Real Number System
Numbers, Numbers, ...

ACTIVITY 1.1

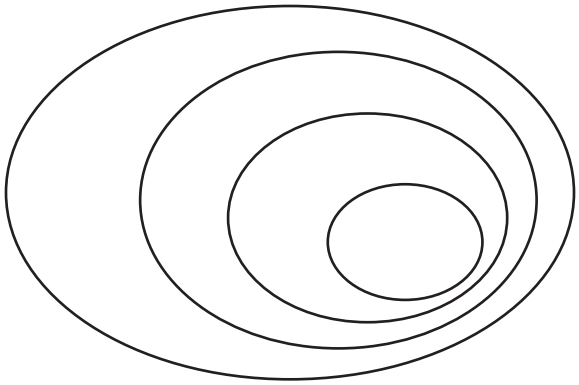
SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Graphic Organizer, Quickwrite, Think/Pair/Share

My Notes

Your teacher will use a Venn diagram to help you understand the relationships between different types of numbers including *rational numbers, integers, whole numbers*, and *natural numbers*.

- Your teacher will ask for volunteers to name numbers to place into the Venn diagram.
- Listen carefully to the numbers chosen. Try to determine how your teacher decides where to place each number in the Venn diagram. (See if you can decide where a number will be placed in the Venn diagram *before* your teacher shows the class.)

1. Your goal is to figure out the description for each part of the Venn diagram. Write the numbers in your diagram as your teacher puts the numbers in the class Venn diagram.



Where did these groups of numbers originate? Archaeological records show that ancient cultures used many different systems of writing numbers. Most of these cultures had symbols for the numbers 1, 2, 3, 4, 5, …. These numbers are called **natural numbers**, or **counting numbers**.

The set of natural numbers can be written as N= {1, 2, 3, 4, 5, …}. A set is a collection of numbers or objects. Writing numbers in brackets and separating them by commas is called **set notation**.

2. Which name, natural numbers or counting numbers, do you think is more descriptive? Why?

WRITING MATH

To show all the numbers in a sequence, an **ellipsis**, which is three periods in a row (for example, 1, 2, 3, 4, 5, ...), is used. An ellipsis indicates a set of numbers that continues on without end.

WRITING MATH

The set of **natural numbers** is represented by the symbol **N**.

ACADEMIC VOCABULARY

set notation

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share

My Notes

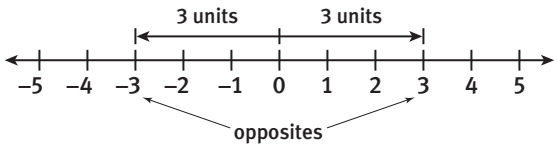
WRITING MATH
The set of **whole numbers** is represented by the symbol **W**.

Notice that 0 is not in the set of natural numbers. Most ancient cultures did not have a concept of the number 0. The Mayans were one of the first to use the concept of 0. Adding 0 to the set of natural numbers forms the set of **whole numbers**.

3. Consider the set of whole numbers and the set of natural numbers. Describe the relationship between the two sets.

4. Use set notation to represent the set of whole numbers.

In mathematics, numbers that are **opposites** are the same distance from zero and have different signs. The numbers −3 and 3 are examples of opposites. On the number line, these two numbers are the same distance from 0 in opposite directions.



MATH TERMS
The set of **integers** is the set containing all natural numbers, their opposites, and zero.

WRITING MATH
The set of **integers** is represented by the symbol **Z**. This symbol was chosen because in German *Zahlen* is the word for numbers.

5. Each element in the set of whole numbers has an opposite. Together, all the whole numbers and their opposites form the set of **integers**.

a. What is the opposite of 52?

b. What is the opposite of –312?

c. What is the opposite of 0?

6. The set of natural numbers was represented using the notation {1, 2, 3, 4, 5, …}. Consider the difference between the set of natural numbers and the set of integers.

a. How would you describe this difference?

b. Write the set of integers using set notation.

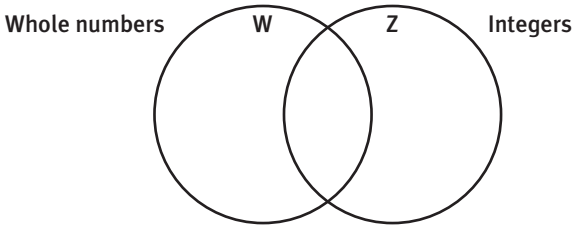
SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Think/Pair/Share, Group Presentation, Quickwrite

My Notes

Venn diagrams can be used to compare the set of whole numbers and the set of integers.

7. Place each number in the appropriate region of the Venn diagram.

5, −1, 0, 3, −12, −11, 15, 23, 2, −9



8. Write one or more sentences summarizing the results in the Venn diagram in Item 7.

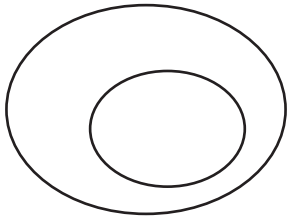
9. Complete this sentence that describes the relationship of the sets in Item 7:

Every _____ is a(n) _____,

but not every _____ is a(n) _____.

10. The Venn diagram below also can be used to compare the set of integers and the set of whole numbers.

a. Place the names of these sets in the Venn diagram and explain why you placed them there.



CONNECT TO HISTORY

The set of numbers known as **integers** was used by ancient Chinese mathematicians more than 2000 years ago. They performed computations by manipulating counting rods—short rods approximately 10 centimeters long—on a table or counting board. Red rods represented positive numbers and black rods represented opposite or negative numbers.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share,

My Notes

b. Compare and contrast the two Venn diagrams. Which do you think is a better representation of the relationship between the two sets? Explain your reasoning.

WRITING MATH

The set of **rational numbers** is represented by the symbol **Q**. The symbol Q was chosen because it is the first letter of *quotient*.

One of the areas in the Venn diagram we used at the beginning of this activity represents the set of **rational numbers**. The set of rational numbers is the set of all numbers that can all be written as a ratio of two integers.

A rational number, x, can be defined using symbols as $x = \frac{a}{b}$, where both a and b are integers and $b \neq 0$. Any rational number may be represented as a fraction, as shown, or as a decimal.

11. Explain why b cannot be 0.

12. If rational numbers are defined as fractions, why can a rational number be represented by a decimal?

MATH TERMS

A **terminating decimal** is a decimal that ends.

A **repeating decimal** is a decimal that has one or more digits following the decimal point that repeat endlessly.

Some decimals are *terminating decimals*. To show that these decimals are rational numbers, it must be possible to express these numbers as the ratio of two integers in fraction form.

13. Write each decimal as a ratio of two integers. Express the answer as a fraction.

a. 0.35 **b.** 0.004

14. Can all terminating decimals be written as fractions? Justify your answer.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Look for a Pattern, Activating Prior Knowledge

My Notes

Some decimals are *repeating* decimals. To show that these decimals are rational numbers, it must be possible to express these numbers as the ratio of two integers in fraction form.

15. Rewrite each rational number as a decimal. Describe any patterns you observe with the quotients.

a. $\frac{1}{3}$ **b.** $\frac{4}{9}$ **c.** $\frac{5}{6}$

16. Now explore some numbers with nines. Study these equalities and describe any patterns that you observe.

$0.\overline{41} = \frac{41}{99}$ $0.\overline{352} = \frac{352}{999}$ $0.\overline{8} = \frac{8}{9}$

17. Consider the numbers in Items 15 and 16. Would these be considered rational numbers? Why or why not?

18. Explain why natural numbers, whole numbers, and integers are all **subsets** of the set of rational numbers?

Irrational numbers are numbers that cannot be written as the ratio of two integers. Some examples are decimals that do not terminate or repeat. These include *pi*, represented by the symbol, π, and the square root of 2, $\sqrt{2}$.

- An approximate decimal value of *pi* is **3.14159265358979323846264338327 95…**
- Pi is often represented by the rounded decimal, 3.14. Pi cannot be expressed as a fraction, $\frac{22}{7}$ is close, but is not exact.
- The approximate decimal value of $\sqrt{2}$ is **1.41421356237309504880168872420 97…**
- This decimal also does not terminate or repeat and a fraction cannot be written to equal its value.

CONNECT TO AP

In advanced math courses you will learn why a repeating decimal can be expressed as a fraction.

MATH TERMS

A **subset** of a set is another set whose elements are all in the original set. Every set is a subset of itself.

TECHNOLOGY TIP

Look at what your calculator shows as a value for π. How is it different from the one on this page?

WRITING MATH

A symbol that is sometimes used to represent the irrational numbers is $\overline{\mathbb{Q}}$. This symbol means "not rational."

My Notes

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share, Graphic Organizer, Group Presentation, Self Revision/Peer Revision

19. Are all square roots irrational? Explain using at least two examples.

ACADEMIC VOCABULARY

The set of **real numbers** is represented by the symbol **R**.

The set of **real numbers** includes all rational and irrational numbers.

20. For each number, place a check in the box for any set of which the number is a member.

Number	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
0.25						
3						
$\sqrt{3}$						
0						
$\sqrt{64}$						
-3						
3.14						
$0.\overline{4}$						
π						
$5\frac{2}{5}$						

21. Create a Venn diagram to illustrate the relationship between real numbers, rational numbers, irrational numbers, integers, whole numbers, and natural numbers.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think/Pair/Share, Self Revision/Peer Revision, Identify a Subtask, Quickwrite, Create Representations

My Notes

Some properties can be helpful when you are solving equations and evaluating expressions with real numbers.

22. Write what you recall about the commutative property. Give an example using only whole numbers in your explanation.

23. Write the meaning of the associative property in your own words. Give an example using only integers in your explanation.

24. Two students, Nick and Nathaniel, used the distributive property to make mental math easier. Look at each student's work and explain what each student did.

Nathaniel: $4\,(100 + 21) = 4 \times 100 + 4 \times 21$

Nick: $3\,(108) = 3 \times 100 + 3 \times 8$

25. Which property states that when you add a number and its opposite, the sum is zero? Write a problem using rational numbers that are not integers to illustrate this property.

26. What do the identity properties state and does this hold true for all real numbers? Give examples to justify your answer.

My Notes

SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share

27. List any properties of real numbers that were used to evaluate each expression.

a. $26 \times 1 = 26$

b. $2(a + b) = 2a + 2b$

c. $14 + 30 + 26 = 30 + 14 + 26$

d. $2 \times (13 \times 5) = (2 \times 5) \times 13$

28. How does using properties of real numbers make it easier for you to do mental math?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Name one number for each description.

1. A whole number but not a natural number.
2. An integer but not a whole number.
3. An integer and a natural number.
4. An irrational number.
5. A rational number but not an integer.

Write the property illustrated in each example.

6. $5 + 0 = 5$
7. $2(3 + 6) = 6 + 12$
8. $2 + 3 + (-2) = 2 + (-2) + 3$
9. $(2 \times 4) \times 5 = 2 \times (4 \times 5)$
10. MATHEMATICAL REFLECTION Describe the set of numbers with which you are most comfortable and explain why. Describe the set of numbers with which you are least comfortable and explain why.