

Railway track settlements - a literature review

Tore Dahlberg

Division of Solid Mechanics, IKP,
Linköping University, SE-581 83 Linköping, Sweden

Preface

This report has been written as part of the BriteEuram project SUPERTRACK (Sustained performance of railway tracks), contract No G1RD-CT-2002-00777. The project started in July 2002 and will continue for three years.

This literature survey consists of two parts:

1. A LITERATURE REVIEW (this report), in which a minor part of the literature found on dynamic train/track interaction, railway ballast, and railway track settlement has been reviewed.
2. A BIBLIOGRAPHY, containing almost 200 references dealing with different aspects of railway ballast, track structure, train/track interaction, and track settlement.

In a recent project, the BriteEuram project EUROBALT II, a literature review was written and a Bibliography was published. The bibliography contains more than 1000 references dealing with different aspects of railway ballast, track structure and components, train/track interaction, and track settlement. The EUROBALT bibliography is available on the home page of Solid Mechanics at Linköping University, address www.solid.ikp.liu.se under the heading Research: Dynamic Train/Track Interaction, see also Dahlberg (1998).

In the review presented here mainly papers published in recent years will be covered. For earlier papers, the reader is referred to the review by Dahlberg (1998) and the EUROBALT Bibliography.

Also the Bibliography appended to this report will be made available on the Internet on the address given above.

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Tore Dahlberg

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by Tore Dahlberg

Division of Solid Mechanics, IKP,
Linköping University, SE-581 83 Linköping, Sweden

Abstract

Railway tracks (rails and sleepers) are normally laid on a sub-structure that consists of two or more layers of different materials. The top layer (below the sleepers) is a layer of railway ballast. Below the ballast there might be layers of sub-ballast, a formation layer and/or the subground (the formation). Historically, the ballast layer performs the function of supporting the sleepers against vertical and lateral forces.

A railway track exposed to train traffic will degenerate. Track alignment and track level will deteriorate. Settlements of the track (loss of track level and alignment) require maintenance of the track; the track is aligned and lifted, and new ballast material is injected under the sleepers.

Explanations why track settlements occur are very scarce. Often, some parts of a track are more prone to settlements than other parts of the same track. So far, mainly the influence of such factors as axle load and train speed have been investigated. Having in mind that tracks subjected to the same load show different settlement behaviour, explanations of track settlements must be sought for in the track itself; not only in the loading of the track.

This review deals with railway ballast and railway track settlements. It also presents some mathematical and numerical methods dealing with the static and dynamic loading of the track due to interaction of train, track, and sub-structure. The report has been synthesized for the BriteEuram project SUPERTRACK (Sustained performance of railway tracks). One aim of the SUPERTRACK project is to contribute to the understanding of the fundamental physical behaviour of railway ballast when used in track sub-structure.

1. Introduction

A railway track normally consists of rails, sleepers, railpads, fastenings, ballast, sub-ballast, and subgrade. Sometimes, for example in tunnels, the ballast bed is omitted and the rails are fastened to concrete slabs resting on the track foundation. Two subsystems of a ballasted track can be distinguished: the superstructure (rails, sleepers, ballast and sub-ballast) and the subgrade (composed of a formation layer and the base). In this paper we will distinguish between the mechanical part of the track structure (rails, sleepers, and railpads between rails and sleepers), and the geotechnical part (the sub-structure below the sleepers).

A railway track sub-structure normally consists of a top layer of railway ballast, an intermediate layer of sub-ballast, and the subgrade, see Figure 1.

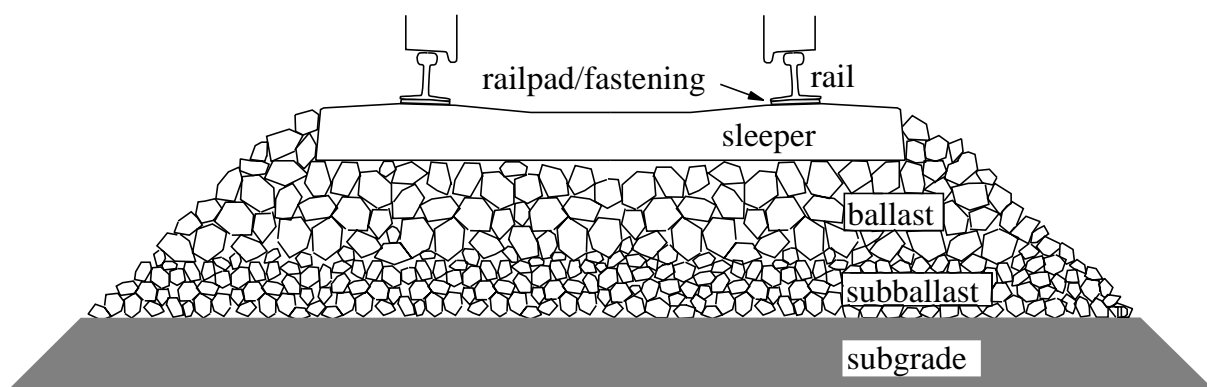


Fig. 1. Track with its different components: rails, railpads and fastenings (fastenings not shown in this figure), sleepers, ballast, sub-ballast, and subgrade.

1.1 Ballast

Ballast is the selected material placed on top of the track subgrade to support the track structure. Conventional ballast is a coarse-sized, non-cohesive, granular material of a uniform gradation. Traditionally, angular, crushed, hard stones and rocks have been considered good ballast materials. Granite, limestone, slag or other crushed stones have been used. Availability and economic motives have often been prime factors considered in the selection of ballast materials.

The ballast layer supports the track structure (the rails and the sleepers) against vertical, lateral, and longitudinal forces from the trains. The sleepers, to which the rails are fastened, are embedded in the ballast, which is tightly compacted or tamped around the sleepers to keep the track precisely levelled and aligned. The standard depth of ballast is 0.3 metres, but it is packed to 0.5 metres around the sleeper ends to ensure lateral stability.

The ballast layer has several important functions:

- it limits sleeper movement by resisting vertical, transverse and longitudinal forces from the trains,
- it distributes the load from the sleepers to protect the subgrade from high stresses, thereby limiting permanent settlement of the track,
- it provides necessary resilience to absorb shock from dynamic loading,
- it facilitates maintenance surfacing and lining operations,
- it provides immediate water drainage from the track structure,
- it helps alleviate frost problems, and
- it retards the growth of vegetation and resists the effects of fouling from surface-deposited materials.

Although the cheapness and practical advantages of ballast make it unlikely that ballasted tracks will be replaced by ballastless tracks, the use of ballasted track for new high speed lines may continue only if a fundamental physical understanding of ballast behaviour is available.

1.2 Sub-ballast

Below the ballast a layer of sub-ballast is placed. The sub-ballast is material chosen as a transition layer between the upper layer of large-particle good quality ballast and the lower layer of fine-graded subgrade. The sub-ballast used in most new construction is intended to prevent the mutual penetration or intermixing of the subgrade and the ballast and to reduce frost penetration. Any sand or gravel materials may serve as sub-ballast material as long as they meet proper filtering requirements. In the following the sub-ballast is included in the discussion even if it is not always mentioned.

1.3 Subgrade

Subgrade is the layer of material on which the ballast and sub-ballast layers rest. The subgrade is a very important component in the track structure and has been the cause of track failure and development of poor track quality (Li and Selig, 1995). Unfortunately, in existing track, the subgrade is not involved in the maintenance operation and little can be done to alter its characteristics. Some investigations are, however, prepared in the SUPERTRACK project.

At present, the state-of-the-art of track design concerning the ballast, sub-ballast, and the subgrade is mostly empirical. The factors that control the performance of these layers are poorly understood. To assess the reasons why a particular section of track requires maintenance, it is necessary to know the characteristics of the ballast and subgrade, the maintenance history, the environmental history, and the traffic history. Usually only the last three items can be estimated from records. Information on the

characteristics of the ballast and subgrade of an existing track is in most cases non-existent. To gain information on the present conditions of a site, field examinations is all that is possible.

As mentioned, factors that control the performance of the ballast (and the other layers) are poorly understood. In Knothe (1998), the long-term behaviour of the railroad track, including the ballast behaviour and the damage mechanisms underlying the ballast settlement, is discussed. Knothe states that there do not exist any generally accepted damage and settlement equations, and hardly any material equations for the ballast itself. Only different suggestions to describe the ballast settlement from a phenomenological point of view are available; the settlement then being a function of number of loading cycles and magnitude of the loading.

1.4 Train/track models

The dynamics of the compound train and track system plays an important role when investigating vehicle and track dynamics. The dynamic train/track interaction forces will give rise to vibrations that lead to track deterioration, such as track settlements, railhead corrugation growth, damage to track components (railpads, sleepers, ballast), and so on. Low-frequency (less than 20 Hz) motion of the train is crucial for assessment of safety and riding quality. High-frequency vibrations cause discomfort to passengers and emit noise and vibration to the surroundings.

Early studies of the train-track interaction problem have been reviewed in the book by Fryba (1972, 3rd edition 1999). In the early 1900's Timoshenko published papers on the strength of rails, and later Inglis was active in this field. The book by Fryba contains investigations on the vibrations of solids and structures under moving loads; the train (or wheel) then being modelled as a moving force. Knothe and Grassie (1993) and Popp *et al.* (1999) have presented state-of-the-art reviews in the field of train-track interaction.

Techniques to study the train-track interaction can be divided into two groups: frequency-domain techniques and time-domain techniques.

In the frequency-domain technique receptances of the track is required. If a stationary (not moving) wheel is loading the track, then the track receptance is needed only at the point where the wheel is situated. The receptance (vertical or lateral, depending on what is studied) may be measured in-situ on the track or it can be calculated using a track model. If a harmonically varying stationary load excites the track, then the direct receptance provides the track response.

When train-track dynamics is investigated in the time domain deflections of the track and displacements of the vehicle are calculated by numerical time integration as the vehicle moves along the track. The vertical motion of the wheelset should then

coincide with the vertical deflection of the rail, while taking the wheel-rail contact deformation into account. The wheel-rail contact force is unknown and has to be determined in the calculations.

The track can be modelled by finite elements and in many cases a modal analysis of the track is performed. The track is then described through its modal parameters, and the physical deflections of the track are determined by modal superposition. Often the vehicle is modelled by use of rigid masses, springs (linear or non-linear) and viscous dampers.

The modal analysis technique requires linear models. A finite element track model may comprise also non-linear track elements. In such a model, the material properties may be selected to display the physical behaviour of the non-linear track elements. Normally, non-linearities can be found in studded rubber railpads and in the ballast-subgrade material, see Oscarsson (2001). Non-linearities in the track have been treated as extra loads giving a force-displacement relationship for the track comparable with the non-linear characteristics of the real track, Oscarsson (2001).

A survey of railway track dynamics and modelling of the train-track interaction is given in Dahlberg (2003).

2. Railway ballast

2.1 Ballast materials, requirements and properties

Ballast must be capable of withstanding the loading from the train traffic, a large number of loading cycles, vibrations of varying frequencies and intensities, repeated weathering and other factors that cause deterioration. The ballast must satisfy the following conditions:

- it must be tough enough to resist breakdown through fracturing, and it must be hard enough to resist attrition through wear with neighbouring ballast particles,
- it must be dense enough so that it will have sufficient mass to withstand lateral forces to anchor the sleepers,
- it must be resistant to weathering so that weakening of the ballast due to crystallization or acidity do not occur, and
- for good mechanical stability in track, ballast particles should be angular and equidimensional in shape with rough surfaces to provide maximum friction (well-rounded particles such as those common in glacial gravel ballasts constitute an unstable track bed).

2.2 Experimental measurements on ballast

Field and laboratory measurements have been performed by many researchers, for example by Tarumi (1994), Raymond (1994), Trevizo (1995), Read *et al.* (1995), Guérin (1996), Guérin *et al.* (1999), Jacobsson (1998), Gotschol (2002), Stöcker (2002), Augustin (2003) and others.

Guérin (1996), Guérin *et al.* (1999) used tests on railway ballast at reduced scale (1/3) to establish a settlement law in the vertical plane. The ballast sample was subjected to a vertical loading that simulated the loading of a TGV (high-speed train) bogie. The settlement law is further discussed in Section 3.3 below.

Jacobsson (1998) reviews and discusses test procedures on ballast materials, with triaxial cells as test devices. The review covers test procedures including monotonic loading, quasistatic loading, cyclic loading with constant amplitude, repeated loading with variable amplitude, repeated loading with variable confining stress, shear tests, and other loading cases. Different mathematical descriptions of the constitutive behaviour of the material are summarized. In particular, descriptions of the material's resilience properties and the evolution of permanent deformations, as function of stress state, stress history and the degree of compaction, are described.

Gotschol (2002) reports an experimental investigation on the constitutive behaviour of non-cohesive soils and railway ballast under cyclic dynamic loading. Both cyclic-viscoplastic and cyclic-viscoelastic models were examined. The knowledge gained through the experiments provided a basis for appropriate constitutive equations for numerical modelling performed by Stöcker (2002). Stöcker implemented the constitutive approach into a FEM program and carried out modelling of long-term deformation behaviour of foundation structures subjected to cyclic loading. Extensive testing on railway ballast provided the basis for description of plastic and elastic strain of the ballast under cyclic loading. A significant difference between the elastic behaviour of non-cohesive soils and ballast materials under static and dynamic loading could be identified. The objectives of the work by Stöcker (2002) concerns the necessity of investigating the long-term deformation behaviour of granular soils and railway ballast subjected to cyclic loading from railway traffic.

Augustin *et al.* (2003) draw several interesting conclusions from their measurements and calculations. The ballast under badly bedded sleepers undergoes greater vertical deformation than under well bedded sleepers. Cyclic loading tests of ballast showed that the stress minimum has a strong influence on the permanent deformation. Simulations with a hypoplastic material law modelled the main features of the mechanical behaviour of ballast correctly, including increase of stiffness, decrease of hysteresis and influence of stress minimum. Using a numerical model, it was shown that the initial state has a strong influence on the long-term behaviour of the track. Variation of density leads to higher permanent deformations and to higher differential changes of ballast height.

2.3 Ballast modelling

A review of research on railroad ballast used as track substructure has been presented by Peplow *et al.* (1996). Evaluation and specification of ballast materials using different methods is examined and reviewed. The ballast materials and their interactions are complex from a physical point of view. Hence appropriate constitutive laws for the response of the materials have been developed including resilient modulus and variable modulus approaches. The laws are verified with respect to laboratory tests.

A historical method for assessing track performance is the use of track modulus. Its value for static and dynamic loading for track structure-ballast interaction is reviewed and discussed in Peplow *et al.* (1996). For static loading a comprehensive review of one approach is given. That approach uses multi-layer linear elastic static theory to represent the ballast and the subgrade layers. A number of finite element models have been developed and compared, and stresses incurred within the ballast and subgrade for various configurations are discussed. For modelling of the dynamic interaction between the track structure and the ballast a simple beam on elastic foundation model (the Winkler foundation) is used in many analyses. In a number of studies a discrete support model with a finite number of parameters describing the rail, railpad, and sleeper combination is used. In the final part of the survey by Peplow *et al.*, modelling of the dynamic interaction between the train and the track structures is reviewed. The review also presents some mathematical and numerical methods dealing with the static and dynamic interaction of the train-track system and the sub-structure.

Janardhanam and Desai (1983) used conventional triaxial equipment to study graded aggregates. As a result two models were proposed. Three different materials were investigated; one with the grain size distribution of an ordinary ballast material, and two scaled down materials. It was found that

- the resilient modulus is dependent on particle size
- the volumetric behaviour is significantly affected by particle size
- the shear behaviour is dependent on particle size

Li and Selig (1995) report on plastic deformation of ballast and substructure layers. Excessive and rapid accumulation of plastic deformation leads to an excessive rail settlement, and thus requires frequent maintenance.

The effect of different vehicles on track deterioration (and consequent maintenance costs) has been examined by Iwnicki *et al.* (2000). A number of track settlement models were investigated. It was noted, for example, that the ORE (Office de Recherches et d'Essais de l'Union Internationale des Chemins de Fer) deterioration model contains no track parameters at all but only loading parameters such as traffic volume, dynamic axle load, and speed. Such a model, containing no track parameters, would imply that two different tracks, one stiff and one soft, would

undergo the same deterioration if they were subjected to the same loading. This can be questioned, of course, since in such a case the quality of the ballast and subgrade material would have no influence on the track deterioration.

In a research programme in Germany, aiming at a better understanding of the dynamic interaction of vehicle and track and the long-term behaviour of the components of the entire system, settlement and destruction of the ballast and the subgrade were examined. Non-linear behaviour of the ballast was investigated experimentally in laboratories and simulated by new material laws and the coupling of the track model and the subgrade was defined using various models, see Gotschol (2002), Stöcker (2002), Kempfert *et al.* (2003), and Popp and Schiehlen (2003).

In Suiker (2002), advanced models were developed in order to provide detailed insight into short-term and long-term mechanical processes in a railway track. One of the purposes of the work was to derive enhanced continuum models from the discrete micro-structure of a granular material (the ballast). The long-term mechanical process concerns the evolution of track deterioration as a result of a large number of train axle passages. A model that simulates the plastic deformation behaviour of the track bed during each loading cycle (wheel passage) would be unattractive. Instead, a model is employed that captures only the envelope of the maximum plastic deformations generated during the cyclic loading of the track.

A constitutive model for coarse-sized granular materials subjected to cyclic loading with a high number of loading cycles was presented by Jacobsson and Runesson (2002). The evolution law of the permanent deformations was based on an analogy with viscoplastic formulation, where the traditional time scale was replaced with the number of cycles. Calibration results based on cyclic CTC-tests and one-dimensional cyclic compression tests were presented. It was shown that the model can reproduce the experimental data in an acceptable way.

Jacobsson (2003) PhD thesis

3. Track settlement

When a track is loaded by the weight of the train and, superimposed to that, high-frequency load variations, the ballast and subground may undergo non-elastic deformations. When unloaded, the track will not return exactly to its original position but to a position very close to the original one. After thousands and thousands of train passages, all these small non-elastic deformations will add, differently in different parts of the track, to give a new track position. This phenomenon is called differential track settlement. The track alignment and the track level will change with time. Depending on the subground, the wavelength of these

track irregularities will be of the order of metres up to hundreds of metres. The uneven track will induce low-frequency oscillations of the train. Succeedingly, the track load variations will increase and so will the track settlement. Especially, the transition area from an embankment to a bridge is a place where track settlements use to occur, see Figure 2. In the lower part of the figure (Figure 2) it can be seen that large track settlements occur at 9.4 km due to a bridge and at 11.4 km and 11.7 km due to embankment and light-weight fill.

Railway track will settle (change its position) as a result of permanent deformation in the ballast and underlying soil. After having been used some time, the track will not be so straight and at so good level as it was when it was new. The settlement is caused by the repeated traffic loading and the severity of the settlement depends on the quality and the behaviour of the ballast, the sub-ballast, and the subgrade.

Track settlement occurs in two major phases:

- directly after tamping, when the track position has been adjusted to a straight level, the settlement is relatively fast until the gaps between the ballast particles have been reduced and the ballast is consolidated,
- the second phase of settlement is slower and there is a more or less linear relationship between settlement and time (or load).

The second phase of settlement is caused by several basic mechanisms of ballast and subgrade behaviour:

- continued (after the first phase) volume reduction, i.e. densification of the ballast and subground, caused by particle rearrangement produced by repeated train loading,
- sub-ballast and/or subgrade penetration into ballast voids. This causes the ballast to sink into the sub-ballast and subgrade and the track level will change accordingly,
- volume reduction caused by particle breakdown from train loading or environmental factors; i.e. ballast particles may fracture (divide into two or more pieces) due to the loading,
- volume reduction caused by abrasive wear. A particle may diminish in volume due to abrasive wear at points in contact with other particles, i.e. originally cornered stones become rounded, thus occupying less space,
- inelastic recovery on unloading. Due to micro-slip between ballast particles at loading, all deformations will not be fully recovered upon unloading the track. In this case the permanent deformation is a function of both stress history and stress state,
- movement of ballast and subgrade particles away from under the sleepers. This causes the sleepers to sink into the ballast and subgrade,

- lateral, and possibly also longitudinal (in the rail direction), movement of sleepers causing the ballast beneath the sleepers to be “pushed away”, and the sleepers will sink deeper into the ballast.

Here, the first four items concern densification of ballast and subgrade, whereas the three last-mentioned items concern inelastic behaviour of the ballast and subgrade materials.

Concerning the volume reduction or densification caused by particle rearrangement produced by repeated train loading, it could be mentioned that the train load also may have an opposite effect. Due to the elastic foundation, the train load will lift the track (rails and sleepers) in front of and behind the loading point, thus reducing or eliminating the preload (the dead load) caused by the rails and sleepers on the ballast. On the same time, due to the dynamic high-frequency train-track interaction forces, waves will propagate from the wheel-rail contact patches, either through the ballast and subgrade or through the track structure, to the region with the unloaded ballast. These waves will normally propagate faster than the train, giving vibrations in the unloaded ballast. This, in turn, may cause a rearrangement of the ballast particles so that the density decreases. As a result, this may cause a lift, at least temporarily, of the track.

3.1 Experiments and measurements on track settlements

Yoo and Selig (1979) have monitored performance of ballast and subgrade layers under repeated traffic loading. Test sections included wooden and concrete sleepers, tangent and curved track, and various types and depths of ballast. Soil-strain gauges were installed in the ballast layers to measure the vertical and horizontal strains. Vertical extensometers were used to determine the settlement, and soil-stress gauges at the ballast/subgrade interface were used to measure vertical stress on subgrade. Monitoring included both long-term measurements of permanent strain and deformation and dynamic measurements of elastic response under wheel loading. The authors found that the most significant features shown by the dynamic records, for each application of transient wheel loads, were that deformations of the track support system appeared mostly elastic in nature and plastic deformations were almost negligible. However, static measurements taken periodically showed that there was a gradual accumulation of permanent strains with traffic.

Morgan and Markland (1981) have tested the stability of a scaled down ballast bed by measuring the settlement of a plate placed on the surface when subjected to several cycles of sustained loading. The authors found that initially loose ballast stabilised noticeably when the ratio of peak table acceleration to that of gravity was greater than unity. Moreover, for a stabilised bed it was found that resonant frequencies occurred at which only a small force produced a fast penetration of the plate into the ballast bed.

Using a small railway model, Augustin *et al.* (2003) investigated the long-term behaviour of ballast track. The test showed that saddles and troughs appeared fixed in place along the track and that ballast under badly bedded sleepers undergoes greater vertical deformation than under well bedded sleepers. It was also shown that the stress minimum at cyclic loading had a decisive influence on the permanent deformation. The accumulated vertical deformation by repeated loading was significantly increased when the stress minimum was reduced. This explains the increased settlements at the troughs at hanging sleepers that was found in the experiments. Also in Baessler and Ruecker (2003) it is shown that the influence of the load minimum at cyclic loading is essential for the track settlement.

3.2 Track stiffness irregularities and track settlement

Track stiffness variation due to the sleeper spacing has already been mentioned. Other places along the track having variable stiffness are at switches and turnouts. The sleepers have different lengths and different spacings at the switches, and this influences the track stiffness. The symmetry of the track is lost at switches, implying that the left and right rail will have different stiffnesses (the stock rail keeps the stiffness of the track, whereas the switch rail becomes stiffer because of the longer sleepers supporting that rail). Especially if a switch is equipped with a manganese crossing (frog), the rail bending stiffness (EI) will change dramatically at the crossing. Such a sudden change of the stiffness will induce transient and high-frequency vibrations in the train and in the track. The mass is also larger at the crossing making inertia forces larger, see Andersson and Dahlberg (1999).

The track superstructure is seldom built on a homogeneous substructure. Due to a variable stiffness of the track subgrade, the track stiffness experienced by the train will vary along the track. As this stiffness is more or less random along the track, it will induce low-frequency random oscillations of the train.

Nowadays it is possible to measure the vertical track stiffness continuously along the track. Banverket, the Swedish National Rail Administration, has developed a trolley by which the track can be loaded and track stiffness measured while moving along the track at a speed of up to 30 km/h, Berggren *et al.* (2002). Measured track stiffness along a distance of 3000 m is shown in Figure 2. It is seen in the figure that there may be rather rapid changes of the track stiffness, and it is also seen that the track subground has a large influence on the track stiffness. It is seen in the figure (km 11.4 to 11.65) that a pile deck and a bridge makes the track very stiff, whereas the light-weight fill makes it very flexible (at km 11.4). A transition area from an embankment to a bridge is also a place where large and rapid changes of the track stiffness may occur.

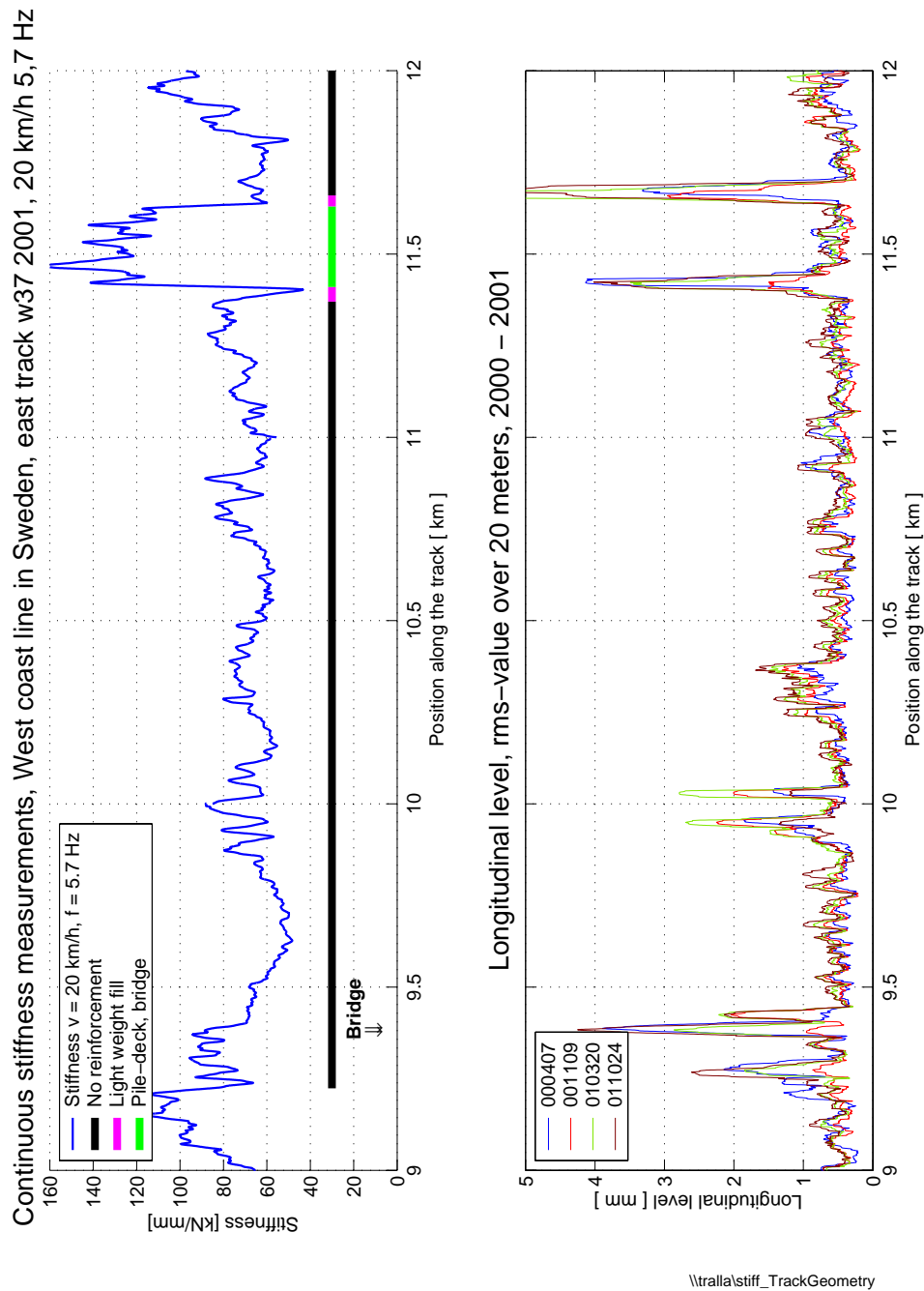


Fig. 2. Track stiffness (axle load divided by track deflection) along railway track as measured by the Banverket track stiffness measurement trolley, and longitudinal level of the track (positive downwards, meaning that a large peak in the curve indicates a local settlement of the track. Figure provided by Eric Berggren at Banverket.

3.3 Modelling ballast and track settlements

The mathematical modelling of railway track settlements caused by inelastic behaviour of ballast and subground will now be looked at in some detail. Railway track will settle as a result of permanent deformation in the ballast and underlying soil caused by repeated traffic loading. Permanent deformation of track structure results from several mechanisms of ballast and subgrade behaviour (see above). Some mechanisms are:

- (1) Volume reduction or densification caused by particle rearrangement under the cyclic shear straining produced by repeated train load.
- (2) Inelastic recovery on unloading or stress removal. Permanent deformation is a function of both stress history and stress state.
- (3) Volume reduction caused by particle breakdown from train loading or environmental factors.
- (4) Subgrade penetration into ballast voids. This causes the ballast to sink into the subgrade.

The first two points apply to both ballast and subgrade, the third applies mainly to ballast and the fourth to subgrade.

Based on the assumptions of ballast densification caused by particle rearrangement and inelastic recovery on unloading of the ballast, Alva-Hurtado and Selig (1981) developed a methodology to calculate the permanent deformation response. If the assumption that the ballast starts from an uncompressed state is made, then a simple model may be formulated. According to the authors, the total permanent strain ε after load cycle N of a series of identical load cycles is given by

$$\varepsilon = \varepsilon_1 (1 + C \log(N))$$

where ε is the total permanent strain, ε_1 is the permanent strain after the first load cycle and C is a dimensionless constant controlling the rate of growth of deformation. Ford (1995) investigated what happens if the load cycles are not identical (i.e. if they do not have the same amplitude). He describes the curve of permanent strain for the first cycle, ε_1 , against the ratio of applied stress $\Delta\sigma$ and failure stress σ_f . He also described the situation of varying the load in the ballast and how the equation above may be modified accordingly. A similar logarithmic form of the settlement function was suggested by Hettler (1984). This and other German models have been discussed and exploited by Mauer (1995).

Another early model trying to simulate the track deterioration was suggested by Shenton (1985). Several factors influencing the track deterioration were investigated. The factors were dynamic forces, rail shape (lack of straightness), sleeper spacing, sleeper support, ballast settlement (due to randomness and inhomogeneity), and substructure. Shenton also makes some comments on the logarithmic settlement law (the settlement is assumed to be proportional to $\log N$). This law may be considered

reasonable over a short period of time, but it might significantly underestimate the settlement in the case of large numbers of loading cycles. Based on laboratory and field experiments, Shenton suggests a settlement law where the settlement is considered to be proportional to the fifth root of the number of axles (number of load cycles). This agrees well with site measurements up to 10^6 load cycles. To obtain an acceptable fit for larger values of load cycles, a linear term is added to the settlement law, giving a settlement equation on the form

$$y = K_1 N^{0.2} + K_2 N$$

where the constants K_1 and K_2 are selected so that the second term becomes significant only for values of N larger than 10^6 .

The numerical values of the factors K_1 and K_2 depend on a number of factors, such as axle load, rail section, sleeper spacing, and track and foundation stiffnesses. The axle load – or rather the wheel/rail contact force – is varying from one sleeper to the next due to dynamic effects from rail and wheel irregularities and from other irregularities in the track. There are also mixed axle loads. Further, the settlement depends on size and type of sleepers, ballast type and condition, sub-ballast and subgrade, type of track maintenance, and lift given to the track during maintenance.

According to Shenton, the axle load is probably one of the most important factors influencing the settlement. It was found (estimated) that at low train speeds a linear relationship between axle load and settlement is a reasonable approximation. Further, tri-axial tests have shown that the higher loads predominate the track settlement. Loads below half the maximum load had no influence on the settlement. This was valid even when the number of low loads was a significant proportion (90 per cent) of the total number, implying that for many railway lines, it might be only the locomotive axles that cause track settlement. For N different axle loads F_i , Shenton suggests an equivalent axle load F_{eq} as

$$F_{eq} = \left(\frac{\sum_{i=1}^N F_i^5}{N} \right)^{0.2}$$

In the PhD thesis by Fröhling (1997), a new approach to determine differential track settlement due to dynamic wheel loading and spatially varying track support conditions is formulated. To account for a mix of wheel loads at a particular sleeper, a procedure for superimposing permanent strains in the ballast is suggested. The accumulated strain giving the track settlement is based on a cumulative relationship similar to the Palmgren-Miner rule for accumulation of fatigue damage in mechanical structures.

Following up earlier works, Fröhling (1998) reports an investigation in which he suggest a relationship between vehicle and track parameters, the dynamic response of the vehicle, and measured differential track settlements. Based on measured results, it was found that the differential settlement of the track was dominated by the spatial variation of the track stiffness. The relationship suggested contains factors depending on measured average track stiffness, calculated track stiffness (calculated by the program GEOTRACK), deviator stress, dynamic and static wheel load, and an experimentally determined exponent. The dependence on number of loading cycles N is set proportional to $\log N$.

In an extensive study by Guérin (1996) (see also Guérin *et al.* (1999)) a scaled-down model (scale 1 to 3) is used to investigate the ballast and subground settlement. The settlement is divided into two phases: first one phase of ballast compaction, where the ballast settlement rate is relatively large, and then the second phase with a steady-state settlement rate. The length of the first phase depends strongly on the quality of the pre-compaction of the sample, and it comprises from 50 000 cycles up to almost one million loading cycles. In the second phase, the settlement τ per loading cycle (N) is expressed as a function of the maximum elastic deflection d of the ballast and subground sample during the loading cycle as

$$\frac{d\tau}{dN} = \alpha d^{\beta}$$

where α and β are material parameters. For the material used in the scaled-down experiments reported, and for τ and d in mm, the parameters were determined (by linear regression) to $\alpha = 0.48 \cdot 10^{-6}$ and $\beta = 2.51$ (the coefficient of correlation was 0.61, indicating some scatter in the data).

It is noticed that this settlement law corresponds to the second term of the law proposed by Shenton (1985), and also to the second term of the law proposed by Sato (1995). Further, there is no coupling between the first phase of rapid settlement (which is measured but not modelled by Guérin) and the second stationary phase. In practice, if there are different physical phenomena involved in the two phases, there should be no coupling in the modelling of them either. The separation of the settlement into two phases also reminds on an early model proposed by Holzlöhner (1978). Holzlöhner suggested a generalized settlement law divided into two loading periods, one preloading period comprising the first n_a cycles, and then the second period comprising the following cycles.

For most tracks there is a non-linear relationship between applied load and track (or ballast) deflection. By using the deflection d as an input to the model, one sees that this non-linearity enters only indirectly (via d) into the track settlement. By using the maximum elastic deflection d as a measure of the loading of the ballast, cycles with different loading amplitudes – and thereby different deflections – may be handled. The ballast settlement increases non-linearly with d (d to the power β). Using the

value of β found by Guérin ($\beta = 2.51$), one obtains for a ten per cent increase of the ballast deflection (not necessarily corresponding to a ten per cent increase of the load) an increase of the settlement rate by a factor $1.1^{2.51} = 1.27$. Thus, in this case (with the ballast material and loading levels used here) a ten per cent increase of the deflection yields a 27 per cent increase of the ballast settlement.

The finding that low loads have no influence on the track settlement (Shenton (1985), Sato (1997), for example) strengthens the assumption that there might exist a threshold value of the loading below which no settlement will occur. In the other extreme end of the loading (very high loads), the settlement is proportional to the fifth power of the loading (and somewhere in-between, there might be a linear relationship between settlement and loading).

Thus, loading at a level below the threshold value would then result in a purely elastic (linear or non-linear) behaviour of the track sub-structure. Loading above the threshold value, but for moderate values of the sleeper/ballast pressure, there is a more or less linear relationship between the loading level and the track settlement per loading cycle. For larger values of the pressure, however, there is a non-linear region where the settlement depends very strongly on the sleeper/ballast pressure; the settlement might be proportional to the fifth power or more of the pressure, see Sato (1997) and Dahlberg (2001). The different behaviour in the different loading ranges might possibly depend on that different degradation or failure mechanisms are involved in the different loading ranges. For example, there might be abrasion and compaction of the ballast particles in the lower loading range and fracture of ballast particles in the higher range.

As mentioned in the Introduction to this paper, different physical phenomena might be involved in the track settlement. It is possible that one physical phenomenon (for example ballast densification) causes the main part of the settlement in the initial part of the loading immediately after tamping, whereas other physical phenomena (for example friction, wear and fracture of the ballast particles) might be involved later. From this point of view, the separation into two loading periods is a promising approach. The different phenomena may then be treated in the different settlement periods.

4. Theoretical modelling of railway track and sub-structure

The cheapness and practical advantages of ballast make it unlikely that ballasted track will be replaced by ballastless track (Eisenmann, 1995). The future use of ballasted track, for example in new high speed lines, may continue, however, only if a fundamental physical understanding of ballast behaviour is available. When determining the net effect of traffic loads on the track structure, the track model has to interrelate the components of the track structure so that their complex interactions

are properly represented. Such a model provides the foundation for predicting track performance and, therefore, the technical and economic feasibility of track design and maintenance procedures. Analyses become complicated, however, by the fact that the physical states of the ballast change with time.

A historical method for assessing track performance is the use of track modulus. Its value for static and dynamic loading for track structure/ballast interaction is reviewed and discussed. A number of finite element models have been developed and compared, and stresses incurred within the ballast and subgrade for various configurations are discussed. Many analyses for dynamic modelling of the track structure/ballast interaction problem use a simple beam on elastic foundation model (the Winkler foundation). In a number of studies a discrete support model with a finite number of parameters describing the rail, railpad, and sleeper combination is used. Modelling of the dynamic interaction between train and track structure is reviewed in the final part of this Section.

4.1 Modelling for static loading

The evaluation of track performance due to traffic loads requires the ability to predict realistic pressure distributions at interfaces between the sleeper and the ballast and between the ballast and the subgrade. Thus, theoretical models must include the effects of sleeper bending, sleeper spacing, rail/fastener stiffness, changes in ballast depth, and subgrade and roadbed material properties.

Currently, the analysis of track structures usually follows one of two paths:

- (1) The track structure is represented very simply, *e.g.*, a beam on an elastic foundation wherein the substructure is represented as a spring-damper system (the damped Winkler foundation).
- (2) The track structure is modelled in detail by using a finite-element model.

In the first case, the system is represented so that individual contributions, such as ballast, subgrade and sleeper bending are not sufficiently detailed or easily evaluated. To inter-relate the components of the track structure to properly represent its complex interactions in determining the net effect of traffic loads on the stresses, strains and deformations, several comprehensive models have been developed, Peplow *et al.* (1996).

4.2 Modelling for dynamic loading

The dynamics of the compound train and track system plays an important role when investigating vehicle and track dynamics. Vibrations may lead to track deterioration, such as railhead corrugation growth, damage to track components (railpads, sleepers, ballast), track settlement, and so on.

Techniques to study the train-track interaction can be divided into two groups: frequency-domain techniques and time-domain techniques.

4.2.1 Frequency domain modelling

Using the frequency-domain technique it is possible to investigate the track and wheel response to a “moving irregularity”. Instead of a having a wheel moving on an un-even rail, one investigates a stationary wheel. The rail and the (stationary) wheel are then excited at the wheel-rail contact patch by a prescribed displacement. One may think of this excitation as if a strip of irregular thickness were inserted between the wheel and the rail. The strip is then forced to move between the wheel and the rail so that the irregularity of the strip will excite both wheel and rail. The response of the wheel and the track is obtained in the frequency domain. If the strip thickness irregularity is sinusoidal, the response is found from a receptance function. A non-sinusoidal irregularity (as from a wheel-flat) must first be transformed into the frequency domain by the Fourier transform. The track and wheel receptances and the wheel-rail contact stiffness are combined to form the appropriate transfer function. Together with the Fourier transform of the irregularity, the Fourier transform of the response is obtained, and the inverse transform provides the time-domain response. Several authors have used this technique to investigate the development of short wavelength corrugation on the railhead.

If a continuously supported rail is excited by a harmonically varying moving load, then the track response can be determined in a coordinate system following the load. The response is then assumed to be stationary. This topics has been thoroughly investigated in the book mentioned above, Fryba (1999). One method to treat a discretely supported rail is to develop the support reactions into Fourier series (making the support continuous but non-uniform) and then the moving load problem is solved with respect to a coordinate system following the load.

In the frequency-domain technique only fully linear systems can be treated. The track responses are also assumed to be stationary, implying that singular events along the track, such as a rail joint, a sleeper hanging in the rail (no support from the ballast), varying track stiffness, and so on, cannot be treated.

4.2.2 Time-domain modelling

When train-track dynamics is investigated in the time domain deflections of the track and displacements of the vehicle are calculated by numerical time integration as the vehicle moves along the track. The vertical motion of the wheelset should then coincide with the vertical deflection of the rail, while taking the wheel-rail contact deformation into account. The wheel-rail contact force is unknown and has to be determined in the calculations.

The track can be modelled by finite elements and in many cases a modal analysis of the track is performed. The track is then described through its modal parameters, and the physical deflections of the track are determined by modal superposition. Often the vehicle is modelled by use of rigid masses, springs (linear or non-linear) and viscous dampers. If a more detailed response of the vehicle is of interest, then it could be convenient to use modal analysis also for the vehicle deformations (the vehicle is no longer composed of rigid bodies). Modal analysis of a wheelset makes it possible to include elastic deformations of the wheelset without a large increase of the number of degrees of freedom of the compound train-track system, Ripke (1995), Andersson (2003).

The modal analysis technique requires linear models. Every so often track models may comprise also non-linear track elements. Normally, non-linearities can be found in studded rubber railpads and in the ballast-subgrade material, see Oscarsson (2001). In such a track models, the material properties may be selected to display the physical behaviour of the non-linear track elements. Non-linearities in the track have been treated as extra loads on a linear track model, Oscarsson (2001). The extra loads give a force-displacement relationship for the track comparable with the non-linear characteristics of the real track.

4.2.3 Track models

Sometimes the ballast deflects in a highly non-linear manner under load. In particular, the ballast and sleeper may not stay in contact (Profillidis and Poniridis, 1986) and the ballast itself deflects in a non-linear fashion. Energy dissipation occurs due to friction and from wave radiation through the substrate. Despite this, most analyses use a simple linear two-parameter model in the vertical direction, *i.e.* the sleepers rest on a damped Winkler foundation. This model may be justified if only the high-frequency dynamic behaviour is of interest when the load is directly over the sleeper of interest. Further, a bogie passing over a particular sleeper can be modelled by such a linear model. Stiffness and damping coefficients for existing models have to be obtained from correlation of calculated and measured results.

In another model the ballast and substrate are considered together as a visco-elastic half-space. It appears that internal damping is insignificant compared to geometric wave radiation. It has been shown (Mohammadi, 1995) that at least five sleepers should be considered when modelling ground-borne vibration.

A third possible sleeper-support model has been used by Zhai and Sun (1994). This model includes additional ballast masses below each sleeper. The ballast masses are interconnected by springs and dampers in shear. The attraction of such a model is that it offers the possibility of obtaining better correlation between calculated and

measured response. A similar track model was used by Oscarsson and Dahlberg (1997), and optimal values of nine track parameters were determined by correlating calculated frequency response functions to measured ones.

Grassie *et al.* (1982) developed two track models: one continuous, with an infinite Timoshenko beam resting on a homogeneous two-layer support of masses and springs representing sleepers, railpads and ballast; the other discrete, with an Euler beam supported at regularly spaced points. At each point, railpad and ballast are modelled as springs and dampers, and the sleepers are modelled as rigid masses. The rail receptance in the frequency range 600 to 1000 Hz is strongly affected by the pinned-pinned mode. A model with rail supports in discrete points is therefore necessary in this range. Nevertheless, the continuous model gives a good description at other frequencies: for instance at ballast resonance, where rail and sleeper move in-phase on the stiffness of the ballast; or at railpad resonance, where rail and sleeper move out-of phase with the deforming railpad in-between. Measurements of rail receptance performed by Grassie *et al.* (1982) provide realistic data for ballast and railpad stiffnesses and appropriate damping values. Correlating calculated receptance values to measured data is, to date, the only method to determine the ballast and railpad properties, Nielsen (1993). Ballast stiffness and damping values increase with increasing load, yielding a reduced receptance at low frequencies. To achieve acceptable correlation, Nielsen allots extra mass to the sleepers for a loaded track.

A general track support model is a layer of ballast on a three-dimensional half-space. From the work by Rücker (1982) it is clear that theoretical investigation of such a model is difficult.

5. Train/track interaction model with track settlement

In many track models the track stiffness and damping and the track mass are discretized. Then the mass of the track (sleepers, ballast etc.) is modelled by use of rigid masses and the track stiffness and damping are modelled by springs and dampers. In a model proposed by Dahlberg (2001) it is assumed that also the track settlement can be “discretized”. Thus, the settlement of the track is collected in a “settlement element” in the track model.

The model contains one rail (symmetry with respect to the centre line of the track is assumed), sleepers, ballast stiffness (spring elements), ballast damping, and an element beneath the ballast stiffness to take care of the permanent deformations in the ballast and subground giving the track settlement, see Figure 3. The rail is modelled by finite elements, the (half-)sleepers are rigid masses, the ballast springs have stiffnesses that may be linear or non-linear, and the ballast damping is linear (modelled by viscous dampers).

The element taking care of the track settlement is modelled as a solid block (3-D finite elements used) made of a linear elastic-ideally plastic material. This means that if the loading is not too high (i.e. below a threshold value, here the yield limit of the material), no settlement will occur. If, on the other hand, the threshold value (the yield limit) is exceeded, then plastic deformation will occur in the solid block. (It should be noted that this simple model was created to explore if it was possible to use a computer software to simulate the dynamic train-track interaction problem and to consider the track settlement in the same model. In a practical situation, a better model for the track settlement should be used, of course.)

The loading of the track comes from the moving wheel, which is loaded by a constant force: the dead load of the car body. The wheel mass and half of the axle mass are included, so the inertia force from the unsprung mass, i.e. the wheel and the axle, is taken into account.

It was found that a large value of the yield limit of the settlement element material (all elements were elastic) would imply that the track returned to its original position after the wheel had passed (as expected, of course). A low value of the yield limit in some of the settlement elements resulted in a permanent deformation of the track after the wheel had passed. The level of the rail thus changed, giving a bump in the track at the place where the elastic-plastic settlement elements were situated.

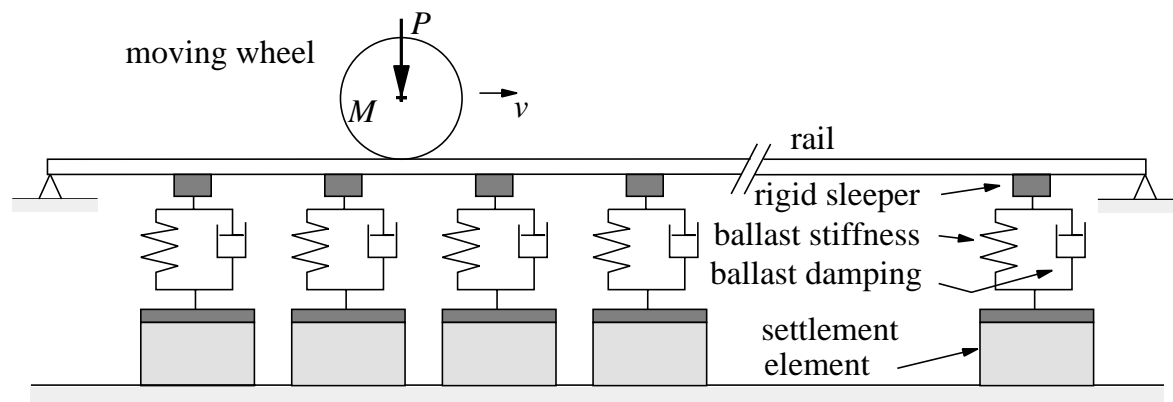


Fig. 3. Model of railroad track containing rail, sleepers, ballast and an element to model the track settlement. The model is loaded by the weight of the track structure (rail and sleepers) and by a moving mass (a wheel, mass M) subjected to a constant force P from the car body. Also inertia forces of wheel and track are taken into account.

In a recent project, the BriteEuram project EUROBALT II, investigations of different track settlement models were made, Dahlberg (2001). Recently the work initiated in the EUROBALT project was continued. The development of a computer program for simulation of the dynamic train/track interaction that was initiated in the EUROBALT project was further developed. This work has been reported by

Månsson (2001). Finite element models of a railway track were developed. Focus was on the performance of the ballast bed. Several finite element models were created and investigated to ensure that a numerical model, suitable for dynamic train/track interaction calculations, could be established. (It could be mentioned here that one modelling problem is the wheel/rail contact; the vertical displacement of the moving wheel should agree with the rail deflection, taking the contact patch deformation into account.) One intention of this project was to implement material models for the ballast and subground so that track settlements could be simulated. The ballast bed was modelled as a continuum with elastic or elastic-plastic material properties. Much work has to be done to obtain a suitable material model for the ballast and the subground. This work continues and will be reported in Lundqvist and Dahlberg (2003).

In another part of this study, linear and non-linear track models were investigated. Calculated results from the two models were compared with in-situ track measurements performed by the Swedish National Rail Administration (Banverket). Important results from that investigation were that linearised track models can be used for one single axle load only and that the wheel load distributes differently into the two track models. Some problems with using viscous damping models in the track were also pointed out, Dahlberg (2002).

In the numerical simulations presented by Augustin *et al.* (2003) the vehicle, the rail, railpads, and sleepers are, respectively, a discrete system, a Timoshenko beam, elastic springs, and rigid masses. The ballast is modelled as a hypoelastic material with intergranular strain; the ballast body is divided into biaxial elements with one element per sleeper (compared to the model in Figure 3, the ballast stiffness and permanent deformation is thus collected in one ballast element per sleeper). The subsoil, finally, is an elastic material with 3D-wave propagation and energy radiation. The initial state of the track is defined by the state of the ballast. The initial ballast height or the initial ballast density (the void rate) were varied.

The simulation results presented show that, beginning from a random height distribution and constant void ratio, track bed waves of characteristic length (8 to 9 sleeper distances) evolved. Once a trough had been established, it continued to grow at the same place. On the other hand, variation of the void ratio between the ballast elements results in variation of ballast stiffness (this variation can arise due to tamping). The simulations show that the influence of the initial variance of the void ratio on the difference of accumulated deformation is considerable. The initial variation of void ratio produces both more mean and more differential height difference than the initial differences in ballast height. This implies that in track construction, particular attention should be paid to a uniform and high ballast density. The authors suggest that maintenance methods should not disturb the stabilized ballast grain skeleton (as tamping do). Thus, the authors recommend that tamping should be replaced by something else.

6. Concluding remarks

Not much work has been done to establish a mathematical model of railway ballast as a continuum whose behaviour can be predicted when excited by the combination of quasi-static and dynamic loadings from the trains. The apparently simple question whether ballast mass (the inertia) has to be introduced into the modelling process or not cannot yet be answered clearly. More research is needed.

7. Summary

Properties of a railway track sub-structure have been surveyed. When a train moves on the track, the train, which is one dynamic system, interacts with the track, that is another dynamic system. Dynamic effects in the compound train/track system become more pronounced when train speed increases and when the axle load gets higher. Oscillations and vibrations of the train/track system then induce a decreased ride comfort, an increased track deterioration and noise emission.

Several mathematical models of the track structure have been presented. These extend from the simple beam on Winkler foundation model to the more sophisticated three-dimensional finite element models, including rails, railpads, sleepers, ballast, and so on.

Most models used to describe the long-term behaviour of a railway track focus on the loading of the track (number of loading cycles and/or passed tonnage) and on the track deflection due to that load. The elastic deflection of a track, however, need not necessarily have any influence on the permanent settlement of the track. Instead of elastic deformations of the track, non-elastic deformations of the ballast and subground should be modelled; a "yield limit" or a "fatigue limit" of the ballast should be sought for. Only little has been found in literature dealing with non-elastic deformations of the track substructure.

To overcome this shortage of models to simulate the long-term behaviour of a track, a computer model has been developed. The track model contains one rail, rigid sleepers, linear or non-linear ballast stiffnesses, and ballast damping. In an element beneath each ballast spring, track settlement can be accumulated. Settlement will occur if the stresses in that element will exceed the yield limit of the material the element is composed of (so far, the material is modelled as linear elastic, ideally plastic). By this model also "hang sleepers" may be obtained as a result of the track settlement. The track model is excited by a moving wheel that is loaded by a constant force (the weight of the car body). Inertia forces from the wheel and the track are included. Dynamic interaction between the wheel and the track is simulated by a time-stepping method. Promising results have been achieved for the train/track dynamic calculations, but further work is needed to improve the track settlement model. This work goes on in the European project SUPERTRACK.

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Focusing on the railway track ballast and subground behaviour, this Bibliography lists literature on the dynamic train/track interaction, railway track dynamics, railway ballast behaviour, and track deterioration (especially track settlements). Also some papers on ballastless tracks are included. Only in exceptional cases literature published earlier than 1995 has been included in this Bibliography.

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