Ratios and Rates

Grade 6 Mathematics

Summary: Students learn that a ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percents. Students apply these concepts to a variety of real world and mathematical situations.

In the culminating performance task, students plan a recipe, using ratios to find the quantities, unit rates, and costs of ingredients for different numbers of servings.

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Stage 1 Desired Results

ESTABLISHED GOALS

6.RP.1

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2

Understand the concept of a unit rate a/b associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* [1]

6.RP.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Transfer

Students will be able to independently use their learning to…

Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems.

Meaning

UNDERSTANDINGS **U**

Students will understand that…

U1
A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities (including unit rates.)

U2
A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.

(See the chart on p.6)

U3
Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.)

ESSENTIAL QUESTIONS

Q1
When is it useful to be able to relate one quantity to another?

Q2
How are ratios and rates used in everyday life? How would life be different without ratios and rates?

Note:
Essential questions provide an important anchor for the unit. They should be posted in the classroom throughout the unit. Students can be redirected to these questions throughout the unit lessons through questions, discussions, and math journal entries.

Acquisition

Students will know…

K1
A **ratio** compares two related quantities.

K2
Ratios can be represented in a variety of formats including *each, to,*

Students will be able to…

S1
Use ratio and rate reasoning to solve real-world and mathematical problems.

S2

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

a. Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* b. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent. c. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **Standards for Mathematical Practice:** **SMP.1** **Make sense of problems and persevere in solving them.** (Students must understand the problem context in order to translate them into ratios/rates.) **SMP.2** **Reason abstractly and quantitatively.** (Students must understand the relationship between two quantities in order to express them mathematically.)	*per, for each, %, 1/5*, etc. **K3** A **percent** is a type of ratio that compares a quantity to 100. **K4** A **unit rate** is the ratio of two measurements in which the second term is 1. **K5** When it is appropriate to use ratios/rates to solve mathematical or real life problems. **K6** Mathematical strategies for solving problems involving ratios and rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.	Make and interpret tables of equivalent ratios. **S3** Plot pairs of values of the quantities being compared on the coordinate plane. **S4** Use multiple representations such as tape diagrams, double number line diagrams, or equations to solve rate and ratio problems. **S5** Solve unit rate problems (including unit pricing and constant speed). **S6** Solve percent problems, including finding a percent of a quantity as a rate per 100 and finding the whole, given the part and the percent.

SMP.3 **Construct viable arguments and critique the reasoning of others.** This supports **ELA 6.W.1: Write arguments to support claims with clear reasons and relevant evidence.** (This will be reinforced in class discussion and in the written requirements of the CEPA.) **SMP.4** **Model with mathematics.** (Students can model a real-life situation using ratios and rates). **SMP.7** **Look for and make use of structure.** (The structure of a ratio is unique and can be used across a wide variety of problem-solving situations.) *Supporting Standards*: **G10 ELA** **RST.4** Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6 – 8 texts and topics. **WHST.2** Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. a. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. b. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts.		

c. Use precise language and domain-specific vocabulary to inform about or explain the topic. d. Establish and maintain a formal style and objective tone. e. Provide a concluding statement or section that follows from and supports the information or explanation presented. **WHST.4** **Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.**		

Stage 2 - Evidence	
Evaluative Criteria	**Assessment Evidence**
See CEPA rubric.	**Curriculum Embedded Performance Assessment** **Title**: Pizza Champions **Overview** For this assessment, students assume the role of school cafeteria chef to create a pizza recipe and plan ingredients to make pizza for 12, 60, and 240 students. Students compute unit prices for the ingredients, prepare a budget, and calculate the cost to feed varying numbers of people. Using ratio/rate language, each student writes a proposal to persuade the cafeteria manager to use the recipe. **List of Large-Scale Tasks** CEPA Task I: Cost to Make One Pizza CEPA Task II: Cost to Feed 12, 60, and 240 Students CEPA Task III: Proposal to the Cafeteria Manager All materials for this CEPA can be found at the end of the unit.

Other Assessments:

Lesson 2	Ticket-to-Leave: *Are All Fractions Ratios?*
Lesson 3	Quiz: *Equivalent Ratios*
Lesson 6	Summative Assessment: *Ratios*
Lesson 7	Formative Assessment: *Rates*
Lesson 9	Formative Assessment: *Unit Rates: Justin's Pizza*
Lesson 10	Summative Assessment: *Unit Prices: Little Red Riding Hood's Grocery Trip*
Lesson 14	Summative Assessment: *Percent Problems: Tim's Tires, Raffle Ticket Sales, and Library Books*

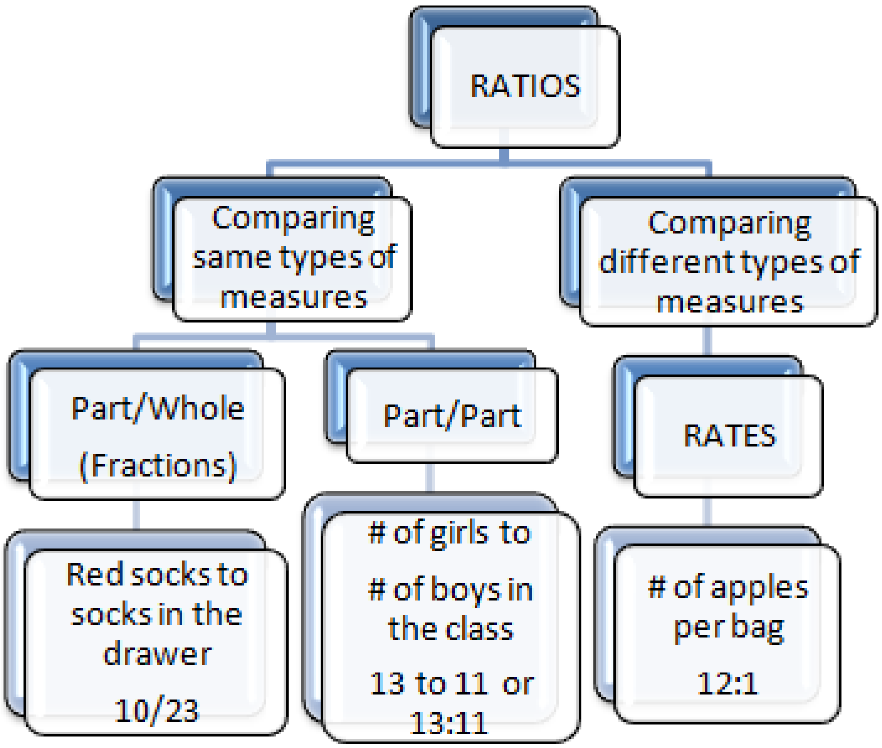
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

Note: In Grade 6, the focus is on ratios, equivalent ratios, and rates. In Grade 7, students will study proportions and proportional reasoning. All sessions are 50 minutes each.

Types of Ratios

(See UNDERSTANDINGS , p.2)



Lesson 1: Introduction to Ratios (50 mins)

Essential Question(s) to be addressed in this lesson:
When is it useful to be able to relate one quantity to another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
6.RP.1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly 3 votes."*

6.SMP.2
Reason abstractly and quantitatively. (Students must understand the relationship between two quantities in order to express them in ratio form.)

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students will be able to multiply fractions.

Students are able to find equivalent fractions without manipulatives.

Academic vocabulary: compare, quantity.

Outcome(s)

By the end of this lesson students will know:

A ratio compares two related quantities.

Ratios can be represented in a variety of formats, including *1 to 5, 1:5, 1/5*.

By the end of this lesson, students will be able to:

Use a ratio to express the relationship between two quantities.

Use ratio language to describe the relationship between two quantities.

Instructional Resources/Tools

Teaching videos (see below) Access to the internet and projector

Anticipated Student Preconceptions/Misconceptions

1. Students may be confused about the order of the quantities. For example, a comparison of 2 wins to 3 losses is written as 2:3, and not 3:2. It is helpful if students begin labeling the quantities of the things they are comparing both in writing and orally. *2.* Students may have difficulty distinguishing a part:part ratio from a part:whole ratio. For example, *"There are 12 girls compared to 11 boys in the class (12:11), but 12 of the 23 students in the class are girls (12:23)."*

Assessment

Pre-assessment/ Formative	Summative (optional)
The first half of the lesson (#1 – 5 below) serves as an informal assessment of student knowledge about ratios.	None.

Lesson Sequence and Description	Content Background for Teachers:
1. Introduce students to the Math Wall for the unit. (see explanation at end of lesson) 2. Probe for student understanding through either a math journal entry or small group discussion opportunities. a. What is a ratio? b. Why do we use ratios? c. What are some examples of real-world ratios? d. What do those ratios mean? 3. Collect student ideas about ratios and post on chart paper or the board without comment or corrections. 4. Explain that a ratio compares two quantities. Ask students to work in pairs to find some ratios in the classroom (without requiring specific ratio language or notation). Give an example or two to get them started. Students write down their ideas, and then share out. Examples: *chairs to desks, students to teachers, books per students, the*	**Ratio:** A ratio expresses the relationship between two quantities. Ratios compare two measures of the same types of things. Examples: *the number of one color of marble to another color of marbles, or the*

number of boys to girls, etc. 5. Ask students what the ratios mean. For example if the ratio of books to students is 1:2, what does this mean? (It might be difficult for two students to share a book.) What are the implications of either a very high teacher:student ratio or a very low teacher:student ratio? 6. Briefly introduce the concept of ratio and the key vocabulary and notation associated with it. 7. View a teaching video on ratios. At the learnalberta.ca website there are a variety of teaching videos. This is a Mathematics-Grade 6 Spy Guys Ratio video. http://www.learnalberta.ca/content/mesg/html/math6web/index.html?page=lessons&lesson=m6lessonsshell03.swf and/or http://www.youtube.com/watch?v=eT1yYqmjHPY or http://www.youtube.com/watch?v=yztq_ELjfSw&feature=related Teaching videos focusing on the definition of ratio and the ways that a ratio can be expressed. 8. Revisit student ideas: a. Are there any ideas that need to be refined based on the activities and the videos? b. Do students want to contribute additional thoughts to the chart about ratios? c. Do they want to delete flawed ideas from the chart?	*number of cats to dogs.* Ratios can compare parts to a whole (part:whole). Example: *12 of the 15 students are playing soccer (12/15).* Ratios can also compare a part of one whole to another part of the same whole (part:part). Example: *The ratio of green marbles in the jar to red marbles in the jar is 4:2.* Ratios can be expressed in following notation: *x:y, x/y, or x to y.* Note: Rates and unit rates are addressed later in this unit. **Rate:** When a ratio compares two different types of measures, it is called a rate. Examples: *5*
Extended Learning/Practice (homework) Students find three examples of ratios in the real world. They can find examples on the internet, in newspapers, or in their own homes. For each, they write down the ratio and discuss its meaning. Example: *The ratio of citizens who voted in the last election compared to those who didn't vote was 1:6. Analysis: Not very many people voted. A few people are making decisions for the whole city.* Example: *Two of my sisters have jobs after school. The ratio of their hourly pay is $7:$10. Analysis: The sister who makes $7 an hour could ask for a raise in her hourly rate, but she is younger and has less experience, so it is probably fair.*	

	gallons of paint are needed to paint 8 walls (5:8). 3 shirts for $20 (3/$20) **Unit Rate:** A unit rate is a rate which compares a quantity to one of the other quantity. Examples: *Miles per hour, cost per foot, eggs per carton.* **Proportion:** (not addressed until Gr. 7) A proportion is an equation written in a form that states that two ratios are equal. *A/B = C/D*

Closure

Review outcomes of this lesson:

- Students use ratios to explain the relationship between two quantities in real-world examples.
- Students use ratio language when talking about ratios (each, to, per, for each, etc.)

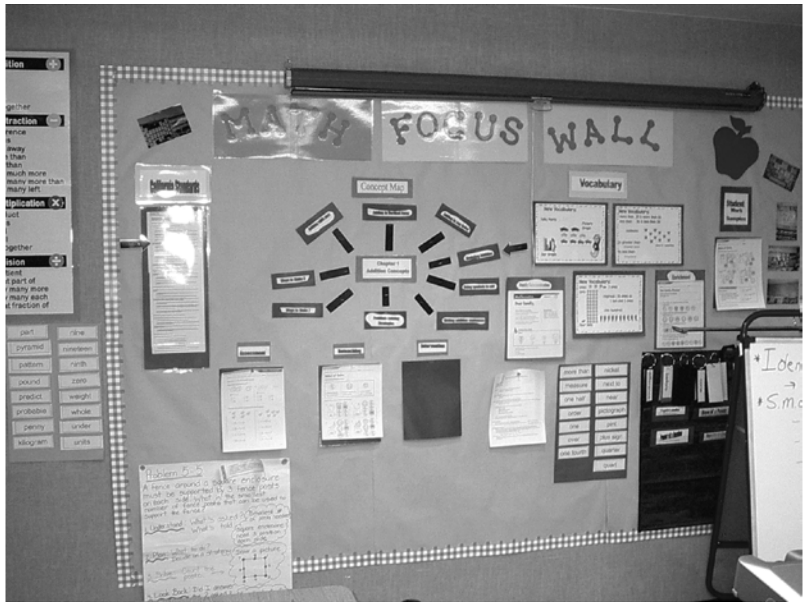
Preview outcomes for the next lesson:

- Students will be able to write ratios to express the relationship between two quantities.
- Students will use appropriate ratio notation and language.

Teacher Reflection (to be completed after lesson)

What went well in this lesson? Did all students accomplish the outcome(s)? What evidence do I have? What would I do differently next time?

<u>Math Walls</u>



At the beginning of this unit, select a wall or bulletin board as the designated space for the Math Wall and display the title, “Ratio and Rates.” The Math Wall will serve as an interactive resource to support students as they develop their mathematical understanding of ratio reasoning.

As the unit unfolds and each lesson is taught, add the following content:

- Vocabulary words, as they are introduced in the day’s lesson.
- Pictures, diagrams, manipulatives …multiple ways to represent the mathematical thinking.
- Strategies, problem solving steps, ways to communicate our thinking, ways to represent a number.
- Sample student work, examples of journal entries.
- Criteria/rubrics

A Math Wall….

→ Assists students in understanding the mathematical language they may come across in written and spoken instructions.
→ Serves as a valuable resource for students to refer to.
→ Demonstrates how mathematics builds on what is already known.
→ Provides a <u>context</u> for the learning taking place in the classroom.
→ Makes the mathematics more **accessible** by allowing students to recall the appropriate vocabulary, work expectations, accurate spelling, representations, and strategies.
→ Provides the support for concepts and processes based upon students’ needs.

<u>Lesson 2: Writing Ratios</u>

(50 mins)

Essential Question(s) to be addressed in this lesson:

When is it useful to be able to relate one quantity to another?

How are ratios used in everyday life?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly 3 votes."*

6.SMP.4 Model with mathematics.   Students model real-life situations with mathematics.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students can multiply fractions.

Students can find equivalent fractions.

Outcome(s)

By the end of this lesson students will know:

A ratio expresses the relationship between two related quantities.

Ratios can be represented in a variety of formats including *1 to 5, 1/5, 1:5*.

By the end of this lesson, students will be able to:

Write a ratio to express the relationship between two quantities.

Use proper ratio notation and language to describe the relationship between two quantities.

Instructional Resources/Tools

Access to the internet and a projector.

Assessment

Pre-assessment/ Formative	Summative (optional)
See #1 below. At the end of the lesson: <u>Ticket to leave</u>: We know that all ratios can be written in fraction form. *Are <u>all</u> fractions ratios? Why or why not?* (Students write a response, explaining their thinking on a card or paper scrap. After putting their names on them, they turn them in on their way out.)	None.

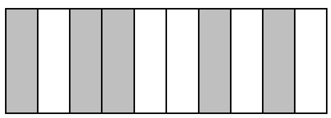
Lesson Sequence and Description	Note to Teachers:

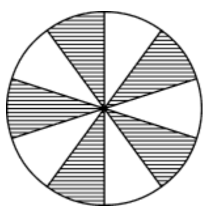
1. Students work in groups of 3 to briefly share their "ratios in the real world homework". Each group shares one good example with the rest of the class. Student 1 reads the example so that the teacher can record it on the board. Student 2 tells which notation to use in the written ratio. Student 3 explains the meaning of the ratio and any inferences that can be made. This activity reviews the previous lesson, and pre-assesses student readiness for writing ratios. 2. Students practice writing ratios based on the following types of problems: • Pictures of objects in scattered arrangements.  Questions should include **part:whole**: Write the ratio that compares the number of ___ to the total number of _____ in the ____. Questions should also include **part:part**: Write the ratio that compares the number of ____ to the number of _____. Questions should request ratios in **different notation** (e.g., write as x:y, write as x/y, express as x to y, etc.)	In this lesson, the focus is on writing ratios that accurately represent mathematical, tabular, or pictorial situations. In the next lesson, students will be asked to express ratios in simplest form. Here are some additional technology resources for all students. The first one may be very useful for students with disabilities or ELLs. 1. http://www.thinkingblocks.com/ThinkingBlocks_Ratios/_TB_Ratio_Main.html Interactive site where students are taught how to use blocks to model ratio problems. Problems may ask students to find one of the two quantities in the ratio, the difference between the two quantities, or the total. Provides a video with step-by-step clear, visual, auditory demonstration of using blocks to solve ratio problems. Teachers can use to guide instruction with block manipulatives, or students can virtually manipulate blocks. 2. http://illuminations.nctm.org/LessonDetail.aspx?_id=L722 Pairs (or groups) of students use a cup of beans to find ratios to express the number of marked beans in the cup compared to the total number of beans in the cup. Theoretically, each sample should be essentially the same. The decimal representation of each ratio confirms that ratios are, indeed, approximately equivalent. 3. http://www.math-aids.com.Ratios/ Practice sheets that use shapes to help students explore ratio relationships.

- Working backward:

  In the diagram, what does the ratio ___:___ represent

- Shapes divided into equal parts with some parts shaded.: Write the ratios for shaded to non-shaded or shaded to total.





2. Ticket to leave (see Formative Assessments above).

We know that all ratios can be written in fraction form.

Are <u>all</u> fractions ratios? Why or why not?

Note: Some students may benefit from using actual manipulatives that they can move around.

Extended Learning/Practice (homework)	
Practice sheet: 1. Students write ratios in the form requested for 6 different situations. 2. Students create a ratio problem for someone else to solve. Include pictures of objects, the question (what ratio the student wants the solver to find), and the form in which the student wants the ratio written.	

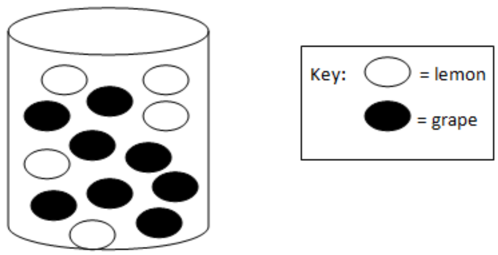
Closure
Review outcomes of this lesson: Students write ratios to express the relationship between two mathematical or real-world quantities. Students use appropriate ratio notation and language. Preview outcomes for the next lesson: Students will learn to make tables of equivalent ratios. Students will identify missing values in a table of equivalent ratios.

Teacher Reflection (to be completed after lesson)

What went well in this lesson? Did all students accomplish the outcome(s)? What evidence do I have? What would I do differently next time?

Lesson 2: In-Class Work

1. The jar below contains grape and lemon jelly beans.



   A. What is the ratio of lemon jelly beans to grape jelly beans? ________________

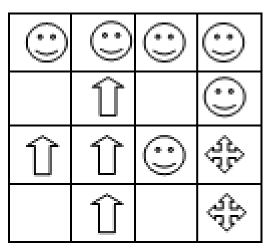
   B. How many grape jelly beans would have to be added so that the ratio was 1 lemon jelly bean for 2 grape jelly beans? ___________________

   C. Show this solution by drawing the added grape jelly beans in the jar. How many jelly beans are in the jar now? __________

   D. With the added jelly beans, write the ratio of lemon jelly beans to the whole jar of jelly beans in fraction form. ______

   E. With the added jelly beans, write the ratio of grape jelly beans to the whole jar of jelly beans in fraction form. ______

2. Here is a grid with different kinds of boxes.



   A. Using the colon form (:), write the ratio of ✥ to ⇧ _______

   B. Using the fraction form, write the ratio of ☺ to the total number of boxes in the grid.

   ____

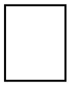
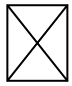
   C. Write the ratio of ⇧ to the total number of boxes in the grid. _______

   D. Write the ratio of empty boxes to the total number of boxes in the grid. _____

   E. Write the ratio of ✥ to ☺ ______

   F. Which two kinds of boxes have a ratio of 2:3? Justify your answer.

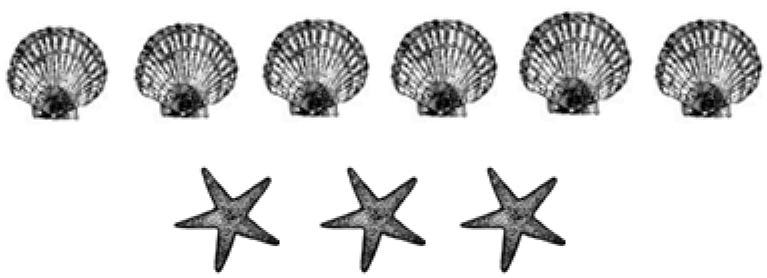
3. Luisa has 12 playing cards. She places 2 out of every 3 cards face-up on the table.

 = face-up  = face-down

A. Draw Luisa's cards, showing how many cards are face-up and how many are face-down.

B. Luisa wants to change the ratio to 2 face-up cards to 3 face-down cards. How many cards does she need to add to the original 12 cards? Draw the solution.

4. Dr. Jin has 6 scallops and 3 sea stars in his aquarium.



A. What is the ratio of sea stars to scallops? ____________

B. Each sea star eats 4 scallops per week. How many scallops does Dr. Jin need to add to his tank to feed the sea stars for a week? ____________

C. One of Dr. Jin's fellow scientists brings him two more sea stars. What will the ratio of sea stars to scallops need to be in order to feed the sea stars for a week? ______________

Lesson 2: Homework

1. X X X X X O O O

X X X X X O O O

X X X X X O O O

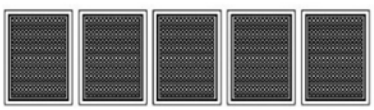
What is the ratio of X to O in this set? ____________________

In one row, what is the ratio of X to O? ____________________

What is the ratio of O to the whole set? ____________________

In one row, what is the ratio of O to X? ____________________

2. Here is a set of cards:



Write each ratio as a fraction:

A. The ratio of face-up cards to the whole set.

B. The ratio of face-down cards to the whole set.

C. The ratio of face-down cards to face-up cards.

D. Add a drawing of some face-down cards so that the ratio of face-down to face-up cards is 3:4. Justify your answer.

3. Samantha drew a 1:4 ratio. Stephan drew a ratio picture that had more than nine objects, but it was still a 1:4 ratio. Draw a picture to show what Stephan could have drawn.

4. Create a ratio problem for someone else to solve. Include pictures of objects.

Your problem should clearly tell the problem-solver what ratio to find, and in what form the answer should be written (to, fraction, colon).

Lesson 3: Equivalent Ratios

(50 mins)

Essential Question(s) to be addressed in this lesson:

Are all fractions ratios? Why or why not?

How can I use what I know about fractions to solve ratio problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, and find missing values in the table.

Use tables to compare ratios.

6.MP.7 Look for and make use of structure.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students can create and complete tables.

Students can multiply multi-digit numbers.

Students understand that equivalent fractions are different names for the same quantity.

Students can use different strategies to find equivalent fractions.

Students understand that a fraction a/b is a multiple of 1/b.

Outcome(s)

By the end of this lesson students will know:

Mathematical strategies for solving problems involving ratios, including tables, and equivalent fractions.

By the end of this lesson, students will be able to: Make tables of equivalent ratios. Find missing values in tables of equivalent ratios.

Instructional Resources/Tools

Access to the internet and a projector. Equivalent ratios quiz printed out from http://www.math-aids.com/Ratios (See description below in Formative Assessment)

Anticipated Student Preconceptions/Misconceptions

Understanding the multiplicative relationship of ratios can be difficult for many students. Teachers should spend significant time with hands-on activities and guided practice, while modeling multiple strategies to help students see this relationship. (See Teacher Content Knowledge section.) In the context of a ratio problem, a ratio is not necessarily a fraction. For example, ¾ relates a quantity of 3 to a quantity of 4. If this is a part-to-whole comparison, then the ratio is a fraction. If it is a part-to-part comparison, it is not a fraction.

Assessment

Pre-assessment/ Formative	Summative (optional)
Formative assessment: Quizzes with equivalent ratio problems in this format: 2 : 3 = 4 : __ = 6 : __ = 8 : __ = 10: __ = 12: __ can be downloaded and printed out from: http://www.math-aids.com/Ratios	None.

Lesson Sequence and Description	Content Background for Teachers:
1. Class discussion based on the Ticket to Leave from the last lesson: *Ratios can be written as fractions. Are all fractions ratios?* (Can students think of counterexamples? Students present arguments based on mathematical thinking as they debate this.) Note: All fractions are indeed ratios which express multiplicative relationships. *For example, 1/2 = 2/4 = 3/6, etc. In this example, you can multiply any numerator x 2 to get the denominator.* 2. Relate the idea of equivalent ratios to equivalent fractions. If 540/630 students like pepperoni on their pizza, what does this mean? To make the relationship easier to understand, find an equivalent ratio by expressing 540/630 in simplest terms: 6/7 students like pepperoni pizza. 3. Game: Students find ratios between two quantities of objects. http://math.rice.edu/~lanius/proportions/rate5.html For example, students choose which picture shows 2 dolls for every 1 camera (4:2) 4. Show a table of ratio values, with a value missing. Here is an example:	A ratio expresses the relationship between two quantities. It is important for middle school students to understand that the relationship is a **multiplicative relationship**. A **multiplicative relationship** is defined as a relationship in which one number is multiplied to get the second number. In addition, the same multiplicative relationship can be applied to other situations. It yields a relative rather than an absolute number. *For example, if the ratio of boys to girls in a school is 2:3, the school could have 20 boys and 30 girls, or 200 boys and 300 girls, or 400 boys and 600 girls. Given any number of boys, you can find the number of girls by multiplying the boys by 3/2. Given any number of girls, you can find the number of boys by multiplying by 2/3. In essence, these ratios can all be*

Cost of Potatoes	
Pounds of Potatoes	Cost
2	\$3.00
4	\$6.00
6	?
8	?

To find the missing value, it is important to establish the ratio first. In this case, the ratio is 2:3. Students discuss how to find the missing value.

For students who need more scaffolding, present a table with a simpler ratio, and no "skipped" quantities in the left-hand column, such as the following:

Cost of Grapes	
Pounds of Grapes	Cost
1	$2.00
2	$4.00
3	$6.00
4	?
5	$10.00

For students who need additional challenge, present a table with no labels and missing values. Ask students to create a context for the table presented, add labels, and find the missing values.

Title: ______________________	
2	30
5	?
?	120
10	150

5. Students work with a partner to create a similar table problem.

6. Quiz with equivalent ratio problems in this format:
   2 : 3 = 4 : __ = 6 : __ = 8 : __ = 10: __ = 12: __
   can be downloaded and printed out from:

expressed as equivalent fractions.

Students learn fractions by viewing shapes that are divided into equal parts with some of the parts shaded. They have an **additive understanding** of fractions.

For example, in a circle divided into fourths with three parts shaded, students think:

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

They see the 3 parts shaded and they see the 4 parts total in the circle, and they think "3 out of 4 parts are shaded". However, this makes it difficult for students to think about quantities such as 5/3. It also makes it difficult for them to apply knowledge of fractions to proportional reasoning.

In middle school, students should view ¾ as a multiplicative relationship: 3 x ¼ . They need to understand a/b as a x 1/b parts.
This multiplicative understanding of fractions begins in grade 4 (4.NF.4a).

If students understand that ratios have a multiplicative relationship, they will be

http://www.math-aids.com/Ratios	able to use tables, equivalent fractions, and tape diagrams to solve ratio problems. When finding missing values in a table, students should be encouraged to establish the ratio first, rather than looking for iterative patterns in the table.
Extended Learning/Practice (homework) Students fill in missing values in a ratio table and make a table of equivalent ratios.	To check student understanding of a multiplicative relationship, give students this problem: Julie swims 1 lap for every 3 laps that Joe swims. Eliza made a table showing this relationship: Julie <table><tr><td>1</td><td>4</td><td>7</td><td>10</td></tr><tr><td>3</td><td>6</td><td>9</td><td>12</td></tr></table>Joe Is Eliza correct? (She is not – she has used a reiterative addition pattern of +3 rather than a multiplicative relationship to solve the problem.) The correct table is: Julie <table><tr><td>1</td><td>4</td><td>7</td><td>10</td></tr><tr><td>3</td><td>12</td><td>21</td><td>30</td></tr></table>Joe Important note: Teachers should avoid teaching the “cross multiply and divide” strategy of solving

	proportions in Gr. 6.

Closure
Review outcomes of this lesson: Students make tables of equivalent ratios. Students fill in missing values in a ratio table. Preview outcomes for the next lesson: Students will learn to plot the values of quantities being compared on the coordinate plane.

Teacher Reflection (to be completed after lesson)

What went well in this lesson? Did all students accomplish the outcome(s)? What evidence do I have? What would I do differently next time?

Lesson 3: Homework

1. Michaela charges $6.50 per hour to babysit. Complete the table below to show how much she would earn.

Babysitting Earnings	
Hours Worked	**Amount Earned**
1	
2	
3	
4	
5	
6	

2. At birth, a baby blue whale weighs 5,000 pounds. The baby whale continues to gain 200 pounds per day. Make a table showing how much the baby whale gains in one week. Remember to include a title and label each column.

3. What will the baby whale weigh at the end of 10 days? How did you find your answer?

4. Explain how to determine how much the baby whale will weigh on any day. How could you find the weight on the 20^{th} day and the 50^{th} day?

5. The drama club mixes paint needed for scenery. To make green, the ratio is 2 pints of blue per 3 pints of yellow. Fill in the missing numbers below. Explain your thinking.

Mixing Green Paint							
BLUE	1	2			20		50
YELLOW		3	9	15		60	

6. It takes Hans 6 hours to read 480 pages. Today, he is going to read for 5 hours. How many pages will he read? Fill out the table below to figure out how many pages Hans will read.

Pages Read				
HOURS	1	2		6
PAGES			240	480

7. For every 3 laps that Julie ran, Joe ran 9 laps.

   A. When Julie finished 1 lap, how many laps had Joe run? ____________

   B. When Julie finished 9 laps, how many laps had Joe run? ____________

   C. When the total number of laps that Julie and John have completed is 16, how many of those laps were John's, and how many were Julie's? ________________________________

   D. Make a table below to show how you got your answers. Remember to include a title and labels for each column.

Lesson 4: Equivalent Ratios (50 mins)

Essential Question(s) to be addressed in this lesson:

How can the relationship between two quantities be represented visually?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.MP.7 Look for and make use of structure.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students should understand that equivalent fractions are different names for the same quantity.

They should be able to use different strategies to find equivalent fractions.

They should be able to plot a point (x,y) on a coordinate plane.

Outcome(s)

By the end of this lesson students will know:

Mathematical strategies for solving problems involving ratios, including tables, equivalent fractions, and graphs.

By the end of this lesson, students will be able to:

Plot pairs of values of the quantities being compared on the coordinate plane

Instructional Resources/Tools

Access to the internet and a projector Graph paper

Anticipated Student Preconceptions/Misconceptions

Translating a ratio into an ordered pair to be plotted on the coordinate plane may be difficult for many students. Some students lack basic graphing skills, such as distinguishing the x axis from the y axis, or plotting points with the coordinates in the correct order. In addition, they may view graphing as entirely unrelated to ratios. Teachers need to make the connection that graphing provides another way of looking at the relationship between ratio values.

Assessment

Pre-assessment/ Formative	Summative (optional)
Prerequisite Skill Check: Check students' graphing skills by doing a quick graphing activity. The teacher gives a series of coordinates in quadrant one: (4,1), (2,3), (4,5) and (6,3). If students plot the points correctly, it will form a square.	None.

Lesson Sequence and Description	Content Background for Teachers:
1. A. Students find at least 3 equivalent ratios for 8/12, and make a simple ratio table. On an overhead projector, graph students' numbers, using the first quantity (the numerator) as the x coordinate, and the second quantity (the denominator) as the y coordinate. In the above example, students might volunteer these coordinates: (16, 24), (2, 3), and (4, 6). What do students notice about the resulting line? (The graph is a straight line, indicating a proportional relationship.) B. Students read the graph to find two additional points on the line. They identify the coordinates of the two points. Can students justify that these points represent equivalent ratios? Extension for advanced students: Can students make a generalization about all of the points on the line?	When equivalent ratios are graphed, the data points always form a straight line. The equation of the line is in the form $y = mx + b$, where m is the slope or the multiplicative factor. It can also be thought of as the rate of change. This will be taught in grade 8 (8.F.4). (b is the y-intercept, indicating where the line crosses the y axis when x = 0)

C.
Use the graph from Part B to evaluate whether their answers to Problem 5 from the previous night's homework are correct.

2. Revisit the completed data tables from the previous lesson. Be sure that data tables have the correct values, as noted in bold below.

Cost of Potatoes	
Pounds of Potatoes	Cost
2	$3.00
4	$6.00
6	**$9.00**
8	**$12.00**

Cost of Grapes	
Pounds of Grapes	Cost
1	$2.00
2	$4.00
3	$6.00
4	**$8.00**
5	$10.00

Title: ____________	
2	30
5	**75**
8	120
10	150

7. Students graph the values in these data tables (or any other examples). What do they notice? (Again, the resulting graph is a straight line.)

8. The teacher shows how to use the graph to find a

In advance, make graphs that can be shown on an overhead projector, document camera, etc. This will allow students to quickly check the accuracy of their work. There are many technology tools that can provide quick graphs. If students do not have correct values in their graphs, the lesson will be ineffective.

Remind students that information can be gained from comparing the different values in the tables. For example, here are three ways to look at a table comparing paint ratios.

missing value. For example, in a graph based on the first data table, students could find an interim data point to estimate how much 5 lbs. of potatoes cost. Or, they could extend the straight line to find out how much 20 lbs. of potatoes cost.

9. Students complete the In-Class Worksheet (2 problems).

Extended Learning/Practice (homework)

See homework problems.

Three ways to compare paint mixtures:

Abby's

red	yellow
1	3
2	6
3	9
4	12
5	15

Zack's

red	yellow
3	5
6	10
9	15
12	20
15	25

Same amount of red

Abby's

red	yellow
1	3
2	6
3	9
4	12
5	15

Zack's

red	yellow
3	5
6	10
9	15
12	20
15	25

Same amount of yellow

Abby's

red	yellow	total
1	3	4
2	6	8
3	9	12

Zack's

red	yellow	total
3	5	8
6	10	16
9	15	24

Same total

Closure

Review outcomes of this lesson:

Students plot values of equivalent ratios on the coordinate plane.

They use graphs to solve ratio problems.

Given a graph of a proportional relationship, students identify the ratio between two quantities.

Preview outcomes for the next lesson:

Students will learn other strategies for solving mathematical and real-life problems with ratios.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

Lesson 4: In-Class Work

1. James can read 12 pages in 4 minutes.

   A. Complete the following table:

Time to Read	
Pages Read	**Time in Minutes**
12	4
24	
36	
48	
60	

   B. Use graph paper to plot the values from the data table. Draw a line that best fits the data points, extending it beyond the first and last data points.

   C. Use the graph to find the number of pages that can be read in 1 minute. Circle this point on the graph.

   D. Use the graph to find the number of pages that can be read in 20 minutes. Put a △ around this point.

   E. Does the graph show a ratio? How do you know? Support your answer with evidence from the table and the graph.

2. Mrs. Baker poured a cup of hot chocolate and set it on the table to cool. She recorded the temperature of the hot chocolate every 10 minutes. Here is her data:

Hot Chocolate Temperature	
Elapsed Time in Minutes	**Temperature**
5	180
10	165
15	140
20	130
25	125
30	120
35	115
40	112

A. Use graph paper to plot the values from the chart.

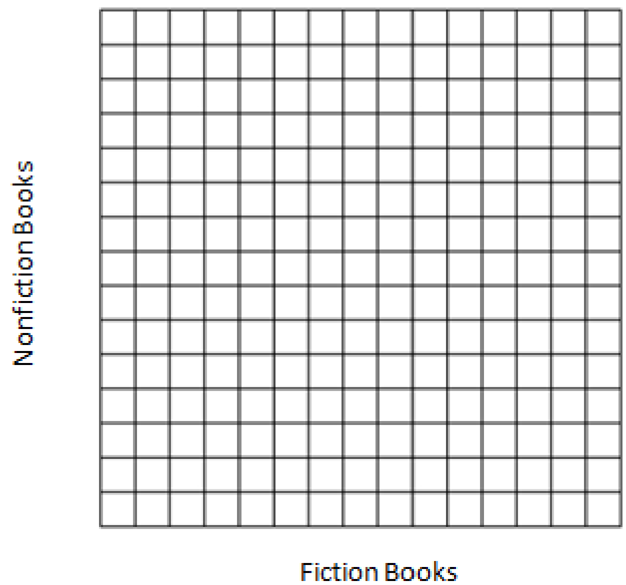
B. Does the graph show a ratio? How do you know? Support your answer from the table and the graph.

Lesson 4: Homework

1. Complete the table and graph for the following situation:

   The library has 5 fiction books for every 3 non-fiction books.

Books in the Library	
Fiction	**Nonfiction**
5	3



2. In April 2012, the U.S. dollar could buy 1.30 Euros. Make a table showing what \$1, \$5, \$10, \$15, \$20, \$50, and \$100 would purchase in Euros. Graph this relationship. (Note: A “Euro” is a unit of money used in many European countries.)

3. Complete the following data table. Then make a graph that shows this relationship. How far can a car travel on 1 gallon of gasoline? Circle this point on your graph.

Gas Mileage	
Miles	Gallons of Gas
	1
50	2
75	
100	6

4. The recipe for Pumpkin Dog Biscuits is 2 eggs, ½ cup canned pumpkin, ½ teaspoon salt, 2 tablespoons dry milk, and 2 ½ cups wheat flour. This recipe makes about 20 treats. Rewrite the recipe for 10 biscuits. 50 biscuits. 100 biscuits. Show your work. What method did you use to rewrite the recipe? (Equivalent fractions, tape diagram, double line diagram?) Why did you choose that method?

Lesson 5: Solving Mathematical and Real-Life Problems with Ratios

(2 lessons, 50 mins each)

Essential Question(s) to be addressed in this lesson:

How are ratios used in everyday life?

What strategies can we use to solve ratio problems?

What is the best way to communicate solutions of ratio problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

6.SMP.4 Model with mathematics. (Students model a real-life situation using ratios.)

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students should know basic unit measurement conversions for length (e.g., 12 inches in a foot; 3 feet in a yard; 100 cm in a meter), mass (e.g. 1000 grams in a kilogram), and volume (e.g., 1000 ml in a liter). For students who are not secure in this knowledge, it is helpful to provide a chart of conversions.

Outcome(s)

By the end of this lesson students will know:

When it is appropriate to use ratios to solve mathematical or real-world problems

Mathematical strategies for solving problems involving ratios, including tape diagrams and double line diagrams.

By the end of this lesson students will be able to:

Use ratio reasoning to solve real-world and mathematical problems.

Use multiple representations such as tape diagrams and double line diagrams to solve ratio problems. Convert measurement units using ratio reasoning.

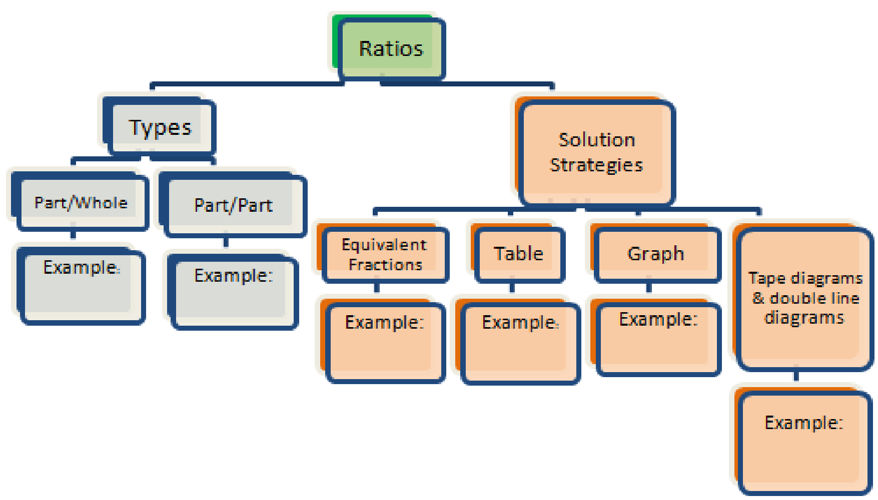
Instructional Resources/Tools
Access to the internet and a projector

Anticipated Student Preconceptions/Misconceptions

Many students have not had enough experience with different types of measurement of length, mass, and volume. The U.S. system is difficult to understand and must be memorized. The metric system makes more sense, but it is easy to confuse with the U.S. system, which is more familiar to most students in terms of daily life experiences.

Assessment

Formative	Summative (optional)
Formative Assessment: See # 6: Given a template with the first three levels provided, students create a top-down web about ratios by providing examples. Students should be encouraged to go back and look at prior work. They work individually at first. Then they work with a partner to complete their webs.	



Lesson Sequence and Description

1. Ratios can be used to convert measurements. With students, the teacher makes a list on the board of common measurement ratios. Examples: 12 inches/foot, 3 feet/yard, 100 cm to a meter, 1000 grams/kilogram, etc.

2. The teacher demonstrates the use of **Double Line Diagrams** for converting measurements.

3. Students are given tables that list various measurements with missing values. Remind students about multiplicative relationships. Ask them to demonstrate that their table entries are correct by creating a double line diagram.

   Example:

Inches	Feet
48	
	12
144	

Grams	Kilograms
100	
	6
1300	

Content Background for Teachers:

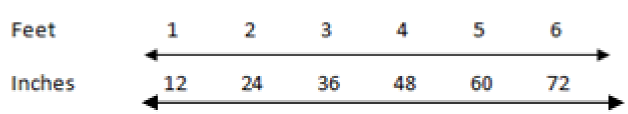
There is lots of new terminology in this lesson. Make sure that students understand and can apply the vocabulary.

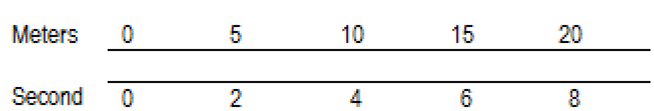
It is important to recognize ratios in many different contexts. To beginners, each proportional reasoning situation presents a different kind of problem, even though they may understand the concept of ratios. More experienced students recognize that although ratios occur in many different settings, the same mathematical idea (multiplicative relationship) underlies all. These students have a toolbox of strategies for solving ratio problems.

Double line diagrams:

A double line diagram compares two quantities that have a multiplicative ratio.

Example:





Tape diagrams:

A tape diagram is a drawing that looks like a

4. The teacher demonstrates the use of **Tape Diagrams** to solve ratio problems. Through guided practice, students apply this strategy to several problems.

5. Students solve a variety of ratio problems, using tables, graphs, equivalent fractions, tape diagrams, and double line diagrams as directed.

6. Given a tape diagram, students create a story problem that matches the diagram, solve it themselves, and then try it out on a friend. (Teachers may need to provide feedback about the wording and the structure of the student-authored problem to ensure clarity before the problems are distributed to others.)

7. Students fill out the top-down web about ratios (see Formative Assessments). The teacher circulates to correct misconceptions.

8. Students are given a sample of student work in which the student has made an error in solving a ratio problem. Ask students to explain the error and correct it.

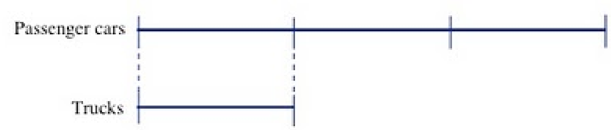
9. Extension: Students view examples of ratios in nature, art, etc. (See websites.)

segment of tape, used to illustrate number relationships. They are also known as strip diagrams, bar models or graphs, fraction strips or length models.

Example:

A sixth grade class counted the number of vehicles that went by the front entrance of the school between 9 o'clock and 10 o'clock. The total number of car counted was 156. There were 3 times as many passenger cars as trucks. How many passenger cars and how many trucks were counted?

Using this tape diagram, students can see that the ratio is 1:3. 156 needs to be divided into 4 equal parts. There were 39 trucks and 117 cars.



A fruit punch recipe has a 3 to 2 ratio of apple juice to grape juice.

Apple Juice			
Grape Juice			

This diagram represents any mixture of apple juice

Extended Learning/Practice (homework) <u>Journal entry</u>: When you are solving a ratio problem, which strategy makes the most sense to you? ☐ Equivalent fractions ☐ Graph ☐ Table ☐ Double Line Diagram ☐ Tape Diagram ☐ Other Explain why this strategy is the best one for you.	and grape juice with a ratio of 3 to 2. The total amount of juice is partitioned into 5 parts of equal size, represented by the 5 shaded boxes. For example, if the diagram represents 5 cups of juice, then each box represents 1 cup. If the diagram represents 15 cups of juice, then each box represents 3 cups. If the total amount of juice is 1 gallon, than each part represents 1/5 gallon and there are 3/5 gallon of apple juice and 2/5 gallon of grape juice. **The Golden Ratio** Extensions: Students may be interested in ratios such as the golden ratio (found in art and nature), common ratios in architecture, and ratios in cooking and nutrition. http://www.geom.uiuc.edu/~demo5337/s97b/art.htm Information on the GOLDEN RATIO. Includes pictures of the Parthenon, Pyramids, and other art works. Gives directions to draw plane figures based on the Golden Ratio. http://www.youtube.com/watch?v=fmaVqkR0ZXg&feature=youtube_gdata_player Overview of the Golden Ratio. Includes examples from math, art, architecture, and real-life objects. ~3 minutes http://www.nytimes.com/2009/04/29/dining/reviews/29book.html An article about the book “Ratio: The Simple Codes Behind the Craft of Everyday Cooking” explains how ratios can be used to simplify cooking. Example: 5:3 ratio of flour to water in bread making.

Closure

Review outcomes of this lesson:

Students recognize mathematical and real-world problems that can be solved using ratio reasoning.

Students use multiple representations such as tables of equivalent fractions, graphs, tape diagrams, and double line diagrams to solve ratio problems.

Students convert measurements using ratio reasoning.

Preview outcomes for the next lesson:

Students will apply knowledge of ratios to propose, justify, and communicate a solution (both orally and in writing).

Students will prepare for a summative assessment on ratios.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

Lesson 5: In-Class Practice

Use equivalent fractions, tape diagrams, or double line diagrams to solve the following problems.

1. Fill in the following table:

Linear Measurement Conversion	
Inches	**Feet**
48	
	5
132	
	12

2. Use the following double line diagram to represent the information in the table above.

Inches: ____

Feet: ____

3. The Green Team Field Trip

   A. Mrs. Timmon's Green Team Club has 39 students. They are taking a field trip to a park to study pond life. If the students travel by car, each car will carry 4 students. Make a table that shows the ratio of cars to students. How many cars will she need?

   B. Mrs. Timmons can also arrange to rent minivans. If the students travel by minivan, each minivan will carry 6 students. Make a table that shows the ratio of minivans to students. How many minivans will she need?

   C. Mrs. Timmons decides to rent a combination of cars and minivans. What is the most efficient combination of cars and minivans? Justify your answer.

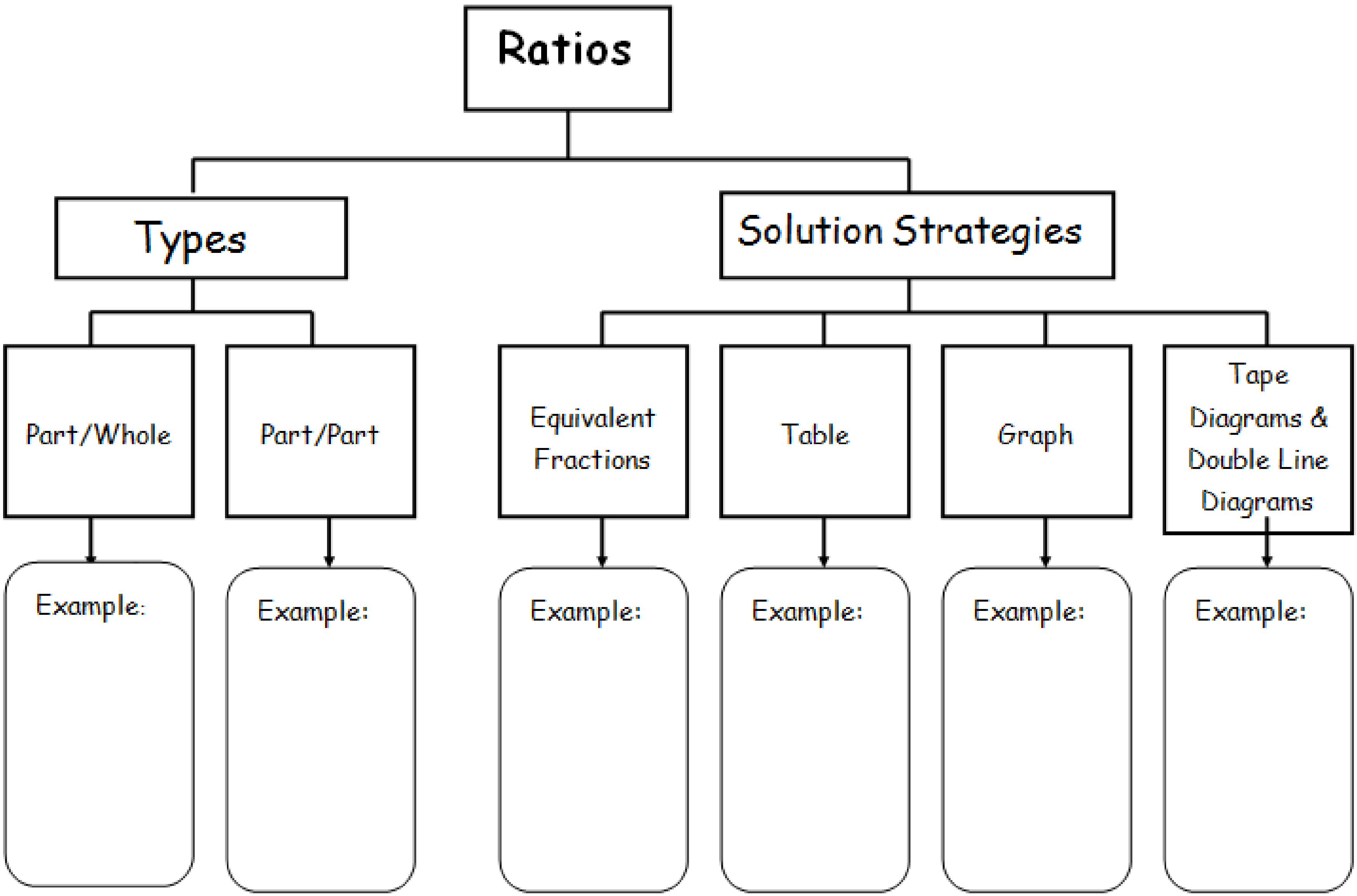
4. Mixing Paints

   A. To mix purple paint, an artist uses 2 quarts of red paint to 3 quarts of blue paint. Make a tape diagram to show how the artist mixes purple paint. Include labels.

   B. The artist mixes up 15 quarts of purple paint. How many quarts are red? How many parts are blue? Show your answer with a tape diagram.

   C. Check your answer with another method, and explain how this method justifies your original answer.

5. A runner and a bike rider leave Calder School at the same time. For every mile that the runner completes, the bike rider will complete 3 miles. If the runner completes 3 miles, how far will the bike rider travel? Show how you got your answer using two different methods.


Ratios
Types
Solution Strategies
Part/Whole
Part/Part
Equivalent Fractions
Table
Graph
Tape Diagrams & Double Line Diagrams
Example:
Example:
Example:
Example:
Example:
Example:

Lesson 6: Ratios – Assessment

(2 lessons, 50 mins each)

Essential Question(s) to be addressed in this lesson:

When is it useful to be able to relate one quantity to another?

What strategies do I know for solving ratio problems?

How do I communicate my solutions to others?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.3 Use ratio reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

SMP.1 Make sense of problems and persevere in solving them. (Students must understand the problem context in order to translate into a ratio).

SMP.2 Reason abstractly and quantitatively. (Students must understand the relationship between two quantities in order to express them mathematically.)

SMP.3 Students construct viable arguments and critique the reasoning of others. (Students justify their solutions and look critically at others' reasoning.)

SMP.4 Model with mathematics (Students model a real-life situation using mathematics.)

SMP.7 Look for and make use of structure. (The structure of a ratio is unique and can be used across a wide variety of problem-solving situations.

ELA

6.RST.4

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students understand ratios and ratio vocabulary, and can use multiple strategies to solve mathematical and real-life ratio problems.

Outcome(s)

By the end of this lesson students will know:

When it is appropriate to use ratios/rates to solve mathematic or real-world problems.

By the end of this lesson students will be able to:

Demonstrate their knowledge of mathematical strategies for solving problems involving ratios, including tables, tape diagrams, double line diagrams, equivalent fractions, graphs, etc.

Propose, justify, and communicate a solution (both orally and in writing).

Assessment

Pre-assessment/ Formative	Summative (optional)
The first day of this two-day lesson is a review. Students are given a review sheet with the following types of problems. These are simple, uncomplicated examples designed to remind students of different concepts and strategies. Put one problem type in each of nine boxes. Review Problem Types 1. Write ratios based on a shaded fraction picture, including part:part and part:whole ratios. 2. Write ratios based on a graphic of scattered objects, including part:part and part:whole ratios. 3. Translate between different forms of ratio notation. 4. Find equivalent fractions.	On the second day of this two-day lesson, students take a summative assessment on ratio reasoning and problem-solving. The assessment consists of the nine types of problems listed at the left. Scaffolding for at-risk students: 1. Vocabulary list with definitions. 2. Table of measurement equivalencies. 3. Allow use of notes and examples from the previous day.

5. Fill in missing values on a data table of equivalent ratios. 6. Graph the proportional relationship of two quantities. 7. Use a graph to find missing values. 8. Use a tape diagram to solve a ratio problem. 9. Use a double line diagram to convert measurements. Students work individually to solve each problem.	

Lesson Sequence and Description	
Day 1 of the two-day lesson: 1. Day 1 is devoted to review. Assign in-class ratio review. After working on the problem sheet individually, students compare their answers with a partner, and take turns explaining the solution to each other. Circulate among students to address misconceptions and to keep partnerships functioning smoothly. 2. Ask, "Which problems were the most difficult?" 3. Reveal the answers one-by-one on the overhead projector, reviewing any that were considered difficult, giving extra practice examples if needed, and noting common misconceptions observed. 4. Students take notes on their problem sheets, which will become their study guides. 5. Ask students to study for the next day's test, including: • Reviewing the nine boxes of examples and notes done in class. • Making up extra practice problems for themselves or classmates, and solving them. Day 2 of the two-day lesson: Students take the assessment.	

Extended Learning/Practice (homework) Day 1: Review for the test, using notes and writing practice problems to solve.	
Closure Review outcomes of this lesson:	
The student has demonstrated knowledge of mathematical strategies for solving problems involving ratios, including tables, tape diagrams, double line diagrams, equivalent fractions, graphs, etc. The student has proposed, justified, and communicated a solution(both orally and in writing).	
Preview outcomes for the next lesson: Students will learn to solve problems about rates and unit rates.	

Teacher Reflection (to be completed after lesson)

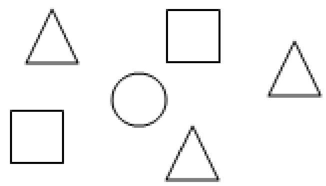
What went well in this lesson? Did all students accomplish the outcome(s)? What evidence do I have? What would I do differently next time?

Lesson 6: In-Class Work: Ratio Review

1. **part: part part:whole**

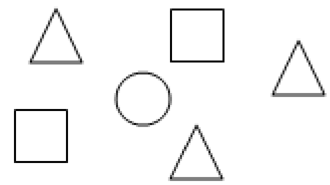
- Draw a rectangle and divide it into 6 equal parts. Shade several parts.
- What is the ratio of shaded parts to white parts?
- What is the ratio of shaded parts to the total of the parts?

2. **part: part part: whole**



- Which shapes are in a ratio of 2:3?
- What is the ratio of the triangles to the total number of shapes?

3. **ratio notation**



- Write the ratio of circles to triangles in these forms:

Colon: _____ : ______

Fraction:

Words:

4. **equivalent ratios**

A cake recipe calls for 1 cup of sugar for every 4 cups of flour.

- How many cups of sugar are needed if you have 64 cups of flour?

$$\frac{1}{4} = \frac{2}{8} = \frac{\square}{64}$$

- How many cakes will that make?

5. **data table of equivalent ratios**

There are 5 toes on one foot.

Fill in the missing values.

Feet	4			6
Toes		15	5	

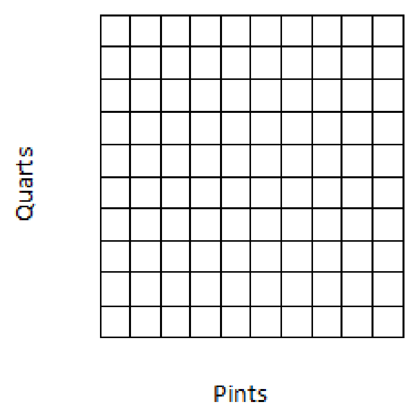
6. **graphing the relationship between two quantities**

There are 2 pints in one quart.

Fill in the missing values.

Pints		4		2
Quarts	5		3	

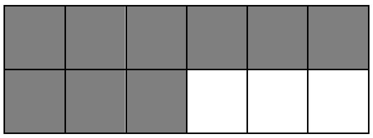
Graph the number of pints compared to the number of quarts.



7. **using a graph to find missing values**	8. **using a tape diagram**	9. **double line diagram**
Use the graph of pints to quarts. • How many quarts equal 8 pints? • How many quarts equal 5 pints? Mark your answers on the graph.	Make a tape diagram to show the relationship of pints to quarts.	Trent can bike 12 miles in one hour. He bikes for 6 hours one Saturday. Make a double line diagram that shows how many miles he will have biked after each hour.

Lesson 6: Ratio Assessment

1.

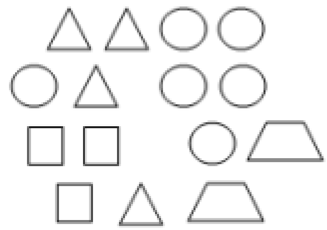


12 squares 9 shaded squares

Write a ratio for the shaded squares to the unshaded squares.

Write a ratio for the unshaded squares to the total number of squares.

2.



Which shapes are in the ratio 3:2?

5:15 is the ratio for ________ to total shapes.

3.
There are 30 birds to 3 cages. Write this ratio using the : form.

Each truck can carry 3 boxes. Write this ratio as a fraction.

Two trucks can carry 6 boxes. Write this ratio using the "to" form.

4.
In a deck of 52 cards, one out of every 4 cards is a heart. How many cards are hearts?

In a deck of cards, there are 3 face cards (Jack, Queen, and King) in each of the 4 suits.

Express in fraction form the ratio of face cards that are diamonds to the total number of diamonds in the deck.

Express in fraction form the ratio of the total number of face cards in the deck to the total number of cards in the deck.

5.
Mrs. Brown buys packages of pencils with 5 pencils per package. Fill in the chart below to show how many pencils she will get, depending on the number of packages that she buys.

Packages	1	2				7
Pencils	5		20	15	100	

6.
The perimeter of a square can be found by multiplying the side S x 4. Fill in the chart below.

Side Length (S)	Perimeter (P)
1	
2	
3	
4	
5	
6	

On a separate piece of paper, graph the side length (*x* axis) compared to the perimeter (*y* axis). Label the graph and axes.

7.

Using the graph that you made for Problem 6, answer the following questions:

A.
What is the perimeter of a square with a side of 3.5?

B.
If the perimeter is 21 units, what would be the length of one side?

C.
Extend the line on your graph. If the perimeter is 36 inches, what will be the length of the side?

D.
Mark and label your answers A, B, and C on your graph.

8.

Use a tape diagram to answer this question:

One recipe makes a batch of 12 cookies. If you need to make 80 cookies, how many batches of cookies do you need to make?

Batches

1									
12									

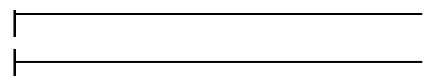
Cookies

9.

Use a double line diagram to answer this question:

The speed of a runner is 5 meters every 2 seconds. How many meters will the runner go in 9 seconds?

Meters
5



2
Seconds

Lesson 7: Understanding Rates and Unit Rates (50 mins)

Essential Question(s) to be addressed in this lesson:

Why is it important to know how to use rates?

How are rates used in everyday life?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar. " "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

SMP.1: Make sense of problems and persevere in solving them.

SMP.7: Look for and make use of structure.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students understand ratios and how they express the relationship between two quantities.

Students understand that division problems can be expressed in fraction form, e.g. ¾ = 3 ÷ 4 (6.NS.1)

Students understand how to visually represent relationships using a tape diagram or a double line diagram.

Outcome(s)

By the end of this lesson students will know:

A rate is a special kind of ratio that compares two types of measurements.

A unit rate is the ratio of two measurements in which one of the measurements is a single unit.

By the end of this lesson students will able to: Find rates and unit rates using tools such as tables, tape diagrams, and double line diagrams; use rate language (*each, to, per, for each*, etc.).

Instructional Resources/Tools

Anticipated Student Preconceptions/Misconceptions

Students may have difficulty understanding that rates compare two types of measurement quantities.

Assessment

Pre-assessment/ Formative	**Summative (optional)**
Discussion: What is a rate? A rate is a comparison of two quantities using different types of measures. *Ex. Miles to gallons; pounds to inches.* What is a unit rate? A unit rate is when one measure of a quantity is compared to a single unit of another.	

<table>
<tr><th>Lesson Sequence and Description</th><th><u>Teacher Background Information</u></th></tr>
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Students learn that a rate is a special kind of ratio.

1. Give students examples of rates. Write them on a poster. Ex. Miles per gallons (125 gallons using 5 gallons), ounces to money (15 ounces for $1.20), words per minute (175 words in 2 minutes).

2. Have students partner up. Challenge each partnership to find three rates to share with the class. Encourage them to use resources in the classroom.

3. As students share with the class, they can record their examples on a poster to be kept up in the classroom during the unit.

4. Present students with the following rate: 125 miles using 5 gallons
   a. Ask students: "If I only had one gallon, how far could I drive?"
   b. Students work with a partner.

   Explain that this is a unit rate.

5. Ask students to convert rates to unit rates, using tape diagrams, double number lines, equivalent ratio, and division methods. (Students need extensive practice with visual models before moving to the division procedure shown below.)

125 gallons using 5 gallons.

$$\frac{125\ miles}{5\ gallons} = \frac{x\ miles}{1\ gallon}$$

- Students write a journal entry:
  - What is a rate and what is a unit rate?

- Ticket to leave:
  Describe a situation where knowing how to find a

</td><td>

A **rate** is a measure, quantity, or frequency, typically one measured against a different type of quantity or measure:

- *A rate of speed of 60 miles an hour.*
- A measure of a part with respect to a whole; a proportion: *the mortality rate; a tax rate, the interest rate.*
- The cost per unit of a commodity or service: *postal rates.*

A **unit rate** is a ratio where a quantity of one measure is compared to a single unit of another: *miles per gallon, cost per pound, miles per hour, feet per second, heartbeats per minute, gallons per minute, etc.*

Show students how to solve problems using **tape diagrams** and double number lines diagrams.

If 2 lbs. of beans cost $5, how much will 15 lbs. of beans cost?

Method 1:

I found that 14 pounds cost $35, and 1 pound is another $2.50, so that is $37.50 in all.

pounds	2	4	6	8	10	12	14
dollars	5	10	15	20	25	30	35

Method 2: I found that 1 pound costs $2.50, so I multiplied 15 lbs. x $2.50 and got $37.50.

</td></tr>
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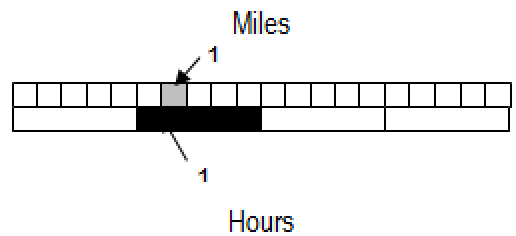
unit rate would be very important.

Example of a tape diagram: On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?



Extended Learning/Practice (homework)

Students will bring two rates or unit rate examples from home/outside of school to school to share.

Closure

Review outcomes of this lesson:

Students can give examples of rates

Students can describe how to find a unit rate.

Preview outcomes for the next lesson:

Students will continue to practice solving word problems/real-life situations involving rates and unit rates.

Teacher Reflection (to be completed after lesson)

What went well in this lesson? Did all students accomplish the outcome(s)? What evidence do I have? What would I do differently next time?

Lesson 7: Formative Assessment: Rates

A rate compares two quantities with different units.

1. There are 3 cars for 15 people. Write this as a rate. How many people per car?

2. The tree is 12 feet tall.   The tree's shadow is 6 yards long. Write this as a rate of tree height to shadow length.

3. Using the rate from Problem 2, make a chart of tree heights compared to shadow lengths for trees that are 1 foot tall, 3 feet tall, 6 feet tall, and 18 feet tall.

4. In a fruit basket, there are 2 apples for every 3 oranges. To make a bigger fruit basket with 20 apples, how many oranges are needed?

Lesson 8: Solving Problems with Unit Rates (50 mins)

Essential Question(s) to be addressed in this lesson:

Why is it important to know how to use unit rates?

How are unit rates used in everyday life?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.2: Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar. "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

SMP.2: Reason abstractly and quantitatively.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students understand that division problems can be expressed in fraction form, e.g. ¾ = 3 ÷ 4 (6.NS.1)

Students understand how to visually represent relationships using a tape diagram or a double line diagram.

Outcome(s)

By the end of this lesson students will know:

Problems-solvers use a variety of strategies to solve rate and unit rate problems.

By the end of this lesson, students will be able to:

Determine when it is appropriate to use unit rates to solve mathematical or real life problems.

Use mathematical strategies for solving problems involving unit rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.

Instructional Resources/Tools

Samples of rate and unit comparisons

Anticipated Student Preconceptions/Misconceptions

Students may have difficulty understanding that rates compare quantities in different measures. Students maybe confused about using division to find a unit rate. Students may have difficulty labeling the different quantities properly.

Assessment

Pre-assessment/ Formative	Summative (optional)
Do Now/Warm Up: Joey is shopping for decks of cards. He finds a deal on cards that is on sale for a special rate of 5 packs for \$5.95. At this rate, what is the cost of one pack of cards? At another store, a deck of cards is 99¢ each. Which is the better deal?	TBD

Lesson Sequence and Description	**Teacher Content Knowledge**

Students learn that a unit rate is a ratio where a quantity of one measure is compared to a single unit of another: *miles per gallon, feet per second, heartbeats per minute, gallons per minute.*

1. Discuss:
    - When is finding a unit rate important?
    - Why do we make the choice to purchase what we do?
    - Are we making the best decision? Is the purchase cost effective?
    - What is important to know when purchasing items at the store?

2. List examples of unit rates found in everyday life.
    - Miles per gallon
    - Miles per hour
    - Cost per pound
    - Passengers per bus load
    - Roses per bouquet
    - Words per minute
    - Pages per minute
    - Heartbeats per minute
    - Laps per hour/minute
    - Dollars per item
    - Feet per second
    - Centimeters per inch

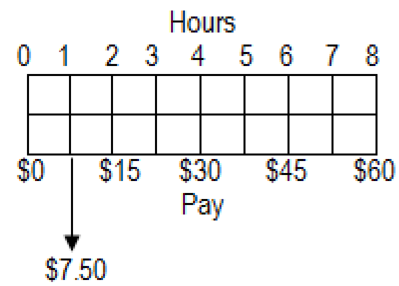
3. Students add unit rates from their homework to this list.

4. Review strategies for finding unit rates, including using tape diagrams, double number lines, equivalent ratios, and division method.

5. Guided Practice:

Different ways to solve a unit rate problem:

Tape diagram:
Michelle earned $60 babysitting for 8 hours. What was her rate per hour?



Equivalent ratios:

$$\frac{\$60}{8\ hours} = \frac{x}{1\ hour}$$

(Note: Students should not cross-multiply here; they need to use equivalent ratio strategies.)

Table:

Hours Worked	Pay
8	$60
4	$30
2	$25
1	?

(Solve this problem using tape diagram, equivalent ratios, and division so that students see that all three methods yield the same result). Which would you choose? Brand A: 1 box of 30 cookies for \$3.00 Brand B: 2 boxes of 20 cookies each selling for \$5.00 for 2 boxes Find the cost per cookie for each brand. Which would you choose? Why? Show/explain your work. 6. Students practice solving rate and unit rate problems (not provided). Ask students to do some problems using two different strategies. 7. Circulate among the class, and choose several students who have particularly unique, efficient, or clever strategies for solving problems to show and explain their work to the class.	*Division:* *\$60 ÷ 8 = \$7.50* Extension: The cookie problem can also be solved by making the amount of money equal (for example, \$15) and figuring out how many cookies you could buy for \$15. Or you could figure out the cost of the same number of cookies (for example, 120 cookies) and then compare the costs.
Extended Learning/Practice (homework) Students solve a unit rate problem. Students write their own unit rate problem. Students may use one of the rates from the class poster.	

Closure

Review outcomes of this lesson:

Students can solve unit rate problems using a variety of strategies in mathematical and real-world problems.

Preview outcomes for the next lesson:

Students use problem-solving skills to apply strategies to mathematical and real-world problems.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

Lesson 9:
Using Rates and Unit Rates to Make Informed Consumer Decisions

(50 mins)

Essential Question(s) to be addressed in this lesson:

Why is it important to know how to use rates and unit rates?

How are rates and unit rates used in everyday life?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

ELA 6.RST.4: Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to Grades 6- 8 texts and topics.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students know how to solve rate and unit rate problems using a variety of strategies.

Outcome(s)

By the end of this lesson students will know:

Knowledge of unit rates and rates can help consumers make informed purchasing decisions.

By the end of this lesson students will be able to:

Determine when it is appropriate to use unit rates to solve mathematical or real life problems. Use mathematical strategies for solving problems involving unit rates, such as tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc.

Instructional Resources/Tools

Sample or actual store flyers (Sam's Market Flyer) Unit pricing tags

Anticipated Student Preconceptions/Misconceptions

Students may make impulsive decisions about which offer is the best deal based on the language of the offer rather than the cost per item or unit.	
Pre-assessment/ Formative	Summative (optional)
Warm Up/Do Now: Students exchange unit rate problems generated from the previous night's homework and solve them. Grocery Store Flyer Scavenger Hunt (see below).	**Justin's Pizza** **(problem-solving with unit rates)** Justin needs to buy four pizzas for a party. He found the following three ads in his local newspaper:

Rosa's Pizza	**Pizza Mart**	**Collins Pizza**
Each 12-inch pizza: $6.35 Buy 3 and get one free!	Each 12 inch pizza: $4.65	Four-pack of 12-inch pizzas: $19.60

Where would Justin get the best deal?

Answer: Pizza Mart ($18.60), Rosa's Pizza

	($19.05), Collins' Pizza ($19.60)

Lesson Sequence and Description	**Teacher Background Information:**
1. Grocery store flyer scavenger hunt: o Students are provided with a worksheet that lists various grocery items (Sam's Market Sale Flyer). o They find the unit rate for a single item. o Use this as a formative assessment. Not all students may be able to find the unit price. 2. Justin's Pizza Assessment	Some students may need guidance in knowing how to read a store flyer (layout, rate language, vocabulary, types of information).
Extended Learning/Practice (homework) Students research sales flyers, grocery, clothing, electronics, etc., and compare prices on two items. Flyers sources: newspapers, stores, internet, or provided by the school. They bring examples into class for sharing.	

Closure

Review outcomes of this lesson:

Students can use unit rates to explain and make informed decisions about purchases.

Preview outcomes for the next lesson:

Students will be assessed on rates and unit rates.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

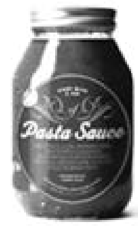
What evidence do I have?

What would I do differently next time?

Sam's Market Weekly Sale Flyer

Oranges: One dozen for $3.60

24 bottles of spring water for $3.60



3 jars of pasta sauce for $9.69

Snack pack of 5 bags for $5.50

2 gallons of milk for $5.98

Here are some items on sale at Sam's Market. Find the unit rate (unit price) of each item, and fill in the chart below.

ITEM	Quantity Listed	Price Listed	Unit	Unit Price
Oranges	12	$3.60	1 orange	

Lesson 10: Rates and Unit Rates Assessment (50 mins)

Essential Question(s) to be addressed in this lesson:

Why is it important to know how to use ratios and rates?

How are ratios and rates used in everyday life?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar. " "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

SMP.1: Make sense of problems and persevere in solving them.

SMP.2: Reason abstractly and quantitatively.

SMP.4: Model with mathematics

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students understand that ratios relate two quantities to one another. They understand that rates are special kinds of ratios that relate two different measurement quantities to each other. Given a rate, students know how to find a unit rate. Students understand that another word for "flyer" is "circular", and that these circulars advertise special money-saving deals at stores.

Outcome(s)

By the end of this lesson students will know:

Several strategies for solving rate and unit rate problems.

By the end of this lesson, students will be able to:

Solve real world and mathematical rate/unit rate problems.

Use rate language to describe the relationship between two quantities. (each, to, per, for, /, each, etc.)

Instructional Resources/Tools

Grocery store circular (looks like an authentic artifact from a grocery store) Written Assessment: Little Red Riding Hood

Anticipated Student Preconceptions/Misconceptions

Students may have trouble distinguishing between rates and unit rates. Students may find it difficult to read and interpret an authentic grocery store flyer that uses rates and unit rates.

Assessment

Pre-assessment/ Formative	Summative (optional)
Warm Up/Do Now: In this week's grocery circular, Giant Food advertises bananas on sale for 5 lbs. for $4.45. Food Mart has them on sale for $.95/lb. Which store has the best deal? *(Giant Foods: $.89/pound)*	Grocery Store Circular (see below)

Lesson Sequence and Description	

• Students complete the formative assessment (Warm Up/Do Now)
 	
• Briefly review different strategies for solving rate/unit rate problems:
  o Equivalent ratios
  o Equations, including division
  o Tape diagrams and double line diagrams
• Assessment:
Students are given two grocery store circulars. They are asked to calculate the unit rate on a variety of items. They will use the unit rates to compare prices to determine the store with the best overall “best value”	

Extended Learning/Practice (homework)	
To prepare for the next lesson, students find examples of percents at home and bring in three examples of percents (including their contexts). Possible sources include newspapers, magazines, store flyers, bills, and internet sources.	

Closure

Review outcomes of this lesson:

Students solve real world and mathematical rate and unit rate problems.

Preview outcomes for the next lesson:

Students study percents.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

Lesson 10: Assessment

Little Red Riding Hood's Grocery Trip

After spending some time phoning and texting with Granny, Little Red Riding Hood decided it was time to visit her beloved grandmother, who lived in the Far, Far Away Woods. She thought it would be nice if she brought some groceries and fixed lunch for her elderly, yet hip, Granny. She remembered that her Granny loved tacos and brownies, so she decided that she would make them for her as a surprise! She quickly made her grocery list. Red decided she needed to buy:

- soft taco shells
- shredded cheese
- ground beef
- salsa
- lettuce
- brownies (for dessert, of course!)

Little Red Riding Hood scanned recent grocery store flyers and found that two different stores, the Very Hungry Wolf's Food Store and the Strong, Brave Woodsman's Food Mart, had all of the ingredients that she needed. Red didn't have a lot of time to shop, so she decided to use the flyers to compare prices to help her decide which store had the best values. After comparing unit prices for each item, she totaled the unit prices and chose the store that would give her the best overall value.

Determine the unit price for each item at each grocery store. Put your answers in the place provided in the chart shown. You must show your work for each item. Once you have decided which grocery store will offer Red the best value for her money, explain why she should shop there. Give specific detail about how you decided whether she should shop at the Very Hungry Wolf's Food Mart or the Strong, Brave Woodsman's.

The Very Hungry Wolf's Food Store

Grocery Item	Price	Unit Price
Soft Taco Shells	8 for $.96	
Shredded Cheese	7 oz. for $1.75	
Ground Beef	1 lb. for $2.50	
Salsa	22 oz. for $3.30	

The Brave Woodsman's Food Mart

Grocery Item	Price	Unit Price
Soft Taco Shells	12 for $1.68	
Shredded Cheese	1 lb. for $3.20	
Ground Beef	5 lbs. for $9.95	
Salsa	10 oz. for $1.98	

Lettuce	1 head for $.89			Lettuce	3 heads for $1.69	
Brownies	12 for $3.60			Brownies	4 for $1.00	

Lesson 11: What is a Percent? (50 mins)

Essential Question(s) to be addressed in this lesson:

What is a percent?

How do percents relate one quantity to another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6 RP.3c Find the percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent

6.SMP.7 Look for and make use of structure.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students will have had experiences with fractions and equivalent ratios that are compatible with decimal tenths and hundredths. (½, ¼, ⅕, $^1/_{10}$, $^1/_{25}$, $^1/_{100}$)

Outcome(s)

By the end of this lesson students will know:

Ratios can be represented in a variety of formats, including percents.

A percent is a type of ratio that compares a quantity to 100.

Percents can be represented graphically in several ways.

By the end of this lesson students will be able to:

Describe a ratio as a percent.

Represent a quantity compared to 100 as a percent on a 10 x 10 grid.

Instructional Resources/Tools

10 x 10 grids

Optional: Base ten blocks (manipulatives or virtual manipulatives) Projector and internet access

Anticipated Student Preconceptions/Misconceptions

Students may have difficulty understanding that 100% = the whole. Students may use the numerator of any fraction as the percent.

Assessment

Pre-assessment/ Formative	Summative (optional)
Discuss: What does “percent” mean? Students may have some real-world experience with percents. Pre-assessment: Draw a sketch of a pizza with 8 slices. How much is 100% of the pizza? 50%? 25%?	

Lesson Sequence and Description

1. In a small group, students share percents from Lesson 10's homework (finding examples of percents at home). They discuss what each percent might mean in its context. Examples can be posted in the classroom.

2. Discuss: When you are at a store and there is a sale that is 20% off, what does this mean?

3. Give each student a grid worksheet with 10 x 10 grids. Have students shade 10 x 10 grids in various amounts. Examples: 30%, 54%, 3%. Have students write each percent as a decimal and as a fraction: 54% = .54 = 54/100

   Example: This grid shows 25%.

.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01

100 squares of .01 25 shaded = 25%

<u>Teacher Background Information</u>:

(Note: The pre-assessment [pizza question] is intended to be a quick dipstick of students' prior knowledge. If students cannot answer questions, move on to the lesson sequence rather than making the pizza question the focus of the day's lesson.)

The concept of percent first appears in Grade 6 in the new standards. Students have had extensive work in fractions and decimals, which will help them to find equivalencies in percents.

Percents are part/whole ratios where the whole is measured in hundredths.

Beginning students can derive rates per 100 from fractions that have denominators of 2, 4, 5, 10, 20, 25, and 50, by using their knowledge of fraction equivalencies.

Significant time should be spent on visual models such as 10 x 10 grids (this lesson), tape diagrams, and double-line diagrams (subsequent lessons) to help students see the comparison between a quantity and 100.

Students should be familiar with the following fractions ½, ¼, ⅕, $^1/_{10}$, and $^1/_{100}$.

Other useful manipulatives include base-ten blocks (hands-on or virtual), pennies, tiles, etc.

NCTM Illuminations *Grid and Percent It* Lesson has good background for teachers.

http://illuminations.nctm.org/LessonDetail.aspx?id=L249

This resource presents a method of solving percent problems that focuses on the basic concept of percent, that of "parts per hundred." A 10 × 10 grid, which is a common model for visualizing percents, is extended to solve various

Another way to show 25%:

.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
.01	.01	.01	.01	.01	.01	.01	.01	.01	.01

100 squares of .01 25 shaded = 25%

4. Partner work. Each student shades an amount on a grid. The other student then reads the amount shaded, giving the decimal, the fraction, and the percent names. Repeat with different examples. Challenge students to find as many different ways as possible to show a percent. For example, 20% can be shown as 2 x 10, 4 x 5, or 5 x 4 grids.

5. Require students to use models and drawings to explain their thinking for the following problems:

   - The paper carrier delivers 100 newspapers every Sunday. If 75% were delivered before noon, how many newspapers still need to be delivered? (Extension: What if the paper carrier delivers 200 newspapers?)

   - 25% of the cars at a car dealership come with a 5-year warranty. The rest

types of percent problems.

Virtual Manipulative Percent grids:

http://nlvm.usu.edu/en/nav/frames_asid_333_g_3_t_1.html?from_category_g_3_t_1.html

Students represent, name, and explore percentages using hundreds grids.

Advanced students would be ready to solve more difficult problems independently shading 6.5%, 33.3%,

have a 2-year warranty. How many have a 2-year warranty?	

Sample representation:

5 yr	5 yr	5 yr	5 yr	5 yr	2 yr	2 yr	2 yr	2 yr	2 yr
5 yr	5 yr	5 yr	5 yr	5 yr	2 yr	2 yr	2 yr	2 yr	2 yr
5 yr	5 yr	5 yr	5 yr	5 yr	2 yr	2 yr	2 yr	2 yr	2 yr
5 yr	5 yr	5 yr	5 yr	5 yr	2 yr	2 yr	2 yr	2 yr	2 yr
5 yr	5 yr	5 yr	5 yr	5 yr	2 yr	2 yr	2 yr	2 yr	2 yr
2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr
2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr
2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr
2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr
2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr	2 yr

100 squares 25 5yr and 75 2yr

- The math class has completed 80 pages of the 100 page geometry unit. What percent have they completed and what percent is left to do? Show on a hundreds grid.

- During the first 100 days of school there were 65 sunny days. What percent of days were not sunny?

- There were 100 pounds of potatoes harvested from the community garden.

o 20% were sweet potatoes o 45 pounds were russet potatoes o The rest were golden potatoes On a 10 x 10 grid represent each type of potato by both pound and percent. Explain how you found the number of pounds and the percent of golden potatoes. 6. Battle of Fractions, Decimals, and Percents Game (see below). 7. Return to the opening problem: If an item costs \$100 but there is a 20% discount, how much will you pay?	
Extended Learning/Practice (homework) Worksheet: Percent problems (where the whole = 100).	

Closure

Review outcomes of this lesson:
Students understand that a percent is a part/whole ratio where the whole is measured in hundredths.
Students are able to use equivalent fractions to represent a percent graphically on a 10 x 10 grid.

Preview outcomes for the next lesson:
Students expand their understanding of percents by representing any ratio as a quantity out of 100.
Example: 4/5 = 80/100 = 80%

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

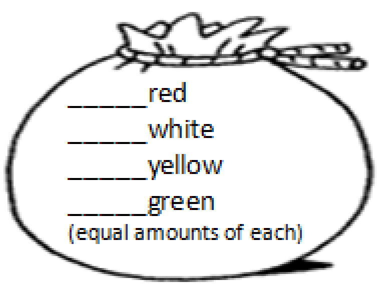
Lesson 11: Homework

1. Each bag of marbles contains 100 marbles. Find the missing number of marbles for each bag.

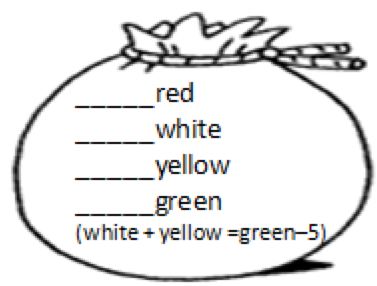
A.

C.

B.



D.



2. There are 100 students in sixth grade. Students in sixth grade can participate in one after-school music program. 22% chose chorus, 55% chose band, and 10% chose orchestra. What percent of sixth graders are not participating in a music program? Show your answer on a 10 x 10 grid.

3. There are 100 songs on Jed's iPod. 70 songs are rock music, 8 songs are rap music, and 10 are pop music. The rest of the songs are country music. How many songs are country music? How do you know? What percent of the total songs is this?

   Challenge: If you could only put 100 songs on an iPod, which types of music would you choose? List the categories and that percent of songs in each type of music that you would choose.

4. A bucket contains 100 blocks. 30% of the blocks are yellow. 15% are red, 12% are blue, and 8% are orange. Show this on a 10 x 10 grid. Write each quantity as a fraction. Which colors could be added to total 50%? Which colors could be added to total 20 blocks?

Lesson 11: Game

Battle of FRACTIONS, DECIMALS, and PERCENTS

Object of the game:

Practice comparing fractions, decimals, and percents.

Number of Players: 2, 3,

Directions:

1. Distribute the cards equally to each player.
2. Players keep their cards face-down.
3. Each player turns over one card.
4. The player who has the card with the greatest value takes all cards.
5. If there is a tie, the players with the two highest cards place three additional cards face-down. Then they turn one more card face-up. The player who has the card with the greatest value takes all the cards.

OPTION 1:

The game continues in this way until all the cards have been played.

The player with the greatest number of cards wins the game.

OPTION 2:

When a player runs out of cards, he or she is eliminated from the game.

The game continues until one player has won all the cards.

Note: If students do not agree on the card with the highest value, have them use classroom resources (visual models, manipulatives, calculator) to determine the winner of the round.

Fraction	Decimal	Percent
1/4		

Fraction	Decimal	Percent
		50%

Fraction	Decimal	Percent
		25%

Fraction	Decimal	Percent
	0.75	

Fraction	Decimal	Percent
	0.1	

Fraction	Decimal	Percent
7/10		

Fraction	Decimal	Percent
1/2		

Fraction	Decimal	Percent
		70%

Fraction	Decimal	Percent
	0.25	

Fraction	Decimal	Percent
3/4		

Fraction	Decimal	Percent
1/10		

Fraction	Decimal	Percent
		75%

Fraction	Decimal	Percent
		10%

Fraction	Decimal	Percent
	0.7	

Fraction	Decimal	Percent
	0.5	

Fraction	Decimal	Percent
1/5		

Fraction	Decimal	Percent
	0.2	

Fraction	Decimal	Percent
80/100		

Fraction	Decimal	Percent
1/8		

Fraction	Decimal	Percent
		80%

Fraction	Decimal	Percent
		12.5%

Fraction	Decimal	Percent
	0.6	

Fraction	Decimal	Percent
	0.3	

Fraction	Decimal	Percent
6/10		

Fraction	Decimal	Percent
		20%

Fraction	Decimal	Percent
	0.8	

Fraction	Decimal	Percent
	0.125	

Fraction	Decimal	Percent
3/5		

Fraction	Decimal	Percent
3/10		

Fraction	Decimal	Percent
	1/3	

Fraction	Decimal	Percent
		30%

Fraction	Decimal	Percent
		30%

Fraction	Decimal	Percent

Fraction	Decimal	Percent

Fraction	Decimal	Percent

Fraction	Decimal	Percent

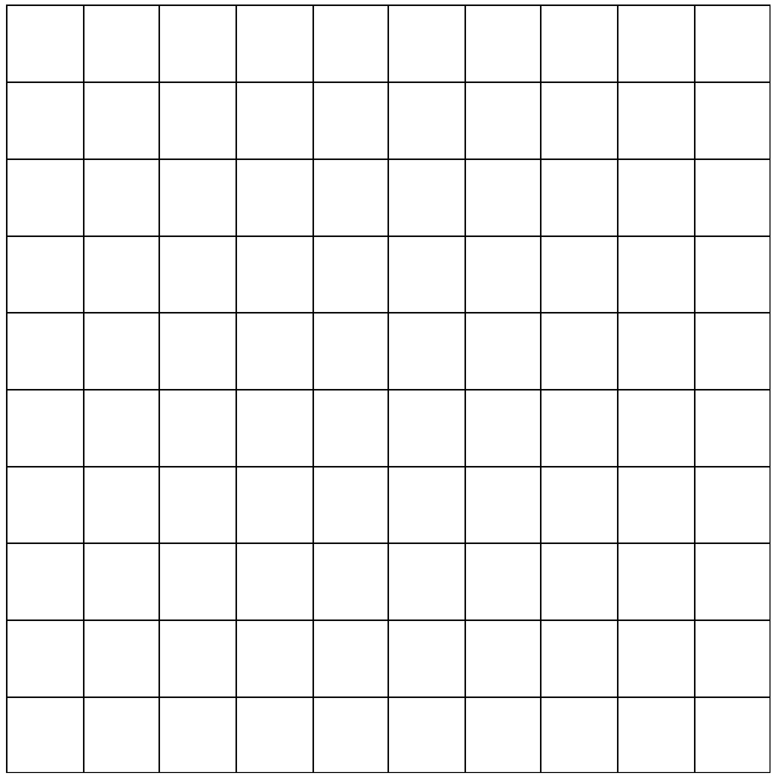
Fraction	Decimal	Percent

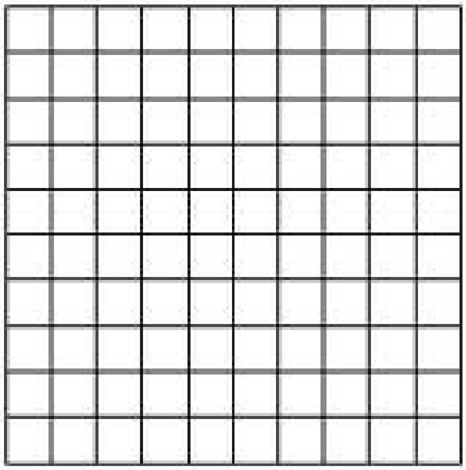
Fraction	Decimal	Percent

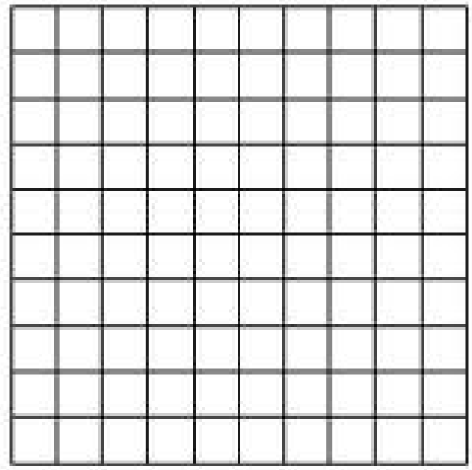
Fraction	Decimal	Percent

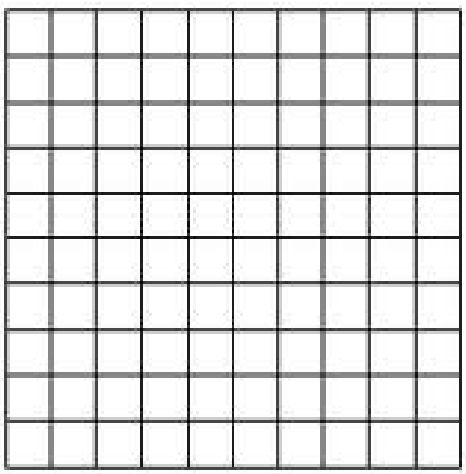
Fraction	Decimal	Percent

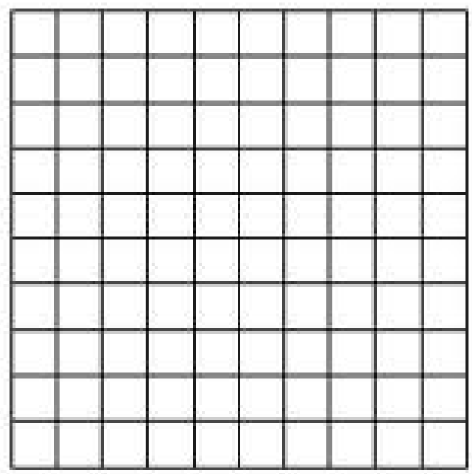
10 x 10 Grid

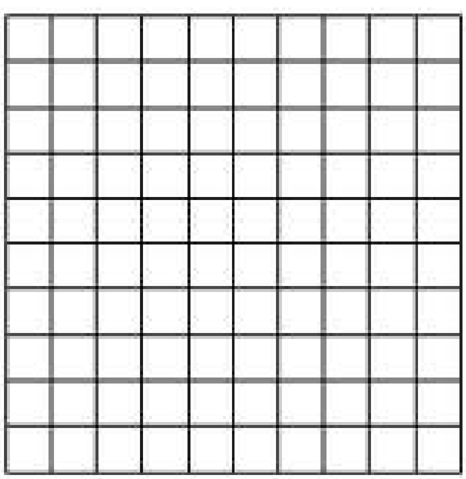


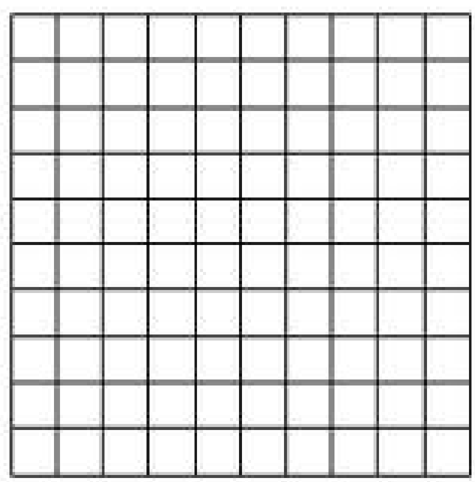












1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Lesson 12: Percents and Tape Diagrams (50 mins)

Essential Question(s) to be addressed in this lesson:

How can we represent percents visually to help us solve problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6 RP.3c Find the percent of a quantity as a rate per 100 (e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent

6.SMP.7 Look for and make use of structure.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students understand how to find a percent when the whole is 100.

Students can find equivalent fractions.

Students are comfortable using tape diagrams to compare quantities.

Outcome(s)

By the end of this lesson students will know:

The quantity represented by a percent depends upon the size of the whole.

Visual representations of percents are useful in solving percent problems.

By the end of this lesson, students will be able to:

Represent a percent of a number using tape diagrams.

Compare a percent of different wholes.

Write a statement in the form of ___% of ____ = _____

Instructional Resources/Tools

Anticipated Student Preconceptions/Misconceptions

Students may have difficulty understanding that the whole can be more than 100 when representing a percent.

Students may have difficulty determining accurate and appropriate segments on tape diagrams.

Assessment

Pre-assessment/ Formative	Summative (optional)
Ticket to Leave: Explain why 25% does not always represent the same amount.	None

Lesson Sequence and Description

1. Discuss with a partner or group: How could a 10 x 10 square grid represent 200 people? Be prepared to share your ideas. (Each square equals 2 people.)

   Discuss with a partner or group: How could a tape diagram represent 200 people?

Percent:	25%	25%	25%	25%
People:	50	50	50	50

   Discuss the similarities and differences between the two visual models.

2. Students shade in 25% on tape diagrams where the whole represents different amounts. The amounts are: 100, 200, 60, 20, and 500. Challenge: 50, 250, 2.

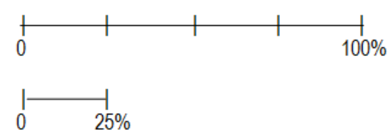
   Record the fractions represented in each tape diagram. What pattern can you see? What conclusion can be drawn?
   - All fractions are equivalent and equal to 25% or ¼.
   - 25% represents different quantities (depending upon the amount of the whole).

3. Students express the quantity in each tape diagram as follows:
   25% of _____ = _______
   (25% of 100 = 25; 25% of 500 = 125, etc.)

<u>Teacher Background Knowledge</u>

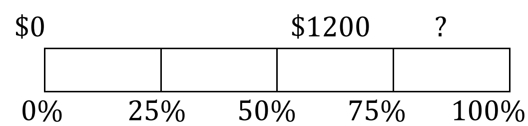
- Students should access their prior knowledge of number lines and tape diagrams to represent fractions. (See previous lessons.)

- Double line segments can be used to show the whole (100%) compared to a part.



If 75% of the budget is $1200, what is the total budget?

Dollars



- Remind students that tape diagram segment divisions will vary depending upon the percent. (Example: To find 20%, it is useful to divide 100 into 5 segments; to find 10%, it is useful to divide 100 into 10 segments, etc.)

4. Students are given another percent (such as 10%, 20%, 50%, 75%, etc.) and are asked to create their own set of five tape diagrams in which they find the assigned percent of the following wholes: 100, 200, 60, 20, and 500. Record the fractions represented in each tape diagram in Step 4. What pattern do they see? What conclusion can be drawn? Students express the quantity in each tape diagram as follows: _____% of _____ = _______ for each tape diagram. 5. Students share their tape diagrams, and conclusions. 6. Ticket to leave: Explain why 25% does not always represent the same amount.	• Many students may be ready to use more challenging numbers such as 240, 175, and 350 as the whole for #4. • Advanced students might use the *Now and Then* PowerPoint and lesson to work more independently and figure out how prices change. They need a good understanding of percent to do these activities. http://illuminations.nctm.org/LessonDetail.aspx?id=L837
Extended Learning/Practice (homework) Worksheet: Tape diagram practice	

Closure

Review outcomes of this lesson:

Students will be able to represent percents of different wholes using tape diagrams.

Preview outcomes for the next lesson:

Students will represent solutions to percent problems with equations in addition to visual representations.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s))?

What evidence do I have?

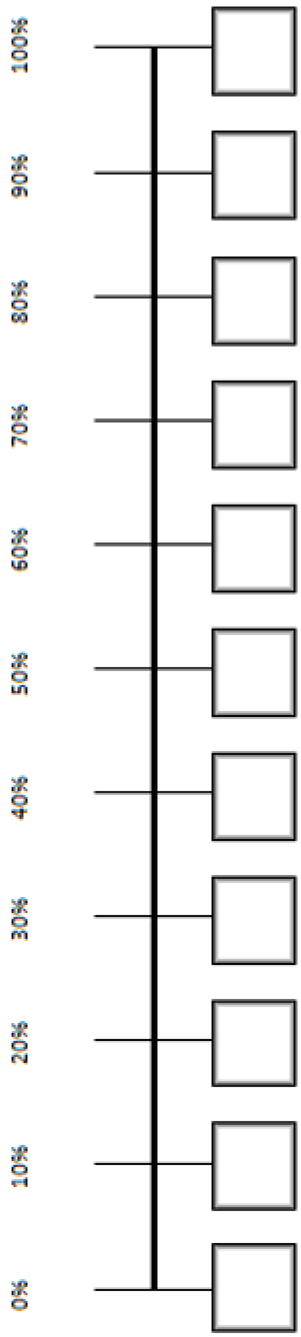
What would I do differently next time?

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

25%	50%	75%	100%

25%	50%	75%	100%


100%
90%
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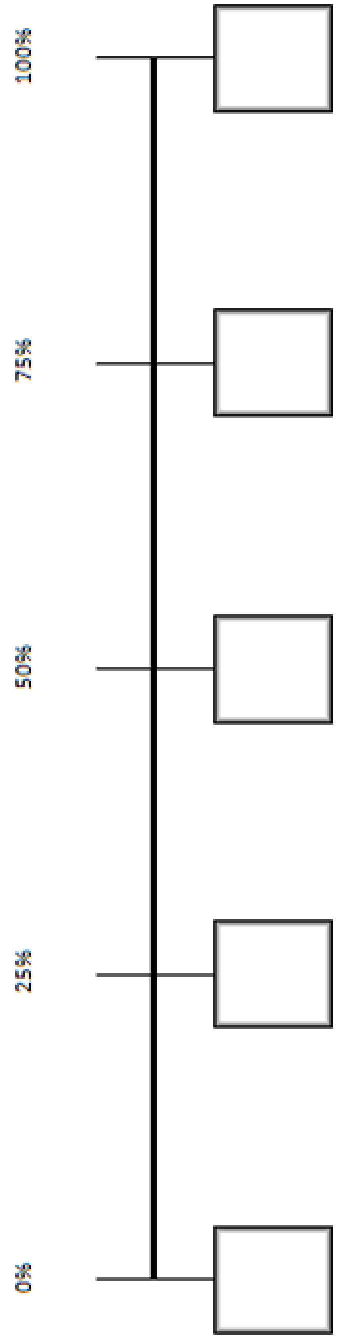
40%
30%
20%
10%
0%


100%
75%
50%
25%
0%

Lesson 13: Solving Percent Problems: Missing Part, Missing Percent

(50 mins)

Essential Question(s) to be addressed in this lesson:

How are percents used to solve problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6 RP.3c Find the percent of a quantity as a rate per 100 (e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent.

6.SMP.2 Reason abstractly and quantitatively.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students see the connections between ratios and percents.

Students can solve for a missing quantity in equivalent ratios.

Outcome(s)

By the end of this lesson students will know:

Percent problems contain three components: the percent, the part and the whole.

Students will be able to:

Solve problems where they must find the unknown part or percent (given the other two values).

Write equations to match percent problem contexts.

Instructional Resources/Tools

Virtual Manipulative site:
http://nlvm.usu.edu/en/nav/frames_asid_160_g_3_t_1.html?open=activities&from=category_g_3_t_1.htm

With this virtual manipulative, students specify any two of the three quantities, "part," "whole," or "percent." When the student clicks the Compute button, the computer calculates the remaining quantity. The percentage relationship is displayed visually in the vertical column of the Percent Gauge and in a pie chart. An equation shows how to compute the unknown quantity and the result is stated verbally.

Anticipated Student Preconceptions/Misconceptions

Students may have difficulty understanding a problem context in order to determine the missing value. Students may invert values when setting up equivalent ratios.

Assessment

Pre-assessment/ Formative	Summative (optional)

Lesson Sequence and Description	Teacher Background Knowledge
Students solve problems to find the missing part. 1. Warm-up problems: Students work independently to solve the following problems, using any strategy that they choose. They share solutions in a small group. • On Saturday, the bicycle tour covered 100 miles. I took my first break at 10 miles. What percent of the day's ride was complete? • The next day, the bicycle tour covered 50 miles. I stopped after 10 miles. What percent of that day's ride was complete? • If 50 students went to the cafeteria and 30 purchased chicken fingers, what percent of students purchased chicken fingers? (Scaffold: Change the fraction 30/50 to 60/100 by using equivalent fractions. Fill this in on the 10 by 10 grid. One student will be represented by 2 squares.) 2. Working with partners solve: A. Mrs. Brown purchased a new $800 computer for 10% off. What was the savings? Students use a 10 x 10 grid to explain their thinking.	Refrain from teaching arbitrary rules such as "To determine 10% of a number, move the decimal point over one place to the left" or cross-multiplying to solve ratio equivalencies. Specifically use the terms part, whole and percent during classroom discussions Make connections between prior knowledge of common fractions and percents:

Prior Knowledge	Connection to %
¾ of 20 = 15	75% of 20 = 15
15 out of 20 = 3/4	15 out of 20 = 75%
¾ = 75%	

Ask students to represent the same problem using a tape diagram.

Ask students to represent this problem with an equation of equivalent ratios.

10/100 = X/800

This can be understood as: 10% is _____ out of 800.

Students should also write the relationship in this way:

10% of $800 = $80

B. Mrs. Brown went to a different store. The same model of computer is usually priced at $1,000 but it is on sale for 25% off. How many dollars would she save? Ask students: Which strategy is best, a tape diagram or a 10 x 10 grid?

25-100 = X/1000

This can be understood as: 25% is _____ out of 1000.

C. From which store should Mrs. Brown purchase the computer?

Students solve problems to find the missing percent.

D. Rose read 60 pages of 300 page book. What percent had she read? What percent does she still have to read?
Have students set up this equivalent ratio and solve using their knowledge of equivalent fractions. Justify the answer using a tape diagram.

60/300 = X/100

Encourage the use of mental math strategies and number sense to estimate solutions and check answers. For example, 40% of 200 must be less than half of 200.

This can be understood as: 60 out of 300 is ____%. E. Robert read 20 pages of a 300-page book. What percent has he read? What percent does he still have to read? Explain how this problem was solved. (Calculators allowed.) F. Mill River Middle School has 320 families with students at the school. For the Open House, 240 families were represented. What percent of families came to Open House? Students solve this problem independently and then pair with another student to compare results and strategies. G. Students write two percent problems (one with a missing part, and one with a missing percent). They solve their own problems, and then exchange problems with a friend.	
Extended Learning/Practice (homework) Students write two additional percent problems: 1) Missing part 2) Missing percent	

Closure

Review outcomes of this lesson:

Students solve problems where they must find the unknown part or percent (given the other two values).

Students write equations to match percent problem contexts.

Preview outcomes for the next lesson:

Students will learn how to find the missing whole, given the part and the percent.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

Lesson 14: Solving Percent Problems: Missing Whole (2 lessons, 50 mins ea)

Essential Question(s) to be addressed in this lesson:

What types of problems can we solve with percents?

How does knowing about percents help to make us to make decisions?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

6 RP.3c Find percent of a quantity as a rate per 100(e.g.30% of a quantity means 30/100 times the quantity); solve problems involving the percent of a whole, given a part and a percent

6.SMP.1 Make sense of problems and persevere in solving them.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Students will know that a percent problem contains three parts: the percent, the part and the whole.

Given the part and the whole, students can determine the percent.

Given the percent and the whole, students can determine the part.

Outcome(s)

By the end of this lesson students will know:

Percent problems have three parts: the part, the percent, or the whole. Given two parts, the student can solve for the missing part.

Percents can be used to solve a variety of mathematical and real-world problems.

By the end of this lesson students will be able to:

Determine the whole, given the part and the percent.

Solve problems involving percents when the part, the percent, or the whole is the unknown quantity.

Make decisions or judgments using their knowledge of percents.

Instructional Resources/Tools

Anticipated Student Preconceptions/Misconceptions

Students may know that 100% represents the whole, but many students may still experience difficulties understanding that this whole can be any number (100, greater than 100, or less than 100).

When setting up equivalent ratios, students may place quantities in the incorrect position.

Assessment

<table>
<tr><td>Pre-assessment/ Formative</td><td>Summative</td></tr>
<tr><td>Review homework examples:

The previous night's homework was to write two problems: one with a missing part, and one with a missing percent. Students ask a friend to solve their problems.</td><td>Summative Percent Assessment:

<u>TIM'S TIRES</u>

Tim needs to buy 4 new tires for his car.

He found the following three ads in his local newspaper:

<table><tr><th>Steve's Tires</th><th>Tire Mart</th><th>Collins Tires</th></tr><tr><td>Each tire: $63.55

Buy 3 and get one free!</td><td>Each tire: $46.25</td><td>Each tire: $56.75

25% off the total purchase!</td></tr></table>
A. Where would Tim get the best deal?
B. If Tim only needed one tire, where would get the best deal?
C. If he were buying two tires, where would he get the best deal?

<u>RAFFLE TICKET SALES</u>

The students at Hunking School were selling raffle tickets to raise money for the tornado victims.

• The eighth grade had 300 tickets to sell.
• The seventh grade had 200 tickets to sell.
• The sixth grade had 100 tickets to sell.</td></tr>
</table>

	A. At the end of the first week, each grade had sold 40% of their tickets. How many tickets had each grade sold? B. With one day left in the fundraiser, the eighth grade had sold 225 tickets, the seventh grade had sold 150 tickets, and the sixth grade had sold 60 tickets. Which grade was had the fewest tickets left to sell? C. Estimate the percentages sold for each grade. Which grade came closest to its goal of selling all of its tickets? D. The teachers also decided to help out and were given raffle tickets to sell. They sold 32 tickets. This was 80% of the tickets the teachers were given. How many raffle tickets were the teachers given to sell? **LIBRARY BOOKS** Brianna and Devin went to the library and checked out some books. 30% of the books that Brianna checked out were fiction. 50% of the books that Devin checked out were fiction books. Brianna and Devin's friend Shannon said that Devin checked out more fiction books than Brianna. Is Shannon correct? Use examples when you explain your thinking.

Lesson Sequence and Description	Teacher Background Information
Note: This sequence of lessons and assessments takes two class periods. 1. Students review the previous night's homework by asking a friend to solve the two problems that they wrote. 2. Introduce finding the missing whole with the following problem: Yesterday, I bought a sweatshirt on sale for 50% off and ended up paying only \$15. I also bought a t-shirt that was on sale for 25% off and ended up paying only \$15. What was the original price for each? Using a quick "think-pair-share", ask students to determine the original prices, and to share their answers and strategies with classmates. 3. Summarize percent problem solving strategies: • Number sense (1/2 of something is \$15, so the whole must be \$30) • Tape diagrams • Equivalent ratios: $\frac{75}{100} = \frac{15}{?}$ 4. Guide students through the solution for the following problem: 36% of the students at Central Middle School are sixth graders. This is 180 students. How many total students are enrolled at Central Middle School? • What percent represents the total enrollment? • What do we need to find out in this problem? • What percent do we know? Is this the part or the whole? • If 180 students or 36% are 6th graders, have students estimate the total number. (Hint: 36% is close to 1/3) • Model the problem with a tape diagram. • Solve the problem with equivalent ratios. • Discuss: does this answer make sense? Can anyone suggest another strategy to solve this problem? 5. Assign the following problem: 60% of the people at the movie theater were adults, and the rest	• Remind students that "percent" is not a new concept. It is a new notation for what they already know … fractions and decimals. • Students should be encouraged to use their number sense and percent benchmarks to judge the reasonableness of answers. (i.e. greater than or less than 50%?) • It is important for students to continue to draw a visual model to represent the problem, so a deeper conceptual understanding is developed. • Make sure that students understand the meaning of key vocabulary in the word problems. Extension: Students conduct a survey and use the results to determine percents. These percents can then be used to construct a circle graph. Students

were children. There were 720 adults. How many people were at the movie theater in all? Students work in pairs or small groups to solve the problem using two different strategies. Record work on chart paper. Conduct a gallery walk to compare and comment on the approaches of each group. 6. Students identify the missing quantity in each of the following three problems: (part, percent, whole). • Harry has 350 songs on his MP3 player. 15% are country, 35% are rock, and the rest are pop. How many songs are pop? • There are 150 students at Whittier school who have been to New York City. This is 30% of the school's total population. How many students attend Whittier School? • Sally had 25 pairs of socks. Seven pairs were striped and the rest were solid. What percent of the pairs of socks were solid? 7. Ask students to solve the problems in #6. 8. Students select additional problems from the percent problem-solving worksheet. (#1-19) (Note: Problems on this sheet are organized by level of difficulty.) 9. Students complete the three problems: **Tim's Tires, Raffle Ticket Sales,** and **Library Books**. (See Summative Assessment section.)	write a summary of the responses.
Extended Learning/Practice (homework) Additional problems from the Percent Problems worksheet can be assigned.	
Closure Review outcomes of this lesson:	

Students can solve all three types of percent problems using a variety of strategies.

Preview outcomes for the next lesson:

Students will use their knowledge of ratios, rates, unit rates, and percents to complete the CEPA.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?

Did all students accomplish the outcome(s)?

What evidence do I have?

What would I do differently next time?

Lesson 14: Percent Problem Solving Worksheet

1. Sidney spent 30% of her monthly salary on a new laptop last month. Then she spent 50% of her salary to pay her monthly bills. She put the remaining $1,200 into her savings account. What was her monthly salary?

2. Mr. Stevenson has 160 cows, goats, horses, and chickens on his farm. 19% of the animals are cows, 28% are goats, 13% are horses, and the rest are chickens. How many chickens are on the farm?

3. Larry had $60. He spent $24 on a beach towel and sunscreen. He spent the rest on a bathing suit. What percent of the $60 did he spend on the new bathing suit?

4. Thirty-five percent of the people who visit Pinewoods State Park take a hike on the Sunset Trail Loop . If 175 visitors hiked the trail last Saturday, how many visitors total did Pinewoods State Park have that day?

5. There were 440 boys and 360 girls at the championship baseball game. What percent of the students at the baseball game were girls?

6. Ms. Spring spent 45% of her savings on a new television set that cost $810. How much money did she have in her savings account before she bought the television set?

7. Shari and Justin are starting a coin collection. The chart below shows how many coins they have collected so far.

   Complete the charts, showing:
   - How many coins they have of each kind
   - The percentages for each type of coin

	SHARI		**JUSTIN**	
	# of Coins	**%**	**# of Coins**	**%**
Silver Dollars	9			40%
Half dollars	6			10%
Quarters	5			50%
TOTAL			30	

8. Joelle and Kyle started a collection of sports trading cards. The chart below shows how many cards they have collected.

   Complete their charts, showing:

   How many cards they have of each kind

   The percentages for each type of card

	JOELLE		**JUSTIN**	
	# of Cards	**%**	**# of Cards**	**%**
Football	10			25%
Baseball	30			10%

Basketball	5			50%
Hockey	15			15%
TOTAL			40	

Use the information in this chart to solve the next three problems:

School	Number of Students
Valley Middle School	450
Woodbury Middle School	600

9. A survey was done at both schools to find out how much television students watch.

   The survey found that 300 Valley School students watch more than 2 hours of television every night, while only 270 Woodbury School students watch more than 2 hours of television every night.

   - What percentage of Valley students watch more than 2 hours of television every night?
   - What percentage of Woodbury students watch more than 2 hours of television every night?
   - Do more Woodbury students or more Valley students watch more than 2 hours of TV each night?

10. Woodbury Middle School has 250 girls and Valley Middle School has 275 girls.
    Which school has a greater percentage of girls? Explain your answer.

11. At each middle school, 75% of the students play on a sports team.

    - How many Woodbury students play on a sports team?

- How many Valley students play on a sports team?
- A third middle school, Lancaster School, has 300 students on a sport team. This is 75% of the total students at Lancaster. How many total students are at Lancaster School?
- Create three other ratios equal to 75%.

12. The students at Whittier Middle School were raising money to help a local family who lost their home due to a fire.

    Eighth grade had a goal of $300.

    Seventh grade had a goal of $250.

    Sixth grade had a goal of $200.

- At the end of the first week, each grade had raised 40% of their goal. How much money had each grade raised?
- With one day left in the fundraiser, the eighth grade had raised $225, the seventh grade had raised $200, and the sixth grade had raised $150. Which grade was closest to their goal?
- Suppose each grade was only $25 short of meeting its original goal. Calculate the percentages raised for each grade. Which grade came closest to its goal of raising all their money?

13. Technologies Are Us has a $900 computer on sale for 30% off.
    Discount Technology has the same computer priced $800 on sale for 20% off.

    Which store has the better computer deal? How do you know?

14. Anna has 85% of her books on bookshelves. The rest of her books have been borrowed by friends. If there are 76 books on the shelves, how many books are in Anna's total book collection?

15. To raise money for charity, the sixth graders at Fisk School want to sell magazines. 26% of the magazine sales will be donated to charity. If the class plans to donate $400, what is the total dollar amount of magazines they will need to sell?

16. Amy's paycheck for Tuesday is 80% of her paycheck for Wednesday. Her total pay for Tuesday and Wednesday is $144. How much did Amy earn on Wednesday?

17. Kelly's monthly salary is 20% more than Lynn's monthly salary. Kelly's monthly salary is $4,800. What is Lynn's monthly salary?

18. During the lacrosse season, Marissa scored 25% more goals than Patty. The two girls scored a total of 63 goals. How many more goals did Marissa score than Patty?

19. Cailey and David went into a department store during a winter clearance sale. Everything was marked down 20%. Cailey and David each have a coupon for an additional 10% discount as well. Cailey wants to buy a sweater for $60 and David wants to buy a pair of jeans for $60.

   - Cailey first took 20% off the price of her sweater. Then she used her coupon and took the additional 10% off.

   - David used the coupon first to take 10% off the price of his jeans, and then he took the markdown discount of 20% off.
     Which is the better deal? Why? In writing, explain your reasoning and justify your answer. If you owned the department store, how would you want the two discounts to be calculated? Why?

Curriculum Embedded Performance Assessment (CEPA)

Pizza Champions

Overview

Students create a pizza recipe and plan ingredients to make pizza for 12, 60, and 240 students. Students compute unit prices for the ingredients, prepare a budget, and calculate the cost to feed varying numbers of people. Using ratio/rate language, each student writes a proposal to persuade the cafeteria manager to use his/her recipe. Students may work in pairs or small groups to discuss and plan the tasks, but all written work should be completed individually in order to assess each student's understanding of proportional reasoning (ratios, rates, and unit rates).

List of Tasks

CEPA Task 1: Cost to Make One Pizza
CEPA Task 2: Cost to Feed 12, 60, and 240 Students
CEPA Task 3: Proposal to the Cafeteria Manager

Teacher Instructions for CEPA Tasks 1, 2, and 3:

Read the instructions on the handouts aloud and verify that all students understand the directions. (*Note: During the pilot phase, please note whether students used calculators or computers, and record any additional directions/instructions given by the teacher.)

Students may plan and discuss this part of the task with other students. However, they should complete the work independently without help from teachers or peers.

The work produced during each task will be incorporated into the final project. Evaluation of each task will occur when the final product is submitted. All work must be completed during class time. CEPA Tasks 1 and 2 will each take one class period. CEPA Task 3 will take 1-2 class periods. Students are not permitted to take this work home until the project is finished and scored.

Standards Addressed

6.RP.2

Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

6.RP.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

G7 SMP.4

Model with mathematics

WHST.2

Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.
Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples.
Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts.
Use precise language and domain-specific vocabulary to inform about or explain the topic.
Establish and maintain a formal style and objective tone.
Provide a concluding statement or section that follows from and supports the information or explanation presented.

WHST.4

Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

This CEPA also addresses the writing portion of guiding principle 5.

Guiding Principle 5: Literacy across the Content Areas
An effective mathematics program builds upon and develops students' literacy skills and knowledge.

Reading, writing, and communication skills are necessary elements of learning and engaging in mathematics, as well as in other content areas. Supporting the development of students' literacy skills will allow them to deepen their understanding of mathematics concepts and help them to determine the meanings of symbols, key terms, and mathematics phrases, as well as to develop reasoning skills that apply across the disciplines. In writing, teachers should consistently support students' ability to reason and achieve deeper understanding of concepts, and to express their understanding in a focused, precise, and convincing manner. Mathematics classrooms should incorporate a variety of written assignments ranging from math journals to formal written proofs.

CEPA Task I: Cost to Make One Pizza: Directions for Teachers

Materials Required

Calculators or computers with spreadsheet software
Directions for students
Price of selected ingredients

Description

Students create a recipe for pizza and use a price sheet to determine the cost of ingredients for their pizzas.

Using a template, students create a table showing the cost to make one pizza. The first line of the template is partially completed as an example. Students use a spreadsheet or calculator to compute unit costs for the pizza ingredients.

CEPA Task 1: Student Directions

You have created an award-winning recipe for pizza. You must write a proposal to convince the cafeteria manager to use your recipe. The proposal will include the cost to feed 12, 60, and 240 students. To help persuade the manager, you must explain all of the mathematics used to create your budget.

This CEPA involves three tasks.

1. First, you will find the cost to make one pizza using your recipe.
2. Second, you will determine the cost to make pizza for 12, 60, and 240 students.
3. Last, you will write a proposal to the cafeteria manager.

Use the price sheet to determine the ingredients for your pizza.

You may use a spreadsheet or calculator to perform the computations.

Use the template below to make a table showing the cost to make one pizza using your recipe.

The first line is partially completed as an example.

Important information:

Your pizza must include dough, sauce, cheese, and three other ingredients.

1 pound (lb) = 16 ounces (oz)

One 16-inch pizza serves 4 students.

Award Winning Pizza Recipe

(Recipe for One Pizza)

Ingredient	Amount Needed	Package Size	Price	Unit Cost (cost per ounce)	Total Cost for 1 Pizza
Dough	20 oz.	60 oz.	$4.20		

Price Sheet

Ingredient	Amount Needed	Package Size	Price
Dough	20 oz.	60 oz.	$4.20
Pizza Sauce	5 oz.	15 oz.	$1.20
Cheese (shredded)	8 oz.	16 oz.	$3.36
Pepperoni	4 oz.	12 oz.	$5.52
Meatballs	8 oz.	16 oz.	$3.84
Sausage	8 oz.	1 lb.	$4.16
Green Bell Peppers	8 oz.	1 lb.	$1.44
Onions	8 oz.	2 lbs.	$2.56
Mushrooms	6 oz.	24 oz.	$4.56
Tomatoes	3 oz.	12 oz.	$2.88
Black Olives	6 oz.	6 oz.	$1.86

CEPA Task 2: Cost to Feed 12, 60, and 240 Students: Directions for Teachers

Materials Required

Calculators or computers with spreadsheet software
Directions for students
Price of selected ingredients

Description

Students determine the cost to make pizza for 12, 60, and 240 students and make a table or series of tables to show the following.

- the number of students
- how many pizzas are needed
- how much of each ingredient is needed
- the price of each ingredient
- the unit cost for each ingredient
- the total cost for each ingredient
- the total cost (based on unit rates) to feed 12, 60, and 240 students

CEPA Task 2: Student Directions

Determine the cost to make pizza for 12, 60, and 240 students. Remember, one 16-inch pizza serves 4 students. Create a table to show the following.

- the number of students
- how many pizzas are needed
- how much of each ingredient is needed
- the price of each ingredient
- the unit cost for each ingredient
- the total cost for each ingredient
- the total cost (based on unit rates) to feed 12, 60, and 240 students

You may use a spreadsheet or calculator to perform the computations.

Ingredient	Unit Cost for 1 pizza	Pizzas Needed for 12 Students	Total Cost for 12 Students	Pizzas Needed for 60 Students	Total Cost for 60 Students	Pizzas Needed for 240 Students	Total Cost for 240 Students

CEPA Task 3: Proposal to the Cafeteria Manager: Directions for Teachers

Materials Required

Directions for students
Computers with word processing software

CEPA Task 3: Student Directions:

Write a proposal to convince the cafeteria manager to add your pizza to the school lunch menu. Your proposal should include the following.

- the tables you created for Tasks 1 and 2
- an explanation of the information provided in each table
- an explanation of the mathematics you used to create the information in each table
- a convincing argument for choosing your recipe.

Student work will be submitted electronically for scoring at the state-level. Teachers may use the scoring guide below or any other rubric that they choose for their classroom-based scoring.

<table>
<tr><th>Possible Points</th><th>Mathematical Work
CEPA Tasks 1 and 2</th></tr>
<tr><td>4</td><td>Response demonstrates thorough understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.
• The table is clear and accurately labeled.
• The unit costs and/or costs for multiple servings are accurate.
• The amounts and total costs are accurate.</td></tr>
<tr><td>3</td><td>Response demonstrates strong understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.
• The table is mostly clear and accurately labeled.
• Most unit costs and/or costs for multiple servings are accurate.
• Minor errors do not interfere with overall accuracy.</td></tr>
<tr><td>2</td><td>Response demonstrates some understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.
• The table may be somewhat unclear.
• Some unit costs and/or costs for multiple servings are accurate.
• Without additional work, it is difficult to present a solid argument for persuading the cafeteria manager to adopt the recipe.</td></tr>
<tr><td>1</td><td>Response demonstrates weak or limited understanding of computing unit costs and/or costs for multiple servings and displaying the information in a table.
• The table is unclear or mostly incorrect.
• Few or no unit costs and/or costs for multiple servings are accurate.
• Without significant support and additional work, it is impossible to present a solid argument for persuading the cafeteria manager to adopt the recipe.</td></tr>
<tr><td>0</td><td>Response is missing, unrelated to the assigned topic, illegible, not of sufficient length to score, or shows no understanding of the standards addressed.</td></tr>
</table>

Score	Written Explanation (CEPA Task 3)
4	**Proposal demonstrates strong ability to use precise mathematical language to write explanations that are focused, precise, and clear. Tables are detailed, well-organized, and informative. All mathematical computations are correct and clearly explained.** • Writing is exceptionally focused and clear. • Use of mathematical language is precise and sophisticated. • Tables are detailed, informative, and well-organized. • All mathematical computations are correct. • The presentation is highly convincing and thorough.
3	**Proposal demonstrates good ability to use mathematical language to write explanations that are focused, precise, and clear. Tables are mostly well-organized and informative. Mathematical computations are mainly correct and clearly explained.** • Writing is focused and clear. • Mathematical language is used correctly. • Tables contain mostly correct information and are easy to interpret. • Mathematical computations are mostly correct. • The presentation is convincing.
2	**Proposal demonstrates some ability to use precise mathematical language to write explanations that are focused, precise, and clear. Mathematical computations contain some errors and need revision. Some additional work is required to transform this proposal into an effective argument for a particular pizza recipe.** • Writing sometimes lacks focus and clarity. • Use of mathematical language is inconsistent. • Tables may contain some inaccurate information or be difficult to interpret. • With some support, the presentation could be transformed into a convincing proposal.
1	**The proposal demonstrates limited ability to use precise mathematical language to write explanations that are focused, precise, and clear. The tables contain inaccurate information. Significant support and revision are required to transform this proposal into an effective argument for a particular pizza recipe.** • Writing lacks focus and clarity. • Mathematical language is missing. • Tables contain inaccurate information or are difficult to interpret. • The student requires significant reteaching and support to develop a viable argument.
0	**Response is unrelated to the assigned topic, illegible, not of sufficient length to score, or shows no understanding of the standards addressed.**