Radical Expressions

Notes

Name: ______________________

Date: ______________________

Keepin' it Real

Jordan is an aerospace engineer for NASA. He has to get a new satellite into orbit around Pluto's moon Hydra. The mathematician has given him different flight paths that include radical expressions and Jordan has to determine which is most likely to be successful. He knows that the smaller the turn angles, the more likely it is to hit orbit. But at the same time, the longer the total distance traveled, the more fuel the satellite requires.

Vocabulary

Write the definition of the term and include an image or example that represents it.

Term	Definition	Example
Radical Expression		
Radicand		
Index		
Surd		
Conjugate		
Radical Function		
Radical Equation		

Key Concept (a)

What are Radicals?	The Conceptualizer!
The radical sign, $\sqrt{\ }$, is used to indicate a root. It has the inverse meaning of raising a value to a power. You have have long referred to this sign as the "square root", but now we will be adventuring into a world where saying "square" is not always appropriate. The *index* is indicates the nth root of the radical. When this number is not written, it is an implicit index of 2. The expression $\sqrt{36}$ is the same as $\sqrt[2]{36}$. It is a square root, the "opposite" of squaring, raising to a power of 2.	

Key Concept (b)

Multiplication of Radicals	The Conceptualizer!
You can find the product of two *radicals*. The product of two radicals is the radical of the product of the radicands. #totallyrad To simplify this way, the radical signs MUST "match". What you are really doing is using an exponent-property rule, in reverse. If you know that $(ab)^c = a^c \cdot b^c$, then you can work backwards, and write $a^c \cdot b^c$ as $(ab)^c$.	

Skill Practice #1

Multiplying Radicals	Notes
Find $\sqrt{5} \cdot \sqrt{20}$	

Key Concept (c)

Simplifying Radical Expressions	The Conceptualizer!
If you can rewrite $\sqrt{3} \cdot \sqrt{27}$ as $\sqrt{3 \cdot 27}$, then you can work backwards, too. Given the square root of a number, you can simplify it by writing it as the square root of its factors. To do so… Choose the largest factor that is a perfect square. And pull it out of the radical. If the original number itself is not a perfect square, then your final simplified expression will contain a radical. It may be a prime number, such as 3, or a number with no factors that are perfect squares, such as 6 or 10.	

Skill Practice #2

Simplifying Radicals	Notes
Simplify $\sqrt{360}$	

Key Concept (d)

Multiplying & Simplifying Radicals	The Conceptualizer!
Now that you know how to simplify radicals, you have another option in finding the products of radicals. You could find $\sqrt{12 \cdot 18} = \sqrt{216}$. Then you can think of perfect squares that are factors of 216. BUT there is an easier way. You can simplify before you multiply.	

Key Concept (e)

Radical Basics: Division with Radicals	The Conceptualizer!
Division of radicals is similar to multiplication of radicals. The quotient of two radicals is the radical of the quotient of the radicands.	

Key Concept (f)

Rationalizing Denominators	The Conceptualizer!
Mathematicians don't leave negative values "in the basement", in the denominator of fractions. Just as you would rewrite $\frac{5}{-3}$ to $-\frac{5}{3}$, you also don't want to have a denominator that that includes a radical. To correct it, the process is called "rationalizing the denominator". You obtain an integer in the denominator. You are able to do this by multiplying the numerator and denominator by the denominator.	

Rationalizing Denominators with Conjugates	The Conceptualizer!
How can you rewrite $\frac{18}{1+\sqrt{6}}$? We want to rationalize the denominator, but how can you get an integer there? We must multiply by the *conjugate* of the denominator: $1-\sqrt{6}$.	

Skill Practice #3

Rationalizing Denominators	Notes
Rewrite $\frac{\sqrt{2}}{\sqrt{6}+4}$	

Key Concept (g)

Solving Radical Equations	The Conceptualizer!
We know to solve equations we work backwards and undo operations.The radical sign is another kind of grouping (which classifies with parenthesis). An equation that includes a radical is a *radical equation*.	

There are basic steps to solve a radical equation.

Step 1	
Step 2	
Step 3	
Step 4	

Skill Practice #4

Solving Radical Equations	Notes
Solve for *x*: $2\sqrt{x+4} = x - 4$	

Key Concept (h)

Solving Algebraically	The Conceptualizer!
How would you solve an equation with radicals on both sides? Carefully. It generally starts with undoing the radicals by raising the power on both sides. It is a journey. You will encounter cases where you must do this multiple times.	

Skill Practice #5

Equations with more than one Radical	Notes
Solve $\sqrt{x+4}+\sqrt{x-3}=7$	

#Observations

Root of a Root of a Root
Now you can also solve something like $\sqrt{x+\sqrt{x+\sqrt{x}}}=4$ Follow the steps -- repeatedly. Use what you know of solving equations, such as applying the Quadratic Formula. Just keep squaring!

Applications

One side of rectangular fence measures $6\sqrt{2}$ m. The other is $2\sqrt{2}$ m. What is the area of the fenced area?

Mike is gazing upward at an airplane. His equipment estimates that the plane is $6\sqrt{96}$ meters above the ground and $2\sqrt{108}$ meters in front of him. What is the rationalized slope of Mike's line of sight to the plane?

An equilateral triangle has side lengths of 4 units. Find the area.