Real Options in Capital Budgeting. Pricing the Option to Delay and the Option to Abandon a Project

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Abstract. *Traditional discounted cash-flows method for assessing projects assumes that investment decision is an irreversible one, which is not correct. Managers can and must reconsider their initial decision as the new information arises during the project life. This is managerial flexibility and it creates strategic value for a project, only if management takes advantage of the opportunities associated with an analyzed project. Real options represent a new approach in capital budgeting, using the theory of pricing financial options for investments in real assets. In this paper, we emphasize the characteristics and valuation methodologies of real options. The objective in the last section is pricing the option to delay and the option to abandon a project in construction materials field.*

Key words: capital budgeting; real options; managerial flexibility, timing options; exit options.

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Introduction

Traditional approaches to capital budgeting, such as discounted cash-flows (from now on DCF), cannot capture entirely the project value, for different reasons: it is assumed that investment decision is irreversible, interactions between today decisions and future decisions are not considered, and investment in assets seems to be a passive one (management doesn't interfere during the life of the project).

Managerial flexibility generates supplementary value for an investment opportunity because of managerial capacity to respond when new information arises, while the project is operated. Investment in real assets includes a set of real options that management can exercise in order to increase assets value (under favorable circumstances) or limit loses (under unfavorable situations).

Managerial flexibility in decision-making process introduces an asymmetry for probability distribution of net present value (from now on NPV) for a project. An investment opportunity value is dependent on future uncertain events, so therefore, it will be greater than forecasted value in the situation of passive management.

From this perspective, a project has a standard value, determined through traditional techniques (DCF, which does not catch adaptability and strategic value), but also a supplementary value, coming from operational and strategic real options held by an active management.

Decision trees are related with real option approach by recognizing their existence, but investment value is calculated as an average of expected cash flows (positive or negative) weighted with probabilities associated with

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each state. Real option appraisal eliminates potential loses by abandoning the project when circumstances are unfavorable (negative cash flows become zero) and adjust discount rate to reflect the new level of risk.

Real options represent an integrated solution used under uncertainty, by transposing the theory of financial options to valuation of real assets, projects or even companies, in an uncertain and dynamic environment, where taking decisions must be a flexible process.

Specific features of real options

Unlike financial options, real options have as underlying asset a real asset, which value is given by discounted cash-flows plus the value of any other options associated with the ownership of that real asset (Bruun, Bason, 2001, p. 1).

Real options are defined and valued by analogy with financial options, but they have some specific features (Trigeorgis,1996, pp. 127-129) that make distinction between the two categories: *not exclusively owned by any investor* (to assure the exclusive rights for a project, someone must identify and use the competitive advantages and raise substantial barriers to entry for other competitors); *nontradable securities* (there are no financial markets where these rights could be traded, the only two alternatives being: exercising option or giving up exercising); *preemption for the investor who undertake the project*, despite that at the beginning the real option was jointly held by all firms involved in that industry; *option compoundness*, consisting of interdependencies inside a project or between projects, depending on exercising other options or taking other projects.

Black and Scholes (1973, pp. 637-657) identified the elements affecting the theoretical price of an option in their formula: stock price (S), exercise price (E), time to expiration (t), variance of returns (σ) and risk free rate (r). Afterwards, Merton (1973, pp. 141-183) completed the Black & Scholes formula (from now on BS) with the sixth element, dividends (δ). Models assessing financial options could be extended for real options because of the analogy between financial and real options, concerning the elements influencing their value (Figure 1)[1].

Traditional methods like DCF and NPV cannot catch the flexibility because they focus only on two components of value creation: discounted payoffs and investment cost. Real options capture the influence of all six elements described before. That means a *reactive management*, consisting of response from the managerial team (through decisions they take) to the cumulated information during operating the investment, under uncertainty.

The *proactive management* of flexibility (Leslie, Michaels, 1997, pp. 12-14) assumes, besides identifying the real options embedded in a project, fairly appraising, exercising them to seize the opportunities and taking decisions for increasing their value, for maximizing project value and shareholders' wealth as well.

A prospective investor (the owner of a real option) cannot control all the components of value creation because of internal or external constraints (such as technical, marketing, competition). Maximizing option price can be obtained by focusing on those elements that indeed can be influenced and confer competitive advantage, keeping in mind the constraints.

Difficulties in using financial options pricing models for real options valuation

There are certain limits for using financial options pricing models for real options because the first category has standard components, which could be easily identified (part of them are clearly specified - maturity, exercise price, or can be observed in the market - price of underlying asset, risk free rate), while the second category do not have such standardization and managerial flexibility determine their pattern. Some of these potential inconveniences, along with solutions proposed in literature on real options are synthetically presented further on (Bruun, Bason, 2001).

Risk neutrality

Also Cox, Ross & Rubinstein (from now on CRR) binomial model and BS model make the assumption that investor creates a replicating portfolio with constant underlying asset price (S), which is risk neutral for an instant of time. A new value for S means changing portfolio structure because hedging ratio is not the same anymore.

Real assets are not frequently traded in a specific market and their price is difficult to observe and is almost impossible to form a replicating portfolio. Under these circumstances, the result of risk neutral valuation is not

Financial option (CALL or PUT)	Variable	Real option (investment opportunity)
Stock price	S	Discounted cash-flows for the project
Exercise price	E	Capital expenditure
Time to expiration	t	Period of time that exercise decision may be deferred
Risk free rate	r	Time value of money
Variance of returns	σ	Risk for assets of the project
Dividend yield	δ	Lose of value by deferring investment decision

Figure 1. *Analogy between financial options and real options*

applicable anymore and it is not possible to use risk free rate as discount rate.

Lander and Pinches (1998, pp. 537-567) proposed either usage of discount factor corresponding to a traded security with similar risk pattern like underlying asset, or estimation of discount factor by applying an asset pricing model such as CAPM.

The underlying asset

Any investor does not exclusively own this. Option price depends on the level of competition in that industry and on the respond of competitors to new challenges. A study by Smit and Ankum (1993, pp. 241-250) concluded that high growth rates for an industry would bring new competitors, so the rate of return will be closer to the cost of capital and the value of held option will be smaller.

The value of underlying asset is represented by present value of projected cash flows and is calculated from NPV for a project (the so-called static or standard value).

Willner (1995) built a model with discontinuous changes in the value of underlying asset:

$$P(t) = P(0) \times e^{\mu \times t} \times \left(\frac{\lambda}{\lambda - 1} \right)^{Q \times t}$$

The model is working under certain assumptions:

- the value of underlying asset increases with a constant exponential rate, as a result of value creation through research activity (except the occasional up jumps owing to new discoveries and down jumps generated by the entry of new competitors on the market);
- the up jumps will not bring the expected payoffs because the new discoveries will attract more competitors and some loses occur from this situation; decrease in value can be projected from historical data for similar projects;
- the new discoveries are not correlated with market or with entire economy, and investors are not confronted with systematic risk;
- the investment cost (exercise price) is fixed.

The model may be successfully used for assessing multistage projects or start-up ventures.

The exercise price

This is not fixed (it is not established from the very beginning) and do not follow a deterministic process. It should be found a mutual probability distribution for value of underlying asset and exercise price to determine the level and direction of correlation between the two variables.

Schwartz și Moon (2000) developed a model with a stochastic process for the exercise price. The level of uncertainty is conversely proportional with investment level, which means that uncertainty about initial cost could be eliminated only by undertaking the project.

Pindyck (1993) identified two sources of uncertainty about investment cost: a technological one, eliminated only in the moment when the decision to invest is taken, and an economic one (related with the changes in costs of entries for the project, such as materials and labour). The author considered a stochastic evolution for investment cost, given by the following equation:

$$dK = -Idt + g(I, K)dz,$$

which means that exercise price (K) decreases as investment proceeds and fluctuates along with the two sources of uncertainty (technical and cost of entries for the project).

Time to maturity

Exercise date for real options is not a priori specified and it is uncertain because of exogenous factors, such as competition or barriers to entry. Optimal exercise time depends on various elements:

- the greater the technological uncertainty, the greater the uncertainty concerning exercise date;
- the stronger the competition and the weaker the barriers to entry for an industry, the sooner the exercising of the option (to take advantage on preemption), although a delay for the project has some benefits, because in the mean time suplimentary information occures;
- the patents or licences owned by investor protect him against competition and permit him not to hurry with exercising the option.

Risk free rate

The discount rate is the rate of return for a riskless security, with the same maturity as the real option. But exercise date for the option is uncertain, so is quite difficult to identify the appropriate discount factor. More than that, rate of return is not constant over time. The solution is similar with previous cases: rather considering a stochastic evolution for discount rate than a deterministic one.

The volatility of the underlying asset

Volatility is the only element not directly observable for financial options and it has to be estimated for the entire period until maturity. If the underlying asset is traded, the historical variance is extrapolated for future periods. If the underlying asset is not traded (the case of real options), volatility is difficult to appraise. Luehrman (1998) suggested some alternative solutions. First, the observed risk for a financial market index could be used as a proxy for the project risk (adjusted with the level of individual, specific risk). Second, the volatility could be estimated from historical data regarding similar projects from related industries. Third, volatility could be obtained from probability distribution of projected cash flows, when applying Monte Carlo simulation.

Another problem is that volatility is not constant over long periods, but this inconvenient could be eliminated by applying a GARCH model, if possible.

The dividend yield

It is represented by lose of cash flows until maturity of the real option (for example, by deferring the project).

Pricing real options in capital budgeting - the option to delay and the option to abandon a project

A various range of real options are identified and used as powerful instruments in capital budgeting: *timing options* assume that investor may postpone the investment decision until specific information arise and help him to understand, even partly, the uncertainty connected with the analyzed project; *staging options* are very useful for assessing multistage projects, when uncertainty is not resolved over time and investor must undertake the project even in small increments, in order to learn about cost pattern and profitability of the project; *exit (abandon) options* allow investor to avoid or at least reduce loses if bad circumstances appear, by turning negative cash-flows into null payoffs; *operating options* enable the firm to organize operations for adjusting its processes to business environment and react to economic changes by scaling up to enhance earnings or scaling down to reduce damages, depending on given circumstances; *flexibility options* consist of purchasing or building a flexible production capacity or asset, so that it has two or more different uses, depending on market conditions; *growth options* are usually associated with strategic investments, which sometimes have negative NPV, but are indispensable for implementing following projects with substantial positive NPV, greater (in module) than loses from the initial project.

The objective of this section is to explain in detail the working mechanism and asses two types of real options from the categories discussed above, namely the *option to delay* and the *option to abandon* a project, with wide applications in capital budgeting.

Pricing the option to delay a project

On the one hand, the possibility of deferring a project is significantly valuable because investor needs more time to learn about uncertain variables of the project. On the other hand, this value is diminished with the lost cash flows for period that investment was deferred. So the option is exercised only if potential earnings from delay exceed loses in such a situation.

Projects with negative NPV (initially rejected from the point of view of traditional investment analysis) could turn to positive NPV in the future. Decision to postpone a project is equivalent to holding a CALL option, which provides the right, but not the obligation, to undertake the project sometimes in the future, when the holder decides to do so. Even profitable projects from the very beginning (with positive NPV) acquire a plus of value by delay of the project, if conditions related to competition, barriers to entry or exclusive rights for a product or technology allow that.

Pricing an option to delay a project requires identification of the variables (Damodaran, 2002) in the model:

- *underlying asset* - is represented by the project itself; its value (S) is calculated with DCF method;
- *exercise price* - is the cost of implementing the project; the model is working under assumption that this is constant in real terms and it is affected only by inflation; its evolution is deterministic, but not stochastic;
- *time to maturity* - is established as the period of time that investor enjoy of exclusivity for the analyzed project or at least has an important competitive advantage which allow him deferring the project without risking its achievement by another firm;
- *risk free rate* - is represented by the expected rate of return for a riskless security (treasury bill or treasury bond), with the same maturity with real option;
- *volatility of the underlying asset* - it appears because of the errors associated with estimation of the financial cash-flows and the value of underlying asset and it is the most difficult element to appraise because the underlying asset is not traded. Monte Carlo simulation is used for

assessing variance, because the distribution of probability for the components of cash flows (size of the market, market share) is defined and random numbers (normally distributed, for example) are generated for the specification errors of the variables in the model. Different scenarios for expected DCF result from here. Rolling a great number of simulations (it is recommended to do at least 3,000 iterations), the standard deviation of DCF is obtained, and then it is used as a proxy for volatility of underlying asset in the option pricing model.

■ *dividend yield* - delay of investment generates lose of cash-flows for each year. There are two situations:
  – if annual cash-flows are evenly distributed in the period until maturity of the option (n years), the lost value for the first year is 1/n and it increases with time (1/(n-1) in the second year, 1/(n-2) in the third year and so on);
  – if annual cash-flows are not evenly distributed, the cost of postpone the project with one year is given by the following formula:

Cost of delay = (PV of CF $_{in\ the\ future}$ −
− PV of CF $_{at\ the\ current\ moment}$) / PV of CF $_{at\ the\ current\ moment}$

After identifying all these six elements, all we need is to apply a pricing option model (BS or CRR), but also keep in mind the difficulties emphasized in the previous section.

Users prefer BS model because it consists in a simple formula:

CALL option price =
$$= S \times e^{-\delta \times t} \times N(d_1) - E \times e^{-r \times t} \times N(d_2)$$

PUT option price =
$$= -S \times e^{-\delta \times t} \times N(-d_1) + E \times e^{-r \times t} \times N(-d_2)$$

where:
$$d_1 = \frac{\ln(S/E) + [(r - \delta) + 1/2 \times \sigma^2] \times t}{\sigma \times \sqrt{t}}$$

and

$d_2 = d_1 - \sigma \times \sqrt{t}$ and N(d) represents the cumulated probability of normal distribution.

Binomial model requires laborious calculus because valuation proceeds iteratively backwards, from the last period to the current time moment (the more periods until option maturity, the more complex the determination of option price).

More than that, binomial model considers only a finite number of periods (it is a model in discreet time). It leads, at limit, to BS model (in continuous time), which unlike the first one, assumes that time to maturity is divided into an infinite number of periods. That is why the value derived from BS formula is always smaller than value provided by binomial model (but close to it). To apply CRR model, when the values of the six variables for BS model are known, Damodaran proposes two formulas[2]:

u (the "up" factor)   = $e^{\sigma \times \sqrt{dt} + (r - \sigma^2/2) \times dt}$

d (the "down" factor) = $e^{-\sigma \times \sqrt{dt} + (r - \sigma^2/2) \times dt}$

where dt = 1/number of periods from a year, until maturity.

We stressed before that option value is reducing as time passes (the loss in value measured by dividend yield is enhancing). Repeating the calculus for delay option price, an equilibrium point can be determined, where strategic NPV (including option premium) decreased and became equal to standard NPV. This is the moment for exercising the option (by investing), only if, of course, NPV is positive. Beyond this point, deferring does not create value anymore, but destroy part of it. Therefore, the real duration of delay option is always shorter than the initially projected one (because of competition, appearance of similar products).

Pricing the option to abandon a project
The possibility to renounce to a project under unfavourable circumstances represents, in fact, another real option for investor. The option to abandon (a PUT option this time) will be exercised if DCF generated subsequently by the project are even negative or positive, but inferior to salvage value gained after project abandonment.

Value of the underlying asset is represented again by estimated DCF and volatility could be assessing with the same tools as before (similar traded companies from industry, simulations). Option maturity is the period for adopting abandonment decision and it may coincide with time remaining for operating the project.

Exercise price is the salvage value obtained from abandonment of investment, estimated at the current

moment of time. Pricing model is applied under the hypothesis that real assets do not depreciate (salvage value is fixed over time). Lose of value is quantified through dividend yield and it is determined[3] as: $\delta = 1/period$ remaining from the project life.

The actual value from abandonment may be inferior to the estimated one, from different reasons: there is not demand for such a technology or an organized second-hand market is not operational.

Until now, we implicitly supposed that liquidation value is positive. There are situations when from abandonment does not result something or worse, there is a cost to cover for. In this case, the investor will renounce to the project only if loses from still operating the project are bigger (in absolute value) than the costs implied by liquidation procedure.

The price of option to abandon a project increases if investor builds a flexible operational structure from the very beginning, which allows him to take easier the decision concerning abandonment. This objective can be accomplished by employing labor for limited time, by renting or taking in leasing the fixed assets, by choosing a more expensive, but flexible technology.

Case study: pricing of real options - option to delay and option to abandon a project

Investment presentation and assumptions[4]

A construction company from Bucharest analyses the opportunity to invest in a project for producing metallic tiles for roofs (a green field project). There will be obtained two types of products (A and B), with different technical features. The firm will install an annual production capacity of about 1,500 thousands pieces (type A) and 500 thousands pieces (type B). The life of the proposed project is ten years (2007 - 2016). The cost required by implementing the project is 3,670 thousands USD, consisting of land, buildings and technology, and it will be entirely financed with equity capital. Salvage value, estimated for the end of exploitation period of investment (including tax shields), is 1,485 thousands USD.

A rigorous and detailed analysis of the project (financial analysis of the company, investment cost, European and Romanian market of metallic tiles and perspectives for growth rates, market share, sale prices, cost components, forecasted inflation rate, fiscal implications, discount rate) was performed before and it doesn't make the object of this study. That's why the hypothesis for assessing free cash-flows are concisely presented, to create the basis for identifying and pricing real options associated with this investment. Estimation of free cash flows is done in American dollars (USD) because some cost elements are specified in USD.

Value of sales for the two categories of products, on internal market, is of approximately 4,000 thousands USD. The annual growth rate, in real terms, is estimated to 20-22% for the first two years, 10% for the next four years, respectively 1-2% for the last four years. This foreseeing represents the combined results of estimations for market size (as quantity) and unit sale prices (in real terms) for the two categories of products, for the entire life of the project. The firm wants to achieve a market share of 20-30% in the first two years, following an increasing of this percent with ten percentage points every year, until production capacity is completely utilized (according to forecasting, this event will occur in the sixth year of operating).

The expenditures were appraised under subsequent assumptions: unit variable cost represents 60% of turnover (in the first two years), then it enhances annually with two percentage points until it reaches 70%, which is kept constant afterwards; fixed costs (others than amortization) are established to a level of 80 thousands USD (in real terms) and amortization is about 166 thousands USD (in nominal terms).

Projected inflation rate is 2% per year (for USD) and nominal discount rate is 18.5%. There are also used in calculations a corporate income tax rate of 16% and an average of 30 days for assessing working capital investment.

Investment ratios under certainty

Keeping in mind the assumptions described in the previous subsection, we have estimated annual turnover and operational expenses (in nominal terms), then financial margins. It was also taken into account the tax shields provided by the recover of financial loses from the first year in the following years (when profits are obtained).

Finally, nominal free cash flows were appraised and discounted with nominal cost of capital (in fact, a cost of equity capital). The result is an NPV of 1,297 thousands USD. Profitability index is greater than one (1.34), indicating that the investor recovered all expenses and gained a net discounted profit of 34% from invested capital. Internal rate of return is 25% and exceeds the cost of capital (18.5%). The discounted payback period is about 4.5 years.

All these calculations lead to conclusion that investor must accept the analyzed project and they are exhibited in Table 1.

Investment ratios for the project under certainty

- thousands USD -

YEAR	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
FCF=NOPAT+Amo−ΔImo−ΔWorkCap	−3,777.6	446.8	575.6	779.5	1,059.7	1,366.0	1,860.9	1,761.3	1,735.7	1,695.0	3,681.5	
Cost of capital (nominal)		18.5%	18.5%	18.5%	18.5%	18.5%	18.5%	18.5%	18.5%	18.5%	18.5%	18.5%
Discount factor		118.5%	140.4%	166.4%	197.2%	233.7%	276.9%	328.1%	388.8%	460.7%	546.0%	647.0%
DCF	5,074.84	377.1	409.9	468.4	537.4	584.6	672.1	536.8	446.4	367.9	674.3	
NPV	1,297.3											
IRR	25.0%											
Profitability index	1.34											
Discounted payback period	4.5 years											
PV of future CF (delay of the project)	4,563.55		420.9	373.3	445.4	510.5	579.9	573,4	446,4	367.9	304.5	541.3

Pricing the option to delay the project

The sensitivity analysis presumes that there are certain intervals for possible values taken by variables from free cash-flows model, and thus investment analysis is transposed in an uncertain environment. Strong arguments (related with projected trend for construction and construction materials market, competition level, evolution of imports and currency exchange rate, using similar products in consumption, fiscal policy regarding corporate income tax, firm policies for marketing, employment and wage system, and evolution of utilities costs) help us to establish the inferior and the superior limits for these intervals. The new values for NPV of the project are pictured in Table 2 and they are graphically illustrated in Figure 2.

Sensitivity analysis – NVP

- thousands USD -

| Variable	-50%	-30%	-20%	-10%	-5%	0%	5%	10%	15%	20%	30%	50%
Market size				921.4	1,122.7	1,297.3	1,440.8	1,570.9	1,700.2			
Market share		−48.1	451.5	921.4	1,122.7	1,297.3	1,440.8	1,570.9	1,700.2	1,829.5	2,084.3	
Unit sale price (real terms)				−177.4	561.1	1,297.3	2,033.4					
Unit variable cost (real terms)					1,792.4	1,297.3	802.1	307.0	−189.7			
Fixed expences (real terms)			1,362.1	1,329.7	1,313.5	1,297.3	1,281.0	1,264.8	1,248.6	1,232.4		
Days for working capital	1,434.6	1,379.7	1,352.2	1,324.7	1,311.0	1,297.3	1,283.5	1,269.8	1,256.0	1,242.3	1,214.8	1,159.9
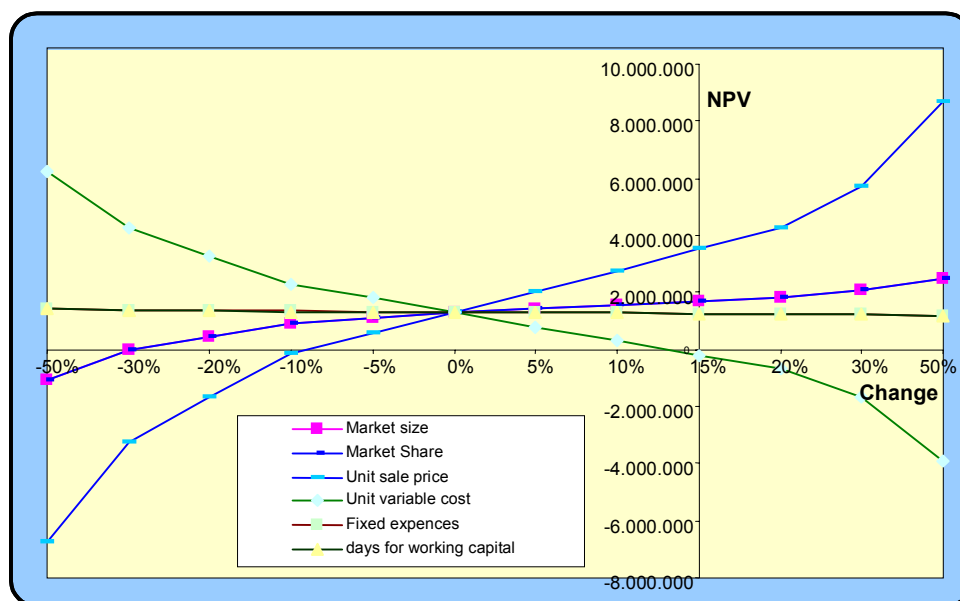
Figure 2. *Sensitivity analysis - Change in NPV*

It is easy to notice that the project is very sensitive to little changes for unit sale price, with a great variance of the results. A decrease for price of only 9% means negative NPV. This situation is possible to achieve, because the value is placed inside the interval of variation, which is (-10%, +5%). A negative NPV is also obtained for a fall of 30% in internal market size or in market share of the new firm. Treated independently, the two variables induce the same effect, because their product represents the quantity sold by the company. The value of 30% is even the inferior limit for market share interval (-30%, +30%), but is outside the interval for market size (-10%, +15%). Increasing the variable costs with at least 13% determines a negative NPV, but the value is situated outside the interval (-5%, +10%), even it is close enough to its superior limit.

To gather supplementary information about unknown variables (sold quantity, market share, unit sale price), the company considers the possibility to delay the investment decision, which means that investor owns a CALL real option, appraised in this section. The first step is to asses the elements from BS and binomial option pricing models. The value of the underlying asset is represented by discounted cash flows of 5,074.84 thousands USD and exercise price is the cost of implementing the project of 3,670 thousands USD (constant in real terms).

Time to maturity is strictly related with the period while investor can maintain the advantage of the first Romanian producer in this market consisting entirely of imports. As we mentioned before, the company does not exclusively own this option, because other investors could be equally interested of such a project. As construction industry is growing, is expected that construction materials industry follow the same trend, so it is a great probability that new competitors entry on this expansive market. Using of top technology and high level of investment needed are significant barriers to entry in this industry, offering a supplementary advantage for investor. Therefore, under given circumstances, we appreciate that deferring the project with more than 3 years could drive to lose of preemption right.

Risk free rate is assimilated to the rate of return for government bonds with maturity in 3 years. Because the Public Finance Ministry in Romania did not issue any more US dollars denominated bonds, we use as a proxy the rate of return for treasury bills with maturity at 3 years, issued by United States Treasury (4.7% at the end of year 2006) plus the country risk premium for Romania. International rating agencies noted Romania, for government bonds denominated in a foreign currency with BBB (Fitch Ratings[4], last modified in august 2006), Baa3 (Moody's[5], improved in October 2006, from Ba1) and BBB- (Standard & Poor's[6]). USA rating is Aaa or AAA that is the greatest note for all agencies. Country default spread is zero for government bonds noted with Aaa and 2% (or 200 points) for Baa3[7]. In conclusion, for the binomial model we use an interest rate (a so-called risk free rate, composed of the two elements) of 6.7% (in discreet time), corresponding to a rate of 6.486% for BS model (in continuous time).

Volatility of the underlying asset is estimated from the distribution of probability of DCF from Monte Carlo

simulation (4,000 scenarios were included). For every rolling of simulation is generated the distribution of probability for NPV and the present value of free cash-flows PV(FCF). Rolling subsequently the simulation for many times, the average for natural logarithm of discounted free cash-flows - E(ln(PV(FCF))) is situated between 15.4 and 15.5, and standard deviation ó(ln(PV(FCF))) falls between 14.56 and 15.54. We could be tempted to use the maximum value for ó (the maximum level of risk), but we know that the greater the volatility of the underlying asset, the greater the value of CALL option. That is why we decide to use an average of the two values, which is $\sigma = 15\%$, corresponding to a dispersion $\sigma^2 = 0.0225$.

Because the financial flows generated by investment are not evenly distributed during the life of project, dividend yield is calculated with the following formula:

Cost of delay = (PV of CF $_{in the future}$ - PV of CF $_{at the current moment}$) / PV of CF $_{at the current moment}$.

Present value of cash-flows (in the future) in the situation of deferring the project with one year is determined under some assumptions: the forecast for exogenous variables is extended with one year to 2017 (market size and unit sale prices remain the same in 2007, even the company doesn't invest, projected inflation rate is still 2%, and corporate income tax is unchanged 16%); for endogenous variables, strictly related to firm features and policies, the foreseeing is lagged with one year, the life of the project falls now between 2008 and 2017 (for instance, market share for the first year is again 25%, but it is applied now to another level for market size, so the quantity sold in the first year is different from the initial scenario). The variable expenses have the same weight, but they are computed from other values for turnover. Amortization has the same value in nominal terms, but the amortization process is also lagged with one year. We do not forget about loses recover procedure (needed for financial lose in the first year of operating) and investment in working capital when we recalculate the discounted cash flows. The result is PV of CF $_{in the future}$ = 4,563.55 thousands USD and dividend yield = -10.075% (the sign indicates that this is a cost).

BS model for pricing the CALL option to delay this project leads to following results: $d_1 = 0.96296$ and $N(d_1) = 0.83222$, $d_2 = 0.70315$ and $N(d_2) = 0.75902$, so *CALL option price is 828.66 thousands USD*.

We can also use binomial model to asses the option, but value of the underlying asset changes this time only once per year, so dt = 1, the "up" factor u = 1.2284 and the "down" factor d = 0.9101. Investor may exercise the option

in each year until maturity (three years), which means that this is an American CALL option. Dividend yield diminishes the price of the underlying asset in each point of the binomial tree. For an example, a value of the underlying asset of 5,074.84 thousands USD become, after one year, S (u - ä) = 5,722.88 or S (d - ä) = 4,107.09 and so on (Figure 3).
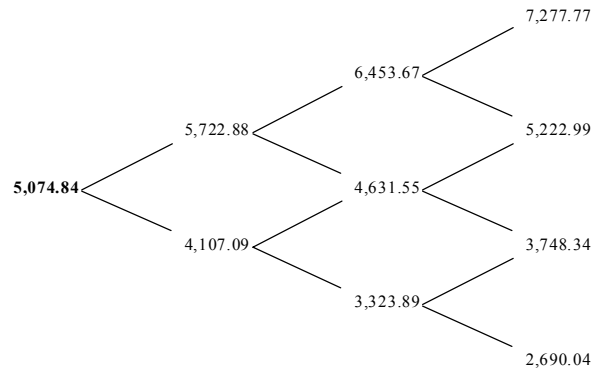
Figure 3. *The binomial tree for the value of the underlying asset*

Risk neutral probabilities are p = $\frac{1+r-d}{u-d}$ = 49.29% and 1-p = 50.71%.

Option pricing is done for every point of the binomial tree, starting with the third year and going backwards to the current moment of time. As an example, we picture the binomial tree for discreet time value of option to delay this project, in Figure 4.
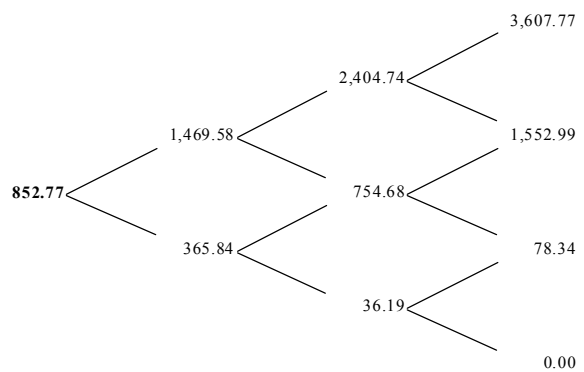
Figure 4. *The binomial tree for the CALL option to delay the project*

CALL option value from the binomial model is *852.77 thousands USD*, bigger than the value obtained from BS model (828.66 thousands USD), because we divided the time until maturity in a very small number of periods (only three). We also built a tree with 12 periods (changing

the value of the underlying asset every trimester). Trimestrial rate of return is 1.68%, u = 1.0726 and d = 0.9232, risk neutral probabilities are p = 0.6262 and 1 - p = 0.3738, and trimestrial dividend yield is 2.51875%. The value of CALL option is now 847.57 thousands USD, smaller than in the situation of annual change for the underlying asset, but still not so close to the value from BS model. Despite of this, unlike other securities traded continuously, the underlying asset for a real option is always represented by discreet value of DCF (usually, estimated once a year).

For pricing financial options, binomial tree is a reliable tool only if year is divided in a large number of periods. However, for real options, a binomial tree with annual change for price of the underlying asset is more appropriate than BS model. For this reason, we say that CALL option to delay the project has a maximum value of 852.77 thousands USD (representing 23.2% of capital investment), but could not be smaller than 828.66 thousands USD (which means 22.6% of project value).

Pricing the option to abandon the project

We can imagine scenarios for correlated evolution of variables from the free cash-flows model with negative NPV for the project. The possibility to abandon the project if one of these pessimistic scenarios occurs becomes valuable. Therefore, the investment has attached a PUT option to abandon.

The price of the underlying asset is again the present value of projected cash flows and that is 5,074.84 thousands USD. Exercise price is represented by the salvage value from the abandonment, estimated at the current moment of time by cumulating market values for land, buildings and technology (the value is fixed over time, in real terms). Experts of real estate industry appraise the market value of land to 360 thousands USD.

Buildings have a long life period (legal amortization period is 40 years) and we can take their production cost (440 thousands USD) as a proxy for the market value after 10 years (no matter that the book value decrease with every passed year). Because the company constructed the buildings itself, the production cost is, for sure, smaller than their market value. Experts estimate that market value is at least 660 thousands USD, with 50% bigger than accounting value.

Market value for equipments and installations is hard to asses, because it is an inflexible and specialized technology, and there is not an active second-hand market for this kind of equipment. That is why we consider (subjectively, of course) that our old technology worth at least 1.600 thousands, approximately half of the price for a similar new technology. Doing so, we do not risk overestimating the salvage value, because for a PUT option, the bigger the exercise price, the bigger the price of the option.

The time to maturity is thought to be 10 years, because investor may decide to abandon the project anytime during the exploitation period.

Risk free rate is equal to rate of return for treasury bonds with 10 years maturity (issued by US Treasury), which was 4.67% at the end of year 2006, plus country risk premium for Romania, estimated to 2%. The result is a discount rate of 6.67% for the binomial model (in discreet time) and 6.457% for BS model (in continuous time).

Volatility of the underlying asset is the same with dispersion obtained in Monte Carlo simulation $\sigma^2 = 0.0225$. The dividend yield is $\delta = 1$/the remained period from the life of the project = 10%.

Pricing the PUT option to abandon the project with BS model generates the following results: $d_1 = 0.88401$ and $N(-d_1) = 0.18835$, $d_2 = 0.40966$ and $N(-d_2) = 0.34103$, and *PUT option value is 116.82 thousands USD*.

We also built a binomial tree for the underlying asset for 10 years, with dt = 1 (annual change for the discounted cash-flows of the project). The multiplying factors are u = 1.2281 and d = 0.9098. This PUT option is also an American option and it may be exercised anytime in the period of 10 years until maturity. Risk neutral probabilities are p = 49.3% and 1-p = 50.7%. Option pricing is done by calculating its value in every single point of the tree, backwards from the tenth year to the current moment of time.

PUT option value from the binomial model is *142.5 thousands USD*, bigger than the value obtained from BS model (116.82 thousands USD), for the same reasons reported before, for the option to delay the project. So we consider that PUT option to abandon the project has a maximum value of 142.5 thousands USD (representing 3.9% of capital investment), but could not be smaller than 116.82 thousands USD (which means 3.2% of project value).

Conclusions

The first conclusion is obvious: pricing real options associated with the investment of producing metallic tiles conduct the investor to recommendation of adopting the project. NPV was positive even before identifying and pricing the real options. Option to delay and option to abandon bring a supplementary value of 26% of invested capital, and represents an enhancement of approximately 75% for NPV, in comparison with situation when real options were ignored.

The option to delay the project has a significant value, because uncertainty associated with this investment is greater than investor thought it would be. Sensitivity analysis reveals that the project is extremely sensitive to changes in unit sale price and unit variable cost. On one hand, to postpone the investment decision is a valuable opportunity because new information arises while time passes. On the other hand, deferring the project exposes the company to potential competition. For such investments with multiple sources of uncertainty, the only solution is to undertake the project.

Option to abandon the project has a small price, because of the influence coming from the reduced value for volatility (ó is only 15%) and from undervalued exercise price (these two elements diminish the price of a PUT option). If abandonment appears in the first two or three years, this value is certainly bigger.

The value for the underlying asset is taken from project appraisal in a certain environment, using the same discount rate of 18.5%. But real options embedded in a project reduce the risk of investment, and we must use a smaller discount rate, so the value for the underlying asset increases, which implies that the price of option to delay enhances, while the price of option to abandon decreases.

Real options are reliable tools for capital budgeting only if they are used for complex strategic projects. Real options are used rather to conceptualize projects than to appraise them, to change organizational structure in a company, needed to gather the maximum potential from a project.

In conclusion, using real options is more related with company management than with valuation methodology. Applying real options successfully assumes training the managerial team in real options spirit. Managers must detach from decisions adopted only with DCF or NPV and to be ready to renounce to a project if it is proven to be more efficiently that way.

Notes

(1) Source: Mauboussin M., 1999, pp. 8 și Bruun S., P. Bason, 2001, „Essay One: What Are Real Options?", pp. 5 (abordare preluată și adaptată din Timothy A. Luehrman, 1998, „Investment Opportunities as Real Options: Getting Started on the Numbers", *Harvard Business Review,* July – August, pp. 51 – 58.

(2) Damodaran A., 2002, Chapter 5: „Option Pricing Theory and Models" și Capitolul 28: „The Option to Delay and Valuation Implications". A similar formula also in Richard A. Brealey and Stewart C. Myers, 1996, „Principles of Corporate Finance",

Fifth Edition, The McGraw–Hill Companies Inc., New York, pp. 598, where $u = e^{\sigma \times \sqrt{dt}}$ și d = $e^{-\sigma \times \sqrt{dt}}$, and d = 1/u.

(3) See Damodaran A., 2002, Chapter 29: „The Option to Expand and Abandon: Valuation Implications"

(4) Source: www.fitchratings.com, Fitch Ratings Ltd., New York

(5) Source: www.moodys.com, Moody's Investors Service

(6) Source: www.standardandpoors.com, Standard & Poor's, The McGraw–Hill Companies, New York

(7) Source: www.sjsu.edu/faculty/watkins/countryrisk, according the data given by Moody's

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