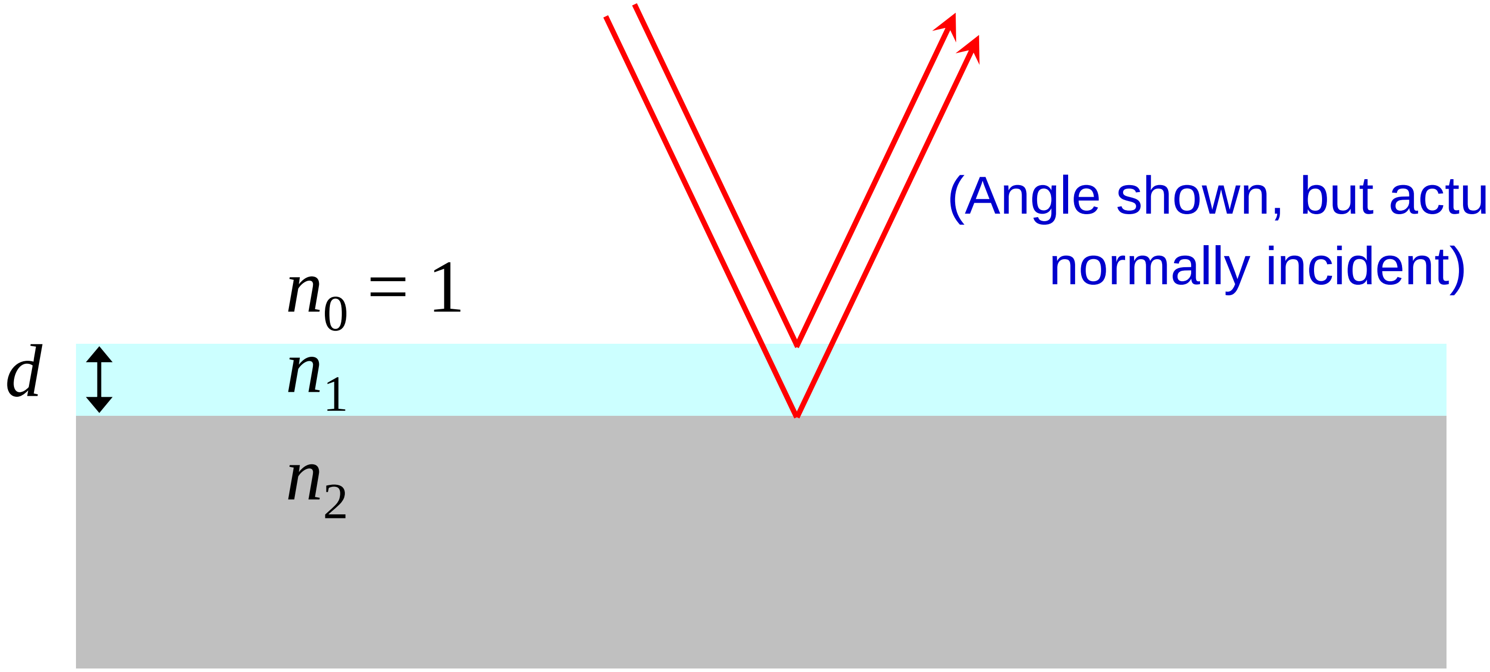
Reflection and Interference from Thin Films

- ➔ Normal-incidence light strikes surface covered by a thin film
  - ◆ Some rays reflect from film surface
  - ◆ Some rays reflect from substrate surface (distance d further)
- ➔ Path length difference = 2d causes interference
  - ◆ From full constructive to full destructive, depending on λ



(Angle shown, but actually normally incident)

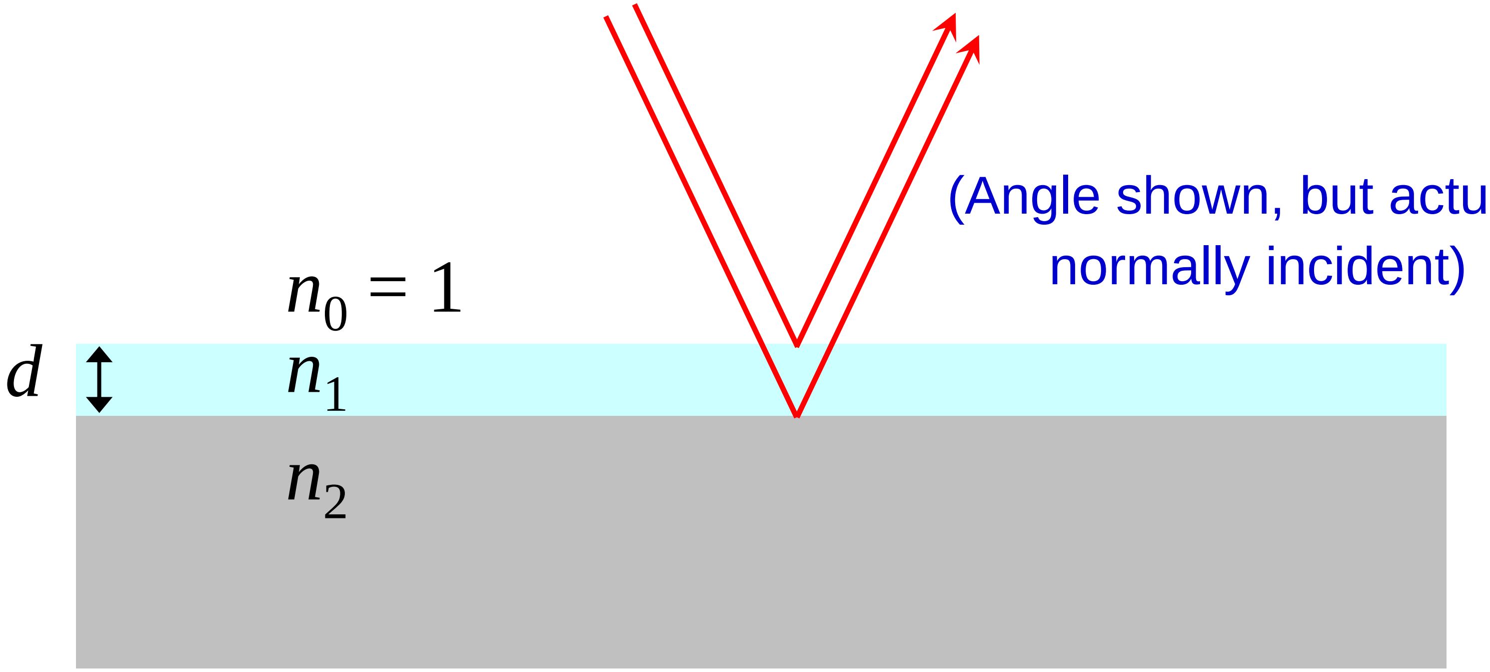
Standard analysis of Thin Film Interference

$2d = m\lambda_{n_1}$ Max (constructive)

$2d = \left(m + \tfrac{1}{2}\right)\lambda_{n_1}$ Min (destructive)

$$\lambda_{n_1} = \frac{\lambda}{n_1}$$

Wavelength inside film!



(Angle shown, but actually normally incident)

Example of Thin Film Interference

Let λ = 500 n_1 = 1.38 (MgF2) n_2 = 1.5 (glass)

$$\lambda_{1.38} = \frac{500}{1.38} = 362.3\,\text{nm}$$

Max intensity $d = \tfrac{1}{2} m \lambda_{1.38} = 181\,\text{nm},\ 362\,\text{nm},\ 543\,\text{nm},\ \ldots$

Min intensity $d = \tfrac{1}{2}\left(m + \tfrac{1}{2}\right)\lambda_{1.38} = 90.6\,\text{nm},\ 272\,\text{nm},\ 453\,\text{nm},\ \ldots$

Must be careful about phase shift at boundary:

- Reflection for $n_{in} < n_{out}$ has ½ λ phase shift, 0 if $n_{in} > n_{out}$
- Since $n_0 < n_1$ and $n_1 < n_2$, the phase shift has no effect here
- But for other cases, there can be an extra ½ λ phase shift
- Soap bubble (next slide)

Thin Film Interference for Soap Bubble

$$2d = m\lambda_n \quad \text{Min (destructive)}$$

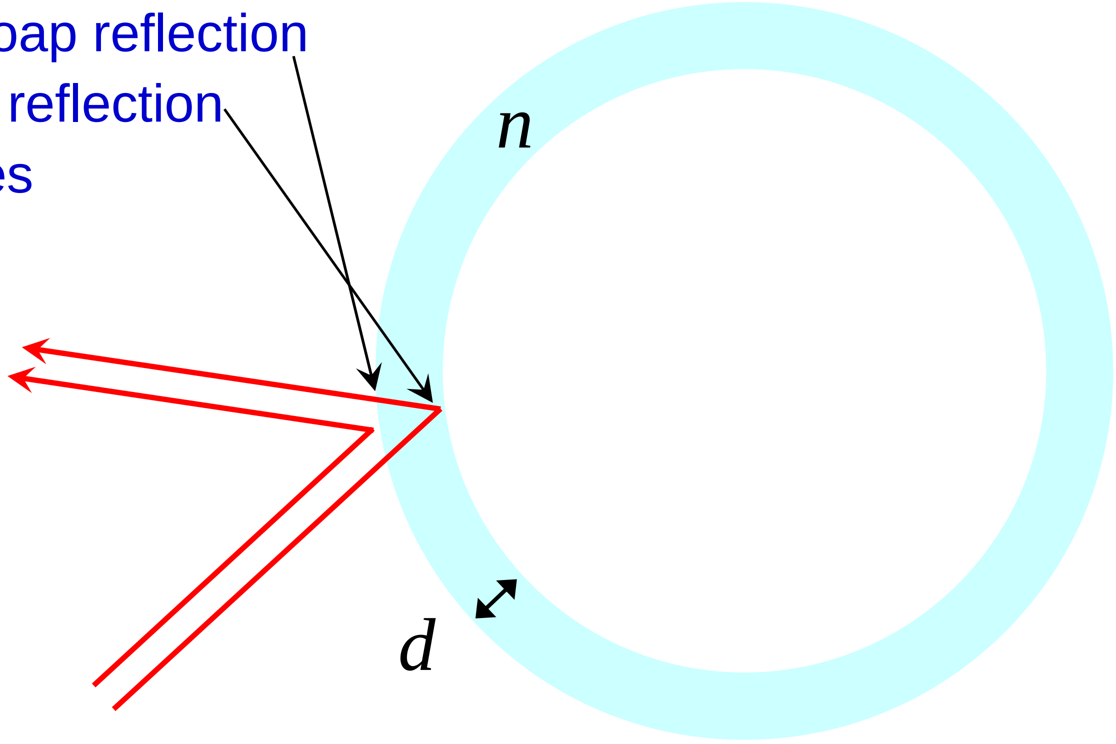
$$2d = \left(m + \tfrac{1}{2}\right)\lambda_n \quad \text{Max (constructive)}$$

$$\lambda_n = \frac{\lambda}{n}$$

Wavelength inside soap!

Similar analysis, but…

- Phase shift of ½ λ for air–soap reflection
- No phase shift for soap–air reflection
- Net ½ λ phase shift switches max ↔ min



Example of Soap Bubble Interference

Let λ = 500   n = 1.32 (soap + water)

$$\lambda_{1.32} = \frac{500}{1.32} = 379\,\text{nm}$$

Min intensity $d = \tfrac{1}{2} m \lambda_{1.32} = 189\,\text{nm},\ 379\,\text{nm},\ 568\,\text{nm},\ \ldots$

Max intensity $d = \tfrac{1}{2}\left(m + \tfrac{1}{2}\right)\lambda_{1.32} = 94.7\,\text{nm},\ 284\,\text{nm},\ 473\,\text{nm},\ \ldots$

Quiz

➔ What is the condition for destructive interference for light reflecting from a soap bubble of thickness d?

- [✓ (1) $2d = m\lambda_n$]
- (2) $2d = \left(m + \frac{1}{2}\right)\lambda_n$

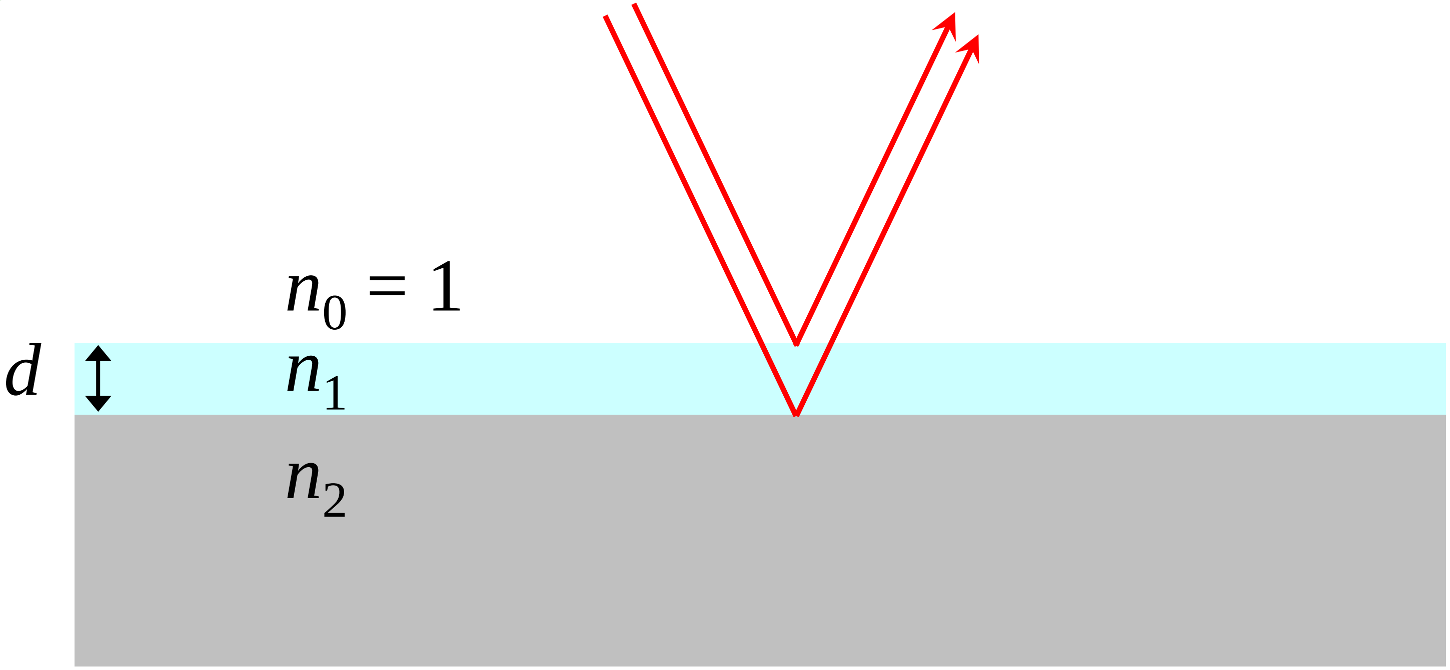
Only 1 reflection has a phase shift, so this switches min and max

Quiz

➔Consider an oil film (thickness d, n = 1.5) on top of water (n = 1.3). Light of λ = 600 nm is normally incident. Which value of d corresponds to destructive interference?

- (1) 300 nm
- (2) 150 nm
- [✓ (3) 200 nm]

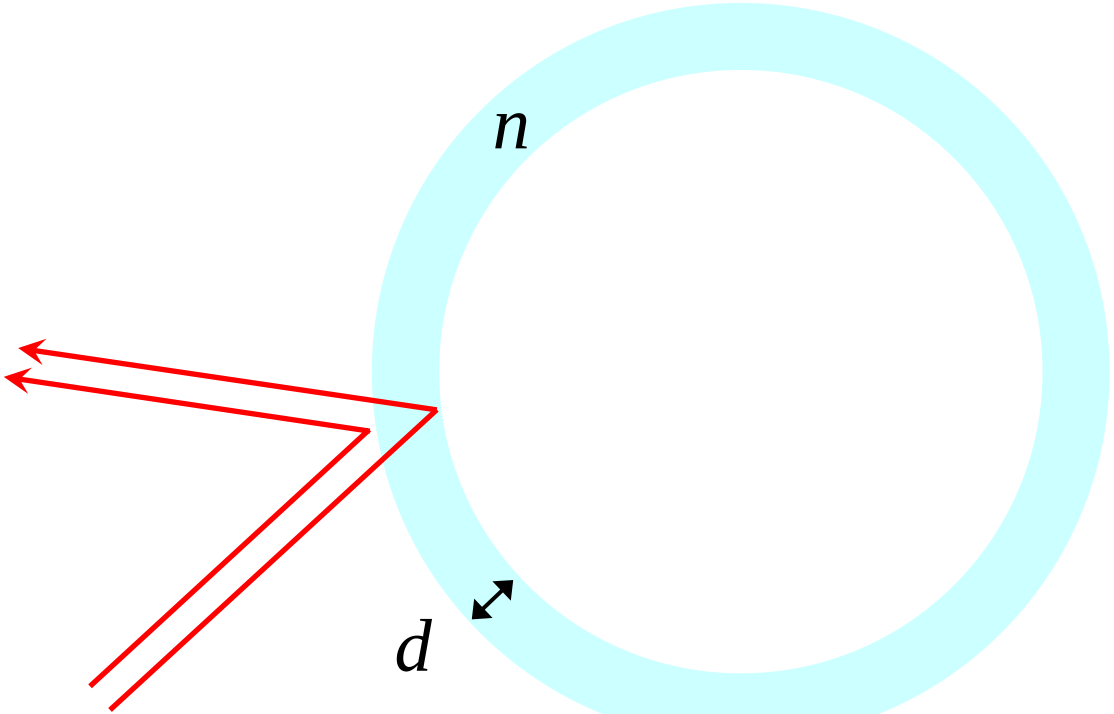
Only 1 reflection has a phase shift, so d = m * 200 nm



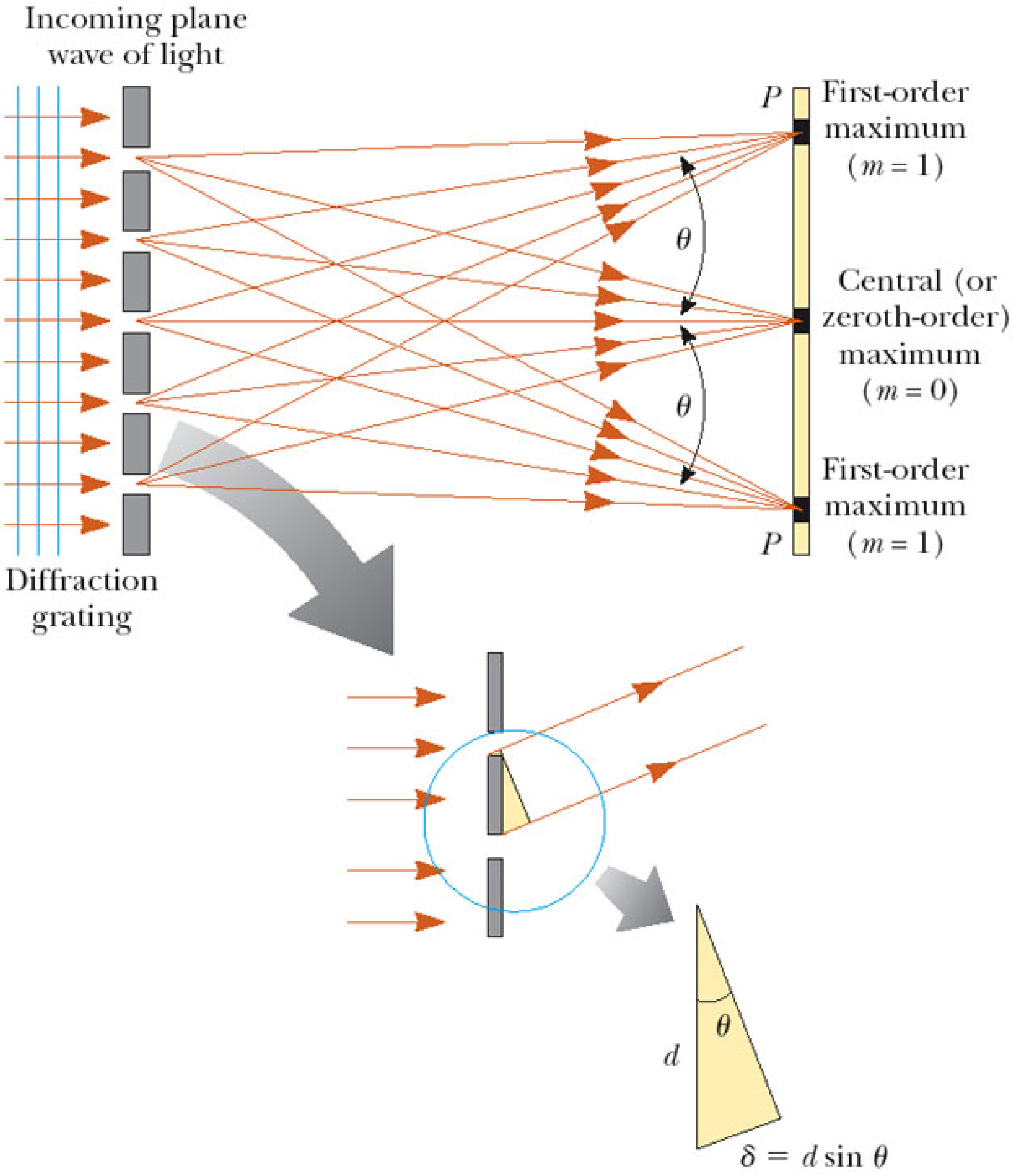
Quiz

- Consider a soap bubble of thickness d and n = 1.5. Light of λ = 600 nm is incident on the bubble. Which value of d corresponds to destructive interference?
  - (1) 300 nm
  - (2) 150 nm
  - [✓ (3) 200 nm]

Only 1 reflection has a phase shift, so d = m * 200 nm

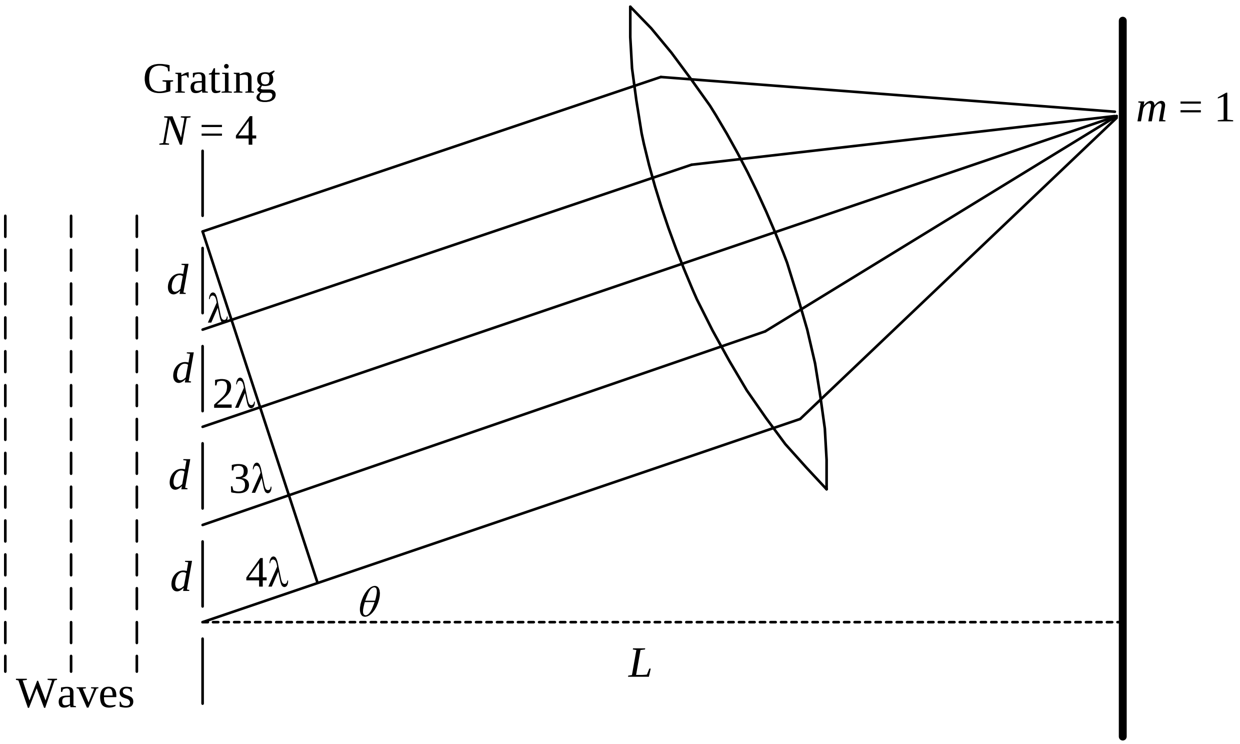


Diffraction Grating: 1000s of Slits!



Diffraction Grating

- Analysis similar to double slit
  - Many slits instead of 2
  - Slits still separated by distance d
  - Maxima again occur only for $d \sin\theta = m\lambda$
  - Maxima are much sharper



Intensity Pattern for Diffraction Grating

- ➔ Calculation of intensity vs θ
  - ◆ Basically a product of single slit and multiple slit formula
  - ◆ Let a = slit width, d = slit separation, N = # of slits

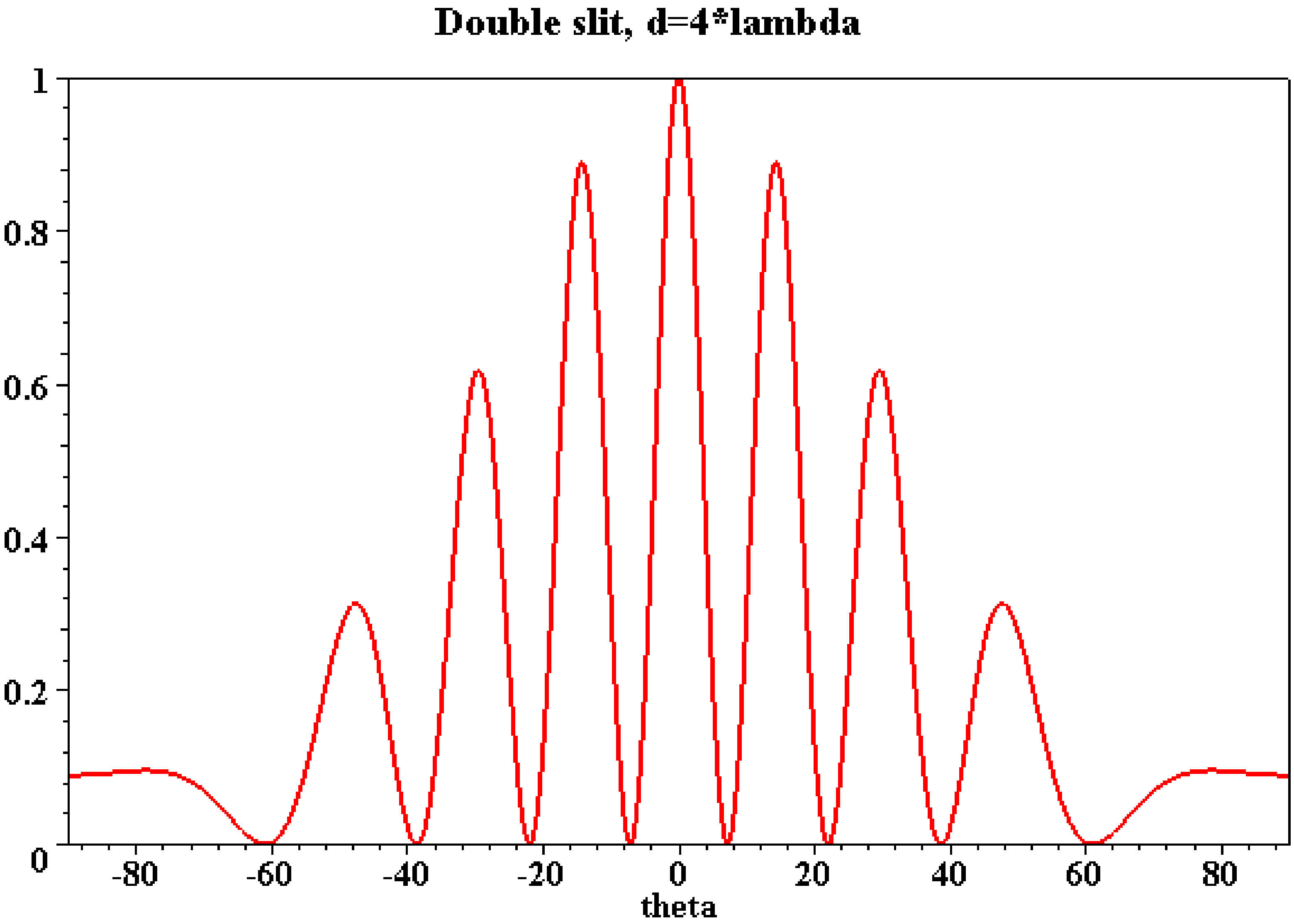
$$I = I_{\max}\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{N\sin\beta}\right)^2$$

$$\alpha = \pi a\sin\theta/\lambda$$

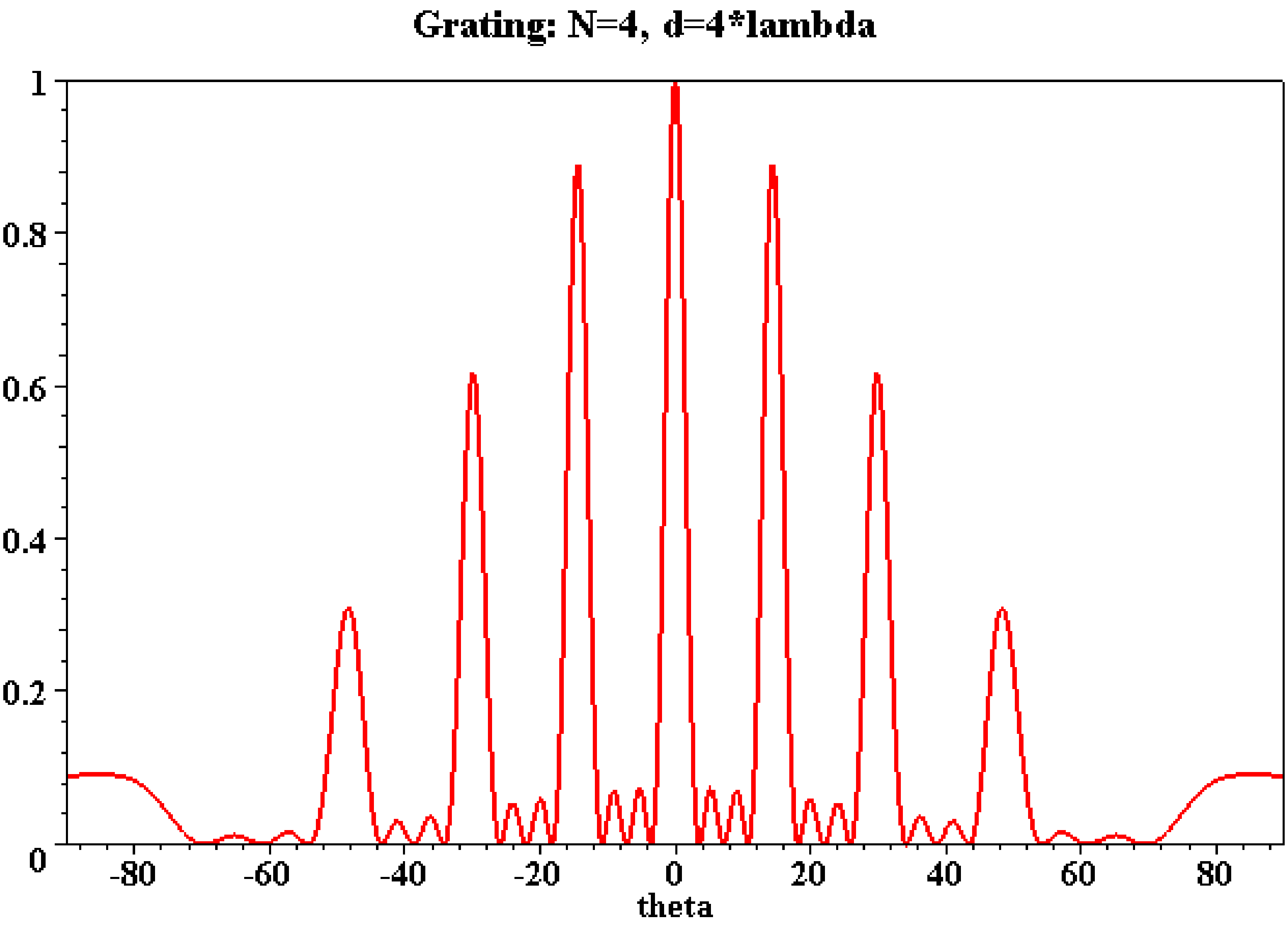
$$\beta = \pi d\sin\theta/\lambda$$

- ➔ When N is large, maxima are extremely sharp
  - ◆ For central peak, angular half-width $\Delta\theta_{hw} \simeq \lambda/Nd$
  - ◆ Imagine N ≅ 10000 – 100000!

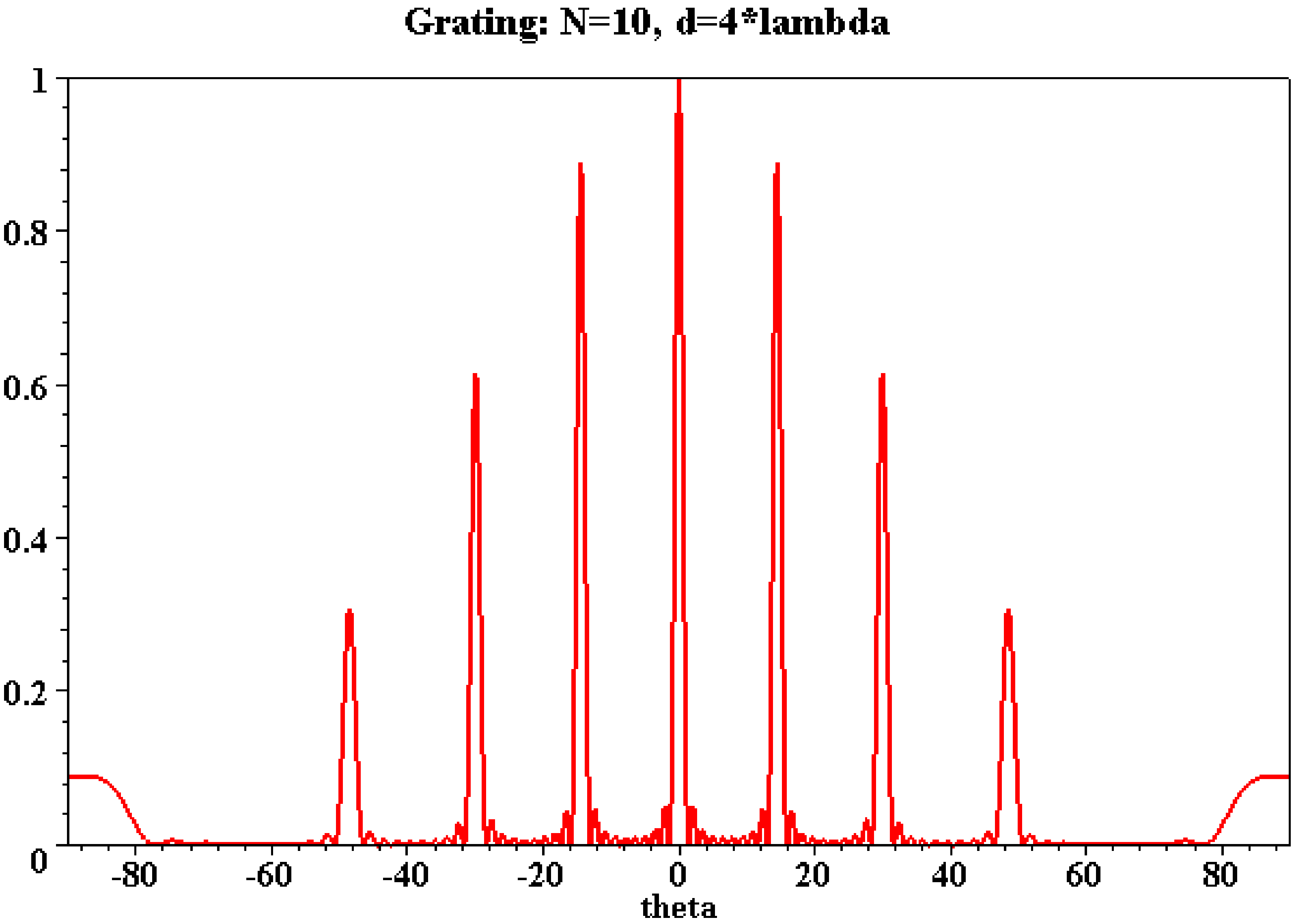
2 Slits: d = 4λ


Double slit, d=4*lambda
1
0.8
0.6
0.4
0.2
0
-80
-60
-40
-20
0
20
40
60
80
theta

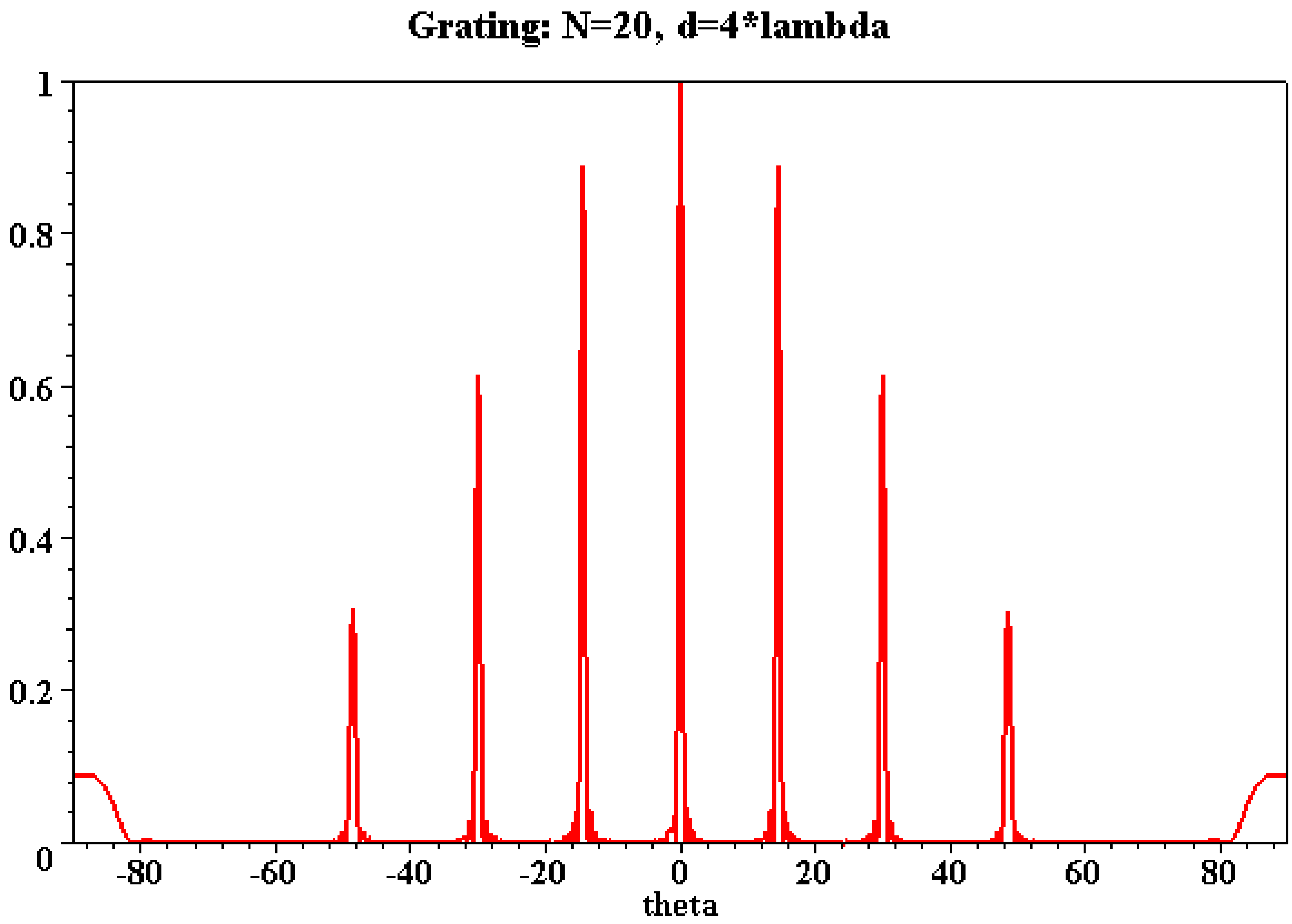
4 Slits: d = 4λ



10 Slits: d = 4λ

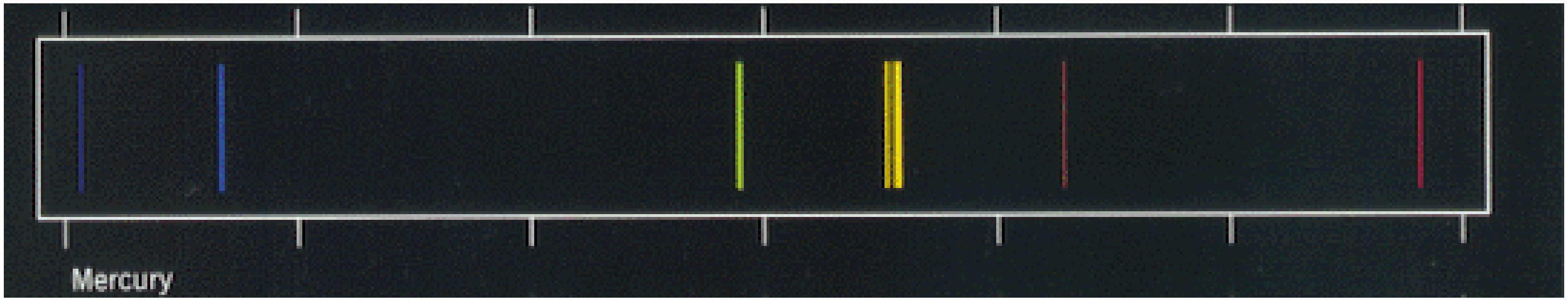

Grating: N=10, d=4*lambda
1
0.8
0.6
0.4
0.2
0
-80
-60
-40
-20
0
20
40
60
80
theta

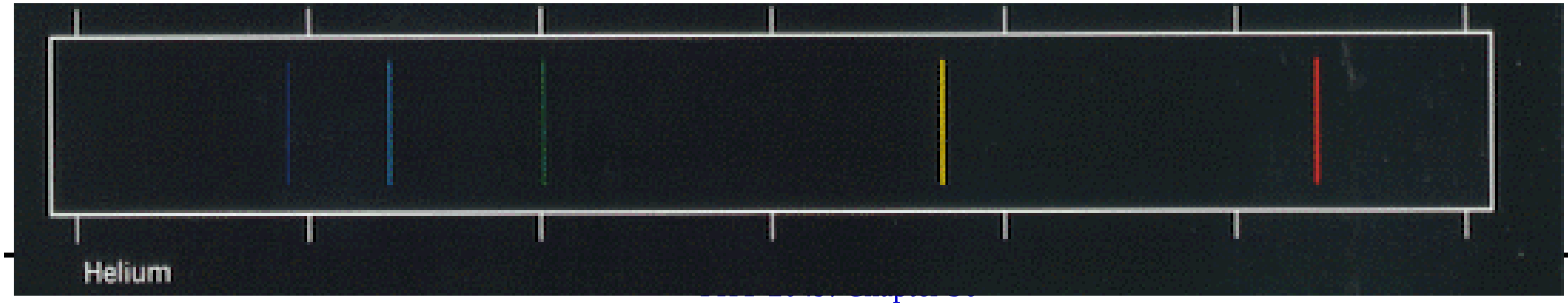
20 Slits: d = 4λ


Grating: N=20, d=4*lambda
1
0.8
0.6
0.4
0.2
0
-80
-60
-40
-20
0
20
40
60
80
theta

Diffraction Gratings in Astronomy

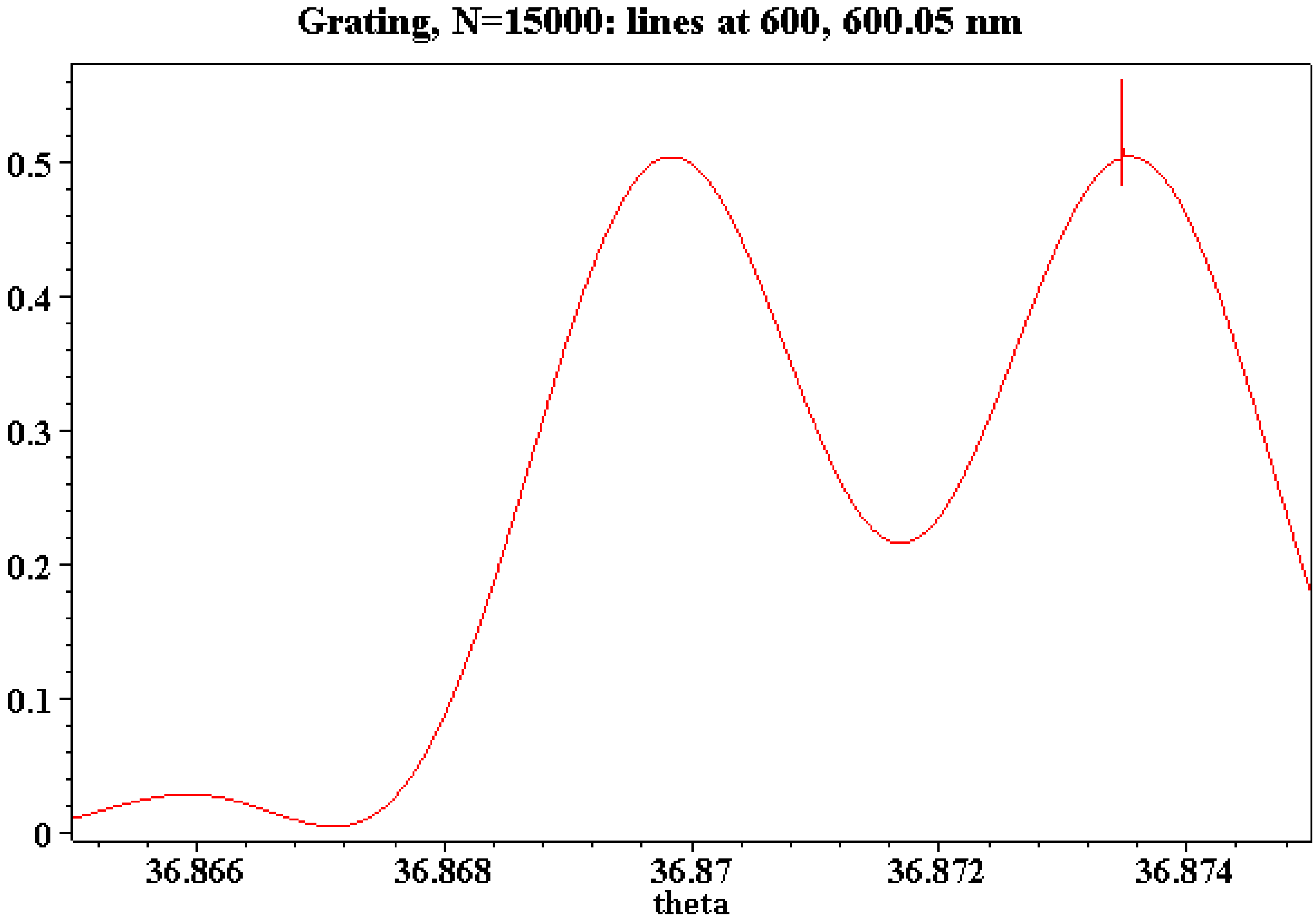
- ➔ Use to determine wavelength (θ measured)
  - ◆ Use $d \sin\theta = m\lambda$ to determine wavelength (θ measured)
  - ◆ Typically have N = 60,000 – 100,000!!
  - ◆ Sharp peaks allow closely spaced wavelengths to be resolved
  - ◆ Accuracy better than 0.001 nm, e.g. 589.605 ± 0.001 nm
- ➔ Important for distinguishing element “signatures”
  - ◆ Find all the elements in a stellar spectrum





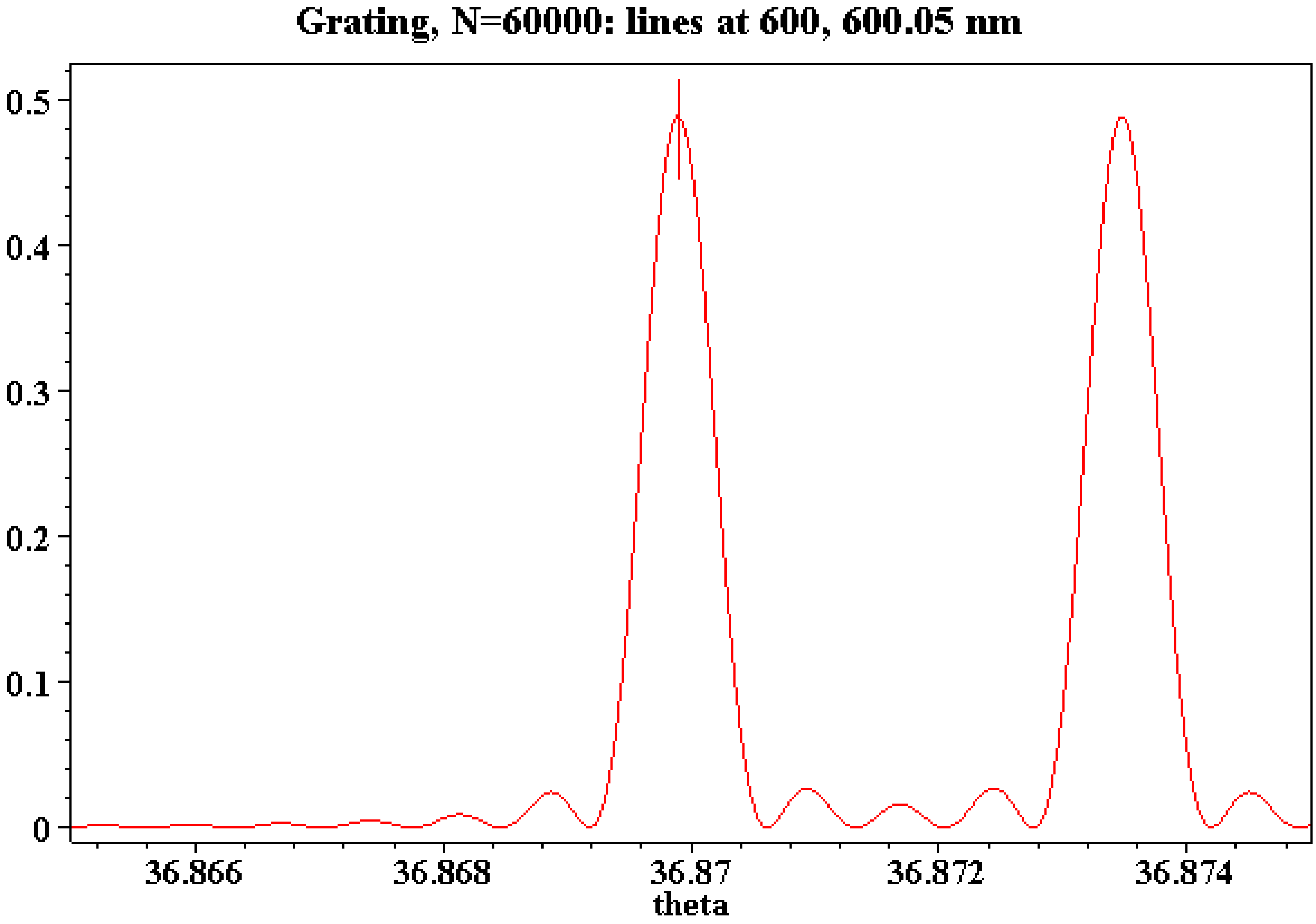
Example: Separate λ = 600, λ = 600.05 nm

- ➔Poorly resolved with 15,000 slit grating
  - ◆ Too few slits ⇒ lines too wide to tell apart



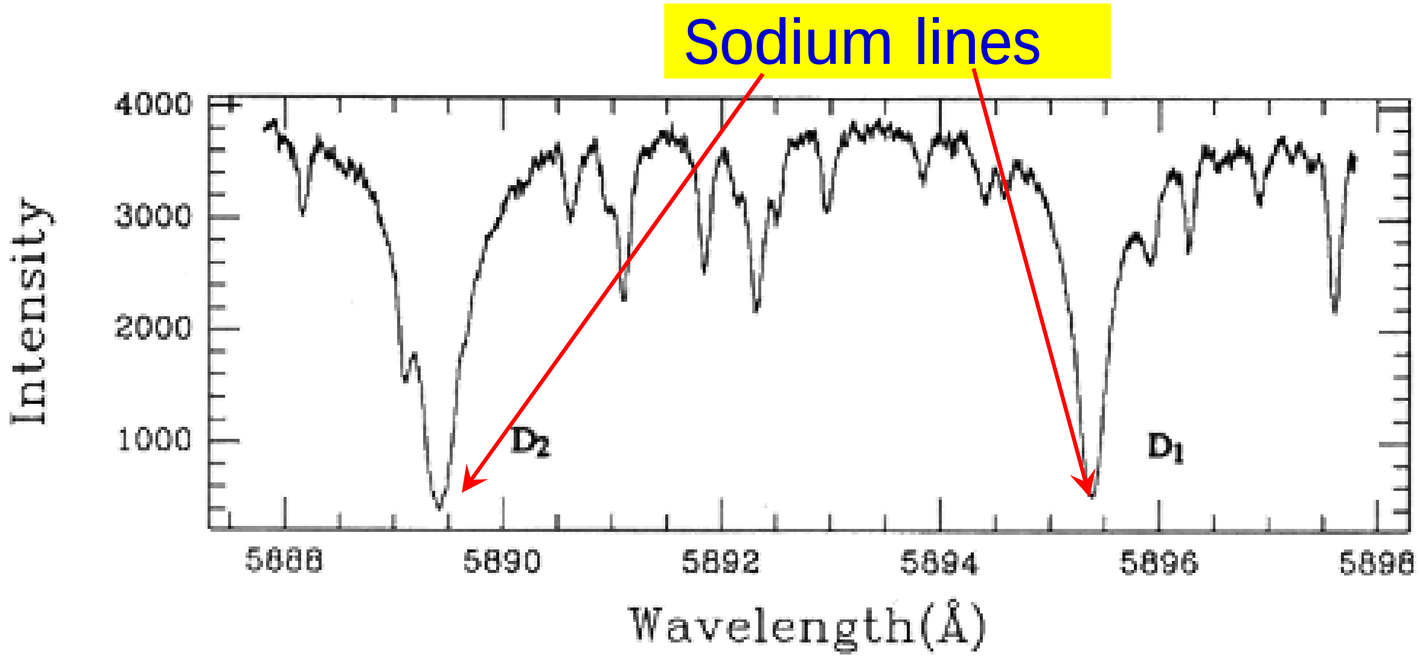
Example (cont.)

- ➔Lines easily resolved with 60,000 slit grating



Example: Resolution of Sodium D_1, D_2 Lines

- ➔High resolution solar spectrum near sodium absorption lines



$$\lambda_{D_1} = 589.54\,\text{nm}$$

$$\lambda_{D_2} = 588.94\,\text{nm}$$

$\Delta\lambda = 0.60$ nm