Rainfall - runoff: Unit Hydrograph

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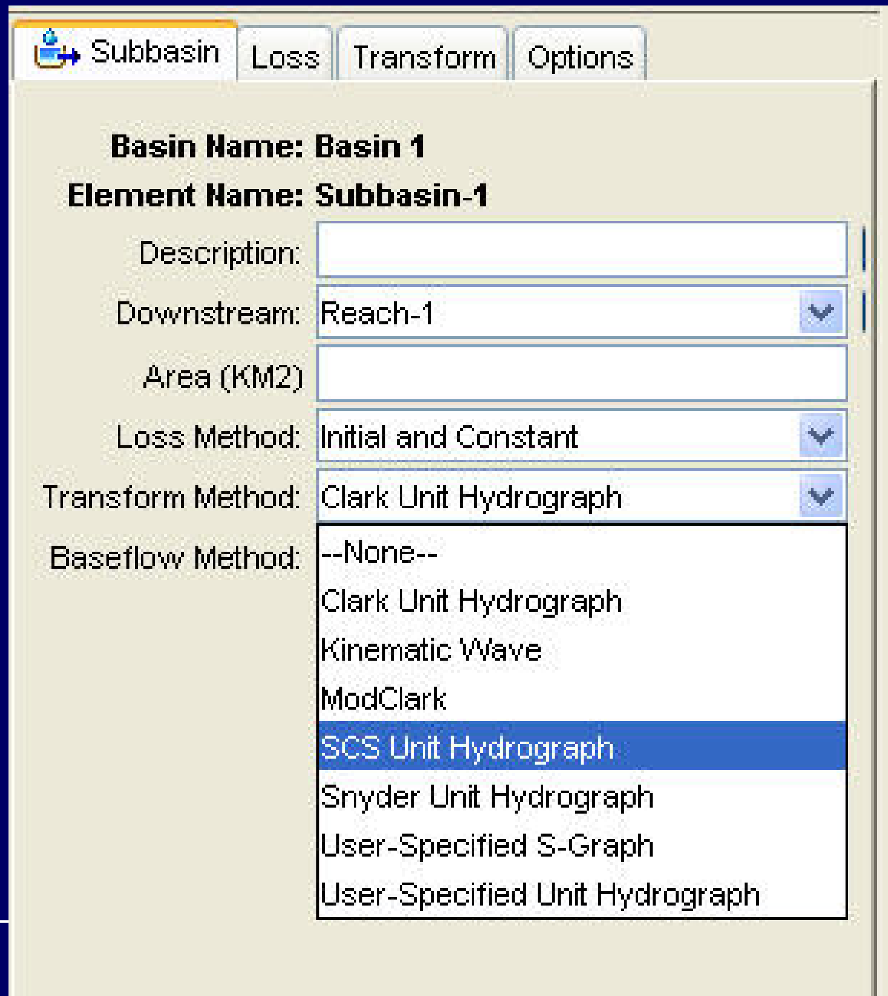
Options in many commercial codes, HMS and others

- HMS Menu
- Transform method, User specified, SCS, etc

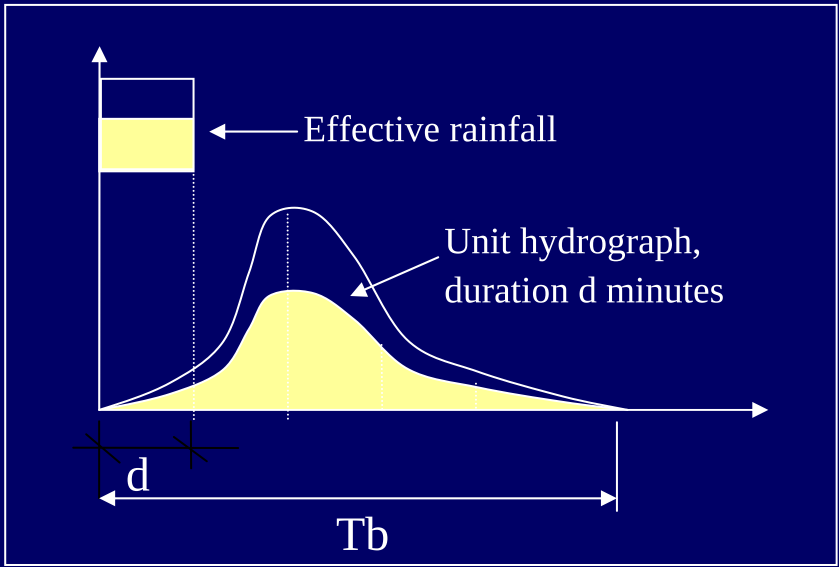


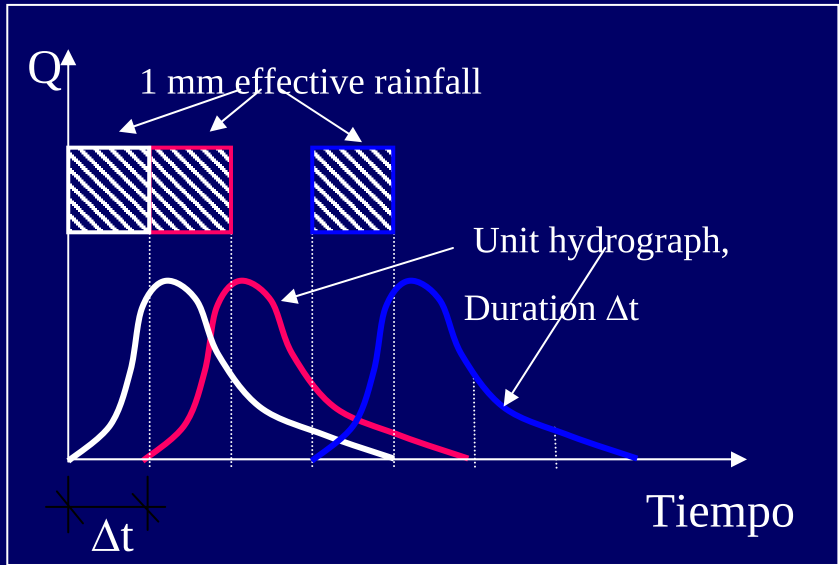
Rainfall - runoff

- Different options
  - **Unit hydrograph (most popular)**
    - **Lumped (global) model**
    - **Hydrological response at the basin outlet from an effective rainfall, 1 mm, duration D minutes, uniformly distributed all over the basin**

UH Hipothesis

- Linear response
- Time invariant (rain event)





Problems in the application

- Lack of real data to obtain it

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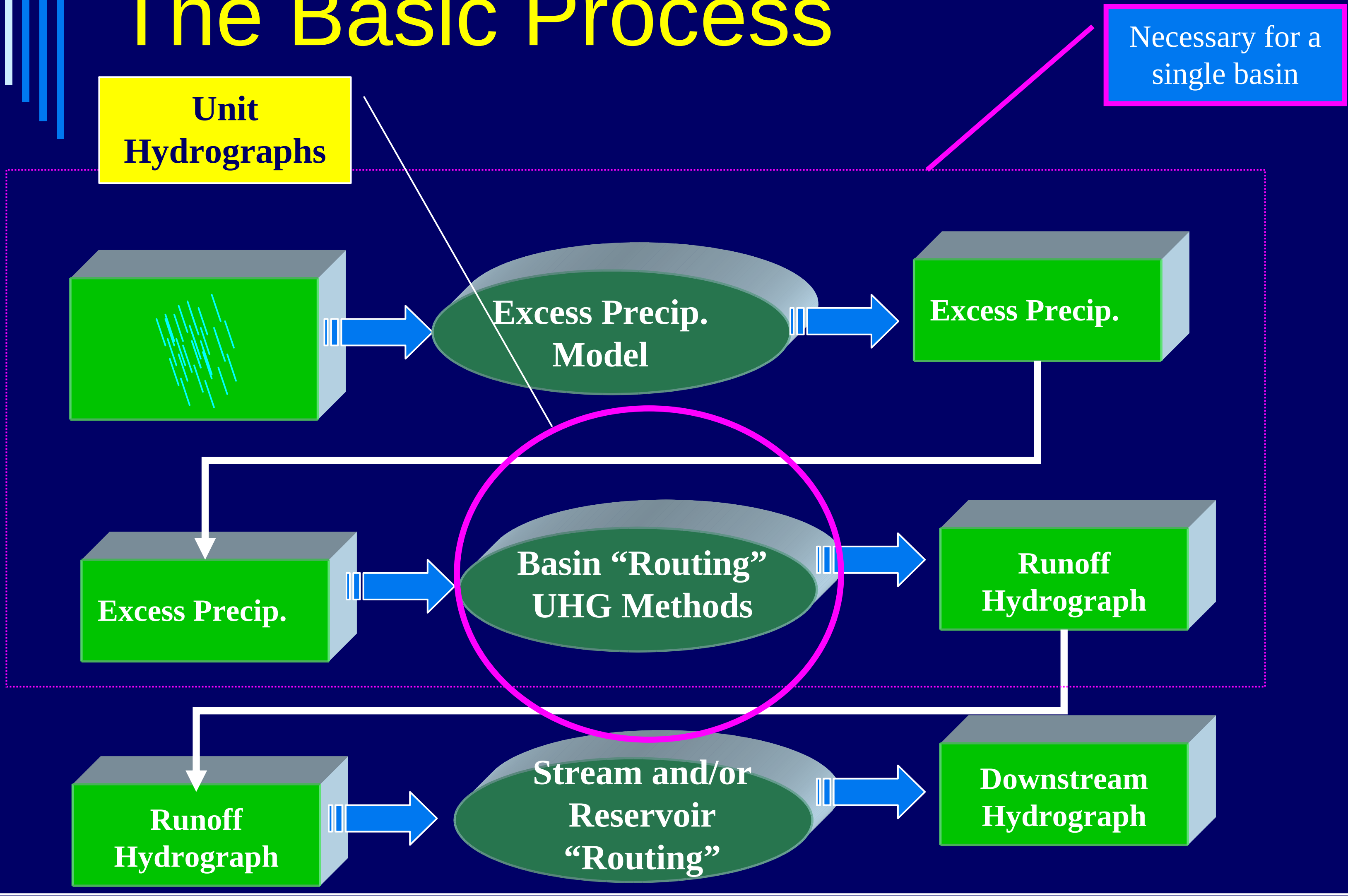
- We need to use synthetic UH

The Basic Process



Proposal of the HU

- Sherman - 1932
- Horton - 1933
- Wisler & Brater - 1949 - “the hydrograph of surface runoff resulting from a relatively short, intense rain, called a unit storm”
- Standardly used in most professional codes for rural basins

Unit Hydrograph "Lingo"

- Duration
- Lag Time
- Time of Concentration
- Rising Limb
- Recession Limb (falling limb)
- Peak Flow
- Time to Peak (rise time)
- Recession Curve
- Separation
- Base flow

Graphical Representation

- Duration
- Lag Time
- Time of Concentration
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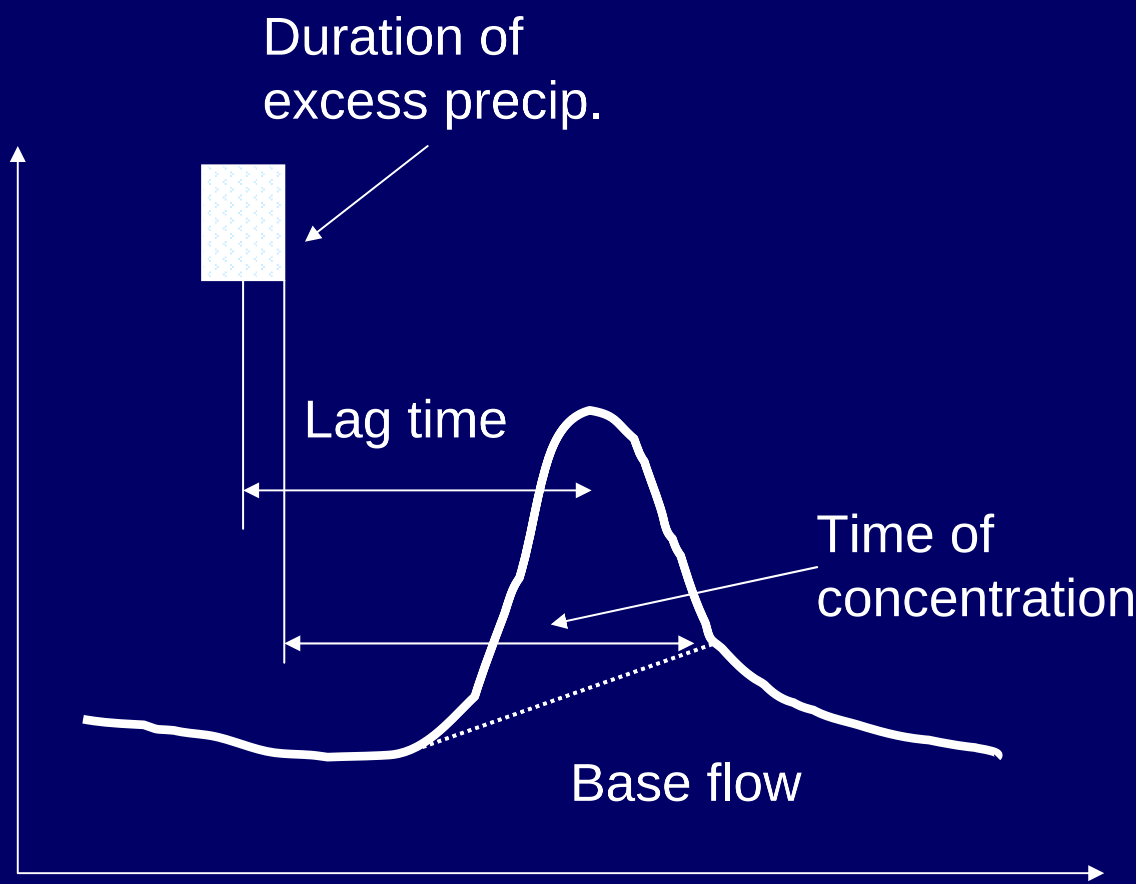


How to get the UH

- Field data measurements I(t), Q(t)
- Approach with synthetic unit hydrographs
  - SCS (NRCS)
  - Time - area curve (Clark, 1945)

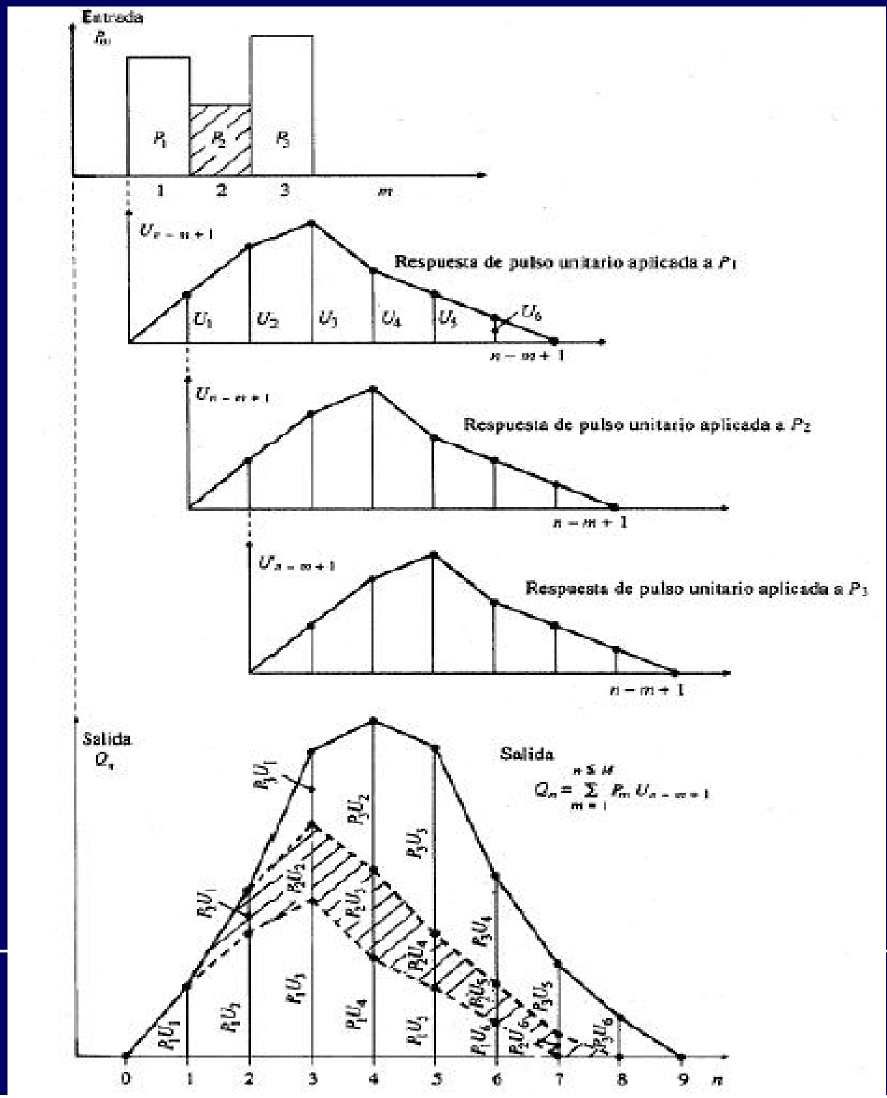
Unit Hydrograph

- ***Hydrological response at the basin outlet from an effective rainfall, 1 mm, duration D minutes, uniformly distributed all over the basin***

- Puntos capitales:
  - ✓ 1-inch 1-mm of effective rainfall
  - ✓ Uniformly distributed in space and time (duration D minutes)
  - ✓ Different hydrographs for different durations

How to use the UH

- Graphical process
- Hyetograph defined with time steps “d”
- Use the unit hydrograph for duration “d”
- Addition of different sub-hydrographs



How to use the UH

- Matrix approach
- Prepare matrix P and U
- Direct operation

$$\begin{bmatrix} I1 & 0 & 0 & 0 \\ I2 & I1 & 0 & 0 \\ I3 & I2 & I1 & 0 \\ 0 & I3 & I2 & I1 \\ 0 & 0 & I3 & I2 \\ 0 & 0 & 0 & I3 \end{bmatrix} \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \end{bmatrix} = \begin{bmatrix} Q1 \\ Q2 \\ Q3 \\ Q4 \\ Q5 \\ Q6 \end{bmatrix}$$

UH obtention

- We need field measurements
  - Rainfall
  - Runoff hydrograph at the basin outlet

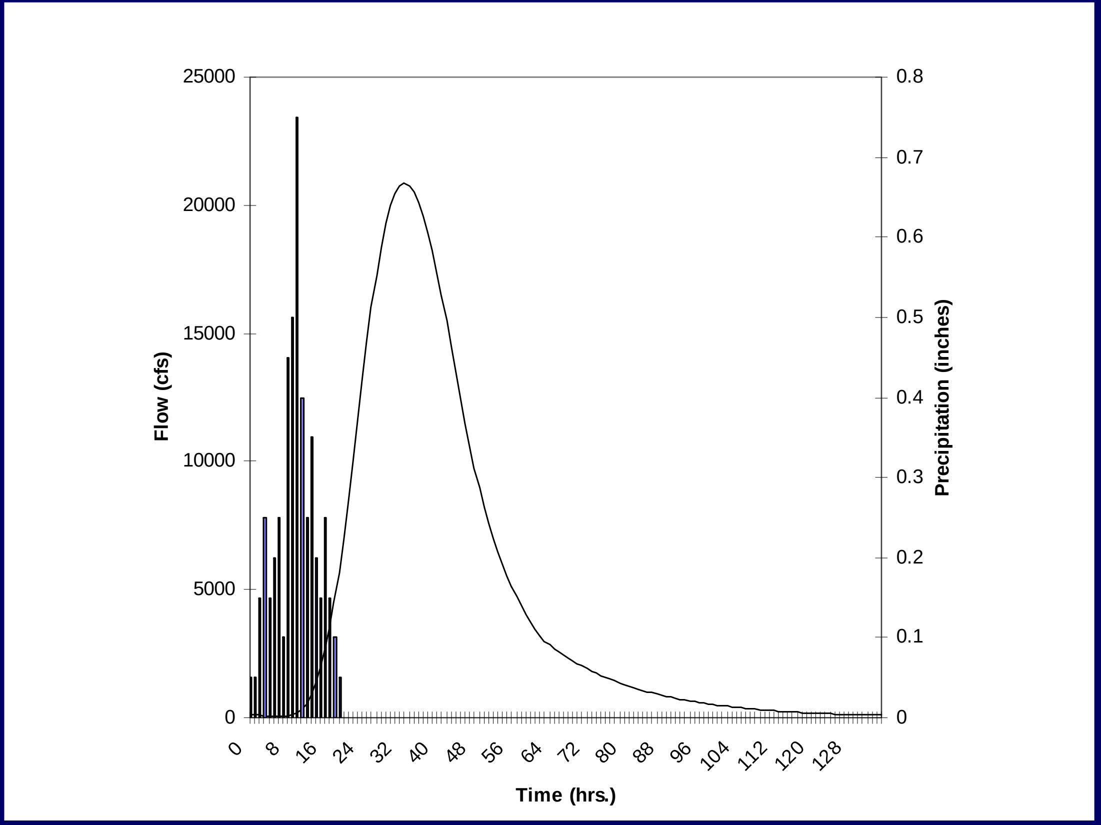
  - We measure the total rainfall (effective rainfall is “estimated”) → SOURCE OF UNCERTAINTY
  - Other problems (errors, spatial distrib.)

UH obtention

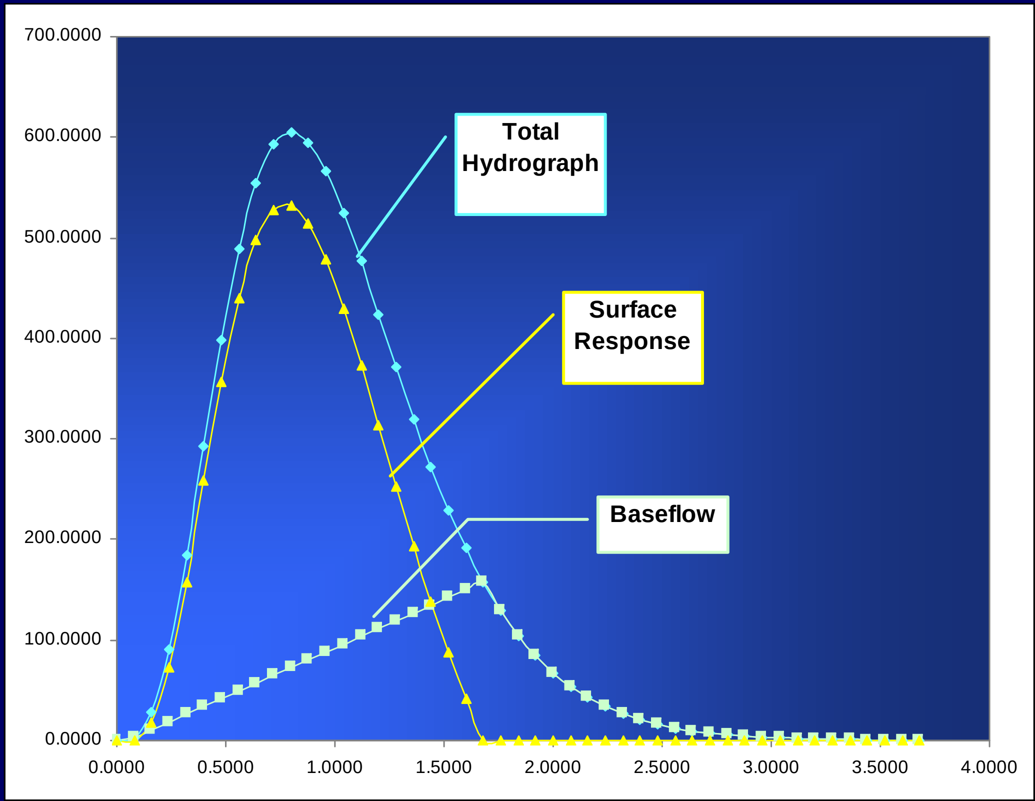
Rules of Thumb :

- … the storm should be fairly uniform in nature and the excess precipitation should be equally as uniform throughout the basin. This may require the initial conditions throughout the basin to be spatially similar.
- … Second, the storm should be relatively constant in time, meaning that there should be no breaks or periods of no precipitation.
- … Finally, the storm should produce at least an inch (1 mm) of excess precipitation (the area under the hydrograph after correcting for baseflow).

Deriving a UHG from a Storm

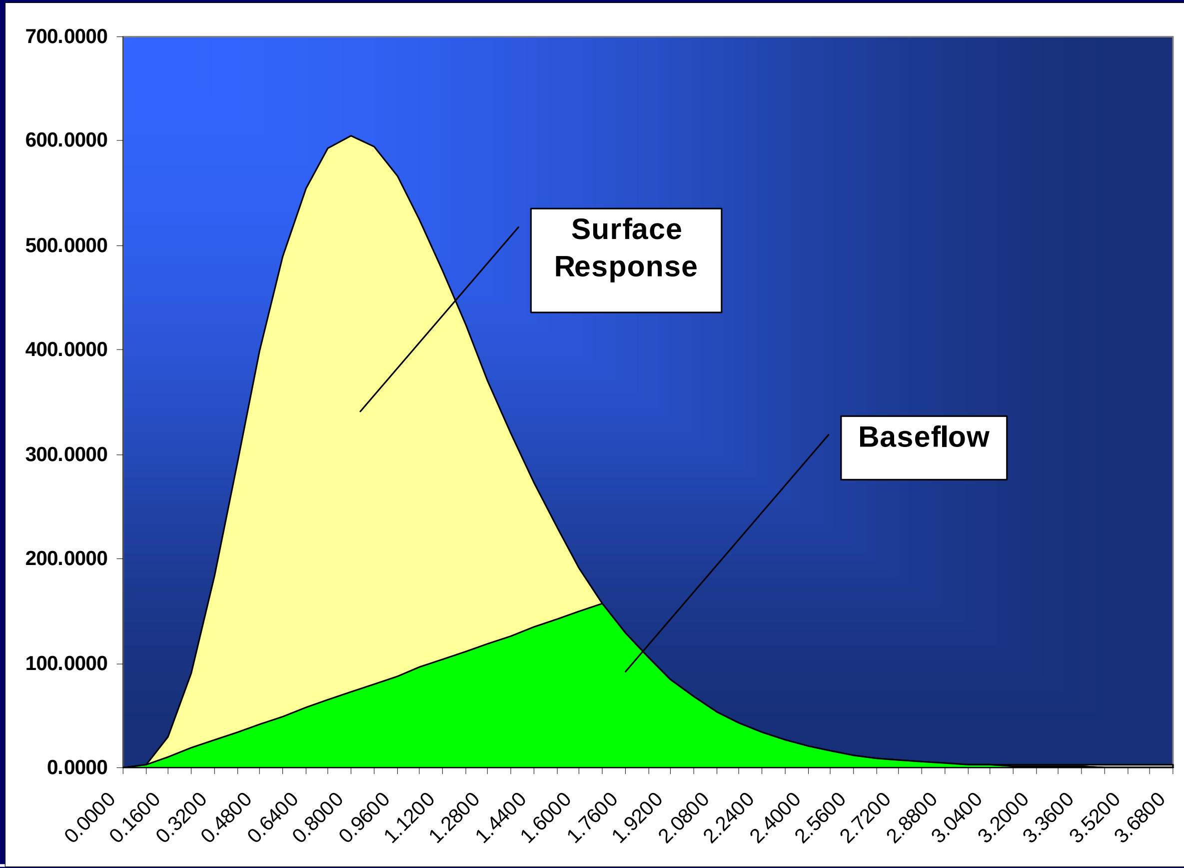


Derived Unit Hydrograph



UH obtention

- Measured Q(t), surface and groundwater response
- We want just the surface response

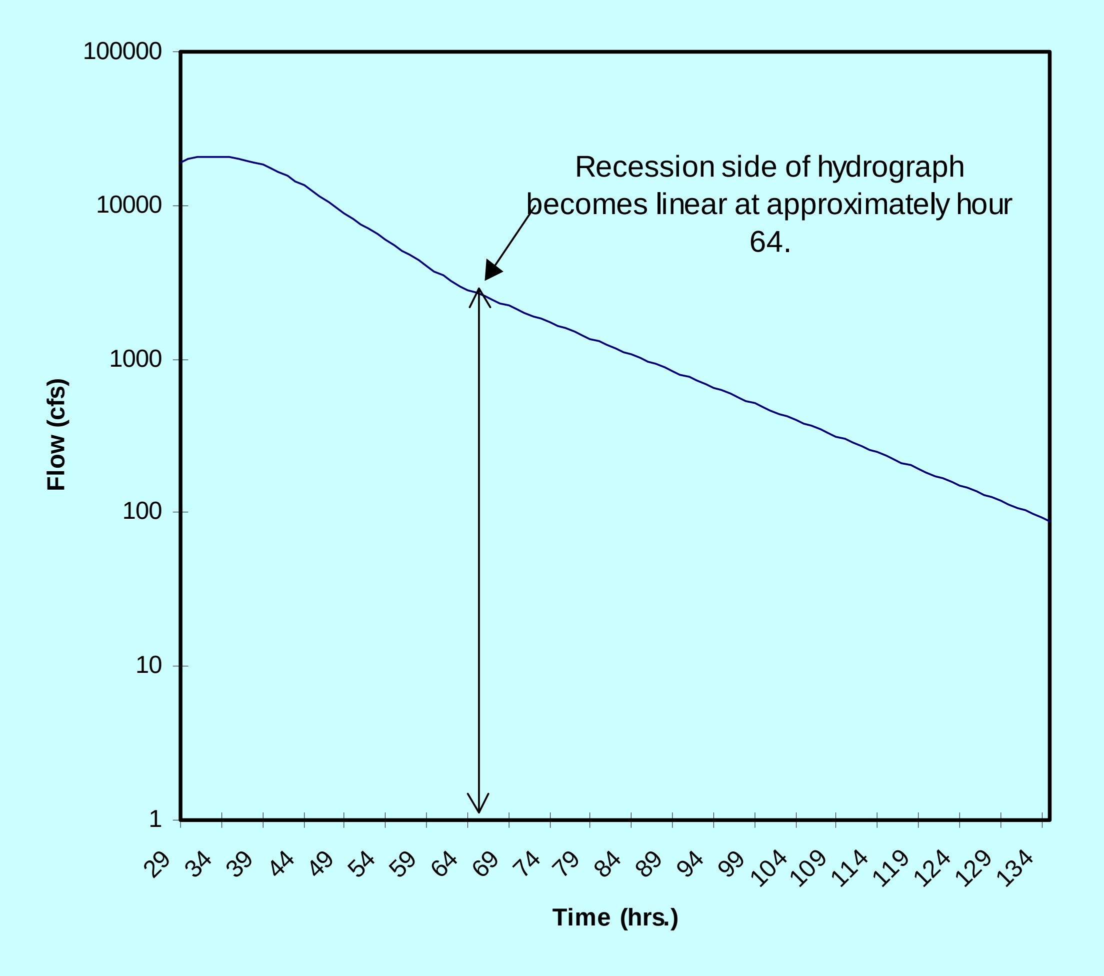


Separation of Baseflow

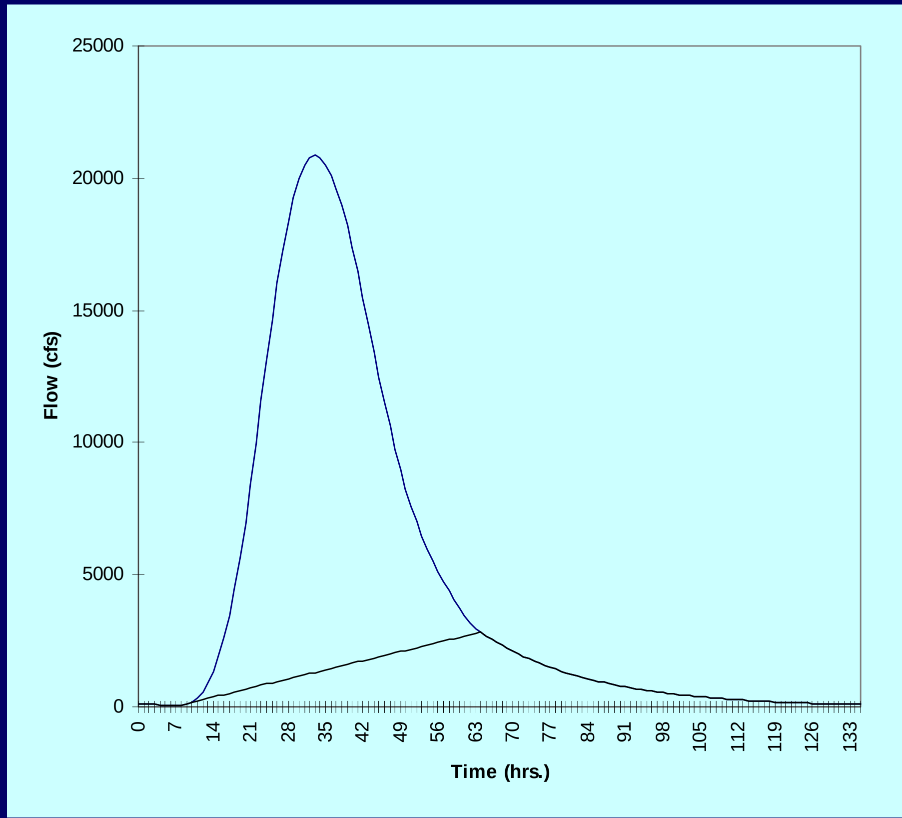
- ... generally accepted that the inflection point on the recession limb of a hydrograph is the result of a change in the controlling physical processes of the excess precipitation flowing to the basin outlet.
- In this example, baseflow is considered to be a straight line connecting that point at which the hydrograph begins to rise rapidly and the inflection point on the recession side of the hydrograph.
- the inflection point may be found by plotting the hydrograph in semi-log fashion with flow being plotted on the log scale and noting the time at which the recession side fits a straight line.

Semi-log Plot

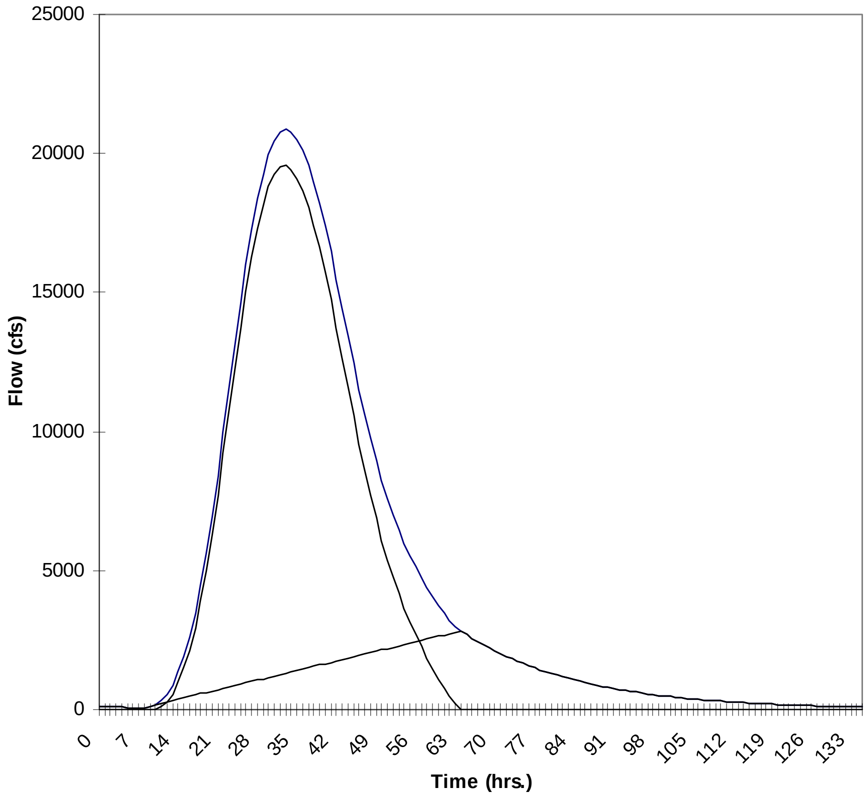
- Groundwater response, exponential
- Use log paper to determine the separation point



Hydrograph & Baseflow



Separate Baseflow



Separation of Baseflow

- If no significant contribution from groundwaters, use a horizontal straight line
  - Constant base flow

UH from field data

- Matrix approach

$$\begin{bmatrix} I1 & 0 & 0 & 0 \\ I2 & I1 & 0 & 0 \\ I3 & I2 & I1 & 0 \\ 0 & I3 & I2 & I1 \\ 0 & 0 & I3 & I2 \\ 0 & 0 & 0 & I3 \end{bmatrix} \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \end{bmatrix} = \begin{bmatrix} Q1 \\ Q2 \\ Q3 \\ Q4 \\ Q5 \\ Q6 \end{bmatrix}$$

$$PU = Q$$

UH from field data

- Considering a matrix algebra, we can obtain the vector U, from vectos Q and matrix P

$PU = Q$

$$PU = Q$$

$$P^T PU = P^T Q$$

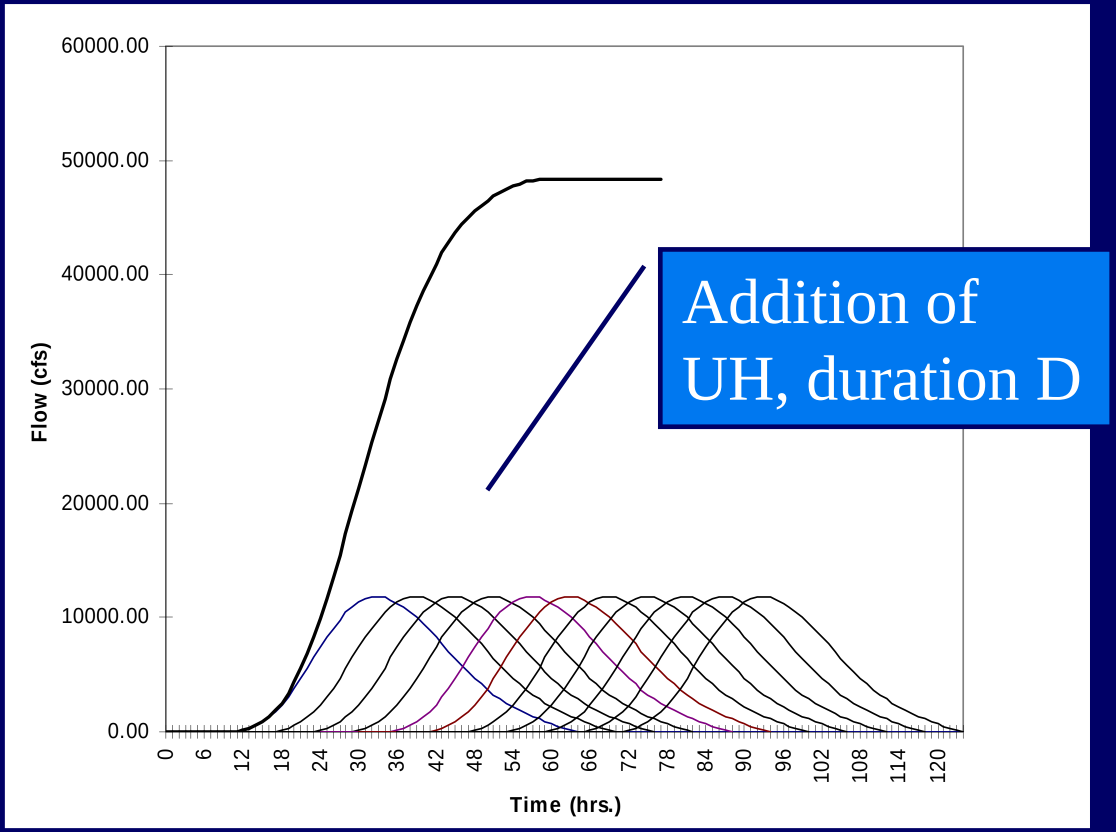
$$U = \left[P^T P\right]^{-1} P^T Q$$

UH of D’ from D minutes UH

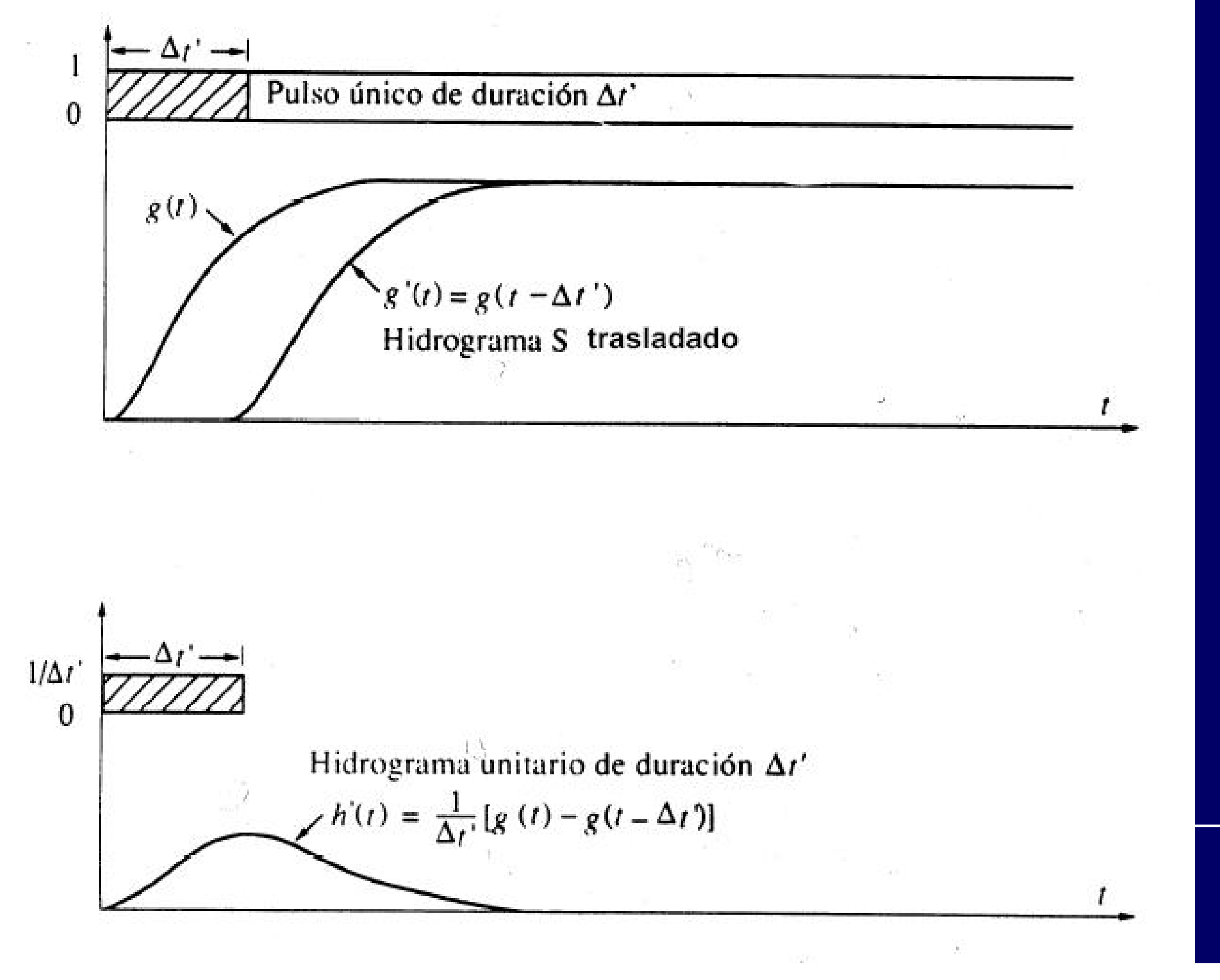
- Sometimes you have UH for duration D, but you need the D’ UH duration
- Use of S-curve
- Just for real UH, not synthetic ones

S hydrograph

- Consider a very long, constant rain event
- Intensity 1/D mm/h
- Hydrological response?



S curve



Δt'
1
0
Pulso único de duración Δt'
g(t)
g'(t) = g(t − Δt')
Hidrograma S trasladado
t
Δt'
1/Δt'
0
Hidrograma unitario de duración Δt'
h'(t) = 1/Δt' [g(t) − g(t − Δt')]
t

UH of D’ from D minutes UH

- From D minutes UH, establish S curve
- Move D’ minutes the S-curve
- Sustract both hydrographs
- Convert to unit rainfall, the obtained hydrograph (1 mm rainfall)

Problems of the UH obtention

- Rain event selection
- Errors in rainfall or runoff measurements
- Non-uniform rain events
- Use of more than 1 event to obtain UH
- Make an average of the UH
- Optimization methods to obtain the UH from several rain event at the same time

Average Several UHG's

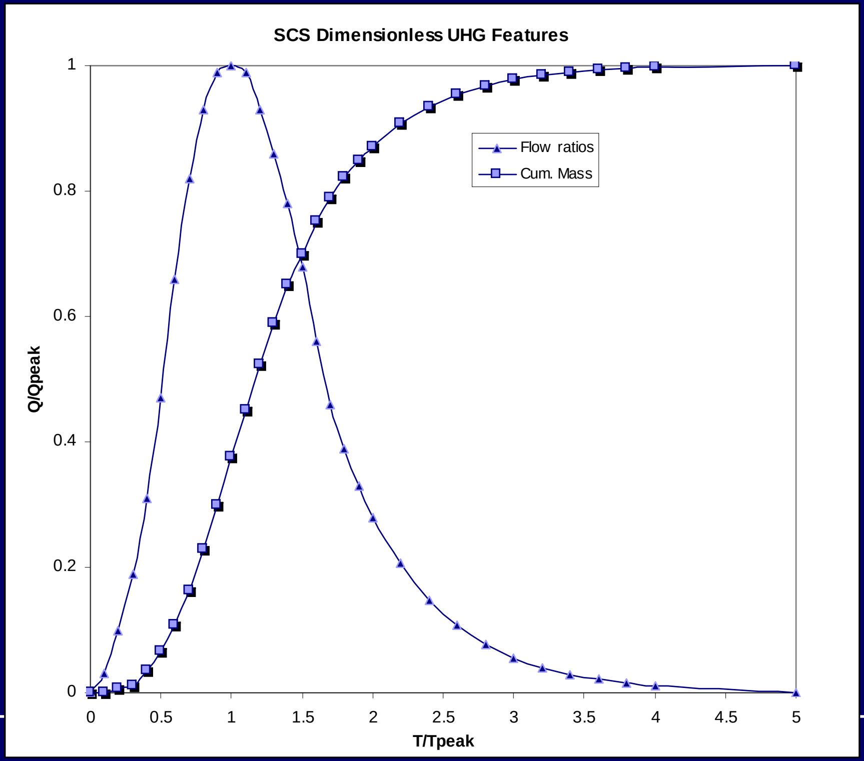
- It is suggested that several unit hydrographs be derived and averaged.
- The unit hydrographs must be of the same duration in order to be properly averaged.
- It is often not sufficient to simply average the ordinates of the unit hydrographs in order to obtain the final unit hydrograph. A numerical average of several unit hydrographs which are different "shapes" may result in an "unrepresentative" unit hydrograph.
- It is often recommended to plot the unit hydrographs that are to be averaged. Then an average or representative unit hydrograph should be sketched or fitted to the plotted unit hydrographs.
- Finally, the average unit hydrograph must have a volume of 1 mm of runoff for the basin.

Synthetic UHG's

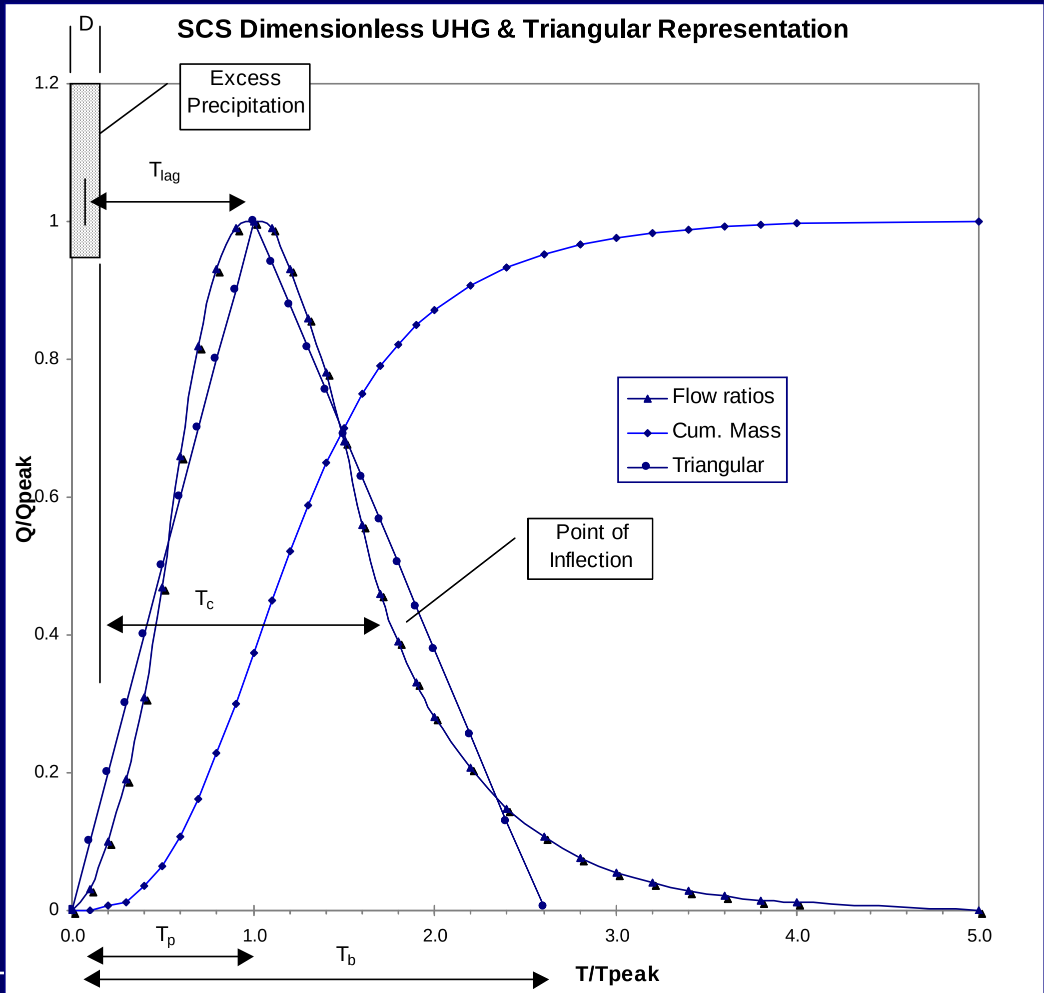
- SCS
- Clark (Time-area method)

SCS SUH

- SCS proposal
- Simple
- Basin of regular shapes
- Single peak



Triangular SHU



Dimensionless Ratios

Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0	.000	.000
.1	.030	.001
.2	.100	.006
.3	.190	.012
.4	.310	.035
.5	.470	.065
.6	.660	.107
.7	.820	.163
.8	.930	.228
.9	.990	.300
1.0	1.000	.375
1.1	.990	.450
1.2	.930	.522
1.3	.860	.589
1.4	.780	.650
1.5	.680	.700
1.6	.560	.751
1.7	.460	.790
1.8	.390	.822
1.9	.330	.849
2.0	.280	.871
2.2	.207	.908
2.4	.147	.934
2.6	.107	.953
2.8	.077	.967
3.0	.055	.977
3.2	.040	.984
3.4	.029	.989
3.6	.021	.993
3.8	.015	.995
4.0	.011	.997
4.5	.005	.999
5.0	.000	1.000

Triangular Representation

$$T_b = 2.67 \text{ x } T_p$$

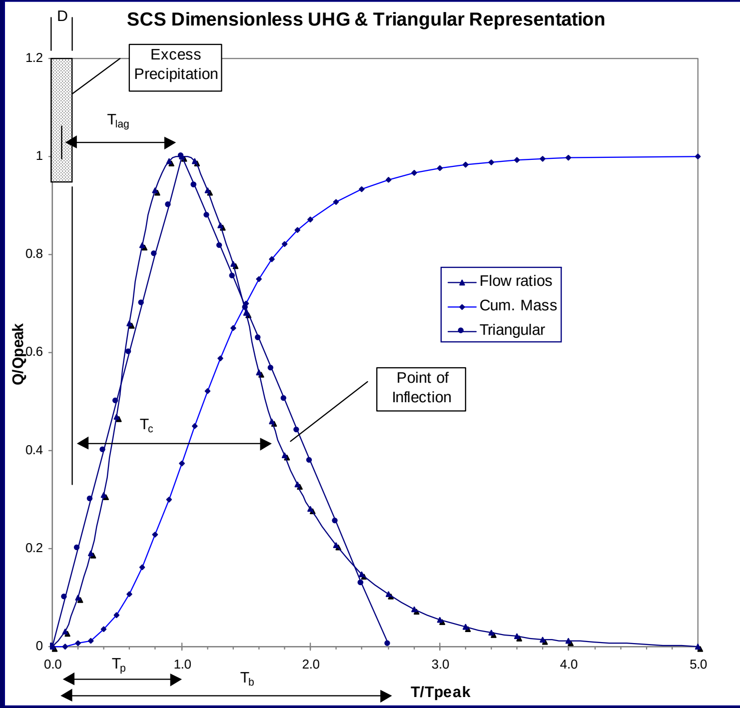
$$T_r = T_b - T_p = 1.67 \text{ x } T_p$$

$$Q = \frac{q_p T_p}{2} + \frac{q_p T_r}{2} = \frac{q_p}{2}(T_p + T_r)$$

$$q_p = \frac{2Q}{T_p + T_r}$$

$$q_p = \frac{654.33 \text{ x } 2 \text{ x } A \text{ x } Q}{T_p + T_r}$$

$$q_p = \frac{484\ A\ Q}{T_p}$$



The 645.33 is the conversion used for delivering 1-inch of runoff (the area under the unit hydrograph) from 1-square mile in 1-hour (3600 seconds).

Duration & Timing?

Again from the triangle

$$T_p = \frac{D}{2} + L$$

L = Lag time

$$L = 0.6 * T_c$$

$$T_c + D = 1.7\, T_p$$

$$\frac{D}{2} + 0.6\, T_c = T_p$$

For estimation purposes should be around : $D = 0.133\, T_c$

To be used with SCS concept and expressions

Time of Concentration

- Regression Eqs.
- Segmental Approach

Time of Concentration

- In Spain, we use the Témez’s formula
- Different concept for Tc than the SCS
- Modify expressions for SHU, SCS

A Regression Equation

$$T_{lag} = \frac{L^{0.8}(S+1)^{0.7}}{1900(\%Slope)^{0.5}}$$

where : T_{lag} = lag time in hours
L = Length of the longest drainage path in feet
S = (1000/CN) - 10 (CN=curve number)
%Slope = The average watershed slope in %

$$T_c = 0.3\left(\frac{L}{J^{0.25}}\right)^{0.76}$$

Tc, is it always the same?

$$T_{lag} = \frac{L^{0.8}(S+1)^{0.7}}{1900(\%Slope)^{0.5}}$$

$$T_c = 0.3\left(\frac{L}{J^{0.25}}\right)^{0.76}$$

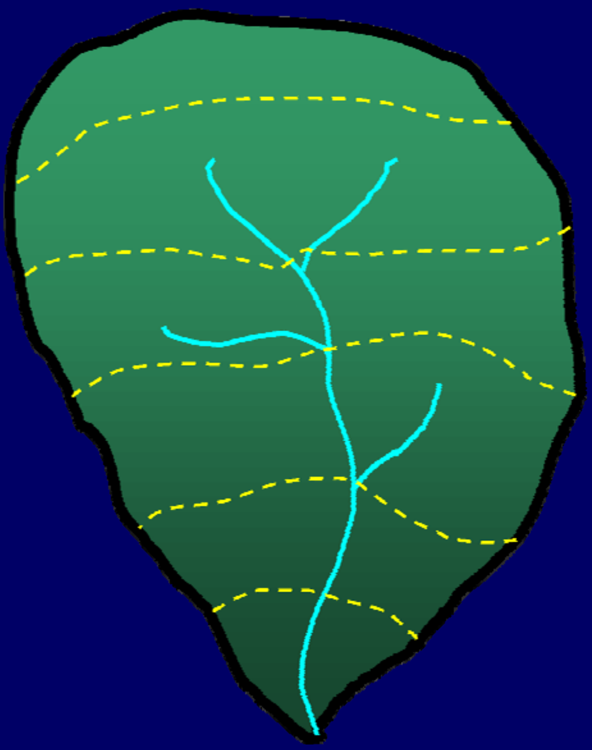
- Tc for SCS and Tc for other expressions are not the same
- T'c for SCS, time to inflexion point of the UH
- Tc as time needed to exit the basin from the farthest point

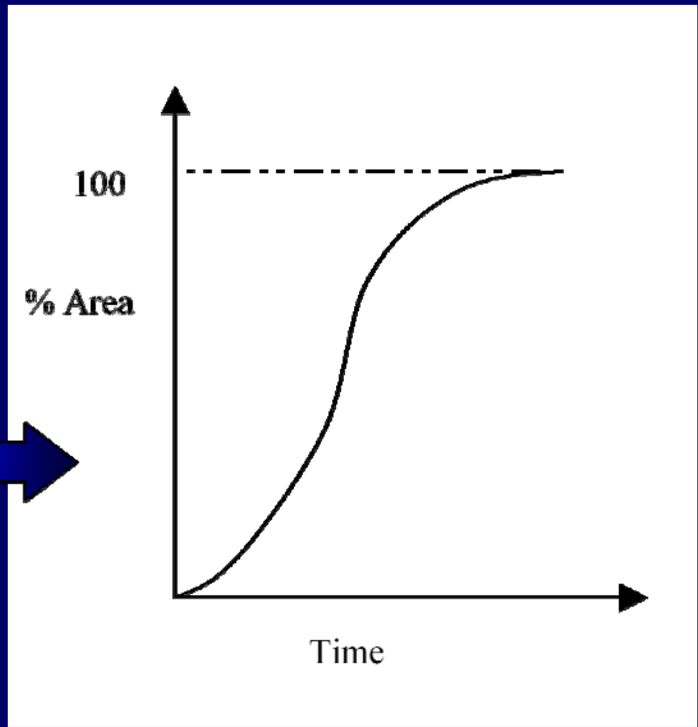
$$T_{lag} = 0.6\ T_c' = 0.35\ T_c$$

Clark, synthetic UH

- Propossed by Clark
- Considering the basin shape, not just the total area
- Consider delays attributed to sub-surface runoff
- Need to be applied in non regular shape basins

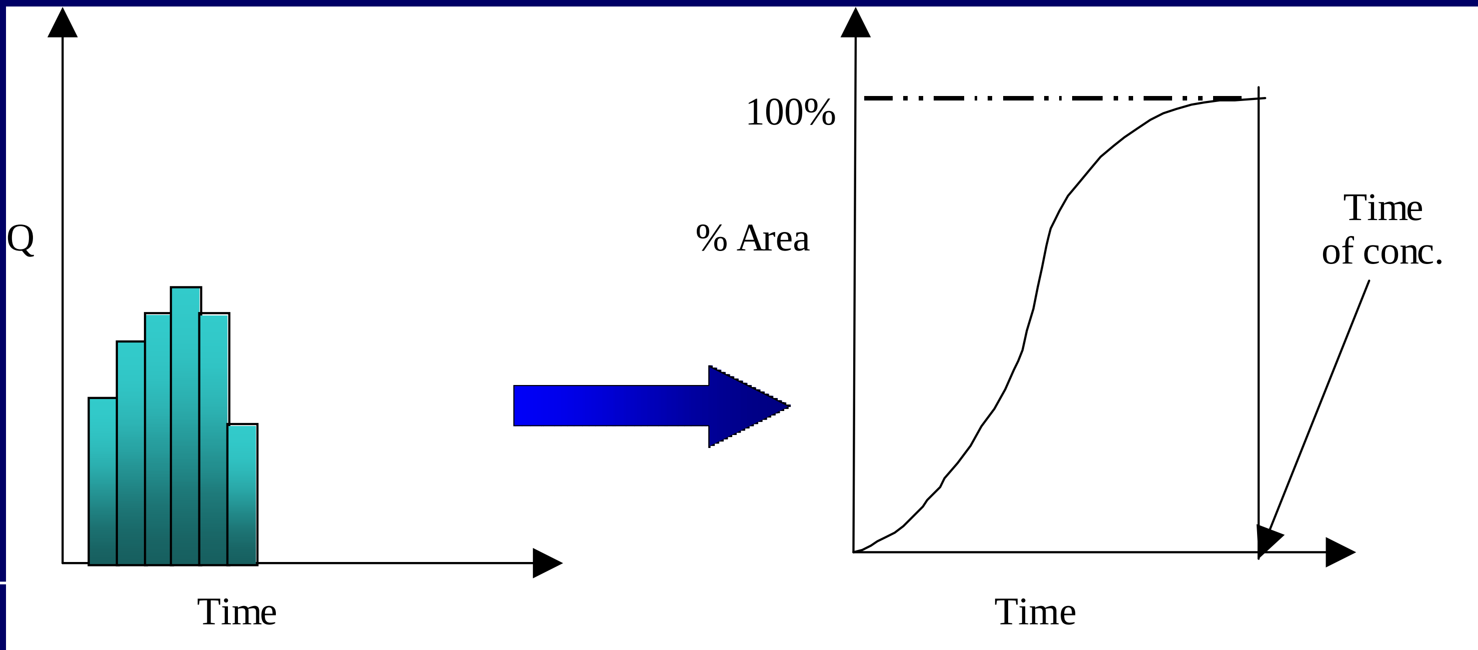
Clark - Time-Area





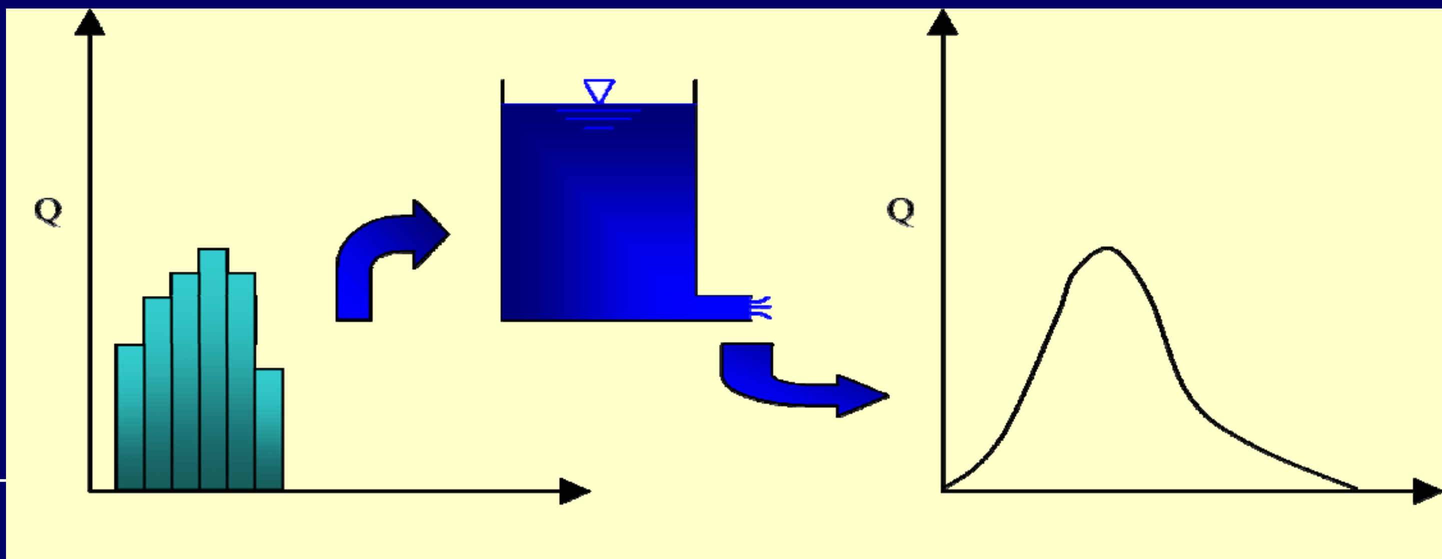
Time-Area

- Synthetic UH, equal form as the time-area curve
- It can show more than one peak



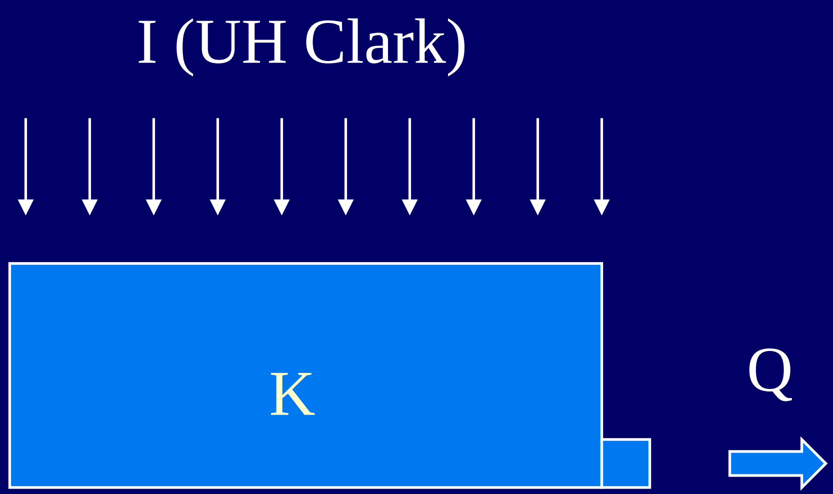
Additional delay

- Presence of sub-surface runoff
- Runoff shows an additional delay, that can not be explained just for surface runoff



Reservoir model for the delay

- Conceptual model
- Assume that additional delay is equal to the produced by a water reservoir



$$I - Q = \frac{dS}{dt}$$

Storage description

- General approach

$$S = K_1 Q + K_2 Q^2 + \cdots K_n Q^n$$

- Simplified to a linear reservoir model

$$S = K\,Q$$

- K has dimensions of time

$$K = [T]$$

Mathematical description

- If K is contant in time

$$I - Q = K \frac{dQ}{dt}$$

⇓

- We can solve the diff. Equation as:

$$Q(t) = \int_0^t \frac{I(\tau)}{K} e^{\frac{\tau - t}{K}} d\tau$$

Practical application

- A finite difference scheme can be used

$$\frac{I_1 + I_2}{2} - \frac{Q_1 + Q_2}{2} = K\frac{Q_2 - Q_1}{\Delta t}$$

- From an initial condition Q1, and from the values of the previous hydrograph we can proceed as:

$$Q_2 = \frac{2\cdot\Delta t}{2K+\Delta t}\left[I_1 - \frac{Q_1}{2} + K\frac{Q_1}{\Delta t}\right]$$

K values

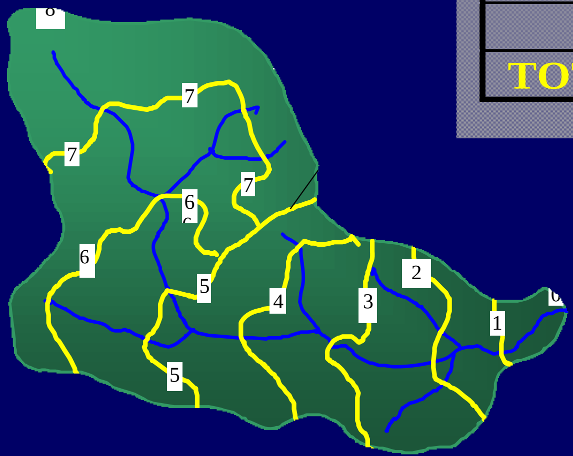
- We need to estimate the K value
- Best approach, field data (I , Q)
- From correlations obtained in other
- Proposed   K = 0.75 Tc

Basin application

- Basin 190 Km^2 , Tc 8 hours
- K= 5.5 hours
- Time step, 1 or 2 hours

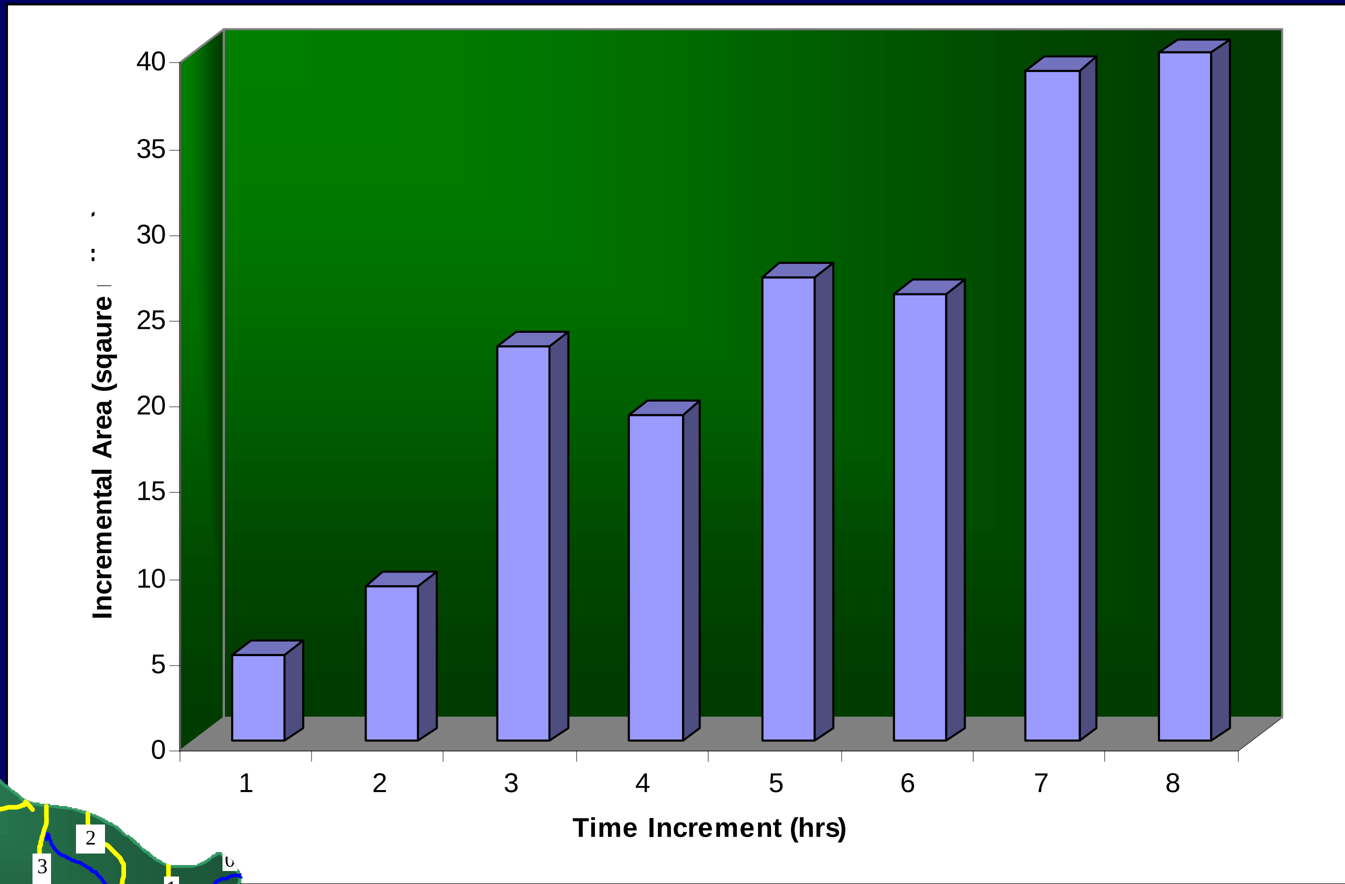
Basin geometry

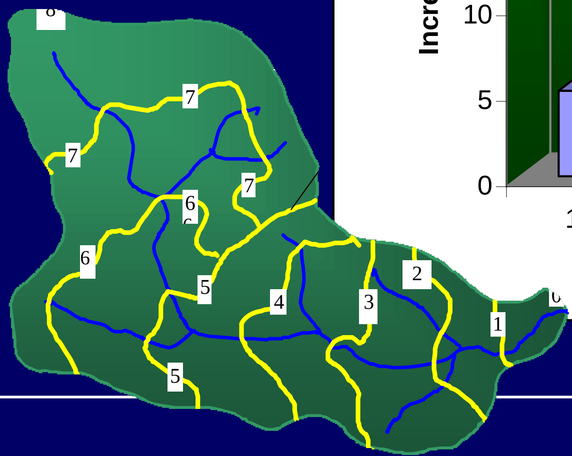
Area #	Isocrones	Area (km2)	Accum. Area (km^2)	Accum Time (hrs)
1	0-1	5	5	1.0
2	1-2	9	14	2.0
3	2-3	23	37	3.0
4	3-4	19	58	4.0
5	4-5	27	85	5.0
6	5-6	26	111	6.0
7	6-7	39	150	7.0
8	7-8	40	190	8.0
TOTAL		**190**	**190**	**8.0**



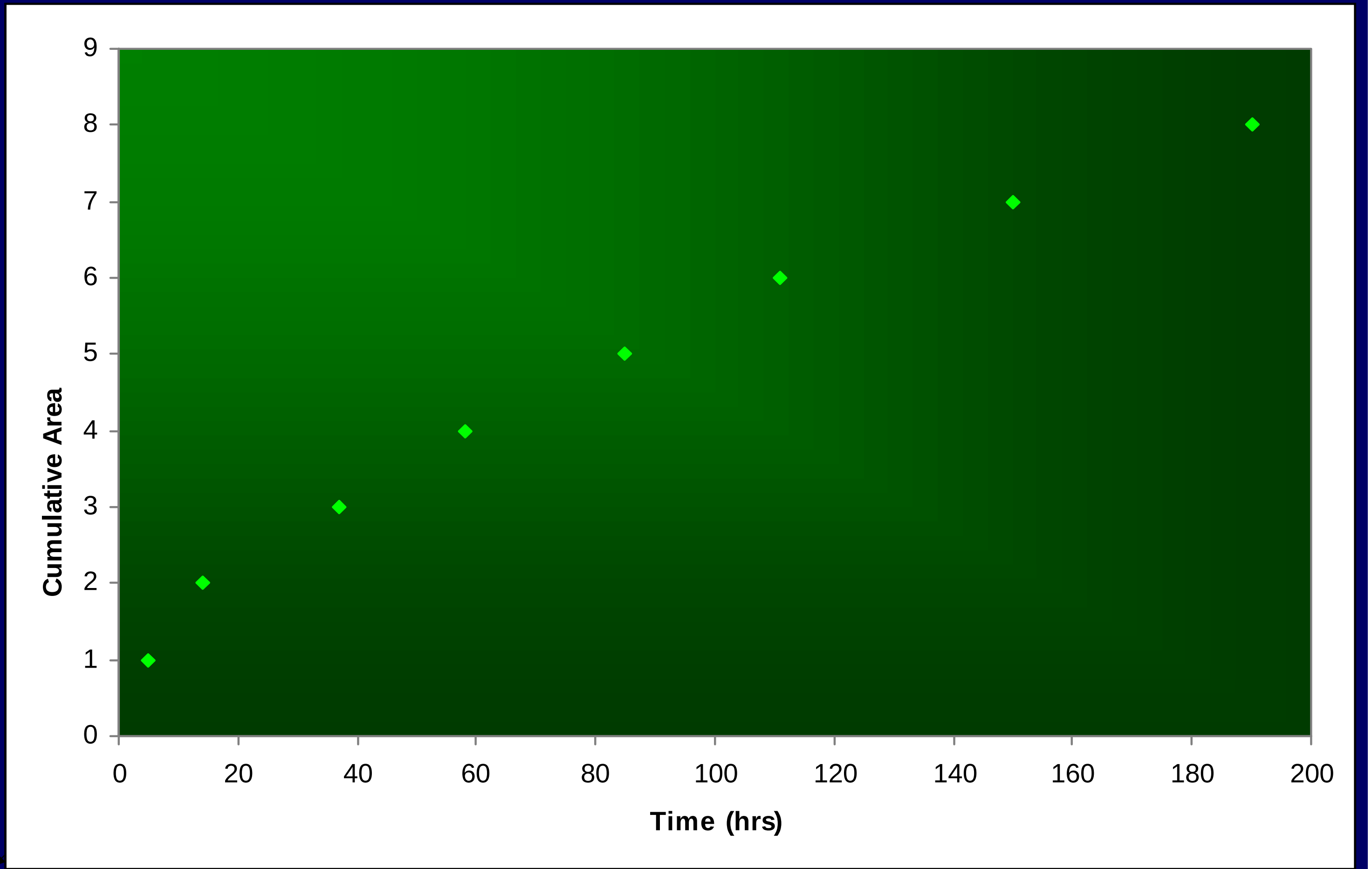
Time area curve

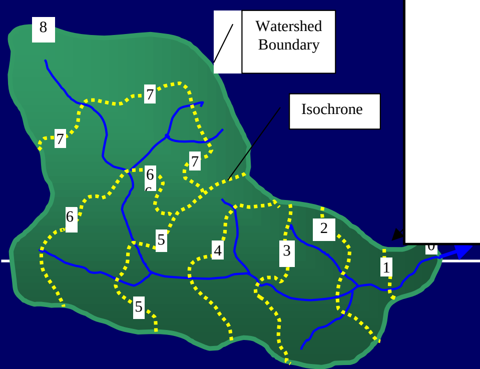
Area (Km2)



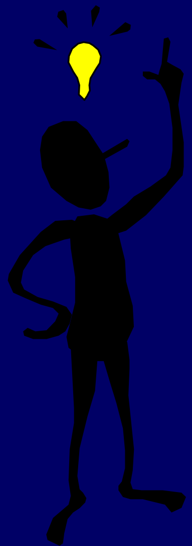


Time accumulated area





No time / area curve ?



$$TA_i = 1.414 T_i^{1.5} \qquad (0 \le \mathrm{Ti} \le 0.5)$$

$$1 - TA_i = 1.414(1 - T_i)^{1.5} \qquad (0.5 \le \mathrm{Ti} \le 1.0)$$

U.S. Army Corps of Engineers (HEC 1990)

- **What about if the synthetic curve does not match the real one?**

SUH, comments

- SUH is an approach to the real UH, could be good or not
- SHU Clark, problems to estimate K
- SHU SCS, one peak value, can only be applied to basins with regular shape
- You must make your choice according the basin characteristics