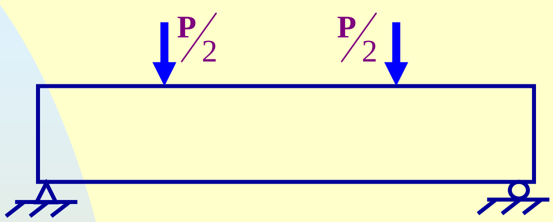
Mechanics of Materials

Reinforced Concrete Beam

Concrete Beam

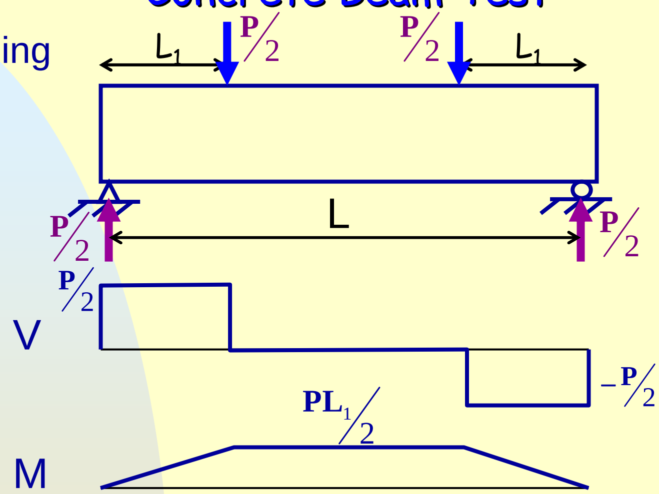
- We will examine a concrete beam in bending



- A concrete beam is what we call a **composite beam**
- It is made of **two materials: concrete and steel**
- Concrete is also a composite

Concrete Beam Test

- Loading



- Maximum Moment is in the center region

Concrete Strength

- We determine the compressive strength of concrete with a concrete cylinder test
- Cylinders are typically 6 inches in diameter
- 12 inches long
- We want the stress-strain results so we measure load and change in length

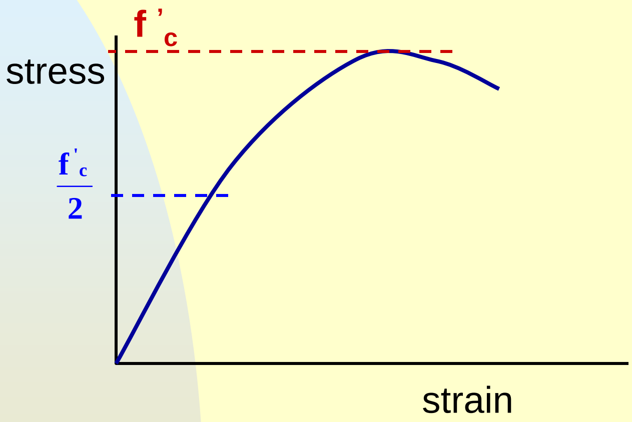
$$\sigma_C = \frac{P}{A} = \frac{P}{\frac{\pi D^2}{4}} = \frac{P}{\frac{\pi 6^2}{4}}$$

$$\varepsilon_C = \frac{\Delta L}{L} = \frac{\Delta/2}{6}$$

Concrete Cylinder Test

Concrete Stress Strain

- The concrete stress-strain diagram is not linear



Composite

- When we examine a beam we generally need a **value of E** to use in the analysis
- **Do we use E for concrete or steel?**
- We will usually pretend that the beam is made of only one material, usually concrete
- We **Transform the steel to an equivalent amount of concrete**
- **How much concrete do we substitute for the steel?**

Modular Ratio

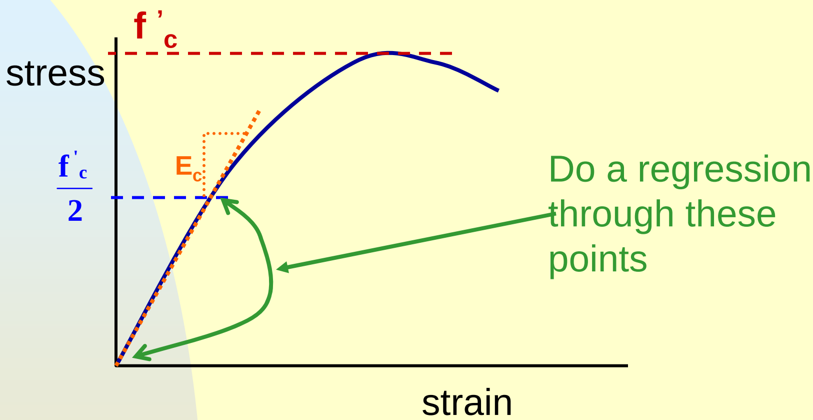
- **Which is stiffer, steel or concrete?**
- E_s for steel is about 29,000 ksi
- E_c for concrete is about 3 to 6,000 ksi, and will be different for every batch of concrete
- We get E_c **from the cylinder tests**
- Let's define **n to be the modular ratio**

$$n = \frac{E_s}{E_C}$$

- It tells you how much stiffer steel is compared to concrete

Modulus of Concrete-E_c

- The concrete stress-strain diagram is not linear



- **E_c is the slope of the stress-strain curve up to about half the strength of the concrete**

Steel is Stiffer

- Since the **steel is stiffer than the concrete,** when we replace the steel with concrete, we will **need more concrete to make it equivalent**
- **How much more?**
- We will **multiply the amount of steel by n** to get the equivalent amount of concrete

$$A_s' = n\ A_s$$

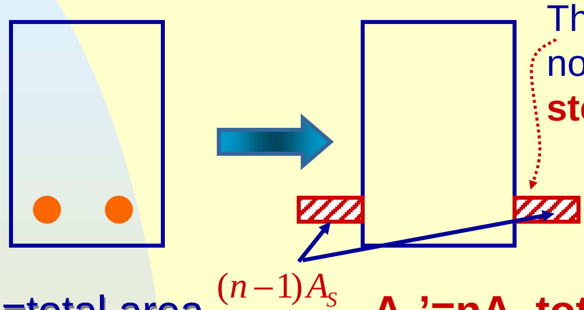
- A_s is the total cross sectional area of the steel for all the bars

Transformed Section

- When we replace the steel with equivalent concrete, we have effectively **transformed everything to concrete**
- We call the resulting all concrete beam the **Transformed Section**
- Let's examine the process . . .

Equivalent Steel

- **Replace A_s by A_s'**



This **concrete** now models the **steel**

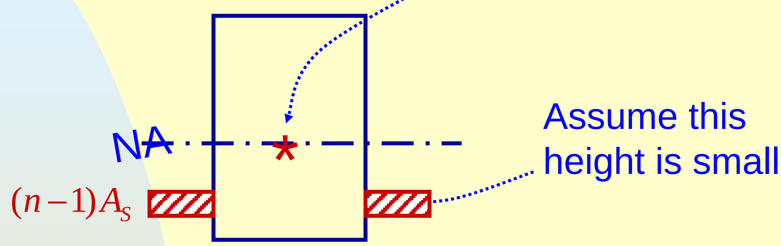
A_s=total area of steel

$A_s'=nA_s$ total

$A_s'=(n-1)\ A_s$ after filling "holes"

Neutral Axis

- The **NA passes through the centroid,** the balance point



- The **moment of the area above the NA is the same as the moment of the area** below the NA (both the real concrete and the equivalent concrete)

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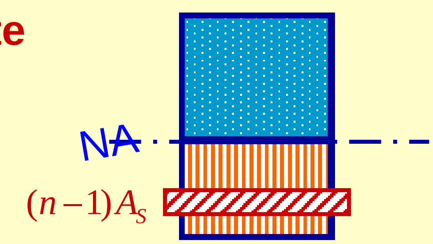
Transformed Section

- We want to find the $\mathbf{I_{tr}}$ of our transformed section **about the NA**
- We first need to **locate the NA**



- The section is composed of **three parts**
  - The part **above the NA**
  - The portion **below the NA**
  - And the transformed area, $\mathbf{(n-1)A_s}$

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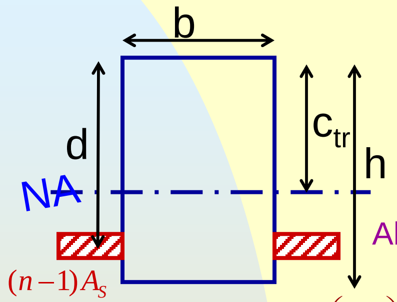
Neutral Axis

- Locate the **NA**



Above NA Below NA Transformed steel

$$bc_{tr}\left(\frac{c_{tr}}{2}\right)=b\left(h-c_{tr}\right)\frac{\left(h-c_{tr}\right)}{2}+(n-1)A_s\left(d-c_{tr}\right)$$

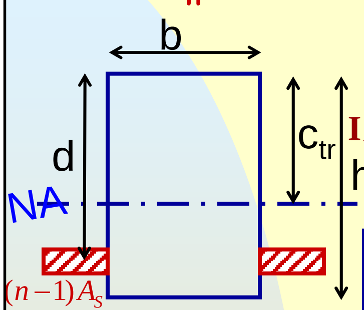
- **Solve for c_{tr}**
- This locates the **NA**

I_{tr} of the Transformed Section

- **Find I_{tr}**

Parallel axis theorem

$$I = I_{cg} + Ad^2$$



$$I_{tr}=\frac{bc_{tr}^{\ 3}}{3}+\frac{b\left(h-c_{tr}\right)^3}{3}+(n-1)A_s\left(d-c_{tr}\right)^2$$

Top, compression concrete

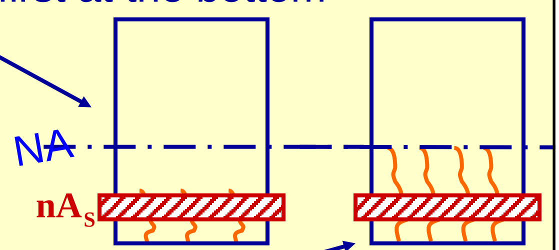
Bottom, Tension concrete

Steel transformed into concrete

- Solve for I_{tr}

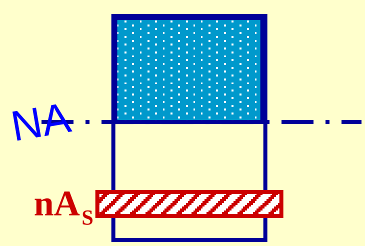
Cracks

- What happens to the tension concrete as the load get higher and higher?
- It **starts to crack,** first at the bottom
- As the load increases, the **cracks progress higher** and higher
- **Finally, all the concrete in tension will crack**
- Once the tension concrete cracks, it cannot hold tension stresses



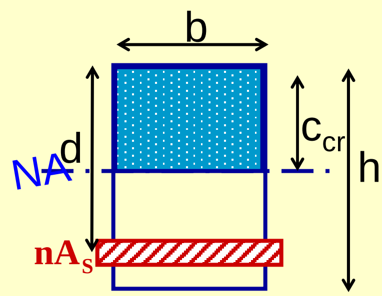
Cracked Section

- We call the resulting section with all the concrete below the NA cracked, the **cracked section**
- Now we effectively have only **two remaining areas,** because the concrete below the NA is cracked
- We must find the **new location of the NA**



Cracked Section NA

- Since there is less concrete toward the bottom, the **NA will move up**
- Calculate the **new centroid** to locate the **NA**



$$bc_{cr}\left(\frac{c_{cr}}{2}\right) = nA_s\left(d - c_{cr}\right)$$

- **Solve for c_{cr}**
- This locates the **NA**

I_{cr} of the Cracked Section

- Now we can find the **cracked moment of inertia**

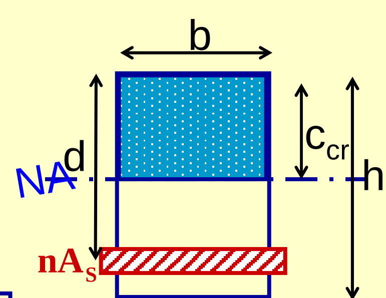
Parallel axis theorem

$$I = I_{cg} + Ad^2$$

$$I_{cr} = \frac{bc_{cr}^{\ 3}}{3} + nA_s\left(d - c_{cr}\right)^2$$

Top, compression concrete

Steel transformed into concrete



- Solve for I_{cr}, the **moment of Inertia of the cracked section**

Load the Beam

- We have **two values** of the moment of Inertia **I, I_{tr} and I_{cr}**
- **Which do we use for the beam bending test?**
- At the start (before cracking) we use I_{tr}
- At a load corresponding to steel yielding, we can use I_{cr} for the beam
- In between I for the beam varies from I_{tr} to I_{cr} as tension cracks propagate upward and along the length of the beam

Nonlinear Beam

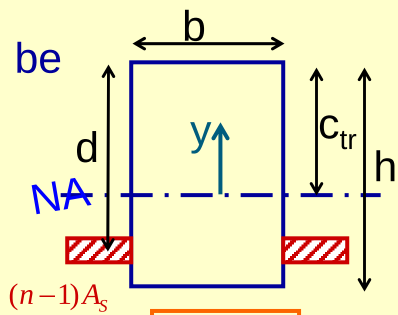
- **The response of the beam to the load is nonlinear in two ways**
  - The stress-strain curve for **concrete is nonlinear, E changes**
  - The **section itself is nonlinear,** since the **I changes** as the load increases

Concrete Stress

- What is the stress in the beam at cracking?
- Stress in the concrete will be

$$\sigma_{conc} = \frac{My}{I}$$

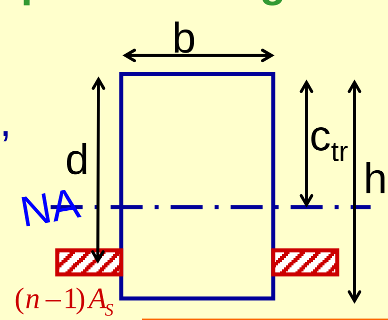
- Which I do we use?
- we use I_{tr}



$$\sigma_c = \frac{Mc_{tr}}{I_{tr}}$$

Steel Stress

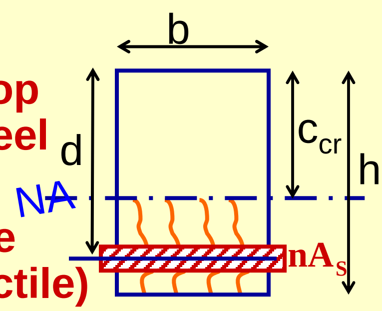
- What is the steel stress up to cracking?
- Since our transformed beam is made of concrete, we must **multiply the stress we calculated in the steel by n**
- we use I_{tr}



$$\sigma_{steel} = n\frac{M(d-c_{tr})}{I_{tr}}$$

Flexural Failure

- How will the beam fail?
- 2 possibilities for flexure:
  - The **concrete on the top crushes before the steel yields (brittle)**
  - The **steel yields before concrete crushes (ductile)**



- The concrete will fail in compression at a concrete strain of ~0.003-0.004.
- The steel will yield at a steel strain of f_y/E_s or a steel stress of f_y.

Cracking of the Concrete in Tension

- As the load is applied to the beam, the tension stress at the bottom of the beam increases
- This is the approximate **cracking stress** for concrete in tension

$$f_r = 7.5\sqrt{f'_c}$$

- Here is the **Bending stress equation** for the tensile stress in the concrete at the bottom of the beam.

$$f_r = \frac{M_{cr} y}{I_{tr}}$$

Why use I_{tr}?

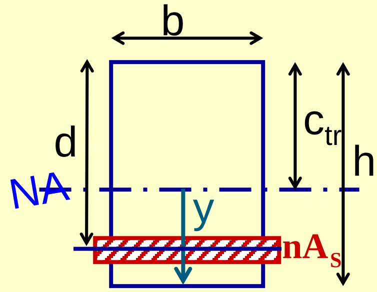
Cracking of the Concrete in Tension

- Use these equations:

$$f_r = 7.5\sqrt{f'_c}$$

$$f_r = \frac{M_{cr}\,y}{I_{tr}}$$

$$y = h - c_{tr}$$



- Find the moment M_{cr} that will cause the concrete to start cracking.
- Then find the load, P_{cr}, that will cause this moment.

$$M_{cr} = \frac{P_{cr}L_1}{2}$$

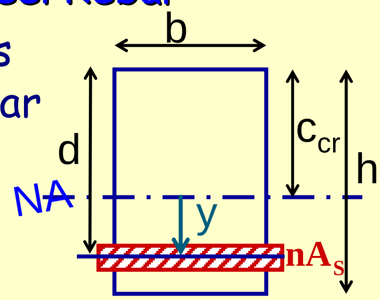
$$P_{cr} = \frac{2M_{cr}}{L_1}$$

Try to locate this point on your graph of load vs deflection as a change in slope

Yielding of the Steel Rebar

- Here is the bending stress equation for the steel rebar with σ_y=68,000psi

$$\sigma_{steel} = 68{,}000 \text{ psi} = \frac{nM_y(d - c_{cr})}{I_{cr}}$$



- Find the moment M_y and the load P_y that will cause the steel to yield

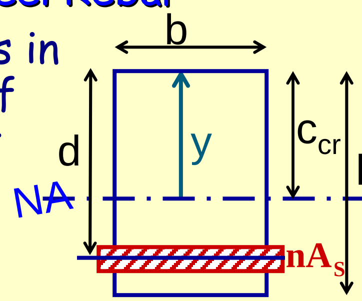
$$M_y = \frac{P_yL_1}{2}$$

$$P_y = \frac{2M_y}{L_1}$$

If you had steel yielding, try to locate this point on your graph of load vs deflection.

Yielding of the Steel Rebar

- Next, calculate the stress in the concrete at the top of the beam at the load that causes the steel to yield

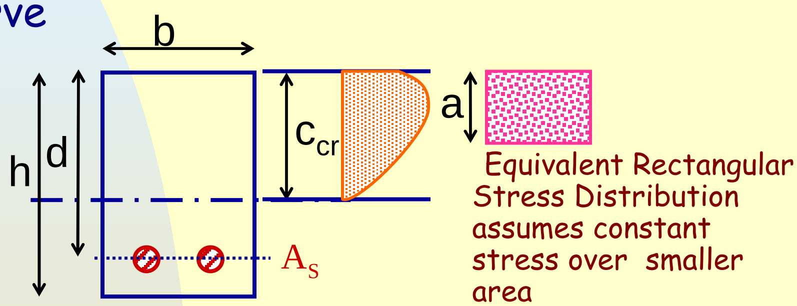


$$f_{c-yield} = \frac{M_y c_{cr}}{I_{cr}}$$

- Determine also the ratio fc-yield /f'c, to see if it is less than approximately 0.7, an upper bound for "roughly" linear concrete stresses
- Was the concrete behaving linearly? If not, more accurate equations can be used.

Ultimate Failure of the Concrete

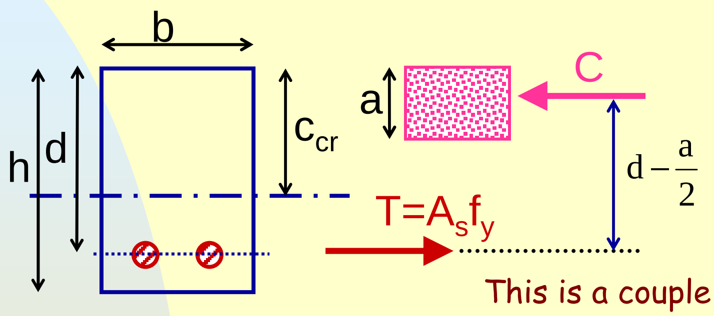
- Here is the nonlinear stress distribution in the compression concrete due to bending
- It is shaped like the concrete stress-strain curve



- To make it easier to model the nonlinear response of the concrete, a simplified stress diagram is used

Ultimate Failure of the Concrete

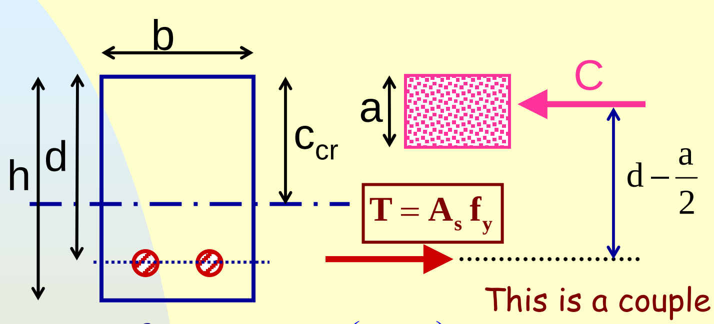
- Here is the resulting force diagram



- For equilibrium: **C=T**

Ultimate Failure of the Concrete

- Here is the resulting force diagram



- Moment of the couple:

$$M_{ult} = T\left(d - \frac{a}{2}\right)$$

$$M_{ult} = A_s f_y \left(d - \frac{a}{2}\right)$$

$$a = A_s * \frac{f_y}{0.85 f'_C\, b}$$

f_y can be replaced by f_{ult} to account for strain hardening

$$f_{ult} \approx 1.25 f_y$$

Ultimate Failure of the Concrete

- Once this ultimate moment for the beam is found, calculate the load, P_{ult}, that would cause this moment

$$M_{ult} = \frac{P_{ult} L_1}{2}$$

$$P_{ult} = \frac{2M_{ult}}{L_1}$$

$$M_{ult} = T\left(d - \frac{a}{2}\right)$$

$$M_{ult} = A_s f_y \left(d - \frac{a}{2}\right)$$

$$a = A_s * \frac{f_y}{0.85 f'_C b}$$

- This is the load that would cause the concrete to crush, usually after the steel yields

Other Types of Ultimate Failure

- Although failure of the concrete usually occurs instead of ultimate fracture of the steel, other final failure modes may occur instead
- This could include concrete bond failure along the plane of the rebar
- Shear failure of the concrete may also occur
- How did your beam ultimately fail?

Shear Failure

- **The beam may fail due to excessive shear stresses** (i.e., diagonal tension), if the shear capacity of the beam is less than its flexural capacity
- The average shear stress at which a diagonal tension crack forms depends on the presence of flexural stresses
- There is a lot of scatter in the data that describes the maximum shear stress that a concrete beam can withstand

Shear Failure

- Here is a range of average shear stresses that can be used to estimate shear failure :

$$\tau_{min} = 2\sqrt{f'_c}$$

$$\tau_{max} = 3.5\sqrt{f'_c}$$

- **The average shear stress that concrete can withstand** can be approximated by this range
- For most cases and **especially regions of high shear and moment,** flexural cracks will form first, reducing shear stress at diagonal cracking to τ_{min}
- In **regions of high shear and small moment** (e.g., simple supports and p.i.), then "web shear" cracks may form at a stress level given by τ_{max}

Shear Failure

- Here is the approximate shear stress equation:

$$V_{concrete} = \tau(area) = \tau(bd)$$

- With the range of τ defined by our equations, this defines a **range of internal shear force, $V_{concrete}$,** that should represent the maximum shear force the beam can withstand.
- P_V represents an estimate for the **maximum load that would cause the concrete to fail in shear.** This failure will occur only if another mode of failure has not occurred prior to reaching this load. You will have a range of P_V based upon the range of $V_{concrete}$ calculated above.

$$V_{concrete} = \frac{P_V}{2}$$

Shear Failure

- You will have a range of P_V based upon the τ_{min} and τ_{max}

$$P_V = 2V_{concrete}$$

- Shear failures are more likely as the amount of steel rebar increases.
- For shear failure, cracking and sudden failure should be shown on the load-deflection plots.

End

Concrete Beam