creative block, of dispersion of creative energy caused by the confrontation, within one's psyche, between (most frequently hidden) antagonistic forces. Aspects of a **violence**, which seemed furthermore (in appearance) "gratuitous", "for sports", had disconcerted me more than once during the Burial, and they brought back within me a host of similar lived experiences. The experience of this violence has been in my life the "hard and irreducible kernel of conflict". I had never before faced heads on the dreadful mystery that is the very existence and universality of this violence in human life in general, and within myself in particular. It is this mystery that is at the center of attention throughout the second half (the "yin" side, or "decline") of the meditation on the yin and the yang. It is during this part of the mediation that a more profound vision emerges progressively regarding the meaning of the Burial and the forces which are expressed within

[38] This "truthfulness to my original nature" was in no way total. For a long time, it concerned only my mathematical work, while in every other respect, and notably in my relationships, I followed the general motion, valuing and giving primacy to those traits within me perceived as "manly", while at the same time repressing "feminine" traits. I write about it in some details in the group of notes "Story of a lifetime: a cycle in three movements" (n^{o}107-110), which practically serves as an opening to "The key to the Yin and the Yang".

it. This is also the part of Récoltes et Semailles which seems to have been the most fruitful in terms of my self-understanding, as it put me in contact with delicate questions and situations, enabling me to actually feel their "delicate" character, when up until last year this character had been eluded.

At the term of this endless "digression" on the yin and the yang, I was still left, more or less, with the "two or three notes" that I still had to write (in addition to one or two more, at most, one of which already had a name: "The four operations"...) before finishing Récoltes et Semailles. The rest of the story is known: the "few last notes" turned into the longest part of Récoltes et

p. L35 Semailles, spanning over nearly five hundred pages: this is therefore the "fourth wave" of the movement. It is also the the third and last part of the Burial, and I named it "The Four Operations", a name which is also used to designate the group of notes constituting the heart of this fourth breath of the reflection ("The four operations (on the remains)". Herein lies the "investigation" segment of Récoltes et Semailles in the strictest sense - with the caveat nonetheless that the investigation is not limited purely to the "technical", or "detective" aspect, and that it is instead carried out in accordance with the desire to know and to understand, as is everything else in Récoltes et Semailles. The tone is noticeably "stronger" than in the first part of the Burial, in which I was still pinching myself, trying to convince myself that I was not dreaming! Regardless, the facts established over the course of the pages often emerge at the perfect time, serving as vivid illustration for many of the things which had until then only been occasionally touched upon in passing, and which as a result find an incarnation into precise and striking examples. It is also in this part that mathematical digressions take a more prominent place, stimulated by a renewed contact (which grew out of the needs of the investigation) with a substance of which I had lost sight for over fifteen years. At the other end of the spectrum, you will also find a timely narrative regarding the misadventures of my friend Zoghman Mebkhout

check that this is correct

(to whom this part is dedicated) at the hands of a high-flying and merciless "mafia" of which he was entirely unaware upon launching into the (passionating, and innocent-looking) subject of the cohomology of varieties of all kinds. For a succinct guiding thread through the intricate maze of notes, sub-notes, and sub-sub-notes... of this "investigation" part, I invite you to refer back to the table of contents (notes 167' through 176_7), as well as to the first note of the bunch, "The detective - or life through rose-colored glasses" (n^{o}167'). I should nonetheless signal that this note, dating from April 22, was later somewhat "overcome by recent events", in the sense that through several twists and turns this investigation which I then believed to be (practically) complete ended up

look up "à brin de zinc"

continuing for two more months.

This fourth breath lasted for four consecutive months, from mid-February through the end of June. It is mostly in this part of the reflection that a concrete and tangible contact with the reality of the Burial is established, day

look up the right translation to "travail sur pièces"

after day and page after page, through a meticulous and obstinate "piece-work". It is also there that I manage to "familiarize" myself with the Burial to some extent, putting to the side the visceral reactions of denial which it had prompted within me (and continues to prompt) and which constituted an obstacle to a

true coming into awareness. This long reflection takes as a starting point a retrospective on Deligne's visit (about which I have already spoken in this), p. L36
and it ends with a "last minute" reflection upon my relationship with Serre and upon Serre's role in the Burial[39]. Until last month, I believe that the most serious shortcoming of my understanding of the Burial was to have tacitly "excluded" Serre from the list of culprits, as a result of the "taboo" which I mentioned earlier - as such, this "last minute" reflection appears to me as the most important thing to have come out of this "fourth breath" of Récoltes et Semailles in contribution to a more substantial understanding of the Burial and of the forces that are expressed within it.

4.11 Movement and structure

I think I am now done enumerating the most important things which I wanted to tell you concerning Récoltes et Semailles, so as to let you know "what it's all about". I have surely said enough for you to be able to determine if **you** consider that the subsequent letter of (over) a thousand pages "is relevant to you" or isn't - and from there decide whether or not you will keep on reading. In case your answer is a "yes", I thought I would add some explanations (notably of a practical nature) regarding the **structure** of Récoltes et Semailles.

This structure is the reflection and expression of a certain **spirit** which I have tried to "convey" in the preceding pages. In comparison to my previous publications, the key novelty in both Récoltes et Semailles as well as "Pursuing Stacks" is **spontaneity**. Granted, you will find guiding threads and wide-ranging interrogations providing coherence and unity to the reflection as a whole. Nonetheless, the reflection is taken up day by day, without any preestablished "program" or "plan", never setting out ahead of time "what is to be demonstrated". My purpose is not to prove,but rather to **discover**, to probe p. L37
further ahead into an unknown substance, to condense what is as of yet only dimly sensed, suspected, or glimpsed. I can truly say, without any exaggeration, that in the course of this work, not a single day or night of reflection passed within the realm of the "planned", as much in terms of the ideas, images, and associations that presented themselves to me at the time when I sat down to write, and in so doing to persistently pursue a tenacious "thread", or to take up a new one that had just appeared. Each time, what ends up appearing during the reflection is different from what I would have predicted had I ventured to

[39] Among parts c, d, and e of the note "The family album" (n^{o}173), the last one dates from June 18 (which was exactly ten days ago). Only one other note or note segment corresponds to a later date (namely, "Five theses for a single massacre - or filial piety", n^{o}176$_7$, dating from the next day, June 19). You will note that in this fourth part of Récoltes et Semailles, or "investigation segment", unlike elsewhere in the reflection, the notes are organized in logical rather than chronological order. Thus, the last two notes of the Burial (constituting the final "De Profundis") date from April 7, two and a half months earlier than the note I just mentioned. I should nonetheless signal that aside from the "investigation" proper of the Burial (3) (notes n^{o}s 167'-176$_7$), constituting the "fifth movement" of the Funereal ceremony ("The Key to the Yin and the Yang" being the second), the notes appear in chronological order, modulo some rare exceptions.

describe in advance what I expected to discover ahead. Most of the time, the reflection ends up taking me down entirely unforeseen paths, eventually leading to new and equally unexpected landscapes. And even when the reflection follows a more or less expected trajectory, the sequence of images that I encounter over time differ from the picture I had at the outset as much as a real landscape, with its interplay of fresh shadows and warm light, its delicate features ever-changing with the trekker's every step, its countless sounds and nameless scents carries around by a breeze which makes the weeds dance and the forest sing... - as much as such a lively, elusive landscape differs from a postal card, however pretty and well-done - however "accurate" it may be.

It is this reflection carried out in one go, over the course of one day or one night, which constitutes the indivisible unit, the living and individual cell of sorts, underlying the entirety of the reflection (Récoltes et Semailles in this case).
p. L38 The reflection as a whole is to each of these units (or "notes"[40], constituting a melody...) as the body of a living organism is to each of its individual cells, the latter of which each fulfill a unique place and function in their infinite diversity. Nonetheless, it can sometimes happen that within a reflection issued from a single outpouring, we are able to perceive in retrospect important divisions, bringing to light several uniting themes or messages, each of which thereafter receives its own name and thus acquires an identity and autonomy. At other times, a reflection which had been cut short for some reason (fortuitously most of the time) is spontaneously continued into the next day and the one after that; or yet a reflection carried over the course of two or more consecutive days appears to use in retrospect as if it had been written in a single stretch - as if the necessity of sleep had forced us to mark some sort of pause (in a sense "physiological"), indicated by a lapidary indication of the date (or of several such) separating consecutive paragraphs of the same "note", with the latter
p. L39 carved out with its own name.

Thus, each of the notes in Récoltes et Semailles has its own individuality, and carries a face and a function distinguishing it from every other. I have attempted to express the particularity of each note by a **name**, supposed to reconstitute or evoke the essential, or at least something essential regarding what that note "has to say". I identify each note by its name before all else, and that is how I choose to refer to a given note every time that it is relevant.

The name of a note often occurred to me spontaneously, before I even paused to think of one. I see such an unprompted appearance as a sign that the note at hand is nearly complete - that it will have said what it had to say by the time I complete the paragraph in progress... Equally as often, the name appears, just as spontaneously, as I am re-reading the notes from the preceding day or the day before that in preparation for the continuation of my reflection. The name may be slightly modified during the days or week following the creation of the new note, or it might even be enriched by a second name which had not occurred to me in the first place. Several notes carry such a double name, shining two different and sometimes complementary lights on their message. The first time

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a double name occurred to me was at the beginning of "Fatuity and Renewal", through "Encounter with Claude Chevalley - or freedom and kind sentiments" (n^{o}11).

I only had a name in mind prior to starting a note at two occasions - and both times, the name was shaken up by the turn of events!

It is only with the hindsight of weeks, and sometimes months, that an **overall movement** becomes visible, together with a **structure** underlying the collection of notes written day after day. I have attempted to delineate these superstructures through various groupings and sub-groupings of the notes, each with its own name, granting it a separate existence as well as a function or message; akin to division of a single body into its organs and limbs (to continue the earlier analogy), as well as the decomposition of a limb into its parts. Thus, within the "Whole" of Récoltes et Semailles are included the five "parts" mentioned earlier, each with its own particular structure: Fatuity and Renewal consists of eight "chapters" I through VIII[41], while the three parts constituting the Burial (which themselves became their own entities over time...) p. L40
consist of one long and solemn Procession involving twelve "Cortg̀es" I through XII. The last of them, **"The Funereal Ceremony"** (as it is named) towards which the preceding eleven Cortèges progressed (surely without suspecting it...), takes truly gigantic proportions, matching the scope of the lifework which is the subject of these solemn Funerals: it occupies the near-totality of RS III (The Burial (2)) and the totality of RS IV (The Burial (3)), spanning nearly 800 pages and 150 notes (even though said ceremony was only expected to fill two notes!). Under the skilled direction (and well-known modesty...) of the great officiating priest himself, the ceremony takes place in nine "acts", or separate liturgical acts, beginning with **The Funereal Ceremony** (naturally) and ending (as is fit) with the final **De Profundis**. Two of these "acts", namely **"The Key to the Yin and the Yang"** and **"The Four Operations"**, each constitute (by far) the largest section of the part of Récoltes et Semailles (III or IV) to which they belong, and as such the latter bear their names.

Throughout Récoltes et Semailles, I took good care of the table of contents (as if it were the apple of my eyes!), ceaselessly restructuring it so as to account for the ever-renewed influx of unexpected notes[42], in the hope that it would reflect as finely as possible the overall movement of the reflection and the delicate structure emerging therein. It is in parts III and mostly IV (just mentioned above), "The Key [...]" and "The Four Operations", that this structure is the most complex and imbricated.

In order to preserve the spontaneous character of the text, and to render the unexpected aspects of the reflection as they were truly experienced, I decided not to append a name to the notes which only appeared after the fact. As such, I recommend that you refer back to the table of contents after reading each note p. L41
so as to discover its name, and to also get a chance to appreciate at a glance how it inserts itself in the reflection thus far, or how it relates to what is yet to come.

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Otherwise, you run the risk of losing yourself in a seemingly indigestible and heteroclite, not to say cumbersome[43], collection of (sometimes strangely numbered) notes; so as to resemble a lost traveler in a foreign city (which sprouted in a bizarre fashion following the whims of the generations over the course of centuries...), with no guide nor even a map to help them orient themselves. In the manuscript destined to publication, I plan on including throughout the text the names of the “chapters” as well as those of other groupings of notes and sections, except for the notes (or sections) themselves. But even then, the occasional recourse to the table of contents seems vital so as not to get lost in an aggregation of hundreds of notes, one following the next for over a thousand pages...

4.12 Spontaneity and rigor

Spontaneity and rigor constitute the “shadow” and “light” side of one indivisible quality. It is only through their marriage that this particular quality can be born in a text or a person - it may be approximately described as the “quality of truth”. Although spontaneity has been played down (if not downright absent) from my past publications, I do not think that its recent blossoming within me has affected my rigor. Rather, the presence of rigor’s yin companion gives it a new dimension and a renewed fecundity.

Rigor has to monitor itself, so as to prevent the careful “sifting” of the multitude of occurrences of the field of consciousness, constantly separating what is significant or essential from what is only fortuitous or accessory, from hardening into bouts of censorship and complacency. Curiosity alone, that thirst for knowledge within us, is capable of stimulating such an effortless alertness and vivacity in the face of the immense and ubiquitous inertia of so-called “natural downward slopes” consisting of ready-made ideas expressing our fears and our conditioning.

The same rigor and alert attention may be directed towards spontaneity and its outgrowths, so as to distinguish once again between these entirely natural
p. L42 “downward slopes” and that which has truly emerged from the depths of one’s being, issued from the original impulse towards knowledge and action which invites us to dive into the world.

In the context of writing, rigor manifests itself in the form of a constant need to capture things as finely and faithfully as possible, using language, thoughts, feelings, perceptions, pictures, or intuitions... So that one is not content merely expressing oneself vaguely or approximatively when the thing at hand has a clearly defined shape; nor is it satisfactory to use an ostensibly precise term (and thereby further deforming the truth) in order to express something which remains immersed in the mist of what has been felt but not yet grasped. Only when we attempt to capture the thing as it is in the moment can we hope to witness its true nature, perhaps even in broad daylight if it is meant to be seen

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in this way and if our desire persuades it to let go of its shadowy veils and enveloping mist. Our role is not to pretend being able to describe and pin down that which we still ignore; rather, it is to humbly and passionately endeavor to understand the mysteries and unknown entities that surround us from all sides.

That is, the role of writing is not to present the results of research, but rather to report on the process of research itself - to document the work of love and the fruits of this work with Mother Nature, the Unknown, who tirelessly calls us to know more about her inexhaustible Body, venturing wherever the mysterious winds of desire carry us.

In the documentation of this process, retracing one's steps so as to add nuance, precisions, supplements and sometimes corrections to the written "first jet" constitute an essential part of the very act of discovery. These backtracks form an essential part of the text and give it its full meaning. This is the reason why the "notes" (or "annotations") placed at the end of Fatuity and Renewal, to which I refer throughout the 50 "sections" constituting the "first jet" of the text, form an inseparable and essential component of the latter. I warmly recommend that you refer to them every now and then, at least whenever you are done reading a given section which makes one or several references to these "notes". The same applies to the footnotes in the other parts of Récoltes et Semailles, or the references made by a given "note" (constituting a part of the "main text") to a later note, which takes on the role of a "flashback", or an annotation, relative to the first note. The present advice, together with my advice not to forget about the table of contents in the course of your reading, are the main reading recommendations which I can think of. p. L43

One last practical point remains to mention before I finally conclude (somewhat prosaically) this letter. There was a bit of a "stampede" at times so as to ensure that the various parts of Récoltes et Semailles issued from the copying Service at the University would be ready (if possible) in time for the summer holidays. As a result of this rush, there remains a whole page of last minute footnotes which "didn't make the cut" to be added to part 2 (The Burial (1) - or The Robe of the Emperor of China). These mostly consist of rectifications of some material errors which only recently became apparent, during the writing of The Four Operations. One of these footnotes is more important than the others, and as such I would like to mention it here. It pertains to an annotation to the note "The victim - or the two silences" (n^{o}78, page 304). In this note, I endeavor, among other things, to collect my (admittedly subjective) impressions regarding the way in which my friend Zoghman Mebkhout was then "interiorizing" the iniquitous plundering of which he was the victim. Zoghman expressed that he felt that the note was unjust towards him, in that I apparently put him "in the same bag" as his plunderers. One thing is for sure, namely that this note, which doesn't pretend to go beyond the recollection of impressions linked to a specific "moment", focuses on a single aspect of the situation, while leaving unsaid (and surely as obvious) other equally real aspects (which are perhaps less conducive to debate). Nonetheless, my reflection surrounding this delicate topic had significantly deepened by the time I wrote the note "Roots and Solitude" (n^{o}171), about a year later. The latter note was not the subject of further comments from Zoghman. Other elements of my reflection on the same topic can be found in the two notes "Three milestones - or innocence" and "The bygone pages" (n^os 171 (x) and (xii)). The aforementioned three notes are part of "The Apotheosis", the segment of The Four Operations centered around the operation of appropriation and hijacking to which Zoghman Mebkhout's work was subjected.

All that remains is for me to wish you a pleasant read - and to look forward to reading you in turn!

Alexandre Grothendieck

Epilogue in postscript - or context and preliminaries to the debate

February 1986

4.13 The bottle spectrograph

p. L44

It has now been seven packed months since this Letter was written, and nearly four months since it was sent, along with the accompanying "tome" - and a handwritten dedication accompanying each[44]. Like "message in a bottle", or rather, like a whole host of vagrant bottles, my message circulated and reached the outermost corners of the mathematical microcosm that I once knew. Through the direct and indirect echoes that are flowing back to me over the course of days, weeks, and months, I have now unexpectedly become the witness of a vast radiography of the mathematical world, which was probed by a sort of tentacular spectrograph, with each of my innocuous "bottles" serving as a separate traveling tentacle. As such (noblesse oblige!), and due to no lack in current occupations, I am now faced with the task of deciphering said radio and producing a report of what I have read therein, to the best of my abilities. This will constitute a sixth (and last, I promise!) part of Récoltes et Semailles. It will thus come to crown, God willing, "the great sociological work of my old days". For now, I shall restrict myself to some initial commentaries.

In response to my modest and artisanal flotilla, the tone which seems to prevail is by far a half-sneering, half-snarling tone along the lines "here comes Grothendieck, who is falling prey to paranoia in his old age", or "he is decidedly taking himself quite seriously" - and voila! And yet, I only received a single letter taking this tone[45], along with two others adopting an attitude of effaced derision and self-satisfaction[46]. By and large, most of the mathematicians I contacted,

[44] With a few exceptions, especially including colleagues whom I do not know personally, and who only received parts 0 through 4 of the preprint, in recognition of their active participation in my Burial.

[45] This letter came from one of my ex-students, who is furthermore a fellow entombed.

[46] Coming from two of my old colleagues from the Bourbaki group, one of whom being the elder who had once welcomed me with warmth and benevolence at my beginnings.

including many of my old students, responded with their silence[47] - a silence that speaks volumes. p. L45

I have nonetheless already received voluminous feedback. The majority of the letters are framed in a tone of polite embarrassment, intended to appear as amicable, as if stemming from a desire to act with decorum. Twice or thrice, I could sense the warmth of an authentic sentiment hiding behind this embarrassment, the latter of which was muted as a result. Most often, when the embarrassment is not expressed in the form of protestations (regarding oneself, or on behalf of another), it comes out through compliments - never in my whole life have I received so many! The likes of “great mathematician”, “superb writing” (in terms of creativity and “all that”...), “indisputable writer”, among others. For good measure, I also received a perceptive (and by no means ironic) compliment regarding the richness of my inner life. Needless to say that in all of these letters, my correspondents kept off the heart of the matter, and wholly refrained from involving themselves personally; rather, the tone assumed was that of someone who has been “solicited to share their opinion” (paraphrasing the language of one of the letters), on a somewhat indecorous story, which is moreover hypothetical, not to say imaginary, and which in any case **does not personally concern them**. When they do consent to touching one of these questions, they only do so with their fingertips, keeping it as far away as possible - be it by providing me with good advice, or prudent conditionals, or by means of commonplace sayings when one is at a loss for something to say, etc... Some even tacitly suggested that some unusual things may have indeed happened - all the while taking great care to remain completely vague as to what it is they are referencing...

I have also received some truly warm echoes on behalf of fifteen or sixteen of my friends old and new. Some shared an emotion with me, without trying to hide it or suppress it. These echoes, as well as other equally warm ones coming from outside of the mathematical world, will have been my reward for a long and solitary endeavor, produced not only for myself, but for all.

And among the hundred something colleagues who received my letter, only three have responded in the full sense of the term, with personal involvement rather than through a detached commentary on the century’s ebbs and flows. I also received another such echo from a non-mathematical correspondent. These were true **responses** to my message - and they constituted the best of rewards.

4.14 Three feet in a single dish

p.L46

Many of the mathematicians among my colleagues and friends have expressed the hope that Récoltes et Semailles would open a large **debate** in the mathematical world, concerning the state of affairs in the milieu, the mathematicians’ ethical code, as well as the meaning and finality of their work. For now, the least

[47] Among the 131 letters mathematicians to whom I sent a copy, only 53 have thus far responded in some way or another, if only to acknowledge receipt. Among those are six of my ex-students - I have yet to hear from the remaining eight.

I could say is that such a debate does not seem to be underway. It presently appears (to use the obvious pun) that the debate around a Burial may well have been replaced by the burial of a debate!

Whether or not the majority remains silent and apathetic, the fact remains that a debate has been started. It is unlikely that it will ever take the scale of a true public debate, or even (God forbid!) the pomp and rigor of an "official" debate. Several people have swiftly gone past it, closing the door to their inner self before even acquainting themselves with the issue, strongly attached to the timeless and immutable consensus that "all is for the best in the best of worlds" (in this case, the mathematical world). Perhaps a reckoning will eventually come from **without**, progressively, through "witnesses" who, in their quality of outsiders, will not be tied down by groupthink, and as such will not perceive themselves (including in their inner core) to be personally targeted.

In almost all of the echoes that I received, I noticed a pervading confusion regarding the above two questions: **what** is the "debate" (tacitly) suggested by Récoltes et Semailles about; and who is in a position to recognize the issue and speak up about it, or to formulate an informed opinion about it.In this regard, I would like to offer **three "orienting points"**. This will not prevent those who hold their confusion dear to continue holding onto it. As to those who would like to know what it is I am talking about, this will perhaps help them not get too distracted by the noises coming from every direction (including from the most well-meaning individuals...).

a) Certain sincere friends of mine are assuring me that "everything with eventually be sorted out" (where the "everything", I suppose, refers to things that have been accidentally damaged...); that I need only make my comeback, "impose myself through new results", speak at conferences, etc - and the others would take care of the rest. They may magnanimously say that "people acted rather unfairly toward good old Grothendieck after all", then discretely adjust
p. L47 their behavior with varying degrees of conviction[48](*);. They may even paternalistically tap his shoulder and throw a few "great mathematician" at him, intent to calm down an individual who remains respectable when all is said and done, but who alas seems to be upset and is making undesirable waves as a result.

Unlike what said friends are suggesting, "letting off steam" or causing steam to be let off is not what this is about. I am neither in need of compliments nor of sincere admirers, and I do not need "allies" for "my" cause, nor for any other cause. This is not about me - I am doing wonderfully well - nor is it about my work, which speaks for itself - even if it lands on deaf ears. If this debate regards my person and my work, among other things, it is in a **revealing** role more than anything else, through the reality of the (most revealing) Burial.

If I had to name "someone" who in my eyes inspires a feeling of alarm, disquietude, and urgency, I would not point to myself, nor to any of my "coburried". Rather, I would point to a collective being, at once elusive and very

[48](*) I have already alluded to several discrete signs of such behavior, indicating that people had taken note of the fact that the lion had come out of his den...

much tangible, about whom we often speak but refrain from ever examining - namely, **"the mathematical community"**.

Over the course of the past weeks, I have come to see it as a person in flesh and blood, whose body is stricken by severe gangrene. Food of the highest quality and the most select of dishes turn into poison when fed to it, only serving to further propagate and entrench the disease. Yet, an irresistible bulimia pushes it to binge on ever more food, as if this were a way to even out the disease which it avoids facing at all costs. Nothing one could say could get to it - even the simplest of words have lost their meaning. They have stopped being communication vehicles, and they now only act as triggers of fear and denial...

b) Most of my colleagues and old friends, even when acting in good faith, only risk voicing an opinion by surrounding it by cautious conditionals such as "if it were true that.... then this would indeed be inadmissible" - after which they may soundly go back to sleep. And here I am thinking I had been clear...

With seven months of hindsight, I am able to confirm that the **quasi-totality of the facts** reported and commented upon in Récoltes et Semailles **are uncontroversially correct**. I will come back to the rare exceptions later, and they will be signaled as such, each in due time. As for all of the remaining fact, following the writing of the preliminary version of Récoltes et Semailles, a p. L48 careful confrontation with some of those principally concerned (namely, Pierre Deligne, Jean-Pierre Serre, and Luc Illusie) allowed me to eliminate technical mistakes, so as to arrive to an unambiguous agreement regarding the material facts proper[49](*).

Thus, the debate does not revolve around the accuracy of the facts at hand, which is not in doubt; rather, it concerns the question of **whether the practices and attitudes illustrated by these facts should or should not be considered to be admissible and "normal"**.

I am referring to practices which I qualify (perhaps wrongly...) in my testimony as scandalous; as breaches of trust, abuses of power, and glaring acts of dishonesty, often displayed with brazenness and iniquitousness. The unimaginable thing that I had yet to discover was that, upon learning about these facts (which would have been unthinkable just fifteen years ago), a large majority of my mathematical colleagues, including some of my ex-students and friends, thought these practices to be normal and perfectly honorable.

c) Another approach to which several of my colleagues and old friends resort to maintain a confusion is by saying a version of the following: "sorry, but we are not qualified to address these matters - stop asking us to take stock of a situation which (providentially...) goes above our head...".

Quite to the contrary, I assert that there is no need for "qualifications" (and for that I am sorry in turn!) in order to become aware of the principal facts - not even the need to know one's multiplication table, or Pythagoras' theorem; nor to have read "Le Cid" or La Fontaine's Fables. A normally developed ten

[49](*) I happily extend my gratitude to all three of them for the good faith which they have demonstrated in this occasion; I hereby recognize their complete good faith in answering questions about material facts.

year old child would be just as qualified as the specialists of highest repute (if not more so...)[50](**).

Allow me to illustrate this claim by an example, the "first to come to mind" from the Burial[51](***). There is no need to understand the ins and outs of the multifaceted and delicate notion of "motive", nor to have even received one's elementary school certificate, in order to take stock of the following facts, and to emit a judgement thereupon.

p. L49

- 1) Between 1963 and 1969, I introduced the notion of "motive" and developed surrounding it a "philosophy" and a "theory" which remained partially conjectural. Rightly or wrongly (it doesn't matter for the present purposes), I consider the theory of motives to be the most profound contribution that I have made to the mathematics of my time. Today, nobody questions the importance and the depth of this "motivic yoga" (and this after ten years of quasi-total silence surrounding this subject, following my departure from the mathematical world).

- 2) In the first and only book (published in 1981) devoted for the most part to the theory of motives (whose name, which I first introduced, appears in the title), the only passage that could indicate to the reader that my humble self had anything to do with a theory that could resemble the one extensively developed in this book can be found on page 261. This passage (two and a half lines long) consists in explaining to the reader that the theory developed in the book has nothing to do with that of a so-called Grothendieck (a theory which is mentioned there for the first and last time, with no further reference or details).

- 3) There is a famous conjecture, called the "Hodge conjecture" (no need to know what exactly it is about), whose validity would imply that the so-called "other" theory of motives developed in this brilliant volume is identical to (a very special case of) the theory which I had developed, out in the open for everyone to see, nearly twenty years earlier.

To these, I could also add 4) that the most prestigious of the four co-signatories of the book was once my student, and that he learned from me the brilliant ideas which he hereby presents as if he had just thought of them this very instant[52](*), 5) that the aforementioned two circumstances are a matter

[50](**) Of course, Récoltes et Semailles is not addressed to said ten year old child - had it been so, I would have chosen a language which he would have found more familiar.

[51](***) This is the first "great operation" pertaining to the Burial which I have discovered, on a fated April 19^{th} 1984, the same day that the title "The Burial" occurred to me. See on this subject the two notes written on the same day, "Memories from a dream - or the birth of motives", and "The Burial - or the New Father" (ReS III, n^{o}51, 52). The complete reference for the book about to be mentioned may also be found there.

[52](*) I am not implying that this book is devoid from ideas, or even beautiful ideas, which are attributed to this author or to the other co-authors. Rather, I am highlighting that the chief concern of the book, and the conceptual context which gives it meaning, including the delicate theory of X-categories (wrongly called "Tannakian categories") which lies at the heart of the book, are issued from my own work.

of public knowledge among the informed public, but that it would be vain to try to find a written acknowledgment in the literature that said brilliant author could have learned anything from me[53](*), and 6) that I was the one who formulated the delicate arithmetic question which (according to the principal author's personal explanation) constitutes the core of the book (without my name ever being mentioned surrounding it) during the 1960s, in the wake of the "yoga of motives", after which the author learned it from me; I could follow-up with a 7), an 8) and so forth (and in fact I do so in due time). p. L50

The above shall suffice to make my point, which is the following: in order to take stock of the above facts and to emit a judgement thereupon, there is no need for any particular "competencies" - **the heart of the matter is not "taking place" at that level**. The faculty which is herein called for, other than a sound mind (possessed in principle by each and every one), is what I will call **a sentiment of decency**.

The book in question is already one of the most cited works of the mathematical literature, and its "principal author" one of the most prestigious mathematicians of the era. That being said, the thing that is by far the most remarkable in my eyes, in this whole story, is that **not a single one of the countless readers of this book**, including those who know firsthand about the state of affairs, being my ex-students or friends **noticed that anything was out of the ordinary**. Or at least, not a single one, up to the very day where I am writing these lines, has come to me to express reserve of any kind[54](**) regarding this prestigious book.

As to those, among my colleagues and old friends, who have never held this book in their hands, and fall back on this fact to plead their incompetence, I would like to tell them the following: there is no need to be a "specialist" in order to ask to see the volume at the nearest mathematical library, skim through it, and thereby see for yourself what is not contested by anyone...

4.15 Gangrene - or the spirit of our times (1)

This "operation motives" is but one of four "great operations" of the same type, which in turn fit into a swarm of other events of lesser scale but of similar spirit. It is neither the most "obnoxious" of the collective mystifications which come to form the "tableau des moeurs" of an era, nor the most iniquitous. It consisted only in pillaging the wealthy man's herd in his absence (or demise...), rather than coming to strangle (in a general climate of indifference) the poor man's lamb, for sports and right before his eyes. Even within the mathematical language now commonly in use, certain apparently anodyne names of books, notions, and theorems cited left and right are themselves indicative of a mystification p. L51

[53](*) An exception should be made for a line included in a handwritten report to Serre from 1977, which will be addressed in due time.

[54](**) All in all, only two colleagues (including Zoghman Mebkhout) have expresses such "reserves" to me. Neither of them may be considered to be among the targeted "readers" of this book. They opened it out of curiosity, so as to see things for themselves...

and an imposture[55], thereby serving as witnesses to the disgrace of an era.

If I have ever made a positive contribution to the "mathematical community", it was by shedding light on a number of disreputable facts which were lurking in the shadows. These were furthermore facts which everybody interacted with on a near-daily basis in some capacity. Yet, how many among us have ever taken the time to stop, take a look, and smell the air surrounding it?

Some of those who have been faced with the arrogance or dishonesty of certain individuals (sometimes one and the same) may have filed away the incident as a strike of bad luck, or as something solely targeted at them. In comparing their experience with my testimony, perhaps they will sense that this "bad luck" is just a name that they have given to the **spirit of our times**, which weighs on them just as much as it weighs on everyone else. And (who knows!) perhaps this realization will encourage them to become involved in a debate, which concerns them just as much as it concerns me.

But if the "dirty laundry" which I am "spreading out in a public place" induces only joyless snickers from some and polite embarrassment from others, amidst general indifference, what was once a confusing situation would become very clear. (At least for those who still care to see things with their own eyes.) p. L52 The traditional consensus of good faith and decency[56] in the relationships between mathematicians and between a mathematician and his art would have become a thing of the past, "outdated". The following standard would have become understood and quasi-official, without the need for some international association of mathematicians to solemnly proclaim it: from now on, **it's a free for all**, without any further reserve or limitations, for the "co-option brotherhood" consisting of the powerful individuals of the mathematical world. Everything is fair game, including the blurring of ideas' origins intended to lead astray the apathetic reader who will readily believe anything, the trafficking of authorship, blank-citations between associates and silent treatments for those condemned to silence, cronyism and falsification of all kinds, all the way to the most heavy-handed plagiarism in full-view - **yes, and amen to it all**, with the benediction, openly voiced or through silent agreement (if not by actively and hastily participating) of all of the "household names" and all of the bosses, big and small, on the mathematical public square. Yes, and amen to the **"new style"** that's all the rage! What was once an art has now become, through (quasi-) unanimous agreement, , under the paternal gaze of the leaders.

There was a time when the exercising of power in the mathematical world was constricted by unanimous and intangibles consensus, indicative of a collective feeling of **decency**. These consensus and this collective feeling would

[55] Here, I am mostly referring to the bizarre acronym "SGA $4\frac{1}{2}$" (how convenient to have access to fractions!), a double imposture all by itself (and one of the most cited acronyms of contemporary mathematical literature), as well as to the names "Verdier duality", "Grothendieck-Deligne conjecture", and "Tannakian categories" (for the latter, Tannaka himself is not to be blamed, as he was never consulted...). I will come back to this topic in more depth at a later time.

[56] In mentioning these "consensus of good faith and decency", I am not suggesting that they were never violated. But even when they were violated, the act was treated as a "violation", and the consensus did not become any less accepted thereafter.

thereafter have become obsolete and outdated, unworthy surely of the glorious era of computers, space shuttles and neutron bombs.

The following principle would become set in stone and taken for granted: the use of power, for the members of the dominating brotherhood, is **discretionary**.

4.16 Honorable amend - or the spirit of our times (2)

In the Letter, I believe I have been sufficiently clear regarding the spirit in which I have written Récoltes et Semailles; in particular, regarding the fact that I did not in any way aim to act as a historian. Récoltes et Semailles is first and foremost a testimony produced in good faith about my lived experience and my reflection thereupon. The testimony and reflection are available to all, including historians, who may use it as one primary source among others. It will be the historians' task to critically examine this primary source, in conformity with the standards of rigor of their art.

It is naturally in order to distinguish between facts in the strict sense (**"raw facts"**, or "material facts") and the "evaluation" or **"interpretation"** of these facts giving them a **meaning**, the latter possibly differing from one observer (or co-actor) to the next. Roughly, one could say that the "testimony" aspect of Récoltes et Semailles concerns the facts, while the "reflection" aspect concerns their interpretation, i.e. the process through which I have assigned meaning to them. Among the "facts" constituting this testimony, I am including the "psychological facts", notably the feelings, associations, and images of all kinds, dating from a more or less distant past or occurring at the time of writing, of which my testimony is the reflection. p. L53

There are three kinds of **sources** for the facts which I describe or relate in Récoltes et Semailles. Some of the facts are obtained from my **memory**, more or less precise or hazy from one instance to the next, and sometimes distorted. I can vouch for my intention to be truthful at the time of writing these facts, but I cannot guarantee the absence of mistakes. Quite to the contrary, I have been able at times to locate a number of mistakes regarding details, and I signal each one in time in subsequent footnotes. Secondly, there are **written documents**, notably letters and mostly scientific publications proper, to which I refer with suitable precision. Finally, there are **testimonies from third parties**. Sometimes, the latter come as complements to my own memories, allowing me to rekindle them, to make them more precise, and sometimes to correct them. In some rare occasions (to which In will soon return), the testimony brings in entirely new information with respect to my understanding at the time. When I choose to echo a given testimony, I do not guarantee that I have been able to thoroughly verify its exactness and well-foundedness, only that it inserted itself sufficiently convincingly in the rich thread of facts which I knew about firsthand, leading me to believe (rightly or wrongly...) that this testimony was indeed mostly true.

I believe that the attentive reader will have no trouble "sorting out" at any time what constitutes a retelling of facts as opposed to an interpretation thereof, and (in the first case) to discern which of the three sources I described is relevant.

*   *
*

p. L54

The testimony from a third party which I just alluded to, and which I have been echoing without having been able to "thoroughly verify its validity", is **Zoghman Mebkhout**'s testimony regarding the vast operation of discrediting surrounding his work. Among the "material facts" which I enumerate in Récoltes et Semailles, the only facts which are currently subject to controversy or which, according to my present judgement, are in need of rectification, are some of the facts supported only by Mebkhout's testimony. Before ending this post-scriptum, I would like to offer some critical commentaries regarding the version of the "Mebkhout affair" presented in the preliminary printing of Récoltes et Semailles. More in-depth commentaries and reflections will be included, each in its place, in the printed edition constituting the definitive text of Récoltes et Semailles.

The "Mebkhout version" for which I have been the interpreter seems to essentially boil down to the following two theses:

- 1. Between 1972 and 1979, Mebkhout was the only one[57] to develop, amidst a general atmosphere of indifference and drawing inspiration from my work, the "philosophy of $\mathcal{D}$-modules", viewed as a new theory of "cohomology coefficients" in my sense of the term.
- 2. There was unanimous consensus, both in France and abroad, to retract his name and his role in this new theory once its scope started being recognized.

This version was extensively documented, on the one hand through Mebkhout's perfectly convincing publications, and on the other hand through several publications by other authors (notably, the **Actes** of the June 1981 Luminy Colloquium), wherein the retraction claim no longer raises any doubts. Finally, the more in-depth details which Mebkhout shared with me at a later date (and which I echo in the part "The Burial (3) - or the Four Operations"), while not being directly verifiable, were entirely aligned with a certain general atmosphere whose reality was no longer in doubt in my eyes.

p. L55

I have just been made aware of several new facts[58], which indicate that the aforementioned point 1) deserves to be strongly nuanced. The isolation under

[57] An exception should be made for Kashiwara's constructibility theorem in 1975, whose importance is in no way contested. But according to Mebkhout's version, this was Kashiwara's one and only contribution to the emerging theory. This (inexact) version was corroborated by the absence of other publications by Kashiwara, in which he would likely have at least alluded to some of the key ideas.

[58] I am grateful to Pierre Schapira and to Christian Houzel for drawing my attention to these facts, and to my tendencious treatment of the Kashiwara-Mebkhout dispute.

which Mebkhout operated[59] was real, but it was only a relative isolation. In France, **J. P. Ramis** was concurrently producing work in the same subject (works which Mebkhout never mentioned to me); secondly and most importantly, it appears that some of the important ideas which Mebkhout developed and carried out, and for which he claims authorship, could be due to Kashiwara[60]. This renders some of the episodes of the Kashiwara-Mebkhout dispute implausible or doubtful in the form in which they are reported in the Mebkhout version, for which I took on the role of (overly) faithful interpreter.

There is no doubt about the fact that, in terms of "full-fledged work", as well as regarding the conception of some of the ideas which he was able to carry out to their end, Mebkhout was one of the principal pioneers of the new theory of $\mathcal{D}$-modules, if not **the** principal pioneer; in any case, he was the only one to have invested himself body and soul into this task, whose true scope was still eluding him, just as it was eluding everyone else. It also remains true that the retraction operation that took place surrounding his work, culminating in the Luminy Colloquium, is in my eyes one of the greatest disgraces of the century in the mathematical world. But it would be wrong to pretend (as I once did in good faith) that Mebkhout was the only one working on the subject. On the other hand, he was the only one with enough honesty and courage to clearly spell out the importance of my ideas and my contributions in his work and in the burgeoning of this new theory.

This post-scriptum is not the place to write about this affair in more details. I shall do the latter in due time, including commentaries attempting to shed light on the psychological context of the "Mebkhout version". If the "contentious Mebkhout-Kashiware dispute" is of interest to me, it is only to the extent that it sheds light on the general atmosphere of an era. In my opinion, the "Mebkhout version" also belongs, along with its deformations and due to the forces that led to its formation, to the list of less contestable materials which I am adding to the "folder of an era", as an eloquent "sign of our times". p. L56

All that remains is for me to make an honorable amend for my flimsiness in presenting the Mebkhout-Kashiwara dispute in a way that only took into account the testimony and documentation provided by Mebkhout, as if this version was beyond any doubt; and for doing so even when this version presented a third party as ridiculous, or even odious, providing me with all the more reasons to exercise caution. For my flimsiness and lack of sane caution, I would like to hereby extend my most sincere excuses to M. Kashiwara.

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[59] This isolation was the result first and foremost of the indifference of my ex-students for Mebkhout's ideas and for his work, the price he had to pay for taking inspiration from an "ancient figure" destined to oblivion by unanimous consensus...

[60] The most important of these ideas is that of the so called "Riemann-Hilbert correspondence" (to employ the trending jargon) for $\mathcal{D}$-modules. The relevant conjecture was proven by Mebkhout, as well as (according to Schapira's correction) Kashiwara (even though Mebkhout assured me that his proof was the only one to have been published). The question of which of the two proofs appeared first remains unclear in my eyes, and I do not intend to spend the rest of my days trying to arrive at an answer...

As for the sister-statement in terms of $\mathcal{D}^{\infty}$-modules, there seems to be no doubt that the authorship for both the idea and the proof belongs to Mebkhout.

Part I

First part: Fatuity and Renewal

Part II

Second part: The Burial (I) - or the Robe of the Emperor of China

Chapter 13

A) Heritage and heir

13.1 Posthumous student

Failure of an instruction (II) - or creation and fatuity

44' [This note was mentioned in section 50 of **VIII The solitary journey** of part **(I) Fatuity and renewal** p. 227] p. 173

This passage "clicked" for the friend who read the previous section "the weight of a past"[1](*) He wrote: "for many of your old students, the aspect, as you put it, of an invasive and borderline destructive "boss" remains strong. Whence the impression you hold." (Namely, I presume, the "impression" which is expressed in certain passages of this section as well as in the Notes n°46, 47, 50 which complete it.) Earlier, he writes: "first of all I think that you did well to leave mathematics for an instant [!]. Because there was a kind of incomprehension between you and your students, except of course for Deligne. They were left a bit dumbfounded. . . ".

This is the first time that I hears about the impression I made in my role as "boss" pre 1970, beyond customary compliments! Even earlier in the same letter: ". . . I have come to realize that your old students [namely: those from "before 1970"] do not really know what a mathematical **creation** is, perhaps in part because of you. . . it must be said that in their time, the problems were clear-cut. . . " [2](**).

My correspondent surely meant that **I** was the one who formulated the "problems" and, with them, the notions that needed to be developed instead of leaving both tasks to my students; and that in so doing I may have prevented

[1](*) (May 10) This friend is none other than Zoghman Mebkhout, who authorized me to reveal his identity, after I thought I should keep it secret upon first writing this letter (on April 2nd 1984).

[2](**) (May 10) The preceding citation was heavily modified, in order to respect the anonymity of my correspondent. See the following note for a complete citation of the relevant passage, as well as for a commentary on its real meaning, which I had missed at first due to a lack of further contextual information.

them from becoming acquainted with what becomes the essential part of a work
p. 174 of mathematical creation. This also aligns with an impression which I formed after talking to two of my students from **after** 1970, about which I wrote in an earlier note (note (23iv)). It is true that I was looking first and foremost, in the students that approached me, for **collaborators** with whom to develop intuitions and ideas which had already formed within me, to "push along", in sum, a carriage that was already there, which they did not have to summon from some kind of void, "something which my correspondent had to do". This summoning - the act of bringing into being a tangible, supple, intense body of work from the intangible mist - had indeed always been, for me, the most fascinating aspect of mathematical work, as well as the part in which I most strongly felt a process of "creation" the "spirit of something more delicate and essential than a mere result".

If I see certain ex-students of mine treating this valuable thing with disdain, letting grow within them this "snobbery" which J.H.C. Whitehead talked about (consisting of disparaging what is "immediately provable")[3](*), I am at least party to blame, for various reasons.

However, I would not go as far as saying that the work which I suggested to my students, or which they produced with me, was of a purely technical nature, strictly a matter of routine, or inept to using their creative faculties. I offered them some starting points which were tangible and sound, among which they were free to choose, and from which they could launch further, just as I had done before them. I do not think I ever suggested a topic to a student which I would not have been happy to work on myself; nor was any of the journeys which they underwent with me more arid than what I have weathered over the course of my mathematical life, without loosing hope or kicking over the traces, when it was clear that the work had to be done and that there was no way
p. 175 around it.

Thus, it seems to me that the failure that I am today confronting rests on subtler causes than the kind of themes which I suggested, or the extent to which said themes remained nebulous or were clearly delineated. My role in this failure seems due rather to attitudes of fatuity within me, in the way I interacted with mathematics; attitudes which I have examined in the course of this reflection. These attitudes were bound to more or less strongly influence, if not the work itself with a given student, at least the atmosphere surrounding my person. Fatuity, even when expressed in the most "discreet" way possible, always points towards close-mindedness, towards insensibility to the delicate essence of things and to their inherent beauty - whether these be "mathematical things", or breathing individuals whom we can welcome and encourage, but also towards whom we can look down from our lofty seat, oblivious to the aura that surrounds us and to the destructive impact it can have on others and on ourselves.

[3](*)See the note (the snobbery of the youth - or the defenders of purity), n°27 p. 247.

A sentiment of injustice and powerlessness

44” [The appearance of this note does not align with the chronological order of writing]

(May 10) Following my friend’s authorization to freely cite excerpts from his work which I may deem useful, I hereby include a more thorough citation[4](*), which situates the earlier truncated citation in its proper context:

> “It is true that I underwent a period of isolation between the years 1975 and 1980, except for rare questions to Verdier. But I don’t blame your old students for that period, because nobody then really understood the importance of this connection [read: between discrete coefficients and continuous coefficients]. Everything changed in October 1980, when the first highly important application of this connection was found to the theory of semisimple groups, namely the discovery of the Kazhdan-Lusztig multiplicity formula, which used in an essential way the equivalence of categories in question. This equivalence took on the name of “Riemann-Hilbert correspondence” without further comment - after all, it is so natural! This is when I understood that your old students do not really know what a mathematical **creation** is, and that perhaps you shared some of the responsibility for this. I still to this day feel a sentiment of injustice and powerlessness. It is true that at the time the problems were already set in stone. The number of applications of this theorem is impressive, in the context of étale topology as well as in the transcendantal context, where it still carries the name of Riemann-Hilbert! I am under the impression that my name is unworthy of this result for many people, p. 176 including your old students. But as you can see clearly in the introduction to my work, it is your “duality” formalism which leads naturally to the result. Like you, I am not worried about the future relevance of this connection between “discrete constructible coefficients” and crystalline coefficients (or holonomic $\mathcal{D}$-modules). It is clearly applicable to several domains, in the cohomology of spaces as well as in analysis.”

The above segment from my friend’s letter inspired (in addition to the present note) the later note “The anonymous worker and the God-given theorem”. Based on the letter’s language, I had not realized (what I am now explaining in his stead) that this “sentiment of injustice and powerlessness” felt by my friend were a reaction, not only to an attitude of disdain which systematically **minimized** his contributions (an attitude that eventually became familiar in some of my old students), but also to a full-fledged operation of embezzlement, consisting in outright **retracting** the authorship of a key theorem. This situation only became clear to me eight days ago - see regarding this subject the note “Unfairness - or a feeling of return” and the subsequent Notes (n^o’s collected under the title ”The Colloquium - of Mebkhout’s sheaves and Perversity”.

[4](*) See second footnote of the preceding note - “Failure of an instruction - or creation and fatuity”, n^o44’.

45 As a result of the changes in my environment and lifestyle, occasions to meet with or otherwise contact my old friends have become rare. The fact remains that many signs of an attitude of "distancing away" have appeared, more or less pronounced depending on the person. However, some people such as Dieudonné, Cartan, or Schwartz - in fact, all of the "elders" who had warmly welcomed me in my first years, have conveyed nothing of the sort. Other than them, I sometimes feel that there are very few people among my old friends or students in the mathematical community with whom my relationship (whether or not it finds the occasion to be expressed) has not become divided, "ambivalent", following my departure from what was once a shared milieu, a common world.

13.2 13.2. II The orphans

13.2.1. My orphans

46 [This note was mentioned in section 50 of chapter **VIII The solitary journey** of part **I (Fatuity and Renewal)**]

ref

p. 177

I would like to take the time to say a few words concerning the mathematical notions and ideas, among those which I have brought to life, which seem (by far) to be the farthest reaching.(46_1)[5](*) I will be mostly speaking about five closely linked key-notions, which I will briefly review in increasing order of specificity, richness, and depth.

The first idea in question is that of **derived categories** in homological algebra (cf. note 48 p. 274), and of their use as a "catch-all" formalism called the "**six operations formalism**" (namely $\otimes^L$, Lf^* , $Rf_!$, $R\underline{\mathrm{Hom}}$, Rf_*, $Lf^!$) (46_2) on the cohomology of the most important kinds of "spaces" introduced to this day in geometry: "algebraic" spaces (such as schemes, schematic multiplicities, etc ...), analytic spaces (i.e. complex analytic as well as rigid analytic, and assimilated), topological spaces ("tempered spaces", pending the context of tempered spaces of all kinds and surely many others, such as that of the category (**Cat**) of small categories, serving as homotopical models...). this formalism accommodates both discrete and "continuous" coefficients.

fix

The progressive discovery of this duality formalism and of its ubiquitousness happened through a solitary, persistent, and exacting reflection which took place between the years 1956 and 1963. It was during the course of this reflection that the notion of derived category slowly appeared, and with it an understanding of the role which it played in homological algebra.

What was still missing from my vision of the cohomological formalism of "spaces" was an understanding of the link which one could conjecture between discrete and continuous coefficients, beyond the familiar case of local systems and their interpretation as modules with a flat connection, or as modules of

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[5](*)Notes n°46_1 through 46_9 contain more technical commentaries on the notions reviewed in the present note. In addition, independently from the particular **notions** which I have introduced, the reader will also find reflections regarding what I consider to be the "core" of my work (within the collection of work which I have "entirely finalized") in note n°88 "The

crystals. This profound link, first formulated in the context of complex analytic spaces was discovered and established (almost 20 years later) by Zoghman Mebkhout, in terms of derived categories obtained on the one hand using "constructible" coefficients, and on the other hand the notion of "$\mathcal{D}$-modules" of "complexes of differential operators" (cf. note 46_3 p.). ref

For almost 10 years, in the absence of the encouragement of those among my old students who were best positioned to offer it, and to support him through their interest and their experience which they had gained through their work with me. Zoghman Mebkhout produced his remarkable work in a near total state of isolation. This did not prevent him from discovering and proving two key theorems[6](*) in the context of a new crystalline theory which was slowly coming into being in the midst of a general indifference. Both theorems were expressed in the language of derived categories (decidedly not a crowd-pleasing topic!): one provided the equivalence of categories mentioned earlier between "discrete constructible" coefficients and crystalline coefficients (subject to certain conditions of "holonomicity" and "regularity") and the other was "**the**" theorem of global crystalline duality for the constant morphism from a smooth complex analytic space (not necessarily compact, thus involving significant additional technical difficulties) to a point. Both are profound theorems,[7](**) which provide a renewed understanding of the cohomology of both analytic as p. 179
well as (in characteristic 0 for now) algebraic spaces, and as such they carry the promise of a far-reaching renewal of the cohomological theory of these spaces. They finally earned the author, following two consecutive denials of job application at the CNRS, a post of research fellow (equivalent to a post of assistant or master-assistant at a university).

Nobody during these ten years cared to tell Mebkhout, while he was wrestling with the significant technical difficulties involved with the transcendental context, about the "formalism of the six variances", well known by my students[8](*), but nowhere to be found "written up". He finally learned about its existence from me last year (in the form of a note, which I was apparently the only one

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remains".

[6](*) (June 7th) Mebkhout mentioned to me that in addition to these two theorems, I should be mentioning a third, also expressed in the language of derived categories, namely what he has called (perhaps a bit improperly) the theorem of biduality for $\mathcal{D}$-modules, which was the hardest of the three. For a sketch of the of Mebkhout's ideas and results, and of their applications, see Le Dung Trang et Zoghman Mebkhout, Introduction to linear differential Systems, Proc. of Symposia in Pure Mathematics, vol.40 (1983) part.2, p. 31-63.

[7](**)(May 30th) The proof of the second theorem required dealing with the usual technical difficulties of the transcendental context, involving the recourse to "évètesque" techniques whence my guess that it ranks among "difficult" demonstrations. The proof of the first theorem is "evident" and profound, using the full force of Hironaka's theorem for the resolution of singularities. As I mention in the penultimate paragraph of the note "solidarity" (n°85), once the theorem is formulated, "anybody" in the loop would be able to prove it. Compare also with J.H.C. Whitehead's observation quoted in the note "The snobbery of the youth - or the defenders of purity", (n°27). I wrote the latter note as if under the silent dictation of a secret prescience as of yet not realizing the extent to which the reality was going to surpass my shy and fumbling suggestions!

[8](*) They learned it first-hand from the seminars SGA 4 and SGA 5, as well as through the intervening text "Residues and Duality" of R. Hartshorne.

to know about…), when he kindly and patiently took the time to explain his work to me, even thought I was out of practice with cohomology… Neither did anybody think to suggest to him that it may be more "profitable" to first to first focus on the context of schemes in characteristic 0, where the difficulties inherent to the transcendental context disappear, while on the other hand the conceptual questions fundamental to the theory appear just as clearly. Nobody thought to mention (or even perceived what I have known ever since I introduced crystals[9](**)) that "$\mathcal{D}$-modules" on smooth (analytic or algebraic) spaces are precisely the same thing as "**modules of crystals**" (once we put aside matters of "coherence" for either of these notions), and that the latter is a versatile notion which works just as well for "spaces" with arbitrary singularities, as it does for smooth spaces (46_4).

In view of the aptitudes (and the rare courage) displayed by Mebkhout it is clear to me that had he evolved in a sympathetic atmosphere, he would have painlessly and even with pleasure established the complete formalism of "the six variances" in the context of crystalline cohomology of schemes in characteristic zero, at a time where all of the essential ideas for a program of such scope (including his own, and those of Sato's school and my own) were already in place, or so it seems to me. For someone of his caliber, this could have been done in the span of a few years, just like the development of the catch-all
p. 180
formalism of étale cohomology a few years earlier (1962-1965), given that the guiding framework of the six-operations was already known (in addition to the two key theorems of base change). It is true that these years were marked by a flow of enthusiasm and sympathy from participants and witnesses, as opposed to a work going upstream relative to the haughty self-importance of those in charge…

I now come to the second pair of notions, namely that of **schemes** and the tightly related notion of **topoi**. The latter is a more intrinsic version of the notion of **site**, which I introduced in order to formalize the topological intuition of "localization". (The term "site" was introduced later Jean Giraud, who greatly contributed by providing the notions of site and topos with the necessary flexibility.) I was led to introduce the notions of scheme and topos one after another in response to the glaring needs of algebraic geometry. This pair of concepts carried within them the potential for a profound renewal of both algebraic and arithmetic geometry and of topology, through a **synthesis** of these "worlds", kept apart for too long, within a common geometric intuition.

The renewal of algebraic and arithmetic geometry through the viewpoint of schemes and the language of sites (or of "descent"), carried over the course of twelve years of foundational work (in addition to the work of my students and other participants of good faith) has been well-established for twenty years; the notion of scheme, and that of étale cohomology of schemes (if not that of étale topos and étale multiplicity) have finally become customary, and have entered the common patrimony.

[9](**) (May 30) Something which I have since forgotten - only to remember it during my second meeting with Mebkhout last year (see the note "Meeting from the grave", n°78).

On the other hand, this vast synthesis that would also encompass topology is still biding its time, even though the essential ideas and principal technical tools appear to have been in place[10](*) for twenty years. During the fifteen years that followed my departure from the world of mathematics, the fertile unifying idea and powerful tool for discovery that is the notion of topoi has been maintained by some customary decree[11](*) outside of the range of notions deemed serious. To this day, few topologists are even aware of the existence of this potentially considerable enlargement of their science, and of the novel resources which it offers. p. 181

Within this renewed framework, topological, smooth, and other type of spaces fit together with schemes (about which they may have heard) as well as topological, differential, **and** scheme-theoretic (seldom-mentioned) multiplicities as various incarnation of a single class of geometric objects, name **ringed topoi** (46_5) which play the role of "spaces", and within which intuition coming from topology, algebraic geometry, and arithmetic come into a single geometric vision. The "modular" multiplicities, which one encounters all over the place (provided one's eyes are open), provide several striking examples of this structure (46_6). The comprehensive study of ringed topoi constitutes a primary guiding thread for the purpose of gaining a deeper understanding of the essential properties of geometric objects (or other objects, if one can find objects which aren't geometric in nature…). In this context, modular multiplicities describe the modalities of variation, degeneration, and generization. This wealth of ideas remains ignored to this day, due to the fact that the notion which allows us to precisely describe it does not fit into the range of currently admitted concepts.

Another unexpected aspect of this recused synthesis[12](**) is the fact that familiar homotopical invariants of some of the most common spaces (46_7) (or rather invariants of their profinite compactifications) come equipped with unsuspected arithmetic structures, such as actions of certain profinite Galois groups… p. 182

Nonetheless, for the past fifteen years, it has been customary within "high society" to look down on those who fancy the word "topos", unless in the context of a joke or if the person happens to be a logician. (For these people are known

[10](*) (May 15) The aforementioned "essential ideas and principal technical tools" were assembled in the vast fresco of seminaries SGA 4 and SGA 5 between 1963 and 1965. The strange vicissitudes that affected the writing and publication of the SGA 5 component of this fresco, which only appeared (unrecognizable, ravaged) eleven years later (in 1977) illustrated what happened to the enveloping vision at the hands of a "certain trend" - or rather, at the hands of certain of my students who were first to instaure it (see following footnote). These vicissitudes and their meaning have been progressively revealed over the course of the past four weeks of reflection, continued in the notes "The accomplice", "Clean slate", "The singular being", "The signal". "The reversal", "Silence", "Solidarity", "Mystification", "The deceased", "The massacre", "The remains", n^{o}s 63"', 67, 67', 68, 68', 84-88.

[11](*) (May 14) The continuation of my reflection during the six weeks that followed the writing of these lines (in late March) revealed this "trend" which was established in the first place by certain of my students - the very students who were best positioned to make theirs a certain vision, as well as a range of ideas and technical tools, and who chose to appropriate certain work instruments, while simultaneously disavowing both the vision that had given rise to these instruments and the person within whom the vision was first born.

[12](**)

to be different, and one must forgive some of their eccentricities. . .) Neither has the yoga of derived categories, serving to express to homology and cohomology of topological spaces, entered the lingo of topologists for whom Künneth's formula (with coefficients in a ring which is not a field) continues to be interpreted as a system of two spectral sequences (or at best a pile of short exact sequences), rather than a unique canonical isomorphism within an appropriate category; just as they continue to ignore the base change theorems (for smooth or proper morphisms for instance) which (in the neighboring context of étale cohomology) constituted the crucial pivot for the "kickoff" of said cohomology (cf note 46_8 p. 470). This comes as no surprise when I realize that the very people who contributed to developing this yoga have long forgotten about it; and that they will not hesitate to strike down anyone who has the misfortune to want to use it! [13](*).

The fifth notion which is close to my heart, perhaps more than any other, is that of **"motives"**. It is distinct from the preceding four ideas in that **"the"** correct notion of motive (be it only over a base field, without even mentioning the case of an arbitrary base scheme) has not been given a satisfactory definition to this day, even if we are to accept all "reasonable" conjectures which one may
p. 183
need to this end. Or rather, visibly, **the** "reasonable conjecture" to be made in the first place, would be that of the **existence** of such a theory, pertaining to certain data and satisfying certain properties. It would not be hard (and entirely fascinating!) for somebody in the know[14](*), to explicitly write such a conjecture down. I was about to do so, shortly before I "left math".

In some ways, the situation resembles that of the quest for the "infinitesimally small" during the heroic era of differential and integral calculus, with two caveats. First, we currently possess an experience in the elaboration of sophisticated mathematical theories, together with an efficient conceptual background, which our predecessors lacked. Second, despite the tools which we have at our disposal, and the twenty years which have elapsed since this visibly essential notion appeared, nobody has cared (or dared in spite of those who didn't care. . .) to get their hands dirty, and to extract the rough features of a theory of mo-

[13](*)(May 13) It appeared upon later reflection that the situation has started to change since the Luminy Colloquium of June 1981: there, some of those who had once "forgotten" (or rather, buried...) these notions were now parading them around, continuing nonetheless to strike down the "poor fellow" without whom this brilliant Colloquium would not have existed. (See notes n^{o}s 75 and 81 for more on this memorable Colloquium.)

[14](*) (May 13) I eventually understood that the only person (other than myself) who to this day meets the "reasonably in the know" criterion is Pierre Deligne, who benefitted for four years, at the same time as he was learning from me "the little I knew about algebraic geometry", from being my day-to-day confidant in the course of my motivic reflections. I did speak about these things to many other colleagues here and there, but it seemed none of them was sufficiently "tuned in" to assimilate the holistic view which had emerged within me over the course of many years, or to take my indications as a starting point for their own development of a vision or program (as I had myself done beginning with two or three "strong impressions" effected upon me by some of Serre's ideas). Although I could be mistaken, it seems to me that the people interested in the cohomology of algebraic varieties where not psychologically disposed to "take motives seriously" for as long as Deligne, who was a figure of authority in cohomology while also being the only one supposed to fully know what these motives were all about, was letting them go unmentioned.

tives, the way our ancestors had done for infinitesimal calculus, without beating around the bush. It is just as clear today for motives as it once was for the infinitesimally small, that such beasts exist, and that they manifest themselves in every corner of algebraic geometry, as long as one is interested in the cohomology of algebraic varieties and families of such varieties, and more specifically in the "arithmetic" properties of such objects. Even more so perhaps than in the case of the four other notions which I have mentioned, the idea of motives which is the most specific and richest of all, naturally associates to a range of intuitions of various kinds, not at all vague and in fact often expressible with p. 184
a perfect precision (provided one is willing, if needed, to admit certain motivic premises). For me, the most fascinating of these "motivic intuitions" was that of a "motivic Galois group", which in a way allows us to "put a motivic structure" on the profinite Galois groups of fields and schemes of finite type (in the absolute sense). (The technical work required to precisely formulate this notion, having admitted the "premises" giving a temporary foundation for the notion of motive, was accomplished in the thesis of Neantro Saavedra on "Tannakian categories".)

The current consensus surrounding the notion of motive is slightly more nuanced than that of its three brothers (or sisters) of misfortune (derived categories, duality formalism of the so-called "six-operations", topoi), in the sense that there hasn't been a case of "swindling" [15](*). Practically speaking, the end-result is nonetheless the same: as long as there hasn't been a proper "definition" of motives and associated "proofs", serious people can only abstain from speaking about them (naturally with the utmost regret, but such is protocol among serious people...). Of course we may never arrive to a theory of motives and "prove" anything regarding them, for as long as it is declared that it isn't serious to even speak about them!

Nonetheless, the few people in the know (and who follow the trends) know that beginning with the premises, which remain secret, one can prove many things. It should be said that as of today, in fact since the notion appeared in the wake of the Weil conjectures (proven by Deligne, which proves a point I guess!), the **yoga of motives** very much exists. But it has the status of a **secret science** with very few initiates [16](**). Even though it is "not serious", it nonetheless allows these few initiates to declare in a range of cohomological situations "what p. 185
one should expect". It thus gives rise to a multitude of intuitions and partial

[15](*) (May 13) As I mentioned in an earlier footnote, derived categories were the subject of an exhumation with great fanfare three years ago (without speaking my name). Topoi and the six operations are still waiting for their turn, as well as motives, except for the small piece thereof which was exhumed two years ago, with a substitute parenthood (see notes n^{o}s 51, 52, 59).

[16](**) (May 13) I now understand that these "few initiates" amounted until 1982 to Deligne and him only. It is true that he revealed the aspects of this "secret science" which are reflected in certain important results included in the yoga, revealing them when he was able to prove then so as to be able to claim credit for them while hiding his source of inspiration, which remained secret. If for the past fifteen years no one has undertaken the development of a wide-ranging theory of motives, it must be because the current times are far from the bold dynamism of the heroic era of infinitesimal calculus!

conjectures, which are sometimes accessible after the fact using tools at hand, in light of the understanding provided by the "yoga". Several works of Deligne are inspired by this yoga, [17](*). notably (if I am not mistaken) his first published work establishing the degeneracy of the Leray spectral sequence for a smooth a projective morphism of algebraic varieties (in characteristic 0 for the needs of the demonstration). This result was suggested by "weight" considerations of an arithmetic nature. This is typical of "Motivic" considerations, which can be formulated in terms of the "geometry" of motives. Deligne proved this statement using the theory of Lefschetz-Hodge and (if I remember correctly) did not say a word concerning the motivation, without which nobody could have guessed such an improbable result.

The yoga of motives was in fact born, in the first place, out of this "yoga of weights" which I learned from Serre [18](**). It is him who showed me all the charm of the "Weil conjectures" (which have become a "theorem of Deligne"). He had explained to me how (modulo a resolution of singularities hypothesis in the characteristic under consideration) one could, using the yoga of weights, associate to every algebraic variety (not necessarily smooth or proper) over an arbitrary base field the so-called "virtual Betti-numbers" - something which I found extremely striking (13.2.9). I believe it was this idea which started my reflection on weights, a reflection which continued (in parallel with my project of writing foundational texts) throughout the following years. (This was also the reflection which I resumed in the 70's, through the notion of a "virtual
p. 186 motive" over an arbitrary base scheme, which the intention of establishing a "six operations" formalism for (at the very least) virtual motives.) If I discussed this yoga of motives for all these years with Deligne (figuring as privileged interlocutor) and to whoever else was interested [19](*), it wasn't so that he and others would keep this subject under the status of secret science, reserved to them, and them alone.($\Longrightarrow$ note 13.2.9 p. 271)

Note 46$_1$ I will make at most the exception of the ideas and viewpoints introduced alongside the formulation which I gave to the Riemann-Roch theorem

[17](*)(May 13) Having become somewhat familiar with said bibliography, I now realize that Deligne's entire line of work is rooted in this yoga. Furthermore, my bibliographical sampling (together with some cross-checking) give me the impression that in Deligne's entire work, the only reference to this source is to be found in one swift line (referencing me and Serre in the same breath) in "Théorie de Hodge I" in 1970. (See notes n^{o}s $78'_1$ and $78'_2$.)

[18](**) What I learned from Serre (in the early 60s?) was an idea or starting intuition, allowing me to understand that there was something to be understood! This contact provided an initial impulse, triggering a reflection which continued into the following years, first around a "yoga" of weight and later around a vaster yoga of motives.

[19](*) (April 10) It seems to me that Deligne was the only one who "listened" - and he took care to keep that privilege to himself. It should also be said that in writing these final lines, I was "delaying" the chain of events: a partial exhumation of the yoga of motives occurred two years ago, without any allusion made to the role I had played! See notes n^{o}s 50, 51, 59 on this subject, resulting from an unexpected discovery which shed a surprising light (at least in my eyes) on the meaning of the funeral that had been taking place for twelve years. Before then, I had been vaguely aware that some kind of funeral was going on, without taking the time to look closer...

(together with the two proofs which I discovered), as well as its various variants. If I remember correctly, such variants appeared in the last exposé of the seminar SGA 5 from 1965/66, which was lost along with several other exposés from the same seminar. The most interesting such variant in my eyes regards discrete constructible coefficients, and I ignore if it has since been made explicit in the literature[20](**). I should observe that there also exists a "motivic" variant which boils down to the statement that the "characteristic classes" (in the Chow ring of a regular scheme Y) associated to constructible χ-adic sheaves for different prime numbers χ (prime to the residual characteristics), in the situation where these sheaves come from a single "motive" (for instance when they are all of the form $R^i f_! \left(\underline{Z}_\chi\right)$ for a given $f\colon X \to Y$) are all equal

Note 46$_2$ This formalism can be viewed as a sort of epitome of a cohomological formalism of "**global duality**"; in its most "efficient" form, freed from any superfluous hypotheses (notably of smoothness for the "spaces" and morphisms under consideration, or properness of morphisms), it can be completed by a formalism of **local duality** in which one distinguishes, among admissible "coefficients", the so-called "**dualizing**" objects or complexes (a notion which is stable under the operation $Lf^!$), i.e. those which give rise to a "**biduality theorem**" (in terms of the operation $R\underline{\mathrm{Hom}}$) with coefficients satisfying appropriate finiteness conditions (on the degrees, together with coherence or "constructibility" conditions on the objects of local cohomology). When I speak of the "formalism of the six variances", I am hinting at this complete duality formalism, including both its "local" and "global" aspects. p. 187

The first step towards a thorough understanding of duality in cohomology was the progressive discovery of the formalism of the six variances in an important special case, namely that of Noetherian schemes and complexes of modules with coherent cohomology. The second step was the discover (in the context of the étale cohomology of schemes) that this formalism also applied to the case of discrete coefficients. These two extreme cases were sufficient to persuasively suggest the **ubiquity** of this formalism in all of the geometric situations giving rise to a Poincaré type "duality" - a conviction which was later confirmed by the works of Verdier, Ramis, and Ruget (among others). It will also surely be confirmed for other types of coefficients when the **block** which for fifteen years has been put in place against the development and large scale use of this formalism will have eroded.

This ubiquity appears to me to be a **fact** of considerable importance. It rendered the feeling of a profound unity between Poincaré duality and Serre duality unescapable; this unity was eventually demonstrated with the required generality by Mebkhout. This ubiquity positions the "formalism of the six variances" as a fundamental structure in homological algebra, serving towards an

[20](**) (June 6) I since found it (in a similar form, under the flattering name of "Deligne-Grothendieck conjecture") in an article of MacPherson which appeared in 1974. See note n^o 87_1 for details.

understanding of "all kinds"[21](*) of phenomena relating to cohomological duality. That this relatively sophisticated structure has not been made explicit in the past (akin to the absence of a "good" notion of "triangulated category", with Verdier's formulation being only temporary and insufficient) does not change this reality; nor does the fact that topologists, and even algebraic geometers who claim to be interested in cohomology, continue as best they can to ignore the very existence of this duality formalism, as well as the language of derived categories upon which it rests.

Note 46$_3$ The framework of $\mathcal{D}$-Modules and of complexes of differential operators was introduced by Sato and first developed by himself and his school, with
p. 188 a perspective quite different (or so it seemed to me) from Mebkhout's, which is closer to my own.

I believe I was the first to formulate the various notions of **"constructibility"** for "discrete" coefficients (in complex analytic, real analytic, or piecewise linear contexts) towards the end of the 1950s (and I later reemployed them in the context of étale cohomology). I had then asked whether this notion was stable under higher direct images with respect to a proper morphism between real or complex analytic spaces, and I do not know if this stability has been established in the complex analytic case[22](*). In the real analytic case, the notion which I had considered was not the right one, as I was lacking Hironaka's notion of real sub-analytic set, which has the essential preliminary property of stability under direct images. As for operations of a local nature such as $\mathbf{R}\underline{\mathbf{Hom}}$, it was clear that the argument which established the stability of constructible coefficients in
clarify: excellent schemes? the context of excellent schemes in characteristic zero (using Hironaka's resolution of singularities) worked just as in the complex analytic case; ditto for the theorem of biduality (see SGA 5 I). In the piecewise linear context, the natural stability theorems and the biduality theorem are "easy exercises", which I happily did in the spirit of verifying the "ubiquity" of the duality formalism, at the beginning of the development of étale cohomology (during which period one of the main surprises was the discovery of this very ubiquity).

Coming back to the semi-analytique case, the "right" context in this direction for establishing stability theorems (for constructible coefficients by the six operations) was visibly that of "tame spaces" (see Esquisse d'un Programme,
cite Esquisse d'un Programme par. 5,6).

Note 46$_4$ Naturally, the framework of $\mathcal{D}$-modules, together with the fact that $\mathcal{D}$ itself is a coherent sheaf of rings, highlights a more hidden notion of "coherence" for crystals of modules than the one which I am used to working with, and which makes sense for possibly singular (analytic or scheme-theoretic) spaces. It would only be fair to call this notion **"M-coherence"** (M as in Mebkhout). It then becomes relatively clear, for somebody in the know (in full possession of

[21](*) The interested reader will find a sketch of this formalism in the Appendix to this volume.

[22](*) (May 25) It was established by J. L. Verdier, see "The right references", note n^o 82.

a healthy mathematical instinct), that the "right category of coefficients" which generalizes the complexes of "differential operators" from the smooth settings is nothing but the "M-coherent" derived category of crystals of modules (where a complex of crystals is called **M-coherent** if it has M-coherent cohomology objects). This category makes sense independently of the smoothness hypothesis, and it should encompass at once the ordinary theory of "continuous" coefficients and the theory of "constructible" discrete coefficients (introducing in the latter case appropriate holonomy and regularity conditions). If my perspective is correct, then the two new conceptual ingredients of this Sato-Mebkhout theory, with respect to the previously known crystalline context, are the notions of M-coherence for crystals of modules as well as the conditions of holonomy and regularity (of a deeper nature) relating to M-coherent complexes of crystals. With these notions in place, one of the first essential tasks would be to develop a formalism of the six variances in the crystalline context, in such a way as to generalize the two special cases (ordinary coherent and discrete) which I have developed over twenty years ago (and which some of my ex-students in cohomology have long ago forgotten in favor of other tasks, surely more important...).

Mebkhout had eventually learned about the existence of a notion of "crystal" through my writings, and he had felt that his viewpoint should provide an appropriate approach for this notion (at least in characteristic zero) - but this idea fell into deaf ears . Psychologically speaking, it was unconceivable to launch into the vast foundational work ahead in his position, surrounded by a climate of haughty indifference on behalf of the very people who acted as figures of authority in cohomology, and who were as such best positioned to encourage him - or discourage him...

verify: appropriate phrase?

Note 46_5 (May 13) The following mostly concerns the notion of ringed topoi associated to a **commutative local ring**. The idea of describing the structure of a "variety" in terms of the data of such a sheaf of rings on a topological space was first introduced by H. Cartan, and was taken up again by Serre in his classical paper FAC (Faisceaux algébriques cohérents). This work was the initial impulsion for a reflection which led me to the notion of "scheme". What was still missing from Cartan's approach reprised by Serre, so as to encompass all types of "spaces" or "varieties" which have appeared to this day, was the notion of topos (meaning "something" on which the notion of "sheaf of sets" makes sense, and has all of the familiar properties).

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Note 46_6 Among notable examples of topoi which are not ordinary spaces, and for which there does not seem to exist a satisfactory substitute in terms of commonly "admitted" notions, are the following: the quotient topos of a topological space under a local equivalence relation (for instance foliations of varieties, in which case the quotient topos is even a "multiplicity" - i.e. is locally a variety); "classifying" topoi associated to essentially any kind of mathematical structure (at least those "expressible in terms of finite projective limits and arbitrary inductive limits"). Given a structure of "variety" (topological, differentiable, real or complex analytic, or Nash, etc ..., or even smooth scheme-theoretic over a given base field), one obtains a particularly interesting topos, which deserves the name of "universal variety" (of the space under consideration). Its homotopical invariants (and notably its cohomology, which deserves the name of "classifying cohomology" for the kind of variety under consideration) should have been studied and determined a long time ago, but as of now no work of the sort seems to be underway...

Note 46_7 I am referring to spaces X whose homotopy type can be described "in a natural way", such as the homotopy type of a complex algebraic variety. The latter may then be defined over a subfield $\mathbb{K}$ of the field of complex numbers, such that $\mathbb{K}$ is an extension of finite type of the prime field $\mathbb{Q}$. The profinite Galois group $\mathrm{Gal}(\bar{\mathbb{K}}/\mathbb{K})$ then acts naturally on the profinite homotopical invariants of X. Often (e.g. in the case where X is a homotopy sphere of odd dimension), one can take $\mathbb{K}$ to equal the prime field $\mathbb{Q}$.

Note 46_8 (May 13) At the time when I took my first steps in algebraic geometry through Serre's FAC article (which was about to "launch" me towards schemes), the very notion of base change was practically unknown in algebraic geometry, except for the particular case of changing the base field. With the introduction of the language of schemes, this operation has surely become the most commonly used in algebraic geometry, where it is now present at all times. The fact that this operation remains practically unknown in topology, with the exception of a few very particular cases, appears to me to be a typical sign (among others) of the isolation of topology from ideas and techniques coming from algebraic geometry, as well as of a tenacious legacy of inadequate foundations from "geometric" topology.

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Note 46_9 (June 5) Serre's idea was that one should be able to associate to any scheme X of finite type over a field k integers

$$h^i(X)(i \in \mathbb{N})$$

which he called the "virtual Betti numbers" of X, in such a way that the following properties hold:

a) given a closed subscheme Y with complementary open U,

$$h^i(X) = h^i(Y) + h^i(U).$$

verify that it is indeed i and not "j" as in the manuscript

b) for smooth projective X,

$$h^i(X) = i^{th} \text{ Betti number of } X$$

(where the RHS is defined for instance via χ-adic cohomology, for χ prime to the characteristic of the base field k).

If one takes for given resolution of singularities for algebraic schemes over $\bar{k}$, then it is immediate that the $h^i(X)$'s are uniquely determined by the above properties. The **existence** of such a function $X \mapsto (h^i(X))_{i\in\mathbb{N}}$ for fixed k can be essentially reduced to the case where k is a finite field using the formalism of cohomology with proper support. Upon passing to the "Grothendieck group" of finite rank vector bundles over $\mathbb{Q}_\chi$, over which $\mathrm{Gal}(\bar{k}/k)$ acts continuously, and taking the χ-adic Euler-Poincaré characteristic (with proper support) $\mathrm{EP}(X, \mathbb{Q}_\chi)$ of X in this group, $h^i(X)$ denotes the virtual rank of the "constituent of weight i" of $\mathrm{EP}(X, \mathbb{Q}_\chi)$, where the notion of weight is deduced from the Weil conjectures together with a weak form of resolution of singularities. Even without resolution of singularities, Serre's idea can be realized using the strong form of the Weil conjectures (established by Deligne in "Conjectures de Weil II").

I have pursued heuristic reflections in this direction, which led me to a formalism of six operations for "virtual relative schemes", where the base field k is replaced by a more or less arbitrary base scheme S, as well as to various notions of "characteristic classes" for such virtual schemes (of finite presentation) over S. I was thus led (coming back for simplicity to the situation over a base field) to considering finer integral numerical invariants than Serre's, denoted $h^{p,q}(X)$, satisfying analogous properties to a) and b) above, and recovering Serre's virtual Betti numbers via the usual formula:

$$h^i(X) = \sum_{p+q=i} h^{p,q}(X).$$

Refusal of an inheritance - or price of a contradiction

47 [This note is the direct continuation of note 46 from section 13.1.1] p. 192

One may note that four out of the five notions which I have just reviewed (corresponding to the ones which are deemed "not serious") concern cohomology, and most prominently, the **cohomology of schemes and algebraic varieties**. In any event, all for of these notions came to me through the needs felt for a cohomology theory of algebraic varieties, first for continuous coefficients, and later for discrete coefficients. That is to say, one of the principal motivations and constant Leitmotiv in my work during the fifteen year period 1955-1970 has been the cohomology of algebraic varieties.

Remarkably, this is also the theme which Deligne regards today as being his principal source of inspiration, based on what is said on this subject in the IHES booklet from last year[23](*) I became aware of this fact with some surprise. Indeed, I was still "at the scene" and in touch with all the trends when Deligne (following his beautiful work on Ramanujan's conjecture) developed his remarkable extension of Hodge theory. This was mostly, for him as for myself, a first step taken towards a construction of the notion of motives over the complex numbers - to begin with! During the first years that followed my "turning point" in 1970, I of course received echoes regarding Deligne's proof of the Weil conjectures (which also implied the Ramanujan conjecture as a result), and, in

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the same stride, of the "hard Lefschetz theorem"[24] in positive characteristic. I expected no less from him! I was even sure that he must have proven at the same time the **"standard conjectures"** which I had formulated towards the end of the 1960s as a first step towards obtaining (at the very least) the notion of "semisimple" motive over a field, and towards translating certain of the expected properties of such motives in terms of χ-adic cohomological properties of groups of algebraic cycles. Deligne later told me that his proof of the Weil conjectures would definitely not imply the standard conjectures (which are stronger), and that he actually had no idea as to how to approach them. This must have happened around ten years ago. Since then, I have heard nothing in the way
p. 193 of decisive progress towards the understanding of "motivic" (or "arithmetic") aspects of the cohomology of algebraic varieties. Knowing Deligne's abilities, I had tacitly concluded that his principal interest must have turned towards other subjects - whence my surprise upon reading that the reality was all to the contrary.

What seems certain to me, it is that for the past twenty years it has no longer been possible to work towards a large scale renewal of our understanding of the cohomology of algebraic varieties without appearing in some sense as a "continuator of Grothendieck". Zoghman Mebkhout learned the lesson the hard way, and (to some extent) the same applied to Carlos Conto-Carrère, who quickly understood that it was in his best interest to change topics (see 47_1). Among the very first things that need doing is the development of the famous "formalism of the six variances" in the context of various coefficients, as close as possible to the context of motives (which currently play the role of a sort of ideal "horizon"): crystalline coefficients in characteristic zero (in the lineage of the Sato school and of Mebkhout, Grothendieck style) or p (mostly studied by Berthelot, Katz, Messing, as well as a group of clearly enthusiastic younger researchers), "stratified promodules" á la Deligne (which appear as a dual or "pro" variant of the "ind"-notion of coherent $\mathcal{D}$-modules or $\mathcal{D}$-coherent crystal), and finally "Hodge-Deligne" coefficients (which appear to be as good as motives, except for the fact that their definition is transcendental and only applies to base schemes of finite type over the complex numbers)... At the other extreme lies the task of extracting the very notion of motives from the mist surrounding it (and for good reason...), as well as, if possible, to attack questions as precise as the "standard conjectures". (For the latter, I had considered, among other things, developing a theory of "intermediary Jacobians" for smooth projective varieties over a field, as a way of maybe obtaining the positivity formula for traces, which was one of the essential ingredients for the standard conjectures.)

The above were burning tasks and questions up until the moment when I "left math" - burning and juicy topics, none of which ever appeared to me as reaching a "wall", or coming to a halt[24](*). They represented a source of
p. 194 inspiration as well as an inexhaustible substance; I needed only pull wherever something was sticking out (and things were "sticking out" all over!) for progress

[24] Translator's note: In French, the theorem is referred to as the "théorème de Lefschetz vache".

[24]

to come, both expected and unexpected. With my limited means, but without being scattered in my work, I know full well what can be achieved once I put my mind to it, in a single day, a year, or ten years. I also know about Deligne's means, having seen him at work at a time when he wasn't scattered in his work, and I am aware of what he can accomplish in a day, a week, or a month, provided he is willing to put his mind to it. But nobody, not even Deligne, can produce fertile work in the long run, work of profound renewal, while towering over the very objects which one is supposed to probe, as well as the language and the assortment of tools which have been developed to this end by one's predecessor (developed moreover with his assistance, among many other people who offered their contribution...) (59).

I am also thinking about the "Deligne-Mumford" compactifications of the moduli multiplicities $M_{g,\nu}$ (over Spec $\mathbb{Z}$) for smooth connected algebraic curves of genus g with ν marked points. These were introduced[25](*) in order to prove the connectedness of the moduli spaces $M_{g,\nu}$ in every characteristic via a specialization argument starting from characteristic zero. The spaces $M_{g,\nu}$ are in my eyes (together with the group SL(2)) the most beautiful and most fascinating objects that I have encountered in mathematics (47_2). Their very existence, with such perfect properties, appear to me as a sort of miracle (which is furthermore perfectly understood) whose scope is immeasurably larger than the connectedness property which was to be established. In my opinion, they quintessentially contain within them that which is most essential in algebraic geometry, namely the totality (more or less) of all algebraic curves (over all possible base fields), these being the ultimate building blocks for all of the other algebraic varieties. Yet, the kind of objects at hand, namely "smooth and proper multiplicities over Spec $\mathbb{Z}$", still falls outside of the "accepted" categories, namely those which the mathematical community is **disposed** (for reasons which are exempt from scrutiny) to kindly "admit". The commonplace etiquette is for people to speak of these things through allusions at most, while taking the sorry air as they engage in yet more "general nonsense", and having taken the precaution to say "stack" or "champ", so as not to utter the taboo word of "topos" or "multiplicity". This is doubtlessly the reason why these unique jewels have not been studied and used (as far as I am aware) since they were first introduced more than ten years ago, other than by myself in seminar notes which have remained p. 195
unpublished. Instead, people persist in working either with "coarse" moduli spaces or with finite coverings of moduli multiplicities which fortuitously happen to be real schemes - even though both of the above are but relatively pale and awkward shadows of the perfect jewels from which they are issued, and which themselves remain practically banned...

Deligne's work on the four topics that are the Ramanujan conjecture, mixed Hodge structures, the compactification of moduli multiplicities (in collaboration with Mumford), and the Weil conjectures, each constitute a renewal in our understanding of algebraic varieties, and with it, the establishment of a new starting point. These fundamental contributions occurred in the space of only

[25](*) In Pub. Math. 36, 1969, pp. 75-110. For comments, see note n$^o 63_1$.

a few years (1968-73). Yet for the past ten years, these great milestones did not serve as launching pads for a new expedition into the unknown, nor as means towards a renewal of vaster scope. They instead ended in a situation of morose stagnation (47_3). This is surely not due to the fact that the "means" which were available ten years ago, possessed by various people, have disappeared as if by magic; nor is it because the beauty of the things within our reach has suddenly vanished. But it is not enough for the world to be beautiful - one still has to rejoice in this fact...

Note 47_1 This note concerns Contou-Carrère's promising start, five or six years ago, on a theory of relative local Jacobians, as well as their link with global Jacobians (so called "generalized Jacobians") for smooth and not necessarily proper (schemes in curves?) over an arbitrary base, and with Cartier's theory
clarify this
of commutative formal groups and (typical curves?). Except for an encouraging reaction by Cartier, the reception of Contou-Carrère's first note, by those who were best positioned to be able to appreciate it, was so cold that the author refrained from ever publishing the second note which he had in reserve, and to thereafter change topics (without quite safeguarding himself from later mis-
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adventures)[26](*). I had suggested the theme of local and global Jacobians to him as a first step towards a program that traces back to the late 1950s, aiming notably for a theory of an "adelic" dualizing complex in arbitrary dimen-
p. 196
sions, constructed via local Jacobians (for local rings of arbitrary dimensions), in analogy with the residual complex of a noetherian scheme (constructed via the dualizing modules of all of its local rings). This part of my cohomological duality program (along with some others) was put on the back burner during the 1960s, as a result of the surge of other tasks which then seemed to be more urgent.

Note 47_2 To be more precise, it is the "Teichmüller tower" into which all of the aforementioned multiplicities fit, and the discrete or profinite paradigm of this tower in terms of fundamental groupoids, which constitutes the richest and most fascinating object that I have encountered in mathematics. The group $S_\chi(2,\mathbb{Z})$, together with the "arithmetic" structure of the profinite compactification of $S_\chi(2,\mathbb{Z})$ (consisting of the $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ action over the latter), can be considered
I think this should actually be SL(2,Z)?
as the main building block leading to the "profinite version" of said tower. For more on this subject, see the indications in "Sketch of a program"[27] (awaiting the one or more volumes of Mathematical Reflections[27] which will be devoted to this theme).

Note 47_3 The observation of a "morose stagnation" is not coming from a carefully weighed opinion from someone in the know about the main events

[26](*) (June 8) See the sub-note 95_1 to the note "Coffin 3 - or the slightly-too-relative Jacobians", n^{o}95.

[27]"Esquisse d'un Programme" in the French text.

[27]"Réflexions Mathématiques" in the French text.

during the past ten years touching on the cohomology of schemes and algebraic varieties. Rather, it is the mere overall **impression** of an "outsider", which I have gleaned from conversations and correspondences with Illusie, Verdier, and Mebkhout in 1982 and 1983, among other things. There are surely several ways to further qualify this impression. For instance, Deligne's work in "Conjectures de Weil II", which appeared in 1980, represents substantial new progress, if not a surprise at the level of the main result. There also seems to have been progress made on crystalline cohomology in characteristic $p > 0$, as well as a "rush" surrounding intersection cohomology, which led some to (reluctantly) return to the language of derived categories, or even to recall some long renounced affiliations.

13.3 III Fashion - or the Life of Illustrious Men

Instinct and fashion - or the law of the strongest

48 [This note is referenced by note 46 from p. 265] ref

As is well known, the theory of derived categories is due to J. L. Verdier. Before he undertook the foundational work which I suggested to him, I had confined myself to working with derived categories in a heuristic way, using a provisory definition (which later turned out to be the right one) and an equally provisory intuition about the essential internal structure (an intuition which turned out to be technically wrong in the expected context, in that the "mapping cone" does **not** depend functorially on the morphism in the derived category within which it is defined, and that it is only defined up to non-unique isomorphism). The theory of duality of coherent sheaves (i.e. the formalism of the "six variances" in the coherent context) which I had developed towards the end of the 1950s[27](*) only made full sense once foundational work on the notion of derived category had been laid down, something which Verdier did at a later time. p. 197

Verdier's thesis (submitted only in 1967), consisting of about twenty pages, appears to me to be the best introduction to the language of derived categories written to this day, in that it situates the language in the context of its key applications (many of which are due to Verdier himself). This text served only as an introduction to a work in progress, which was completed later. I am proud of being, if not the only, at least one of very few people who can testify to having held this work in their hands, a text which warranted the title of doctor es Sciences to its author by means of an introduction alone! This work is (or was - I do not know if a copy can still be found somewhere...) the only text, to this day, which develops systematic foundations for homological algebra from the viewpoint of derived categories. p. 198

Perhaps I am the only one to regret that neither the introductory text not the foundations per se have been published[28](*), so that the essential technical prerequisites to using the language of derived categories find themselves

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scattered across three different sources in the literature[29](**). This absence of a systematic reference of a caliber comparable to Cartan-Eilenberg's classical book appears to me as both **a cause and a sign**, typical of the disaffection which took hold of the formalism of derived categories following my departure from the mathematical world in 1970.

Admittedly, it was already clear in 1968 (in light of the needs of a cohomological theory of traces, developed in SGA 5) that the notion of derived category in its primitive form, as well as the corresponding notion of triangulated category, were insufficient to satisfy certain needs, and that foundational work remained to be done. A useful but still modest step in this direction was taken by Illusie (mostly for the needs of the theory of traces), through the introduction he gave in his thesis to "filtered derived categories". It seems that my 1970 departure signaled a sudden and definitive stop to all reflections touching to the foundations of homological algebra, along with the intimately linked reflections touching to a theory of motives (48_1). Yet, regarding the first of the above themes, all of the essential ideas needed for a large scale foundational work seemed to have been established during the years leading to my departure (48_2). (Including the key idea of "derivator", or "machine producing derived categories", which seems to be the richest object in common to the triangulated categories encountered to this day; this idea was eventually developed to some extent in the non-additive context, nearly twenty years later, in a chapter of volume 2 of Pursuing Stacks[30] .) Moreover, the foundational work at hand was in large part already done by Verdier, Hartshorne, Deligne, and Illusie, and their work could be used as is for a synthesis bringing the acquired ideas to the vaster framework of derivators.

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It is true that the disaffection for the very notion of derived category during the past fifteen years[30](*), which for some was connected to a disavowal of a time past, is in line with a certain fashion which suggests that we look down upon any reflection touching to foundations, however urgent it may be[31](**). On the other hand, it is also clear to me that the development of étale cohomology, which "everybody" uses nowadays without thinking twice (if only implicitly via the now proven Weil conjectures...) would not have been possible without the conceptual background consisting of derived categories, the six operation formalism, and the language of sites and topoi (developed in the first place to this very end), without including SGA 1 and SGA 2. And it is just as clear that the stagnation which is to be found today in the cohomological theory of algebraic varieties would not have appeared, let alone settled, if some of my ex-students had known to follow their sane instinct as mathematicians during these past years, instead of following the fashion which they were among the first to establish, and which through their support has gained force of law.

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30 "Poursuite des Champs" in the French text.

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Note 48_1 The same thing can be said (modulo some reserves) about the entirety of my foundational program in algebraic geometry, of which only a small part has been realized: it stopped immediately following my departure. This halt stroke me the most concerning the duality program, which I considered to be particularly juicy. Zoghman Mebkhout's work, which he carried in counter-current, nonetheless fall in line with this program (and renew it through the incorporation of unexpected ideas). The same applies to Carlos Contou-Carrère's work in 1976 (about which I spoke in note 47_1, p. 273) - a work which he carefully paused sine die. There was also work done on duality in the context of the fppf cohomology of surfaces (by Milne). This is all the work of which I have been made aware.

It is true that I never considered writing the sketch of the long term program whose contours had appeared to me during the years 1955 through 1970, just as I did for the past twelve years in Sketch of a Program. The reason, I think, is simply that there was never any particular occasion (such as presently my candidacy for entrance into the CNRS) to motivate such a work of exposi- p. 200
tion. One may find some indications regarding certain theories (notably duality theories) relevant to my pre-1970 agenda, and which still await ground work in order to enter into the common patrimony, in letters to Larry Breen (from 1975), which are reproduced in appendices to Chapter I of the History of Models (Mathematical Reflections 2) [32] .

Note 48_2 The same is true for the theory of motives as well, except for the fact that the latter will most likely remain conjectural for quite some time.

The anonymous worker and the God-given theorem"

48' [This note is referenced by note 46 p. 178]

Although it is customary to name the key theorems of a theory after the people who have done the work to extract and established them, it seems that Zoghman Mebkhout's name has been deemed unfit to be attached to his fundamental theorem, the culmination of four years of obstinate and solitary work (1975-79), in counter-current to the fashion of the day and despite his elders' disdain. The same elders, when there came the day that the scope of the theorem could no longer be ignored, took fancy in calling it "Riemann-Hilbert theorem", and I trusted (even though neither Riemann nor Hilbert would have demanded as much...) that they had excellent reasons to do so. After all (once a need has been felt - the need for an understanding of the precise relationship between general discrete coefficients and continuous coefficients, which appeared in spite of a widespread indifference; after this impression was refined and made precise through a delicate and patient process, so that the right statement was finally extracted after successive stages; after it was stated clearly and proven; and, at last, after this theorem, the fruit of solitude, had proven its worth in places where it was least expected - only then) this theorem appeared to be so evident

[32] "Histoire de Modèles" in the French text.

(not to say "trivial", for those who "would have been in a position to prove it"...) that there really was no need to burden ourselves with the name of some vague anonymous worker!

Encouraged by this precedent, I suggest that we henceforth begin calling "theorem of Adam and Eve" any theorem that is truly natural and fundamental to the development of a theory, or even to trace things back even further and to attribute recognition where it pertains, by simply calling it a **"God-given** p. 201 **theorem"**[32](*).

As far as I know, Deligne was the only person other than myself who before Mebkhout sensed the interest in understanding the relationship between discrete coefficients and continuous coefficients in a larger context than that of stratified modules, so as to be able to interpret in "continuous" terms arbitrary "constructible" coefficients. The first attempt in this direction formed the main theme of a (yet unpublished) seminar of Deligne at the IHES in 1968 or 1969, during which he introduced the viewpoint of "stratified promodules" and produced a comparison theorem (over the complex numbers) between transcendental discrete cohomology and the associated de Rham-type cohomology, the latter of which makes sense for schemes of finite type over any base field of characteristic zero. (Apparently, he was not aware at that time of the remarkable result of his distant predecessors Riemann and Hilbert...) Even more so than Verdier[33](*) or Berthelot[34](**), Deligne was therefore particularly well positioned to appreciate all the interest underlying the direction which Mebkhout's research took in 1975, as well as the value of Mebkhout's subsequent results, notably the "God-given theorem" which provides a more delicate and deeper apprehension of discrete coefficients in terms of continuous coefficients than that which he had achieved himself. None of this takes away from the fact that Mebkhout had to pursue his work in trying moral isolation, and that the credit is his (all the more so, I would say) for his pioneering work which remains hidden to this day, five years after the fact[35](***).

[32](*) I have never had in my life as a mathematician the pleasure of inspiring, or even encouraging, a student in producing a thesis containing such a "God-given theorem" - at least not of a depth or scope comparable to the one in question.

[33](*) It seems that Verdier, as official doctoral advisor of Zoghman Mebkhout (and who in this role has even "granted him a few discussions"), was the principal suspect (except for Mebkhout himself) in the cover-up which took place surrounding the authorship of this fundamental theorem, as well as in the attribution of the credit that his "student" deserved for the consequent renewal that took place in the cohomological theory of algebraic varieties - through the viewpoint on $\mathcal{D}$-modules developed by Mebkhout. I nonetheless am not under the impression that the situation moved him anymore than it moved Deligne.

[34](**) (May 25) In writing these lines, I have refrained (with some hesitation) from including the name of my friend Luc Illusie in this list of students who were "best positioned" to provide Zoghman Mebkhout with the encouragements which were naturally in order. I wasn't mindful at the time of a certain uneasiness within me, which would have indicated to me that I was favoring someone for whom I had affection, in appearing to discharge him from a responsibility which falls on him just as it falls on my other "cohomologist students".

[35](***) (May 25) In fact, this cover-up is first and foremost the act of Deligne and Verdier themselves. For more on this subject, see the note "The Inequity - or a feeling of return", n^{o}75.

Canned weights and twelve years of secrecy

49 [This note is referenced in note 46 p. 185] p. 202

After cross-checking (in Publications Mathématiques 35, 1968), I observe that towards the end of the article "Th'eorème de Lefschetz et crtières de dégénerescence de suites spectrales", there is an allusion made in three lines concerning "weight considerations" which had led me to conjecture (in a slightly less general form) the principal result of the article. I doubt that this sibylline allusion could have been of use to anybody, nor understood at the time by anyone other than Serre or myself, to either of whom it would have come as no news[36](*).

On this topic, I should also mention that I was well aware (and hence so was Deligne) of a very precise "yoga of weights", including the behavior of weights with respect to operations such as $R^i f_*$ and $R^i f_!$, starting from the end of the 1960s, in the wake of the Weil conjectures. Part of this yoga was finally established (in the context of l-adic coefficients, as an intermediary to the more natural context of motives) in Deligne's paper "Conjectures de Weil II" (Publications Mathématiques 1980). If I am not mistaken, for the twelve years that have elapsed between these two periods[37](**), there was not a single trace in the literature of an exposition, however partial or succinct, of the yoga of weights (still completely conjectural), which as such remained within the exclusive reach of a few (two or three?) initiates[38](***). However, this yoga constitutes an essential first key in the endeavor of understanding the "arithmetic" properties of algebraic varieties, thus acting at once as a **means** of orienting oneself in p. 203

[36](*) (April 29) For a more careful study of this paper, instructive in more than one way, see the note "The eviction" (n^{o}63).

[37](**) (April 19) I realize upon looking at a list of Deligne's papers which I have just received and read with interest, that the notion of "weights" already appears in 1974 in a communication of Deligne at the Vancouver Congress - this thus brings the twelve years of "secrecy surrounding weights" down to six. This secret nonetheless appears to me to be inseparable from the similar secret surrounding the notion of motives (during the period 1970-1982). The meaning of this secrete has just come to a new light during today's reflection, while writing the long double-note that follows n^o$51-52$).

[38](***) (May 25) In light of all of the pieces of information that have appeared during the reflection, it seems that these "two or three initiates" may actually boil down to Deligne and him only, as he seems to have carefully kept exclusive privilege of access to this yoga he learned from me until 1974 (see preceding footnote), at which point the time was ripe to present these ideas as his own, without making reference to me or today Serre (see notes n^o$78'_1, 78'_2$).

(April 18 1985) Since the time when these lines were written, I also became aware of Deligne's paper "Théorie de Hodge I", published in the Congrs̀ Int. Math. de Nice (1970) (Actes, t.1, pp. 425-430). Unlike what I had first believed based on the partial information in my possession, this paper exposes as earl as in 1970 a substantial part of the yoga of weights. Regarding the origin of these ideas, Deligne only makes a sibylline and strictly formal reference to a paper of Serre (which doesn't address the question), and to "Grothendieck's conjectural theory of motives". (Compare with notes n^o$78'_1, 78'_2$.) The crucial question of the behavior of the notion of weights under operations such as $R^i f_!$ and $R^i f_*$ is not even mentioned, and it won't be mentioned until the aforementioned paper "La Conjecture de Weil II" from 1980, in which my name is not mentioned in relation to the main theorem of the article; neither is Serre's name or mine mentioned in the communication "Poids dans la cohomologie des variétés algébriques" ("Weights in the cohomology of algebraic varieties") mentioned in the preceding footnote (published exactly one year ago).

a given situation and for making predictions with an accuracy that has never failed, as well as on of the most urgent and fascinating **tasks** which lie ahead in the cohomological theory of algebraic varieties. The fact that this yoga has remained nearly ignored until the time when it was finally established (at least in regards to some of its important aspects), appears to me to be a particularly striking example of the **information blocking** practiced by some of the very people whose functions and privileged situation call for them to oversee the wide broadcast of said information[39](*).

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There is no stopping progress!

50 [This note is referenced in section 50 of chapter **VIII The solitary adventure** of part **(I) Fatuity and Renewal** p.]

add page

My first experience in this sense was the unexpected outcome of my unsuccessful attempts to have Yves Ladegaillerie's thesis on isotropy theorems for surfaces published - even though his work was as good as that of any of the eleven other works submitted in the context of a doctorat d'état ("pre-1970", admittedly!) under my "directorship". If I recall correctly, these efforts continued over the course of at least a year, and they involved as protagonists several of my old friends (as well as one of my ex-students, naturally)[40](**). The principal events still appear to me to this day as a handful of vaudeville acts!

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This episode also constituted my first encounter with a new spirit and with new customs (which had become customary among my friends of yesteryear), about which I have already alluded here and there over the course of my reflection. It was during that year (namely, 1976) that I learned for the first time, but not the last, that the act of properly demonstrating things which everybody uses and that have been taken for granted by one's predecessors (in this case, the non-existence of wild phenomena in the topology of surfaces)[41](***) is considered (at least when done by a firstcomer...) a mark of a lack of seriousness. The same goes for the proof of a result which encompasses as special cases or corollaries several known deep theorems (which evidently indicates that the new result can only be a special case or an easy consequence of the already

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[39](*) See on this topic sections 32 and 33, "The ethics of a mathematician" and "The note - or the new ethics (1)", as well as the two related notes "Deontological consensus and control of information" and "The snobbery of the youth - or the defenders of purity", n^{o}s 25, 27.

[40](**) See on this topic the note "Coffin 2 - or the chain-sawn cuts", n^{o}94.

[41](***) See also on this topic the episode "The note - or the new ethics" (section 33). This famous "note" had made the very mistake of explicitly presenting notions and statements which had hitherto remained vague, even though I had implicitly used them to establish results which bear my name and which everybody had been using unabashedly for almost twenty-five years (something which the two illustrious colleagues in question knew full well).

(June 8) For more details, see the note "Coffin 4 - or the unceremonious topoi" (n^{o}96). The "results which bear my name" are results on the (engendrement?) and finite presentation of certain global and local profinite fundamental groups, "established" among other things in SGA I via descent techniques that remained heuristic in the absence of a careful theoretical justification, until the latter was produced in the (apparently "unpublishable") work of Olivier Leroy on Van Kampen-type theorems for fundamental groups of topoi.

established results). Ditto for taking the time to carefully formulate the natural hypotheses (sign of a regrettable rambling bout) relevant to stating a given result or to describing one situation in terms of another, rather than limiting oneself to discussing a special case which suits the high-flying individual voicing their opinion. (Just last year, I saw Contou-Carrère be criticized for not limiting himself in his thesis to work over a base field rather than over a general scheme - while nonetheless conceding the extenuating circumstance that is having me as an advisor, so that he must surely have only acted in compliance to his current boss' request. This occurred even though the person who voiced such criticism was sufficiently in the know to realize that even in limiting oneself to working over the complex numbers, the needs of the proof inevitably called for introducing more general base schemes...)

This fashion of the day goes so far as to hold in contempt not only careful demonstrations (if not demonstrations altogether), but sometimes even proper statements and definitions. Given the cost of paper and with the reader's stamina nearing exhaustion, it will soon be time to part ways with such costly luxury! Extrapolating upon the current tendencies, we should be able to project an era when a publication will no longer be required to explicitly state definitions and statements; instead, we shall henceforth content ourselves with naming things using code words, leaving to the tireless and brilliant reader the task of filling the blanks in accordance to their own wits. The referee's task will become all the easier, as it will suffice for them to consult the "Who is Who" directory to determine whether the author is recognized as credible (so that in any case nobody could possibly refute the blanks and dotted lines composing their brilliant article), or on the contrary an unspeakable nobody who will be (as is already the case today, and in fact has been the case for quite some time) rejected from the get-go...

Chapter 14

B) Pierre and Motives

14.1 IV Motives (burial of a newborn)

Memory from a dream - or the birth of motives

51 [This note is referenced in note 46 p. 186] p. 205

(April 19) Since writing the above lines (ending with the note "My orphans", n^{o}46) less than a month ago, I have come to realize that they delay the chain of events to a certain extent! I have just received "Hodge Cycles, Motives and Shimura Varieties" (LN 900) by Pierre Deligne, James S. Milne, Arthur Ogus, and Kuang-Yen Shih, which Deligne kindly transmitted to me along with a list of his publications. This collection of six texts, published in 1982, constitutes an interesting new piece of information since 1970, in that motives are mentioned in the title and present in the text, however modestly, especially through the notion of "motivic Galois group". Of course, we are still far from a panoramic picture of a theory of motives, which for the past fifteen to twenty years has been awaiting the impetus of a bold mathematician who will be willing to "paint it, in a vast enough way to serve as a source of inspiration, as a golden thread and a horizon, for one or several generations of arithmetic geometers who will have the privilege of establishing its validity (or in any case of discovering the final word on the reality of motives...) (53).

1982 also seems to mark the year since which[1](*) the changes in fashion are beginning to slowly turn in favor of derived categories; Zoghman Mebkhout (in a perhaps euphoric rush) already sees them as being on the brink of "invading all domains of mathematics". If their utility, which was made evident by mere mathematical instinct (for well-informed individuals) since the beginning of the 1960s, is now starting to be recognized, it is (or so it seems to me) thanks to Mebkhout's solitary efforts and his willingness to take on the thankless role of

[1](*) (May 25) I am delaying the events once again, by one year this time - the turning point took place in June 1981 with the Luminy Colloquium, see the note "The Inequity - or a feeling of return", n^{o}75.

guinea-pig for seven years, with the courage of those who continue to trust their
p. 206 instinct in the face of tyrannical customs...

Remarkably, in reading this first publication which marks (twelve years after my departure from the mathematical world) a modest re-entry of the notion of motive into the apparatus of admissible mathematical notions, I could find nothing which would indicate to the uninformed reader that my humble person was involved in any way in the origins of this notion, long considered a taboo. Nor is there any allusion to an authorship of some form (51_1) behind the development of a rich and precise "yoga", which appears in the article (in piecemeal form) as if it came out of the void.

When, just three weeks ago, I laid down my thoughts on the yoga of motives in a page or two, qualifying it as one of the "orphans" whom I held closer to my heart than any other, I must have been sorely mistaken! Surely was I dreaming, when I seemed to remember years of gestation of a vision, tenuous and elusive at first, and growing richer and more precise over the course of the months and the years, as a result of a persistent effort to grasp the common "motive", the common quintessence of which the several cohomology theories known at the time (54) were but various incarnations, each telling us in its own language about the nature of the "motive" of which it was one of the directly tangible manifestations. Surely I was still dreaming when I remembered the strong impression that Serre's intuition had made upon me, regarding his conception of a profinite Galois group, an object which appeared to be of a discrete nature (or at least could be tautologically reduced to simple systems of **finite** groups) yet gave rise to an immense projective system of l-adic **analytic** or even **algebraic** groups over $\mathbb{Q}_l$ (by passing to appropriate (algebraic envelopes?)), groups which even had a tendency to be reductive - hence lending themselves to the panoply of intuitions and methods (Lie style) of analytic and algebraic groups. This construction made sense for any prime number l, and I felt (or dreamt that I felt...) that there lay a mystery to be probed, regarding the relationship between these algebraic groups associated to different prime numbers; I felt that they must all come from a single projective system of algebraic groups over the only natural common sub-field to all of these base fields, namely the field $\mathbb{Q}$, the "absolute" field of characteristic zero. And since I like dreaming, I continue dreamed that I remember having gained access to this glimpsed mystery, through work which surely was but a dream since I did not "prove" anything; and I eventually understood how the notion of motives provided the key to understanding this mystery - that, through the mere presence of a category (here, the category of
p. 207 "smooth" motives over a given base scheme, for instance motives over a given ground field), possessing internal structures similar to those which can be found on the category of linear representations of an algebraic pro-group over a field k (the charm of the notion of algebraic pro-group having been revealed to me by Serre as well at an earlier time), one would be able to reconstruct such a pro-group (given the data of an appropriate "fiber functor"), and to interpret the "abstract" category as the category of its linear representations.

This approach towards a "motivic Galois theory" was suggested to me by the approach which I had found, years earlier, to describe the fundamental group of a topological space or scheme (or even of an arbitrary topos - but here I risk offending delicate ears to whom "topoi are no fun"...) in terms of the category of étale coverings of the "space" under consideration, and fiber functors on this category. The very language of **"motivic Galois groups"** (which I could also have called motivic "fundamental groups", the two intuitions amounting to the same thing in my view since the end of the 1950s...) and of "fiber functors" (corresponding precisely to the "manifest incarnations" mentioned earlier, namely the different "cohomology theories" which may be applied to a given category of motives) is tailor-made to express the profound nature of these groups, and to suggest their intimate ties with ordinary Galois groups and fundamental groups.

I still remember the pleasure and awe which I felt, in playing this game with fiber functors and (torsors under Galois groups?) which send them to one another by "twisting", upon retrieving in a particularly concrete and fascinating situation the entire panoply of tools from non-commutative cohomology developed in Giraud's book, with the gerbe of fiber functors (here taken over the étale topos or, better, the fpqc topos of $\mathbb{Q}$ - interesting and non-trivial topoi if there ever were any!), as well as the ("link"?) (in algebraic groups or pro-groups") connecting this gerbe to the avatars of this link, all being realized by various algebraic groups or pro-groups, corresponding to the different "sections" of the gerbe, corresponding to the different cohomological functors. The various complex points (for instance) of a scheme over characteristic zero each give rise (via the corresponding Hodge functors) to sections of the gerbe, and to torsors providing a transition from one to the next; and both these torsors and the pro-groups acting on them possess remarkable algebro-geometric properties, expressing specific structures of Hodge cohomology - but now I am getting ahead p. 208 of myself and speaking about another chapter of my dream about motives... This was a time when those who issued the decrees of fashion had not yet declared that topoi, gerbes, and the like did not entertain them an that as such it was stupid to speak of them (not that this would have prevented me from recognizing topoi and gerbes when I saw them...). Now that twelve years have elapsed, these very people are pretending to be discovering and are teaching to others the fact that gerbes (if not topoi for now) are indeed relevant to the study of the cohomology of algebraic varieties, and even to that of periods of abelian integrals...

I could also evoke the dream of another memory (or the memory of another dream...) surrounding the dream about motives, which was also born from a "strong impression" (I am decidedly in full subjective mode!) which some comments by Serre regarding a certain "philosophy" behind the Weil conjectures had made upon me. Their translation into cohomological terms, for l-adic coefficients with varying l, led one to suspect the existence of remarkable structures on the corresponding cohomology theories - the structure of a "weight filtration"[2](*). Surely, the common "motive" to the various l-adic cohomology

[2](*) (January 24 1985) For a rectification of this distorted memory, see note n^{o}164 (I4), as well as sub-note n^{o}164$_1$, which give more details on the filiation of the "yoga of weights".

theories had to be the ultimate support for this essential arithmetic structure, which as such took on a geometric aspect, that of the remarkable structure of the geometric object that is a “motive”. It would once again be inaccurate of me to speak of a “work” when the task was to “guess” (with the internal coherence of a vision in progress as only guide, using the sparse known or conjectured elements lying here and there...) the specific structure of the various cohomological “avatars” of a motive, and how the weight filtration was expressed therein [3](**), beginning with the Hodge avatar (at a time where Hodge-Deligne theory had not yet been developed, and for good reason...[4](***)). This allowed me (in a dream) to view within a single vast painting the Tate conjecture on algebraic cycles (a third “strong impression” which inspired the Dreamer in his dream about motives!) and the Hodge conjecture (55), and to formulate two or three additional conjectures of the same type, about which I spoke to certain people who must have forgotten as I have never heard anyone mention them since, p. 209 subjected to the same silence as the “standard conjectures”. In any event, these were only conjectures (unpublished on top of that...). One of them did not concern a specific cohomology theory; rather, it gave a direct interpretation of the weight filtration on the motivic cohomology of a non-singular projective variety over a field in terms of the geometric filtration of this variety by closed subsets of given codimension (with codimension playing the role of “weight”)[5](*).

There was also the work (I should be putting quotation marks around “work”, but I can’t find the resolve to do so!) I did towards “guessing” the behavior of weights with respect to the six operations (completely lost since then...). Here again, I never felt that I was inventing anything, but discovering - or rather listening to what things were telling me whenever I sat down to listen to them with a pen in hand. What they were saying was of a peremptory, and as such unmistakable, precision.

Then there was a third “motive-dream”, which was in a sense the wedding of the preceding two dreams - regarding the problem of interpreting, in terms of structures imposed on the motivic Galois groups and on the torsors under these groups which can be used to “twist” a fiber functor to (canonically) obtain any other fiber functor [6](**), the various additional structures exhibited by the category of motives, with the weight filtration being one of the very first such structures. I seem to remember that this process was less guesswork than at any other point, but rather consisted of accurate mathematical translations. The work involved several new “exercises” on linear representations of algebraic group, which I spent days and weeks solving with great pleasure and the feeling

[3](**) (February 28 1985) There was a slight confusion in my mind. I was actually referring to the closely linked filtration by “levels”.

[4](***) This was at a time when the young Deligne had probably not yet heard the word “scheme” be said in a mathematical context, nor the word “cohomology”. (He learned these notions from me starting in 1965.)

[5](*) (February 28 1985) The filtration in question here is actually the “level” filtration (see preceding footnote).

[6](**) Just as the fundamental groups $\pi_1(x), \pi_1(y)$ of some “space” X at two “points” x and y can deduced from each other using the torsor $\pi_1(x, y)$ of homotopy classes of paths from x to y...

that I was at last getting closer to a mystery that had been fascinating me for years! Perhaps the most subtle notion that I had to apprehend and formulate in terms of representations was that of “polarization” of a motive, wherein I drew inspiration from Hodge theory, trying to extract the ideas that still made sense in a motivic context. This reflection must have taken place at the same time as my reflection around formulating the “standard conjectures”, with both events inspired by Serre’s idea (as always!) of establishing a “Kähler analogue” p. 210
to the Weil conjectures. In such a situation, in which the things themselves whisper about their secret nature and the ways in which we will be best able to delicately and faithfully express it, even though several essential facts seem to lie outside of the immediate scope of a proof, instinct dictates that we simply write down on paper what the things are insistently whispering, a message which furthermore grows clearer when we take the time to write it down! There is no need to worry about obtaining complete proofs or constructions - to burden oneself with such expectations at this stage of the process is to bar oneself from accessing the most delicate and essential step of a large scale work of discovery: that of the birth of a vision, gaining shape and substance as it emerges from an apparent void. The simple act of **writing**. **naming**, and of **describing** - if only to describe elusive intuitions or mere “hunches” reticent about taking concrete form - possesses a **creative power**. There lies the most important instrument in enacting the passion for knowledge, when the latter is invested into things which can be apprehended by the intellect. In the process of discovery for such matters, this work is the most creative step of all; it always precedes proof and enables it - or rather, without it, the question of “proving” something would not even arise, as nothing pertaining to the heart of the matter would have yet been formulated and seen. Through simple virtue of the effort of formulating, that which was amorphous takes form and lends itself to examination, in process of separating the visibly wrong from the possible, and above all from that which is so perfectly in accordance with the collection of things known, or guessed, that it itself becomes a tangible and reliable component of the vision being born. The latter grows richer and more precise over the course of the formulation process.

Ten things which are suspected of being true, none of which with certainty (say for instance the Hodge conjecture), through the process of mutually shedding light on one another, completing each other and concurring with a common mysterious harmony, acquire through this harmony the strength of a vision. Even in the event all ten of these things would turn out to be false, the work which resulted in this provisory vision was not done in vain, in that the harmony which it allowed us to glimpse and to ever-so-slightly penetrate is not an illusion, but rather a reality which it is urging us to unravel. Through this work, and only this way, we were able to establish intimate contact with this reality, this hidden and perfect harmony. Once we realize that things are the way they are for a reason, and that our vocation is to know them, rather than p. 211
to dominate them, we are able to see the day when a mistake is highlighted as a day for celebration (56) - just as much as the day when a proof shows us beyond all doubt that such a thing which we were imagining is indeed the true and faithful expression of reality itself.

In either case, such a discovery comes as a reward to labor, and could not have been reached without it. But even though the reward may only come after years at the task, or even in the event that we never reach the final word, an achievement relayed to our successors, the work itself is its own reward, rich in every instant of that which it reveals to us in that instant.

Note 51_1 (June 5) Zoghman Mebkhout just pointed out to me a reference to "Grothendieck motives" on page 261 of the volume in question, in a paper of Deligne which "resumes and completes a letter to Langlands". It reads: "we will not be working with Grothendieck motives, as he defined them in terms of algebraic cycles, but with **absolute Hodge motives**, defined analogously in terms of absolute Hodge cycles". "Grothendieck motives" (not underlined) are thus not mentioned as a source of inspiration, but in order to create a demarcation, insisting that the paper is treating of **something else** (the latter having been carefully underlined). This distancing is all the more remarkable given that the validity of the Hodge conjecture (a conjecture known to Deligne, I suppose, as well as to any reader of his paper-letter, beginning with its primary addressee Langlands) would imply that these two notions are **identical**!!

Of course, beginning in 1964 when I formulated the notion of motivic Galois group, I knew full well that a notion of "Hodge motive" could be developed along the same lines, leading to a corresponding notion of "motivic Galois-Hodge group", which was introduced independently by Tate (whether at an earlier or later time, I cannot recall) and thereafter has been known as the Hodge-Tate group (associated to a Hodge structure). The crude scam (which doesn't seem to inconvenience anybody, since it comes from such a prestigious character) consists in outright obfuscating the filiation of a novel and profound notion - that of motive - as well as the rich web of intuitions which I have developed surrounding this notion, under the derisory pretext that the technical approach taken to study this notion (via absolute Hodge cycles instead of algebraic cycles) is (maybe, if the Hodge conjecture is false) different from the one which I had
p. 212 (provisionally) adopted. This yoga, which I had developed over the course of nearly a ten-year period, was the principal source of inspiration in Deligne's work from his very beginnings in 1968. Its fecundity and power as a tool for discovery were clear well ahead of my departure in 1970, and its identity is independent of the particular technical approach chosen to establish the validity of such or such limited part of this yoga. Deligne deserves credit for finding two such approaches, independently of any conjecture. On the other hand, he did not have the honesty to name his source of inspiration, persisting since 1968 in hiding it from everyone so as to maintain exclusive benefits, awaiting the opportunity to (tacitly) claim the credit for himself in 1982.

The burial - or the New Father

52 Coming back to my dream about motives, I should also mention that I remember dreaming out loud. Granted, a dreamer's work is solitary in nature

- yet, the ebbs and flow of this unrelenting journey that took course over the years, in parallel with a vast project of foundations which occupied the majority of my time, had a witness on a daily basis, someone paying more close attention than Serre, who contented himself with observing things from a distance...[7]() I wrote about this day-to-day confidant in my reminiscence, saying that he "had taken on a bit of a student role" around the middle of the 1960s, and that I had taught him "the little I knew about algebraic geometry". I have also added that I had even told him about what I did not "know" in the common sense of the word - these mathematical "dreams" (on the theme of motives as on other topics) which he always welcomed with attentive ears and an alert mind, as eager to understand as I was myself.

It is true that, at the time of writing, when I said that Pierre Deligne had "taken on a bit of a student role", I was only referring to a wholly subjective impression (57), uncorroborated (as far as I am aware) by any written - or at least by any published - source which would suggest that Deligne may have learned something from me - even though I gladly remember presently that I never once discussed mathematics with him without learning something from the conversation. (And even after I stopped discussing mathematics with him, I continued learning from him about other things which are perhaps more difficult and more important, including on this very day in the writing of these words...) p. 213

I was told about the existence of a text by Deligne and others regarding the question of motives, or at the very least of "tannakian categories", by a third party who surmised (I wonder how!) that I could be interested. Upon reaching out to Deligne thereafter, I was met by his sincere surprise that something of the sort could be of interest to me. Reading through the copy which he kindly sent me nonetheless, I realized that his surprise was indeed completely well-founded. Visibly, I had never had anything to do with the subject in question. There is at most an allusion made in passing, in the introduction, regarding the fact that certain "standard conjectures" (which I had once formulated, heaven knows why) would have consequences for the structure of the category of motives over a field... The reader wishing to know more would be hard-pressed to do so, as no further precision or reference regarding these conjectures is made in the entire book; nor is any mention made to the one and only published paper in which I explain the way in which the category of motives over a field may be constructed in terms of the standard conjectures; nor is the only other text on the topic of motives published pre-1970 cited, an article by Demazure (produced in the context of a Séminaire Bourbaki, if I remember correctly) which followed my construction principle ad hoc from a slightly different perspective...[8]() p. 214

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[7]() (May 25) The beginnings of my reflection surrounding motives nonetheless took place before Deligne's appearance. My handwritten notes on motivic Galois theory are dated to the year 1964.

[8]() Upon verification, I now realize that other than a few pages on the standard conjectures (Algebraic Geometry, Bombay, 1968, Oxford Univ. Press (1969), pp. 193-199), I have never published a mathematical text on the topic of motives. In Demazure's talk (Séminaire Bourbaki n^{o}365, 1969/70) following Manin's talk in Russian, there are references made to a series of talks which I had given at the IHES in 1967 and which were meant (I suppose) to serve as a first broad sketch of a vision on motives. Kleiman also gave a talk on the standard

Nonetheless, Neantro Saavedra, who was lucky to be one of my "pre-1970 students", was duly cited. He had written a thesis under my supervision about what I remember calling "rigid tensor categories", and which he named "tannakian categories". One may again wonder through what miraculous coincidence Saavedra's thesis had foreseen in advance just the needs of Deligne's theory of motives, which was only to emerge ten years later! In fact, the work that Saavedra does in his thesis is precisely **the** key step needed for the development of a motivic Galois theory, just as J. L. Verdier's thesis was in principle **the** key step needed for the development of the formalism of the six operations in cohomology. One difference (among others) to Saavedra's credit is that he actually published his work; granted, he did not have the combined penmanship of Hartshorne, Deligne, and Illusie to exempt him from such a formality. Yet, ten years after the fact, Saavedra's thesis is now reproduced ab ovo and nearly in toto in a remarkable book, written this time around by Deligne and Milne.

Writing this book was perhaps not strictly necessary, if all that needed to be done was to rectify two particular points in Saavedra's work (58). But everything happens for a raison, and I think I understand why Deligne himself took the trouble to do this[9](), going against his own extremely stringent criteria when it comes to publications, which he is known to apply with exemplary rigor when it comes to authors other than himself...[10](*).

Regarding the paternity of the notions involved and of the yoga of motives, the answer goes without saying in the eyes of the uninformed reader (at a time when informed readers are getting rarer by the day and will one day have run their course...) - and this without having to disturb ancestral figures such as Riemann and Hilbert or even the good Lord. If the prestigious author does not say a word about filiation, letting the pretty result on absolute Hodge cycles and abelian varieties appear as a starting point, or even as the birth, of the
p. 215 theory of motives, it is as an honorable act of modesty, fully in line with the customs and ethics of the profession, which advise that we let others (if needs be) give credit where credit is visibly due: to the legitimate Father...

53 Affected by the vicissitudes of the orphan at hand, and doubting that someone else will do the work whose need and scope I am apparently the only one to perceive to this day, I presume that the "bold mathematician" in question will

conjectures and their connection to the Weil conjectures, in more details than was given at the Bombay congress announcement (Algebraic Cycles and the Weil conjectures, in Dix exposés sur la cohomologie des schémas, Masson-North Holland, 1968, p. 359-386). I am not aware of any reflection on the standard conjectures, notably involving steps taken towards a proof thereof, other than my own pre-1970. Based on the echoes which I have received, it seems to me that the deliberate decision to ignore these key-conjectures (which I considered, as I mentioned in my Bombay sketch, as one of the most important open problems in algebraic geometry, together with the resolution of singularities of (excellent?) schemes) has a lot to do with the impression of stagnation which is currently emanating from the cohomology theory of algebraic varieties.

[9]() See on this topic my reflections in the note "Clean Slate", n^{o}67.

[10](*) (June 8) All the more so when it comes to publications which bear the sign of my influence - see on this topic the episode "The note - or the new ethics", section 33.

be none other than myself, once I have reached the end of Pursuing Stacks (a project which I expect to last for about another year).

54 Since then, two new cohomology theories for algebraic varieties have appeared (other than Hodge-Deligne theory, which is a natural outgrowth in the "motivic" spirit of Hodge theory) - namely, Deligne's theory of "stratified pro-modules", and most notably the theory of crystals, "$\mathcal{D}$-modules-style" á la Sato-Mebkhout, together with the new light shed on the later by the God-given theorem (alias Mebkhout's theorem) which was discussed earlier. This approach towards constructible discrete coefficients is probably destined to replace Deligne's older approach, due to the fact that it better lends itself to the expression of the connections with de Rham cohomology. These new theories do not produce new fiber functors on the category of smooth motives over a given scheme, but rather (modulo a more extensive work on foundations than has been done as of yet) they provide a way of grasping more precisely the "Hodge" incarnation of a (not necessarily smooth) motive on a scheme of finite type over the complex numbers, or the "de Rham" incarnation on a scheme of finite type over a field of characteristic zero. It is possible that the theory (apparently still unwritten) of Hodge-Deligne coefficients on a scheme of finite type over $\mathbb{C}$ will eventually appear as embedded into the (equally unwritten) theory of crystalline coefficients á la Sato-Mebkhout (with the key added data of a filtration), or, put more precisely, as the intersection of Hodge-Deligne theory with the theory of constructible discrete coefficients in $\mathbb{Q}$-vector spaces... There also remains the elucidation of the relationships between crystalline theory á la Mebkhout and the theory developed in positive characteristic by Berthelot and others, a task perceived by Mebkhout before 1978, in the midst of a completely disinterested environment, and which appears to me to be one of the most fascinating questions currently posed in the endeavor of understanding "the" (unique and indivisible, i.e. motivic!) cohomology of algebraic varieties.

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55 Even though I was only dreaming, my dream about the relationship between motives and Hodge structures led me to inadvertently notice an incoherence in the "generalized" Hodge conjecture, such as it was initially formulated by Hodge, and to replace it by a rectified version which itself should be (or so I would wager) no more nor less false than the "usual" Hodge conjecture about algebraic cycles.

Prelude to a massacre

56 I am notably thinking about Griffith's discovery, in the context of the cohomology of algebraic varieties, regarding the falsity of a tantalizing idea that was circulating for a long time concerning algebraic cycles, namely that a cycle homologous to zero admitted a multiple which is algebraically equivalent to zero. This discover of a brand new phenomenon was so striking that I spent a whole week trying to wrap my head around Griffith's example, by transposing his construction (which was transcendental over the field $\mathbb{C}$) into a "maximally general" construction, making sense over base fields of arbitrary characteristic. This extension was not entirely obvious, involving (if I remember correctly) Leray spectral sequences and the Lefschetz theorem.

(June 16) This reflection lead me to develop the cohomology theory of "Lefschetz pencils" in the étale context. My notes on this topic were developed during the SGA 7 II seminar (by P. Deligne and N. Katz) as well as in the exposeés XVII, XVIII, XX by N. Katz (who took the care to reference these notes, which he closely followed). On the other hand, in the volume's introduction by P. Deligne, where it is said that the key results of the volume are exposé XV (the Picard-Lefschetz formula for étale cohomology) and XVIII (the theory of Lefschetz pencils), the author abstains from indicating that I had anything to do with this "key theory" of Lefschetz pencils. In reading the introduction, one gets the impression that I played no part in the development of the volume's themes.

The long seminar SGA 7, which took place in the years 1967-69, continued the seminars SGA 1 through SGA 6 which were developed under my impetus between 1960 and 1967, was organized by Deligne and myself, after I kicked
évanescents
off a development of a systematic theory of groups of vanishing cycles. Since the write-up of the talks by various volunteers dragged for some time, the two volumes of the seminar (SGA 7 I and SGA 7 II) were only published in 1973,
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by Deligne. Even though it was agreed during the seminar that we would be presenting it as a common endeavor, Deligne made the (strange) request after my departure that we **cut the seminar in half**, with a part I presented as being directed by me, and the other half by himself and Katz. I now view this event as part of an "operation" foreshadowing the operation "SGA $4\frac{1}{2}$", which aims (among other things) to present the entirety of the foundational series SGA 1 through SGA 7 - which in his mind and point of view were inseparable from my person, as well as the series EGA, i.e. Eléments de Géométrie Algébrique, as a compendium of texts with a variety of authors, where I myself only play an episodic, if not superfluous, role. This tendency appears very clearly, if not brutally, in the volume SGA $4\frac{1}{2}$ and most notably in the seminar SGA 5, which is inextricably linked to that volume. See the note "the clean slate", and "the massacre", n°s 67 and 87, and most of all "the remains…" (n°88), among others.

(June 17) I was responsible for the overall structure of the seminar SGA 7 (for which I did not see a need for a separation into parts "I" and "II", and still do not to this day) while Deligne made important contributions (mentioned in my report on Deligne's work written in 1969, see n°13, 14 of this report), the most crucial for the needs of the seminar being the Picard-Lefschetz formula, proven via a specialization argument starting from the already known case in the transcendental setting. The cutting of the seminar in two parts was unjustified mathematically, as well as regarding our respective contributions - both Deligne and I brought substantial contributions to each of the two "pieces" of SGA 7.

I would of course have been delighted if Deligne had continued the foundational series SGA, which I had started and was far from reaching its end!

However, this "operations SGA 7" is not at all a continuation but rather a sort of brutal "saw cut" (or chainsaw cut…) **putting an end** to the SGA series, with a volume which ostentatiously is distinct from my person, even though it is linked to my work and bears its mark as much as any other. Even though my person is obscured as much as it can be, the tone taken with respect to my work is not yet the barely disguised tone of disdain that characterizes the "operation SGA $4\frac{1}{2}$" - the latter represents an even more brutal saw cut, affecting the unity of the seminars SGA 4 and 5, and serving as a means and pretext to the lawful plunder of the unpublished part of SGA 5, whose stolen pieces were equitably shared by Deligne and Verdier…

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57 I should quickly mention that the same remark applied to the other gifted mathematician about whom I hazard to say (in note n°19) that he took on a bit of a student's role, ten years after Deligne.

58 This reminds me that the (publication?) Notes (which had already published six or seven "pre-1970" PhD theses produced under my supervision) never accepted to publich Yves Ladegaillerie's thesis from "post-1970" (stated reason: they do not publish theses!). It should be said that they did on the other hand publish Saavedra's thesis for a second time... I had also told Deligne about Ladegaillerie's beautiful result on isotopy which was refused by every journal (with the secret hope that he would lend a hand to help him publish it) - but it unfortunately did not interest him (stated reason: his incompetency in subject of the topology of surfaces...).

End scene...

The new ethics (2) - or the free-for-all fair

59 (April 20) During the few weeks separating me from the writing of the above lines, which identify a contradiction and its cost, I learned with surprise that the person in question had already found a most simple way to "resolve" said contradiction two years ago - one only had to think about it! This solution could be called "the method of the preemptive burial" (about which the reader can learn in the double note (50), (51) written yesterday, while still freshly affected by the discovery). I apologize in advance if the unexpected reappearance of the preemptively deceased individual in the famous "mathematical world" (which sometimes takes on the airs of a free-for-all fair...) risks introducing technical complications to the flawless execution of this brilliant method! In an earlier note ("deontological consensus - and control of information", n^{o}6) I felt (still confusedly) that the most universally admitted deontological rule in the scientific profession "went unheeded" as long as the individuals who have control over the circulation of scientific information did not respect the right that any scientist has to make their ideas and results known. Around this stage of the reflection, I also took the time to thoroughly describe a case study in which the disregard shown for this right was flagrant in my eyes - so much so that I felt that the disregard displayed was bordering on disdain for the number one rule, which itself is admitted by general consensus. (See "The note - or the new ethics", section 30).

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This wasn't the only time that I felt this very particular sense of unease, witnessing the **spirit** of the number one rule being disregarded while the very perpetrator was displaying a "thumbs up" through his position (above all suspicion!), his means, and the casualness of the execution. I attempt to pin down this uneasiness in the note "The snobbery of the youth - or the defenders of purity", in connection with the aforementioned section. Once we begin disregarding the "obvious" things about which I am speaking, as well as (I should add) the (possibly deep) things which are neither proven nor patented as "conjectures" published and known by all, we may as well (given what little there is!) consider them to be public property (trivial, of course)[11](*), and as such, when the time comes, as "one's own", with the greatest nonchalance and the most unaffected conscience - because it goes without saying that we would never claim ownership over a difficult proof of ten or a hundred pages (or even ten lines) which establishes a result that "we would not have been able to prove" (59'). I did not think I was being so sensitive or accurate (regarding the "unheeded rule") .[12](**)

missing sentence

I fortunately have the ability to defend myself - being able to express with some accuracy what I feel and want to say, having acquired (rightly or wrongly) a certain credibility, and having the chance of being heard when I have something to say, or having the possibility to publish if I feel the need to do so. On the other hand, I now vividly realize the "feeling of injustice and powerlessness" to which aggrieved individuals without my privilege are subjected, feeling like their hands are tied in the face while "those who own everything" may dispose of them arbitrarily - a power which they use however they please.

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It is true that I have at times displayed such condemnable behavior in my own life as a mathematician, in equally good conscience - and I have had the occasion in my reflection to speak about some of these instances as my conscience brought them out of my subconscious, where they had been buried together with their unexamined ambiguity. In probing these events I finally understood that I had no reason to be surprised at the fact that today (and for quite some time) the student has surpassed the master, and that I shouldn't repudiate anyone towards whom I feel sympathy or affection. It is nonetheless healthy for me and for others to call a cat a cat, whether that cat be from our home or someone else's home.

[11](*) Such was the fate of the "God-given theorem" (aka Mebkhout's theorem),

(June 8) And this, as for the yoga of motives, while also deftly creating an impression of filiation, without ever saying so explicitly! See on this topic (as a case study) the note "The Prestidigitator" n^{o}75, and for the brilliant general method or style, the note "Thumbs up!" n^{o}77, as well as the note to follow "Appropriation and disdain", n^{o}59$'$.

[12](**)

Appropriation and contempt

!59′ (June 8) I am no longer convinced of the above, concerning my friend Pierre Deligne, as I have witnessed that he eventually gave in to the game of "tacit filiation" vis-á-vis the tool of l-adic cohomology, i.e. what I call the "mastery" of étale cohomology. A remarkable evolution occurred between the "operation SGA 4 $\frac{1}{2}$" (where my name is still spoken, albeit with an attitude of flippant disdain towards this central component of my work, from which his own draws its origins) and the "The Funeral Service" in which any reference to even the word "cohomology" is scratched in relation to my name. (See the notes "Clean slate" and "One of a kind" for the initial phase, and the notes "The Funeral Service (1), (2)" for the final phase.)

Among the intermediary phases in this escalation, there was the "memorable paper" of 1981 on so-called "perverse" sheaves (see on this topic the notes "The inequity - or a feeling of return" and "Thumbs up!", n^{o}75 and 77), and the exhumation of motives in LN 900 the following year (the Funeral Service taking place the year after that, in 1983). In all of these cases and others of lesser scale, I was able to realize that the internal attitude and "method" which allowed Deligne to claim credit for others' ideas with flawless good conscience was that of **disdain** (one which remains partially tacit, all the while being deftly suggested) towards the "little" which we are about to appropriate - so "little" in fact that there is no need to even speak about it, especially given that we are about to use it right away to do truly powerful things - think Weil conjectures, theory of so-called "perverse" sheaves, ... After the operation is finished, and the appropriation is complete and accepted by all, there is always time to rectify the situation and to modestly show off that which has been appropriated. The same contribution is treated with offhand disdain while it remains attached to the name of one of those who are to be buried, only to be highlighted once it has p. 221
been appropriated by himself (l-adic cohomology, motives, Mebkhout's yoga) or by a good friend (yoga of derived categories and yoga of duality, appropriated by Verdier under Deligne's active encouragement).

14.2 V My friend Pierre

The child

p. 223

60 (April 21) Coming back to my dream about a memory, which concerns more than the birth of a vision... I remember (even though I have forgotten so many things!) the ever-renewed pleasure which I took in discussing with the person who had quickly become my confidant on everything which captivated me, as well as each step forward and encthanting discovering in my day-to-day love story with mathematics - and as such he never really was a "student". His perennial receptiveness and the ease with which he learned about each thing ("as if he had always known it. . . ") acted as a constant source of enchantment. He was an ideal listener, moved by the same thirst for understanding as my own - he has an extremely sharp ear, signaling a communion between us. His comments always ran ahead of my own intuitions or restraint, or shed some new light on the reality I was painstakingly trying to grasp through the mist that still surrounded it. As I have said elsewhere, he often has answers to the questions which I was asking, sometimes even on the spot, while other times he would reach the answer in the days or weeks that followed. The role of the listener was reciprocated when he took his turn in sharing the answers which he had found, which he presented as no more than the nature of things, always appearing with perfect naturality, and presented with the same ease which had tantalized me and certain of my elders such as Schwartz and Serre (as well as Cartier). This was the same simplicity, the same "evidence" which I had always pursued in my quest to understand mathematical things. Without needing to mention it, it was clear through our shared approach and standards we both belonged to "the same family".

Ever since we first met, I had felt that his "abilities", as we say, were of a very rare quality, and far exceeded the modest abilities which I myself possessed, even though we were of the same breed when it came to our passion for understanding and out exigency regarding the comprehension of mathematical things. I also sensed, dimly, without yet being able to put my finger on it that this "strength" which I noticed in him (and which I also noticed in myself, although present to a lesser degree) of "seeing" the obvious things which nobody else could see was the faculty of a child as well as the innocence of a child's eyes. He held within him something of a child, much more visibly than other mathematicians whom I have known, and this was surely not an accident. He once told me that one day, while he was still in high school I believe, he independently took the time to verify the multiplication table (as well as the addition table along the
p. 224
way) for the numbers 1 through 9 from the definitions. Not that he expected to find anything surprising - other than possibly the (pleasant as always...) surprise that the proof could be accomplished so beautifully and completely in a matter of a few pages, and in just about half an hour. I could sense, while he cheerily related this anecdote, that this had been a half hour well spent, and that is something I understand today even better than I did then. This story of his had marked me, even impressed me (even though I don't recall making that known to him) - because I saw it as the sign of a **self autonomy**, and of a certain freedom with regards to received knowledge, both of which had accompanied my relationship to mathematics since childhood, at the very first contact. (69)[13]().

This status of privileged interlocutor which we shared with one another, at a time when we saw each other nearly every day[14](*), continued over a period of

[13]() In fact, I believe that this freedom has never entirely disappeared over the course of my mathematical life, and that it is now as present as it had been during my childhood. Two or three years ago, I reminded my friend about the multiplication table episode. He seemed embarrassed by this evocation of a childhood memory, which visibly no longer corresponded to the image he had of himself. I wasn't really surprised by this embarrassment, but it pained me to see once again a confirmation of something which I had by then understood but struggled to admit...

[14](*) Such was the case while I was in Bures, and he was housed in an IHES accommodation. Starting from 1967 (when I moved to Massy), I think that we still saw each other once or

5 years - from 1965 (if I remember correctly) to 1969 included. I still remember the pleasure I took, during that year, in writing a comprehensive report of his works as part of my proposal to appoint him as professor at the institution at which I had been working since its creation (in 1958), and where I produced the largest part of my mathematical work. I no longer possess a copy of that report (64), in which I had reviewed around a dozen papers by my friend. Almost all of them are still unpublished (in fact, many remain unpublished), and most if not all of them individually contained, in my opinion, enough substance to constitute a solid Ph.D. thesis. It made me prouder and happier to present this eloquent report on his behalf than if I had been presenting a report on my own works (something which I have only done twice in my life, and even then only by forcing myself...). Many of his papers brought answers to questions which I had asked (the only published one being the aforementioned paper on the degeneracy of the Leray spectral sequence for a smooth and proper morphism of schemes (63)). Nonetheless, the two most important papers constituted answers p. 225 to questions that Deligne had asked himself, and in this case it was clear that their scope far surpassed that of a “solid PhD thesis”. I am referring to his work on the Ramanujan conjecture (which appeared in the Bourbaki seminar), as well as his work on mixed Hodge structures, also known as “Hodge-Deligne theory”.

At the time of writing of this shining report, the thought that I would be leaving the institution at which I was about to appoint my young and impressive friend, and where I was hoping to dwell for the foreseeable future, would have appeared to me as strange and very much unsuspected. Likewise (as I am now able to connected these two pairs of double-events), it seems strange, and probably not “coincidental”, that this same (no longer young!) friend of mind has communicated to me a month or two ago his own departure from that very institution, just about a year after I had resumed a regular mathematical activity in the form of an unexpected “return” to the mathematical world (if not to its “high spheres”...).

I expressed myself about this departure - this “salutary disconnection” - more than once in Récoltes et Semailles, and I wrote even more often about the “awakening” that happened soon thereafter, rendering this episode a crucial turning point in my life. During the intense years that followed, the mathematical world, as well as the members whom I had loved and the components of mathematics itself which had most fascinated me, became very distant from my mind - as if drowned into the mist, memories of another “me” who had been dead for ages. . .

Well before this episode, as well as during the years that followed this first great turning point, I knew that he who had been my students (a little[15](*)) as well as a confidant and a friend (a lot) needed only follow discontinuous impulse of the child in him, at play and in the pursuit of knowledge, to discover nd bring forth new and unexpected worlds, probing them and understanding their

twice a week, at least during the time when I was still invested in mathematics.

[15](*) For more on this hesitation to consider the (overly!) brilliant Deligne as one of my students, see the note “One of a kind” (n^o67).

intimate nature - and by so doing revealing them to his peers and to himself. As such, if I could envision a "bold mathematician" busy laying out a rough outline (to begin with…) after my departure (with no intention to return!) of the vast scenery which I had glimpsed, and of which I had only made a few cursory and provisional sketches. It would definitely have to be him - as he had everything in place to do so! To paint this far reaching tableau, a "master plan" uniting in a common vision the essential points of what was known and already
p. 226 guessed about the cohomology of algebraic varieties, required the work of a few months rather than years, for a person within whom this vision was already in place and ready to take form out of the mist of the yet unwritten. (Even if it involves further developing it over the course of years or generations if needed - until the end word of the reality of motives has been fully understood and established.) At that time, I had no doubts that this work, which felt "burning hot" would be accomplished at any moment, at the very least during the two or three following years. After my departure, there remained a single person who was called, by his very impetus towards knowledge, to pursue this pressing and fascinating work, even if it meant letting others further pursue this work. Once the master plan has been written and proofread, one can always launch into other adventures within the world of mathematical things, where every twist of the road brings the promise of a new and limitless world, for those who come with fresh and open eyes…

At the time when my life was still unraveling in the warm scientific cocoon, isolated from the noises of the world, and when Deligne was developing his extension of Hodge theory (the year must have been 1968 or 69) it went without saying that this work was a first step that needed to be taken in order to realize, test, and make more precise a certain **part** of this "tableau of motives", which had not been written out in its entirety.[16](*) In the years following my departure from the cocoon, at a time when mathematics seemed very distant from me, I was not surprised upon learning that the Weil conjectures had been finally proven. (If anything surprised me it was that the "standard conjectures" had not been proven in the same stride, even though they had been formulated as a possible approach towards the Weil conjectures, as well as a way to establish, at the very least, a theory of semisimple motives over field.[17](**).) I knew that
p. 227 neither this first jet towards a general theory of Hodge-type coefficients, nor through this proof of certain key conjectures (among many others that were more or less known) he was still not operating at full measure - he was in fact falling significantly short of that. And I was impatiently waiting for him to reach his full potential, even when the bulk of my attention was taken up by

[16](*) The fact that this Hodge-Deligne theory never (as far as I am aware) developed beyond the stage it reached through this first jet, to grow into a theory of "Hodge-Deligne coefficients" (and of the associated "six functor formalism") on schemes of finite type over the complex numbers is indissociable from another strange fact: namely, that this vast "tableau of motives" has never been painted, its very existence having been carefully silenced to this day...

[17](**) It is only in the past few years that I began to vaguely realize (albeit with more precision as of late!) that the "standard conjectures", as well as the very notion of motives to which they provided a "constructive" approach, had been **buried**, for reasons that now appear to me as particularly clear. (Compare also with the preceding footnote).

other things. (-> 61)

The burial

61 I had the privilege to witness his first youthful spirit carrying the promise of a vast expansion to come. During the following fifteen years, I started realizing that this promise was being repeatedly delayed. There was something delicate within him that I had been able to sense and recognize (even at a time when I was insensitive to so many things!) something altogether different than shear intellectual horsepower (which is as smothering as it is penetrating. . .) - the most essential thing of all for any truly creative work. I had sensed this in others at times, but I had never felt it to such a degree in any other mathematician. I had expected (as is natural) that this force would continue blossoming and developing within him, to later effortlessly express itself in the form of a unique work of which I would have been the modest precursor. Equally as strangely (and there surely must be a deep and simple link connecting all of these "strange phenomena"), I saw this delicate thing, this "force", which pertains neither to muscle nor to brain go progressively dimmer over the course of the years as if **buried** under successive layers, ever thicker layers of **something else** that I know too well - the most common thing in the world! Said thing doesn't necessarily collide with intellectual horsepower, nor would it impede a consummate experience or a skilled flair in a particular field, and both of these skills would force the admiration of some or fear of others or both at once through the accumulation of accomplishments that are potentially brilliant and would surely project force and beauty. Yet it is not **this** which I had in mind when I spoke of "expansion" or of "blossoming". The blossoming I had in mind was to be the fruit of innocence, hungry for knowledge, and always ready to rejoice at the beauty of things both small and large in this inexhaustible world of ours, or in some particular region of this world (such as the vast universe universe of mathematical things. . .). Only a blossoming of this kind holds the potential for a profound renewal, being the renewal of oneself or that of ones understanding of things in the world. This potential was fully realized, I believe, in the humble man Riemann[18](*). This true blossoming removes one from disdain; p. 228 be it disdain towards others (whom one feels are far below oneself. . .) or disdain towards things that are too "small" or too obvious to warrant attention, or which are deemed below ones legitimate expectations; or even disdain for a given **dream**, which is insistently communicating things which we claim to love. . . Disdain thereby becomes as foreign as the self-satisfaction which feeds it.

Granted, because of his impressive "means" as well as this delicate things which impresses no one yet **creates**, the "student" was destined to far surpass the "master". I did not doubt that right after my departure from this milieu where I had witnessed such a beautiful ascent, Deligne would give his full measure to the development of a vast and profound work which I will have helped initiate. The echoes of such work would surely reach me over the course of the

[18](*)

years, and as I will be engaged in separate quests far from mathematics, I will only be able to partially grasp the full scope and beauty of the worlds that he was about to discover.

However, the student cannot surpass the master while simultaneously inwardly **disavowing** him, secretly laboring towards the suppression of any trace of what he had contributed (whether that contribution had been for the better or the worse…), both from his own eyes and the eyes of others - in the same way that the son cannot truly surpass the father while simultaneously disavowing him. This is something that I have learned mostly through my relationship with my own children, but also (later) through my relationships with certain past students of mine; mostly the one student, among others, whom I had always refrained from calling a "student", having felt since the day we met that I was about to learn from him as much as he would be learning from me[19](*). Only ten years after that day, post 1975, and especially since I began meditating upon the meaning of my experiences, have I begun sending this **hindrance** within this person who nonetheless remained dear to me. I also sensed, obscurely, that this secret disavowal of my person and of the role which I had played during these crucial years of his life, were also, on a deeper level, a disavowal **of himself**,
p. 229
(so it goes, godlessly, every time that we disavow and wish to erase something which has happened to us, when it is our role to harvest the fruit…).

Having failed to remain "plugged into" "what has been happening in mathematics", including what he had been doing himself[20](*), I had never realized, until I stopped to reflect a couple of weeks ago, the extent to which this hindrance had weighed equally on that which he had given his whole: his mathematical work. Admittedly, more than once in the last eight or nine years, I have seen his mathematical instinct and simple common sense be erased through a deliberate manifestation of disdain (towards me) or of contempt (towards other whom he had the power to discourage) (66). He also wasn't the only one of my ex-students - with or without quotation marks - in whom I witnessed similar attitudes towards people whom I held dear (or towards others). None of these other occurrences affected me as painfully. More than once over the course of the past two months I have alluded to this experience in my reflection, "the most bitter experience of my life as a mathematician" - and I have also shared what it taught me as part of my reflection. This pain was so acute, it taught me something profound about a person whom I had always held dear (while I was also busy excavating what he taught me about myself and my past…), that the question of whether or not this event had an impact on mathematical creativity whether within him or within those who were discouraged or humiliated, became entirely accessory not to say derisory.

The note "refusal of a heritage - or the premise of contradiction" is the first written reflection in which I present a summary of what had occurred to me bit by bit, over the course of the years, both regarding the "state of the art" as well as regarding the work of the person whom I had known so well yet so

[19](*)
[20](*)

little. This marks the first time that I finally saw, at a glance, the **"price"**, or the weight within his mathematical work, of this refusal that he must have been carrying within him for over fifteen years. Even then, in writing this note, I was "delaying" the chain of events, in that for more than two years already (without anyone deeming it useful to update me), motives had become public knowledge and removed from the secretive state in which they had been kept for twelve years... Today, as I write the ultimate section (I think) of my reflection on my mathematical past, and just two days after having learned the broad p. 230
strokes of the memorable volume consecrating this furtive "introduction", I am stricken by the crushing weight that transpires. By this I mean the weight that is carried around, day after day and through circuitous paths, by a being born to fly - lightly and delicately, with joy and confidence towards the unknown, out of shear joy for oneself and for the carrying wind...[21](*)

If he does not fly, instead contenting himself with being admired and feared by others, by accumulating proofs of his superiority, I need not worry. He must surely find satisfaction in carrying around this weight - just as I in turn carried some weights and continue to this day to carry those which I did not know how to leave behind. He took what he liked from what I had to offer, both good and bad, and left the rest. I need not worry about such choices which pertain only to himself; nor need I hereby decree which choices are better than others (62). What is "better" for one is worth for the other, or sometimes even for the same person (who may have changed over time, albeit this is a rare occurrence...).

The choices I do make and the acts that follow (even when our words deny them), are made at my own risk. If they often bring us the expected rewards (which we receive as "the goods"), such rewards often end up having repercussions (which we recuse as "bad" and often react with outrage). Once we understand that the repercussions cannot be recused, we start considering them as a price to pay, and we so oblige with reluctance. Yet we sometimes understand that these repercussions need not be seen as cashiers who must be paid for the good time we had. Rather, they can be seen as patient and persistent messengers who tirelessly come back to us with the same message; an unwelcome message constantly recused - because this humble message, even moreso than the repercussion itself, is what appears to us as "the worst"; worse than a thousand repercussions, worse than a thousand deaths, or the destruction of the entire universe, about which we no longer give an f... p. 231

The day we finally decide to welcome this message is the day when our eyes suddenly open to see that what was feared as "the worst" is really a **liberation**, an immense salvation - and that the crushing weight from which we are suddenly relieved is the very thing to which we were holding on just yesterday as "the goods".

[21](*)

The event

62 (April 21) If there is no reason for me to worry, one may ask why I am elaborating for page after page about a personal relationship, when this relationship only concerns me and the person in questions!

If I felt the need to launch into this retrospective reflection on certain important aspects of this relationship, it is due to the impact of a specific event which hit close to home (even though I am only learning about it two years after the fact). This event is furthermore public information, even more so than the behaviors and routine actions of renowned mathematicians (such as Deligne and myself) towards lesser known or beginner mathematicians (even though the impact of these acts on people's lives is often far greater than in the case at hand). The even in question (namely the publication of the "memorable volume" that is the Lecture Notes LN 900, aka "the burial volume") as well as the circumstances surrounding it appeared to me as **toxic**, at least in my eyes. As such, it seemed healthy for everybody, starting with the "person of interest" himself, to give a more circumstantial testimony regarding certain ins and outs of the matter, so as to lay things down as I perceive them today.

Through this testimony and this reflection, I am not trying to convince anybody of anything (something far too tiring, not to mention hopeless!)[22](*) But rather to understand the events and situations in which I found myself involved. If this leads others to a true reflection, beyond the usual formalities, then this testimony will not have been published in vain.

The eviction

p. 232

63 This article[23](*) in the Publications Mathématiques in 1968, two years after my departure from the mathematical world. Its starting point had been a conjecture which I had shared with Deligne, regarding a degeneracy property of certain spectral sequences, which at the time could have seemed rather incredible, yet which nonetheless became plausible through the "arithmetic" viewpoint, as a consequence of the Weil conjectures. This motivation was of great interest in itself, as it showed just how much one could gain from using the "yoga of weights" implicitly contained in the Weil conjectures (a yoga that was first glimpsed by Serre, in certain important aspects of it). Already then, I currently applied it to all kinds of analogous situations, in order to extract conclusions about the "geometric" nature (for the cohomology of algebraic varieties) using "arithmetic" arguments. This happened on a heuristic level for as long as the Weil conjectures had not yet been established, but it nonetheless had a great power as a proving mechanism, representing a **tool of discovery** of the highest order. Deligne's "geometric" proof for the particular conjecture in question, using the Lefschetz theorem (which had at the time only been established in the characteristic zero setting) was interesting for a different reason, in addition to being independent of any conjecture. The link indicated by these two approaches

[22](*)
[23](*)

between two things which could at first sight seem to be unrelated, namely the Weil conjectures on the one hand (and the associated yoga of weights, which for me constituted the most fascinating aspect), and the Lefschetz theorem on the other hand, was extremely instructive in itself.

Most interesting for the present purposes, is something which only fully appeared to me today, namely that the reader of this article would be given little reason to think that I had something to do with the initial motivation of the main result, and no reason at all to learn what exactly this motivation was. (See also the beginning of note (49.)) The **spontaneous** approach (including, I am sure, for the author himself) in introducing such a result would have been to begin with the (striking) conjecture, then to indicate the first piece of motivation, equally striking, which would have constituted a good occasion to finally “sell” this famous yoga of weights, a much farther reaching accomplishment than the principal result of the article [24](*) then to follow up with the “Lefschetz the- p. 233
orem”[25] viewpoint, allowing one to prove the initial conjecture under slightly more general conditions, over an arbitrary base scheme, not necessarily smooth and proper over a field, but only in zero characteristic. On the other hand, the chosen approach begins with homological algebra generalities (very pretty of course, and presented with the customary elegance of the author), generalities which he has probably since forgotten, like everyone else, as a sort of axiomatization of the Lefschetz theorem. The main result (the only one of course which everybody remembers) appears around corollary X in the middle of the paper, while the word “weight” and my name are only pronounced in “remark 3.9” somewhere near the end (without the reader really knowing why)...

I don't recall my reaction upon first discovering the paper - as I was still in the game, I probably just took a brief glance at it. I probably felt an intention from him to “distance himself”, but at the same time brushed it off as a natural decision from my friend so as not to risk appearing as a disciple (or “protégé”) of a “master”.[26](***) Nonetheless, if he had had a quiet confidence in his own p. 234
strength, he wouldn't have hesitated to write a further reaching and more universally useful (including for himself) work without fearing that it be seen for what it is... (65).

The situation was fairly analogous upon the publication of his first far-reaching work in the following year, concerning mixed hodge theory. (I then considered this work to have a comparable scope to Hodge theory itself, as I conceived of it as a starting point for a theory of “Hodge-Deligne” coefficients which unfortunately never saw the light of day...) As I mentioned earlier, it was obvious for both of us that this work took its “motivation” from the yoga of motives, which I had developed over the course of the preceding years - namely, it constituted a first approach towards a tangible incarnation of this yoga. Highlighting such a link in his work would have, I believe (and as I probably believed then as well), would have made his work even more wide-ranging than they already were by virtue of their own merit. At the same time,

[24](*)
[25]
[26](***)

this would also have been an occasion to call the attention of the reader towards the world of motives which could be felt at every step underlying that of Hodge structures (63_1).

Only in hindsight do one of these omissions take on their full meaning, overlaid upon six years of silence regarding the yoga of weights[27], twelve years of silence (not to say of prohibition) regarding motives[28], their surprising reentry in the burial-volume LN 900, the stagnation of Hodge-Deligne theory following dazzling beginnings... But one can't do great things while also serving as an undertaker!

In any event, had I been more mature at the time of my departure from the IHES in 1970, I would have clearly felt the presence of a profound ambivalence towards me, coming from the person who had been my closest friend during the preceding five years. Furthermore, behind the friendly facade of good company inside that same muted institution, the fact was that my departure arranged everybody for reasons which I believe I can perceive in retrospect, and which
p. 235 varied from one person to the next. Visibly, this departure was wonderfully convenient for my young friend, who had recently arrived at the institution, and who would only have had to express solidarity towards me (to counterbalance the hesitating indifference of the other three permanent colleagues) in order to reverse an irresolute situation. If I did not understand the meaning of what was happening then, it must have been that I decidedly did not want to; what was being said was loud and clear! As was often the case at other times in my life, I felt within me an anxiety (without ever quite naming it!) signaling to me a "detachment" between a tangible and straightforward reality, and a picture of reality of which I could not separate myself from: namely, the picture I had of what my role had been in the institution which I was living, and perhaps more importantly, the picture I had of my relationship with my friend. It was this refusal to face an undeniable reality, together with the anxiety signaling this contradiction to which I was clinging, that made this "salutory exit" so painful at the time.[29](*)

To tell the truth, having never before engaged in a written reflection surrounding this relationship (except for some brief jets of reflection included in a few of the occasional letters I sent to my friend, never to receive a reply...), I had not realized that the first signs (admittedly discreet, but nonetheless unmistakeable) of the ambivalence that affected my friends relationship with me can be traced back to at least 1968, two years before "the great turning point". This was still a time when our relationship appeared as perfect, a flawless communion at the mathematical level, in the context of a simple and effectuous friendship. One could therefore be tempted to criticize my previous "tributes" regarding innocence, the creative energy of the child, and all the rest!

Yet I know that this communion was a **reality** and not an illusion; just as this "delicate thing" was a reality - namely, the creative force which is only lightly reflected in the resulting work. "The innocence" and "the conflict" are

[27]

[28]

[29](*)

tangible realities, rather than mere concepts, visible to anyone who cares to look carefully; and they appear to me as natural opposites, with one excluding the other. Yet, there is no doubt that these two realities existed jointly within my relationship with my friend, at varying levels.[30] During the period about which I am writing, it does not seem that the "conflict" interfered with mathematical creativity - at least not with work done in solitude, or during one-on-one meet- p. 236
ings. It is also true, that in the two aforementioned papers, which are some of the most tangible fruits of this work, the footprint of the conflict was already clearly visible. With fifteen years of hindsight, and at the turn of my reflection over the past days and weeks, I now see that this footprint (however discreet) strikingly foreshadows the particular hold that this growing conflict was about to have on the initial impetus, stripping it of its core essence, over the course of years, that essence which leads to great destinies(*).

Note 63_1 (May 26) Compare also with the remark made in the footnote[31](*) made at the end of note 60, commenting on the "block" faced by the natural development of Hodge-Deligne theory, caused by an attitude of denial towards some of the key idea which I had produced (namely the six operations, to which motives are inexplicably linked), a denial of the same nature as what is being presently examined, and which was therefore visible ever since the publication of Théorie de Hodge I and II.

That same attitude, striving to erase each and every trace of my influence to the extent that it was possible (or even beyond!) can also be found in the work (already mentioned in note 47) which he wrote in collaboration with Mumford, p. 237
on Mumford-Deligne compactification of moduli multiplicities. (This work also came out before my departure.) This paper uses a technique which transports topological results over $\mathbb{C}$ (proven using transcendental methods) to results in characteristic $p > 0$ - I had introduced these results at the end of the 1950s for the theory of the étale fundamental group. Ever since the early 1960s I had been suggesting that this method could be used to prove the connectedness of moduli varieties in every characteristic[32]. This idea nonetheless presented technical difficulties which stopped Mumford, which were elegantly overcome in their paper via the introduction of moduli **multiplicities** along with a "compactification" of the latter possessing perfect properties. The very idea of moduli multiplicities can be found at least "between the lines" in my Teichmüller exposés at the Cartan seminar, produced at a time when the language of sites and topoi didn't exist. Deligne's very language "algebraic stack", at a time when there was an entire language of sites, topoi, multiplicities, tailor-made to describe this very kind of situation, shows rather clearly, in hindsight, and in light of much more crude operations (at a later time) his intension to obscure the origin of some of the main ideas used in this brilliant paper. It is surely the same attitude (as I anticipate for the first time in the note "Refusal of an inheritance - or the price

ref

moduli stacks

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31 (*)

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of contradiction" n°47) which had a "chainsaw effect", excluding any further reflection on moduli multiplicities, objects which appear to me as being some of the most beautiful and fundamental of all "concrete" mathematical objects discovered to this day.

I should signal in passing, that the arguments which I had introduced at the end of the 1950s enable one (via Mumford-Deligne compactification) not only to prove the connectedness of moduli multiplicities in every characteristic, but also to determine their fundamental group prime to p, as being the "profinite compactification prime to p" of the ordinary Teichmüller group.

The ascension

63' (May 10) With the further hindsight of three weeks' time, I now realize that my desire to show "understanding" towards this "entirely natural" intention to keep his distance was in fact hinting at a lack of clarity, as well as a complacency towards my young and brilliant friend. If I had trusted my perceptive abilities rather than putting on blinders and lying to myself by means of vague clichés parading as "understanding" or even "generosity", ("I am not going to criticize him just because he doesn't put my name under the spotlight…"), I would have realized then what I see now, sixteen years after the fact. I could have called out his lack of probity towards the reader, myself, and him. Seeing things
p. 238 simply and unafraid to call them by their name, I would have been able to express myself plainly, as I am doing now, leaving to my friend the opportunity to learn a lesson - at the very least he would have understood that, even with his unique abilities, his elders (or at least one of them) expected him to work with the same level of integrity as they did. I thus realize that on this occasion, situated prior to my departure from the mathematical scene, at a time when I was still "in the game" and in possession of a certain moral ascendancy over my young friend, I did not fulfill my responsibility to him due to the **laxity**.[33] This impression was confirmed following the publication of "Théorie de Hodge II", which was Deligne's thesis work in which he makes no reference to motives or myself. It is true that already at that time, mathematics as well as my friend's person, appeared to me as distant, as if separated from me by a mist!

In light of the my friend's evolution as I see it, both spiritually and mathematically (two aspects which are tightly linked), I realized that upon meeting him and being impressed by his intellectual abilities, his acute vision, and his rapid comprehension of mathematics, I perceived no lack of maturity within him; nor could I anticipate (in what followed) the effects which his vertiginous social ascension would have on him, in the span of nearly four years, rising from the status of unknown student, to start of the mathematical world and permanent professor, possessing considerable privilege and power in an already prestigious institution. I do not regret facilitating and accelerating this ascent - but I do acknowledge that through my own lack of discernment and maturity, I mistakenly thought that I was doing him a "service". It was not a "service",

33

for as long as my friend had not reached the term of the harvest which he had set out for himself, with the help of my care-free assistance.

The ambiguity

63" (June 1) In the three weeks following my acknowledgement of a "laxity" (or the "complacency", to use what has since become the more appropriate expression) within my relationship with my friend Pierre, I have had the occasion to realize more clearly a certain lack of rigour and complacency within myself. The latter appeared first and foremost with the context of my relationship with the one who I considered to be "one of a kind", but they also appeared in relation to other mathematicians who viewed me as an elder. Namely, what I have been able to detect so far is a certain ambiguity, both on my end and on p. 239 behalf of the student, in situations wherein the student appropriated ideas and methods which they had learned from me, or even a detailed maître d'oeuvre of one of their project without clearly indicating their source or even alluding to it. Situations of this sort were rather frequent during the 1960s, after my departure, and even up to the last few years. I believe that I already felt the ambiguity at a certain level in all of these situations, in the form of a lurking feeling of unease, never examined up until the past few days. What made me play along with such a complacency and led me to overcome this uneasiness without paying it too much attention, was the desire to **conform** to a certain self-image I held, which revolved around a so-called "generosity". But true generosity does not come from a need to conform, or a desire to be (and appear, both to oneself and to others) "generous". The repressed uneasiness was a clear sign that this "generosity" was artificial, that it was an **attitude**, and not the spontaneous and unqualified gifting which characterizes true generosity.

I can distinguish two components of a different origin within this uneasiness. One comes from the "boss", the "me" who remained frustrated, because he was unable to win at both games, that is, to share the credit for a work in which he was involved in (a more or less large) part, while at the same time rising to the standards of a certain brand name, which involves (among many other things) the label of "generosity". The other component comes from the "child", that is the being within me that is not fooled by attitudes and facades, and who is candid enough to see what was unnatural about the situation[34](*) - unnatural for myself as well as for others. All in all, my "generosity" boiled down to playing p. 240 a game where one person collects, as his own, ideas which come from another, thereby projecting an image of himself and of certain reality which we both pertinently know is false. The two of us are therefore stand in solidarity in what may be called trickery, wherein both parties benefit. This was at least a "trickery" according to the consensus which prevailed "in my day", and which seem to still be professed as lip-service. I would surely not have played along, had the ideas involved come from someone other than myself and my "portége"[35](*)

[34](*)(June 5)
[35](*)

used them as if he had discovered them himself. Nonetheless, the fact that I gave my tacit agreement for ideas of mine to be presented as somebody else's does not change anything, in my eyes, about the nature of the situation - the only difference being that in this case there are two cheaters instead of just one. And even if we were to set aside this aspect of my person (namely, that I am myself participating in a scam, and therefore that my behavior is contrary to the very consensus which I pretend to uphold), it is clear that there is nothing generous about encouraging others to cheat (even if it only costs the person doing it - which is not even the case), nor about holding an ambiguous attitude towards a consensus, to which I was pretending to adhere whilst breaching it. True generosity should be well-meaning towards all, starting with the person admitting it, and the one on the receiving end. My ambiguous attitude, inducing or encouraging ambiguity in the recipient, and allowing me to pose as "generous" while the recipient should logically appear as borderline cheating (when in fact we are both engaged in fraud) is neither a blessing for myself nor for the recipient.

One only needed to look at the situation for the evidence to make itself known, without needing to refer to past experience or some "life lesson". Yet, it is experience which eventually led me to this self-examination, making me
p. 241 discover evidence which I would have been just as able to find thirty years ago, even before the appearance of a student coming to learn a trade with me and to absorb a certain spirit or philosophy underlying the practice of this trade through his interactions with me. I have spoken about the "rigour" involved in the work itself at an earlier occasion, something which I believe I have upheld
ref (see the section "Rigour and rigour"). But I am now realizing that outside of the "work" proper, I showed a lack of rigour, as expressed by the ambiguity and the complacency aforementioned. I don't recall any of my elders pointing out this ambiguity in me to me, even though (I think) they all had the same expectations for me as they did for themselves. Beyond ambiguity pertaining to a specific attitude, I can detect ambiguity in my very person, about which I speak more than once in the first part of Récoltes et Semailles This ambiguity began resolving itself with my discovery of meditation in 1976, but certain aspects of this ambiguity, expressed in what have now become habitual attitudes and behaviors (notably in my relationships to my students) have probably persisted to this day.

Visibly, my ambiguity was favourably received by some of my students. What used to be done tacitly even became, or so it seems, a backdrop within the mores of the mathematical high society of today. Namely, to go fishing in troubled waters (with or without the consent of the "relevant party"), or even downright looting (legitimate as long as the partaker is part of the intangible elite), seem to have become so common, that nobody appears surprised anymore, and in fact everybody refrains from speaking about it. The "boss" within me would like to stand up, denounce, take offence - yet in so doing I would only be perpetuating this same ambiguity within myself which has already yielded such a prolific harvest.

The accomplice

63"' (April 24)[36](*) While going through a private article which I just received from Mebkhout two days ago, I stumbled upon a reference to J.L. Verdier called "Catégories Dérivées, Etat 0" which appeared in SGA $4\frac{1}{2}$ (Lectute notes n°569 p. 262-311). I can be excused for having taken so long to become aware of this publication, having never had the honor of holding this book in my hands to this day, since neither Verdier nor Deligne (the author) deemed it useful to send me a p. 242 copy, be it after the first print run, or afterwords. I ignore if either C. Chevalley or R. Godemont, the other members of the jury, who attributed to J.L. Verdier the title of "docteur es sciences" on the basis of a 17 page introduction (still unpublished) had the chance yen years later to receive "L'état 0" (50 pages long this time around) of this rather one-of-a-kind thesis! I think I can remember once holding in my hands a serious foundational work of about one-hundred pages, which could have reasonably stood as a good Ph.D. thesis, and which roughly corresponded to the foundational work that I had proposed to Verdier around 1960 - with the caveat that it had already become clear by then, that the context of "triangulated categories" which he had developed so as to express the internal structure of derived categories) was insufficient.

It goes without saying that my name is nowhere to be seen in this "Etat 0" of a thesis. One could wonder what I could have possibly had to do with this work: it is well-known that derived categories were introduced by Verdier to allow him to develop the "Poincaré-Verdier duality" of topological spaces, as well as the so-called "Serre-Verdier" duality of analytic spaces, before some unknown author [37](*) developed a synthesis of the two for him, rightly called "Poincaré-Serre-Verdier duality" (the Unknown Author could not have been less recognized!). After all this, I only had to follow the motions and make the few necessary adjustments in order to develop Poincaré-Verdier duality, as well as Serre-Verdier duality in the particular settings of étale or coherent cohomology of schemes. . .

I have finally browsed through that SGA $4\frac{1}{2}$ (libraries can be useful!) [38](**), where I am kindly included as coauthor, or rather as "collaborator" (sic) of Deligne (without deeming it useful to inform me of this fact, let alone consult me). This was visibly a precursor to the memorable "burial volume" which appeared five years later, and whose existence I just heard about a few days ago (see notes **??**, and onwards, inspired by this event). But in any case, I did not need to hold this pre-burial volume in my hands, and thereby see for myself the evidence for this ghost-thesis whose name must not be spoken, in order to realize already a year ago, that the next stage of this "thesis" would never be written by anyone other than myself. This is why I set out to work on **Pursing Stacks** seventeen years ago, picking where my illustrious ex-student chose to p. 243 leave off.

[36] (*)
[37] (*)
[38] (**)

The investiture

64 (April 25) I nonetheless found a copy of it on my desk yesterday at the university. It actually consists of two surveys written one year apart, the first in April (?) 1968, and the second in April 1969. There, I review, in the span of seventeen pages, fifteen projects that he had pursued during three years of scientific activity at the IHES. Among them is his work on the Ramanuyam conjecture, on the compactification of moduli sites, and on an extension of Hodge theory. The collection of projects reviewed in this survey (or even merely the ones I just named) is a testament to a prodigious creativity deployed with perfect ease, as if at play. Putting aside the proof of the Weil conjectures, which occurred in the wake of his first expedition into the unknown, it appears to me that the subsequent work gives only a pale image of the unique search for a young spirit gifted with exceptional means, and benefiting from equally exceptional conditions to his growth. Yet, it appears that something in these "exceptional conditions" nourished another force, disjoint from the impulse to understand, which eventually took over the latter, diverting and absorbing the initial impetus. And visibly this "something" was tied to my person...[39](*)

This brief annotated survey (which I think I should include as an appendix to the present volume) seems interesting to me for more than one reason, including from the mathematical viewpoint (in that some of the projects reviewed therein remain unpublished to this day). At several points in the survey, I predict that certain projects, which Deligne had only sketched broadly, and for which he had addressed some of the crucial points would be developed by future students. These student never appeared, in light of the changes that took place thereafter in his relationship with mere mortals.[40](**) Among the ideas that I review, p. 244 the only one which to my knowledge was developed by somebody else (who would therefore appear to have been a student of Deligne) is the theory of cohomological descent, developed by Saint Donat in SGA 4 (that is, during the period of the initial impetus), a theory which has since become one of the most commonly used tools in the cohomological arsenal.

As an amusing and characteristic detail, I should mention that in three out of the four projects which have since been featured in papers of Deligne[41](*), I take great care to indicate, in passing, the relationship between these works and ideas which I had introduced or questions which I had raised - as if to counterbalance, it would seem, the silence which the author was to keep on these matters in his papers (none of which had been published, nor even redacted I think, at the time when I was writing this survey).

The knot

65 (April 26) It is also clear that keeping a large scale "yoga" to himself (namely that of weights and beyond, that of motives), a yoga which I mentioned to

[39](*)
[40](**)
[41](*)

others, but which Deligne was the only one to have intimately assimilated, and fully understood its reach, therefore giving him an additional "superiority" as the exclusive owner of a uniquely potent tool of discovery for the purpose of understanding the cohomology of algebraic varieties. I do not think that such a temptation played a decisive role back when I was still present and active in the mathematical world, without anything hinting at my departure *sine die*. It must have appeared at or soon after my departure, the latter of which presenting itself as the unexpected "occasion" to seize an inheritance (to which he had full rights!), while hiding both the inheritance and its origin.

I hereby can see once again, in an extreme and particularly striking case, the heart of a contradiction, far more telling than any other case study. I am referring to the ignorance, the disdain, and the deeply buried doubt surrounding the creative force that resides within our own being - this unique and invaluable inheritance which cannot be transmitted by anyone. It is this ignorance, this insidious alienation of what is most precious, and more rare within us, which p. 245
makes it so that we could feel envious of the force perceived in an other, and covet for ourselves the fruits and external signs of this force which we have forgotten within ourselves. If this desire to **supersede** then takes root, and finds a way to proliferate, thereby focalizing away the available energy that could be used towards a creative endeavor, the alienation grows more profound and settles in for the long haul. And the closer we get to the coveted "goal" to supersede, to oust, and to dazzle, the farther we get and the more we take away from this delicate force within us, thereby chopping the wings off of our own creative impulse. In our persistent efforts to drag ourselves upwards, we have long forgotten how to fly, and that we are made to fly.

In the context of my relationship, ever since the day we met, I have felt my friend to be perfectly at ease, not showing any signs that would have made me suspect that he was in any way impressed or dazzled by my reputation or by my person, or that he held some unexpressed doubt regarding his gifts or abilities in the field of mathematics, or regarding any other topic. It is also true, or so it seems to me, that he had received a friendly and warm welcome from myself and the milieu which I occupied, as well as by my family, something which contributed to putting him at ease. This appearance of a simple and visibly problem-free individual which attracted me to him, just as it attracted others, had surely not waited until our encounter to appear and develop. The aura which emanated from his person and which made him so endearing was that of a harmonious equilibrium, wherein his inclination towards mathematics did not in any way look like the workings of an all consuming goddess. Next to him, I looked a bit like an unrepentant "purist", not to say an "oaf" - and I still recall his quiet surprise upon realizing my profound lack of awareness of the nature around me, and of the rhythm of the seasons, which I passed through without seeing or saying anything. . .

Yet, this profound "doubt" which I would not have been able to perceive at the time (nor even today, were I to be put in similar circumstances) must have been present within my friend well before we encountered one another. In hindsight, I can unambiguously see its first signs starting in 1968, followed

by other, more clear, signs throughout the subsequent years [42](*). These are nonetheless "indirect" signs - none of the ones I witnessed first-hand took the form of a doubt or lack of confidence - rather, they took the opposite form as the years went on, manifesting themselves as self-importance and deliberately
p. 246 disdainful, if not contemptuous, comments. But such an "opposite" only reveals its counterpart, with which it is paired and of which it is the shadow.

I also learned from a third party that in the expectation of meeting such a prestigious (and famously austere) mathematician for the first time he may have felt a great tension, and entertained an irrational fear not to be considered good enough for the great man. This testimony was so opposed to what I had seen myself in my young friend that I had a hard time believing it (this was back in 1973). Yet in hindsight, it aligns with the signs of division which I have observed elsewhere and which all converge.

This division, and the role that I played as a sort of mediator in a conflict that remained vague before our encounter. These two things would probably have remained concealed in the normal circumstances of the evolution of a relationship with somebody who was (in one way or another) a "master" or at the very least somebody who transmits or entrusts. Thus my departure will have caused the **revelation** of a conflict ignored by all, and which I may have been the only one to know.

My "return" today acts as a second revealing factor, probably more inopportune. I cannot imagine what it will reveal to me, beyond what it had already taught me about my own past and present, and about people whom I have loved, and to which I remain linked to this day. Nor can I guess what it will reveal to the person who, for the past week, has been at the heart of this ultimate phase of the reflection, a phase which I called last month "**the weight of the past**" (a remarkably accurate title in hindsight. . .).

Two turning points

66 (April 25) The occasionally disdainful and antagonistic conduct of my friend Pierre towards me was strictly limited to the mathematical and professional level. Our personal relationship remains, to this day, an affectionate and friendly one, punctuated by several delicate gestures which struck me as indicative of genuine sentiments with no ulterior motive.

During the intense years that following my departure from the IHES, this conduct slipped from my mind and so did the long-misunderstood lesson that it carried. Furthermore, for ten consecutive years, my friend had remained (naturally) my preferred mathematical interlocutor; more precisely, he was the only interlocutor (with a single exception) between 1970 and 1981 whom I addressed
p. 247 during my periods of sporadic mathematical activity whenever the need for an interlocutor was felt.

It was also him, as the closest mathematician to me, to whom I addressed myself at the first occasions (between 1975 and 1978) when I had to seek guar-

[42](*)

antorship for students working with me. The first such occasion was for the defense of Mrs. Sinh's thesis in 1975. A thesis which she had prepared in Vietnam, under exceptionally difficult positions. Deligne was the first person whom I invited to be on the thesis committee. He refused, tacitly implying that this was surely a perfunctory thesis, for which he could not possibly be a guarantor. (I was nonetheless able to elicit the good faith of Cartan, Schwartz, Deny, and Zisman in support of this sham, and the defence took place in an atmosphere of interest, and warm sympathy.) It took three or four more experiences of this kind over the following three years for me to finally understand that my prestigious and influential friend had a deliberately antagonistic attitude towards my "post 1970" students, just as he did towards works that bare even the slightest mark of my influence (at least when these works were produced "post 1970"). I do not know if this attitude of manifest destiny which I have noticed on several of these occasions are also present in his relationship to other mathematicians whom he considers to be far below him. The very spirit of a boundless elitism which he prides himself in upholding suggests to me that he might. In any event, since 1978, I have been abstaining from asking anything of him. The fact remains that his discouraging influence has continued to efficiently influence others.

It was around the same year that I noticed the first signs, initially discrete, of a disdainful attitude towards my own mathematical activity. The first episode surrounded my reflections about cellular charts, following a discovery about the subject which had astounded me (see on this topic: Esquisse d'un Programme, par. 3: "Number fields associated to a dessin d'enfant"). This discovery (admittedly "trivial" and which presented nothing that would interest, let alone move my prestigious friend) was the starting point and the first material evidence for another mathematical **dream**, whose dimensions are comparable to that of motives, which began taking shape three years later (January-June 1981), through "The Long March Through Galois Theory". These notes and others from the same period (two thousand handwritten pages all in all) constitute a very first tour of this "new continent" that a trivial observation about a dessin d'enfant had allowed me to glimpse. p. 248

During this intense period of work, I wrote to my friend two or three times to share some of my ideas with him, and occasionally ask him questions of a technical nature. When he fancied answering my questions, his comments were just as clear and relevant as they had always been, testifying of the same "means" that had already impressed me in his young age. But a smugness had blunted the thirst for knowledge in him that had once enchanted me, as well as his ability to approach complex things, such as large enterprises, by means of thinking about how "small" things converse with one another. This ability does not pertain to intellect, and is not a matter of mere "efficiency" or of the "mastery" of a pre-established discipline or of known techniques. Rather, it is the reflection in the realm of the intellect, of an entirely different thing, namely the **capacity for awe** of the child. This gift seemed to be extinguished within him, as if it had never been present. At least, this was the impression I gathered from his relationship with me, and as I had already gleaned from his relationships with my "later" students. He had become an important man, and his approach to mathematics had become strictly limited to the "sportsmanship" attitude which I examined for the first time just a month or two ago, and to which I was no stranger...

Perhaps I could have gotten used to the absence of the communion in a common passion and the profound link which had once connected us. I could have contented myself, probably, with asking (whenever the need was felt) more or less technical questions or simple requests for information from my friend, directed at his vast knowledge of the world of mathematical things. But in that year (1981) the signs of his disdainful attitude certainly became so brutal
cite [43](*), that I lost all interest in continuing to communicate with him regarding mathematical questions, even occasionally. ($\Longrightarrow$ 67)

Clean slate

67 (April 26) While writing the above lines, yesterday, I connected this turning point in our relationship with the publication in 1982 (practically coinciding with this radical turn of events) of the "remarkable volume" of the lectures Notes,
reference and unceremoniously consecrating my mathematical burial! Having been left for "dead" mathematically speaking, it was therefore a form of grace for my friend to continue occasionally responding to mathematical questions which, after all,
p. 249 had no reason to exist...

In trying to learn from the chain of events, I get the feeling that it also was not a coincidence that the first appearance of his disdain and mathematical neglect (towards things that were burning and fertile as his "sane mathematical instinct" must have told him), at least in the context of his relationship to me, occurred right around the publication of the pre-burial volume SGA $4\frac{1}{2}$ five years earlier.[44](*) The circumstances surrounding the publication of this volume already speak to a deliberate attitude of disdain, at once discreet and ostentatious. The mere act of citing me as a "collaborator" of Deligne, without thinking to inform let alone consult me, and refraining from sending me a copy, appears to me as more eloquent than a proper testimony. Add to this the fact that this work of Deligne was supposed to, essentially, make work which I had done more than fifteen years earlier more accessible to a wide audience, at a time when I had never even heard anyone utter the name of my brilliant friend being pronounced! His disdain, and later his arrogance, must have been fueled in part by my passivity, which made it so that I was completely oblivious and unknowingly "withstanding blows"; but also in part by a certain climate which allowed for such aberrations to "pass" without raising any comment. To this day I have never received any echo regarding said volume (not even from the many friends whom I thought I had in the mathematical world), nor regarding
Hodge Cycles, Motives, and Shimura Varieties Ch. 1(?) the burial-volume for which the first book laid the groundwork.

[43](*) (May 28) For a new light on this second turning point, see also the note "Perversity" n°76.

p. 250 [44](*)

In the introduction, the author announced is upfront about the purpose of the book. It was written to save the non-expert from having to "read the bulky expositions in SGA IV and SGA V", "prune unnecessary details", and to "allow the user to forget about SGA V which can be considered to be a series of digressions, some of which are very interesting" (how kind were they about these "digressions"!). The existence of SGA $4\frac{1}{2}$ "will soon allow for the publication of SGA 5 as is" - a mysterious assertion which may leave one wondering in which sense this publication (of a volume which we are encouraged to forget anyway), which had already been in the works for twelve years, and presented a perfect coherent collection of results (which did not need Deligne's intervention to be formulated and proven) could be subordinate to the existence of SGA $4\frac{1}{2}$(*).

In asking this question, I already anticipate a simple answer, and a possible explanation regarding vicissitudes of poor seminar SGA 5 (68) (which I had thoroughly developed in 1965/66, eleven years before the publication of Deligne's volume SGA $4\frac{1}{2}$)[45](*). One can already detect the beginning of such an explanation on page 2 of the original version of SGA 5, where it is said that "the Lefschetz-Verdier formula was only conjecturally established" (a low blow towards Verdier, who presumably knew how to prove this theorem, the latter of which preceded SGA 5 [46](**)) and "furthermore, the local terms had not been computed". This could appear to be a regrettable shortcoming to a non-expert reader (to whom this volume is addressed in the first place). However, the reader in the know is well-aware that said local terms are not always computed "nowadays" and that the brilliant and peremptory author himself may be stumped if questioned about what exactly he means (in the general case) by "computing"[47](***) (although nobody apparently thought to ask him this indiscrete question).

An ambiguous sentence, "this seminar (?) contains another proof, this one complete, in the particular case of the Frobenius morphism" suggests that SGA 5 does not provide, after all, a complete proof of the main "result" that it announces, a trace formula implying the rationality of Weil-style L-functions (which is no surprise coming from a volume of digressions!); fortunately, this "seminar" saves the day by redressing this compromised situation - better late than never. . . p. 251

On page 4, we learn that the goal of the "Arcata" lectures was to "provide proofs of the fundamental theorems of étale cohomology, stripped from the gangue of nonsense[48](*) which surrounds them in SGA IV". He was charitable enough not to expand upon the regrettable nonsense that plagues SGA IV (such as Topoi and other similar horrors - the reader can celebrate having been spared from this ordeal thanks to the providential appearance of this brilliant volume, finally establishing a clean slate, following the regrettable gong that had preceded it. . .) $(67')(61_1)$ cite

As I browsed through the introduction to the volume and the introductions

[45](*)
[46](**)
[47](***)
[48](*)

to the various chapters, I wrote down the positive comments and declarations of intention which seemed to set the tone most clearly, among two or three others (along the lines of: digressions, indeed, albeit "very interesting") which seemed mostly destined to "help me swallow the bill" (and I did indeed effortlessly swallow it). For instance, the author has the integrity to clearly state at the start of the volume that "for complete results and detailed proofs, SGA 4 remains indispensable". This book, however ambiguous in its spirit and motivations, does not qualify as theft [49](**). Its role rather seems to be that of a probe,
p. 252 visibly conclusive at that: they did not need to go to such lengths!

Hodge cycles, motives, and Shimura varieties. LNM 900, Springer, 1982

There is a kind of **escalation of absurdity** (which apparently went completely unnoticed!) in this volume and the one he is preparing (SGA $4\frac{1}{2}$, and LN 900). In both texts, we witness a man with remarkable abilities, born to discover, traverse, and probe vast new worlds, grow attached to the "remaking" of a predecessor's work, starting with my own and later followed by that of an ex-student of mine (Saavedra), even though he had nothing essential to contribute to these works that had been produced with care and studied things in depth. (It seems to me that, the totality of what he brought to the table could be exposed in 20-30 pages.) In the first case, the reason provided was plausible; namely, to provide to the non-expert user a painless access to étale cohomology [50](*), without having to rely on the voluminous seminars SGA 4 and 5. (This was the first time that the author ever expressed such solicitude for mere mortals, at the expense of pleasurable time spent doing math. . .) The second time, his work almost entirely consisted of **copying** in substance the thesis which was produced under my supervision! Said thesis constituted a perfect reference, and the fact that the proof of a statement therein was mistaken and that another statement contained an unnecessary hypothesis was surely not reason enough to rewrite the entire article. Of course, no "reason" was given for such an undertaking.

I did not have to wait until I could hold SGA $4\frac{1}{2}$ in my hands in order to guess the meaning of this manifestly absurd undertaking: Deligne was "rewriting" Saavedra's thesis, ten years after it was originally written! It was surely the same reason which led him to take on the ever-so-slightly less absurd task which had proceeded it: Deligne's writing (twelve years after the fact) of a "Reader's Digest" (occasionally verging on condescending) of a certain part of Grothendieck's published work. It was precisely the part which he could not avoid, were he to continue pursuing his interest in the cohomology of algebraic varieties (a topic which he can't seem to let go of). Even more so, Saavedra is the work among all, published and carrying the mark of my influence, which he strictly cannot do without if he intends to reappropriate "to his own ac-
p. 253 count" the notion of motivic Galois groups which I had developed, to finally exploit (fifteen years later!) this evidently crucial notion. Through the writing of SGA $4\frac{1}{2}$ in the first place, and five years later the expansive Milne-Deligne (aka Saavedra) article in LN 900, my friend found a way to give himself an

[49](**)
[50](*)

illusory sensation of liberation from a feeling which he surely experienced as a burdensome obligation: that of having to constantly refer to the very individual which was being supplanted and repudiated, or any other individual that responds to him.

In order to arrive at my intimate conviction about the common meaning of these two absurd "acts", I did not need to read through the entirety of the (fifty one) publications of my prolific friend, a list of which I have received (for the first time) about ten days ago. To tell the truth, I did not even think to browse once again the four printouts in my possession[51](*) to seek confirmation for what I believe is true. If in the future I were to once again consult the works of my friend, it will be in order to find something new that I did not already know from someplace else. I will then surely have the opportunity to discover beautiful mathematical things, which I once had the even greater pleasure to hear in person!

Note 67_1 (June 14) I have caught two more micro-swindlings (of a smaller scale) in SGA $4\frac{1}{2}$. The first occurs in the "Ariadne's thread for SGA 4, SGA $4\frac{1}{2}$, SGA 5" (behold the suggestive sequence!) where the author writes (p. 2) that in order to establish, within the framework of étale cohomology, an "analogous duality formalism to that of coherent duality... Grothendieck made use of the resolution of singularities and the purity conjecture", thereby giving the impression that this formalism was only conclusively established by himself, Deligne, in the case (sufficient for many applications) of schemes of finite type over a regular scheme of dimension zero or one (or even alinéa). He knows full well that I had been alinéa
the one to establish the six-functor formalism (and hence the theory of global duality) without using any "conjecture", and that his qualifying statement only applied to the theorem of biduality (or "local duality") which in SGA 5 suddenly became (under Illusie's pen) "a theorem of Deligne"!

Next, there is a section on page 100 called "the Nielsen-Wecken method", the latter being a method which I introduced in algebraic geometry in order to prove a Nielsen-Wecken-type formula mimicking a result established by said others (in the transcendental settings) using triangulation methods which cannot be transported to the algebraic context. Deligne learned about this method (and p. 254
about MM Nielsen and Wecken, whose beautiful article, written in German, he did not have to read) from me during the SGA 5 seminar, aka the seminar of "technical digressions" which SGA $4\frac{1}{2}$ was destined to erase from memory! In this section, no allusion is made to SGA 5 nor to myself, and the reader is given a choice regarding the authorship of this method between Nielsen-Wecken (if he is very poorly informed) or the brilliant and modest author of the present volume.

Interestingly, in the entire volume, Verdier's "Woodshoie" proof of a trace Woodshoie
formula including the case that I needed (for Frobenius morphisms) is not mentioned. This proof (which apparently fell into oblivion for the benefit of the more general method developed in SGA 5) was the missing link in order to completely

[51](*)

justify my cohomological interpretation of L-functions. Visibly, there was "probably tacit" agreement between Deligne and Verdier - with Verdier yielding to Deligne the credit for the trace formula involved in the Weil conjectures, in exchange for the part of SGA 5 which he had appropriated the preceding year (in 1976). (See on this topic the note "The good references" n°82 .) There was further compensation, namely the appearance in SGA $4\frac{1}{2}$ of "the Etat 0" on derived and triangulated categories, in which my name is equally as absent. Incidentally, four years later, under Deligne's pen, étale duality in algebraic geometry takes the name of "Verdier duality" - Verdier had not stricken a bad deal (see the end of note 75 "The Iniquity or a feeling of return".)

One of a kind

67' (May 27)[52](*) The excerpts cited above, as well as the circumstances as a whole that surrounded the publication of the remarkable volume called SGA $4\frac{1}{2}$, testify a deliberate attitude of derision and disdain that my friend has taken vis-a-vis the centerpiece of my work, presented by the two intimately linked seminars SGA 4 and SGA 5. Among the "circumstances" that have revealed themselves to me over the course of my reflection, between April 24 (see the "The accomplice" note 63") and May 18 (see the notes "The remains…", "…and the body", n°88, 89), the sacking of the original seminar SGA 5 concretized in the massacre edition of 1977 is no minor one. (See notably "the massacre" n°87.)

p. 255

This deliberate attitude of derision in my friend gains its full meaning if one recalls that the oral seminar SGA 5 constituted the first contact of young Deligne with schemes, cohomological techniques, and notably the duality formalism, as well as χ-adic cohomology. When he arrived at the IHES in 1965 at age 21, it was with the well-defined goal of learning algebraic geometry with me. It was during this oral seminar, and from the notes of the SGA 4 seminar that took place two years earlier, that he had the privilege of learning first-hand about the ideas and techniques that have dominated his work to this very day. [53](*)

This essential aspect of the context surrounding "the operation SGA $4\frac{1}{2}$-SGA 5" and, beyond it, the relationship between my friend Pierre and myself, was something I visibly omitted while writing the previous note ("Clean slate (1)", n°67), just as I omitted it from the part of my reflection concerning the Burial that precedes it. The memory of this "young Deligne", arriving at the seminar SGA 5 with everything yet to learn, something that he did well (and very fast), only re-emerged in the latest stages of the reflection, as if against its will. My deliberate decision, since the very year that young Deligne appeared in my mathematical "microcosm", not to count him as one of my students (as if by so doing I would have failed to show adequate modesty towards such a brilliantly gifted person), made me minimize, or rather completely ignore up until these last few weeks, an evident and tangible reality which may be

[52](*)
[53](*)

commonly expressed by the double appellation (which I recused) of "master-student"[54](**). I somehow fancied to forget and ignore that there had indeed been a "transmission" of something from my person to his, something which held great **value** for both of us, albeit probably in very different ways. What I transmitted to him during this four years of close mathematical contact between the two of us, was something into which I had poured the best of myself, and that I had nourished with my strength and my love - a love which (believe) I offered unreservedly and without measure, perhaps without feeling the cost of having such an attitude.

Surely, what I was giving was feeding a passion for understanding within him which was in tune with the one invigorating me - along with **something** p. 256 **else** which I only perceived much later without yet tying it to the "transmission" that took place and which I tended to ignore. In other words, what I gave was **also** received, at a level which remained hidden from my view, as **instruments** serving to (first) supersede, rather than tools to probe a fascinating and inexhaustible Unknown, and to later assert a dominance, a merciless superiority upon others. Without keeping tabs on what was received by the "child" in my friend, eager to discover, and what was received by the "boss" eager to supersede and to dominate (or even to crush), looking superficially at the extent to which certain ideas, techniques, and tools appear in his work led me to an unexpected discovery over the course of the past six weeks, as I realized how much of my friend's body of work, which took off starting from the year of our first encounter, was nourished to this day by that which I had transmitted to him. I had imagined, upon leaving the mathematical scene fifteen years ago, that the "little" that I had shared with my non-student-friend (a little whose role I did nonetheless perceive in his impressive first foray) would serve as a first launching pad to a flight that was to lead him far beyond his starting point, thereby **distancing** him from my work and myself. What actually happened, on the other hand, is that my friend remained to this day attached to said starting point, **attached** to the very work which he took it upon himself to repudiate, to subject, to derision, or to oblivion, and "to use". There lies a typical case of one's conflictual link to a father or mother figure, whereby one indefinitely remains within the orbit of those whom he was destined to leave and to surpass, all the while finding pleasure in cultivating this conflict within him rather than launching towards the discovery of the world...

I now realize that through my deliberate choice to treat my young friend as being "one of a kind" rather than simply as one of my students, who happened to have more means than the others - and through my equally deliberate decision to minimize and forget in the context of my relationship to him (the value of what I was transmitting as well as the **power** which I consequently was placing within his young hands...), I was feeding, against my will, a self-satisfaction and conflict within him which both remained hidden from my sight. At the same time, I was entering into a sort of game - or rather, there was a two player-game, in perfect harmony, for which I would be at a loss to determine who started

[54](**)

it (assuming the question even makes sense): myself displaying modesty in pretending that my young friend was much too brilliant to be anybody's student, and that the little which I was able to share with him was not even worth mentioning, and himself marking a distance (right after my departure) from my
p. 257
person and my work, repudiating (under my indulgent eye) the breading ground which had definitively nurtured him.

It is only in writing the present note that I can finally see through his game, whose dim awareness must have been present for the past week or two. I can also see that the "modesty" or "humility" which I displayed was false: it marked a lack of forthrightness in seeing things as they are. This game involved a leniency towards my young friend - sowings which yielded one-hundred fold! There was also, more subtly, a leniency toward myself in putting such a privileged, exceptional, extraordinary, and so on and so forth relationship on a pedestal[55](*). (After all, perhaps it is the case that any lack of forthrightness is more or less a form of leniency...)

The green light

68

The reversal

!68′

Squaring the circle

69

The funeral

70

The coffin

71

[55](*)

Chapter 15

C) High Society

15.1 VII The Colloquium - or Mebkhout's sheaves and Perversity

The Iniquity or a feeling of return

75

The colloquium

75'

The prestidigitator

!75"

Perversity

76

Thumbs up!

77

The robe of the emperor of China

77'

Encounters from beyond the grave

78

Note 78_1

The victim - or the two silences

78'

Note 78_1'

Note 78_2'

The Boss

!78"

My friends

79

The tome and high society - or moon and green cheese...

80

15.2 VIII The Student - a.k.a. the Boss

Thesis on credit and risk-proof insurance

81

Note 81_1

Note 81_2

Note 81_3

The right references

82

The jest, or the "weight complexes"

83

15.3 IX My students

Silence

84

Note 84_1

Solidarity

85

Mystification

!85'

Note 85_1

Note 85_2

The defunct

86

The massacre

87

Note 87_1 (May 31) This closing talk, probably one of the most interesting and substantial, along with the opening talk, were visibly not lost on everyone, as I now realize upon learning about MacPherson's paper "Chern classes for singular algebraic varieties" (Chern classes for singular algebraic varieties, p. 362
Annals of Math. (2) 100, 1974, p. 423-432) (submitted in April 1973). There, cite
I found under the name of "Deligne-Grothendieck conjecture" one of the main conjectures which I had introduced in said talk in the context of schemes. The conjecture was reformulated by MacPherson in the transcendental context of algebraic varieties over the complex numbers, where the Chow ring is replaced by the homology group. Deligne had learned about this conjecture[1](*) during my 1966 talk, the same year that he joined the seminar and started familiarizing himself with the language of schemes and cohomological methods (see the note "One of a kind", n^o 67'). It is nonetheless kind to have included me in the name of this conjecture - a few years later this would have been out of the question...

(June 6) I would like to use this occasion to explicitly write down the conjecture which I had announced in the context of schemes, while also probably

[1](*) In a slightly different form admittedly, see the rest of the note dated May 31.

hinting at the obvious analogue in the complex analytic (or even rigid analytic) context. I viewed it as a "Riemann-Roch"-type theorem, albeit with discrete coefficients instead of coherent coefficients. (Zoghman Mebkhout also told me that his viewpoint on $\mathcal{D}$-modules should enable one to consider both Riemann-Roch theorems as contained in a single crystalline Riemann-Roch theorem, which in zero characteristic would constitute the natural synthesis of the two Riemann-Roch theorems that I have introduced in mathematics, the first in 1957 and the second in 1966). Start by fixing a coefficient ring Λ (not necessarily commutative, but noetherian for simplicity and furthermore with torsion prime to the characteristic of the schemes under considerations, to meet the needs of étale cohomology...). Given a scheme X, write

$$\mathrm{K}.(X, \Lambda)$$

to denote the Grothendieck group associated to constructible étale sheaves of Λ-modules. This group is functorial in X with respect to the functors $\mathbf{R}f_!$ when restricting our attention to separated scheme morphisms of finite type. For regular X, I claimed that there exists a canonical group homomorphism, playing
p. 363 the role of the "Chern character" in the coherent Riemann-Roch theorem,

$$\mathrm{ch}_X : \mathrm{K}.(X, \Lambda) \to \mathrm{A}(X) \otimes_{\mathbb{Z}} \mathrm{K}.(\Lambda), \tag{15.1}$$

where $\mathrm{A}(X)$ is the Chow ring of X and $\mathrm{K}.(\Lambda)$ is the Grothendieck group associated to Λ-modules of finite type. This homomorphism was supposed to be completely determined by the presence of a "discrete Riemann-Roch formula" for **proper** morphisms between regular schemes $f : X \to Y$, whose form is analogous to the Riemann-Roch formula in the coherent context, except that the Todd "multiplier" is replaced by the total relative Chern class:

$$\mathrm{ch}_Y(f_!(x)) = f_*(\mathrm{ch}(x)c(f)),$$

where $c(f)$ denotes the total Chern class of f. It isn't hard to see that, in a context where one has access to a resolution of singularities theorem in the strong sense of Hironaka's, this Riemann-Roch formula does indeed uniquely determine the ch_X's.

Of course, we are supposing that we are working in a context in which there is a notion of Chow ring. (I am not aware of any attempt to develop a theory of Chow rings for regular schemes that are not of finite type over a field.) Otherwise, we could also work with the graded ring associated to the usual "Grothendieck ring" $K^0(X)$ in the coherent context, equipped with the usual filtration (see SGA 6); or we could replace $\mathrm{A}(X)$ by the even ℓ-adic cohomology ring, given by the direct sum $\bigoplus_i \mathrm{H}^{2i}(X, \underline{\mathbb{Z}_\ell}(i)$. This comes with the added baggage of an artificial parameter ℓ and produces coarser, "purely numerical" formulas, whereas the Chow ring has the added charm of having a continuous structure which is destroyed upon passing to cohomology.

Already in the case where X is a smooth algebraic curve over an algebraically closed field, computing ch_X involves studying delicate Artin-Serre-Swan-type local invariants. This hints at the depth of the general conjecture, whose pursuit would involve understanding the analogues of these invariants in higher dimensions.

Remark. Writing $\mathrm{K}^{\cdot}(X, \Lambda)$ to denote the "Grothendieck ring" associated to constructible complexes of étale sheaves of Λ-modules of finite Tor-dimension (which acts on $\mathrm{K}_{\cdot}(X, \Lambda)$ when Λ is commutative...), we also expect to have a homomorphism

$$\mathrm{ch}_X : \mathrm{K}^{\cdot}(X, \Lambda) \to \mathrm{A}(X) \otimes_{\mathbb{Z}} \mathrm{K}^{\cdot}(\Lambda), \tag{15.2}$$

giving rise (mutatis mutandis) to the same Riemann-Roch formula. p. 364

Now, let $\mathrm{Cons}(X)$ denote the ring of constructible integral-valued functions on X. We can define more or less tautologically canonical homomorphisms:

$$\mathrm{K}_{\cdot}(X, \Lambda) \to \mathrm{Cons}(X) \otimes_{\mathbb{Z}} \mathrm{K}_{\cdot}(\Lambda), \text{ and} \tag{15.3}$$

$$\mathrm{K}^{\cdot}(X, \Lambda) \to \mathrm{Cons}(X) \otimes_{\mathbb{Z}} \mathrm{K}^{\cdot}(\Lambda). \tag{15.4}$$

If we restrict our attention to schemes in **zero characteristic**, then (using Euler-Poincaré characteristics with proper support) we see that the group $\mathrm{Cons}(X)$ is a covariant functor with respect to morphisms of finite-type between noetherian schemes (in addition to being contravariant as a ring-functor, and this independently of the characteristic), compatibly with the above tautological morphisms. (This corresponds to the "well-known" fact, which I don't recall being proven in the oral seminar SGA 5, that in **zero characteristic**, a locally constant sheaf of Λ-modules F over an algebraic scheme X has image $d_X(X)$ under the map

algebraic space?

$$f_! : \mathrm{K}^{\cdot}(X, \Lambda) \to \mathrm{K}^{\cdot}(e, \Lambda) \simeq K^{\cdot}(\Lambda),$$

where d is the rank of F, $e = \mathrm{Spec}\, k$, and k is the algebraically closed base field...) This suggests that the Chern homomorphisms 15.1 and 15.2 should be deducible from the tautological homomorphisms 15.3 and 15.4 upon composition with a "universal" Chern homomorphism (independent of the choice of coefficient ring Λ)

figure out what $d_X(X)$ means

$$\mathrm{ch}_X : \mathrm{Cons}(X) \to \mathrm{A}(X),$$

in such a way that the two versions of the Riemann-Roch formula "with Λ-coefficients" appear as formally enclosed in a RR formula at the level of constructible functions, with the latter always taking the same form.

When working with schemes over a fixed base field (this time in arbitrary characteristic), or more generally over a fixed **regular** base scheme S (such as for instance $S = \mathrm{Spec}\, \mathbb{Z}$), the form of the Riemann-Roch formula closest to the traditional notation (in the coherent context, familiar since 1957) can be obtained by introducing the product

$$\mathrm{ch}_X(x) c(X/S) = c_{X/S}(x) \tag{15.5}$$

(where x is in either $\mathrm{K}_{\cdot}(X, \Lambda)$ or in $\mathrm{K}^{\cdot}(X, \Lambda)$), which could be called the **Chern class of x relative to the base S**. When x is the unit of $\mathrm{K}^{\cdot}(X, \Lambda)$, i.e. the p. 365

class of the constant sheaf with value Λ, we recover the image of the relative total Chern class of X with respect to S under the canonical homomorphism $\mathrm{A}(X) \to \mathrm{A}(X) \otimes \mathrm{K}^{\cdot}(\Lambda)$. With this notation in place, the RR formula becomes equivalent to the fact that the formation of these relative Chern classes

$$c_{X/S} : \mathrm{K}_{\cdot}(X, \Lambda) \to \mathrm{A}(X) \otimes \mathrm{K}(\Lambda) \tag{15.6}$$

make sure the right equation is being referenced

for fixed S and varying regular scheme X of finite type over S is functorial with respect to proper morphisms, and likewise for the variant 15.2. In zero characteristic, this can be reduced to the functoriality (with respect to proper morphisms) of the corresponding map

$$c_{X/S} : \mathrm{Const}(X) \to \mathrm{A}(X). \tag{15.7}$$

It is under this form that the existence and uniqueness of an absolute "Chern class" 15.7 in the case $S = \operatorname{Spec} \mathbb{C}$ is conjectured in the work of MacPherson, the relevant conditions being (here as in the general case in zero characteristic) (a) functoriality of 15.7 with respect to proper morphisms and (b) the identity $c_{X/S}(1) = c(X/S)$ (in this case, the "absolute" total Chern class). The form of the conjecture presented and proven by MacPherson differs from my initial conjecture in two ways. The first is a "negative", namely that he is not working in the Chow ring, but rather in the integral cohomology ring, or more precisely the integral homology group, defined by transcendental methods. The other is a "positive" - and this is possibly where Deligne contributed to my initial conjecture (unless this contribution is due to MacPhersson[2](*)). Namely, the observation is that in order to prove existence and uniqueness for 15.7, we don't need to restrict ourselves to regular schemes X, as long as we replace $\mathrm{A}(X)$ by the integral homology group. As such, it is probable that the same holds in the general case, if we write $\mathrm{A}(X)$ (or better $\mathrm{A}(X)$) to denote the **Chow group** (which is no longer a ring in general) of a noetherian scheme X. Said differently: while the heuristic definition of the invariants $\mathrm{ch}_X(x)$ (for x in either $\mathrm{K}_{\cdot}(X, \Lambda)$ or in $\mathrm{K}^{\cdot}(X, \Lambda)$) uses in an essential way the hypothesis that the ambient scheme is regular, upon multiplying by the "multiplier" $c(X/S)$ (for X of finite type over a fixed regular scheme S), the product obtained in [**?**] seems to still make
p. 366 sense regardless of any regularity hypothesis onX, as an element of the tensor product

$$\mathrm{A}_{\cdot}(X) \otimes K_{\cdot}(\Lambda) \text{ or } \mathrm{A}_{\cdot}(X) \otimes K^{\cdot}(\Lambda),$$

where $\mathrm{A}_{\cdot}(X)$ denotes the Chow group of X. The spirit of MacPherson's proof (which does not use resolution of singularities) seems to suggest that it it possible to exhibit a "constructive" and explicit construction of the homomorphism 15.6, by "making do" with the singularities of X as they are, as well as with the singularities of the sheaf of coefficients F (whose class is x), so as to "collect" a cycle on X with coefficients in $\mathrm{K}_{\cdot}(\Lambda)$. This would fit in the circle of ideas which I had introduced in 1957 with the coherent Riemann-Roch theorem, where I

[2](*) (March 1985) That was indeed the case, see note n^{o}164 referenced in the previous footnote.

notably computed self-intersections, without quite "moving around" the cycle under consideration. An initial obvious step (obtained by immersing X in an S-scheme) would be to reduce to the case where X is a closed subscheme of a regular S-scheme.

make sure this is what Grothendieck meant

The idea that it should be possible to develop a **singular** (coherent) Riemann-Roch theorem was already familiar to me, although I couldn't say for how long, but I never seriously put it to the test. It was in part this idea (other than the analogy with the "cohomology, homology, cap-product" formalism) which had led me to systematically introduce $\mathrm{K}_\cdot(X)$, $\mathrm{K}^\cdot(X)$, $\mathrm{A}_\cdot(X)$, and $\mathrm{A}^\cdot(X)$ in SGA 6 (in 1966/67), instead of choosing to work only with $\mathrm{K}^\cdot(X)$. I can't remember if I had thought about something along those lines in the SGA 5 seminar in 1966, or if I made mention of it in my talk. As my handwritten notes have disappeared (perhaps while moving?), I may never know...

(June 7) In reading through MacPherson's article, I was stricken by the fact that the word "Riemann-Roch" is never used - this is also the reason why I did not immediately recognize the conjecture which I had made in the SGA 5 seminar in 1966, the latter having always been (and still is) a "Riemann-Roch"-type theorem in my view. It seems that at the time of writing his article, MacPherson did not notice this evident filiation. I am guessing that the reason behind this is that Deligne, who circulated this conjecture in the form he liked best after my departure, took care to "erase", insofar as possible, the evident filiation with the Riemann-Roch-Grothendieck theorem. I think I understand his motivation behind this. On the one hand, this weakens the link between the conjecture and myself, making more plausible the name currently under p. 367 circulation, "Deligne-Grothendieck conjecture". (N.B. I ignore where this conjecture is currently circulating in the scheme context, and is so, I would be curious to know under which name). But the deeper reason seems to be his obsession with denying and destructing, to the extent possible, the fundamental unity of my work and mathematical vision[3](*). This is a striking example of the way in which a fixed idea entirely foreign to any mathematical motivation can obscure (if not downright seal) what I have called the "sane mathematical instinct" of a mathematician whose abilities are nonetheless exceptional. Such a mathematical instinct would not fail to perceive the analogy between the two statements, one "continuous" and the other "discrete", of a "single" Riemann-Roch theorem, an analogy which I had of course spelled out during my talk. As I indicated yesterday, this filiation will probably be confirmed in the near future by a formal statement (conjectured by Zoghman Mebkhout), at least in the complex analytic context, enabling us to deduce both theorems from a common result. Clearly, given Deligne's "grave-digging" attitude towards the Riemann-Roch theorem[4](**), he was not positioned to discover the common

[3](*) Compare with the comment made in the note "The remains" (n^o88) regarding the profound significance of the SGA $4\frac{1}{2}$ operation, similarly aiming to break out into an amorphous collection of "technical digressions" the profound unity of my work on étale cohomology via the "violent insertion" of the outlandish text SGA $4\frac{1}{2}$ between the two indissoluble parts SGA 4 and SGA 5 in which this work is carried out.

[4](**) This attitude towards the Riemann-Roch-Grothendieck theorem are manifested par-

statement connecting them in the analytic context, nor to think to look for an analogous statement in the general context of schemes. In the same way, this attitude prevented him from unearthing the fruitful viewpoint on $\mathcal{D}$-modules in studying the cohomology theory of algebraic varieties, which followed too naturally from a circle of ideals that needed to be buried; neither was he able to recognize Mebkhout's fertile work for years on end, a work which had been successful where he had failed.

figure out a citation

Note 87$_2$ (May 31) That was also the year when I gave my Bourbaki talk on the rationality of L-functions, heuristically using Verdier's result (mostly the expected form of local terms in the case at hand) thirteen years before
p. 368
Illusie proved it upon Deligne's request. I seem to recall that the super-general formula Verdier showed me, which came as a surprise, was proven using the "six operations" formalism in a few lines - it was the kind of formula for which writing it was (nearly) proving it! If there was any "difficulty", it could only have been regarding the verification of one or two compatibility conditions[5](*). Furthermore, both Illusie and Deligne knew full well that the proofs I had given during the seminar regarding various explicit trace formulas **were complete**, and that they did not depend in any way upon Verdier's general formula, the latter having only played the role of a "catalyst" inciting me to formulate and prove trace formulas in the most general possible settings. Both showed a patent lack of good faith at this occasion. In the case of Deligne, this was already clear to me while writing the note "Clean slate" (n^{o}67) - but it was probably not so clear to an uninformed reader, or to an informed reader who renounced the use of their sane faculties.

(June 6) As for Illusie, he fully played along in trying to muddy the waters so as to project the appearance of a hyper-technical oral seminar which did not even provide complete proofs of all the results, and notably of the trace formulas. These were nonetheless proven (for the first time) in 1965/66, and this was also the place where Deligne had the privilege to learn about them and
p. 369
about all of the delicate machinery that accompanies them[6](**).

ticularly clearly in the "Funeral Rite"; see the note "The Funeral Rite (1) - or the compliments", n^{o}104.

[5](*) (June 6) It furthermore seems that the initial proof of the Lefschetz-Verdier formula via the biduality theorem (now promoted to "Deligne's theorem") depended on a hypothesis regarding the resolution of singularities, which Deligne was able to circumvent in the case of schemes of finite type over a field. This constituted a good opportunity to turn the situation to their advantage, by giving the impression that SGA 5 is relying to the (sic) seminar SGA $4\frac{1}{2}$ which "precedes" it (and which was indeed published earlier!).

[6](**) In the second paragraph of the Introduction to the volume titled SGA 5, Illusie presents the three exposés III, III B, and XII, on the Lefschetz formula in étale cohomology, as the "heart of the seminar", even though it is also said in the introduction to exposé III B that (contrary to reality) "this exposé does not correspond to an oral talk from the seminar", while in the introductions to both exposés III and III B he tries to give the impression that they rely on SGA $4\frac{1}{2}$, with exposé III being presented as "conjectural"!! In reality, the totality of the seminar SGA 5 was technically independent of the exposé III (Lefschetz-Verdier formula), which acted as heuristic motivation, and exposé III B was nothing but the "hole" (exposé

This reminds me that I naturally had taken the time to prove the Lefschetz-Verdier formula during the seminar - this was the least I could do, and it provided a particularly striking application of the local-global duality formalism which I was setting out to develop. A question recently occurred to me as to why on earth Deligne and Illusie had chosen to cover the theorem of their good friend Verdier, whose name the result carried, similarly to the derived and triangulated categories which he never took the time to develop in full (or at least to make such developments available to the public), when there were ten or so other exposés begging to be written by one of my students, so that they were not short of choices when it came to naming the technical "obstacle" to the publication of SGA 5. Therein lies a **challenge** of sorts in the absurdity displayed (or in the collective cynicism exhibited by my ex-cohomology students, whom I consider to have been unified in this operation-massacre) which reminds me of the "weight-complex" brilliantly invented by Verdier in the preceding year (see the note with the same name, n^{o}83), as well as (in the iniquitous register) the choice by Deligne to call "perverse" the sheaves which should have been called "Mebkhout sheaves" (see the note "Perversity", n^{o}76). I see all of these inventions as acts of dominance and contempts towards the mathematical community as a whole - as well as a **bet** which had visibly been won until the unexpected reappearance of the deceased, almost appearing as the only person awake amidst a community in slumber...

Note 87$_3$ (June 5) In light of this summary of the massacre, one can fully appreciate the following declaration of Illusie, written on line 2 of his introduction to the volume titled SGA 5:

> "Compared to the primitive version, the only important changes regard exposé II [generic Künneth formulas], which is not reproduced, as well as exposé III [Lefschetz-Verdier formula], which has been entirely rewritten and augmented by an appendix numbered III B[7](*). Except for some detail modifications and added footnotes, the other p. 370 exposés were left **tel quel**" (emphasis mine).

Here as elsewhere, Illusie complaisantly echoes one of his ineffable friend's charades, namely that the existence of SGA $4\frac{1}{2}$ "will enable the publication of SGA 5 **tel quel** in the near future" (see the note "Clean slate", n^{o}67). Illusie tries his best throughout his exposés and introductions to add credit to this

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XI) created by Bucur's moving, which turned into a welcomed excuse for this additional dismemberment.

In order to add to the impression of a seminar of "technical digressions" (whispered to him by his friend Deligne), Illusie took care to remove the introductory exposé in which I had brushed a preliminary outline of the principal great themes which were to be developed during the seminar - an outline in which trace formulas only account for a small (which is of particular importance due to their arithmetic implications, towards the Weil conjectures). For an overview of these "great themes", see the sub-note n^{o}87$_5$ later.

[7](*) With this appendix presented as belonging to the "heart of the seminar"! (see preceding footnote.)

imposture (the fact that SGA 5, where he and his friend first learned about their trade, depends on the pirate-volume SGA $4\frac{1}{2}$ which consists of bits and pieces gleaned or pillaged over the course of the twelve following years) by generously sprinkling references to SGA $4\frac{1}{2}$ at every corner of the volume...

The final word comes from Deligne (as is appropriate), who wrote to me a month ago (on May 3rd) in response to a laconic request for information (for more on this topic see the beginning of the note "The Funeral Rites", n^{o}70):

> "In summary, the fact that seven years had elapsed since you last did mathematics [?!] at the time when SGA $4\frac{1}{2}$ appeared corresponds [?] to the long delay to which the edition of SGA 5 was subjected, **as it was too incomplete to be usefully published tel quel.**
>
> In the hope that you will approve of these explanations."

If I did not "approve" of these explanations, they will have at least edified me...

Note 87$_4$ (June 6) Now might be a good time to list the principal themes that were developed during the oral seminar, of which the published text only paints a disjointed picture.

I) Local aspects of the theory of duality, whose key technical ingredient is (as in the coherent case) the theorem of biduality (together with a theorem of "cohomological purity"). I am under the impression that the geometric meaning of the latter theorem as a form of local Poincaré duality has been entirely forgotten by my ex-students[8](*), even though I had explained it during the oral seminar.

II) Trace formulas, including "non-commutative" trace formulas which were subtler than the usual trace formulas (where both sides are integers, or more generally elements of the coefficient ring - such as $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}_l$, or even $\mathbb{Q}_l$) in that
p. 371
they take values in the group algebra of a finite group acting on the scheme under consideration, with coefficients in a suitable ring (such as the ones listed in the previous parenthesis). This generalization came naturally from the fact that even in the usual Lefschetz-type formulas for "twised" sheaves of coefficients, one was led to replace the initial scheme with a Galois covering (possibly ramified) so as to "untwist" the coefficients, while keeping track of a Galois group action. In this way, "Nielsen-Wecken"-type formulas are naturally introduced into the schematic context.

III) Euler-Poincaré formulas. This consisted of a detailed study of an "absolute" formula for algebraic curves using Serre-Swan modules (generalizing the case of tamely ramified coefficients which gave rise to the more naive Ogg-Chafarévitch-Grothendieck formula), as well as new and profound conjectures regarding a "discrete" Riemann-Roch formula, one of which reappeared seven years later in hybrid form under the name of "Deligne-Grothendieck conjecture" and was proven by MacPherson via transcendental methods (see note n^{o}87$_1$).

[8](*) Upon verification, its turns out that this geometric interpretation was at least preserved in Illusie's write-up.

The comments which I had inevitably made regarding the profound connections between these two themes (Lefschetz formulas and Euler-Poincaré formulas) have also disappeared without a trace. (As was usual, I had left all of my handwritten notes to the sic volunteer writers, so that I do not have access to any written trace of the oral seminar, even though I once naturally had a complete if at times succinct set of handwritten notes.)

IV) Detailed formalism of the homology and cohomology classes associated to a cycle, following naturally from the general duality formalism and from the key idea of working with cohomology "with supports" in the cycle under consideration, using the theorems of cohomological purity.

V) Finiteness theorems (including generic finiteness theorems) and generic Künneth theorems for cohomology with arbitrary support.

The seminar also developed a technique for passing from torsion coefficients to ℓ-adic coefficients (exposés V and VI). This was the most technical part of the seminar, with the latter generally worked with torsion coefficients, with the possibility to "take the limit" in order to deduce the corresponding l-adic results. This viewpoint was a temporary trade-off, awaiting for Jouanolou's p. 372
thesis (unpublished to this day) which was to develop the appropriate formalism to work directly in the ℓ-adic settings.

In this list of the main "themes", I am not including the computations for certain classical schemes and the cohomological theory of Chern classes, which Illusie highlights in his introduction as "one of the most interesting themes" of the seminar. As the program was already packed, I did not think it was necessary to spend time on these computations and this construction during the oral seminar, given that one needed only follow, almost verbatim, the arguments which I had produced ten years earlier in the context of Chow rings for the purposes of the Riemann-Roch theorem. It was clear on the other hand that these ideas needed to be included in the written seminar, so as to provide a useful reference to the user of étale cohomology. Jouanolou had taken charge of this work (exposé VIII), and instead of seeing it as a service done for the mathematical community as well as an opportunity to learn essential basic techniques for his own use, he must have viewed it as a chore, since the write-up lagged for years[9](*). The same probably held true for his thesis, which was to forever remain a phantom reference like Verdier's... The "taking the limit" section should not be included as one of the "main themes" of the seminar either, in that it does not correspond to a particular geometric idea. Rather, it reflects a technical complication particular to the context of étale cohomology (distinguishing it from the transcendental contexts), namely the fact that the main theorems in étale cohomology pertain in the first place to **torsion** coefficients (prime to the residual characteristics), and that in order to obtain a theory corresponding to rings of coefficients in zero characteristic (as needed for the Weil conjectures), one needs to take the limit over the rings of coefficients $\mathbb{Z}/\ell^n\mathbb{Z}$ in order to obtain "ℓ-adic" results.

[9](*) (June 12) After reading the exposé in question, I convinced myself of a perfect complicity between Jouanolou and my other cohomology students.

Having described all of this, the only main theme of the oral seminar which seems to appear in complete form in the published text is theme I. Themes IV and V have downright disappeared; they have been incorporated into SGA $4\frac{1}{2}$ with the added benefit of being able to make numerous references to the latter, so as to give the impression that SGA 5 depends on a text of Deligne framed as an earlier work. Themes II and III appear in the published volume in a mutilated form, always maintaining the same counterfeited appearance of a dependency on the text SGA $4\frac{1}{2}$ (which in reality was issued in its entirety from the mother-seminars SGA 4 and SGA 5).

The remains

p. 373 **88** (May 16) The seminars SGA 4 and SGA 5 (which in my eyes constitute a **unique** "seminar") taken together develop from scratch the **language** of topoi, as a powerful instrument of synthesis and discovery, as well as the fully sharpened **tool** of perfect efficacy that is étale cohomology - whose essential formal properties were henceforth better understood than even the cohomology theory of ordinary spaces[10](*). Among the projects that I carried out in full, this work represents the most profound and innovative contribution that I have made to mathematics. At the same time - and without intending to, in the sense that things simply followed their self-evident natural course at every moment - this work represents the vastest technical "tour de force" that I have accomplished in my mathematical career[11](**). In my eyes, these two seminars are inextricably linked. They represent in their unity both the **vision** and its **tools** - namely, topoi and the complete formalism of étale cohomology.

Even though the vision remains rejected to this day, the tools have in their twenty years of existence brought about a profound renewal of algebraic geometry in its most fascinating aspect - namely, its "arithmetic" aspect, apprehended by means of an intuition as well as a conceptual and technical toolkit of a "geometric" nature.

It was surely not the intention to suggest that his cohomological "digest" was **anterior** to SGA 5 which motivated Deligne to deceivingly call it SGA $4\frac{1}{2}$ - after all, he might as well have called it SGA $3\frac{1}{2}$! I see the "SGA $4\frac{1}{2}$ operation" as an attempt to frame the work when all of his own work is issued (that very work from which he cannot detach himself!) - a work whose unity and depth is clearly visible in SGA 4 and (the true) SGA 5, as a **divided** entity (just as he himself is divided...), **cut in half** by the violent insertion of a foreign and
p. 374 disdainful text; the latter pretends to be the living heart, the quintessence of a

[10](*) And that is so even if we restrict ourselves to the spaces closest to "varieties", such as spaces admitting a triangulation.

[11](**) Some of the difficult or unexpected results were obtained by others (Artin, Verdier, Giraud, Deligne), and certain parts of my work were carried out in collaboration with others. This doesn't affect (at least in my mind) how strongly I feel about the position of this work in the totality of my mathematical work. I believe I come back to this point in more details in an appendix to the "Esquisse Thématique", dotting the i's where it had visibly become necessary.

school of thought and of a vision in which it in truth played no part[12](*), while the surrounding two "quarters" appear as vaguely grotesque appendices of sorts, a hodgepodge of "digressions" and "technical complements" to a work framed as central and essential, written by Deligne, with my name being graciously included (prior to its final burial) in the list of "collaborators"[13](**).

As chance would have it, these "remains left at their mercy", this "unfortunate seminar" always brushed to the side by its "writers" and left in the hands of my cohomology students after my departure, was **not just any** part of my masterwork! It was neither SGA 1 and SGA 2 (where I quietly developed the tools which I was yet to discover would constitute the two indispensable technical auxiliaries to the "take-off" of my main work to come), nor SGA 3 (where my contribution consisted chiefly in a series of scales and arpeggios - sometimes difficult - meant to refine the theory of schemes "in all directions"), nor SGA 6 (in which I systematically developed my ten-year old ideas around a Riemann-Roch theorem and the formalism of intersection theory), nor SGA 7 (which, through the logic internal to the reflection, follows from the wielding of a single central tool, namely the mastery of cohomology). It truly was the **masterpiece** of my work, whose write-up had remained incomplete (by their own fault...), which I had left, at least in part, in the hand of my cohomology students. They decided to destroy this masterpiece of my work, appropriating the pieces while forgetting the unity which gives them their meaning and beauty, as well as their creative virtue (90).

It is also not a coincidence if, equipped with a motley of tools whilst denying the spirit and vision that brought them into being, they were unable to discern the innovative work which was being reborn in front of their indifferent and disdainful gaze. Neither is it an accident that after six years, when the new tool was at last accessed by Deligne, there was a unanimous agreement to bury the person who had created it in solitude - namely, Zoghman Mebkhout, p. 375
the posthumous student of the disavowed master! Finally, it is not by chance that following Deligne's initial impetus (which led him over the span of a few years toward the start of a new version of Hodge theory and the proof of the Weil conjectures), and despite his astounding abilities as well as my cohomology students' great means, I nowadays witness a "morose stagnation" in their domain, despite its prodigious richness and the multitude of things yet to be done. This is not surprising in light of the fact that for nearly fifteen years the main source of inspiration as well as some of the "great open problems"[14](*), which

[12](*) This school of thought had reached full maturity, both in terms of key ideas and essential results, before Deligne entered the stage as a young man wishing to learn algebraic geometry and cohomological methods in my contact, between 1965 and 1969.

(May 30) See on this topic the note "One of a kind", n^{o}67'.

[13](**) See the notes "Green light", "The reversal", n^{o}s 68, 68'.

[14](*) This "main source of inspiration" is of course the "yoga of motives". It has been active in Deligne alone, who kept it for his only "benefit" in a restricted form lacking a large part of its strength, in that he rejected some of the essential aspects of this yoga. Among the "great open problems" inspired by this yoga, many of which were ignored or discretely discredited, I can state right away (however much of an outsider I may currently be) the standard conjectures, as well as the development of the "six functor formalism" for all of the

have been present in the background and relevant at every step of the way, have been carefully circumvented and hidden away, in line with the treatment of the messengers of the person to be buried.

... and the body

89 (May 17) I view the philosophy and viewpoint which lived within me and which I thought I had communicated as a breathing, healthy, and harmonious body, moved by the power of renewal of all living things: the power to conceive and to engender. This body has now turned into **remains**, divided between several people - one of the limbs was duly embalmed and now features as somebody's trophy; another has been skinned and now serves as a club or boomerang for somebody else; yet another, who knows, may have been used for some home cooking (why not!) - everything else may as well be left to rot in a landfill...

Such is the scene that was eventually revealed to me, admittedly presented in colorful terms, but which I nonetheless believe accurately express a certain reality. The club may occasionally be used to fracture a few skulls[15](*) - but none of these disparate pieces, be they trophies, clubs, or homemade soups, will ever inherit the simple and obvious power of the living body: that of the loving embrace at the origin of new beings...

(May 18) This picture of the living body whose "remains" were scattered to p. 376 the four winds must have formed within me during this past week. The quirky form in which it materialized itself through my typewriter should not be taken as an indication that this image is in any way a (slightly morbid) **invention** or as a burlesque improvisation spontaneously generated by the needs of a speech. The image is expressing a **reality** which was profoundly felt at the time when it took on a material form through my writing. I must have already grappled with this reality in bits and pieces during the fourteen years that elapsed since my "departure", or perhaps even earlier than that. These fragments of information were at first registered at an informal level, while my distracted attention was absorbed in something else; these pieces were nonetheless all congruent, and they must have assembled into a coherent image at some deeper level, even though I was too busy to pay attention to it. This image grew considerably richer and more precise over the course of the reflection which has been taking place since late March, beginning six or seven weeks ago. More precisely, the disparate pieces of information slowly assembled into **another** image which appeared at the more superficial level of the examining and probing mind through a process which may appear to be independent from the presence of the first image, lodged in a deeper layer. This conscious process culminated six days ago in my sudden vision of the "slaughter" that took place - I could sense the "wind", the "odor" of **violence** for what is I believe the first time in the entire reflection[16](*). This

usual kinds of coefficients, more or less close to "motives" themselves (the latter playing the role of "universal" coefficients, giving rise to every other). Compare with my comments on this topic in the note "My orphans", n^{o}46.

[15] (*)

[16] (*)

was also the time when the awareness of a living and harmonious body which had been “slaughtered” must have started rising to the surface of my consciousness; whilst the deeper and diffuse image began surfacing as well, adding to the image in the making a carnal dimension which thought alone cannot produce.

This “carnal” aspect manifested itself once more in a dream yesterday night - and it is under the impulse of this dream that I am choosing to return to the lines I wrote yesterday. In this dream, I had rather deep wounds at several locations of my body. The most glaring were cuts in my lips and inside my mouth, which were bleeding profusely as I rinsed my (blood-reddened) mouth abundantly in front of a mirror. There were also cuts in my stomach, bleeding just as profusely. One of them was particularly severe, and blood was flowing as p. 377 if out of an artery (the Dreamer did not bother with anatomical accuracy). The thought even occurred to me that I may be done for if I kept bleeding at this rate; I applied my hand to the wound and huddled so as to stop the bleeding. The maneuver was successful: the bleeding slowed down until the blood clotted and formed a large crust. Later, I carefully lifted the crust to find that a delicate cicatrization had already begun. I also had a cut at one of my fingers, which was already wrapped in a large bandage...

I do not intend to launch into a more careful and detailed description of this dream, nor to thoroughly examine it (here or elsewhere). What the dream “as is” already reveals to me with striking power is that the “body” about which I was speaking yesterday as something detached from my being, like a child I would have conceived and who later left off to trace his or her own path in the world, had in fact been an intimate piece of my person: it is **my** body, in flesh and blood, endowed with a life force allowing it to survive and recover from profound wounds. And my body is in turn, doubtlessly, the thing in the world to which I am most profoundly and inextricably linked...

The Dreamer disagreed with the image I had of the “slaughter” and of the sharing of the remains. The image I had in mind depicted a set of intentions and dispositions in **others** which I had sharply perceived, rather than the way in which I myself experienced this aggression, this mutilation to which I was tied through the thing I held close. The Dreamer allowed me to realize the extent to which I was still tied to this mutilation. This aligns with what I perceived (less strongly) in the note “What goes around comes around - or spilling the beans” (n^o73), where I try to articulate the feeling of a “profound link between the conceiver of something and the thing itself” which had appeared during that day’s reflection. Prior to the reflection of April 30^{th} (just three weeks ago), and throughout my entire life, I had pretended to ignore this link, or at least to minimize it, thereby following the path pre-determined by the norms of the time. To become preoccupied with a work that is no longer in one’s hands, and especially to wonder whether one’s name remains somewhat attached to them, is perceived as a form of pettiness and narrowness - all the while deeming it perfectly natural for someone to be profoundly affected by the fact that a child one raised (and believes to have loved) suddenly chooses to repudiate the name he or she was given at birth.

The heir

p. 378

90 (May 18)

The co-heirs

91

Note 91_1

Note 91_2

Note 91_3

Note 91_4

… and the chainsaw

92

Part III

Third part: The Burial (II) - or the Key to the Yin and the Yang

Chapter 16

The defunct (who still lives...)

16.1 The incident - or body and mind

p. 421

98 (September 22) The latest Burial note (with the exception of a few footnotes) dates from May 24 - that is, four months ago. The two weeks that followed, up until June 10, were mostly devoted to re-reading, completing, and adjusting the already written notes here and there, putting aside a visit by Zoghman Mebkhout for a day or two, having come to read the Burial notes in their entirety and to share his comments with me, before I was to entrust him with the typing. I thought that the definitive manuscript would be ready around early June, and that it would be typeset and printed before the summer holidays (although that might have been overly optimistic...). I liked the idea of sending out my “five hundred page letter” before the commotion of the start of the holidays!

In actuality, the text of the Burial is still not complete as I am writing these lines: today as was the case four months ago, two or three last notes remain to be written - plus an additional one[1] whose need started being felt since then: this is the note that I am now writing, and which is intended to serve as a brief summary of what has happened since then.

On June 10^{th}, a new unforeseen event intervened during the writing of Récoltes et Semailles - a process that had already been full of unexpected turns: I fell ill! A stitch suddenly appeared (catching me completely unaware) and peremptorily forced me to lie in bed, leaving me with no other choice. The very act of standing or sitting up became arduous, and only when lying down could I feel relatively at ease. It was very silly, and occurred right at a time when I was about to conclude and file away an urgent project! There was no way for me to use the typewriter while lying down, and handwriting in this position was not

[1] (September 23) In fact, it appears that this “note” eventually split into three distinct ones (n^os 99-101)

much of a sinecure either...

It took me nearly two weeks to face the obvious, all the while trying to continue working against all odds: my body was exhausted and insistently demanded complete rest, but it kept on falling on deaf ears.

I had such a difficult time hearing my body's plea because my mind had remained fresh and alert throughout, eager to carry its momentum forward, acting as if it had a life of its own, entirely independent of the rest of my body. It was so fresh and nimble that it struggled mightily to accept my body's need for sleep, pushing me to the limits of exhaustion by persistently refusing the takeover of sleep, the annihilator of the spinning mind!

16.2 The trap - or ease and exhaustion

99 (September 23) I had to force myself to stop working yesterday night, so as to avoid following my natural course until two or three a.m. and thereby becoming caught in a cycle that I know only too well. I felt fresh and receptive, and had I followed my natural inclination, I could have kept going until sunrise! The trap laid by intellectual work - at least when such work is pursued with passion, in a subject in which we eventually feel like fish in water, as a result of a prolonged familiarity - is that it is so incredibly **easy**. We pull, and pull, and it keeps coming; at most, we occasionally feel some amount of effort, some friction, indicating a little bit of resistance...

Yet, I remember the persistent feeling of heaviness and gravity which I used to feel during my early years as a mathematician, a feeling which I had to surmount through obstinate effort, only to be left with a sensation of fatigue. This mostly corresponded to a period in my life during which I was working with an incomplete, or even inadequate, toolbox; as well as to a later period during which I had to more or less laboriously acquire new tools "left and right", under the pressure of an environment (essentially, that of the Bourbaki group) where they were routinely used - and I had to do so even though I did not perceive these tools' "raison d'être" until a later point, up to several years later in some cases. I have spoken on occasion about these rather arduous years (see "The welcome stranger" s.9, and "One hundred irons in the fire - or: there is no use in being stuck!", note n^{o}10) in the first part of Récoltes et Semailles. This period
p. 429 mostly took place between the years 1945 and 1955, coinciding with my time as a functional analyst. (It seems to me that the students whom I later supervised, between 1960 and 1970, experienced much less resistance than I did regarding having to learn new things without sufficient motivation and to absorb notions and techniques based on the elders' authority, taken on faith - in fact, I did not perceive any resistance at all.)

Coming back to my original point, it was from the year 1955 onward that I started having the impression that I was "flying" - in that doing mathematics felt like play, unencumbered by any sensation of effort - the same way my elders did, exhibiting a quasi-miraculous ease for which I had once envied them, considering that such facility was out of reach for my modest and heavy person! Today, I understand that such a "facility" is not a privilege one acquires as some exceptional gift (as seemed to be the case for certain individuals, at a time when such a "gift" appeared to be entirely absent in my case); rather, this facility emerges on its own as the fruit of the union of a passionate interest for a given subject (such as mathematics) together with a more or less lengthy familiarization with said subject. If a "gift" does play a role in the appearance of such ease, it is through the length of time it takes to reach perfect fluency in one's work on a given subject[2](*), something which can vary from one person to the next (as well as from one occasion to the next for the same individual, admittedly...).

The fact remains that the more time passes - the more years I have accumulated as a mathematician, the more I experience this feeling of "ease" when doing mathematics - the sensation that things are pleading to be revealed to us, if we only take the time to look at them and inspect them a little bit. This ease is not a matter of technical virtuosity - it is clear to me that, in this respect, I am in much worse shape than I was in 1970, at the time when I "left math". Since then, I have mostly had the occasion to unlearn what I had learned, and I only "do math" sporadically, on my own, in a very different spirit and on themes different from the ones that I used to work on (at least at first glance). I am also not saying that I would need only assign myself a given famous problem (such as, say, Fermat's last theorem, the Riemann hypothesis, or the Poincaré conjecture) so as to launch on a geodesic path towards its solution, in a year or two, or even[3](*) three! The ease that I am speaking of does not apply to a process whereby we set out to reach a given **goal**, fixed ahead of time - such as proving some conjecture or finding a counter-example... Rather, it applies to excursions into the unknown, following a direction which some obscure instinct tells us will be fruitful, and supported by the inner confidence, never misplaced, that each day and each hour of our journey shall bring us its own fresh harvest of new understandings. **Which** understanding we shall reach in the morrow, or even in the following hour, is something we can indeed anticipate - yet, it is the constant rectification of this "anticipation", and the suspense that results, which constantly draw us forward, as the very things which we are probing are inviting us to come closer. What is eventually understood always surpasses what had been anticipated in terms of precision, flavor, and richness; at which point the known readily turns into a new starting point and as raw material

The footnote seems to be repeated twice - can we doube check this with any other edition of ReS?

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[2](*) I nonetheless know several mathematicians having each made profound contributions, yet having never seemed to produce the impression of ease and "facility" which I am writing about - they seem to be constantly grappling with a ubiquitous gravity, which they must surmount with effort at every step. For one reason or another, the aforementioned "natural fruit" did not "appear on its own" for these eminent figures the way it was supposed to. This goes to show that not every union is destined to bear the fruits that we may have expected...

[3](*) I nonetheless know several mathematicians having each made profound contributions, yet having never seemed to produce the impression of ease and "facility" which I am writing about - they seem to be constantly grappling with a ubiquitous gravity, which they must surmount with effort at every step. For one reason or another, the aforementioned "natural fruit" did not "appear on its own" for these eminent figures the way it was supposed to. This goes to show that not every union is destined to bear the fruits that we may have expected...

allowing us to form a renewed anticipation, causing us to launch further into the unknown that is avidly awaiting to become understood. In this game of discovery, the **direction** which we follow at every moment is known, but the **goal** is forgotten - supposing we had even started with a goal which we intended to reach. This "goal" actually constituted a **starting point**, issued from some combination of our ambition and ignorance; it served its purpose by motivating "the boss", fixing an initial direction, and setting the game in action; but the game itself has no use for a goal. As long as the journey we undertake lasts for longer than a day or two, so that we are in it for the long haul, the things which will be revealed to us over the course of days and months, and the places we will be reaching at the term of a series of unknown twists and turns, are a complete mystery to us as travelers; in fact, they form such a distant and inaccessible mystery so as to be immaterial! If the traveler does look out at the horizon, it is not to guess what the unpredictable endpoint shall be, and even less so to decide on a chosen endpoint, but rather to take stock of where he currently stands, and to choose, among the directions available to him in the continuation of his journey, the one which he feels to be most burning...

Such is the "incredible ease" which I alluded to earlier, regarding intellectual work proceeding entirely in an intellectual direction, such as mathematics. It is neither **held back** by inner **resistance**[4](*) (as is often the case with meditation
p. 431
in the way that I practice it) nor by a **physical effort** that needs to be supplied and which would generate a feeling of fatigue, eventually culminating in an unequivocal stop signal. **Intellectual** effort (if we can even speak of an "effort", once we have reached a stage where the only remaining "resistance" is the time factor...) does not seem to generate either intellectual or physical tiredness. More precisely, if physical "tiredness" does occur, it is not experienced as such, other than through occasional soreness resulting from having remained in a fixed sitting position for too long, or other incidental annoyances of this kind. Such soreness can easily be remedied by simply changing position. Horizontal positions have the unfortunate virtue of alleviating the soreness, so as to enable a continuation of the intellectual work instead of some much needed sleep!

There exists nonetheless, as I eventually came to realize, a subtler and more insidious physical "tiredness" that muscular or nervous tiredness, which manifests itself through an inescapable need for rest and sleep. The word "exhaustion" (rather than "tiredness") would be more accurate in this case, although it should be understood that I am not using this word in its common sense, namely that of an extreme feeling of fatigue, manifesting itself notably through a sensation of great effort accompanying one's attempt to even stand up, walk a few steps, etc... Rather, I am talking about an "exhaustion" of the body's energy for the benefit of the brain, manifesting itself through a gradual degradation of the body's general "tonicity" and of its level of vital energy. By exhaustion as

[4](*) I nonetheless know a remarkably gifted mathematician whose relationship to mathematics is typically conflictual, impeded at every step by powerful resistances such as the fear that a given expectation (in the form, say, of a conjecture) would turn out to be false. Such resistances can sometimes culminate in a state of full-out intellectual paralysis. Compare this with the preceding footnote.

a result of excessive intellectual work I mean: work that is not compensated by sufficient physical activity, the latter of which generates bodily fatigue and a need for rest. This type of exhaustion is gradual and **cumulative**. Its effects must depend on both the **intensity** and the **duration** of the intellectual activity over the course of a given period. At the level of intensity at which I pursue intellectual work, and at my current age and constitution, it seems that the cumulative exhaustion in question reaches a critical and dangerous threshold after one or two years of uninterrupted activity uncompensated by a regular physical activity

In a sense, the "facility" about which I am talking is only apparent. Intense intellectual activity clearly requires considerable energy: and this energy has to be drawn from somewhere, so as to be "spent" on one's work. It appears that this "somewhere" pertains to the body, which "endures" (or rather **forks out**) as much as it can the (sometimes vertiginous) expenses which the head indulges in carelessly. The normal path towards the recovery of the energy provided by the body is sleep. But when the brain's bulimia begins impinging on sleep, one p. 432
begins digging into the energy-capital without renewing it. The trap and the danger of the "ease" of intellectual work is that it endlessly incites us to cross this threshold, or to remain past it once it has been crossed, and that all the while this crossing is not brought to our attention by the usual and clear-cut signs of tiredness, or even exhaustion. I now realize that a great vigilance is required so as to detect the moment at which one is approaching and crossing the threshold in question, when one's entire being is engaged in the pursuit of a thrilling adventure. To be able to perceive this energy shortage in one's body requires a state of attentiveness which I often lacked and which few people have. I even doubt that such a state of communion between one's conscious activity and one's body could blossom in anybody during a period that is dominated by a purely intellectual activity and which excludes all physical activity.

Many intellectuals by profession instinctively feel the need for such physical activity, and adapt their life in consequence: gardening, handiwork, mountaineering, boating, sports... Those who, like me, neglected this healthy instinct for the benefit of an overly invasive passion (or an overpowering lethargy), eventually have to pay the price. I have had to pay for my neglect three times over in the past three years, and each time I did so without complaining, or rather with indebtedness, realizing with each episode of sickness that I was only harvesting the fruits of my own negligence, and, moreover, that the episode brought with it a lesson that it alone could provide me. Perhaps the main lesson that the latest episode, which just ended, brought with it, is that it is high time for me to take the lead and make such wake-up calls unnecessary - or, more concretely: that it is high time for me to tend to my garden!

Part IV

Fourth part: The Burial (III) - or the Four Operations