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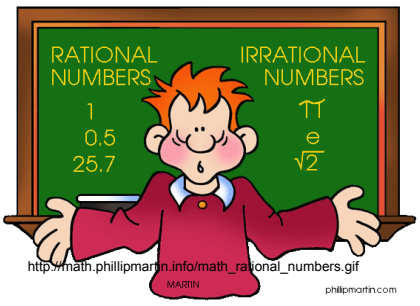
# Rational Numbers and Equations



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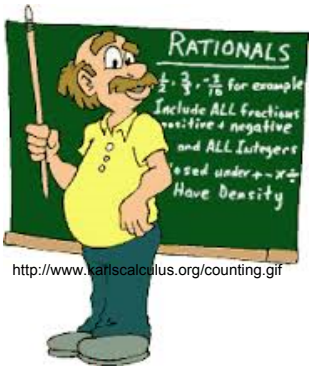

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# Rational Numbers

Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers then -(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts. CC.7.NS.2b

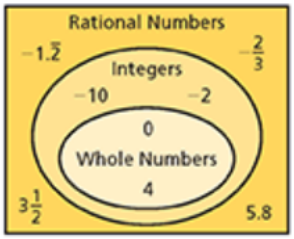
Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. CC.7.NS.2d

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. CC.7.EE.3

Lesson objectives | Teachers' notes

## Rational Numbers

**Rational number** is a number that can be written as $\frac{a}{b}$ where a and b are integers and b ≠ 0.




**Terminating decimal** is a decimal that ends

**Repeating decimal** is a decimal that has a pattern that repeats.

## Converting Fractions to Decimals

To write a fraction as a decimal, divide the numerator of the fraction by the denominator of the fraction.

Convert $\frac{5}{8}$ to a decimal. Round to the nearest thousandth, if necessary.

___)5.0 Set up the division. Divide ___ into 5.

8) 5.000 How many times does 8 go into 50?

___

−16

40

0

Is there a remainder? _____

$\frac{5}{8}$ = _____

## Writing Fractions as Decimals

**Write each fraction as a decimal. Round to the nearest hundredth, if necessary.**

1) $\frac{1}{4} =$

$4\overline{)1.00}$

2) $\frac{9}{5} =$

$5\overline{)9.0}$

3) $\frac{5}{3} \approx$

$3\overline{)5.000}$

4)

## Converting Decimals to Fractions

Convert 0.52 to a fraction in simplest form.

0.52 can be written as fifty-two _____ __.

$0.52 = \frac{\quad}{100}$ Write the decimal as a fraction with a denominator of 100.

$0.52 = \frac{52 \div \_}{100 \div}$ Divide the numerator and denominator by their greatest common factor. GCF = ___

$0.52 = \frac{\quad}{25}$ Simplify.

## Writing Decimals as Fractions

**Write each decimal as a fraction in simplest form.**

5) **0.018**

6) **−0.6**

7) **1.55**

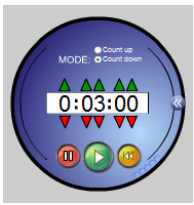
1.55 = ——

= ————

= —, or —

# Check Point

8) Write $\frac{2}{5}$ as a decimal. $\frac{2}{5}$ = _____




**Write each fraction as a decimal. Round to the nearest thousandth, if necessary.**

9) $\frac{7}{8}$ _____

10) $\frac{5}{3}$ _____

# Adding and Subtracting Rational Numbers

Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. CC.7.NS.1b

Understand subtraction of rational numbers as adding the additive inverse, p - q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. CC.7.NS.1c

Apply properties of operations as strategies to add and subtract rational numbers. CC.7.NS.1d

Lesson objectives | Teachers' notes

Warm up:

1) $\frac{5}{7} + \frac{1}{7}$

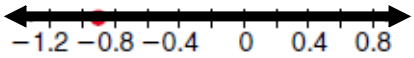
2) $-\frac{1}{8} - \frac{5}{8}$

3) $\frac{5}{12} - \frac{1}{12}$

**Use a number line to find each sum.**

4) **0.3 + (−1.2)**

*Move right units.*
*From , move left units.*




So, 0.3 + (-1.2) = -0.9

Which way is correct?

0.3
+ (-1.2)

-1.2
+ 0.3

Key Idea

To add and Subtract rational numbers, use the same rules for signs as you used for integers.

**Add or subtract.**

5) -1.2 + 8.4

6) 2.5 + (-2.8)

7) $-\frac{2}{9} - \frac{5}{9}$

— —

—

5) $\frac{1}{8} + \frac{2}{7}$

$\frac{1}{8}(\ ) + \frac{2}{7}(\ )$ Find a common denominator

— + — Rewrite with a common denominator

— Simplify if possible.

8) $1\frac{1}{6} + \frac{5}{8}$

$1\frac{1}{6}\left(\ \right) + \frac{5}{8}\left(\ \right)$ Find a common denominator

$1\frac{\ }{\ } + \frac{\ }{\ }$ Rewrite with a common denominator

$\frac{\ }{\ }$ Simplify if possible.

7) $4\frac{2}{11} - 1\frac{7}{11}$

=

# Check Point

**Add or subtract. Write answer in simplest form.**


9) $\frac{1}{5} + \frac{1}{2}$

10) $-\frac{3}{14} - \frac{4}{7}$

11) $-2\frac{3}{5} - 1\frac{7}{10}$

12) $2\frac{3}{4} - 4$

12) 27.3 + (-9.5)

13) 92.7 - (4.8)

# Multiplying and dividing rational numbers

Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. CC.7.NS.2a

Apply properties of operations as strategies to multiply and divide rational numbers. CC.7.NS.2c

Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) CC.7.NS.3

Lesson objectives | Teachers' notes

## Multiplying and dividing rational numbers

**Multiply.**

1) $-3 \cdot 0.03$

$$\begin{array}{r} -3 \\ \times\ 0.03 \\ \hline \end{array}$$

2) $2.45 \cdot 35$

$$\begin{array}{r} 2.45 \\ \times\ 35 \\ \hline \\ + \\ \hline \end{array}$$

3) $2.4 \cdot 1.8$

$$\begin{array}{r} 2.4 \\ \times\ 1.8 \\ \hline \\ + \\ \hline \end{array}$$

$-3.84 \cdot 0.9$

$$\begin{array}{r} -3.84 \\ \times\ 0.9 \\ \hline \end{array}$$

$-12 \cdot \frac{3}{4}$

$=$ $\cdot$ *Write $-12$ as a fraction.*

$=$ *Simplify.*

$=$ $=$ *Multiply numerators. Multiply denominators.*

4) $\frac{1}{3} \cdot \frac{3}{8}$

$=$ *Simplify.*

$=$ *Multiply numerators. Multiply denominators.*

5) $\left(-\frac{2}{3}\right) \cdot \frac{4}{5}$

$=$

$=$ $=$ *Multiply numerators. Multiply denominators.*

6) $\frac{2}{5} \cdot 1\frac{2}{3}$

$= \frac{2}{5} \cdot$ *Write the mixed number as an improper fraction.*

$=$ $\cdot$ *Simplify.*

$=$ *Multiply numerators. Multiply denominators.*

7) $4\frac{1}{5} \cdot 2\frac{1}{7}$

$= \frac{\quad}{5} \cdot \frac{\quad}{7}$ *Write mixed numbers as improper fractions.*

$=$ $\quad \cdot \quad$ *Simplify.*

$=$ — or *Multiply numerators. Multiply denominators.*

Divide.

8) **8.28 ÷ 4.6**

$46\overline{)82.8}$

9) **18.48 ÷ (−1.75)**

$175\overline{)1{,}848.00}$

10) $\frac{3}{7} \div \frac{2}{5}$

$= \frac{3}{7} \cdot$ —  *Multiply by the reciprocal of $\frac{2}{5}$.*

$= \frac{3 \cdot}{7 \cdot}$

$=$ — or

11) $\frac{3}{8} \div 12$

$= \frac{3}{8} \cdot$ —  *Multiply by the reciprocal of 12.*

$=$ — $\cdot$ —  *Simplify.*

$=$ —

12) $5\frac{2}{3} \div 1\frac{1}{4}$

$= \frac{\quad}{\quad} \div \frac{\quad}{\quad}$ *Write mixed numbers as improper fractions.*

$= \frac{\quad}{\quad} \cdot \frac{\quad}{\quad}$ *Multiply by the reciprocal of $\frac{5}{4}$.*

$= \frac{\quad}{\quad}$ or $\frac{\quad}{\quad}$

13) $\frac{3}{4} \div 2\frac{1}{2}$

$= \frac{3}{4} \div \frac{\quad}{\quad}$ *Write $2\frac{1}{2}$ as an improper fraction.*

$= \frac{3}{4} \cdot \frac{\quad}{\quad}$ *Multiply by the reciprocal of $\frac{5}{2}$.*

$= \frac{\quad}{\quad}$ *Simplify.*

$= \frac{\quad}{\quad}$

# Solving equations by adding or subtracting

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. CC.7.EE.1

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." CC.7.EE.2

Solve real-life and mathematical problems using numerical and algebraic expressions and equations. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. CC.7.EE.4

Lesson objectives | Teachers' notes

## Solving equations by adding or subtracting

**Equation :** A mathematical sentence that shows that two expressions are equivalent.

**Solve:** To find an answer or solution

**Solution:** A value or values that make a statement true.

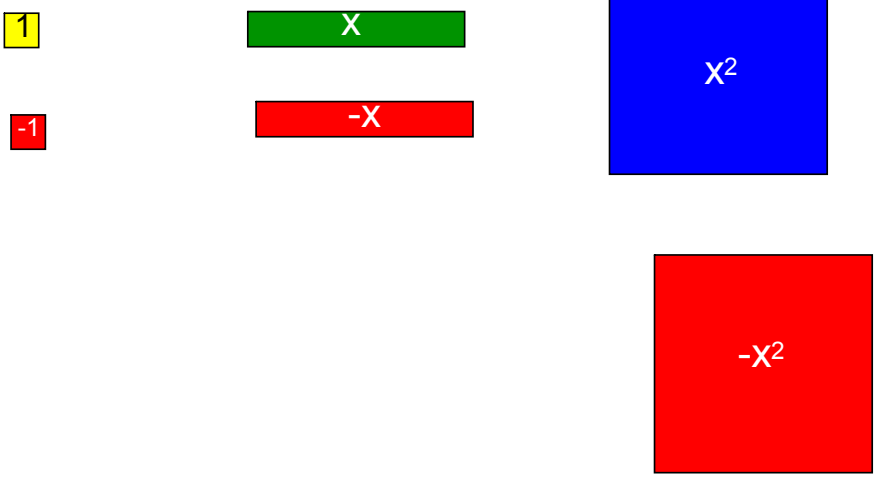
**inverse operation:** Operations that undo each other.

**isolate the variable:** To get a variable alone on one side of an equation or inequality in order to solve the equation or inequality.

**addition property of Equality:** The property that states that if you add the same number to both sides of an equation, the new equation will have the same solution.

**Subtraction property of equality**
The property that states that if you subtract the same number from both sides of an equation, the
new equation will have the same solution.

# Using Algebra Tiles

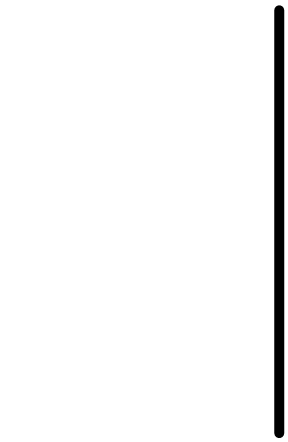


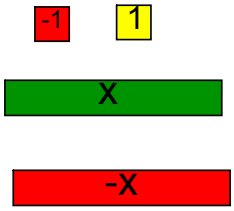
Solving Equations by Adding or Subtracting

**Show how to solve both mathematically and using the Algebra Tiles.**

1) $x + 5 = 12$

Algebra Tiles



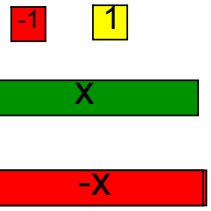


**Show how to solve both mathematically and using the Algebra Tiles.**

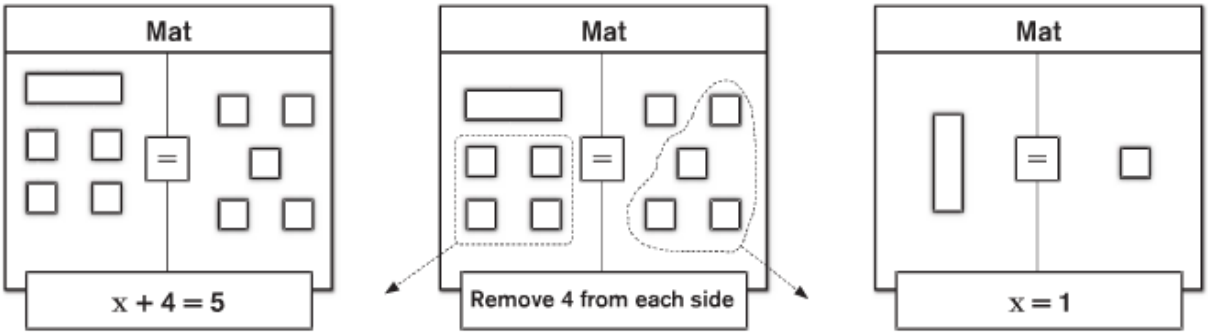
**Solve**

2) -9 = w - 3

Algebra Tiles



In the second method, we just subtract the same quantity from both sides of the linear equation $x + 4 = 5$.

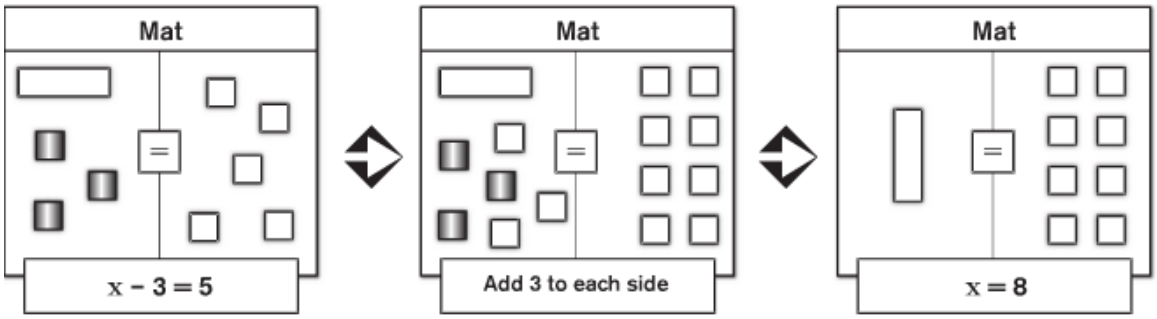



Write Mathematically how to isolate the Variable.

ACTIVITY 4.1 **Solving Linear Equations** Name:

*When* **solving linear equations**, *we find a value for the variable that satisfies the equation. Sometimes they are easy and we might be able to guess the value. For example,* $x - 3 = 5$. In this equation, $x = 8$.
**Two different methods to solve a linear equation are shown below.**

In the first method, we add the same quantity to both sides of the linear equation $x - 3 = 5$.



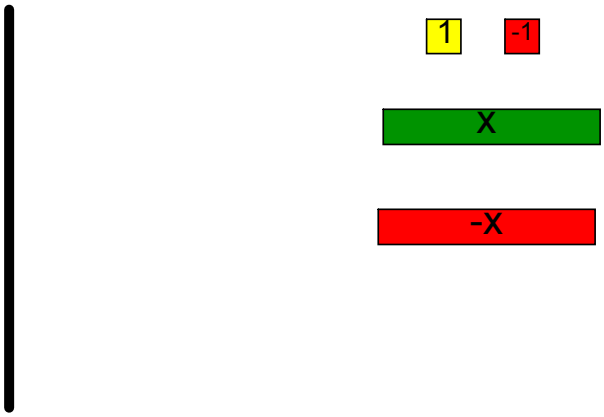

Write Mathematically how to isolate the Variable.

**Complete to solve the equation. check the solution.**

**Show how to solve both mathematically and using the Algebra Tiles.**

3) $x + 2 = -15$

Algebra Tiles



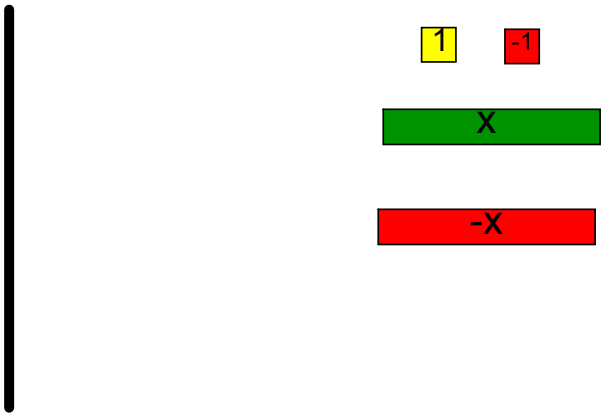

**Check:**

**Complete to solve the equation. check the solution.**

**Show how to solve both mathematically and using the Algebra Tiles.**

4) $z + (-6) = 14$

Algebra Tiles




**Check:**

**Complete to solve the equation. check the solution.**

5) $n - 2.75 = 8.30$

_____ _____ *Add to isolate n.*

$n$ =

$a + 32.66 = 42.00$

_____ _____ *Subtract to isolate a.*

$a$ =

## Solving Equations with Fractions

6) $n + \frac{2}{7} = -\frac{3}{7}$

Subtract $\frac{2}{7}$ from both sides.

$n =$

5) $y - \frac{1}{6} = \frac{2}{3}$

Add $\frac{1}{6}$ to both sides of the equation.

Find a common denominator, 6.

$y =$

$y =$

## Check Point

**Complete to solve the equation. check the solution.**

7) $52 = x + 45$

8) $b - 140 = 55$

9) $7.5 + c = 10.6$

## Solving Word Problems Using Equations

**Mr. Rios wants to prepare a casserole that requires $2\frac{1}{2}$ cups of milk. If he makes the casserole, he will have only $\frac{3}{4}$ cup of milk left for his breakfast cereal. How much milk does Mr. Rios have?**

**Verbal Expression:**

| *Original amount of milk* | – | *Milk for casserole* | = | *Milk for cereal* |
|---|---|---|---|---|

**Write an equation, then solve:**

c - = 

c =

**Solution:**

Mr. Rios has cups of milk.

# Solving Equations by Multiplying or Dividing

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. CC.7.EE.1

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." CC.7.EE.2

Solve real-life and mathematical problems using numerical and algebraic expressions and equations. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. CC.7.EE.4

Lesson objectives | Teachers' notes

## Solving Equations by Multiplying or Dividing

**The Properties of Equality:**

- **Addition Property of Equality:**
  adding the same number to both sides of an equation does not change the equality

- **Subtraction Property of Equality:**
  subtracting the same number from both sides of an equation does not change the equality

- **Multiplication Property of Equality:**
  multiplying both sides of an equation by the same number does not change the equality

- **Division Property of Equality:**
  dividing both sides of an equation by the same number does not change the equality

## Solving Equations

Why:

To find the value of a variable that makes the statement true

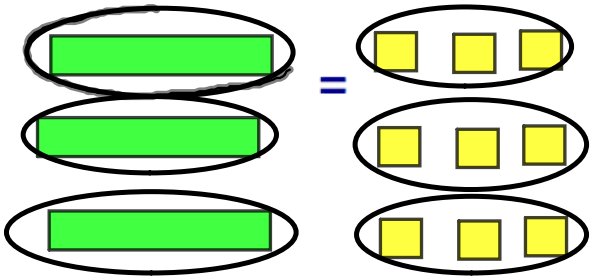
How: Isolate the variable to get it by itself on one side of the equation

How: Perform inverse operations to each side of the equation, then simplify to find the value of the variable.

## Use algebra tiles to model and solve equations

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$



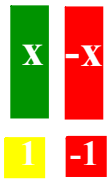
Check the answer: $3 \cdot 3 = 9$

$9 = 9$ ✓

BACK

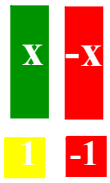
2) How can the remaining squares bedistributed among the bars?

$2x = 4$



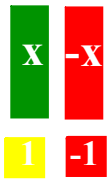
3) How can the remaining squares bedistributed among the bars?

-4x = 12



4) How can the remaining squares bedistributed among the bars?

$-2x = -10$



## Solving equations using multiplication or division

5) $\frac{h}{2} = 13$

$h =$

Think: h is divided by 2, so multiply both sides by 2 to isolate h.

6) $51 = 17x$

$= x$

Think: x is multiplied by 17, so divide both sides by 17 to isolate x.

**Solve the equation. Check your solution.**

7) $4x = 24$

x =

**Check:**

$4x = 24$

4 ( ) = 24

=

8) $-7x = 77$

x =

**Check:**

$-7x = 77$

-7 ( ) =

=

9) -8 • $\frac{x}{-8} = 6$ • -8

x = -48

**Check:**

$\frac{x}{-8} = 6$

$\frac{-48}{-8} = 6$

6 = 6

10) $9 = 3.6d$

*Divide to isolate d.*

*Think: 9 ÷ 3.6 = 90 ÷ 36.*

$9 = d$

$= d$

11) $\frac{x}{4.8} = 5.4$

$\frac{x}{4.8} = 5.4$

*Multiply to isolate x.*

$x =$

# Check Point

Solving Equations by Multiplying or Dividing


12) $\frac{x}{6} = 3$

13) $\frac{c}{10} = 7$

14) $8y = 93.6$

15) $\frac{h}{0.4} = -7.2$

16) $\frac{3}{4}d = 5$

17) $6y = \frac{2}{3}$

# Solving two step equations

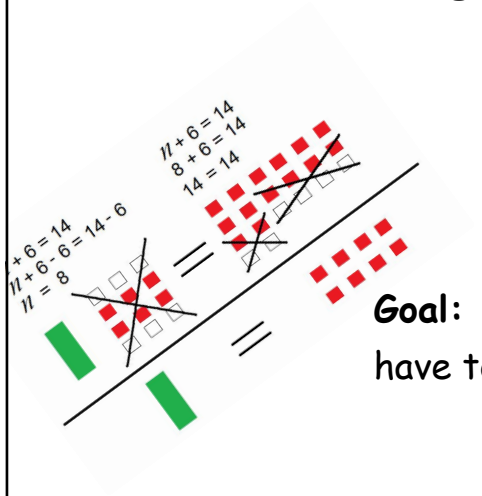
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. CC.7.EE.1

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." CC.7.EE.2

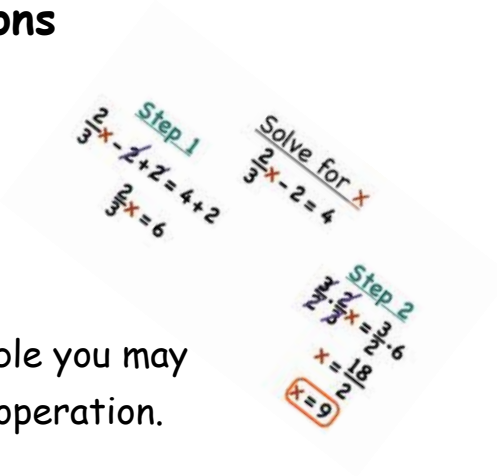
Solve real-life and mathematical problems using numerical and algebraic expressions and equations. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. CC.7.EE.4

Lesson objectives | Teachers' notes

## Solving two step equations




**Goal:** To isolate the variable you may have to use more than one operation.



$3x - 2 = 4$

The tiles model the equation. A green tile represents $x$.

$3x - 2 \quad = 4$

to each side.

$3x = 6$

Simplify by removing zero pairs.

$3x \_ 6$

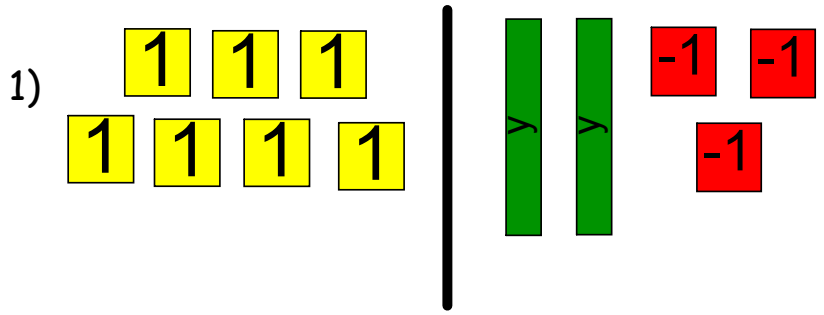
each side into three equal groups.

$x =$

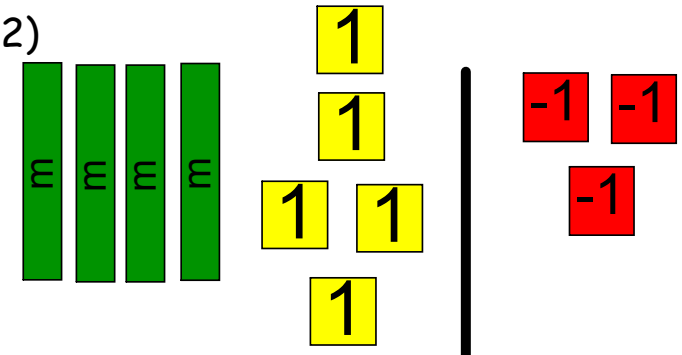
Each green tile equals two yellow tiles, so $x =$ .

Write each two-step equation represented below:

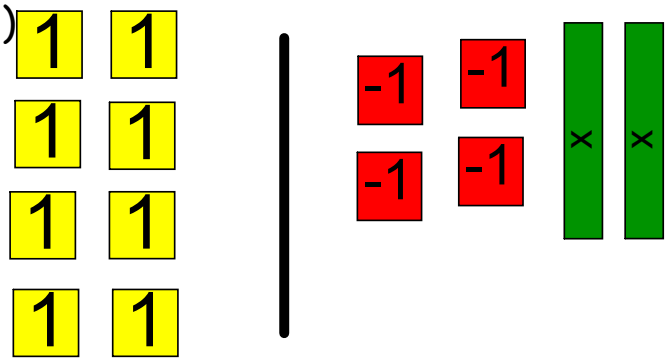
1)




2)



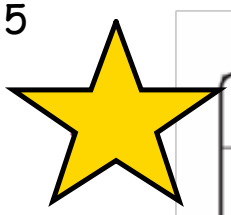

3)

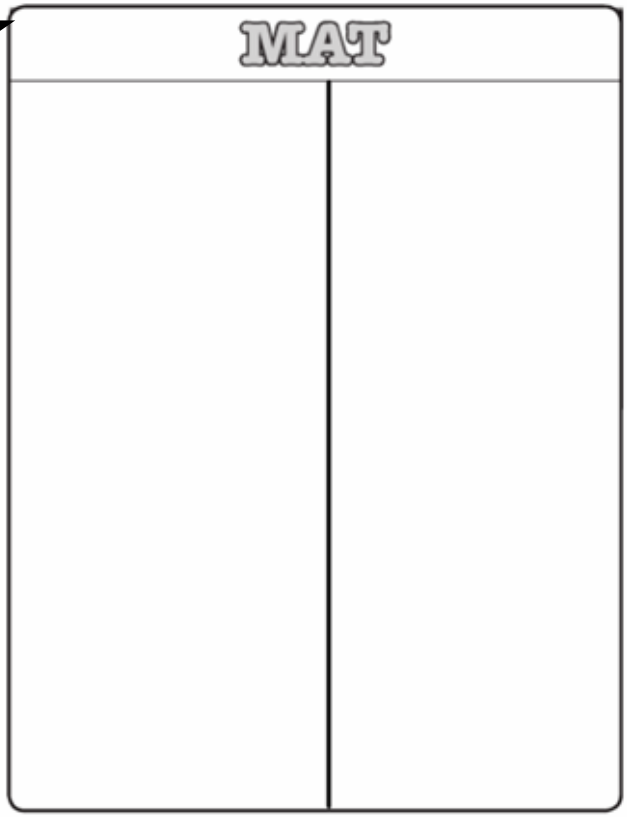


4) 2x + 3 = 5

Work:

2x + 3 = 5



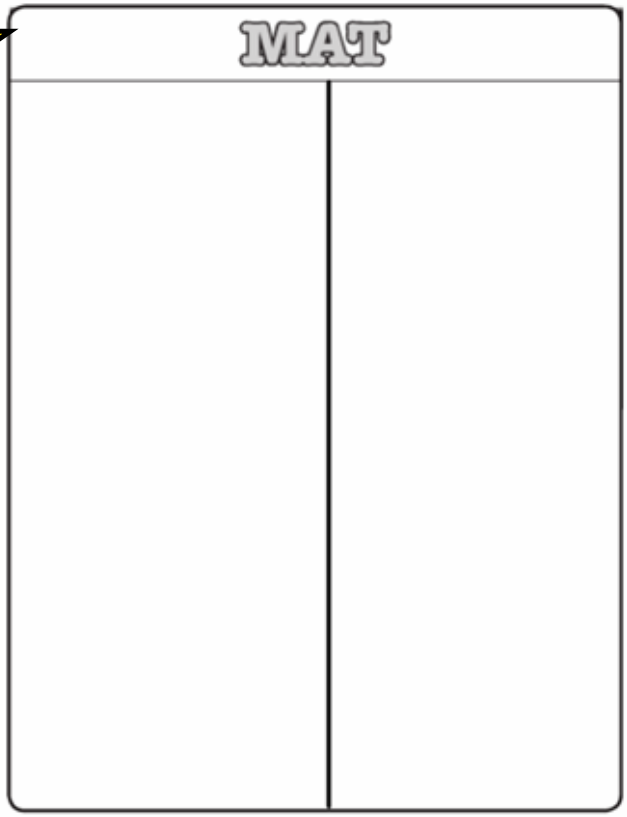




5) $3x + 3 = -3$

Work:

$3x + 3 = -3$

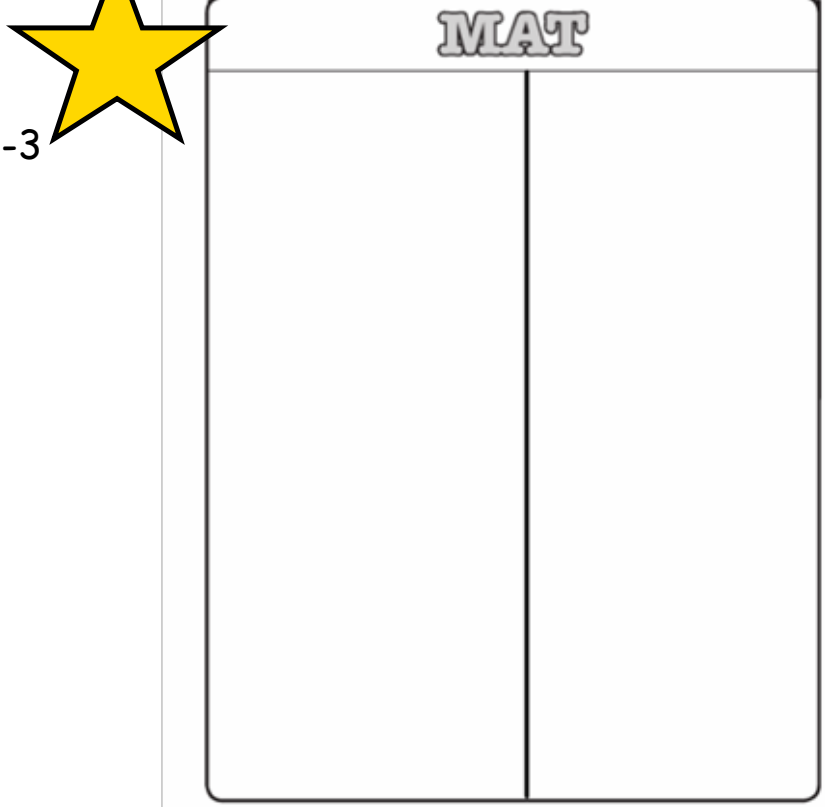


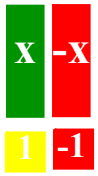


6) -2x +(-5) = -3

Work:

-2x +(-5) = -3



**Complete to solve and check each equation.**

7) $3t + 7 = 19$

To undo addition, .

$3t = $

To undo multiplication, .

$t = $

**Check:** $3t + 7 = 19$

$3(\quad) + 7 \stackrel{?}{=} 19$ Substitute for $t$.

$\quad + 7 \stackrel{?}{=} 19$

$= \quad$ ✓

8) $\frac{w}{3} - 7 = 5$

To undo subtraction, .

$\frac{w}{3} =$

To undo division, .

$\cdot \frac{w}{3} = \cdot 12$

$w =$

**Check:** $\frac{w}{3} - 7 = 5$

$\frac{}{3} - 7 \stackrel{?}{=} 5$ Substitute.

$- 7 \stackrel{?}{=} 5$

$=$ ✓

9) $\frac{z-3}{2} = 8$

$\_\_ \cdot \frac{z-3}{2} = \_\_ \cdot 8$ To undo division, .

$z - 3 =$

To undo subtraction, .

$z \quad =$

**Check:** $\frac{z-3}{2} = 8$

$\frac{-3}{2} \stackrel{?}{=} 8$ Substitute.

$\frac{}{2} \stackrel{?}{=} 8$

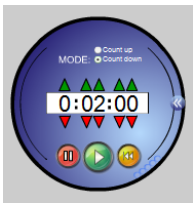
$=$ ✓

# Check Point

Solve.

10) $2x + 3 = 9$

11) $\frac{x}{3} - 1 = 5$




12) $\frac{1}{2}x + 2 = -3$