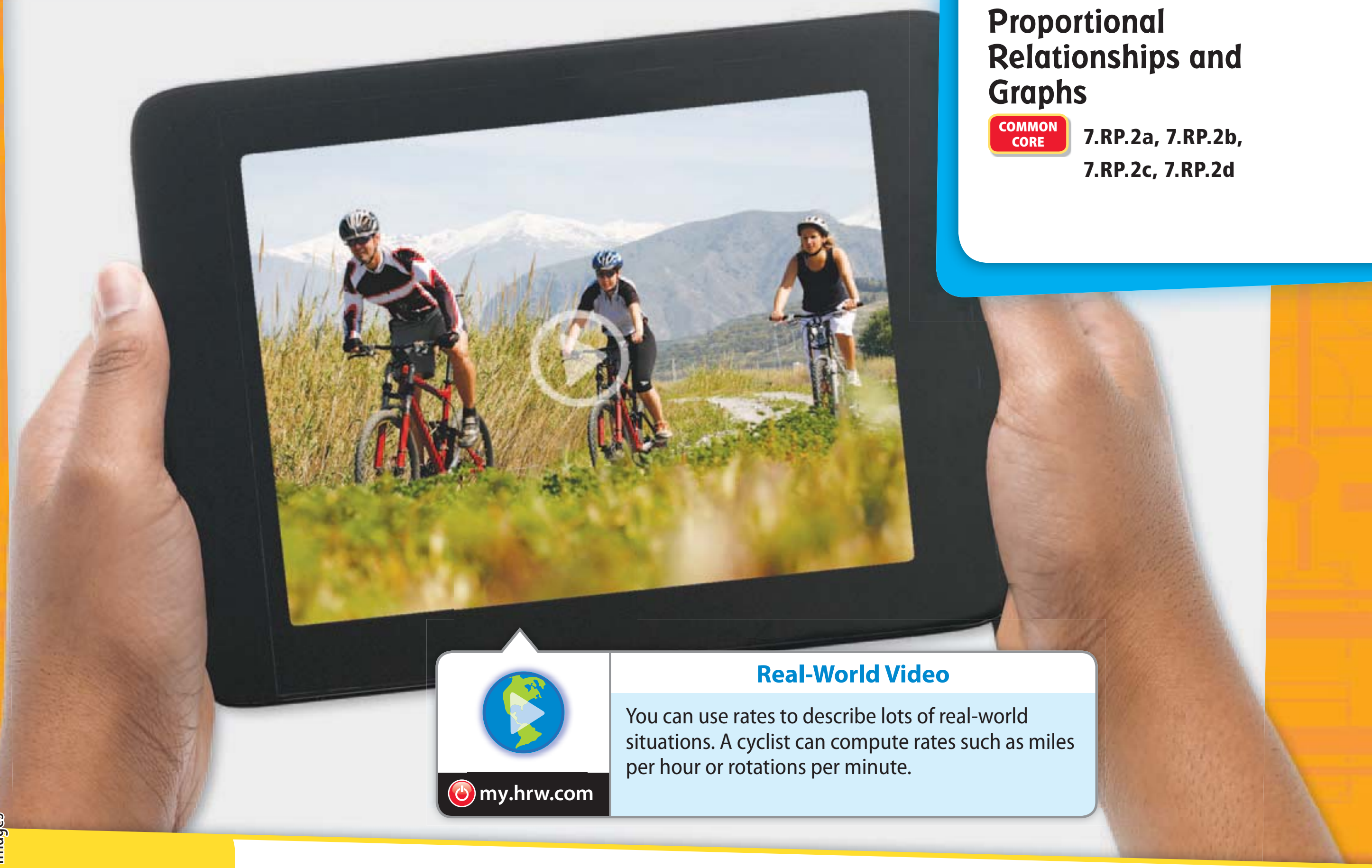
Rates and Proportionality

MODULE 4

COMMON CORE

ESSENTIAL QUESTION

How can you use rates and proportionality to solve real-world problems?



Real-World Video

You can use rates to describe lots of real-world situations. A cyclist can compute rates such as miles per hour or rotations per minute.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Operations with Fractions

EXAMPLE

$$\frac{3}{10} \div \frac{5}{8} = \frac{3}{10} \times \frac{8}{5}$$ Multiply by the reciprocal of the divisor.

$$= \frac{3}{\cancel{10}_{5}} \times \frac{\cancel{8}^{4}}{5}$$ Divide by the common factors.

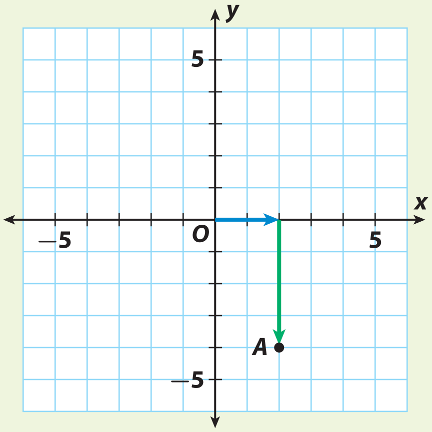
$$= \frac{12}{25}$$ Simplify.

Divide.

1. $\frac{3}{4} \div \frac{4}{5}$ ________ **2.** $\frac{5}{9} \div \frac{10}{11}$ ________ **3.** $\frac{3}{8} \div \frac{1}{2}$ ________ **4.** $\frac{16}{21} \div \frac{8}{9}$ ________

Ordered Pairs

EXAMPLE



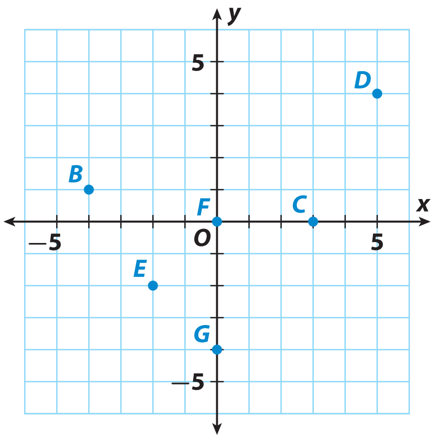
To write the ordered pair for A, start at the origin.
Move 2 units right.
Then move 4 units down.
The ordered pair for point A is $(2, -4)$.

Write the ordered pair for each point.

5. B ____________ **6.** C ____________

7. D ____________ **8.** E ____________

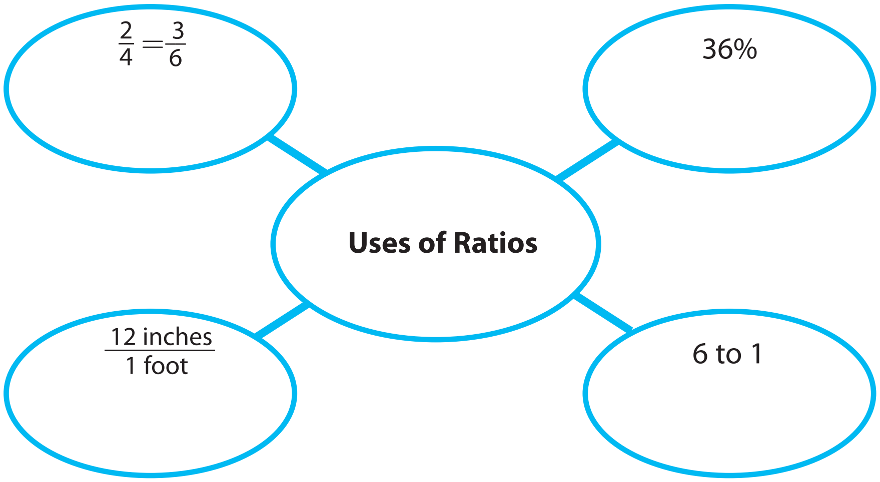
9. F ____________ **10.** G ____________



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You can put more than one word in each bubble.



Understand Vocabulary

Match the term on the left to the definition on the right.

1. rate of change	**A.** Statement that two rates or ratios are equivalent.
2. proportion	**B.** A rate that describes how one quantity changes in relation to another quantity.
3. unit rate	**C.** Rate in which the second quantity is one unit.

Vocabulary

Review Words

constant *(constante)*
✔ conversion factor *(factor de conversión)*
✔ equivalent ratios *(razones equivalentes)*
✔ percent *(porcentaje)*
rate *(tasa)*
✔ ratio *(razón)*

Preview Words

complex fraction *(fracción compleja)*
constant of proportionality *(constante de proporcionalidad)*
proportion *(proporción)*
proportional relationship *(relación proporcional)*
rate of change *(tasa de cambio)*
unit rates *(tasas unitarias)*

Active Reading

Three-Panel Flip Chart Before beginning the module, create a three-panel flip chart to help you organize what you learn. Label each flap with one of the lesson titles from this module. As you study each lesson, write important ideas like vocabulary, properties, and formulas under the appropriate flap.

COMMON CORE

MODULE 4

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE 7.RP.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Key Vocabulary

rate *(tasa)*
A ratio that compares two quantities measured in different units.

unit rate *(tasa unitaria)*
A rate in which the second quantity in the comparison is one unit.

What It Means to You

Given a rate, you can find the equivalent unit rate by dividing the numerator by the denominator.

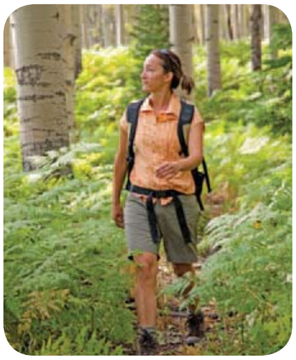
UNPACKING EXAMPLE 7.RP.1

Lisa hikes $\frac{1}{3}$ mile every $\frac{1}{6}$ hour. How far does she hike in 1 hour?

$$\frac{\frac{1}{3}}{\frac{1}{6}} = \frac{1}{3} \div \frac{1}{6}$$

$$= \frac{1}{\cancel{3}_1} \cdot \frac{\cancel{6}^2}{1}$$

$$= 2 \text{ miles}$$



COMMON CORE 7.RP.2b

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Key Vocabulary

constant *(constante)*
A value that does not change.

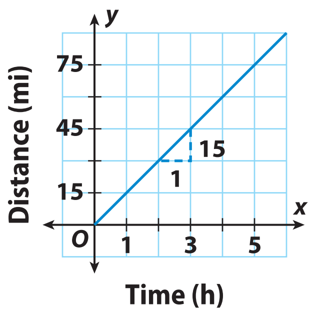
constant of proportionality *(constante de proporcionalidad)*
A constant ratio of two variables related proportionally.

What It Means to You

You will determine the constant of proportionality for proportional relationships.

UNPACKING EXAMPLE 7.RP.2b

The graph shows the distance a bicyclist travels over time. How fast does the bicyclist travel?



$$\text{slope (speed)} = \frac{\text{rise (distance)}}{\text{run (time)}}$$

$$= \frac{15}{1}$$

The bicyclist travels at 15 miles per hour.

The bicyclist's speed is a unit rate. It is indicated on the graphed line by the point (1, 15).

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LESSON

4.1 Unit Rates

COMMON CORE 7.RP.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

ESSENTIAL QUESTION

How do you find and use unit rates?

EXPLORE ACTIVITY Real World

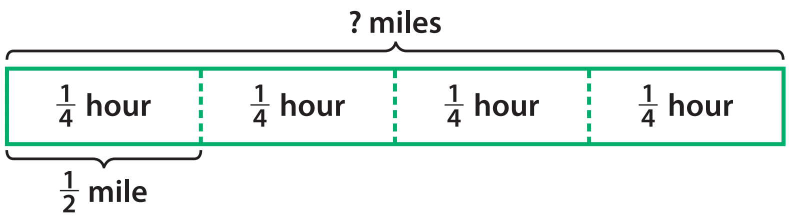
COMMON CORE 7.RP.1

Exploring Rates

Commonly used rates like miles per hour make it easy to understand and compare rates.

Jeff hikes $\frac{1}{2}$ mile every 15 minutes, or $\frac{1}{4}$ hour. Lisa hikes $\frac{1}{3}$ mile every 10 minutes, or $\frac{1}{6}$ hour. How far do they each hike in 1 hour? 2 hours?

A Use the bar diagram to help you determine how many miles Jeff hikes. How many $\frac{1}{4}$-hours are in 1 hour? How far does Jeff hike in 1 hour?

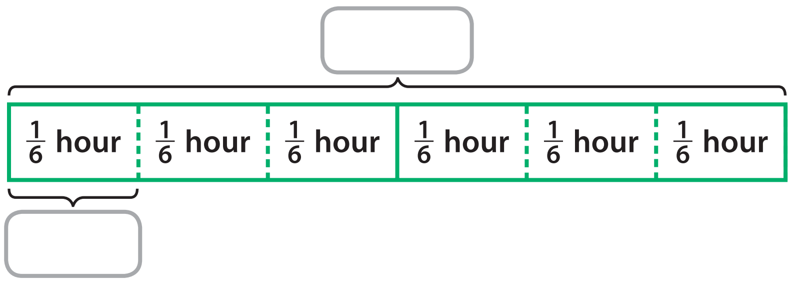


____

B Complete the table for Jeff's hike.

Distance (mi)	$\frac{1}{2}$				
Time (h)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	2

C Complete the bar diagram to help you determine how far Lisa hikes. How many miles does she hike in 1 hour?



____

D Complete the table for Lisa's hike.

Distance (mi)	$\frac{1}{3}$				
Time (h)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. How did you find Jeff's distance for $\frac{3}{4}$ hour?

____

2. Which hiker walks farther in one hour? Which is faster?

____

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Finding Unit Rates

A rate is a comparison of two quantities that have different units, such as miles and hours. Rates are often expressed as **unit rates**, that is, with a denominator of 1 unit.

$$\frac{60 \text{ miles} \div 2}{2 \text{ hours} \div 2} = \frac{30 \text{ miles}}{1 \text{ hour}}$$ This means 30 miles per hour.

When the two quantities being compared in the rate are both fractions, the rate is expressed as a *complex fraction*. A **complex fraction** is a fraction that has a fraction in its numerator, denominator, or both.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$$

EXAMPLE 1 Real World

COMMON CORE 7.RP.1

While remodeling her kitchen, Angela is repainting. She estimates that she paints 55 square feet every half-hour. How many square feet does Angela paint per hour?

STEP 1 Determine the units of the rate.

The rate is **area in square feet** per **time in hours**.

STEP 2 Find Angela's rate of painting in area painted per time.

area painted: 55 sq ft **time:** $\frac{1}{2}$ hour

$$\frac{\text{area painted}}{\text{time}} = \frac{55 \text{ square feet}}{\frac{1}{2} \text{ hour}}$$

The fraction represents area in square feet per time in hours.

STEP 3 Find Angela's unit rate of painting in square feet per hour.

$$\frac{55}{\frac{1}{2}} = 55 \div \frac{1}{2}$$ Rewrite the fraction as division.

$$= \frac{55}{1} \times \frac{2}{1}$$ Multiply by the reciprocal.

$$= \frac{110 \text{ square feet}}{1 \text{ hour}}$$ The unit rate has a denominator of 1.

Angela paints 110 square feet per hour.

YOUR TURN

3. Paige mows $\frac{1}{6}$ acre in $\frac{1}{4}$ hour. How many acres does Paige mow per hour?

____

4. Greta uses 3 ounces of pasta to make $\frac{3}{4}$ of a serving of pasta. How many ounces of pasta are there per serving? _______________

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Using Unit Rates

You can use unit rates to simplify rates and ratios that appear complicated, such as those containing fractions in both the numerator and denominator.

EXAMPLE 2 Real World

COMMON CORE 7.RP.1

Two pools are leaking. After 15 minutes, pool A has leaked $\frac{2}{3}$ gallon. After 20 minutes, pool B has leaked $\frac{3}{4}$ gallon. Which pool is leaking faster?

STEP 1 Find the rate in volume (gallons) per time (hours) at which each pool is leaking. First convert minutes to hours.

Pool A

$$\frac{\frac{2}{3}\text{ gal}}{15\text{ min}} = \frac{\frac{2}{3}\text{ gal}}{\frac{1}{4}\text{ h}}$$

$15\text{ min} = \frac{1}{4}\text{ h}$

Pool B

$$\frac{\frac{3}{4}\text{ gal}}{20\text{ min}} = \frac{\frac{3}{4}\text{ gal}}{\frac{1}{3}\text{ h}}$$

$20\text{ min} = \frac{1}{3}\text{ h}$

STEP 2 To find the unit rates, first rewrite the fractions.

Pool A

$$\frac{\frac{2}{3}\text{ gal}}{\frac{1}{4}\text{ h}} = \frac{2}{3} \div \frac{1}{4}$$

Pool B

$$\frac{\frac{3}{4}\text{ gal}}{\frac{1}{3}\text{ h}} = \frac{3}{4} \div \frac{1}{3}$$

STEP 3 To divide, multiply by the reciprocal.

Pool A

$$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1}$$

$$= \frac{8}{3}, \text{ or } 2\tfrac{2}{3}\text{ gal per h}$$

Pool B

$$\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1}$$

$$= \frac{9}{4}, \text{ or } 2\tfrac{1}{4}\text{ gal per h}$$

STEP 4 Compare the unit rates.

Pool A **Pool B**

$$2\tfrac{2}{3} > 2\tfrac{1}{4}$$

So, Pool A is leaking faster.

My Notes

Math Talk
Mathematical Practices
How do you compare mixed numbers?

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YOUR TURN

5. One tank is filling at a rate of $\frac{3}{4}$ gallon per $\frac{2}{3}$ minute. A second tank is filling at a rate of $\frac{5}{8}$ gallon per $\frac{1}{2}$ minute. Which tank is filling faster?

____

Guided Practice

1. Brandon enters bike races. He bikes $8\frac{1}{2}$ miles every $\frac{1}{2}$ hour. Complete the table to find how far Brandon bikes for each time interval. (Explore Activity)

Distance (mi)	$8\frac{1}{2}$				
Time (h)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$

Find each unit rate. (Example 1)

2. Julio walks $3\frac{1}{2}$ miles in $1\frac{1}{4}$ hours.

____

3. Kenny reads $\frac{5}{8}$ page in $\frac{2}{3}$ minute.

____

4. A garden snail moves $\frac{1}{6}$ foot in $\frac{1}{3}$ hour.

____

5. A fertilizer covers $\frac{5}{8}$ square foot in $\frac{1}{4}$ hour.

____

Find each unit rate. Determine which is lower. (Example 2)

6. Brand A: 240 mg sodium for $\frac{1}{3}$ pickle
or Brand B: 325 mg sodium for $\frac{1}{2}$ pickle

____

7. Ingredient C: $\frac{1}{4}$ cup for $\frac{2}{3}$ serving
or Ingredient D: $\frac{1}{3}$ cup for $\frac{3}{4}$ serving

____

ESSENTIAL QUESTION CHECK-IN

8. How can you find a unit rate when given a rate?

____

Name ____________ Class ____________ Date ____________

4.1 Independent Practice

COMMON CORE 7.RP.1

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9. The information for two pay-as-you-go cell phone companies is given.

On Call	Talk Time
3.5 hours: $10	$\frac{1}{2}$ hour: $1.25

a. What is the unit rate in dollars per hour for each company?

b. **Analyze Relationships** Which company offers the best deal? Explain your answer.

c. **What If?** Another company offers a rate of $0.05 per minute. How would you find the unit rate per hour?

d. **Draw Conclusions** Is the rate in part **c** a better deal than On Call or Talk Time? Explain.

10. **Represent Real-World Problems** Your teacher asks you to find a recipe that includes two ingredients with a rate of $\frac{2\text{ units}}{3\text{ units}}$.

a. Give an example of two ingredients in a recipe that would meet this requirement.

b. If you needed to triple the recipe, would the rate change? Explain.

11. A radio station requires DJs to play 2 commercials for every 10 songs they play. What is the unit rate of songs to commercials?

12. **Multistep** Terrance and Jesse are training for a long-distance race. Terrance trains at a rate of 6 miles every half hour, and Jesse trains at a rate of 2 miles every 15 minutes.

a. What is the unit rate in miles per hour for each runner?

b. How long will each person take to run a total of 50 miles at the given rates?

c. Sandra runs at a rate of 8 miles in 45 minutes. How does her unit rate compare to Terrance's and to Jesse's?

13. **Analyze Relationships** Eli takes a typing test and types all 300 words in $\frac{1}{10}$ hour. He takes the test a second time and types the words in $\frac{1}{12}$ hour. Was he faster or slower on the second attempt? Explain.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

14. **Justify Reasoning** An online retailer sells two packages of protein bars.

Package	10-pack of 2.1 ounce bars	12-pack of 1.4 ounce bars
Cost ($)	15.37	15.35

a. Which package has the better price per bar?

b. Which package has the better price per ounce?

c. Which package do you think is a better buy? Justify your reasoning.

15. **Check for Reasonableness** A painter painted about half a room in half a day. Coley estimated the painter would paint 7 rooms in 7 days. Is Coley's estimate reasonable? Explain.

16. **Communicate Mathematical Ideas** If you know the rate of a water leak in gallons per hour, how can you find the number of hours it takes for 1 gallon to leak out? Justify your answer.

LESSON **4.2**

Constant Rates of Change

COMMON CORE **7.RP.2**

Recognize and represent proportional relationships between quantities. *Also 7.RP.2a, 7.RP.2b, 7.RP.2c*

ESSENTIAL QUESTION

How can you identify and represent proportional relationships?

EXPLORE ACTIVITY Real World

COMMON CORE **7.RP.2a**

Discovering Proportional Relationships

Many real-world situations can be described by *proportional relationships*. Proportional relationships have special characteristics.

A giant tortoise moves at a slow but steady pace. It takes the giant tortoise 3 seconds to travel 10.5 inches.

A Use the bar diagram to help you determine how many inches a tortoise travels in 1 second. What operation did you use to find the answer?

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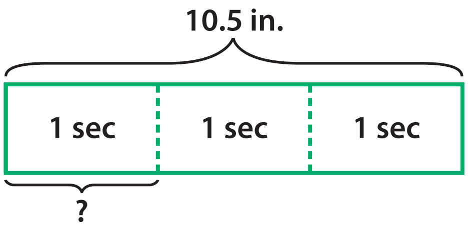

B Complete the table.

Time (sec)	1	2	3	4	5
Distance (in.)			10.5		

C For each column of the table, find the distance and the time. Write each fraction as a decimal. Put distance in the numerator and time in the denominator.

$\frac{\square}{\square} = \square$ $\frac{\square}{\square} = \square$ $\frac{\square}{\square} = \square$ $\frac{\square}{\square} = \square$ $\frac{\square}{\square} = \square$

D What do you notice about the decimal forms of the fractions?

____

E **Conjecture** How do you think the distance a tortoise travels is related to the time?

____

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. Suppose the tortoise travels for 12 seconds. Explain how you could find the distance the tortoise travels.

____

2. How would you describe the rate of speed at which a tortoise travels?

____

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Proportional Relationships

A **rate of change** is a rate that describes how one quantity changes in relation to another quantity. A **proportional relationship** between two quantities is one in which the rate of change is constant, or one in which the ratio of one quantity to the other is constant.

Any two rates or ratios based on a given proportional relationship can be used to form a *proportion*. A **proportion** is a statement that two rates or ratios are equivalent, for example, $\frac{6\text{ mi}}{2\text{ h}} = \frac{3\text{ mi}}{1\text{ h}}$, or $\frac{2}{4} = \frac{1}{2}$.

My Notes

EXAMPLE 1 Real World

COMMON CORE 7.RP.2a, 7.RP.2b

Callie earns money by dog sitting. Based on the table, is the relationship between the amount Callie earns and the number of days a proportional relationship?

Number of Days	1	2	3	4	5
Amount Earned ($)	16	32	48	64	80

STEP 1 Write the rates.

$$\frac{\text{Amount earned}}{\text{Number of days}} = \frac{\$16}{1\text{ day}}$$

Put the amount earned in the numerator and the number of days in the denominator.

$$\frac{\$32}{2\text{ days}} = \frac{\$16}{1\text{ day}}$$

$$\frac{\$48}{3\text{ days}} = \frac{\$16}{1\text{ day}}$$

Each rate is equal to $\frac{\$16}{1\text{ day}}$, or \$16 per day.

$$\frac{\$64}{4\text{ days}} = \frac{\$16}{1\text{ day}}$$

$$\frac{\$80}{5\text{ days}} = \frac{\$16}{1\text{ day}}$$

STEP 2 Compare the rates. The rates are all equal. This means the rate is constant, so the relationship is proportional.

The constant rate of change is $16 per day.

Math Talk
Mathematical Practices
How can you use the constant rate to find how much Callie earns for 10 days of dog sitting?

YOUR TURN

3. The table shows the distance Allison drove on one day of her vacation. Is the relationship between the distance and the time a proportional relationship? Did she drive at a constant speed? Explain.

Time (h)	1	2	3	4	5
Distance (mi)	65	120	195	220	300

____

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Writing an Equation for a Proportional Relationship

If there is a proportional relationship between x and y, you can describe that relationship using the equation $y = kx$. The variable k is called the **constant of proportionality**, and it represents the constant rate of change or constant ratio between x and y. The value of k is represented by the equation $k = \frac{y}{x}$.

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EXAMPLE 2 Real World

COMMON CORE 7.RP.2c, 7.RP.2b

Two pounds of cashews shown cost \$19, and 8 pounds cost \$76. Show that the relationship between the number of pounds of cashews and the cost is a proportional relationship. Then write an equation for the relationship.

STEP 1 Make a table relating cost in dollars to pounds.

Number of Pounds	2	3	8
Cost (\$)	19	28.50	76

STEP 2 Write the rates. Put cost in the numerator and pounds in the denominator. Then simplify each rate.

$$\frac{\text{Cost}}{\text{Number of Pounds}} \longrightarrow \frac{19}{2} = 9.50 \qquad \frac{28.50}{3} = 9.50 \qquad \frac{76}{8} = 9.50$$

The rates are all equal to \$9.50 per pound. They are constant, so the relationship is proportional. The constant rate of change is \$9.50 per pound.

STEP 3 To write an equation, first tell what the variables represent.

- Let x represent the number of pounds of cashews.
- Let y represent the cost in dollars.
- Use the numerical part of the constant rate of change as the constant of proportionality.

So, the equation for the relationship is $y = 9.5x$.

Math Talk
Mathematical Practices
How can you use your equation to find the cost of 6 pounds of cashews?

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YOUR TURN

4. For a school field trip, there must be 1 adult to accompany 12 students, 3 adults to accompany 36 students, and 5 adults to accompany 60 students. Show that the relationship between the number of adults and the number of students is a proportional relationship. Then write an equation for the relationship.

Number of students	12	36	60
Number of adults	1	3	5

____

____

Guided Practice

1. Based on the information in the table, is the relationship between time and the number of words typed a proportional relationship? (Explore Activity and Example 1)

Time (min)	1	2	3	4
Number of words	45	90	135	180

$\frac{\text{Number of words}}{\text{Minutes}}$: $\frac{45}{1} = \square$ $\frac{\square}{\square} = \square$ $\frac{\square}{\square} = \square$ $\frac{\square}{\square} = \square$

The relationship is / is not proportional.

Find the constant of proportionality *k*. Then write an equation for the relationship between *x* and *y*. (Example 2)

2.

x	2	4	6	8
y	10	20	30	40

____

3.

x	8	16	24	32
y	2	4	6	8

____

ESSENTIAL QUESTION CHECK-IN

4. How can you represent a proportional relationship using an equation?

____

____

Name ________________ Class ________________ Date ________________

4.2 Independent Practice

COMMON CORE 7.RP.2, 7.RP.2a, 7.RP.2b, 7.RP.2c

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Information on three car-rental companies is given.

Rent-All				
Days	3	4	5	6
Total Cost ($)	55.50	74.00	92.50	111.00

A-1 Rentals
The cost *y* of renting a car for *x* days is $10.99 for each half day.

Car Town
The cost of renting a car from us is just $19.25 per day!

5. Write an equation that gives the cost *y* of renting a car for *x* days from Rent-All. ______________

6. What is the cost per day of renting a car from A-1? ______________

7. **Analyze Relationships** Which company offers the best deal? Why?

____

____

8. **Critique Reasoning** A skydiver jumps out of an airplane. After 0.8 second, she has fallen 100 feet. After 3.1 seconds, she has fallen 500 feet. Emtiaz says that the skydiver should fall about 187.5 feet in 1.5 seconds. Is his answer reasonable? Explain.

____

____

____

Steven earns extra money babysitting. He charges $31.25 for 5 hours and $50 for 8 hours.

9. Explain why the relationship between how much Steven charges and time is a proportional relationship.

____

10. **Interpret the Answer** Explain what the constant rate of change means in the context.

____

11. Write an equation to represent the relationship. Tell what the variables represent.

____

____

12. How much would Steven charge for 3 hours? ______________

A submarine dives 300 feet every 2 minutes, and 6,750 feet every 45 minutes.

Work Area

13. Find the constant rate at which the submarine dives. Give your answer in feet per minute and in feet per hour.

14. Let *x* represent the time of the dive. Let *y* represent the depth of the submarine. Write an equation for the proportional relationship using the rate in feet per minute.

15. **Draw Conclusions** If you wanted to find the depth of a submarine during a dive, would it be more reasonable to use an equation with the rate in feet per minute or feet per hour? Explain your reasoning.

H.O.T. FOCUS ON HIGHER ORDER THINKING

16. **Make a Conjecture** There is a proportional relationship between your distance from a thunderstorm and the time from when you see lightning and hear thunder. If there are 9 seconds between lightning and thunder, the storm is about 3 kilometers away. If you double the amount of time between lightning and thunder, do you think the distance in kilometers also double? justify your reasoning.

17. **Communicate Mathematical Ideas** A store sells 3 ears of corn for $1. They round prices to the nearest cent as shown in the table. Tell whether you would describe the relationship between cost and number of ears of corn as a proportional relationship. Justify your answer.

Ears of corn	1	2	3	4
Amount charged ($)	0.33	0.67	1.00	1.34

LESSON 4.3

Proportional Relationships and Graphs

COMMON CORE 7.RP.2a

Decide whether two quantities are in a proportional relationship, e.g., by...graphing on a coordinate plane and observing whether the graph is a straight line through the origin. *Also 7.RP.2b, 7.RP.2c, 7.RP.2d*

ESSENTIAL QUESTION

How can you use graphs to represent and analyze proportional relationships?

EXPLORE ACTIVITY Real World

COMMON CORE 7.RP.2a

Graphing Proportional Relationships

You can use a graph to explore proportional relationships.

Most showerheads that were manufactured before 1994 use 5 gallons of water per minute. Is the relationship between the number of gallons of water and the number of minutes a proportional relationship?

Each minute, 5 gallons of water are used. So for 2 minutes, $2 \cdot 5$ gallons are used.

A Complete the table.

Time (min)	1	2	3		10
Water Used (gal)	5			35	

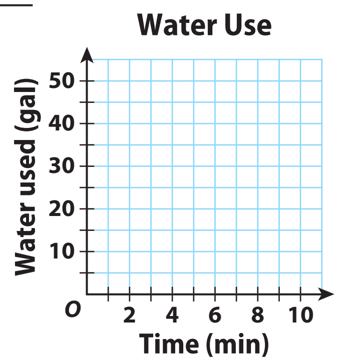
B Based on the table, is this a proportional relationship? Explain your answer.

____

C Write the data in the table as ordered pairs (time, water used).

(1, 5), (2, ____), (3, ____), (__ , 35), (10, ____)

D Plot the ordered pairs.



E If the showerhead is used for 0 minutes, how many gallons of water will be used? What ordered pair represents this situation? What is this location called?

____

F **Draw Conclusions** If you continued the table to include 23 minutes, would the point (23, 125) be on this graph? Why or why not?

____

____

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Identifying Proportional Relationships

In addition to using a table to determine if a relationship is proportional, you also can use a graph. A relationship is a proportional relationship if its graph is a straight line through the origin.

EXAMPLE 1 Real World

COMMON CORE 7.RP.2a

A house cleaning company charges $45 per hour. Is the relationship a proportional relationship? Explain.

STEP 1 Make a table.

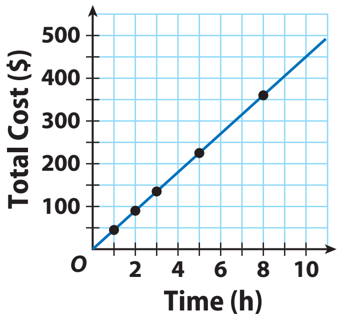
Each hour costs $45. So for 2 hours, the cost is $2 \cdot \$45 = \$90$.

Time (h)	1	2	3	5	8
Total cost ($)	45	90	135	225	360

STEP 2 Write the data in the table as ordered pairs (time, cost).

(1, 45), (2, 90), (3, 135), (5, 225), (8, 360)

STEP 3 Graph the ordered pairs.



Place time on the x-axis and total cost on the y-axis.

Plot each point.

Connect the points with a line.

The graph is a line that goes through the origin.

So, the relationship is proportional. The point (1, 45) on the graph shows that the unit rate is $45 for 1 hour.

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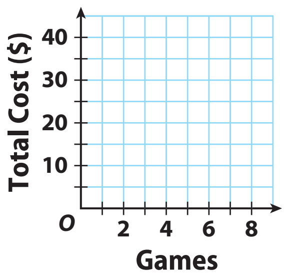
YOUR TURN

1. Jared rents bowling shoes for $6 and pays $5 per bowling game. Is the relationship a proportional relationship? Explain.

Games	1	2	3	4
Total Cost ($)	11	16	21	26



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Analyzing Graphs

Recall that you can describe a proportional relationship with the equation $y = kx$. The constant of proportionality k tells you how steep the graph of the relationship is. The greater the absolute value of k, the steeper the line.

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EXAMPLE 2 Real World

COMMON CORE 7.RP.2d, 7.RP.2b, 7.RP.2c

The graph shows the relationship between time in minutes and the number of miles Damon runs. Write an equation for this relationship.

STEP 1 Choose a point on the graph and tell what the point represents.

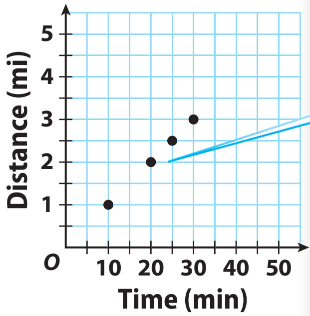
The point (25, 2.5) represents the distance (2.5 miles) that Damon runs in 25 minutes.

STEP 2 What is the constant of proportionality?

Because $\frac{\text{distance}}{\text{time}} = \frac{2.5\text{ mi}}{25\text{ min}} = \frac{1}{10}$, the constant of proportionality is $\frac{1}{10}$.

STEP 3 Write an equation in the form $y = kx$.

$y = \frac{1}{10}x$



The points appear to form a line through the origin so the relationship is proportional.

Reflect

2. **Communicate Mathematical Ideas** What does the point (0, 0) on the graph represent?

____

3. **What If?** Suppose you drew a graph representing the relationship $y = \frac{1}{8}x$ between time in minutes and the number of miles Esther runs. How would the graph compare to the one for Damon? Explain.

____

Math Talk
Mathematical Practices
What is the meaning of the point on the graph in Exercise 4 with x-coordinate 1?

YOUR TURN

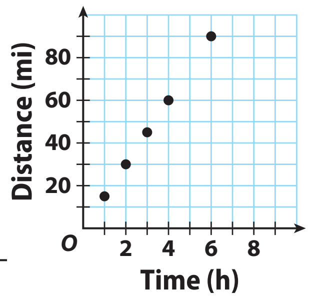
4. The graph shows the relationship between the distance a bicyclist travels and the time in hours.

a. What does the point (4, 60) represent?

____

b. What is the constant of proportionality? ____________

c. Write an equation in the form $y = kx$ for this relationship. ____________



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Guided Practice

Complete each table. Tell whether the relationship is a proportional relationship. Explain why or why not. (Explore Activity)

1. A student reads 65 pages per hour.

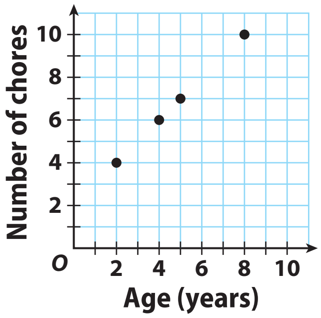
Time (h)	3	5		10
Pages			585	

2. A babysitter makes $7.50 per hour.

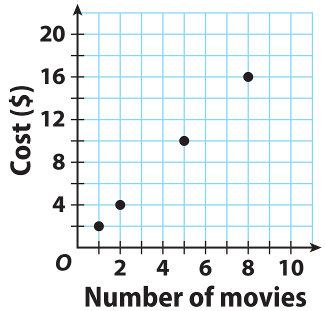
Time (h)	2		5	
Pages		22.50		60

Tell whether the relationship is a proportional relationship. Explain why or why not. (Explore Activity and Example 1)

3. Chores

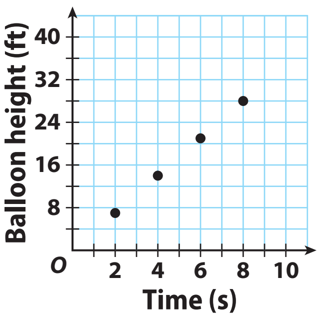


4. Movie Rentals

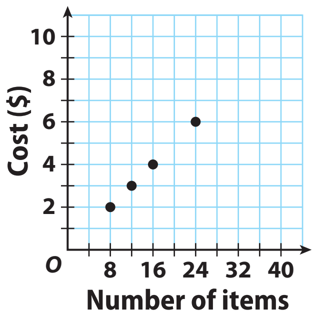


Write an equation of the form $y = kx$ for the relationship shown in each graph. (Example 2)

5.



6.



ESSENTIAL QUESTION CHECK-IN

7. How does a graph show a proportional relationship?

Name ____________ Class ____________ Date ____________

4.3 Independent Practice

COMMON CORE 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d

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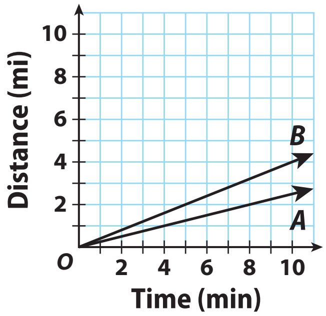
For Exercises 8–12, the graph shows the relationship between time and distance run by two horses.



8. Explain the meaning of the point (0, 0).

____

____

9. How long does it take each horse to run a mile?

____

____

10. **Multiple Representations** Write an equation for the relationship between time and distance for each horse.

____

11. **Draw Conclusions** At the given rates, how far would each horse run in 12 minutes?

____

____

12. **Analyze Relationships** Draw a line on the graph representing a horse than runs faster than horses A and B.

____

____

13. A bullet train can travel at 170 miles per hour. Will a graph representing distance in miles compared to time in hours show a proportional relationship? Explain.

____

____

____

____

14. **Critical Thinking** When would it be more useful to represent a proportional relationship with a graph rather than an equation?

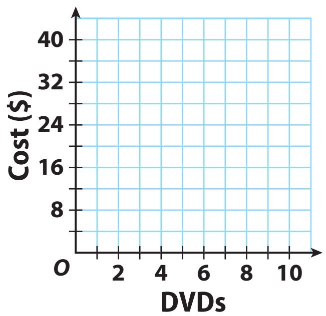
____

____

____

15. **Multiple Representations** Bargain DVDs cost $5 each at Mega Movie.

a. Graph the proportional relationship that gives the cost *y* in dollars of buying *x* bargain DVDs.

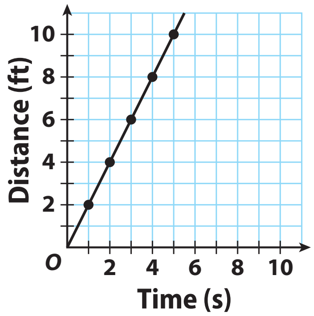


b. Give an ordered pair on the graph and explain its meaning in the real world context.

____

____

The graph shows the relationship between distance and time as Glenda swims.



16. How far did Glenda swim in 4 seconds? ______________________

17. **Communicate Mathematical Ideas** Is this a proportional relationship? Explain your reasoning.

____

18. **Multiple Representations** Write an equation that shows the relationship between time and distance. ______________________

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

19. **Make a Conjecture** If you know that a relationship is proportional and are given one ordered pair that is not (0, 0), how can you find another pair?

____

____

____

____

The tables show the distance traveled by three cars.

Car 1	
Time (h)	Distance (mi)
0	0
2	120
3	180
5	300
6	360

Car 2	
Time (h)	Distance (mi)
0	0
5	200
10	400
15	600
20	800

Car 3	
Time (h)	Distance (mi)
0	0
1	65
2	85
3	105
4	125

20. **Communicate Mathematical Ideas** Which car is not traveling at a constant speed? Explain your reasoning.

____

____

21. **Make a Conjecture** Car 4 is traveling at twice the rate of speed of car 2. How will the table values for car 4 compare to the table values for car 2?

____

____

____

MODULE QUIZ

Ready to Go On?

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4.1 Unit Rates

Find each unit rate. Round to the nearest hundredth, if necessary.

1. \$140 for 18 ft^2 ________________

2. 14 lb for \$2.99 ________________

Circle the better deal in each pair. Then give the unit rate for the better deal.

3. $\frac{\$56}{25\text{ gal}}$ or $\frac{\$32.05}{15\text{ gal}}$ ________________

4. $\frac{\$160}{5\text{ g}}$ or $\frac{\$315}{9\text{ g}}$ ________________

4.2 Constant Rates of Change

5. The table shows the amount of money Tyler earns for mowing lawns. Is the relationship a proportional relationship? Why or why not?

Number of Lawns	1	2	3	4
Amount Earned (\$)	15	30	48	64

____

6. On a recent day, 8 euros were worth \$9 and 24 euros were worth \$27. Write an equation of the form $y = kx$ to show the relationship between the number of euros and the value in dollars.

________________, where y is dollars and x is euros

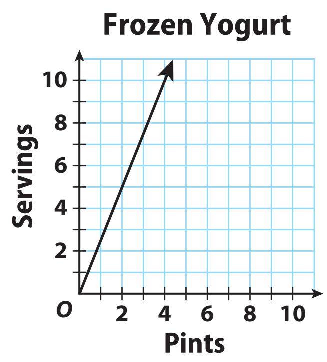
4.3 Proportional Relationships and Graphs

7. The graph shows the number of servings in different amounts of frozen yogurt listed on a carton. Write an equation that gives the number of servings y in x pints.

____

8. A refreshment stand makes 2 large servings of frozen yogurt from 3 pints. Add the line to the graph and write its equation.

____



ESSENTIAL QUESTION

9. How can you use rates to determine whether a situation is a proportional relationship?

____

____

COMMON CORE

MODULE 4 **MIXED REVIEW**

Assessment Readiness

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Selected Response

1. Kori spent \$46.20 on 12 gallons of gasoline. What was the price per gallon?

Ⓐ \$8. 35 Ⓒ \$2.59

Ⓑ \$3.85 Ⓓ \$0.26

2. A rabbit can run short distances at a rate of 35 miles per hour. A fox can run short distances at a rate of 21 miles per half hour. Which animal is faster, and by how much?

Ⓐ The rabbit; 7 miles per hour

Ⓑ The fox; 7 miles per hour

Ⓒ The rabbit; 14 miles per hour

Ⓓ The fox; 14 miles per hour

3. A pet survey found that the ratio of dogs to cats is $\frac{2}{5}$. Which proportion shows the number of dogs if the number of cats is 140?

Ⓐ $\frac{2 \text{ dogs}}{5 \text{ cats}} = \frac{140 \text{ dogs}}{350 \text{ cats}}$

Ⓑ $\frac{2 \text{ dogs}}{5 \text{ cats}} = \frac{140 \text{ cats}}{350 \text{ dogs}}$

Ⓒ $\frac{2 \text{ dogs}}{5 \text{ cats}} = \frac{28 \text{ dogs}}{140 \text{ cats}}$

Ⓓ $\frac{2 \text{ dogs}}{5 \text{ cats}} = \frac{56 \text{ dogs}}{140 \text{ cats}}$

4. What is the cost of 2 kilograms of flour if 3 kilograms cost \$4.86 and the unit price for each package of flour is the same?

Ⓐ \$0.81 Ⓒ \$3.24

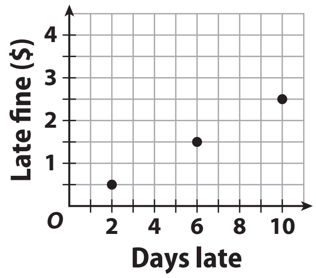
Ⓑ \$2.86 Ⓓ \$9.72

5. One gallon of paint covers about 450 square feet. How many square feet will 1.5 gallons of paint cover?

Ⓐ 300 ft^2 Ⓒ 675 ft^2

Ⓑ 451.5 ft^2 Ⓓ 900 ft^2

6. The graph shows the relationship between the late fines the library charges and the number of days late.



What is an equation for the relationship?

Ⓐ $y = 0.25x$ Ⓒ $y = 0.50x$

Ⓑ $y = 0.40x$ Ⓓ $y = 0.75x$

Mini-Task

7. School is 2 miles from home along a straight road. The table shows your distance from home as you walk home at a constant rate.

Time (min)	10	20	30
Distance from home (mi)	1.5	1	0.5

a. Is the relationship in the table proportional?

____

b. Find your distance from school for each time in the table.

____

c. Write an equation representing the relationship between the distance from school and time walking.

____