



Linear Regression

(ver. 6.0)

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http://dss.princeton.edu/training/

Regression: a practical approach (overview)

We use regression to estimate the <u>unknown effect</u> of changing one variable over another (Stock and Watson, 2003, ch. 4)

When running a regression we are making two assumptions, 1) there is a linear relationship between two variables (i.e. X and Y) and 2) this relationship is additive (i.e. Y = x1 + x2 + ... + xN).

Technically, linear regression estimates how much Y changes when X changes one unit.

In Stata use the command regress, type:

```
regress [dependent variable] [independent variable(s)] regress y x
```

In a multivariate setting we type:

```
regress y x1 x2 x3 ...
```

Before running a regression it is recommended to have a clear idea of what you are trying to estimate (i.e. which are your outcome and predictor variables).

A regression makes sense only if there is a sound theory behind it.

Regression: a practical approach (setting)

Example: Are SAT scores higher in states that spend more money on education controlling by other factors?*

- Outcome (Y) variable SAT scores, variable csat in dataset
- Predictor (X) variables
 - Per pupil expenditures primary & secondary (expense)
 - % HS graduates taking SAT (percent)
 - Median household income (income)
 - % adults with HS diploma (high)
 - % adults with college degree (college)
 - Region (region)

Regression: variables

It is recommended first to examine the variables in the model to check for possible errors, type:

use http://dss.princeton.edu/training/states.dta
describe csat expense percent income high college region
summarize csat expense percent income high college region

. describe csat expense percent income high college region

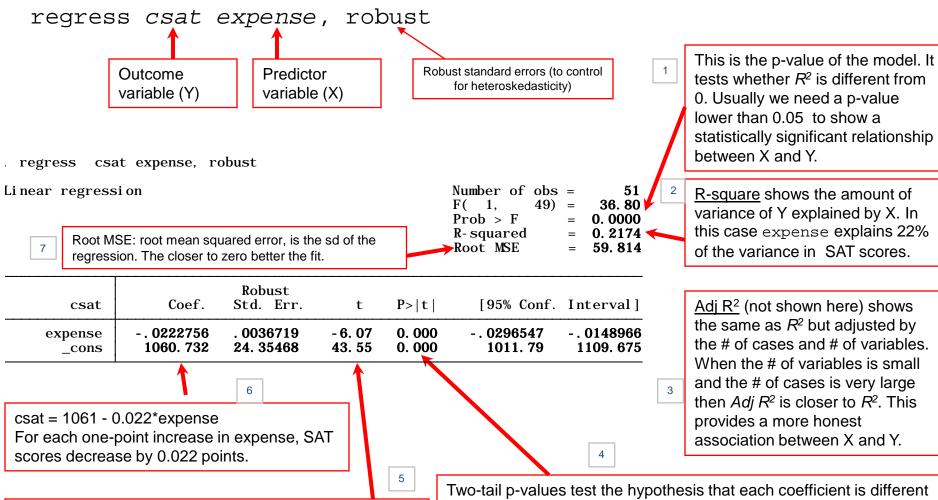
variable name		display format	val ue l abel	vari able label
csat expense percent i ncome hi gh col l ege regi on	int int byte double float float byte	%9. 0g %9. 0g %9. 0g %10. 0g %9. 0g %9. 0g %9. 0g	regi on	Mean composite SAT score Per pupil expenditures prim&sec % HS graduates taking SAT Median household income, \$1,000 % adults HS diploma % adults college degree Geographical region

. summarize csat expense percent income high college region

Vari abl e	0bs	Mean	Std. Dev.	Mi n	Max
csat	51	944. 098	66. 93497	832	1093
expense	51	5235. 961	1401. 155	2960	9259
percent	51	35. 76471	26. 19281	4	81
i ncome	51	33. 95657	6. 423134	23. 465	48. 618
hi gh	51	76. 26078	5. 588741	64. 3	86. 6
college	51	20. 02157	4. 16578	12. 3	33. 3
region	50	2. 54	1. 128662	1	

Regression: what to look for

Lets run the regression:



The t-values test the hypothesis that the coefficient is different from 0. To reject this, you need a t-value greater than 1.96 (for 95% confidence). You can get the t-values by dividing the coefficient by its standard error. The t-values also show the importance of a variable in the model.

from 0. To reject this, the p-value has to be lower than 0.05 (you could choose also an alpha of 0.10). In this case, expense is statistically significant in explaining SAT.

Regression: what to look for

Robust standard errors (to control Adding the rest of predictor variables: for heteroskedasticity) regress csat expense percent income high college, This is the p-value of the model. It Output variable (Y) Predictor variables (X) indicates the reliability of X to predict Y. Usually we need a pvalue lower than 0.05 to show a regress csat expense percent income high college, robust statistically significant relationship between X and Y. Linear regression Number of obs =51 50. 90 F(5. 45) =Prob > F0.0000 R-square shows the amount of R-squared 0.8243 Root MSE: root mean squared error, is the sd of the variance of Y explained by X. In Root MSE 29. 571 regression. The closer to zero better the fit. this case the model explains 82.43% of the variance in SAT Robust [95% Conf. Interval] Coef. Std. Err. P > |t|t csat scores. 0.70 0.487 -.0062766 .0129823 . 0033528 . 004781 expense Adj R² (not shown here) shows 0.000 . 2288594 - 11. 44 -3.079123 -2.15723 percent - 2. 618177 1.207246 0.931 -2.325933 2.537104 the same as R² but adjusted by i ncome . 1055853 0.09 0.091 hi gh 1. 630841 . 943318 1. 73 - . 2690989 3. 530781 the # of cases and # of variables. college 0.96 0.342 6.28829 2. 030894 2. 113792 - 2. 226502 When the # of variables is small 851. 5649 57. 28743 14.86 0.000 736. 1821 966. 9477 _cons and the # of cases is very large then Adj R² is closer to R². This 6 provides a more honest association between X and Y. csat = 851.56 + 0.003*expense- 2.62*percent + 0.11*income + 1.63*high

The t-values test the hypothesis that the coefficient is different from 0. To reject this, you need a t-value greater than 1.96 (at 0.05 confidence). You can get the t-values by dividing the coefficient by its standard error. The t-values also show the importance of a variable in the model. In this case, percent is the most important.

+ 2.03*college

Two-tail p-values test the hypothesis that each coefficient is different from 0. To reject this, the p-value has to be lower than 0.05 (you could choose also an alpha of 0.10). In this case, expense, income, and college are not statistically significant in explaining SAT; high is almost significant at 0.10. Percent is the only variable that has some significant impact on SAT (its coefficient is different from 0)

Regression: with dummies

Region is entered here as dummy variable. The easy way to add dummy variables to a regression is using "xi" and the prefix "i." (interpretation is the same as before). The first category is always the reference:

xi: regress csat expense percent income high college i.region, robust

	csat	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	Interval]
_	ovnonce	002021	. 0035883	- 0. 56	0. 576	0092676	. 0052256
	expense percent	-3. 007647	. 2358047	- 0. 36 - 12. 75	0. 000	- 3. 483864	- 2. 53143
	income	1674421	1. 196409	- 0. 14	0. 889	- 2. 583638	2. 248754
	, hi gh	1. 814731	1. 02694	1. 77	0. 085	2592168	3. 888679
	college	4. 670564	1. 599798	2. 92	0.006	1. 439705	7. 901422
	_I regi on_2 _I regi on_3	69. 45333 25. 39701	17. 99933 12. 52558	3. 86 2. 03	0. 000 0. 049	33. 10295 . 101086	105. 8037 50. 69293
	_Iregion_3 _Iregion_4	34. 57704	9. 44989	2. 66 3. 66	0. 043	15. 4926	53. 66149
	_cons	808. 0206	67. 86418	11. 91	0.000	670. 9661	945. 0751

NOTE: By default xi excludes the first value, to select a different value, before running the regression type: char region[omit] 4

xi: regress csat expense percent income high college i.region, robust This will select Midwest (4) as the reference category for the dummy variables.

NOTE: Another way to create dummy variables is to type:

tab region, gen(region)

Regions:
1 West
2 N. East
3 South

4 Midwest

This will create four new variables (or a many a categories in the variable), one for each region in this case.

Regression: ANOVA table

If you run the regression without the 'robust' option you get the ANOVA table

xi: regress csat expense percent income high college i.region

	Source	SS	df	MS	Number of obs =	50
_	Model	(A) 200269. 84	9	22252. 2045 (D)	F(9, 40) = Prob > F =	
	Resi dual	(B) 12691. 5396		317. 28849 (E)	R-squared =	0. 9404
	Total	(C) 212961. 38	49	4346. 15061 (F)	Adj R-squared = Root MSE =	

$$F = \frac{\frac{MSS}{(k-1)}}{\frac{RSS}{n-k}} = \frac{\frac{200269.84}{9}}{\frac{12691.5396}{40}} = \frac{22252.2045}{317.28849} = \frac{D}{E} = 70.13 \qquad AdjR^2 = 1 - \frac{n-1}{n-k}(1-R^2) = 1 - \frac{49}{40}(1-0.9404) = 1 - \frac{E}{F} = 1 - \frac{317.28849}{4346.15061} = 0.9270$$

$$R^{2} = \frac{MSS}{TSS} = 1 - \frac{\sum e_{i}^{2}}{\sum (y_{i} - \overline{y})^{2}} = \frac{200269.84}{12691.5396} = \frac{A}{C} = 0.9404$$

$$RootMSE = \sqrt{\frac{RSS}{(n-k)}} = \sqrt{\frac{12691.5396}{40}} = \sqrt{\frac{B}{40}} = 17.813$$

A = Model Sum of Squares (MSS). The closer to TSS the better fit.

B = Residual Sum of Squares (RSS)

C = Total Sum of Squares (TSS)

D = Average Model Sum of Squares = MSS/(k-1) where k = # predictors

E = Average Residual Sum of Squares = RSS/(n - k) where n = # of observations

F = Average Total Sum of Squares = TSS/(n-1)

 R^2 shows the amount of observed variance explained by the model, in this case 94%.

The *F-statistic*, F(9,40), tests whether R^2 is different from zero.

Root MSE shows the average distance of the estimator from the mean, in this case 18 points in estimating SAT scores.

Regression: estto/esttab

To show the models side-by-side you can use the commands estto and esttab:

regress csat expense, robust eststo model1

regress csat expense percent income high college, robust eststo model2

xi: regress csat expense percent income high college i.region, robust eststo model3

esttab, r2 ar2 se scalar(rmse)

	esttab,	r2	ar2	\mathbf{se}	scal	l ar	(rmse)	1
--	---------	----	-----	---------------	------	------	--------	---

	(1) csat	(2) csat	(3) csat
expense	-0. 0223*** (0. 00367)	0. 00335 (0. 00478)	-0.00202 (0.00359)
percent		-2. 618*** (0. 229)	-3. 008*** (0. 236)
i ncome		0. 106 (1. 207)	-0. 167 (1. 196)
hi gh		1. 631 (0. 943)	1. 815 (1. 027)
col l ege		2. 031 (2. 114)	4. 671** (1. 600)
_I regi on_2			69. 45*** (18. 00)
_I regi on_3			25. 40* (12. 53)
_I regi on_4			34. 58*** (9. 450)
percent2			
_cons	1060. 7*** (24. 35)	851. 6*** (57. 29)	808. 0*** (67. 86)
N R-sq adj. R-sq rmse	51 0. 217 0. 201 59. 81	51 0. 824 0. 805 29. 57	50 0. 911 0. 894 21. 49

Type help eststo and help esttab for more options.

Regression: correlation matrix

Below is a correlation matrix for all variables in the model. Numbers are Pearson correlation coefficients, go from -1 to 1. Closer to 1 means strong correlation. A negative value indicates an inverse relationship (roughly, when one goes up the other goes down).

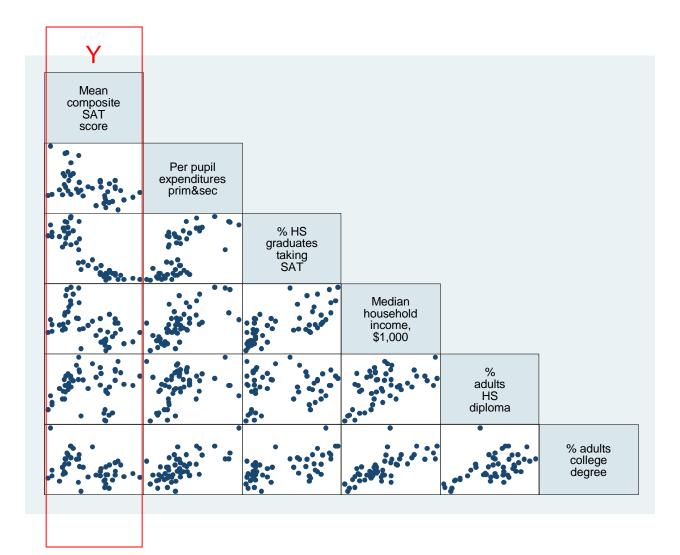
. pwcorr csat expense percent income high college, star(0.05) sig

	csat	expense	percent	income	hi gh	college
csat	1. 0000					
expense	- 0. 4663* 0. 0006	1. 0000				
percent	- 0. 8758* 0. 0000	0. 6509* 0. 0000	1. 0000			
income	- 0. 4713* 0. 0005	0. 6784* 0. 0000	0. 6733* 0. 0000	1. 0000		
hi gh	0. 0858 0. 5495	0. 3133* 0. 0252	0. 1413 0. 3226	0. 5099* 0. 0001	1. 0000	
col l ege	- 0. 3729* 0. 0070	0. 6400* 0. 0000	0. 6091* 0. 0000	0. 7234* 0. 0000	0. 5319* 0. 0001	1. 0000

Regression: graph matrix

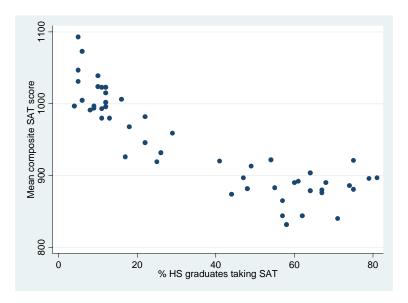
Command graph matrix produces a graphical representation of the correlation matrix by presenting a series of scatterplots for all variables. Type:

graph matrix csat expense percent income high college, half maxis(ylabel(none) xlabel(none))

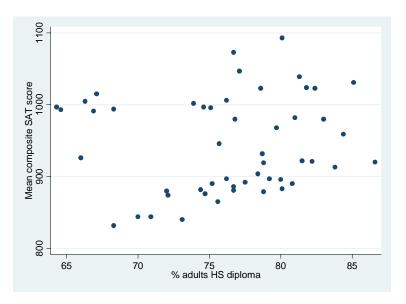


Regression: exploring relationships

scatter csat percent



scatter csat high



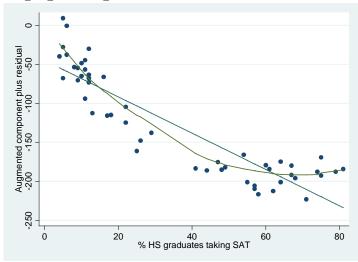
There seem to be a curvilinear relationship between csat and percent, and slightly linear between csat and high. To deal with U-shaped curves we need to add a square version of the variable, in this case percent square

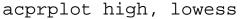
Regression: functional form/linearity

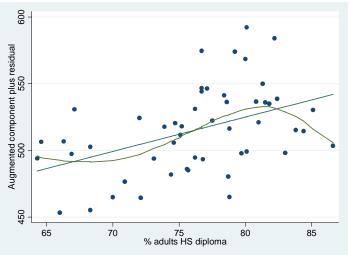
The command acprplot (augmented component-plus-residual plot) provides another graphical way to examine the relationship between variables. It does provide a good testing for linearity. Run this command after running a regression

regress csat percent high /* Notice we do not include percent2 */
acprplot percent, lowess
acprplot high, lowess

acprplot percent, lowess







The option lowess (locally weighted scatterplot smoothing) draw the observed pattern in the data to help identify nonlinearities. Percent shows a quadratic relation, it makes sense to add a square version of it. High shows a polynomial pattern as well but goes around the regression line (except on the right). We could keep it as is for now.

The model is:

xi: regress csat expense percent percent2 income high college i.region, robust

Regression: models

xi: regress csat expense percent percent2 income high college i.region, robust eststo model4 esttab, r2 ar2 se scalar(rmse)

. esttab, r2 ar2 se scalar(rmse)

	(1) csat	(2) csat	(3) csat	(4) csat
expense	-0. 0223*** (0. 00367)	0. 00335 (0. 00478)	-0.00202 (0.00359)	0. 00141 (0. 00372)
percent		-2. 618*** (0. 229)	-3.008*** (0.236)	-5.945*** (0.641)
i ncome		0. 106 (1. 207)	-0. 167 (1. 196)	-0. 914 (0. 973)
hi gh		1. 631 (0. 943)	1. 815 (1. 027)	1. 869 (0. 931)
col l ege		2. 031 (2. 114)	4. 671** (1. 600)	3. 418** (1. 145)
_I regi on_2			69. 45*** (18. 00)	5. 077 (20. 75)
_I regi on_3			25. 40* (12. 53)	5. 209 (10. 42)
_I regi on_4			34. 58*** (9. 450)	19. 25* (8. 110)
percent2				0. 0460*** (0. 0102)
_cons	1060. 7*** (24. 35)	851. 6*** (57. 29)	808. 0*** (67. 86)	874. 0*** (58. 13)
N R-sq adj. R-sq rmse	51 0. 217 0. 201 59. 81	51 0. 824 0. 805 29. 57	50 0. 911 0. 894 21. 49	50 0. 940 0. 927 17. 81

Standard errors in parentheses * p<0.05, ** p<0.01, *** p<0.001

How good the model is will depend on how well it predicts *Y*, the linearity of the model and the behavior of the residuals.

There are two ways to generate the *predicted values of Y* (usually called *Yhat*) given the model:

Option A, using generate after running the regression:

```
xi: regress csat expense percent percent2 income high college i.region, robust
generate csat_predict = _b[_cons] + _b[percent]*percent + _b[percent]*percent + _b[percent2]*percent2
+ _b[high]*high + ...
```

Option B, using predict immediately after running the regression:

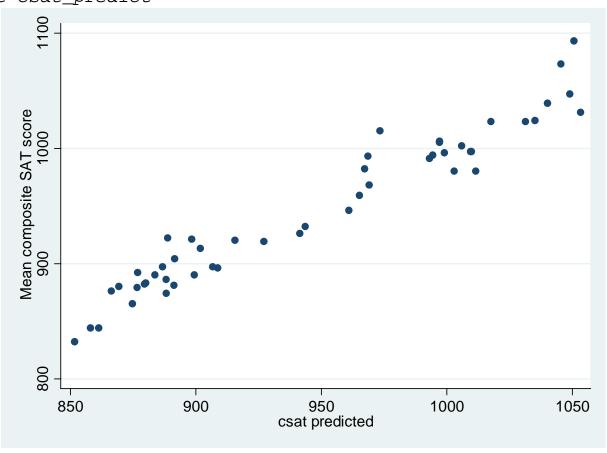
```
xi: regress csat expense percent percent2 income high college i.region, robust predict csat_predict label variable csat_predict "csat predicted"
```

```
. predict csat_predict
(option xb assumed; fitted values)
(1 missing value generated)
```

. label variable csat_predict "csat predicted"

For a quick assessment of the model run a scatter plot

scatter csat csat_predict



We should expect a 45 degree pattern in the data. Y-axis is the observed data and x-axis the predicted data (*Yhat*).

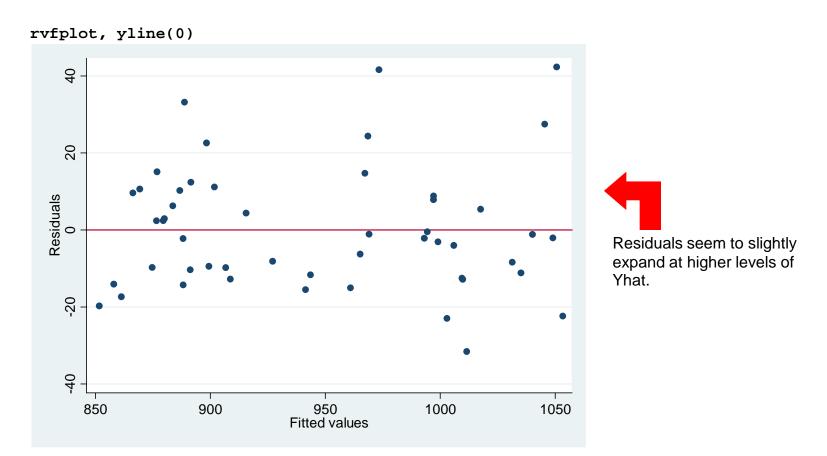
In this case the model seems to be doing a good job in predicting csat

Regression: testing for homoskedasticity

An important assumption is that the variance in the residuals has to be homoskedastic or constant. Residuals cannot varied for lower of higher values of X (i.e. fitted values of Y since Y = Xb). A definition:

"The error term [e] is homoskedastic if the variance of the conditional distribution of $[e_i]$ given X_i [var $(e_i|X_i)$], is constant for i=1...n, and in particular does not depend on x; otherwise, the error term is heteroskedastic" (Stock and Watson, 2003, p.126)

When plotting residuals vs. predicted values (Yhat) we should not observe any pattern at all. In Stata we do this using rvfplot right after running the regression, it will automatically draw a scatterplot between residuals and predicted values.



Regression: testing for homoskedasticity

A non-graphical way to detect heteroskedasticiy is the Breusch-Pagan test. The null hypothesis is that residuals are homoskedastic. In the example below we fail to reject the null at 95% and concluded that residuals are homogeneous. However at 90% we reject the null and conclude that residuals are not homogeneous.

```
estat hettest
. estat hettest
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
    Ho: Constant variance
    Variables: fitted values of csat

    chi 2(1) = 2.72
    Prob > chi 2 = 0.0993
```

The graphical and the Breush-Pagan test suggest the possible presence of heteroskedasticity in our model. The problem with this is that we may have the wrong estimates of the standard errors for the coefficients and therefore their t-values.

There are two ways to deal with this problem, one is using heteroskedasticity-robust standard errors, the other one is using weighted least squares (see Stock and Watson, 2003, chapter 15). WLS requires knowledge of the conditional variance on which the weights are based, if this is known (rarely the case) then use WLS. In practice it is recommended to use heteroskedasticity-robust standard errors to deal with heteroskedasticity.

By default Stata assumes homoskedastic standard errors, so we need to adjust our model to account for heteroskedasticity. To do this we use the option robust in the regress command.

```
xi: regress csat expense percent percent2 income high college i.region, robust
```

Following Stock and Watson, as a rule-of-thumb, you should always assume heteroskedasticiy in your model (see Stock and Watson, 2003, chapter 4).

Regression: omitted-variable test

How do we know we have included all variables we need to explain Y?

Testing for omitted variable bias is important for our model since it is related to the assumption that the error term and the independent variables in the model are not correlated (E(e|X) = 0)

If we are missing variables in our model and

- "is correlated with the included regressor" and,
- "the omitted variable is a determinant of the dependent variable" (Stock and Watson, 2003, p.144),

...then our regression coefficients are inconsistent.

In Stata we test for omitted-variable bias using the ovtest command:

xi: regress csat expense percent percent2 income high college i.region, robust ovtest

. ovtest

```
Ramsey RESET test using powers of the fitted values of csat Ho: model has no omitted variables F(3, 37) = 1.25 Prob > F = 0.3068
```

The null hypothesis is that the model does not have omitted-variables bias, the p-value is higher than the usual threshold of 0.05 (95% significance), so we fail to reject the null and conclude that we do not need more variables.

Regression: specification error

Another command to test model specification is linktest. It basically checks whether we need more variables in our model by running a new regression with the observed Y(csat) against Yhat (csat_predicted or $X\beta$) and Yhat-squared as independent variables¹.

The thing to look for here is the significance of <code>_hatsq</code>. The null hypothesis is that there is no specification error. If the p-value of <code>_hatsq</code> is not significant then we fail to reject the null and conclude that our model is correctly specified. Type:

xi: regress csat expense percent percent2 income high college i.region, robust linktest

. linktest

Source	SS	df	MS		Number of obs	
Model Resi dual	200272. 359 12689. 0209	2 47	100136. 18 269. 979169		F(2, 47) Prob > F R-squared Adj R-squared	= 370. 90 = 0. 0000 = 0. 9404 = 0. 9379
Total	212961. 38	49 4	4346. 15061		Root MSE	= 16. 431
csat	Coef.	Std. Eı	rr. t	P> t	[95% Conf.	Interval]
_hat _hatsq _cons	1. 144949 0000761 -68. 69417	1. 5018 . 000788 712. 38	85 - 0. 10	$0 \boxed{0.923}$	- 1. 876362 0016623 - 1501. 834	4. 166261 . 0015101 1364. 446

¹ For more details see http://www.ats.ucla.edu/stat/stata/webbooks/reg/chapter2/statareg2.htm, and/or type help linktest.

Regression: multicollinearity

An important assumption for the multiple regression model is that independent variables are *not perfectly multicolinear*. One regressor should not be a linear function of another.

When multicollinearity is present standard errors may be inflated. Stata will drop one of the variables to avoid a division by zero in the OLS procedure (see Stock and Watson, 2003, chapter 5).

The Stata command to check for multicollinearity is vif (variance inflation factor). Right after running the regression type:

•	_
T 7 T	-
\ / I	•
v 1	

Vari abl e	VI F	1/VI F
percent2 percent _Iregion_2 income _Iregion_3 high college expense _Iregion_4	70. 80 49. 52 8. 47 4. 97 4. 89 4. 71 4. 52 3. 33 2. 14	0. 014124 0. 020193 0. 118063 0. 201326 0. 204445 0. 212134 0. 221348 0. 300111 0. 467506
Mean VIF	17. 04	

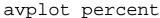
A vif > 10 or a 1/vif < 0.10 indicates trouble.

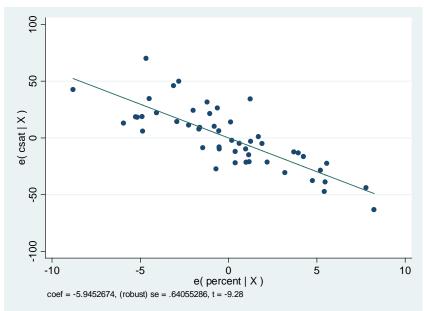
We know that percent and percent 2 are related since one is the square of the other. They are ok since percent has a quadratic relationship with *Y*, but this would be an example of multicolinearity.

The rest of the variables look ok.

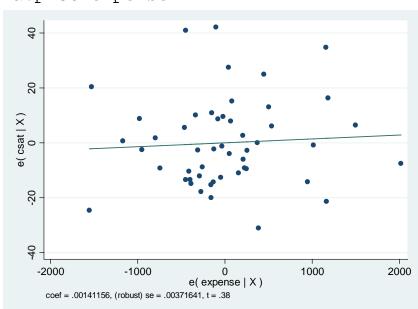
Regression: outliers

To check for outliers we use the avplots command (added-variable plots). Outliers are data points with extreme values that could have a negative effect on our estimators. After running the regression type:





avplot expense



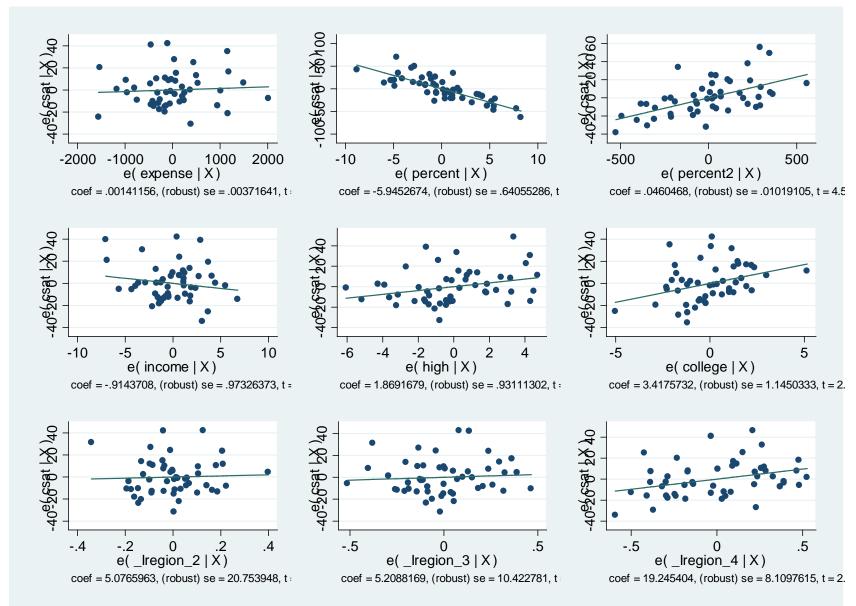
These plots regress each variable against all others, notice the coefficients on each. All data points seem to be in range, no outliers observed.

For more details and tests on this and <u>influential</u> and <u>leverage</u> variables please check <u>http://www.ats.ucla.edu/stat/stata/webbooks/reg/chapter2/statareg2.htm</u>

Also type help diagplots in the Stata command window.

Regression: outliers

avplots



		Regression	on: summary of <i>influe</i>	ence indicators
	Measures the influence of each observation on the	A case is an influential outlier	In Stata type: reg y x1 x2 x3 dfbeta x1	In SPSS: Analyze-Regression- Linear; click Save. Select under "Influence Statistics" to add as a new variable (DFB1_1) or in syntax type REGRESSION
	coefficient of a particular independent variable (for example, x1). This is in standard errors terms. An observation is influential if it has a significant effect on	ID(D / 1 0/00DT(1))	Note: you could also type: predict <i>DFx1</i> , dfbeta(x1)	/MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05)
DfBeta		Where N is the sample size.	To estimate the dfbetas for all predictors just type:	POITT(10)
		Note: Stata estimates standardized DfBetas.	dfbeta	/METHOD=ENTER X1 X2 X3 /CASEWISE PLOT(ZRESID) OUTLIERS(3) DEFAULTS
1			To flag the cutoff	DFBETA
			<pre>gen cutoffdfbeta = abs(DFx1) > 2/sqrt(e(N)) & e(sample)</pre>	/SAVE MAHAL COOK LEVER DFBETA SDBETA DFFIT SDFIT COVRATIO .
	Indicator of leverage and			

High influence if

|DfFIT| >2*SQRT(k/N)

intercept) and N is the

|COVRATIO-1| ≥ 3*k/N

intercept) and N is the

Where k is the number of

parameters (including the

sample size.

High impact if

sample size.

Where k is the number of parameters (including the After running the regression type:

gen cutoffdfit=

covratio

predict dfits if e(sample), dfits

To generate the flag for the cutoff type:

In Stata after running the regression type

predict covratio if e(sample),

abs(dfits)>2*sqrt((e(df_m)

+1)/e(N)) & e(sample)

	An observation is influential if it has a significant effect on the coefficient.
	Indicator of leverage and high residuals.
DfFit	Measures how much an observation influences the regression model as a whole.

Covariance ratio

How much the predicted

including and excluding a particular observation.

Measures the impact of an

errors

observation on the standard

values change as a result of



Same as DfBeta above

24

(COV_1)

Regression: summary of *distance* measures In SPSS: Analyze-Regression-Linear;

In Stata after running the

predict lev, leverage

Same as above (LEV_1)

Same as above (MAH_1)

25

regression type:

Not available

				III SPSS: Analyze-Regression-Linear;
				click Save. Select under "Distances" to add as a new variable (COO_1) or
Cook's distance	Measures how much an observation influences the overall model or predicted values. It is a summary measure of leverage and high residuals.	High influence if D > 4/N Where N is the sample size. A D>1 indicates big outlier problem	In Stata after running the regression type: predict D, cooksd	in syntax type REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT Y /METHOD=ENTER X1 X2 X3 /CASEWISE PLOT(ZRESID) OUTLIERS(3) DEFAULTS DFBETA /SAVE MAHAL COOK LEVER DFBETA SDBETA DFFIT SDFIT COVRATIO.
		High influence if		
		leverage h > 2*k/N		

Where k is the number of

parameters (including the

intercept) and N is the sample size.

A rule-of-thumb: Leverage goes from 0 to 1. A value closer to 1 or over 0.5 may indicate problems.

Higher levels indicate higher distance from average values.

The M-distance follows a Chisquare distribution with k-1 df and

alpha=0.001 (where k is the

number of independent variables).

Any value over this Chi-square value may indicate problems.

Leverage

Mahalanobis distance

Measures how much an

observation influences

regression coefficients.

It is rescaled measure of

M = leverage*(N-1)

Where N is sample size.

leverage.

Sources for the summary tables: influence indicators and distance measures

- Statnotes:
 http://faculty.chass.ncsu.edu/garson/PA765/regress.htm#outlier2
- An Introduction to Econometrics Using Stata/Christopher F. Baum, Stata Press, 2006
- Statistics with Stata (updated for version 9) / Lawrence Hamilton, Thomson Books/Cole, 2006
- UCLA http://www.ats.ucla.edu/stat/stata/webbooks/reg/chapter2/statareg2.htm

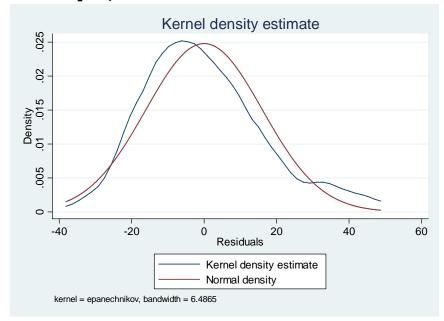
Regression: testing for normality

Another assumption of the regression model (OLS) that impact the validity of all tests (p, t and F) is that residuals behave 'normal'. Residuals (here indicated by the letter "e") are the difference between the observed values (Y) and the predicted values (Yhat): e = Y - Yhat.

In Stata you type: predict e, resid. It will generate a variable called "e" (residuals).

Three graphs will help us check for normality in the residuals: kdensity, pnorm and qnorm.

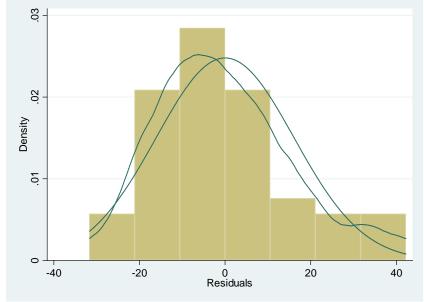
kdensity e, normal



If residuals do not follow a 'normal' pattern then you should check for omitted variables, model specification, linearity, functional forms. In sum, you may need to reassess your model/theory. In practice normality does not represent much of a problem when dealing with really big samples.

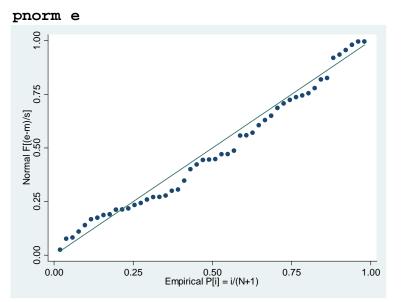
A kernel density plot produces a kind of histogram for the residuals, the option normal overlays a normal distribution to compare. Here residuals seem to follow a normal distribution. Below is an example using histogram.

histogram e, kdensity normal

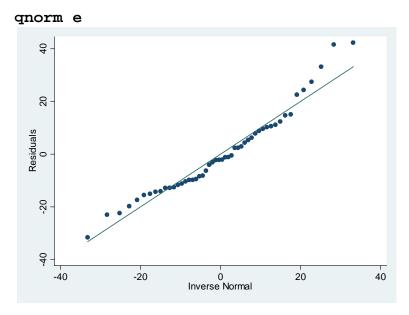


Regression: testing for normality

Standardize normal probability plot (pnorm) checks for non-normality in the middle range of residuals. Again, slightly off the line but looks ok.



Quintile-normal plots (qnorm) check for non-normality in the extremes of the data (tails). It plots quintiles of residuals vs quintiles of a normal distribution. Tails are a bit off the normal.



A non-graphical test is the Shapiro-Wilk test for normality. It tests the hypothesis that the distribution is normal, in this case the null hypothesis is that the distribution of the residuals is normal. Type

swilk e . swilk e

Shapiro-Wilk W test for normal data

Vari abl e)	0bs	W	V	Z	Prob>z
e)	50	0. 95566	2. 085	1. 567	0. 05855

The null hypothesis is that the distribution of the residuals is normal, here the p-value is 0.06 we failed to reject the null (at 95%). We conclude then that residuals are normally distributed, with the caveat that they are not at 90%.

Regression: joint test (*F-test*)

To test whether two coefficients are jointly different from 0 use the command test (see Hamilton, 2006, p.175).

xi: quietly regress csat expense percent percent2 income high college i.region, robust Note 'quietly' suppress the regression output

To test the null hypothesis that both coefficients do not have any effect on csat ($\beta_{high} = 0$ and $\beta_{college}$ = 0), type:

test high college

- . test high college

 - (1) high = 0 (2) college = 0

$$F(2, 40) = 17.12$$

 $Prob > F = 0.0000$

The p-value is 0.0000, we reject the null and conclude that both variables have indeed a significant effect on SAT.

Some other possible tests are (see Hamilton, 2006, p.176):

```
test income = 1
test high = college
test income = (high + college)/100
```

Stata temporarily stores the coefficients as _b[varname], so if you type:

```
gen percent_b = _b[percent]
gen constant_b = _b[_cons]
```

You can also save the standard errors of the variables _se[varname]

```
gen percent_se = _se[percent]
gen constant_se = _se[_cons]
```

summarize percent_b percent_se constant_b constant_se

Vari abl e	0bs	Mean	Std. Dev.	Mi n	Max
percent_b percent_se constant_b	51 51 51	- 5. 945267 . 6405529 873. 9537	0 0 0	- 5. 945267 . 6405529 873. 9537	- 5. 945267 . 6405529 873. 9537
constant_se	51	58. 12895	0	58 . 12895	58 . 12895

Regression: saving regression coefficients/getting predicted values

You can see a list of stored results by typing after the regression ereturn list:

```
. xi: quietly regress csat expense percent percent2 income high college i.region, robust
                  _I regi on_1-4
                                      (naturally coded; _Iregion_1 omitted)
i . regi on
. ereturn list
scal ars:
                  e(N) = 50
               e(df_m) = 9
               e(df_r) = 40
                  e(F) = 76.92400040408057
                 e(r2) = .9404045015031877
               e(rmse) = 17.81259357987284
                e(mss) = 200269.8403983309
                e(rss) = 12691.53960166909
               e(r2 \ a) = .9269955143414049
                 e(11) = -209.3636234584767
               e(11\ 0) = -279.8680043669825
macros:
            e(cmdline): "regress csat expense percent percent2 income high college _Iregion_*, robust"
              e(title): "Linear regression"
                e(vce) : "robust"
             e(depvar) : "csat"
                e(cmd) : "regress"
         e(properties) : "b V"
            e(predict) : "regres_p"
              e(model):
                         "ols"
          e(estat_cmd) : "regress_estat"
            e(vcetype) : "Robust"
matrices:
                  e(b) : 1 \times 10
                  e(V) : 10 x 10
functions:
             e(sample)
```

The following are general guidelines for building a regression model*

- 1. Make sure all relevant predictors are included. These are based on your research question, theory and knowledge on the topic.
- Combine those predictors that tend to measure the same thing (i.e. as an index).
- 3. Consider the possibility of adding interactions (mainly for those variables with large effects)
- 4. Strategy to keep or drop variables:
 - 1. Predictor not significant and has the expected sign -> Keep it
 - 2. Predictor not significant and does not have the expected sign -> Drop it
 - 3. Predictor is significant and has the expected sign -> Keep it
 - 4. Predictor is significant but does not have the expected sign -> Review, you may need more variables, it may be interacting with another variable in the model or there may be an error in the data.

^{*}Gelman, Andrew, Jennifer Hill, *Data Analysis Using Regression and Multilevel/Hierarchical Models*, 2007, p. 69

Regression: publishing regression output (outreg2)

The command outreg2 gives you the type of presentation you see in published papers. If outreg2 is not available you need to install it by typing ssc install outreg2

Let's say the regression is regress csat percent percent 2 high, robust

The basic syntax for outreg2 is: outreg2 using [pick a name], [type either word or excel]

After the regression type the following if you want to export the results to excel*

outreg2 using results, excel

. outreg2 using results, excel

"results. xml" seeout

Click here to see the

Or this if you want to export to word

outreg2 using results, word

. outreg2 using results, word
"results.rtf"
seeout

Click here to see the file

In e	excel	
	Α	В
1	v1	v2
3		(3)
3	COEFFICIENT	csat
4		
5	percent	-6.520***
6		(0.49)
7	percent2	0.0537***
8		(0.0056)
9	high	2.987***
10		(0.55)
11	Constant	844.8***
12		(38.8)
13	Observations	51
14	R-squared	0.93
15	Robust standard errors in parentheses	
16	*** p<0.01, ** p<0.05, * p<0.10	

iii word	
COEFFICIENT	csat
percent	-6.520***
	(0.49)
percent2	0.0537***
	(0.0056)
high	2.987***
	(0.55)
Constant	844.8***
	(38.8)
Observations	51
R-squared	0.93
4 4 4	

In word

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.10

³³

Regression: publishing regression output (outreg2)

You can add more models to compare. Lets say you want to add another model without percent 2:

regress csat percent high, robust

Now type to export the results to word (**notice** we add the append option)

outreg2 using results, word append

In	excel				
3	COEFFICIENT	csat	csat		
4					
5	percent	-6.520***	-2.315***		
6		(0.49)	(0.17)		
7	percent2	0.0537***			
8		(0.0056)			
9	high	2.987***	2.561***		
10		(0.55)	(0.72)		
11	Constant	844.8***	831.6***		
12		(38.8)	(53.5)		
13	Observations	51	51		
14	R-squared	0 .93	0 .81		
15	Robust standard errors in parentheses				
16	*** p<0.01, ** p<0.05, * p<0.1				

- 1	n	WO	rd

COEFFICIENT	csat	csat
percent	-6.520***	-2.315***
	(0.49)	(0.17)
percent2	0.0537***	
	(0.0056)	
high	2.987***	2.561***
	(0.55)	(0.72)
Constant	844.8***	831.6***
	(38.8)	(53.5)
Observations	51	51
R-squared	0.93	0.81

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

NOTE: If you run logit/probit regression with odds ratios you need to add the option eform to export the odd ratios

Type help outreg2 for more details. If you do not see outreg2, you may have to install it by typing ssc install outreg2. If this does not work type findit outreg2, select from the list and click "install".

Note: If you get the following error message (when you use the option append or replace it means that you need to close the excel/word window.

Regression: publishing regression output (outreg2) continue

For a customized look, here are some options:

```
*** Excel
```

outreg2 using results, bdec(2) tdec(2) rdec(2) adec(2) alpha (0.01, 0.05, 0.10) addstat(Adj. R-squared, e(r2_a)) excel

*** Word

outreg2 using results, bdec(2) tdec(2) rdec(2) adec(2) alpha (0.01, 0.05, 0.10) addstat(Adj. R-squared, e(r2_a)) word

For excel outreg2 using results, bdec(2) tdec(2) rdec(2) adec(2) alpha(0.001, 0.01, 0.05) addstat(Adj. R-squared, e (r2_a)) excel results.xml" eeout Click here to see the output, a excel/word window will open For word outreg2 using results, bdec(2) tdec(2) rdec(2) adec(2) alpha(0.001, 0.01, 0.05) addstat(Adj. R-squared, e(r 2 a)) word results.rtf" eeout Set # of Set # of decimals Name of Include some additional statistic, in this decimals for for added statistics the file for case adj. R-sgr. You can select any auxiliary (addstat option) statistics on the return lists (e-class, rthe output statistics class or s-class). After running the regression type ereturn list for a list Click on secout. Set # of Set # of Levels of of available statistics. decimals to browse the decimals significance for the R² results for coefficients

Regression: interaction between dummies

Interaction terms are needed whenever there is reason to believe that the effect of one independent variable depends on the value of another independent variable. We will explore here the interaction between two dummy (binary) variables. In the example below there could be the case that the effect of student-teacher ratio on test scores may depend on the percent of English learners in the district*.

- Dependent variable (Y) Average test score, variable testscr in dataset.
- Independent variables (X)
 - Binary hi_str, where '0' if student-teacher ratio (str) is lower than 20, '1' equal to 20 or higher.
 - In Stata, first generate hi_str = 0 if str<20. Then replace hi_str=1 if str>=20.
 - Binary hi_el, where '0' if English learners (el_pct) is lower than 10%, '1' equal to 10% or higher
 - In Stata, first generate hi_el = 0 if el_pct<10. Then replace hi_el=1 if el_pct>=10.
 - Interaction term str_el = hi_str * hi_el. In Stata: generate str_el = hi_str*hi_el

We run the regression

regress testscr hi_el hi_str str_el, robust

```
regress testscr hi_el hi_str str_el, robust
inear regression
                                                         Number of obs =
                             Robust
                                                 P>|t|
                                                            [95% Conf. Interval]
                    coef.
                            Std. Err.
    testscr
                            2.345952
      hi_el
                -18.16295
                                                 0.000
                                         -7.74
                                                           -22.77435
     hi_str
                                                  0.324
     str_el
                                        478.46
                                                 0.000
```

The equation is testscr_hat = 664.1 - 18.1*hi_el - 1.9*hi_str - 3.5*str_el

The effect of hi_str on the tests scores is -1.9 but given the interaction term (and assuming all coefficients are significant), the net effect is -1.9 -3.5*hi_el. If hi_el is 0 then the effect is -1.9 (which is hi_str coefficient), but if hi_el is 1 then the effect is -1.9 -3.5 = -5.4. In this case, the effect of student-teacher ratio is more negative in districts where the percent of English learners is higher.

See the next slide for more detailed computations.

Regression: interaction between dummies (cont.)

You can compute the expected values of test scores given different values of hi_str and hi_el. To see the effect of hi_str given hi_el type the following right after running the regression in the previous slide.

```
. predict yhat1 if hi_str==0 & hi_el==0
(option xb assumed; fitted values)
(271 missing values generated)
. predict yhat2 if hi_str==1 & hi_el==0
(option xb assumed; fitted values)
(341 missing values generated)
. predict yhat3 if hi_str==0 & hi_el==1
(option xb assumed; fitted values)
(331 missing values generated)
. predict yhat4 if hi_str==1 & hi_el==1
(option xb assumed; fitted values)
(317 missing values generated)
```

These are different scenarios holding constant hi_el and varying hi_str. Below we add some labels

- . label variable yhat1 "Low str/Low el"
- . label variable yhat2 "High str/Low el"
- . label variable yhat3 "Low str/High el"
- . label variable yhat4 "High str/High el"

We then obtain the average of the estimations for the test scores (for all four scenarios, notice same values for all cases).

. summarize y	hat1 yhat2 y	hat3 yhat4			
Variable	Obs	Mean	Std. Dev.	Min	Max
yhat1 yhat2 yhat3 yhat4	149 79 89 103	664.1433 662.2355 645.9803 640.5782	0 0 0	664.1433 662.2355 645.9803 640.5782	664.1433 662.2355 645.9803 640.5782

```
. display 664.1 - 662.2
1.9
. display 645.9 - 640.5
5.4
. display 5.4 - 1.9
3.5
```

Here we estimate the net effect of low/high student-teacher ratio holding constant the percent of English learners. When hi_el is 0 the effect of going from low to high student-teacher ratio goes from a score of 664.2 to 662.2, a difference of 1.9. From a policy perspective you could argue that moving from high str to low str improve test scores by 1.9 in low English learners districts.

When hi_el is 1, the effect of going from low to high student-teacher ratio goes from a score of 645.9 down to 640.5, a decline of 5.4 points (1.9+3.5). From a policy perspective you could say that reducing the str in districts with high percentage of English learners could improve test scores by 5.4 points.

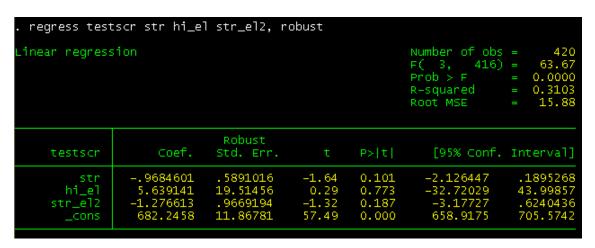
Regression: interaction between a dummy and a continuous variable

Lets explore the same interaction as before but we keep student-teacher ratio continuous and the English learners variable as binary. The question remains the same*.

- Dependent variable (Y) Average test score, variable testscr in dataset.
- Independent variables (X)
 - Continuous str, student-teacher ratio.
 - Binary hi_el, where '0' if English learners (el_pct) is lower than 10%, '1' equal to 10% or higher
 - Interaction term str_el2 = str * hi_el. In Stata: generate str_el2 = str*hi_el

We will run the regression

regress testscr str hi_el str_el2, robust



The equation is testscr_hat = 682.2 - 0.97*str + 5.6*hi_el - 1.28*str_el2

The effect of str on testscr will be mediated by hi_el.

- If hi_el is 0 (low) then the effect of str is 682.2 0.97*str.
- If hi_el is 1 (high) then the effect of str is 682.2 0.97*str + 5.6 1.28*str = 687.8 2.25*str

Notice that how hi_el changes both the intercept and the slope of str. Reducing str by one in low EL districts will increase test scores by 0.97 points, but it will have a higher impact (2.25 points) in high EL districts. The difference between these two effects is 1.28 which is the coefficient of the interaction (Stock and Watson, 2003, p.223).

*The data used in this section is the "California Test Score" data set (caschool.dta) from chapter 6 of the book Introduction to Econometrics from Stock and Watson, 2003. Data can be downloaded from http://wps.aw.com/aw_stock_ie_2/50/13016/3332253.cw/index.html. For a detailed discussion please refer to the respective section in the book.

Regression: interaction between two continuous variables

Lets keep now both variables continuous. The question remains the same*.

- Dependent variable (Y) Average test score, variable testscr in dataset.
- Independent variables (X)
 - Continuous str, student-teacher ratio.
 - Continuous el_pct, percent of English learners.
 - Interaction term str_el3 = str * el_pct. In Stata: generate str_el3 = str*el_pct

We will run the regression

regress testscr str el_pct str_el3, robust

```
regress testsor str el_pot str_el3, robust
inear regression
                                                         Number of obs =
                                                                  416) =
                                                         R-squared
                             Robust
                    coef.
                                                 P>|t|
                                                            [95% Conf. Interval]
                            Std. Err.
    testscr
                -1.117018
                                                  0.058
                             .5875135
                                         -1.90
                                                           -2.271884
                                                                         .0378468
        str
                                         -1.80
                                                 0.073
     el_pct
                -.6729116
                            .3741231
                                                           -1.408319
    str_el3
                 .0011618
                            .0185357
                                          0.06
                                                  0.950
                                                           -.0352736
                 686.3385
                            11.75935
                                         58.37
                                                  0.000
                                                             663.2234
                                                                         709.453
      _cons
```

The equation is testscr_hat = $686.3 - 1.12*str - 0.67*el_pct + 0.0012*str_el3$

The effect of the interaction term is very small. Following Stock and Watson (2003, p.229), algebraically the slope of str is

- -1.12 + 0.0012*el_pct (remember that str_el3 is equal to str*el_pct). So:
 - If el_pct = 10, the slope of str is -1.108
 - If el_pct = 20, the slope of str is -1.096. A difference in effect of 0.012 points.

In the continuous case there is an effect but is very small (and not significant). See Stock and Watson, 2003, for further details.

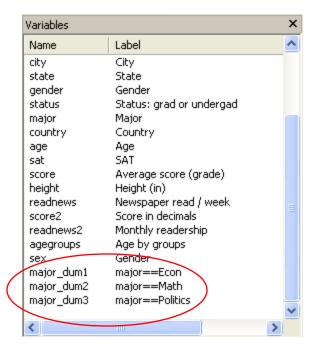
*The data used in this section is the "California Test Score" data set (caschool.dta) from chapter 6 of the book Introduction to Econometrics from Stock and Watson, 2003. Data can be downloaded from http://wps.aw.com/aw_stock_ie_2/50/13016/3332253.cw/index.html. For a detailed discussion please refer to the respective section in the book.

Creating dummies

You can create dummy variables by either using recode or using a combination of tab/gen commands: tab major, generate(major_dum)

. tab major,	generate(major_dum)			
Major	Freq.	Percent	⊂um.	
Econ Math Politics	10 10 10	33.33 33.33 33.33	33.33 66.67 100.00	
Total	30	100.00		

Check the 'variables' window, at the end you will see three new variables. Using tab1 (for multiple frequencies) you can check that they are all 0 and 1 values



. tab1 major_dum1 major_dum2 major_dum3				
-> tabulation	of major_du	m1		
major==Econ	Freq.	Percent	⊂um.	
0 1	20 10	66.67 33.33	66.67 100.00	
Total	30	100.00		
-> tabulation	n of major_du	m2		
major==Math	Freq.	Percent	⊂um.	
0 1	20 10	66.67 33.33	66.67 100.00	
Total	30	100.00		
-> tabulation	n of major_du	m3		
major==Poli tics	Freq.	Percent	Cum.	
0 1	20 10	66.67 33.33	66.67 100.00	
Total	30	100.00	40	

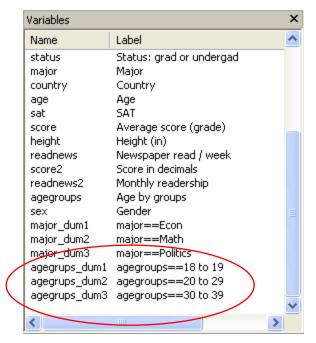
Creating dummies (cont.)

Here is another example:

tab agregroups, generate(agegroups_dum)

. tab agegrou	ıps, generate((agegrups_dum))
Age by groups	Freq.	Percent	Cum.
18 to 19 20 to 29 30 to 39	10 9 11	33.33 30.00 36.67	33.33 63.33 100.00
Total	30	100.00	

Check the 'variables' window, at the end you will see three new variables. Using tab1 (for multiple frequencies) you can check that they are all 0 and 1 values



. tab1 agegr	ups_dum1 ageq	grups_dum2 a	gegrups_dum3
-> tabulation	of agegrups	_dum1	
agegroups== 18 to 19	Freq.	Percent	cum.
0 1	20 10	66.67 33.33	66.67 100.00
Total	30	100.00	
-> tabulation	of agegrups	_dum2	
agegroups== 20 to 29	Freq.	Percent	cum.
0 1	21 9	70.00 30.00	70.00 100.00
Total	30	100.00	
-> tabulation	of agegrups	_dum3	
agegroups== 30 to 39	Freq.	Percent	cum.
0 1	19 11	63.33 36.67	63.33 100.00
Total	30	100.00	

Type help [command name] in the windows command for details

Frequently used Stata commands

Category	Stata commands
Getting on-line help	help
	search
Operating-system interface	pwd
	cd
	sysdir
	mkdir
	dir / Is
	erase
	сору
	type
Using and saving data from disk	use
	clear
	save
	append
	merge
	compress
Inputting data into Stata	input
	edit
	infile
	infix
	insheet
The Internet and Updating Stata	update
	net
	ado
	news

Basic data reporting Data manipulation Formatting Keeping track of your work

Convenience

Source: http://www.ats.ucla.edu/stat/stata/notes2/commands.htm

describe

codebook

inspect

browse

count assert

summarize
Table (tab)
tabulate

generate

replace

egen
recode
rename
drop
keep
sort
encode
decode
order

by

reshape

format label

display

42

log notes

list

Is my model OK? (links)

Regression diagnostics: A checklist

http://www.ats.ucla.edu/stat/stata/webbooks/reg/chapter2/statareg2.htm

Logistic regression diagnostics: A checklist

http://www.ats.ucla.edu/stat/stata/webbooks/logistic/chapter3/statalog3.htm

Times series diagnostics: A checklist (pdf)

http://homepages.nyu.edu/~mrg217/timeseries.pdf

Times series: dfueller test for unit roots (for R and Stata)

http://www.econ.uiuc.edu/~econ472/tutorial9.html

Panel data tests: heteroskedasticity and autocorrelation

- http://www.stata.com/support/faqs/stat/panel.html
- http://www.stata.com/support/faqs/stat/xtreg.html
- http://www.stata.com/support/faqs/stat/xt.html
- http://dss.princeton.edu/online_help/analysis/panel.htm

I can't read the output of my model!!! (links)

Data Analysis: Annotated Output

http://www.ats.ucla.edu/stat/AnnotatedOutput/default.htm

Data Analysis Examples

http://www.ats.ucla.edu/stat/dae/

Regression with Stata

http://www.ats.ucla.edu/STAT/stata/webbooks/reg/default.htm

Regression

http://www.ats.ucla.edu/stat/stata/topics/regression.htm

How to interpret dummy variables in a regression

http://www.ats.ucla.edu/stat/Stata/webbooks/reg/chapter3/statareg3.htm

How to create dummies

http://www.stata.com/support/faqs/data/dummy.html http://www.ats.ucla.edu/stat/stata/faq/dummy.htm

Logit output: what are the odds ratios?

http://www.ats.ucla.edu/stat/stata/library/odds_ratio_logistic.htm

Topics in Statistics (links)

What statistical analysis should I use?

http://www.ats.ucla.edu/stat/mult_pkg/whatstat/default.htm

Statnotes: Topics in Multivariate Analysis, by G. David Garson

http://www2.chass.ncsu.edu/garson/pa765/statnote.htm

Elementary Concepts in Statistics

http://www.statsoft.com/textbook/stathome.html

Introductory Statistics: Concepts, Models, and Applications

http://www.psychstat.missouristate.edu/introbook/sbk00.htm

Statistical Data Analysis

http://math.nicholls.edu/badie/statdataanalysis.html

Stata Library. Graph Examples (some may not work with STATA 10)

http://www.ats.ucla.edu/STAT/stata/library/GraphExamples/default.htm

Comparing Group Means: The T-test and One-way ANOVA Using STATA, SAS, and SPSS

http://www.indiana.edu/~statmath/stat/all/ttest/

Useful links / Recommended books

- DSS Online Training Section http://dss.princeton.edu/training/
- UCLA Resources to learn and use STATA http://www.ats.ucla.edu/stat/stata/
- DSS help-sheets for STATA http://dss/online_help/stats_packages/stata/stata.htm
- Introduction to Stata (PDF), Christopher F. Baum, Boston College, USA. "A 67-page description of Stata, its key features and benefits, and other useful information." http://fmwww.bc.edu/GStat/docs/StataIntro.pdf
- STATA FAQ website http://stata.com/support/faqs/
- Princeton DSS Libguides http://libguides.princeton.edu/dss

Books

- Introduction to econometrics / James H. Stock, Mark W. Watson. 2nd ed., Boston: Pearson Addison Wesley, 2007.
- Data analysis using regression and multilevel/hierarchical models / Andrew Gelman, Jennifer Hill.
 Cambridge; New York: Cambridge University Press, 2007.
- Econometric analysis / William H. Greene. 6th ed., Upper Saddle River, N.J.: Prentice Hall, 2008.
- Designing Social Inquiry: Scientific Inference in Qualitative Research / Gary King, Robert O. Keohane, Sidney Verba, Princeton University Press, 1994.
- Unifying Political Methodology: The Likelihood Theory of Statistical Inference / Gary King, Cambridge University Press, 1989
- Statistical Analysis: an interdisciplinary introduction to univariate & multivariate methods / Sam Kachigan, New York: Radius Press, c1986
- Statistics with Stata (updated for version 9) / Lawrence Hamilton, Thomson Books/Cole, 2006