RADIATED EMISSIONS AND SUSCEPTIBILITY

A basic understanding of the fundamentals of electromagnetic radiation is required in EMC design and testing in order to isolate the source(s) of radiated emissions. Given an understanding of which device components are capable of radiating efficiently at a given frequency, we may couple this understanding with the knowledge of the spectral content of the signals located on these components in order to determine the source of the radiated emission. Basic EMC design principles used to minimize radiation can be implemented in the product design phase to prevent radiated emissions problems. If the radiated emissions problem arises in the testing phase of production, EMC "fixes" must be implemented which also require an understanding of the principles of radiation.

When EMC measurements are conducted, the radiation characteristics of the DUT are dictated by the separation of the DUT and the measurement antenna. Specifically, if the measurement antenna is in the *far field* of the DUT, the radiated fields behave according to the inverse distance rule which states that the far field decreases as $1/R$ where R is the separation distance between the DUT and the measurement antenna. The radiated fields do not follow the inverse distance rule in the near field of the DUT. This factor can lead to errors when measured results at one value of R are translated to a different value of R at frequencies where the far field assumption is not valid. In general, we assume that the measurement antenna is always in the far field of the DUT so that we may employ the *far field approximation* in all calculations for radiated fields.

The actual circuit geometry for a particular DUT may be quite complicated and thus require an extensive numerical model to simulate the radiated fields accurately. However, the EMC engineer does not necessarily require a highly accurately model to pinpoint the source of a radiated emission. Nor is a highly accurate model necessary to implement the EMC fix once the problem component has been identified. Therefore, it is useful to develop simple first-order models of commonly encountered geometries such as common-mode and differential mode currents on parallel conductors.

EMISSION MODELS FOR WIRES AND PCB LANDS

A commonly encountered geometry in EMC problems responsible for radiated emissions is that of common-mode or differential-mode currents flowing on parallel wires or on parallel PCB lands. The far fields radiated by this geometry are easily determined using the *pattern multiplication theorem* for antenna arrays.

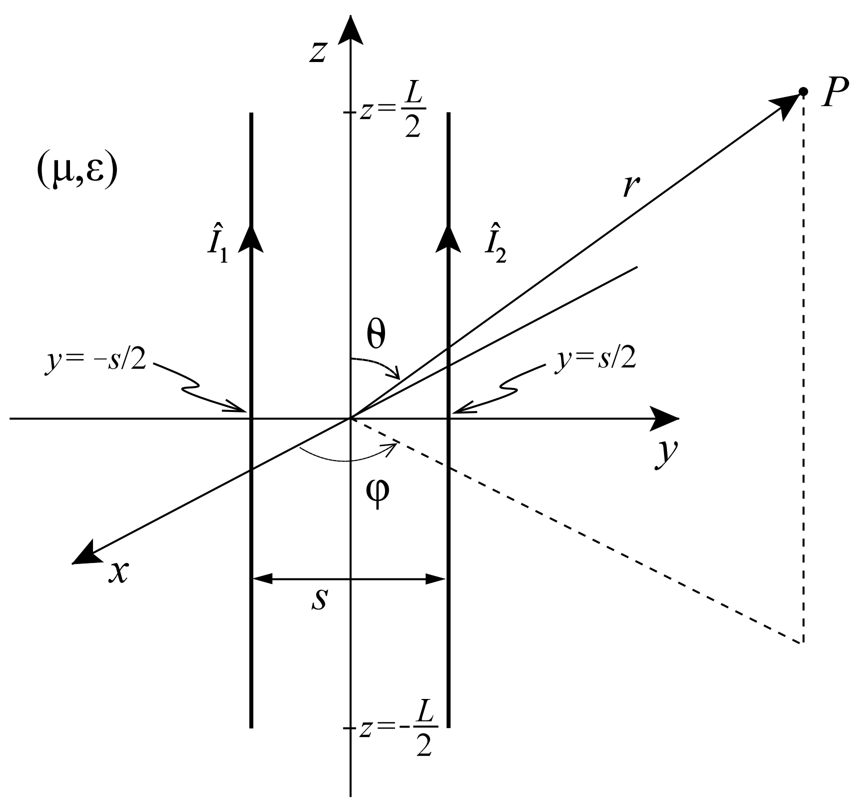
<u>Pattern multiplication theorem</u>

$$\textit{Array far fields} = \textit{Element far fields} \times \textit{Array factor} \; (\mathrm{AF})$$

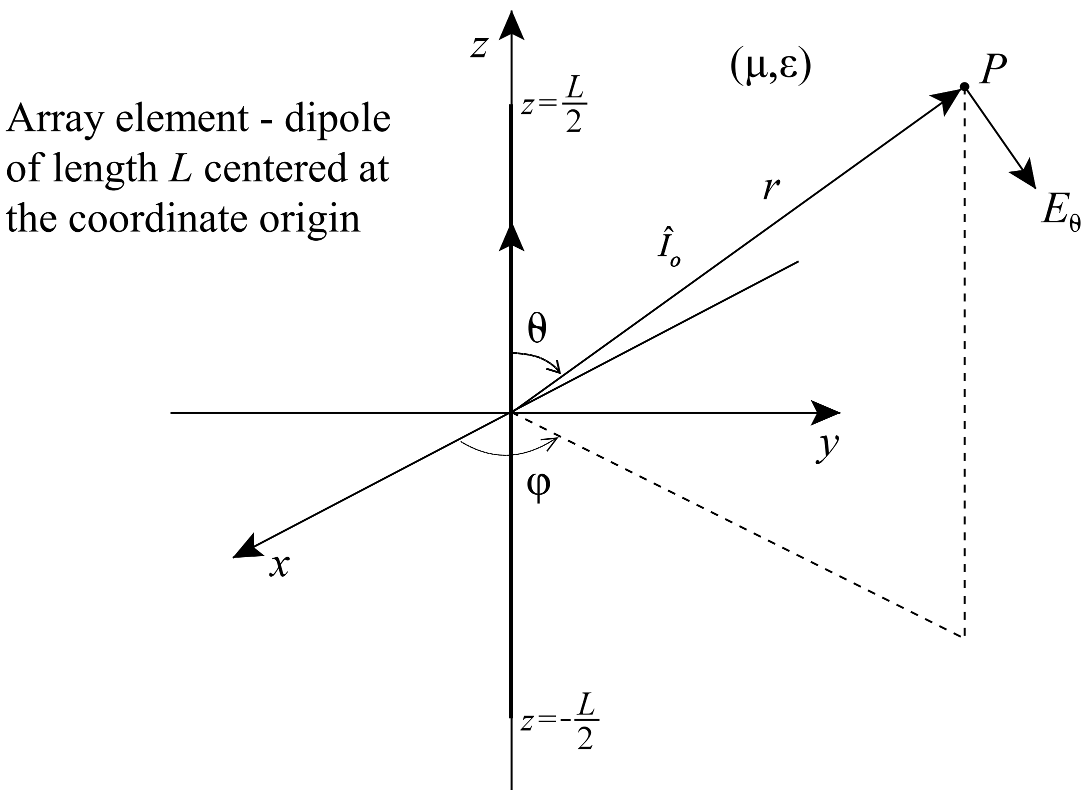
Element - the antenna type that composes the array.

Array factor - a function dependent only on the position and excitation (amplitude, phase) of the array elements.

Consider the pair of parallel current carrying conductors separated by a distance s in a medium characterized by (μ,ε) as shown below.



The parallel conductors form a 2-element array of dipole antennas of length L (the element far field is that of a dipole of length L centered at the coordinate origin).



At the field point P, the far field electric field (in spherical coordinates) of a dipole of length L centered at the coordinate origin is given by

$$\hat{E}_\theta \approx j\eta \frac{e^{-j\beta r}}{2\pi r} \hat{I}_o \left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta} \right] = j\eta \frac{e^{-j\beta r}}{2\pi r} \hat{I}_o F(\theta)$$

where β is the phase constant of the medium and η is the wave impedance of the medium given by

$$\beta = \frac{2\pi}{\lambda} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

and a sinusoidal current distribution is assumed along the conductors with the current going to zero at the conductor ends.

If the conductors are located in air, then $\mu = \mu_o$, $\varepsilon = \varepsilon_o$ and

$$\lambda = \frac{c}{f} \qquad \beta = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

$$\eta = \sqrt{\frac{\mu_o}{\varepsilon_o}} = \eta_o = 120\pi = 377\ \Omega$$

The shape of the dipole pattern function $F(\theta)$ in the radiated field expression varies with the electrical length of the antenna. For dipole lengths such that $0 \leq L \leq \lambda$, the maximum value of $F(\theta)$ occurs at $\theta = \pi/2$. Thus, the direction of maximum radiation is perpendicular to the antenna if the dipole length is less than one wavelength. For dipoles longer than one wavelength, the direction of maximum radiation may occur at some other value of θ.

The current distribution assumed for a dipole antenna is not often encountered in EMC problems. The dipole antenna is constructed such that the current must go to zero at the ends of the dipole conductors. The conductors or PCB lands in an EMC problem do not have this restriction. A more accurate current distribution on the parallel conductors of an EMC problem would be that of a *Hertzian dipole*. A Hertzian dipole is basically a current segment over which the magnitude of the current is assumed to be constant. The Hertzian dipole is an accurate model of an electrically short segment of a current carrying conductor ($L \ll \lambda$).

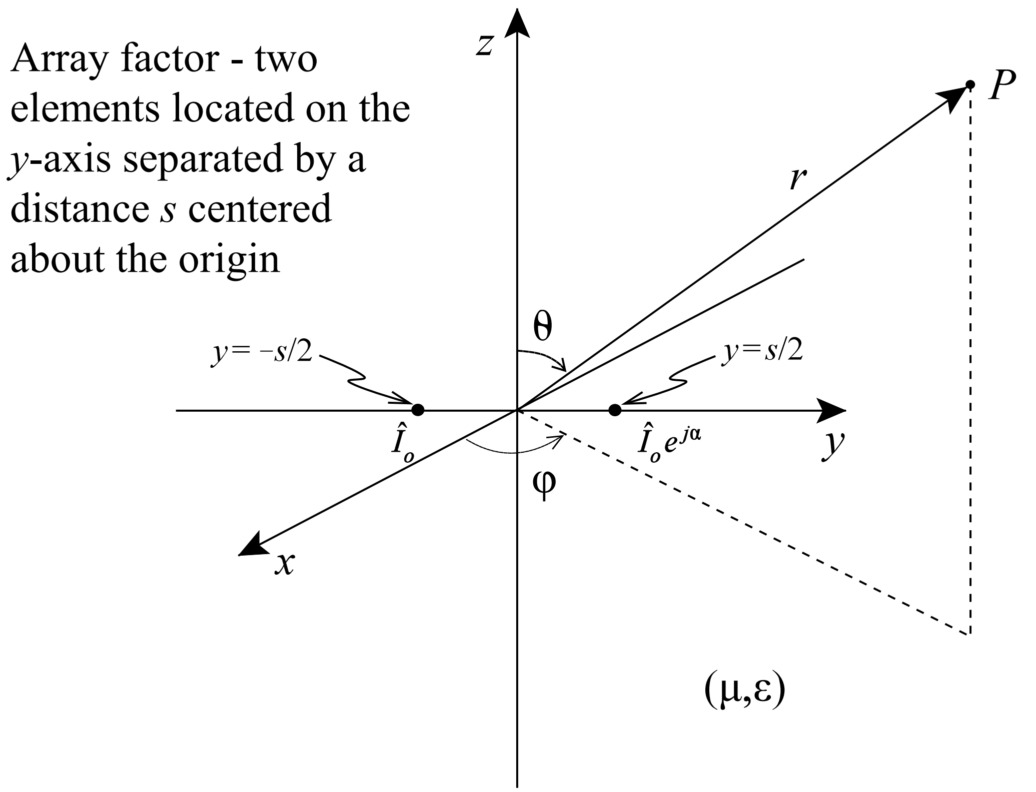
At the field point P, the far field electric field of a Hertzian dipole of length L centered at the coordinate origin is given by

$$\hat{E}_\theta \approx j\eta \frac{e^{-j\beta r}}{4\pi r} \hat{I}_o \beta L \sin\theta$$

Note that the direction of maximum radiation from a Hertzian dipole is also in the direction of $\theta = \pi/2$. If the Hertzian dipole is located in air ($\eta = 120\pi$, $\beta = 2\pi f/c$), the far field electric field as a function of frequency is

$$\hat{E}_\theta \approx j\frac{\pi}{5}\frac{e^{-j\beta r}}{r} \hat{I}_o f L \sin\theta \qquad (\mu\text{V/m})$$

To determine the fields radiated by a pair of Hertzian dipoles, the appropriate array factor is required. The array factor for this configuration is that of the 2-element array shown below.



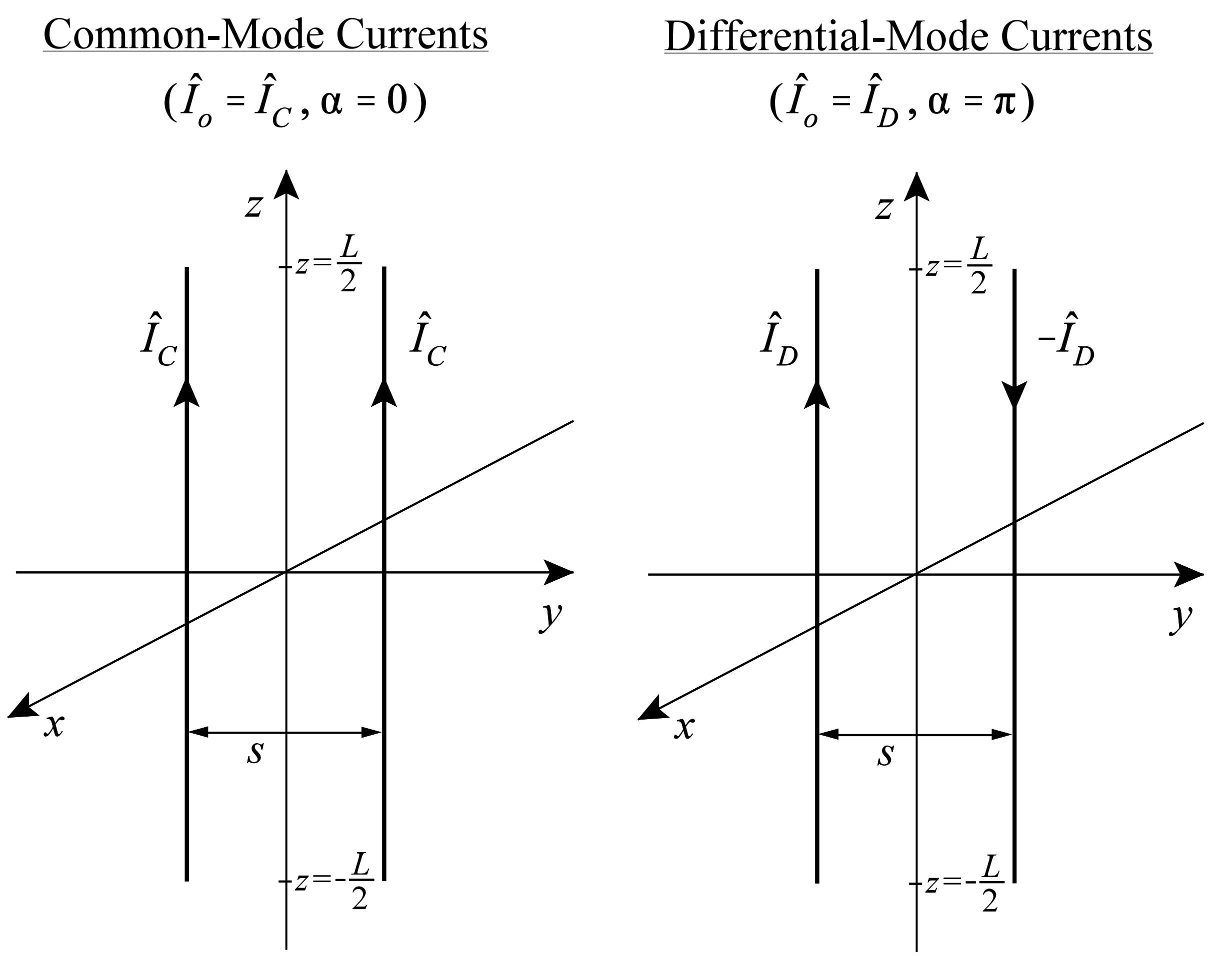
The array factor of this configuration is

$$AF = e^{-j\frac{\beta s}{2}\sin\theta\sin\varphi} + e^{j\alpha}\, e^{j\frac{\beta s}{2}\sin\theta\sin\varphi}$$

where α is the phase difference between the phasor currents for the two antennas in the array. The far fields of the Hertzian dipole can be combined with the two-element array factor as defined by the pattern multiplication theorem to determine the approximate far field electric field radiated by the parallel conductors.

$$\hat{E}_\theta \approx j\frac{\pi}{5}\frac{e^{-j\beta r}}{r} fL\hat{I}_o \sin\theta \left[e^{-j\frac{\beta s}{2}\sin\theta\sin\varphi} + e^{j\alpha} e^{j\frac{\beta s}{2}\sin\theta\sin\varphi} \right] \qquad (\mu\text{V/m})$$

Note that the radiated field depends on the electrical length and spacing of the conductors along with the phasing of the currents on the conductors. The array factor term in the radiated electric field expression can be simplified for the two special cases of common-mode and differential-mode currents.



The array factor for common-mode currents (AF_{cm}) can be written as

$$AF_{cm} = e^{-j\frac{\beta s}{2}\sin\theta\sin\varphi} + e^{j0}\, e^{j\frac{\beta s}{2}\sin\theta\sin\varphi}$$

$$= 2\cos\left(\frac{\beta s}{2}\sin\theta\sin\varphi\right)$$

while the array factor for differential-mode currents (AF_{dm}) can be written as

$$AF_{dm} = e^{-j\frac{\beta s}{2}\sin\theta\sin\varphi} + e^{j\pi} e^{j\frac{\beta s}{2}\sin\theta\sin\varphi}$$

$$= -2j\sin\left(\frac{\beta s}{2}\sin\theta\sin\varphi\right)$$

If we restrict the location of the field point P to the x-y plane ($\theta = 90^o$), the common-mode and differential-mode array factors reduce to

$$AF_{cm} = 2\cos\left(\frac{\beta s}{2}\sin\varphi\right)$$

$$AF_{dm} = -2j\sin\left(\frac{\beta s}{2}\sin\varphi\right)$$

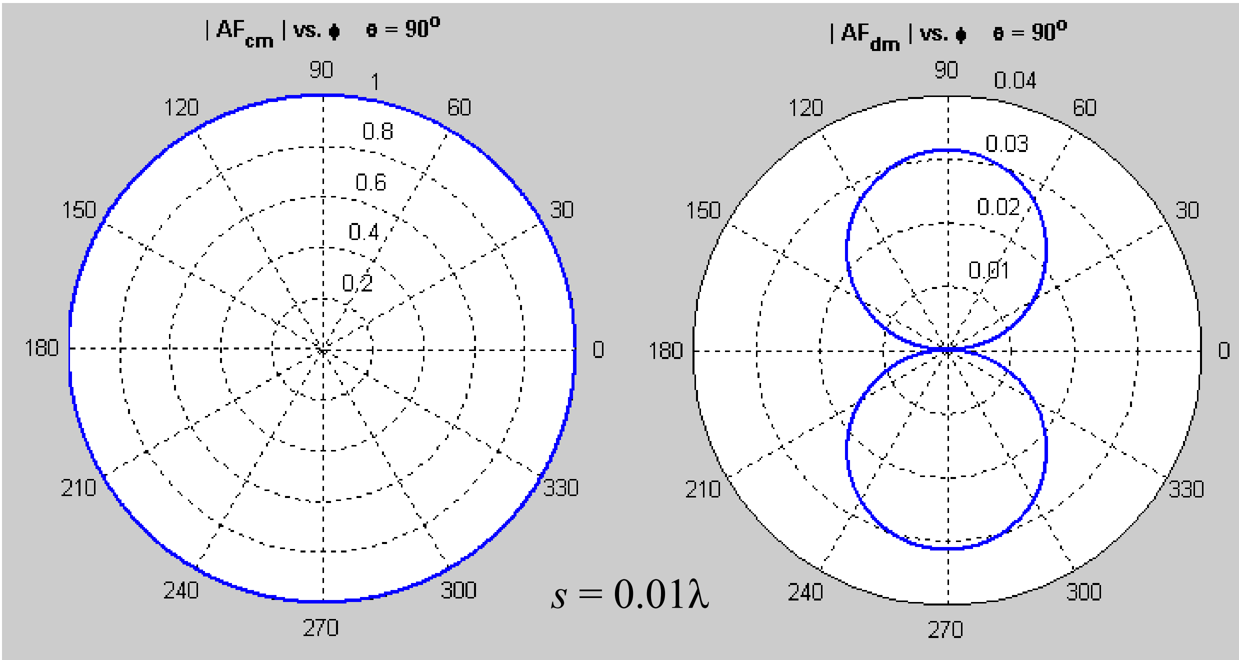
The spacing of the parallel conductors is typically electrically small ($s \ll \lambda$) so that

$$\frac{\beta s}{2} = \frac{2\pi}{\lambda}\frac{s}{2} = \pi\frac{s}{\lambda} = \pi s\frac{f}{c} \ll 1$$

$$AF_{cm} = 2\cos\left(\pi\frac{s}{\lambda}\sin\varphi\right)$$

$$AF_{dm} = -2j\sin\left(\pi\frac{s}{\lambda}\sin\varphi\right)$$

The directional characteristics of common-mode currents and differential-mode currents are quite different in the φ direction according to the differences in the array factors for these currents. The common-mode array factor is near constant with respect to φ while the differential-mode array factor varies significantly. To illustrate these directional radiation properties, consider the magnitude of the common-mode and differential-mode array factors for parallel conductors that are separated by $s = 0.01\lambda$.



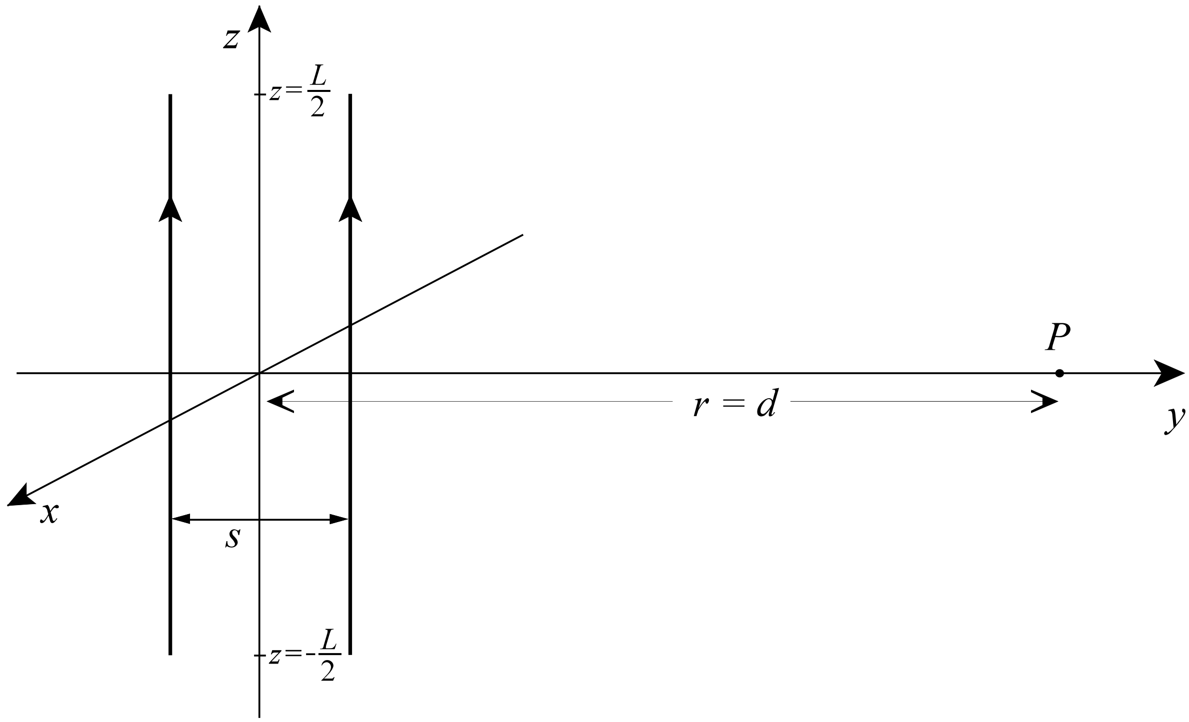
The plots above show that the common-mode array factor is practically omnidirectional in φ for closely-spaced conductors while the differential-mode array factor contains nulls at φ = 0° and φ = 180°. Thus, the radiation characteristics of differential-mode conductor pair are quite sensitive to rotation of the cable while the radiation characteristics of a common-mode conductor pair are quite insensitive to cable rotation.

Inserting the simplified form of the array factor for common-mode and differential-mode currents into the radiated field expression gives the following results.

$$\hat{E}_{\theta cm} \approx j\frac{2\pi}{5}\frac{e^{-j\beta r}}{r} fL\hat{I}_C \sin\theta \cos\left(\frac{\beta s}{2}\sin\theta\sin\varphi\right) \qquad (\mu\text{V/m})$$

$$\hat{E}_{\theta dm} \approx \frac{2\pi}{5}\frac{e^{-j\beta r}}{r} fL\hat{I}_D \sin\theta \sin\left(\frac{\beta s}{2}\sin\theta\sin\varphi\right) \qquad (\mu\text{V/m})$$

The equations above can be evaluated at any arbitrary spherical coordinate point (r,θ,φ) in the far field of the parallel conductors. Consider a distant point ($r = d$) in the direction of maximum radiation for the Hertzian dipoles ($\theta = 90°$) in the plane of the conductors ($\varphi = 90°$).



The radiated common-mode and differential-mode far fields at this distant point ($r = d$, $\theta = 90^{o}$, $\varphi = 90^{o}$) are

$$\hat{E}_{\theta cm} \approx j\frac{2\pi}{5}\frac{e^{-j\beta d}}{d} fL\hat{I}_C \cos\left(\frac{\beta s}{2}\right) \qquad (\mu\text{V/m})$$

$$\hat{E}_{\theta dm} \approx \frac{2\pi}{5}\frac{e^{-j\beta d}}{d} fL\hat{I}_D \sin\left(\frac{\beta s}{2}\right) \qquad (\mu\text{V/m})$$

If we assume that the conductor spacing s is small relative to wavelength ($s \ll \lambda$), then arguments of the sine and cosine functions are small.

$$\frac{\beta s}{2} = \pi\frac{s}{\lambda} \ll 1$$

For small arguments, the sine and cosine functions in the far field expressions may be approximated by

$$\sin x \approx x \qquad \text{for small } x$$

$$\cos x \approx 1 \qquad \text{for small } x$$

Inserting the small argument approximations for sine and cosine into the far field electric field expressions gives

$$\hat{E}_{\theta cm} \approx j\frac{2}{5}\pi f \frac{L}{d}\hat{I}_C e^{-j\beta d} \qquad (\mu\text{V/m})$$

$$\hat{E}_{\theta dm} \approx \frac{2}{5}\frac{\pi^2 f^2}{c}\frac{Ls}{d}\hat{I}_D e^{-j\beta d} \qquad (\mu\text{V/m})$$

Taking the magnitude of both sides of the previous equations yields the relationship between the magnitude of the current on the conductor pair to the magnitude of the radiated far field.

$$|\hat{E}_{\theta cm}| \approx \frac{2}{5}\pi f \frac{L}{d}|\hat{I}_C| = 1.257\, f\frac{L}{d}|\hat{I}_C| \qquad (\mu\text{V/m})$$

$$|\hat{E}_{\theta dm}| \approx \frac{2}{5}\frac{\pi^2 f^2}{c}\frac{Ls}{d}|\hat{I}_D| = 1.316\times 10^{-8} f^2 \frac{Ls}{d}|\hat{I}_D| \qquad (\mu\text{V/m})$$

Note that the approximate far field electric field magnitude of the common-mode conductor pair is directly proportional to the conductor length L and the frequency, but is independent of the conductor spacing. The electric field magnitude of the differential-mode pair is directly proportional to the conductor length L and spacing s (this would define the loop area $A = Ls$ for a closed loop) and the square of the frequency.

In practice, the common-mode currents (noise signals) are typically smaller than the differential-mode currents (desired signals). However, given the orientation of the currents, the common-mode currents radiate much more effectively than differential-mode currents. This radiation effectiveness can be quantified by considering a conductor pair containing both common-mode and differential-mode currents. If the common-mode and differential-mode currents radiate far field electric fields that are equal in magnitude, then

$$|\hat{E}_{\theta cm}| = 1.257\, f\frac{L}{d}|\hat{I}_C| = 1.316\times 10^{-8} f^2 \frac{Ls}{d}|\hat{I}_D| = |\hat{E}_{\theta dm}|$$

Solving this equation for the ratio of differential-mode current to common-mode current gives

$$\frac{|\hat{I}_D|}{|\hat{I}_C|} = \frac{9.552\times10^7}{sf}$$

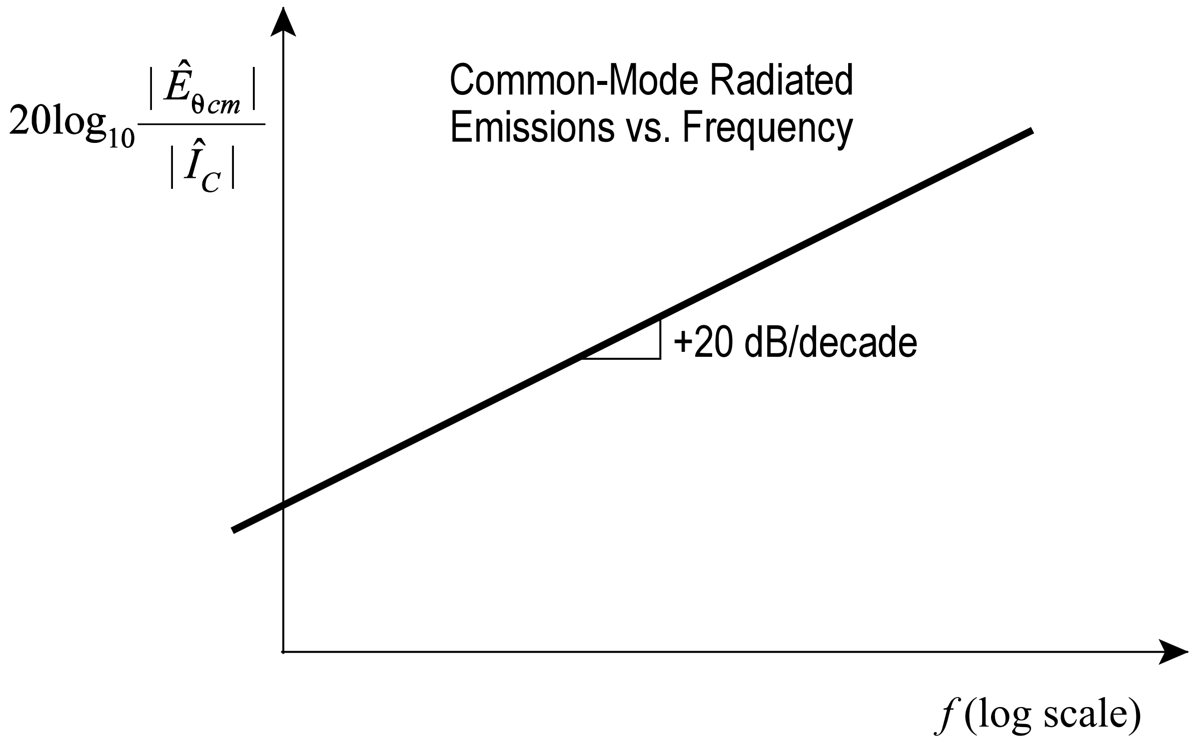
As an example, consider a pair of #28 AWG wires spaced by 50 mils (1.27 mm) operating at 30 MHz. The ratio of currents is

$$\frac{|\hat{I}_D|}{|\hat{I}_C|} = \frac{9.552\times10^7}{(1.27\times10^{-3})(30\times10^6)} = 2507$$

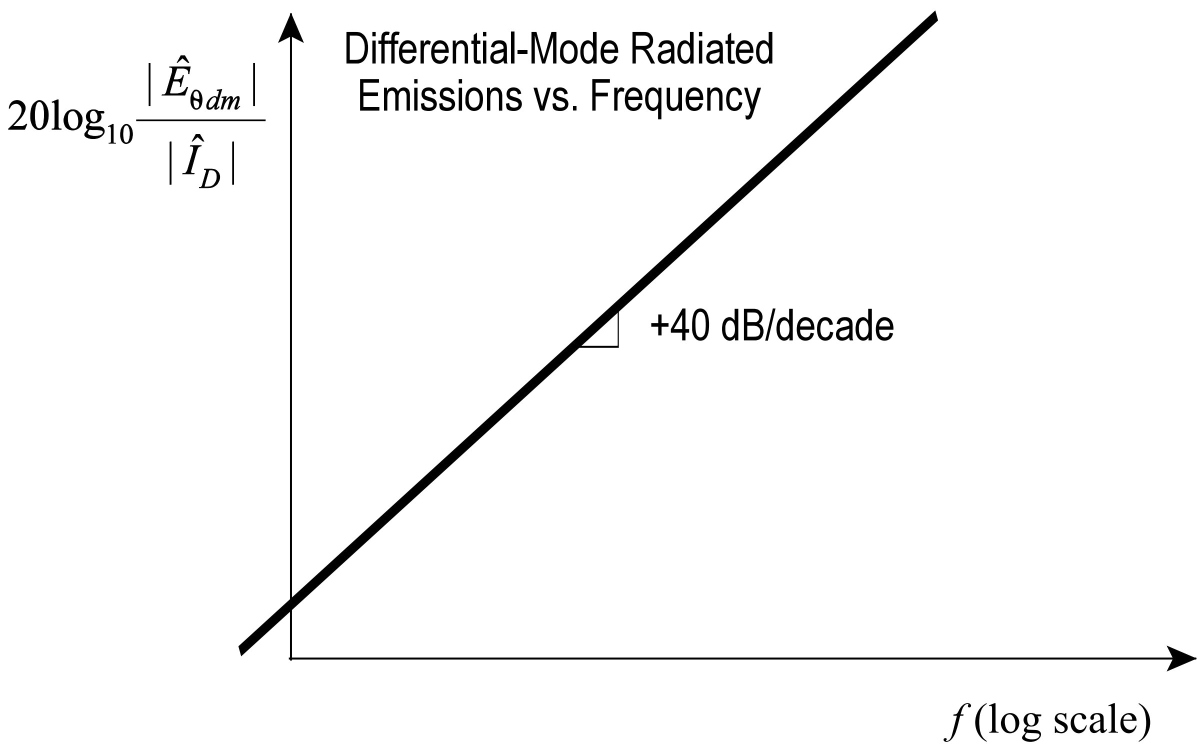
Thus, the differential-mode current would have to be more than 2500 times as large as the common-mode current to produce the same far-field magnitude.

The techniques used to reduce the radiated emissions due to common-mode and differential-mode currents at a particular frequency are dictated by the far field electric field equations for each current type. The radiated emissions for both types of current can be reduced by simply shortening the length of the conductors. Minimizing conductor lengths by careful placement of components is sound EMC design. The radiated emissions of differential-mode currents were found to also be dependent on the conductor spacing. Thus, minimizing loop areas helps to minimize differential-mode emissions. The radiated emissions for both types of current can also be reduced by simply reducing the magnitude of the phasor current at the frequency of interest. This may be done by reducing the amplitude of the time-domain current, if this is an option. Or, in the case of periodic pulse trains, we may adjust the two break frequencies (f_d and f_r) in order to reduce the spectral content of the signal on the conductor pair at the frequency of interest.

It is useful to consider the frequency dependence of common-mode and differential-mode emissions on the standard Bode plot format. Since common-mode conductor pair emissions are directly proportional to frequency, a plot of the radiated field to current ratio in dB vs. frequency on a log scale gives a straight line with a slope of +20 dB/decade. Differential-mode conductor pair emissions, being directly proportional to the square of the frequency, yield a line with a slope of +40 dB/decade.



$20\log_{10}\frac{|\hat{E}_{\theta cm}|}{|\hat{I}_C|}$
Common-Mode Radiated Emissions vs. Frequency
+20 dB/decade
f (log scale)



$20\log_{10}\frac{|\hat{E}_{\theta dm}|}{|\hat{I}_D|}$
Differential-Mode Radiated Emissions vs. Frequency
+40 dB/decade
f (log scale)

Signal Spectra and the Spectra of the Resulting Radiated Emissions

The radiated emissions of common-mode and differential-mode signals on parallel conductor pairs (Hertzian dipoles, far-field point in the plane of the conductors) have been shown to exhibit distinct characteristics with regard to the frequency variation of these signals. The radiated emissions of common-mode currents are linearly proportional to frequency and the magnitude of the common-mode current. The radiated emissions of differential-mode currents are linearly proportional to the square of the frequency and the magnitude of the differential-mode current.

$$|\hat{E}_{\theta cm}(f)| = K_C \, f \, |\hat{I}_C(f)|$$

$$|\hat{E}_{\theta dm}(f)| = K_D \, f^2 \, |\hat{I}_D(f)|$$

The constants K_C and K_D are dependent on the geometry of the conductors (length L, spacing s) and the distance d from the conductors to the field point. For electric field units of μV/m, the constants are

$$K_C = 1.257 \, \frac{L}{d}$$

$$K_D = 1.316 \times 10^{-8} \, \frac{Ls}{d}$$

Thus, the shape of the radiated emissions spectrum is governed by the spectrum of the current producing the emission and the mode of current producing the emission. The radiated field magnitude equations may be expressed in units of dB (taking $20\log_{10}$ of both sides of the equations) which yields

$$|\hat{E}_{\theta cm}(f)|_{\text{dB}\mu\text{V/m}} = \underbrace{20\log_{10} K_C}_{\text{Constant}} + \underbrace{20\log_{10} f}_{\text{+20 dB/decade}} + \underbrace{20\log_{10}|\hat{I}_C(f)|}_{\text{Signal spectrum}}$$

$$|\hat{E}_{\theta dm}(f)|_{\text{dB}\mu\text{V/m}} = \underbrace{20\log_{10} K_D}_{\text{Constant}} + \underbrace{40\log_{10} f}_{\text{+40 dB/decade}} + \underbrace{20\log_{10}|\hat{I}_D(f)|}_{\text{Signal spectrum}}$$

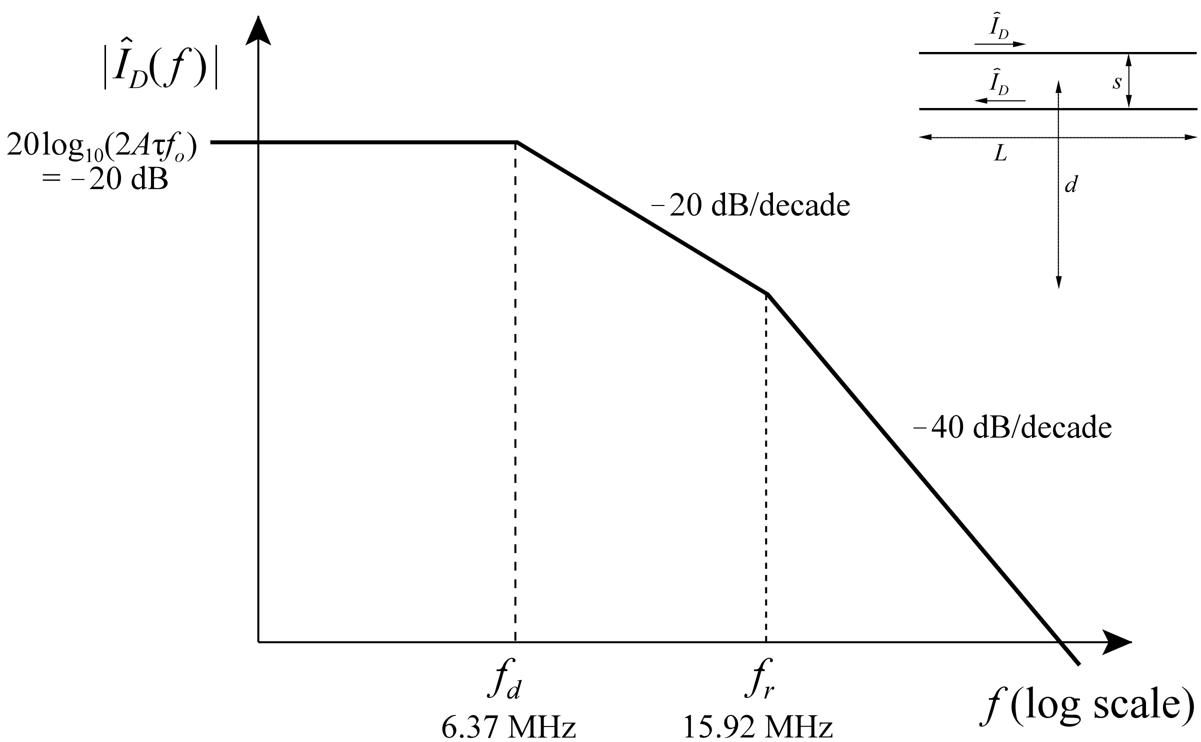
Thus, the Bode plot for the radiated field magnitude is found by adding the appropriate constant and increasing the slopes of the straight lines in the signal spectrum by 20 dB/decade for common-mode signals and 40 dB/decade for differential-mode signals.

Example (Radiated emissions/differential-mode clock signal)

Determine the approximate spectrum (Bode plot) of the radiated emissions at a distance of 10m from a pair of conductors (L = 20 cm, s = 1cm) carrying a 10 MHz differential-mode clock signal (100 mA amplitude, 50% duty cycle, $\tau_r = \tau_f = 20$ ns).

$$f_d = \frac{1}{\pi\tau} = 6.37 \text{ MHz} \qquad f_r = \frac{1}{\pi\tau_r} = 15.9 \text{ MHz}$$

$$20\log_{10}(2A\tau f_o) = 20\log_{10}[2(0.1)(50\times10^{-9})(10^7)] = -20 \text{ dB}$$



$$20\log_{10}K_D = 20\log_{10}\left[1.316\times10^{-8}\frac{Ls}{d}\right]$$

$$= 20\log_{10}\left[1.316\times10^{-8}\frac{(0.2)(0.01)}{10}\right] = -231.59 \text{ dB}$$

At $f = f_d = 6.37$ MHz,

$$|\hat{E}_{\theta dm}(f)|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_D + 40\log_{10}(6.37\times10^6) + 20\log_{10}|\hat{I}_D(f)|_{f=6.37\times10^6}$$

$$= -231.59 + 272.16 - 20$$

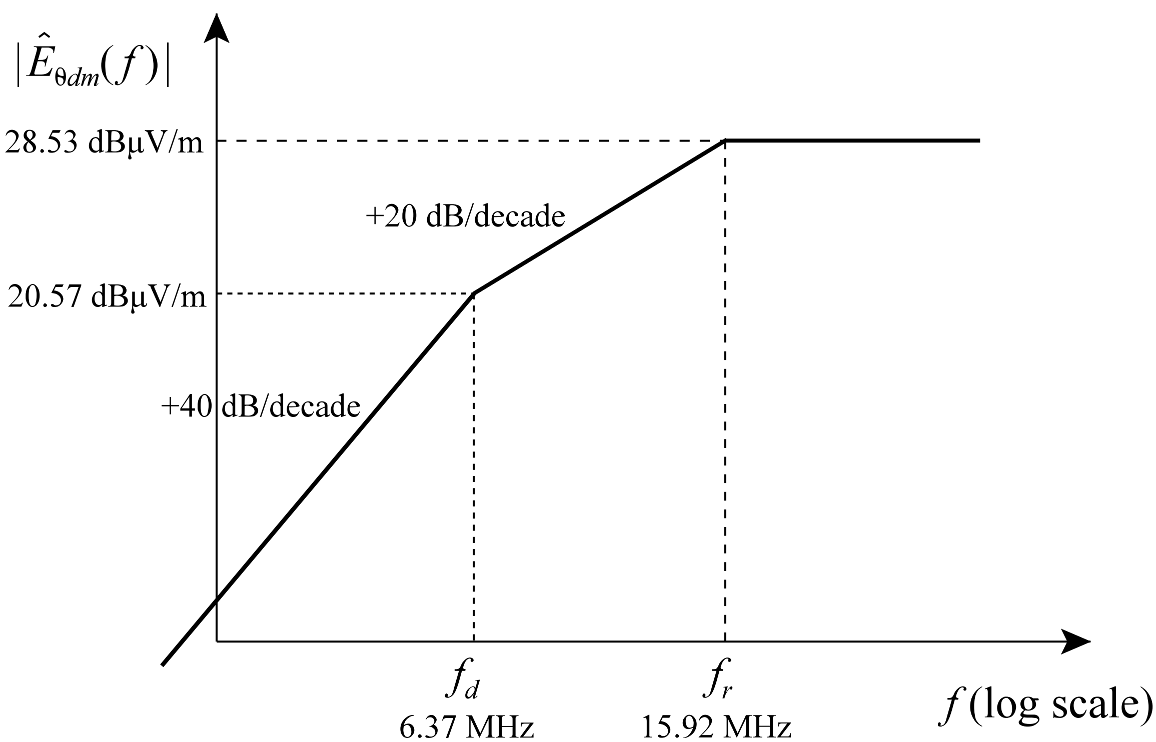
$$= 20.57 \text{ dB}\mu\text{V/m}$$

At $f = f_r = 15.92$ MHz,

$$|\hat{E}_{\theta dm}(f)|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_D + 40\log_{10}(15.92\times10^6) + 20\log_{10}|\hat{I}_D(f)|_{f=15.92\times10^6}$$

$$= -231.59 + 288.08 + \left(-20 - 20\log_{10}\frac{15.92}{6.37}\right)$$

$$= 28.53 \text{ dB}\mu\text{V/m}$$

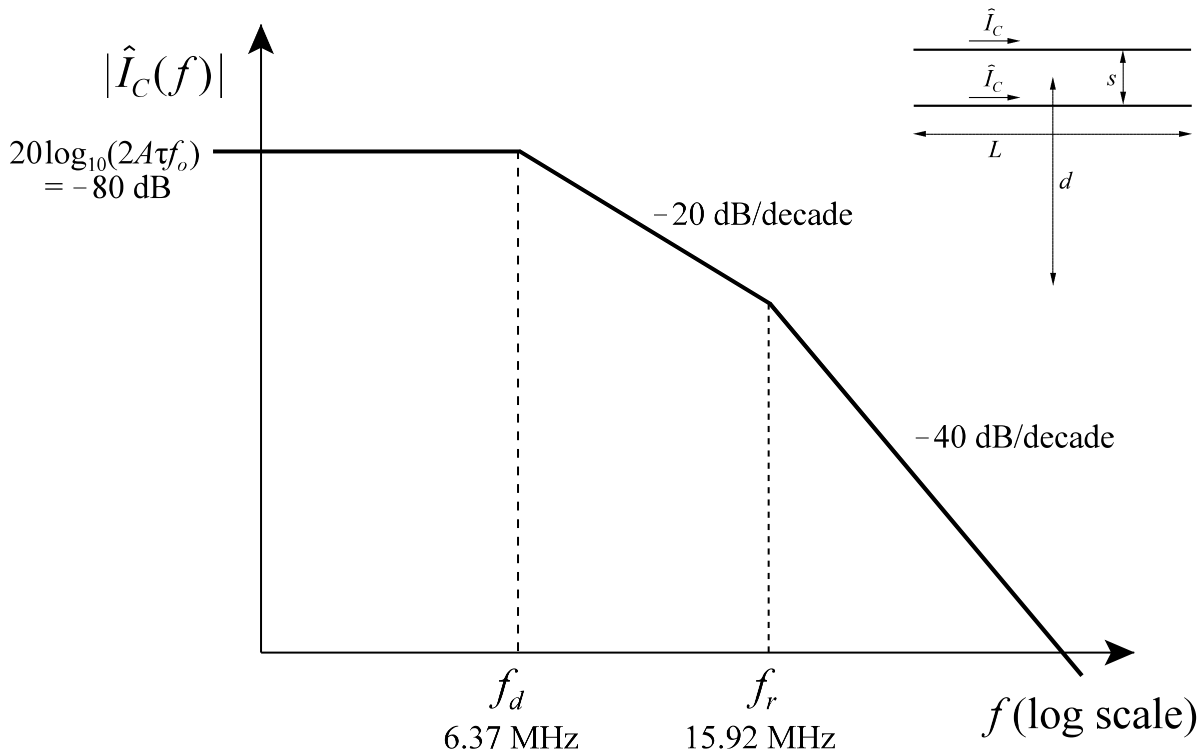


<u>Example</u> (Radiated emissions/common-mode clock signal)

Determine the approximate spectrum (Bode plot) of the radiated emissions at a distance of 10m from a pair of conductors (L = 20 cm, s = 1cm) carrying a 10 MHz common-mode clock signal (100 μA amplitude, 50% duty cycle, $\tau_r = \tau_f$ = 20 ns).

$$f_d = \frac{1}{\pi\tau} = 6.37 \text{ MHz} \qquad\qquad f_r = \frac{1}{\pi\tau_r} = 15.9 \text{ MHz}$$

$$20\log_{10}(2A\tau f_o) = 20\log_{10}[2(10^{-4})(50\times10^{-9})(10^7)] = -80 \text{ dB}$$



$$20\log_{10}K_C = 20\log_{10}\left[1.257\,\frac{L}{d}\right]$$

$$= 20\log_{10}\left[1.257\,\frac{0.2}{10}\right] = -31.99 \text{ dB}$$

At $f = f_d = 6.37$ MHz,

$$|\hat{E}_{\theta cm}(f)|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_C + 20\log_{10}(6.37\times10^6) + 20\log_{10}|\hat{I}_D(f)|_{f=6.37\times10^6}$$

$$= -31.99 + 136.08 - 80$$

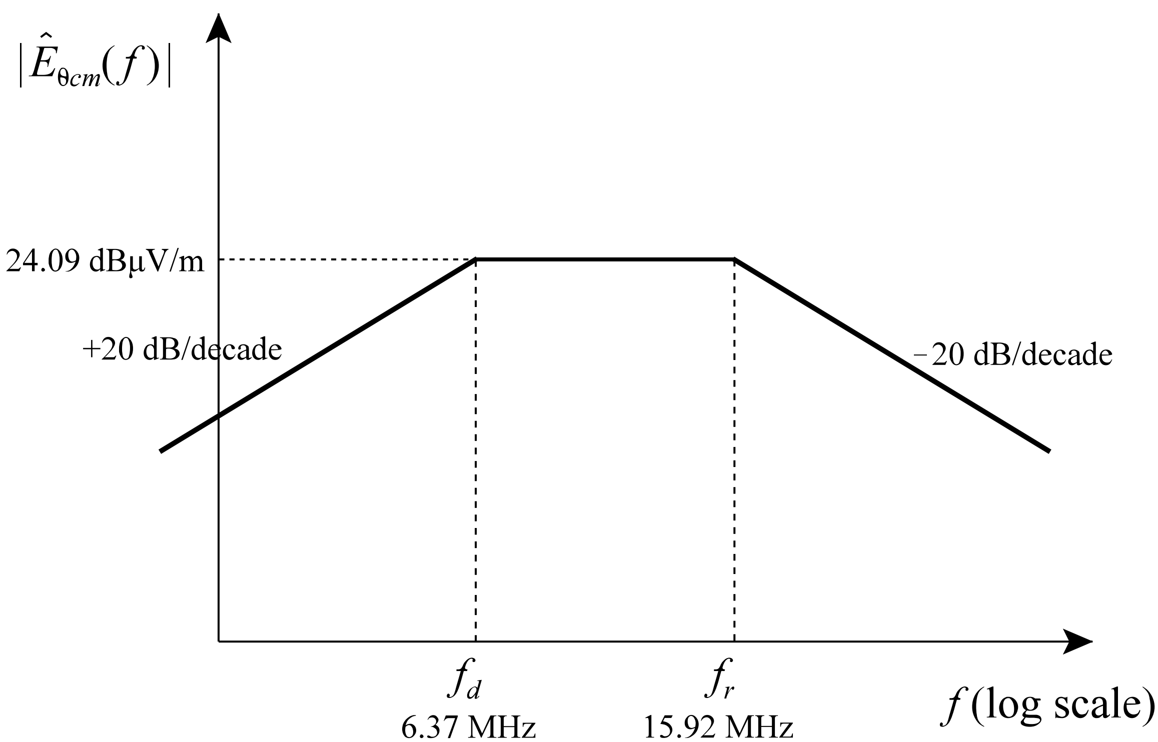
$$= 24.09 \text{ dB}\mu\text{V/m}$$

At $f = f_r = 15.92$ MHz,

$$|\hat{E}_{\theta cm}(f)|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_C + 20\log_{10}(15.92\times10^6) + 20\log_{10}|\hat{I}_D(f)|_{f=15.92\times10^6}$$

$$= -31.99 + 144.04 + \left(-80 - 20\log_{10}\frac{15.92}{6.37}\right)$$

$$= 24.09 \text{ dB}\mu\text{V/m}$$



The general shapes of the radiated emission spectra obtained for the previous differential-mode and common-mode clock signal examples illustrate that differential mode currents radiate most effectively at higher frequencies ($f > f_r$) while common-mode currents radiate most effectively within the mid-frequency range of ($f_d < f < f_r$). We may summarize the characteristics of radiated emissions from common-mode and differential-mode signals as follows.

<u>Radiated emissions of common-mode signals</u>

1. Linearly proportional to:
   - (a.) frequency (f)
   - (b.) conductor length (L)
   - (c.) current ($\hat{I}_C$)
2. Independent of cable rotation.
3. Low and high frequency roll-off.

<u>Radiated emissions of differential-mode signals</u>

1. Linearly proportional to:
   - (a.) the square of frequency (f^2)
   - (b.) conductor length (L) and spacing (s) (loop area)
   - (c.) current ($\hat{I}_D$)
2. Sensitive to cable rotation.
3. Low frequency roll-off only.

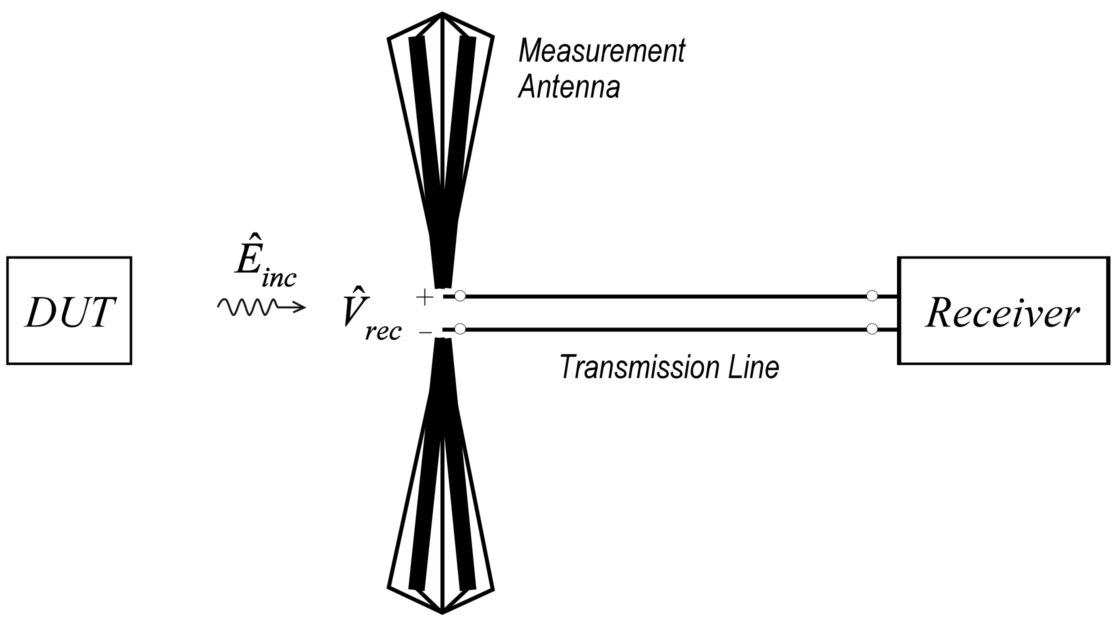
MEASURED SPECTRA AND THE EFFECT OF ANTENNA FACTOR

When the radiated emissions of a DUT are measured with a receiving antenna (biconical, log-periodic, etc.), the frequency response of the antenna will affect the shape of the received signal spectrum. The ratio of the incident electric field to the voltage received at the antenna terminals is defined as the *antenna factor* (*af*) which is a function of frequency.

$$af(f) = \frac{|\hat{E}_{inc}(f)|}{|\hat{V}_{rec}(f)|}$$

$|\hat{E}_{inc}(f)|$ – incident electric field (V/m)

$|\hat{V}_{rec}(f)|$ – received voltage at antenna terminals (V)



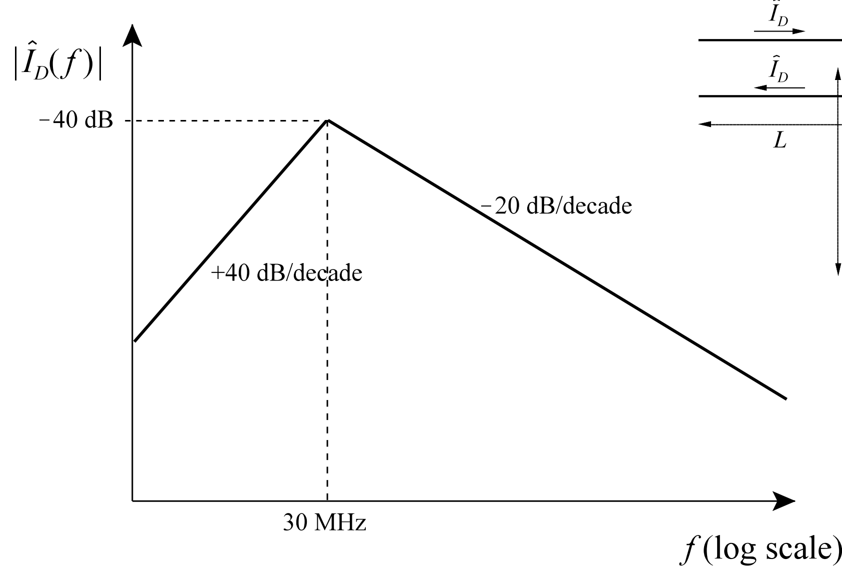
Expressing the received voltage (dBμV) in terms of the antenna factor gives

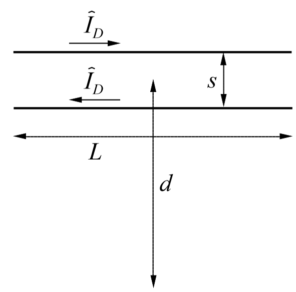
$$|\hat{V}_{rec}(f)|_{\text{dB}\mu\text{V}} = |\hat{E}_{inc}(f)|_{\text{dB}\mu\text{V/m}} - af_{dB}(f)$$

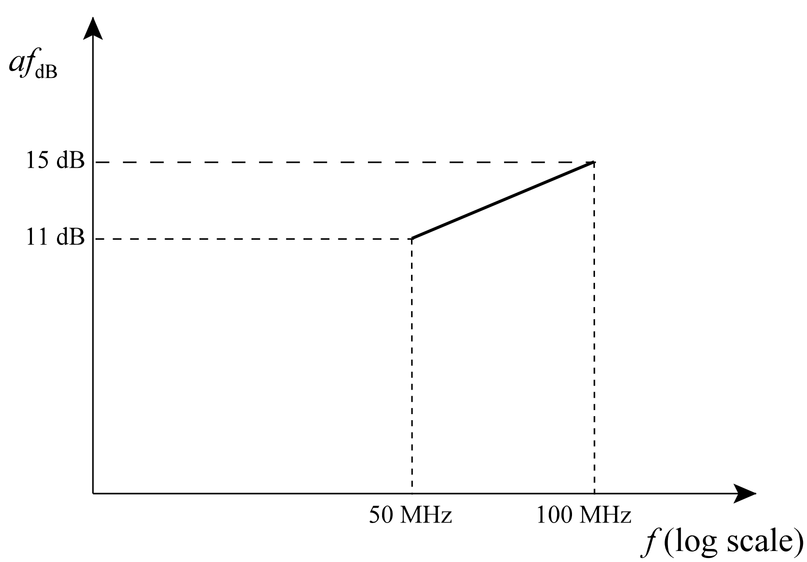
Example

(Measured emissions / signal spectrum / current mode / antenna factor)

Differential mode currents, with spectrum shown below, flow in a conductor pair with length L = 1m and spacing s = 1 cm. The radiated emissions from the differential mode currents are measured in the direction of maximum radiation at a distance of d = 3 m. The variation of the antenna factor with frequency is also shown below. Draw the envelope of the received voltage magnitude at the terminals of the measurement antenna over the range of 50 to 100 MHz.







The frequency range of interest for the radiated emissions measurements (50−100 MHz) lies on the −20 dB/decade segment of the differential current spectrum. The magnitude of the current spectrum at any point on this segment is given by

$$|\hat{I}_D(f)|_{\text{dB}} = -40 - 20\log_{10}\left(\frac{f}{30\times 10^6}\right)$$

At the endpoints of the frequency range, we find

$$|\hat{I}_D(50\text{ MHz})|_{\text{dB}} = -40 - 20\log_{10}\left(\frac{50}{30}\right) = -44.44\text{ dB}$$

$$|\hat{I}_D(100\text{ MHz})|_{\text{dB}} = -40 - 20\log_{10}\left(\frac{100}{30}\right) = -50.46\text{ dB}$$

The general equation for the magnitude of the electric field (dBμV/m) radiated by the differential mode currents in terms of the current spectrum is given by

$$|\hat{E}_{\theta dm}(f)|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_D + 40\log_{10}f + 20\log_{10}|\hat{I}_D(f)|$$

where

$$\begin{aligned} 20\log_{10}K_D &= 20\log_{10}\left[1.316\times 10^{-8}\,\frac{Ls}{d}\right] \\ &= 20\log_{10}\left[1.316\times 10^{-8}\,\frac{(1)(0.01)}{3}\right] \\ &= -207.16\text{ dB} \end{aligned}$$

Evaluating the differential mode radiated fields at the endpoints of the frequency range gives

$$|\hat{E}_{\theta dm}(50\text{MHz})|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_D + 40\log_{10}(50\times10^6) + 20\log_{10}|\hat{I}_D(f)|_{f=50\times10^6}$$

$$= -207.16 + 307.96 - 44.44$$

$$= 56.36 \text{ dB}\mu\text{V/m}$$

$$|\hat{E}_{\theta dm}(100\text{MHz})|_{\text{dB}\mu\text{V/m}} = 20\log_{10}K_D + 40\log_{10}(100\times10^6) + 20\log_{10}|\hat{I}_D(f)|_{f=100\times10^6}$$

$$= -207.16 + 320 - 50.46$$

$$= 62.38 \text{ dB}\mu\text{V/m}$$

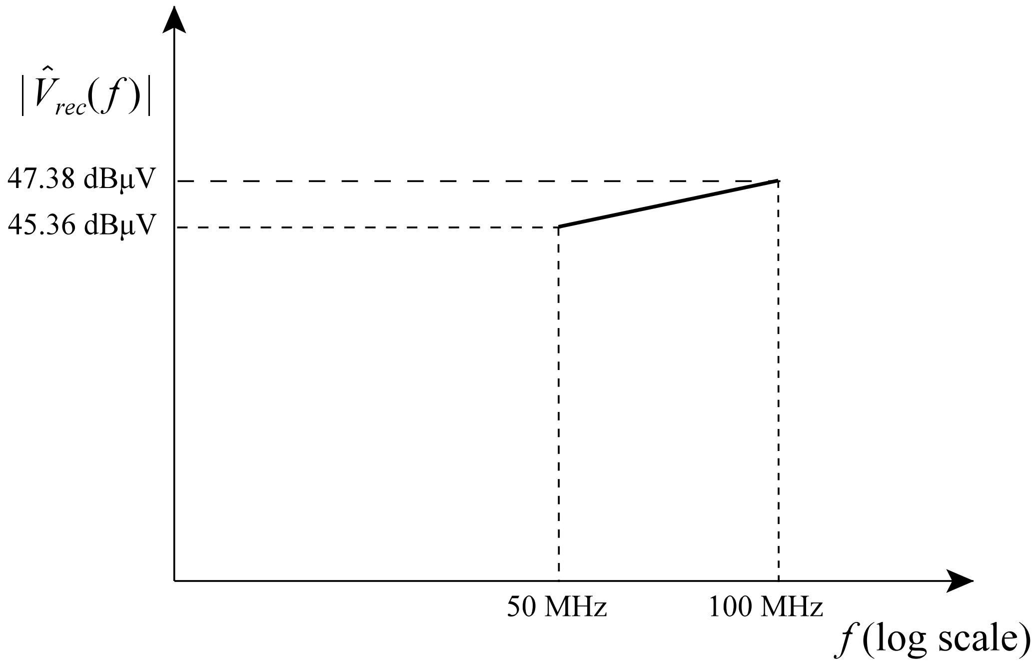
The receive antenna terminal voltage (dBμV) is related to the incident field (dBμV/m) by

$$|\hat{V}_{rec}(f)|_{\text{dB}\mu\text{V}} = |\hat{E}_{inc}(f)|_{\text{dB}\mu\text{V/m}} - af_{dB}(f)$$

so that

$$|\hat{V}_{rec}(50\text{MHz})|_{\text{dB}\mu\text{V}} = 56.36 - 11 = 45.36 \text{ dB}\mu\text{V}$$

$$|\hat{V}_{rec}(100\text{MHz})|_{\text{dB}\mu\text{V}} = 62.38 - 15 = 47.38 \text{ dB}\mu\text{V}$$


$|\hat{V}_{rec}(f)|$
47.38 dBμV
45.36 dBμV
50 MHz
100 MHz
f (log scale)