3-3 Proving Lines Parallel

Teks Focus

TEKS (5)(C) Use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(F), (6)(A)

Vocabulary

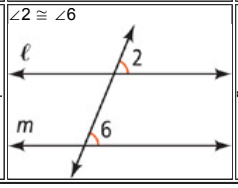
- **Flow proof** – a form of proof in which arrows show the logical connections between the statements
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.
- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim

ESSENTIAL UNDERSTANDING

You can use certain angle pairs to decide whether two lines are parallel.

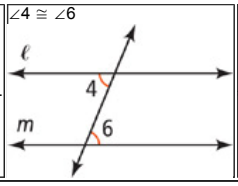
take note

Theorem 3-4 Converse of the Corresponding Angles Theorem

Theorem	If …	Then …
If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.	$\angle 2 \cong \angle 6$ 	$\ell \parallel m$

You will prove Theorem 3-4 in Lesson 5-6.

Theorem 3-5 Converse of the Alternate Interior Angles Theorem

Theorem	If …	Then …
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.	$\angle 4 \cong \angle 6$ 	$\ell \parallel m$

For a proof of Theorem 3-5, see the Reference section on page 683 .

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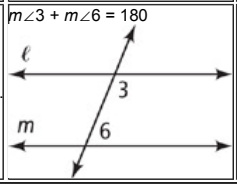
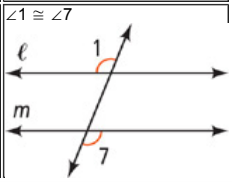
take note

Theorem 3-6 Converse of the Same-Side Interior Angles Postulate

Theorem	If …	Then …
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.	$m\angle 3 + m\angle 6 = 180$ 	$\ell \parallel m$

You will prove Theorem 3-6 in the Got It for Problem 2.

Theorem 3-7 Converse of the Alternate Exterior Angles Theorem

Theorem	If …	Then …
If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 7$ 	$\ell \parallel m$

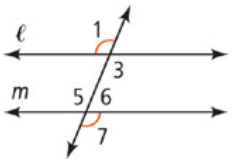
For a proof of Theorem 3-7, see Problem 1.

Problem 1

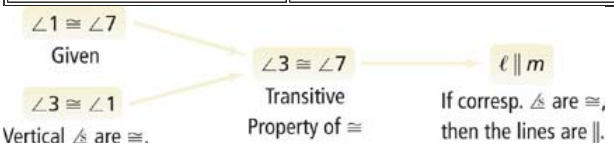
Proof Using a Flow Chart to Prove Theorem 3-7

Given: $\angle 1 \cong \angle 7$

Prove: $\ell \parallel m$



Know	Need	Plan
• $\angle 1 \cong \angle 7$ From the diagram you know • $\angle 1$ and $\angle 3$ are vertical • $\angle 5$ and $\angle 7$ are vertical • $\angle 1$ and $\angle 5$ are corresponding • $\angle 3$ and $\angle 7$ are corresponding	One pair of corresponding angles congruent to prove $\ell \parallel m$	Use a pair of congruent vertical angles to relate either $\angle 1$ or $\angle 7$ to its corresponding angle.

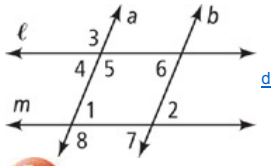


Problem 2

Identifying Parallel Lines

Which lines are parallel if ∠1 ≅ ∠2? Justify your answer.

∠1 and ∠2 are corresponding angles. If ∠1 ≅ ∠2, then *a* ∥ *b* by the Converse of the Corresponding Angles Theorem.



d

Think
Which line is the transversal for ∠1 and ∠2?

Line *m* is the transversal because it forms one side of both angles.

Problem 3

TEKS Process Standard (1)(G)

Determining Whether Lines Are Parallel

The fence gate at the right is made up of pieces of wood arranged in various directions. Suppose ∠1 ≅ ∠2. Are lines *r* and *s* parallel? Explain.

Yes, *r* ∥ *s*. ∠1 and ∠2 are alternate exterior angles. If two lines and a transversal form congruent alternate exterior angles, then the lines are parallel (Converse of the Alternate Exterior Angles Theorem).

Think
How do ∠1 and ∠2 relate to each other in the diagram?

∠1 and ∠2 are both exterior angles and they lie on opposite sides of *t*.

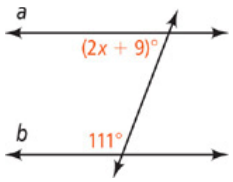
Problem 4

TEKS Process Standard (1)(F)

Using Algebra

Algebra What is the value of *x* for which *a* ∥ *b*?

The two angles are same-side interior angles. By the Converse of the Same-Side Interior Angles Postulate, *a* ∥ *b* if the angles are supplementary.



Think
Work backward. Think about what must be true of the given angles for *a* and *b* to be parallel.

(2*x* + 9) + 111 = 180	Def. of supplementary angles
2*x* + 120 = 180	Simplify.
2*x* = 60	Subtract 120 from each side.
x = 30	Divide each side by 2.

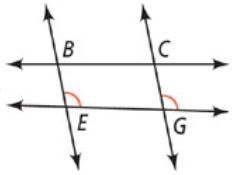
PRACTICE and **APPLICATION EXERCISES**

Which lines or segments are parallel? Justify your answer.

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to PearsonTEXAS.com .

1.



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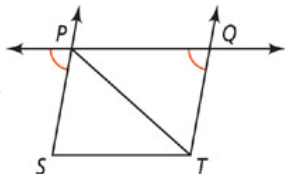

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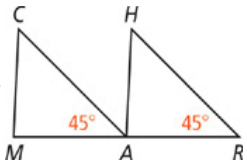

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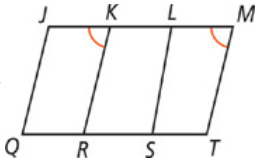
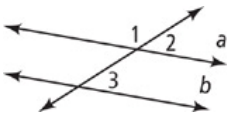


5. Justify Mathematical Arguments (1)(G) Complete the flow chart below.

Given: ∠1 and ∠3 are supplementary.

Prove: $a \parallel b$



∠1 and ∠3 are supplementary.
a. ?

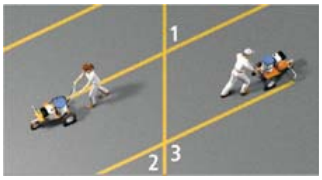
d. ?
Supplements of the same ∠ are ≅.

$a \parallel b$
e. ?

b. ?
Def. of linear pair

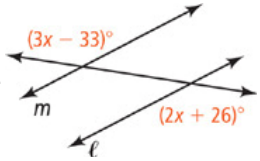
∠1 and ∠2 are supplementary.
c. ?

6. Apply Mathematics (1)(A) Two workers paint lines for angled parking spaces. One worker paints a line so that $m\angle 1 = 65$. The other worker paints a line so that $m\angle 2 = 65$. Are their lines parallel? Explain.



Analyze Mathematical Relationships (1)(F) Find the value of *x* for which $\ell \parallel m$.

7.



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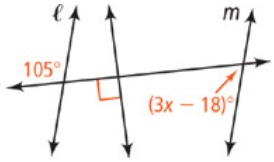


Page 104

Justify Mathematical Arguments (1)(G) Use the given information to determine which lines, if any, are parallel. Justify each conclusion with a theorem or postulate.

9. ∠2 is supplementary to ∠3.

10. ∠1 ≅ ∠3

11. ∠6 is supplementary to ∠7.

12. ∠9 ≅ ∠12

13. $m\angle 7 = 65$, $m\angle 9 = 115$

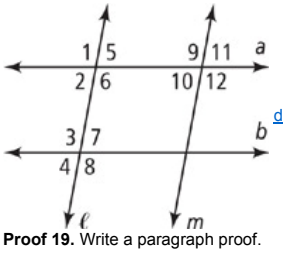
14. ∠2 ≅ ∠10

15. ∠1 ≅ ∠8

16. ∠8 ≅ ∠6

17. ∠11 ≅ ∠7

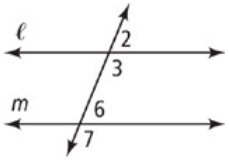
18. ∠5 ≅ ∠10



Proof 19. Write a paragraph proof.

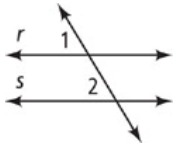
Given: ∠2 is supplementary to ∠7.

Prove: $\ell \parallel m$



20. Explain Mathematical Ideas (1)(G) If the rowing crew at the right strokes in unison, the oars sweep out angles of equal measure. Explain why the oars on each side of the shell stay parallel.

Analyze Mathematical Relationships (1)(F) Determine the value of *x* for which *r* ∥ *s*. Then find *m*∠1 and *m*∠2.



21. $m\angle 1 = 80 - x$, $m\angle 2 = 90 - 2x$

22. $m\angle 1 = 60 - 2x$, $m\angle 2 = 70 - 4x$

23. $m\angle 1 = 40 - 4x$, $m\angle 2 = 50 - 8x$

24. $m\angle 1 = 20 - 8x$, $m\angle 2 = 30 - 16x$

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Page 105

Use the diagram at the right below for Exercises 25–27.

25. Justify Mathematical Arguments (1)(G) If $\angle 1 \cong \angle 7$, what theorem or postulate can you use to show that $\ell \parallel n$?

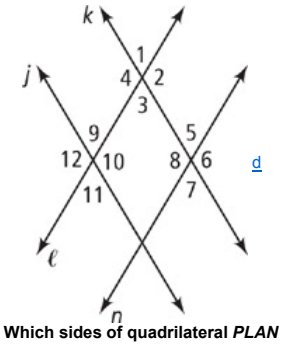
Use a flow chart to write a proof.

Proof 26. Given: $\ell \parallel n$, $\angle 12 \cong \angle 8$

Prove: $j \parallel k$

Proof 27. Given: $j \parallel k$, $m\angle 8 + m\angle 9 = 180$

Prove: $\ell \parallel n$



d

Which sides of quadrilateral *PLAN* must be parallel? Explain.

28. $m\angle P = 72$, $m\angle L = 108$, $m\angle A = 72$, $m\angle N = 108$

29. $m\angle P = 59$, $m\angle L = 37$, $m\angle A = 143$, $m\angle N = 121$

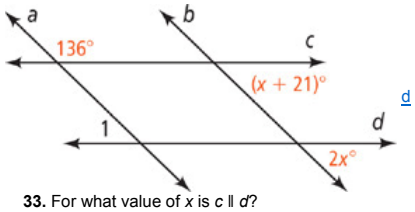
30. $m\angle P = 67$, $m\angle L = 120$, $m\angle A = 73$, $m\angle N = 100$

31. $m\angle P = 56$, $m\angle L = 124$, $m\angle A = 124$, $m\angle N = 56$

Proof 32. Write a two-column proof to prove the following: If a transversal intersects two parallel lines, then the bisectors of two corresponding angles are parallel.

TEXAS Test Practice

Use the diagram for Exercises 33 and 34.



d

33. For what value of x is $c \parallel d$?

A. 21

B. 23

C. 43

D. 53

34. If $c \parallel d$, what is $m\angle 1$?

F. 24

G. 44

H. 136

J. 146

35. Which of the following is always a valid conclusion for the hypothesis? If two angles are congruent, then _?_.

A. they are right angles

B. they share a vertex

C. they have the same measure

D. they are acute angles

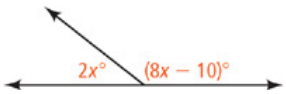
36. What is the value of x in the diagram at the right?

F. $1.\overline{6}$

G. 10

H. 17

J. 19



Page 106