Two-Way Independent ANOVA

- Rationale of factorial ANOVA
- Partitioning variance
- Interaction effects
  - Interaction graphs
  - Interpretation

What is Two-Way Independent ANOVA?

- Two independent variables
  - Two-way = 2 Independent variables
  - Three-way = 3 Independent variables
- Several independent variables is known as a factorial design.

Benefit of Factorial Designs

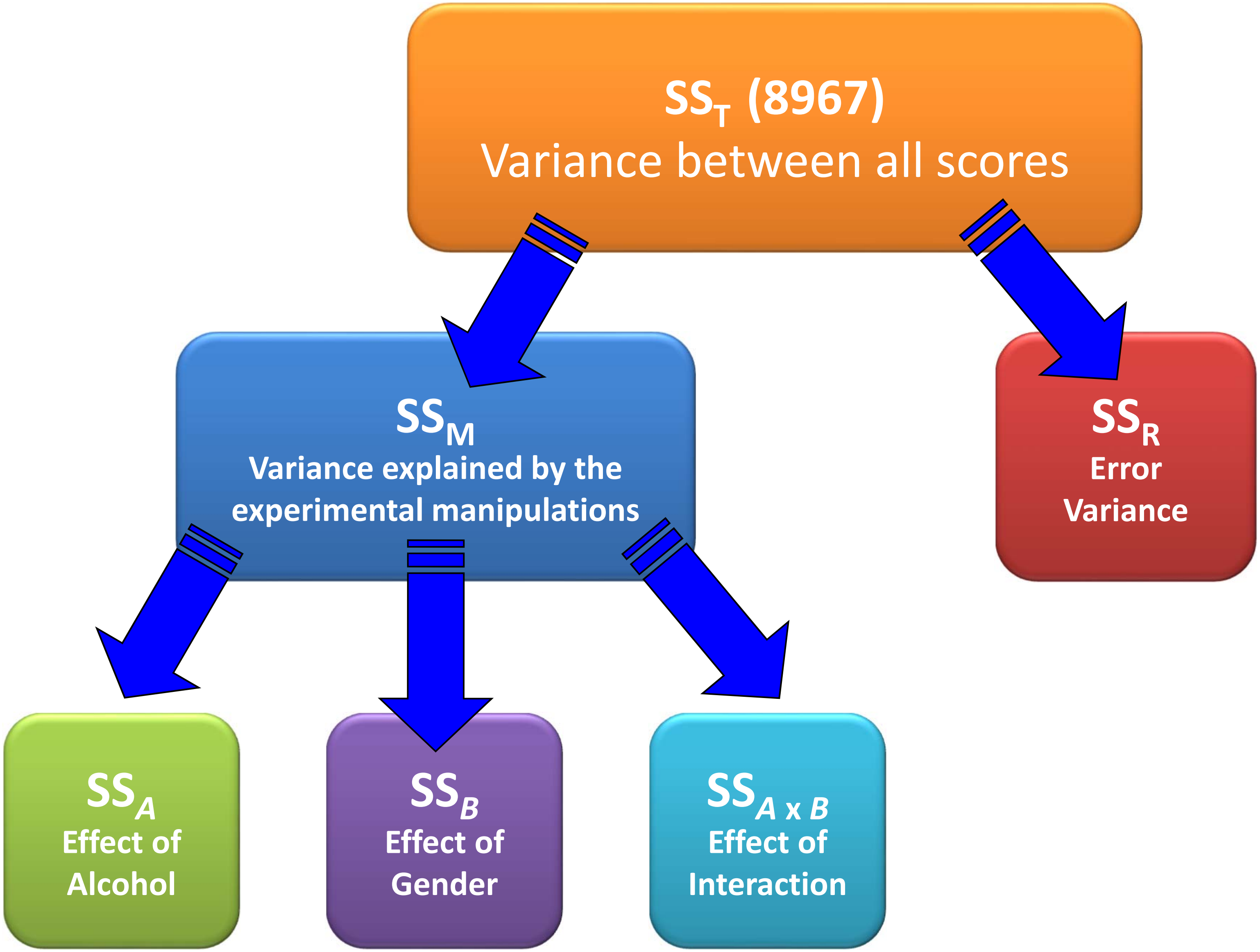
- We can look at how variables *interact*.
- Interactions
  - Show how the effects that one IV might depend on the effects of another
  - Are often more interesting than main effects.
- Examples
  - Interaction between hangover and lecture topic on sleeping during lectures.
    - A hangover might have more effect on sleepiness during a stats lecture than during a clinical one.

An Example

- Field (2009): Testing the effects of alcohol and gender on ‘the beer-goggles effect’:
  - IV 1 (**Alcohol**): none, 2 pints, 4 pints
  - IV 2 (**Gender**): male, female
- Dependent variable (DV) was an objective measure of the attractiveness of the partner selected at the end of the evening.

Table 12.1: Data for the beer-goggles effect

Alcohol	**None**		**2 Pints**		**4 Pints**	
Gender	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**
	65	50	70	45	55	30
	70	55	65	60	65	30
	60	80	60	85	70	30
	60	65	70	65	55	55
	60	70	65	70	55	35
	55	75	60	70	60	20
	60	75	60	80	50	45
	55	65	50	60	50	40
Total	485	535	500	535	460	285
Mean	60.625	66.875	62.50	66.875	57.50	35.625
Variance	24.55	106.70	42.86	156.70	50.00	117.41



SS_T (8967)
Variance between all scores
SS_M
Variance explained by the experimental manipulations
SS_R
Error Variance
SS_A
Effect of Alcohol
SS_B
Effect of Gender
$SS_{A \times B}$
Effect of Interaction

Step 1: Calculate SS_T

65	50	70	45	55	30
50	55	65	60	65	30
70	80	60	85	70	30
45	65	70	65	55	55
55	70	65	70	55	35
30	75	60	70	60	20
70	75	60	80	50	45
55	65	50	60	50	40

Grand Mean = 58.33

$$\begin{aligned} SS_T &= s^2_{\text{grand}}(N-1) \\ &= 190.78\,(48-1) \\ &= 8966.66 \end{aligned}$$

Step 2: Calculate SS_M

$$SS_M = \sum n_i \left(\bar{x}_i - \bar{x}_{\text{grand}}\right)^2$$

$$\begin{aligned} SS_M &= 8(60.625 - 58.33)^2 + 8(66.875 - 58.33)^2 + 8(62.5 - 58.33)^2 \\ &\quad + 8(66.875 - 58.33)^2 + 8(57.5 - 58.33)^2 + 8(35.625 - 58.33)^2 \\ &= 8(2.295)^2 + 8(8.545)^2 + 8(4.17)^2 + 8(8.545)^2 + 8(-0.83)^2 + 8(-22.705)^2 \\ &= 42.1362 + 584.1362 + 139.1112 + 584.1362 + 5.5112 + 4124.1362 \\ &= 5479.167 \end{aligned}$$

Factorial Design

		Toy Color	
		Blue (1)	Pink (2)
Sex	Boy (1)	7 6 5	2 3 4
	Girl (2)	4 5 6	12 10 11

Factorial Design

$\bar{X}_{1,1}$

		Toy Color	
		Blue (1)	Pink (2)
Sex	Boy (1)	7 6 5	2 3 4
	Girl (2)	4 5 6	12 10 11

Factorial Design

Sex		Toy Color: Blue (1)	Toy Color: Pink (2)
	Boy (1)	7 6 5	2 3 4
	Girl (2)	4 5 6	12 10 11

$\bar{X}_{1,1}$

$\bar{X}_{2,1}$

Factorial Design

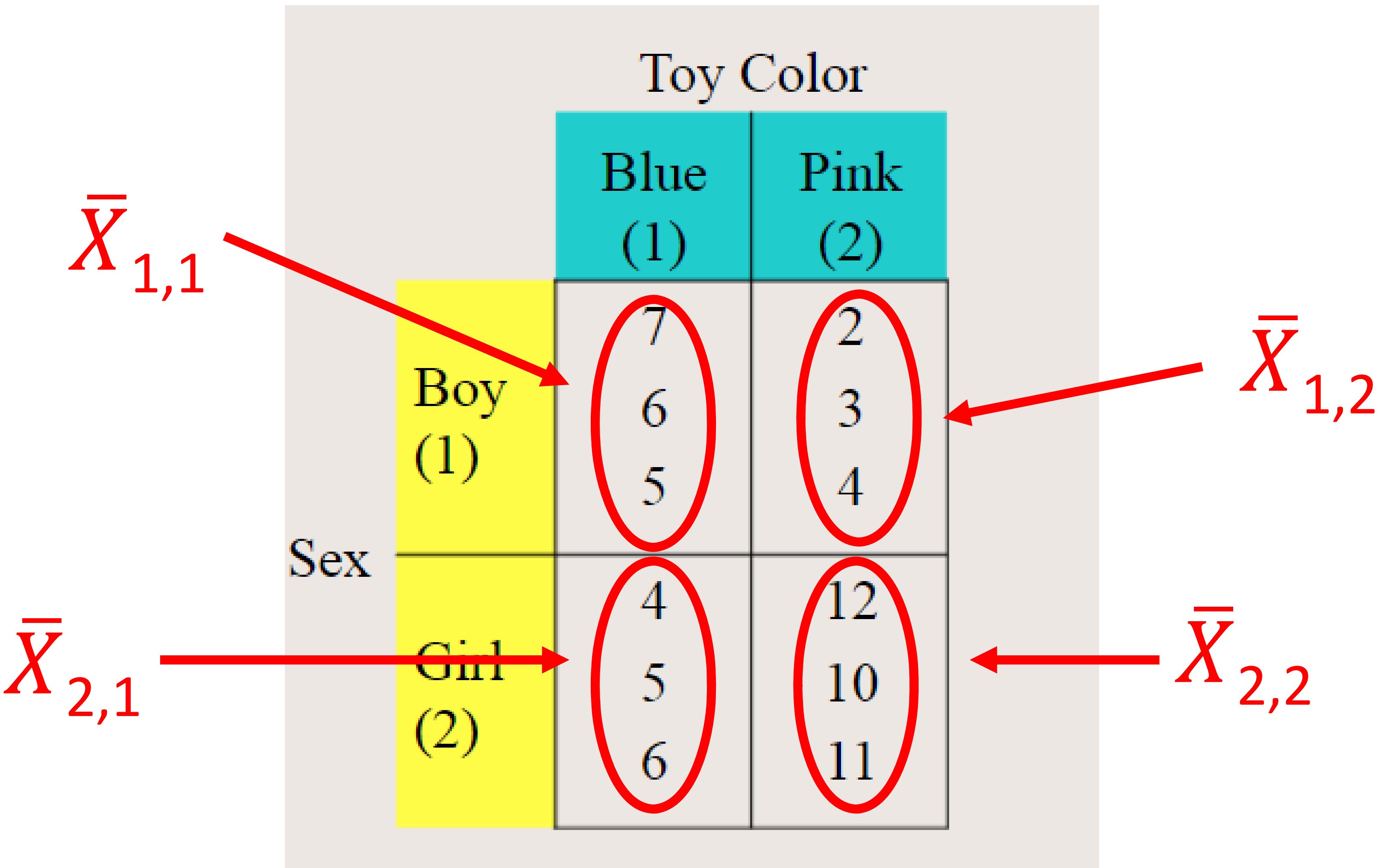
		Toy Color	
		Blue (1)	Pink (2)
Sex	Boy (1)	7 6 5	2 3 4
	Girl (2)	4 5 6	12 10 11

$\bar{X}_{1,1}$

$\bar{X}_{1,2}$

$\bar{X}_{2,1}$

Factorial Design



$$SS_M = \sum_i (\bar{X} \quad - \bar{X}_{Grand})n$$

$n_{1,1}(\bar{X}_{1,1} - \bar{X}_{Grand})$

$n_{1,2}(\bar{X}_{1,1} - \bar{X}_{Grand})$

$n_{2,1}(\bar{X}_{2,1} - \bar{X}_{Grand})$

$n_{2,2}(\bar{X}_{2,2} - \bar{X}_{Grand})$

		Toy Color	
		Blue (1)	Pink (2)
Sex	Boy (1)	7 6 5	2 3 4
	Girl (2)	4 5 6	12 10 11

Step 2a: Calculate SS_A

A_1: Female		
65	70	55
70	65	65
60	60	70
60	70	55
60	65	55
55	60	60
60	60	50
55	50	50

Mean Female = 60.21

A_2: Male		
50	45	30
55	60	30
80	85	30
65	65	55
70	70	35
75	70	20
75	80	45
65	60	40

Mean Male = 56.46

$$SS_A = \sum_1^k (\bar{X}_k - \bar{X}_{Grand}) n_k$$

Where k is the number of levels of factor A

Step 2a: Calculate SS_A

A_1: Female		
65	70	55
70	65	65
60	60	70
60	70	55
60	65	55
55	60	60
60	60	50
55	50	50

Mean Female = 60.21

A_2: Male		
50	45	30
55	60	30
80	85	30
65	65	55
70	70	35
75	70	20
75	80	45
65	60	40

Mean Male = 56.46

$$\begin{aligned} SS_{Gender} &= 24(60.21 - 58.33)^2 + 24(56.46 - 58.33)^2 \\ &= 24(1.88)^2 + 24(-1.87)^2 \\ &= 84.8256 + 83.9256 \\ &= 168.75 \end{aligned}$$

Step 2b: Calculate SS_B

B_1: None	
65	50
70	55
60	80
60	65
60	70
55	75
60	75
55	65

Mean None = 63.75

B_2: 2 Pints	
70	45
65	60
60	85
70	65
65	70
60	70
60	80
50	60

Mean 2 Pints = 64.6875

B_3: 4 Pints	
55	30
65	30
70	30
55	55
55	35
60	20
50	45
50	40

Mean 4 Pints = 46.5625

$$SS_B = \sum_1^k (\bar{X}_k - \bar{X}_{Grand}) n_k$$

Where k is the number of levels of factor B

Step 2b: Calculate SS_B

B_1: None	
65	50
70	55
60	80
60	65
60	70
55	75
60	75
55	65

Mean None = 63.75

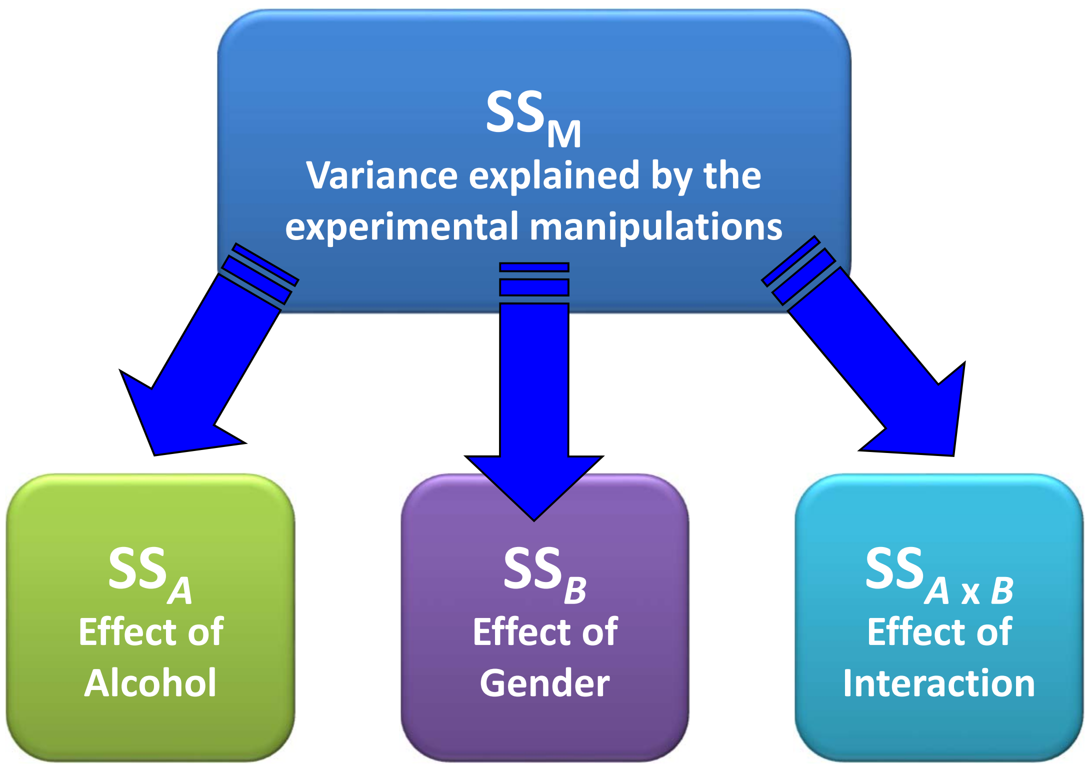
B_2: 2 Pints	
70	45
65	60
60	85
70	65
65	70
60	70
60	80
50	60

Mean 2 Pints = 64.6875

B_3: 4 Pints	
55	30
65	30
70	30
55	55
55	35
60	20
50	45
50	40

Mean 4 Pints = 46.5625

$$
\begin{aligned}
SS_{\text{Alcohol}} &= 16(63.75-58.33)^2 + 16(64.6875-58.33)^2 + 16(46.5625-58.33)^2 \\
&= 16(5.42)^2 + 16(6.3575)^2 + 16(-11.7675)^2 \\
&= 470.0224 + 646.6849 + 2215.5849 \\
&= 3332.292
\end{aligned}
$$


SS_M
Variance explained by the experimental manipulations
SS_A
Effect of Alcohol
SS_B
Effect of Gender
$SS_{A \times B}$
Effect of Interaction

Step 2c: Calculate $SS_{A \times B}$

$$SS_{A \times B} = SS_M - SS_A - SS_B$$

$$\begin{aligned} SS_{A \times B} &= SS_M - SS_A - SS_B \\ &= 5479.167 - 168.75 - 3332.292 \\ &= 1978.125 \end{aligned}$$

Step 3: Calculate SS_R

- The residual sum of squares is calculated in the same way as for one-way ANOVA
- Represents individual differences in performance or the variance that can't be explained by factors that were systematically manipulated.
- We saw in one-way ANOVA that the value is calculated by taking the squared error between each data point and its corresponding group mean.
- So, we use the individual variances of each group and multiply them by one less than the number of people within the group (n).
- We have the individual group variances: there were eight people in each group (therefore, $n = 8$).
- The degrees of freedom for each group will be one less than the number of scores per group (i.e. 7). Therefore, if we add the sums of squares for each group, we get a total of 6 X 7 = 42.

Step 3: Calculate SS_R

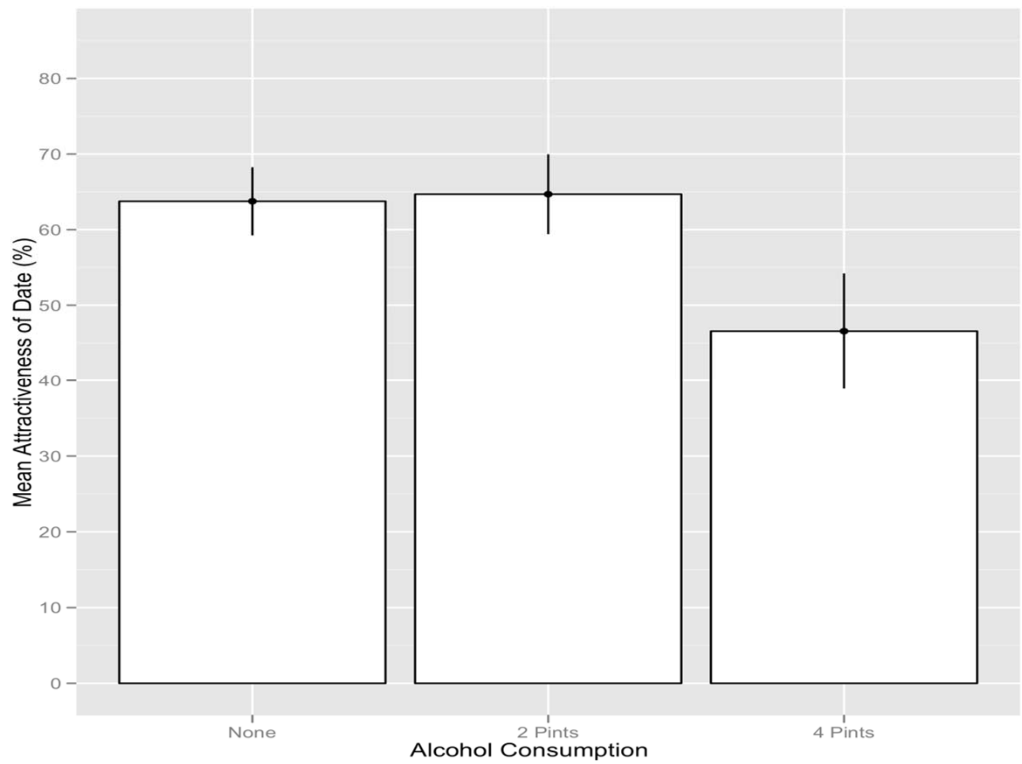
$$SS_R = s^2_{\text{group1}}(n_1 - 1) + s^2_{\text{group2}}(n_2 - 1) + s^2_{\text{group3}}(n_3 - 1) + s^2_{\text{group } n}(n_n - 1)$$

$$\begin{aligned}
SS_R &= s^2_{\text{group1}}(n_1 - 1) + s^2_{\text{group2}}(n_2 - 1) + s^2_{\text{group3}}(n_3 - 1) \\
&\quad + s^2_{\text{group4}}(n_4 - 1) + s^2_{\text{group5}}(n_5 - 1) + s^2_{\text{group6}}(n_6 - 1) \\
&= (24.55 \times 7) + (106.7 \times 7) + (42.86 \times 7) \\
&\quad + (156.7 \times 7) + (50 \times 7) + (117.41 \times 7) \\
&= 171.85 + 746.9 + 300 + 1096.9 + 350 + 821.87 \\
&= 3487.52
\end{aligned}$$

Interpreting Factorial ANOVA

```
Response: attractiveness
                Sum Sq Df   F value    Pr(>F)    
(Intercept)     163333  1 1967.0251 < 2.2e-16 ***
gender             169  1    2.0323    0.1614    
alcohol           3332  2   20.0654 7.649e-07 ***
gender:alcohol    1978  2   11.9113 7.987e-05 ***
Residuals         3488 42                        
```

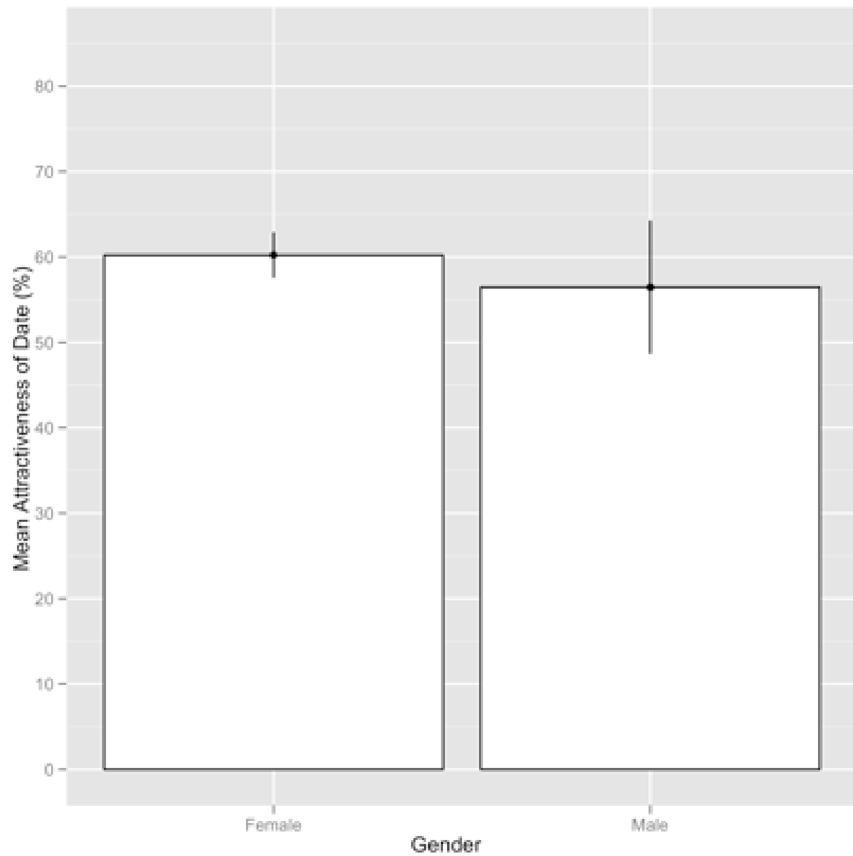
Interpretation: Main Effect Alcohol



There was a significant main effect of the amount of alcohol consumed at the nightclub, on the attractiveness of the mate that was selected,

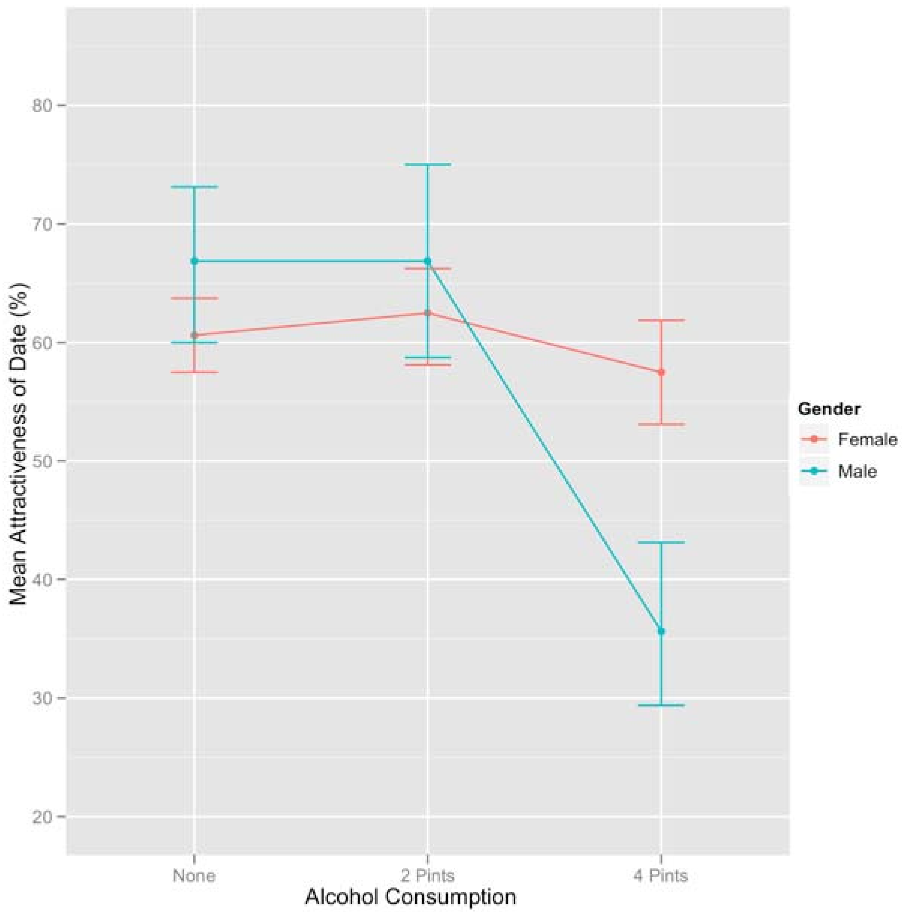
$F(2, 42) = 20.07$, $p < .001$.

Interpretation: Main Effect



There was a non-significant main effect of gender on the attractiveness of selected mates, $F(1, 42) = 2.03$, $p = .161$.

Interpretation: Interaction



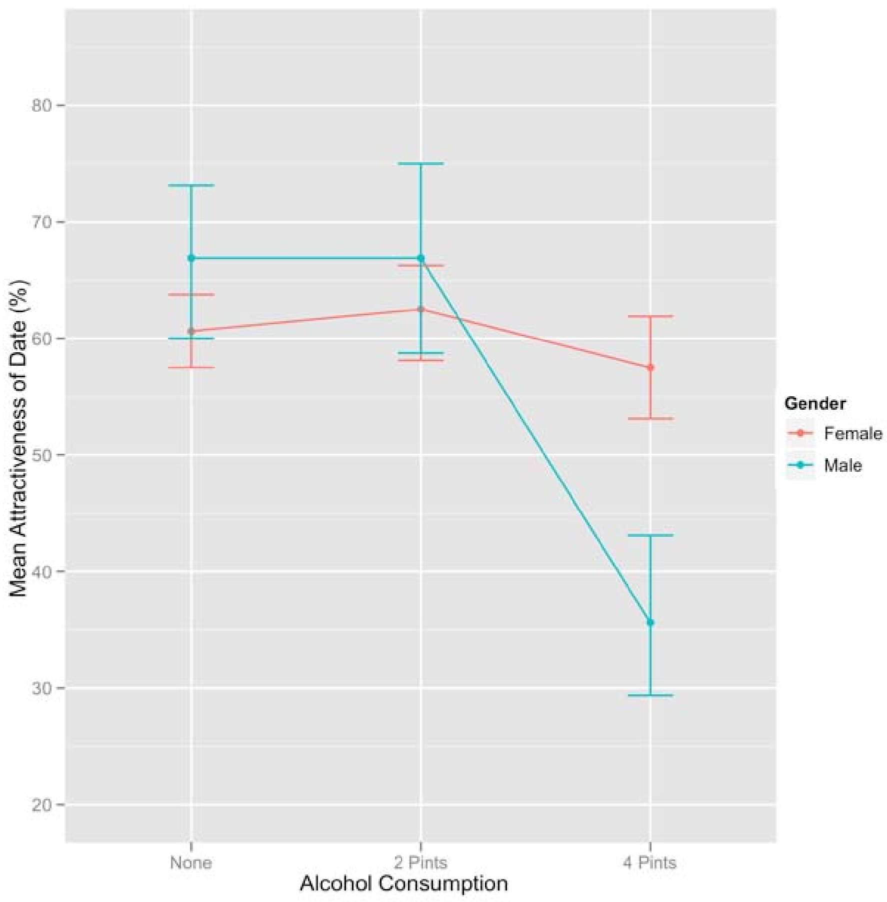
There was a significant interaction between the amount of alcohol consumed and the gender of the person selecting a mate, on the attractiveness of the partner selected, $F(2, 42) = 11.91$, $p < .001$.

Interpretation: Interaction

There was a significant interaction between the amount of alcohol consumed and the gender of the person selecting a mate, on the attractiveness of the partner selected, $F(2, 42) = 11.91$, $p < .001$.

Non-parallel lines indicate such an interaction: For low does of alcohol male and female scores do not change much.

At a high dose (4 pints) male scores plummet but female scores remain fairly high. So, the interaction is caused by a difference between males and females in the effect of alcohol.



Anova Part II
Comparison of means test

- Multi-factorial ANOVA as a linear model
- Hypotheses being tested
- Interaction effects
- Post-hoc tests
- Non-parametric

Factorial ANOVA as regression

Table 12.1 Data for the beer-goggles effect

Alcohol	None		2 Pints		4 Pints	
Gender	Female	Male	Female	Male	Female	Male
	65	50	70	45	55	30
	70	55	65	60	65	30
	60	80	60	85	70	30
	60	65	70	65	55	55
	60	70	65	70	55	35
	55	75	60	70	60	20
	60	75	60	80	50	45
	55	65	50	60	50	40
Total	485	535	500	535	460	285
Mean	60.625	66.875	62.50	66.875	57.50	35.625
Variance	24.55	106.70	42.86	156.70	50.00	117.41

Factorial ANOVA as regression

$$\text{outcome}_i = (\text{model}) + \text{error}_i$$

$$\text{attractiveness}_i = (b_0 + b_1\text{gender}_i + b_2\text{alcohol}_i) + \varepsilon_i$$

Factorial ANOVA as regression

$$\text{outcome}_i = (\text{model}) + \text{error}_i$$

$$\text{attractiveness}_i = (b_0 + b_1\text{gender}_i + b_2\text{alcohol}_i) + \varepsilon_i$$

$$\text{attractiveness}_i = (b_0 + b_1 A_i + b_2 B_i + b_3 AB_i) + \varepsilon_i$$

$$\text{attractiveness}_i = (b_0 + b_1\text{gender}_i + b_2\text{alcohol}_i + b_3\,\text{interaction}_i) + \varepsilon_i$$

- How do we code the interaction term?
- *Multiply the variables*
- *A x B*

Factorial ANOVA as regression

- How do we code the interaction term?
- *Multiply the variables*
- *A x B*

Gender	Alcohol	Dummy (Gender)	Dummy (Alcohol)	Interaction	Mean
Male	None	0	0	0	66.875
Male	4 Pints	0	1	0	35.625
Female	None	1	0	0	60.625
Female	4 Pints	1	1	1	57.500

Factorial ANOVA as regression

Gender	Alcohol	Dummy (Gender)	Dummy (Alcohol)	Interaction	Mean
Male	None	0	0	0	66.875
Male	4 Pints	0	1	0	35.625
Female	None	1	0	0	60.625
Female	4 Pints	1	1	1	57.500

$$\text{attractiveness}_i = (b_0 + b_1\text{gender}_i + b_2\text{alcohol}_i + b_3\ \text{interaction}_i) + \varepsilon_i$$

$$\bar{X}_{\text{Men, None}} = b_0 + (b_1 \times 0) + (b_2 \times 0) + (b_3 \times 0)$$

$$b_0 = \bar{X}_{\text{Men, None}}$$

$$b_0 = 66.875$$

Factorial ANOVA as regression

Gender	Alcohol	Dummy (Gender)	Dummy (Alcohol)	Interaction	Mean
Male	None	0	0	0	66.875
Male	4 Pints	0	1	0	35.625
Female	None	1	0	0	60.625
Female	4 Pints	1	1	1	57.500

$$\text{attractiveness}_i = (b_0 + b_1\text{gender}_i + b_2\text{alcohol}_i + b_3\text{ interaction}_i) + \varepsilon_i$$

$$\bar{X}_{\text{Men, None}} = b_0 + (b_1 \times 0) + (b_2 \times 0) + (b_3 \times 0)$$

$$b_0 = \bar{X}_{\text{Men, None}}$$

$$b_0 = 66.875$$

The cell mean

Factorial ANOVA as regression

Gender	Alcohol	Dummy (Gender)	Dummy (Alcohol)	Interaction	Mean
Male	None	0	0	0	66.875
Male	4 Pints	0	1	0	35.625
Female	None	1	0	0	60.625
Female	4 Pints	1	1	1	57.500

$$\bar{X}_{\text{Women, None}} = b_0 + (b_1 \times 1) + (b_2 \times 0) + (b_3 \times 0)$$

$$\bar{X}_{\text{Women, None}} = b_0 + b_1$$

The cell mean

$$\bar{X}_{\text{Women, None}} = \bar{X}_{\text{Men, None}} + b_1$$

$$b_1 = \bar{X}_{\text{Women, None}} - \bar{X}_{\text{Men, None}}$$

$$b_1 = 60.625 - 66.875$$

$$b_1 = -6.25$$

Factorial ANOVA as regression

Gender	Alcohol	Dummy (Gender)	Dummy (Alcohol)	Interaction	Mean
Male	None	0	0	0	66.875
Male	4 Pints	0	1	0	35.625
Female	None	1	0	0	60.625
Female	4 Pints	1	1	1	57.500

$$\bar{X}_{\text{Men, 4 Pints}} = b_0 + (b_1 \times 0) + (b_2 \times 1) + (b_3 \times 0)$$

$$\bar{X}_{\text{Men, 4 Pints}} = b_0 + b_2$$

The cell mean

$$\bar{X}_{\text{Men, 4 Pints}} = \bar{X}_{\text{Men, None}} + b_2$$

$$b_2 = \bar{X}_{\text{Men, 4 Pints}} - \bar{X}_{\text{Men, None}}$$

Factorial ANOVA as regression

Gender	Alcohol	Dummy (Gender)	Dummy (Alcohol)	Interaction	Mean
Male	None	0	0	0	66.875
Male	4 Pints	0	1	0	35.625
Female	None	1	0	0	60.625
Female	4 Pints	1	1	1	57.500

$$\bar{X}_{\text{Women, 4 Pints}} = b_0 + (b_1 \times 1) + (b_2 \times 1) + (b_3 \times 1)$$

$$\bar{X}_{\text{Women, 4 Pints}} = b_0 + b_1 + b_2 + b_3$$

$$\bar{X}_{\text{Women, 4 Pints}} = \bar{X}_{\text{Men, None}} + (\bar{X}_{\text{Women, None}} - \bar{X}_{\text{Men, None}}) + (\bar{X}_{\text{Men, 4 Pints}} - \bar{X}_{\text{Men, None}}) + b_3$$

The cell mean

$$\bar{X}_{\text{Women, 4 Pints}} = \bar{X}_{\text{Women, None}} + \bar{X}_{\text{Men, 4 Pints}} - \bar{X}_{\text{Men, None}} + b_3$$

$$b_3 = \bar{X}_{\text{Men, None}} - \bar{X}_{\text{Women, None}} + \bar{X}_{\text{Women, 4 Pints}} - \bar{X}_{\text{Men, 4 Pints}}$$

$$b_3 = 66.875 - 60.625 + 57.500 - 35.625$$

$$b_3 = 28.125$$

Two-factor analysis of variance
Hypotheses being tested

- Simultaneous analysis of two factors and measurement of mean response
- Case. 1: equal replication
- Terminology:
- One factor termed *A* and one factor termed *B*
- We use this notation *a* number of levels in *A*
- *b* is the number of levels in *B*

Two-factor analysis of variance
Hypotheses being tested

Researchers have sought to examine the effects of various types of music on agitation levels in patients in early and middle stages of Alzheimer's disease.

Patients were selected based on their form of Alzheimer's disease. Three forms of music were tested: easy listening, Mozart, and piano interludes. The response variable agitation level was scored.

Two-factor analysis of variance
Hypotheses being tested

- What is (are) the null hypothesis(ese) being tested?

Group	Piano Interlude	Mozart	Easy Listening
Early Stage Alzheimer's	21 24 22 18 20	9 12 10 5 9	29 26 30 24 26
Middle Stage Alzheimer's	22 20 25 18 20	14 18 11 9 13	15 18 20 13 19

Two-factor analysis of variance
Hypotheses being tested

- Plot these data (means) on a single figure such that cell-level means can be evaluated.

Group	Piano Interlude	Mozart	Easy Listening
Early Stage Alzheimer's	21 24 22 18 20	9 12 10 5 9	29 26 30 24 26
Middle Stage Alzheimer's	22 20 25 18 20	14 18 11 9 13	15 18 20 13 19

Three-factor analysis of variance
Hypotheses being tested

Evaluate respiratory rate of crabs (ml O_2 hr^{-1})

Factors:

- Sex
- Species
- Temperature

Species 1

Low temp.		*Med. temp.*		*High temp.*	
♂	♀	♂	♀	♂	♀
1.9	1.8	2.3	2.4	2.9	3.0
1.8	1.7	2.1	2.7	2.8	3.1
1.6	1.4	2.0	2.4	3.4	3.0
1.4	1.5	2.6	2.6	3.2	2.7

Species 2

Low temp.		*Med. temp.*		*High temp.*	
♂	♀	♂	♀	♂	♀
2.1	2.3	2.4	2.0	3.6	3.1
2.0	2.0	2.6	2.3	3.1	3.0
1.8	1.9	2.7	2.1	3.4	2.8
2.2	1.7	2.3	2.4	3.2	3.2

Species 3

Low temp.		*Med. temp.*		*High temp.*	
♂	♀	♂	♀	♂	♀
1.1	1.4	2.0	2.4	2.9	3.2
1.2	1.0	2.1	2.6	2.8	2.9
1.0	1.3	1.9	2.3	3.0	2.8
1.4	1.2	2.2	2.2	3.1	2.9

Multiway Factorial ANOVA

Hypotheses being tested

Popcorn	Oil amt.	Batch	Yield
plain	little	large	8.2
gourmet	little	large	8.6
plain	lots	large	10.4
gourmet	lots	large	9.2
plain	little	small	9.9
gourmet	little	small	12.1
plain	lots	small	10.6
gourmet	lots	small	18.0
plain	little	large	8.8
gourmet	little	large	8.2
plain	lots	large	8.8
gourmet	lots	large	9.8
plain	little	small	10.1
gourmet	little	small	15.9
plain	lots	small	7.4
gourmet	lots	small	16.0

Three-factor analysis of variance
Hypotheses being tested

- Three factor ANOVA:
- H_O: Yield is the same in all three **Batch** sizes
- H_O : Yield is the same in all three **Oil** amounts
- H_O : Yield is the same in all three **Popcorn** types
- H_O : The mean yield is the same for all levels of **batch**, independent of oil amount (Batch X Oil)
- H_O : The mean yield is the same for all levels of **Oil** amount, independent of popcorn type (Oil X Type)
- H_O : The mean yield is the same for all levels of batch, independent of popcorn type (Batch X Type)
- H_O : Differences in mean Yield among the batch, oil amount, and popcorn type are independent of the other factors

  (Batch X Type X Oil)

Three-factor analysis of variance
Hypotheses being tested

Response: yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
popcorn.type	1	3.062	3.062	0.1731	0.6883
oil.amount	1	0.062	0.062	0.0035	0.9541
batch.size	1	52.562	52.562	2.9717	0.1230
popcorn.type:oil.amount	1	27.562	27.562	1.5583	0.2472
popcorn.type:batch.size	1	14.062	14.062	0.7951	0.3986
oil.amount:batch.size	1	0.063	0.063	0.0035	0.9541
popcorn.type:oil.amount:batch.size	1	1.563	1.563	0.0883	0.7739
Residuals	8	141.500	17.687		

Interaction effects

- Experiment: we are interested in oxygen consumption of two species of limpets in different concentration of seawater.
  - Factor A is the species of limpet (levels, a)
  - Factor B is the concentration of SW as a function of maximum salinity – 100, 75, and 50 % (levels, *b*)

Interaction effects

Completed anova

Source of variation	*df*	*SS*	*MS*
Species	1	16.6380	16.638 *ns*
Salinities	2	10.3566	5.178 *ns*
Sp × Sal	2	194.8907	97.445**
Error	42	401.5213	9.560
Total	47	623.4066	

Response: respiratory rate of limpets (ml O_2 hr^{-1})

Interaction effects

Completed anova

Source of variation	*df*	*SS*	*MS*
Species	1	16.6380	16.638 *ns*
Salinities	2	10.3566	5.178 *ns*
Sp × Sal	2	194.8907	97.445**
Error	42	401.5213	9.560
Total	47	623.4066	

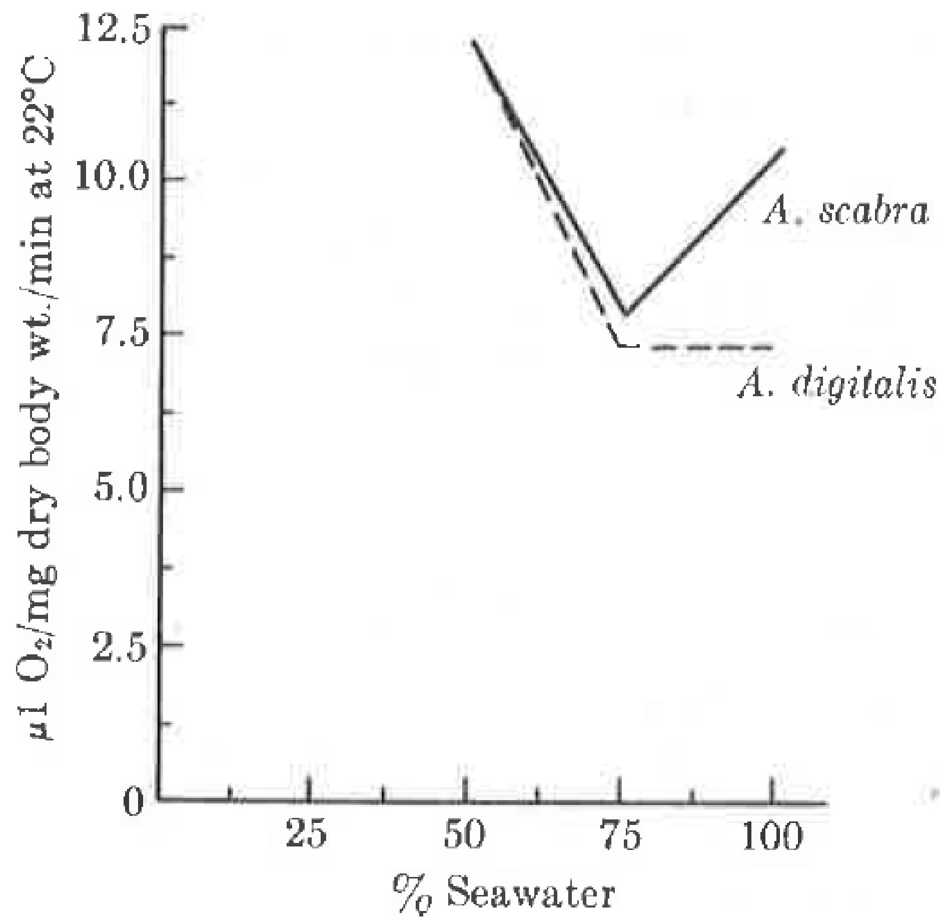
- When the two factors are identified as A and B, the interaction is identified as the A X B interaction.
- Variability not accounted for by A and B alone.
- Interaction: The effect of one factor in the presence of a *particular level* of another factor.
- There is an interaction between two factors if the effect of one factor depends on the levels of the second factor.

Interaction effects

Seawater concentration	Species: *A. scabra*	*A. digitalis*	*Mean*
100%	10.56	7.43	9.00
75%	7.89	12.33	10.11
50%	12.17	7.34	9.76
Mean	10.21	9.03	9.62

Response: respiratory rate of limpets (ml O_2 hr^{-1})

Interaction effects



Interaction effects

- The response to salinity differs between the two species
- At 75% salinity *A. scabra* consumes the least oxygen and *A. digitalis* consumes the most.
- Therefore a simple statement about the species response to salinity is not clear; all we can really say is:

The pattern of response to changes in salinity differed in the two species.

Interaction effects

- The difference among levels of one factor is not constant at all levels of the second factor
- *"it is generally not useful to speak of an individual factor effect – even if its F is significant – if there is a significant interaction effect" Zar*

Post-hoc tests

- Tukey test – balanced, orthogonal designs
  - Step one: is to arrange and number all five sample means in order of increasing magnitude
  - Calculate the pairwise difference in sample means.
- We use a t-test "analog" to calculate a q-statistic

Post-hoc tests

- Scheffe’s test
- Examine multiple contrasts:
  - ideas is to compare combinations of samples to each other instead of the comparison among individual k levels.

Post-hoc tests

- Scheffe's test
- Compare the mean outflow volume of four different rivers: 5 vs 1,2,3,4

$$H_0\colon \mu_2/3 + \mu_4/3 + \mu_3/3 - \mu_5 = 0.$$

$$H_0\colon (\mu_2 + \mu_4 + \mu_3)/3 = \mu_5$$

$$c_2 = \frac{1}{3},\ c_4 = \frac{1}{3},\ c_3 = \frac{1}{3}, \text{ and } c_5 = -1$$

Post-hoc tests

- Scheffe's test
- Compare the mean outflow volume of four different rivers: 1 vs. 2,3,4,5
- Alternatives multiple contrasts:

$$H_0: (\mu_1 + \mu_5)/2 - (\mu_2 + \mu_4 + \mu_3)/3 = 0$$

$$H_0: \mu_1 - (\mu_2 + \mu_4 + \mu_3)/3$$

Post-hoc tests

- Scheffe's test
- Test Statistic:

$$S = \frac{\left|\sum c_i \overline{X}_i\right|}{\text{SE}},$$

where

$$\text{SE} = \sqrt{s^2\left(\sum \frac{c_i^2}{n_i}\right)},$$

and the critical value of the test is

$$S_\alpha = \sqrt{(k - 1)F_{\alpha(1),k-1,N-k}} \ .$$

Non-Parametric tests

- Violations of the assumptions
- We assume equality of variance – ANOVA is a robust test.
- Robust to unbalanced design.
- How to deal with outliers:
  - use in analysis if they are valid data.
- Test of normality: Shapiro Wilks
- Test of equality of variance: Bartletts test.

Non-Parametric tests

- Nonparametric analysis of variance.
- If $k > 2$
- Kruskal-Wallis test – analysis of variance by rank
- Power increases with sample size.
- If $k = 2$ the Kruskal-Wallis is equivalent to the Mann-Whitney test.

Non-Parametric tests

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1),$$

- If there are tied ranks
  - H needs to be corrected using a correction factor C.

$$C = 1 - \frac{\sum t}{N^3 - N},$$

$$H_c = \frac{H}{C}.$$

Non-Parametric tests

If there are tied ranks

H needs to be corrected using a correction factor C.

$$C = 1 - \frac{\sum t}{N^3 - N},$$

$$H_c = \frac{H}{C}.$$

$$\sum t = \sum (t_i^3 - t_i)$$

t_i is the number of tied ranks.

Non-Parametric tests

A limnologist obtained eight containers of water from each of four ponds. The pH of each water sample was measured. The data are arranged in ascending order within each pond. (One of the containers from pond 3 was lost, so $n_3 = 7$, instead of 8; but the test procedure does not require equal numbers of data in each group.) The rank of each datum is shown parenthetically.

H_0: pH is the same in all four ponds.
H_A: pH is not the same in all four ponds.

Non-Parametric tests

Pond 1	*Pond 2*	*Pond 3*	*Pond 4*
7.68 (1)	7.71 (6*)	7.74 (13.5*)	7.71 (6*)
7.69 (2)	7.73 (10*)	7.75 (16)	7.71 (6*)
7.70 (3.5*)	7.74 (13.5*)	7.77 (18)	7.74 (13.5*)
7.70 (3.5*)	7.74 (13.5*)	7.78 (20*)	7.79 (22)
7.72 (8)	7.78 (20*)	7.80 (23.5*)	7.81 (26*)
7.73 (10*)	7.78 (20*)	7.81 (26*)	7.85 (29)
7.73 (10*)	7.80 (23.5*)	7.84 (28)	7.87 (30)
7.76 (17)	7.81 (26*)		7.91 (31)

*Tied ranks.

Non-Parametric tests

Pond 1	*Pond 2*	*Pond 3*	*Pond 4*
7.68 (1)	7.71 (6*)	7.74 (13.5*)	7.71 (6*)
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7.73 (10*)	7.80 (23.5*)	7.84 (28)	7.87 (30)
7.76 (17)	7.81 (26*)		7.91 (31)
*Tied ranks.			
$n_1 = 8$	$n_2 = 8$	$n_3 = 7$	$n_4 = 8$
$R_1 = 55$	$R_2 = 132.5$	$R_3 = 145$	$R_4 = 163.5$

Non-Parametric tests

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1),$$

$$\begin{aligned} H &= \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1) \\ &= \frac{12}{31(32)} \left[\frac{55^2}{8} + \frac{132.5^2}{8} + \frac{145^2}{7} + \frac{163.5^2}{8} \right] - 3(32) \\ &= 11.876 \end{aligned}$$

Non-Parametric tests

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$$\sum t = \sum(t_i^3 - t_i)$$

Number of groups of tied ranks $= m = 7$.

$$\begin{aligned}\sum t &= \sum(t_i^3 - t_i) \\ &= (2^3 - 2) + (3^3 - 3) + (3^3 - 3) + (4^3 - 4) \\ &\quad + (3^3 - 3) + (2^3 - 2) + (3^3 - 3) \\ &= 168\end{aligned}$$

Non-Parametric tests

Pond 1	Pond 2	Pond 3	Pond 4
7.68 (1)	7.71 (6*)	7.74 (13.5*)	7.71 (6*)
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Non-Parametric tests

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$R_1 = 55$	$R_2 = 132.5$	$R_3 = 145$	$R_4 = 163.5$

$$\sum t = \sum(t_i^3 - t_i)$$

Number of groups of tied ranks = $m = 7$.

$$\sum t = \sum(t_i^3 - t_i)$$
$$= (2^3 - 2) + (3^3 - 3) + (3^3 - 3) + (4^3 - 4) + (3^3 - 3) + (2^3 - 2) + (3^3 - 3)$$
$$= 168$$

Non-Parametric tests

$$C = 1 - \frac{\sum t}{N^3 - N} = 1 - \frac{168}{31^3 - 31} = 1 - \frac{168}{29760} = 0.9944$$

$$H_c = \frac{H}{C} = \frac{11.876}{0.9944} = 11.943$$

$$\nu = k - 1 = 3$$

$$F = \frac{(N - k)H_c}{(k - 1)(N - 1 - H_c)} = \frac{(31 - 4)(11.943)}{(4 - 1)(31 - 1 - 11.943)} = 5.95$$

$$F_{0.05(1),3,26} = 2.98$$

Reject H_0.