

## Reinforced Concrete Design

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### Notation:

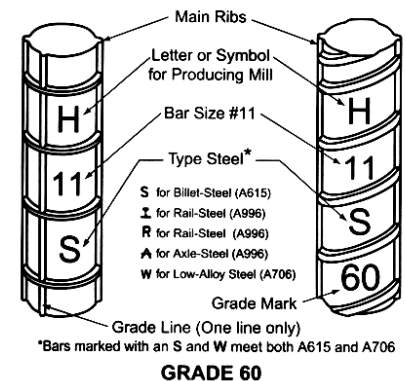
$a$	= depth of the effective compression block in a concrete beam	$E_s$	= modulus of elasticity of steel
$A$	= name for area	$f$	= symbol for stress
$A_g$	= gross area, equal to the total area ignoring any reinforcement	$f'_c$	= concrete design compressive stress
$A_s$	= area of steel reinforcement in concrete beam design	$f'_s$	= compressive stress in the compression reinforcement for concrete beam design
$A'_s$	= area of steel compression reinforcement in concrete beam design	$f_y$	= yield stress or strength
$A_{st}$	= area of steel reinforcement in concrete column design	$f_{yt}$	= yield stress or strength of transverse reinforcement
$A_v$	= area of concrete shear stirrup reinforcement	$F$	= shorthand for fluid load
$ACI$	= American Concrete Institute	$G$	= relative stiffness of columns to beams in a rigid connection, as is $\Psi$
$b$	= width, often cross-sectional	$h$	= cross-section depth
$b_E$	= effective width of the flange of a concrete T beam cross section	$H$	= shorthand for lateral pressure load
$b_f$	= width of the flange	$h_f$	= depth of a flange in a T section
$b_w$	= width of the stem (web) of a concrete T beam cross section	$I_{transformed}$	= moment of inertia of a multi-material section transformed to one material
$c$	= distance from the top to the neutral axis of a concrete beam ( <i>see x</i> )	$k$	= effective length factor for columns
$cc$	= shorthand for clear cover	$\ell_b$	= length of beam in rigid joint
$C$	= name for centroid	$\ell_c$	= length of column in rigid joint
	= name for a compression force	$l_d$	= development length for reinforcing steel
$C_c$	= compressive force in the compression steel in a doubly reinforced concrete beam	$l_{dh}$	= development length for hooks
$C_s$	= compressive force in the concrete of a doubly reinforced concrete beam	$l_n$	= clear span from face of support to face of support in concrete design
$d$	= effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel	$L$	= name for length or span length, as is $l$ = shorthand for live load
$d'$	= effective depth from the top of a reinforced concrete beam to the centroid of the compression steel	$L_r$	= shorthand for live roof load
$d_b$	= bar diameter of a reinforcing bar	$LL$	= shorthand for live load
$D$	= shorthand for dead load	$M$	= internal bending moment
$DL$	= shorthand for dead load	$M_n$	= nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
$e$	= eccentricity	$M_u$	= maximum moment from factored loads for LRFD beam design
$E$	= modulus of elasticity or Young's modulus	$n$	= modulus of elasticity transformation coefficient for steel to concrete
	= shorthand for earthquake load	$n.a.$	= shorthand for neutral axis (N.A.)
$E_c$	= modulus of elasticity of concrete		

$P_o$	= maximum axial force with no concurrent bending moment in a reinforced concrete column	$w_u$	= load per unit length on a beam from load factors
$P_n$	= nominal column load capacity in concrete design	$W$	= shorthand for wind load
$P_u$	= factored column load calculated from load factors in concrete design	$x$	= distance from the top of beam to the neutral axis ( <i>see c</i> )
$R$	= shorthand for rain or ice load = radius of curvature in beam deflection relationships ( <i>see <math>\rho</math></i> )	$y$	= vertical distance
$R_n$	= concrete beam design ratio = $M_u/bd^2$	$\beta_1$	= coefficient for determining stress block height, $a$ , based on concrete strength, $f'_c$
$s$	= spacing of stirrups in reinforced concrete beams	$\epsilon$	= strain
$S$	= shorthand for snow load	$\epsilon_t$	= strain in the steel
$t$	= name for thickness (as is $h$ )	$\epsilon_y$	= strain at the yield stress
$T$	= name for a tension force = shorthand for thermal load	$\lambda$	= modification factor for lightweight concrete
$U$	= factored design value	$\phi$	= resistance factor
$V_c$	= shear force capacity in concrete	$\phi_c$	= resistance factor for compression
$V_n$	= nominal shear force	$\gamma$	= density or unit weight
$V_s$	= shear force capacity in steel shear stirrups		= ratio of reinforcement width to width of column
$V_u$	= shear at a distance of $d$ away from the face of support for reinforced concrete beam design	$\rho$	= radius of curvature in beam deflection relationships ( <i>see R</i> ) = reinforcement ratio in concrete beam design = $A_s/bd$
$w_c$	= unit weight of concrete	$\rho_{balanced}$	= balanced reinforcement ratio in concrete beam design
$w_{DL}$	= load per unit length on a beam from dead load	$\sigma$	= engineering symbol for normal stress
$w_{LL}$	= load per unit length on a beam from live load	$\nu_c$	= shear strength in concrete design
$w_{self\ wt}$	= name for distributed load from self weight of member	$\Psi$	= relative stiffness of columns to beams in a rigid connection ( <i>see G</i> )

## Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-14)* published by the American Concrete Institute International, and uses strength design (also known as *limit state* design).

$f'_c$  = concrete compressive design strength at 28 days (units of psi when used in equations)



## Materials

Concrete is a mixture of cement, coarse aggregate, fine aggregate, and water. The cement hydrates with the water to form a binder. The result is a hardened mass with “filler” and pores. There are various types of cement for low heat, rapid set, and other properties. Other minerals or cementitious materials (like fly ash) may be added.

ASTM designations are

Type I:	Ordinary portland cement (OPC)
Type II:	Moderate heat of hydration and sulfate resistance
Type III:	High early strength (rapid hardening)
Type IV:	Low heat of hydration
Type V:	Sulfate resistant

The proper proportions, by volume, of the mix constituents determine strength, which is related to the water to cement ratio (w/c). It also determines other properties, such as workability of fresh concrete. Admixtures, such as retardants, accelerators, or superplasticizers, which aid flow without adding more water, may be added. Vibration may also be used to get the mix to flow into forms and fill completely.

*Slump* is the measurement of the height loss from a compacted cone of fresh concrete. It can be an indicator of the workability.

Proper mix design is necessary for durability. The pH of fresh cement is enough to prevent reinforcing steel from oxidizing (rusting). If, however, cracks allow corrosive elements in water to penetrate to the steel, a corrosion cell will be created, the steel will rust, expand and cause further cracking. Adequate cover of the steel by the concrete is important.

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8” up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be  $150 \text{ lb/ft}^3$ . It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

## Construction

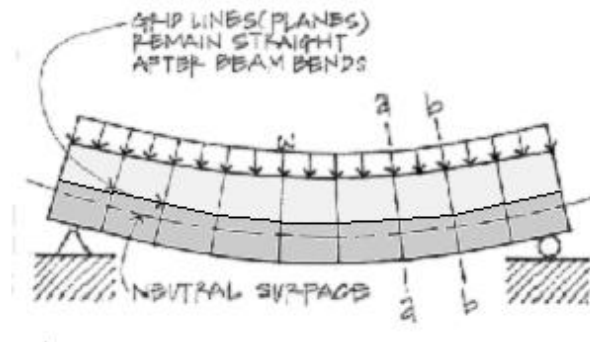
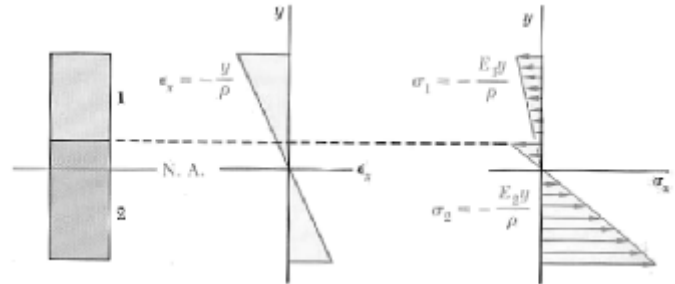
Because fresh concrete is a viscous suspension, it is cast or placed and *not poured*. Formwork must be able to withstand the hydraulic pressure. *Vibration* may be used to get the mix to flow around reinforcing bars or into tight locations, but excess vibration will cause segregation, honeycombing, and excessive *bleed* water which will reduce the water available for hydration and the strength, subsequently.

After casting, the surface must be worked. *Screeding* removes the excess from the top of the forms and gets a rough level. *Floating* is the process of working the aggregate under the surface and to “float” some paste to the surface. *Troweling* takes place when the mix has hydrated to the point of supporting weight and the surface is smoothed further and consolidated. *Curing* is allowing the hydration process to proceed with adequate moisture. Black tarps and curing

compounds are commonly used. *Finishing* is the process of adding a texture, commonly by using a broom, after the concrete has begun to set.

Behavior

Plane sections of composite materials can still be assumed to be plane (strain is linear), *but* the stress distribution is *not* the same in both materials because the *modulus of elasticity* is different. ( $f=E \cdot \epsilon$ )



$$f_1 = E_1 \epsilon = -\frac{E_1 y}{R} \qquad f_2 = E_2 \epsilon = -\frac{E_2 y}{R}$$

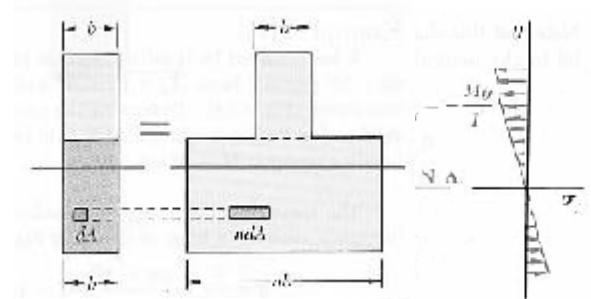
where R (or  $\rho$ ) is the radius of curvature

In order to determine the stress, we can define  $n$  as the ratio of the elastic moduli:  $n = \frac{E_2}{E_1}$

$n$  is used to transform the width of the second material such that it sees the equivalent element stress.

*Transformed Section  $y$  and  $I$*

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.



ex: When material 1 above is concrete and material 2 is steel

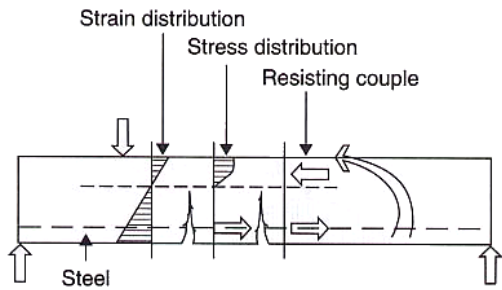
to transform steel into concrete  $n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$

to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by  $n$

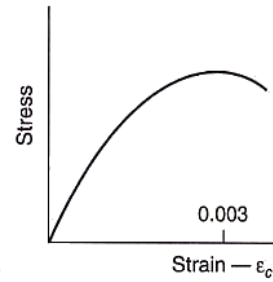
to find the moment of inertia of the equivalent concrete member,  $I_{transformed}$ , use the new geometry resulting from transforming the width of the steel

concrete stress:  $f_{concrete} = -\frac{My}{I_{transformed}}$       steel stress:  $f_{steel} = -\frac{Myn}{I_{transformed}}$

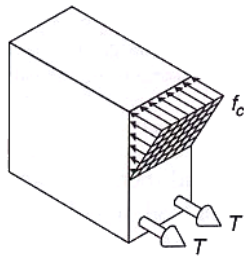
Reinforced Concrete Beam Members



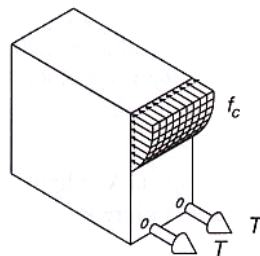
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



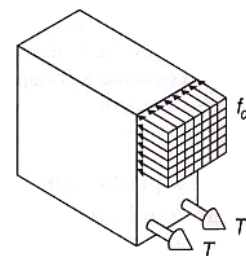
Typical stress-strain curve for concrete.



Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)



Actual stress distribution near ultimate strength (nonlinear).



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

Strength Design for Beams

The strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor  $\phi$  which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular “stress” block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress,  $f_y$

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

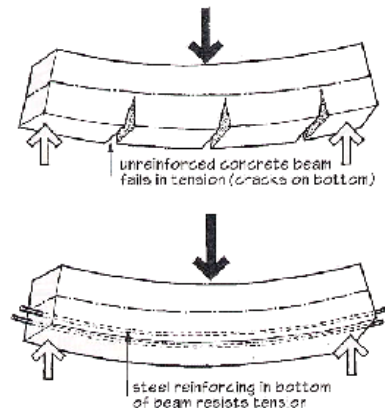
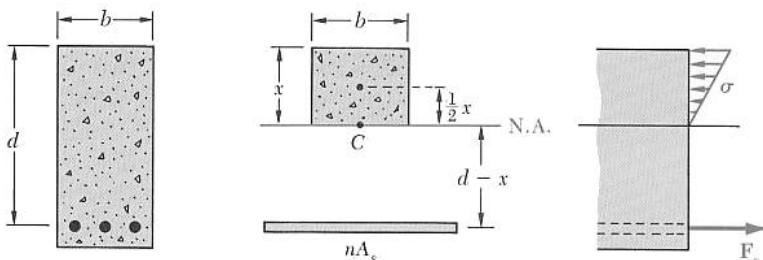


Figure 8.5: Bending in a concrete beam without and with steel reinforcing.

The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. (shown as  $x$ , but also sometimes named  $c$ ) is considered ineffective. The steel below the n.a. is in tension.

Because the n.a. is defined by the moment areas, we can solve for  $x$  knowing that  $d$  is the distance from the top of the concrete section to the centroid of the steel:

$$bx \cdot \frac{x}{2} - nA_s(d - x) = 0$$

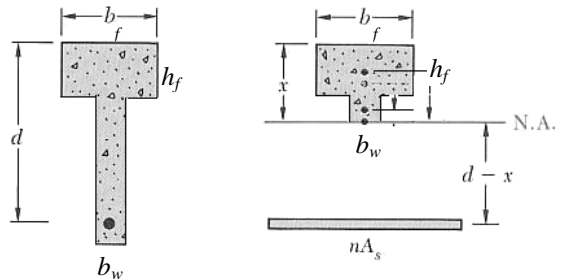
$x$  can be solved for when the equation is rearranged into the generic format with  $a$ ,  $b$  &  $c$  in the

binomial equation:  $ax^2 + bx + c = 0$  by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

*T-sections*

If the n.a. is *above* the bottom of a flange in a T section,  $x$  is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section,  $x$  is found by including the flange and the stem of the web ( $b_w$ ) in the moment area calculation:



$$b_f h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0$$

*Load Combinations (Alternative values are allowed)*

1.4D

1.2D + 1.6L + 0.5(L<sub>r</sub> or S or R)

1.2D + 1.6(L<sub>r</sub> or S or R) + (1.0L or 0.5W)

1.2D + 1.0W + 1.0L + 0.5(L<sub>r</sub> or S or R)

1.2D + 1.0E + 1.0L + 0.2S

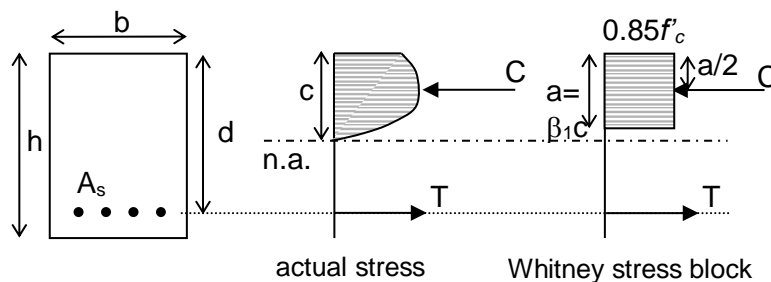
0.9D + 1.0W

0.9D + 1.0E

**ASTM STANDARD REINFORCING BARS**

Bar size, no.	Nominal diameter, in.	Nominal area, in. <sup>2</sup>	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

*Internal Equilibrium*



$C$  = compression in concrete = stress x area =  $0.85 f'_c b a$

$T$  = tension in steel = stress x area =  $A_s f_y$

$C = T$  and  $M_n = T(d-a/2)$

where  $f'_c$  = concrete compression strength

$a$  = height of stress block

$\beta_1$  = factor based on  $f'_c$

$c$  = location to the neutral axis

$b$  = width of stress block

$f_y$  = steel yield strength

$A_s$  = area of steel reinforcement

$d$  = effective depth of section

= depth to n.a. of reinforcement

$$\beta_1 = 0.85 - \left( \frac{f'_c - 4000}{1000} \right) (0.05) \geq 0.65$$

With  $C=T$ ,  $A_s f_y = 0.85 f'_c b a$  so  $a$  can be determined with  $a = \frac{A_s f_y}{0.85 f'_c b} = \beta_1 c$

## Criteria for Beam Design

For flexure design:

$M_u \leq \phi M_n$   $\phi = 0.9$  for flexure (when the section is tension controlled)

so for design,  $M_u$  can be set to  $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

## Reinforcement Ratio

The amount of steel reinforcement is *limited*. Too much reinforcement, or **over-reinforcing** will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be **under reinforced**.

The reinforcement ratio is just a fraction:  $\rho = \frac{A_s}{bd}$  (or  $p$ ). The amount of reinforcement is limited to that which results in a concrete strain of 0.003 and a minimum tensile strain of 0.004.

When the strain in the reinforcement is 0.005 or greater, the section is **tension controlled**. (For smaller strains the resistance factor reduces to 0.65 because the stress is less than the yield stress in the steel.) Previous codes limited the amount to  $0.75 \rho_{balanced}$  where  $\rho_{balanced}$  was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain ( $\epsilon_y$ ) of 0.002.

The strain in tension can be determined from  $\epsilon_t = \frac{d-c}{c}(0.003)$ . At yield,  $\epsilon_y = \frac{f_y}{E_s}$ .

The resistance factor expressions for **transition** and **compression controlled** sections are:

$$\phi = 0.75 + (\epsilon_t - \epsilon_y) \frac{0.15}{(0.005 - \epsilon_y)} \text{ for spiral members} \quad (\text{not less than } 0.75)$$

$$\phi = 0.65 + (\epsilon_t - \epsilon_y) \frac{0.25}{(0.005 - \epsilon_y)} \text{ for other members} \quad (\text{not less than } 0.65)$$

*Flexure Design of Reinforcement*

One method is to “wisely” estimate a height of the stress block,  $a$ , and solve for  $A_s$ , and calculate a new value for  $a$  using  $M_u$ .

1. guess  $a$  (less than  $n.a.$ )

$$2. A_s = \frac{0.85 f'_c b a}{f_y}$$

3. solve for  $a$  from

setting  $M_u = \phi A_s f_y (d - a/2)$  :

$$a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right)$$

4. repeat from 2. until  $a$  found from step 3 matches  $a$  used in step 2.

Maximum Reinforcement Ratio  $\rho$  for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which  $\phi$  is permitted to be 0.9

$f_y$	$f'_c = 3000$ psi	$f'_c = 3500$ psi	$f'_c = 4000$ psi	$f'_c = 5000$ psi	$f'_c = 6000$ psi
	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.80$	$\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239

$f_y$	$f'_c = 20$ MPa	$f'_c = 25$ MPa	$f'_c = 30$ MPa	$f'_c = 35$ MPa	$f'_c = 40$ MPa
	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.81$	$\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Design Chart Method:

1. calculate  $R_n = \frac{M_n}{bd^2}$
2. find curve for  $f'_c$  and  $f_y$  to get  $\rho$
3. calculate  $A_s$  and  $a$ , where:

$$A_s = \rho b d \text{ and } a = \frac{A_s f_y}{0.85 f'_c b}$$

Any method can simplify the size of  $d$  using  $h = 1.1d$

*Maximum Reinforcement*

Based on the limiting strain of 0.005 in the steel,  $x(or c) = 0.375d$  so

$$a = \beta_1 (0.375d) \text{ to find } A_{s-max}$$

( $\beta_1$  is shown in the table above)

*Minimum Reinforcement*

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

Minimum required:  $A_s = \frac{3\sqrt{f'_c}}{f_y} (b_w d)$

but not less than:  $A_s = \frac{200}{f_y} (b_w d)$

where  $f'_c$  is in psi.

This can be translated to  $\rho_{min} = \frac{3\sqrt{f'_c}}{f_y}$  but not less than  $\frac{200}{f_y}$

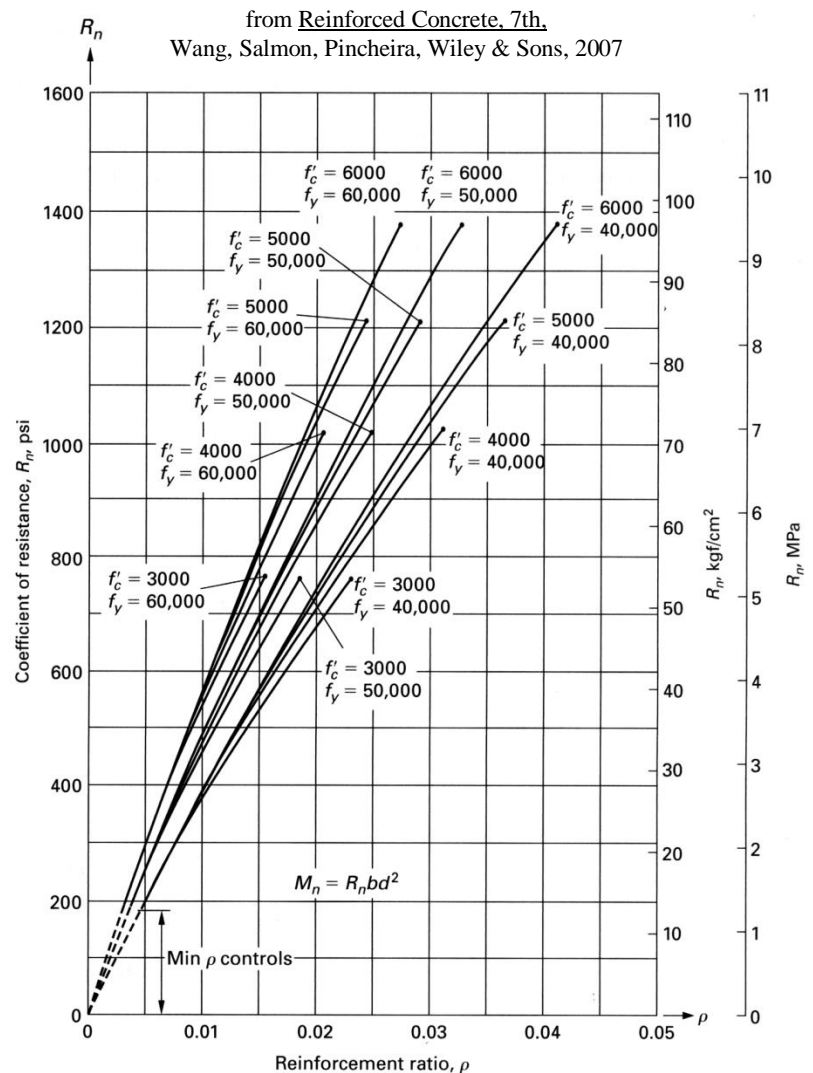


Figure 3.8.1 Strength curves ( $R_n$  vs  $\rho$ ) for singly reinforced rectangular sections. Upper limit of curves is at  $\rho_{max}$ . (tensile strain of 0.004)

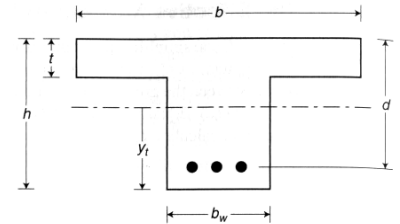


*Lightweight Concrete*

Lightweight concrete has strength properties that are different from normalweight concretes, and a modification factor,  $\lambda$ , must be multiplied to the strength value of  $\sqrt{f'_c}$ . for concrete for some specifications (ex. shear). Depending on the aggregate and the lightweight concrete, the value of  $\lambda$  ranges from 0.75 to 0.85, 0.85, or 0.85 to 1.0.  $\lambda$  is 1.0 for normalweight concrete.

*Cover for Reinforcement*

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 1.5 inch is typical for beams and columns, 0.75 inch is typical for slabs, and for concrete cast against soil, 3 inch minimum is required.



*Bar Spacing*

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement. The minimum spacing is the maximum of 1 in, a bar diameter, or 1.33 times the maximum aggregate size.

*T-beams and T-sections (pan joists)*

Beams cast with slabs have an effective width,  $b_E$ , that sees compression stress in a wide flange beam or joist in a slab system with positive bending.

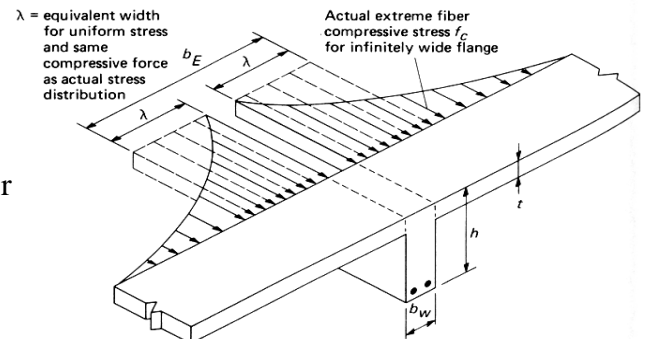


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

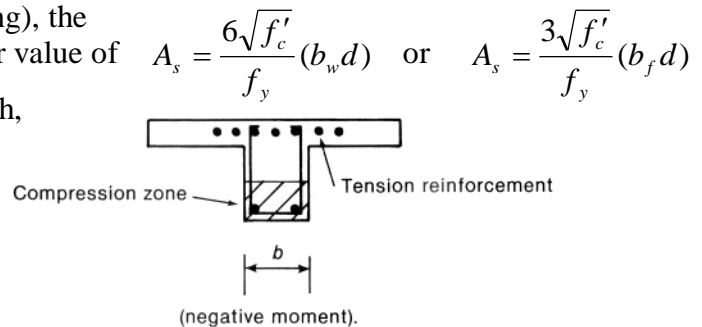
For *interior* T-sections,  $b_E$  is the smallest of  $L/4$ ,  $b_w + 16t$ , or *center to center of beams*

For *exterior* T-sections,  $b_E$  is the smallest of  $b_w + L/12$ ,  $b_w + 6t$ , or  $b_w + 1/2(\text{clear distance to next beam})$

When the **web** is in tension the minimum reinforcement required is the same as for rectangular sections with the web width ( $b_w$ ) in place of  $b$ .  $M_n = C_w(d-a/2) + C_f(d-h_f/2)$  ( $h_f$  is height of flange or  $t$ )

When the **flange** is in tension (negative bending), the minimum reinforcement required is the greater value of  $A_s = \frac{6\sqrt{f'_c}}{f_y}(b_w d)$  or  $A_s = \frac{3\sqrt{f'_c}}{f_y}(b_f d)$

where  $f'_c$  is in psi,  $b_w$  is the beam width, and  $b_f$  is the effective flange width



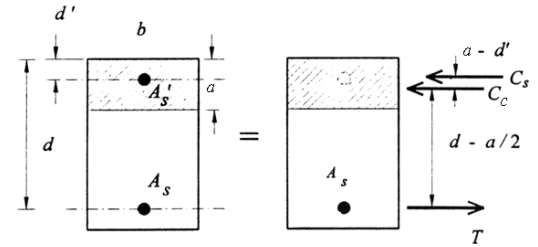
*Compression Reinforcement*

If a section is *doubly reinforced*, it means there is steel in the beam seeing compression. The force in the compression steel that *may not be yielding* is

$$C_s = A_s'(f'_s - 0.85f'_c)$$

The total compression that balances the tension is now:

$$T = C_c + C_s.$$



And the moment taken about the centroid of the compression stress is  $M_n = T(d-a/2) + C_s(a-d')$  where  $A_s'$  is the area of compression reinforcement, and  $d'$  is the effective depth to the centroid of the compression reinforcement

Because the compression steel may not be yielding, the neutral axis  $x$  must be found from the force equilibrium relationships, and the stress can be found based on strain to see if it has yielded.

*Slabs*

One way slabs can be designed as “one unit”-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric.

Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18”.

For required flexure reinforcement the spacing limit is three times the slab thickness not exceeding 18”.

**Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs**

Support condition	Minimum $h^{(1)}$
Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

<sup>(1)</sup>Expression applicable for normalweight concrete and  $f'_c = 60,000$  psi. For other cases, minimum  $h$  shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

**7.3.1.1.1** For  $f'_c$  other than 60,000 psi, the expressions in Table 7.3.1.1 shall be multiplied by  $(0.4 + f'_c/100,000)$ .

**7.3.1.1.2** For nonprestressed slabs made of lightweight concrete having  $w_c$  in the range of 90 to 115 lb/ft<sup>3</sup>, the expressions in Table 7.3.1.1 shall be multiplied by the greater of (a) and (b):

- (a)  $1.65 - 0.005w_c$
- (b) 1.09

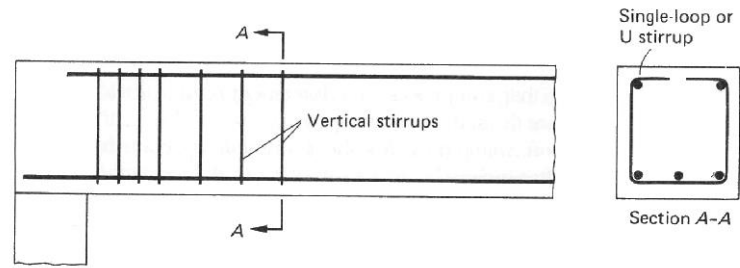
Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars:  $\rho = \frac{A_s}{bt} = 0.002$  or  $A_{s-min} = 0.002bt$

Minimum for slabs with grade 60 bars:  $\rho = \frac{A_s}{bt} = 0.0018$  or  $A_{s-min} = 0.0018bt$

### Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called **shear stirrups** (or **stirrups**).



The maximum shear for design,  $V_u$  is the value at a distance of  $d$  from the face of the support.

### Nominal Shear Strength

The shear force that can be resisted is the shear stress  $\times$  cross section area:  $V_c = v_c \times b_w d$

The shear stress for beams (one way)  $v_c = 2\lambda\sqrt{f'_c}$  so  $\phi V_c = \phi 2\lambda\sqrt{f'_c} b_w d$

where  $b_w$  = the beam width or the minimum width of the stem.

$\phi = 0.75$  for shear

$\lambda$  = modification factor for lightweight concrete

One-way joists are allowed an increase to  $1.1 \times V_c$  if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control):  $V_s = \frac{A_v f_y d}{s} \leq 8\sqrt{f'_c} b_w d$  (max)

where  $A_v$  = area of all vertical legs of stirrup

$f_{yt}$  = yield strength of transvers reinforcement (stirrups)

$s$  = spacing of stirrups

$d$  = effective depth

For shear design:

$$V_u \leq \phi V_c + \phi V_s \quad \phi = 0.75 \text{ for shear}$$

### Spacing Requirements

Stirrups are required when  $V_u$  is greater than  $\frac{\phi V_c}{2}$ . A minimum is required because shear failure of a beam without stirrups is sudden and brittle and because the loads can vary with respect to the design values.

Economical spacing of stirrups is considered to be greater than  $d/4$ . Common spacings of  $d/4$ ,  $d/3$  and  $d/2$  are used to determine the values of  $\phi V_s$  at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_{yt} d}{s}$$

Table 3-8 ACI Provisions for Shear Design\*

		$V_u \leq \frac{\phi V_c}{2}$	$\phi V_c \geq V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, $A_v^{**}$		none	greater of $\frac{50 b_w s}{f_{yt}}$ and $\frac{0.75 \sqrt{f'_c} b_w s}{f_{yt}}$	$\frac{(V_u - \phi V_c) s}{\phi f_{yt} d}$
Stirrup spacing, $s$	Required	—	smaller of $\frac{A_v f_{yt}}{50 b_w}$ and $\frac{A_v f_{yt}}{0.75 \sqrt{f'_c} b_w}$	$\frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$
	Recommended Minimum <sup>†</sup>	—	—	4 in.
	Maximum <sup>††</sup> (ACI 9.7.6.22)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4 \sqrt{f'_c} b_w d$ $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4 \sqrt{f'_c} b_w d$

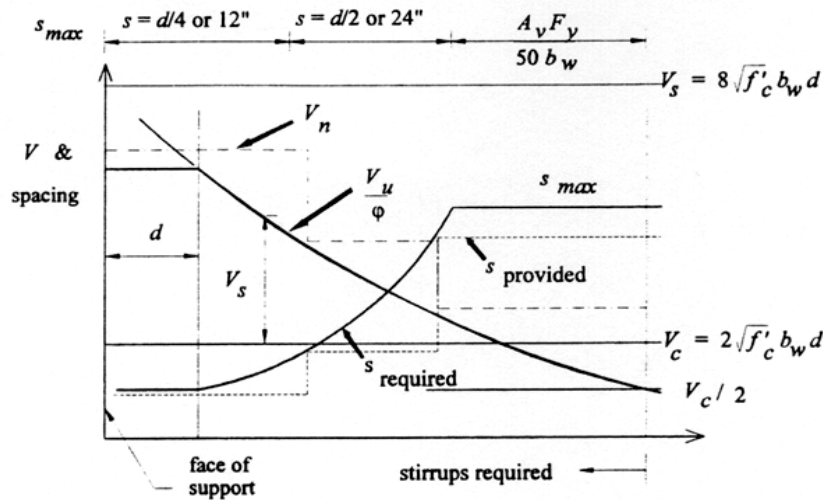
\*Members subjected to shear and flexure only;  $\phi V_c = \phi 2 \lambda \sqrt{f'_c} b_w d$   $\phi = 0.75$  (ACI 22.5.5.1)

\*\* $A_v = 2 \times A_b$  for U stirrups;  $f_y \leq 60$  ksi (ACI 20.2.2.4)

†A practical limit for minimum spacing is  $d/4$

††Maximum spacing based on minimum shear reinforcement ( $= A_v f_{yt} / 50 b_w$  or  $A_v f_{yt} / 0.75 \sqrt{f'_c} b_w$ ) must also be considered (ACI 9.6.3.3)

This figure shows the size of  $V_n$  provided by  $V_c + V_s$  (long dashes) exceeds  $V_u/\phi$  in a step-wise function, while the spacing provided (short dashes) is at or less than the required  $s$  (limited by the maximum allowed). (Note that the maximum shear permitted from the stirrups is  $8\sqrt{f'_c} b_w d$ )



The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

*Torsional Shear Reinforcement*

On occasion beam members will see twist along the axis caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

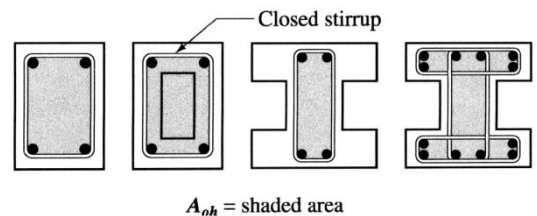


Fig. R11.6.3.6(b)—Definition of  $A_{oh}$

*Development Length for Reinforcement*

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (*both sides*) so it won't slip. This is referred to as the development length,  $l_d$ . Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. *Detailing reinforcement is a tedious job.* The equations for development length must be modified if the bar is epoxy coated or is cast with more than 12 in. of fresh concrete below it. Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations for splices are not provided here.

Development Length in Tension

With the proper bar to bar spacing and cover, the common development length equations are:

#6 bars and smaller:  $l_d = \frac{d_b f_y}{25 \lambda \sqrt{f'_c}}$  or 12 in. minimum

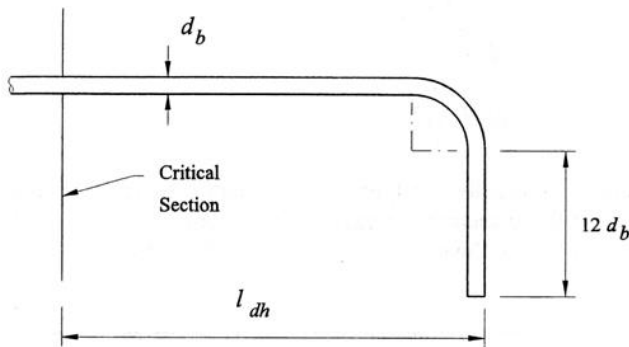
#7 bars and larger:  $l_d = \frac{d_b f_y}{20 \lambda \sqrt{f'_c}}$  or 12 in. minimum

Development Length in Compression

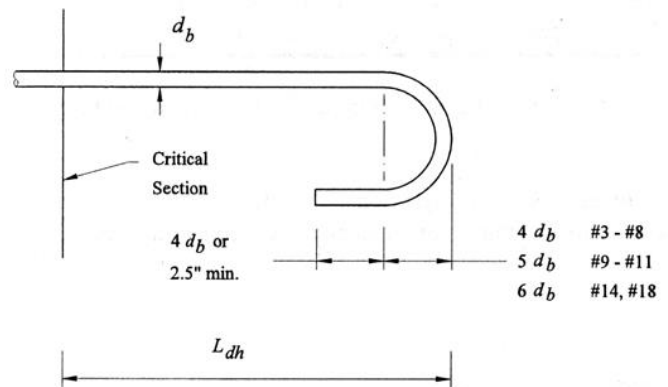
$$l_d = \frac{d_b f_y}{50 \lambda \sqrt{f'_c}} \leq 0.0003 f_y d_b \text{ or 8 in. minimum}$$

Hook Bends and Extensions

The minimum hook length is  $l_{dh} = \frac{d_b f_y}{50 \lambda \sqrt{f'_c}}$  but not less than the larger of  $8d_b$  and 6 in.



**Figure 9-17:** Minimum requirements for 90° bar hooks.



**Figure 9-18:** Minimum requirements for 180° bar hooks.

*Modulus of Elasticity & Deflection*

$E_c$  for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and  $E_c$  is adjusted.

Code values:

$$E_c = 57,000\sqrt{f'_c} \text{ (normal weight)} \quad E_c = w_c^{1.5} 33\sqrt{f'_c}, \quad w_c = 90 \text{ lb/ft}^3 - 160 \text{ lb/ft}^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 9.3.1.1 and 7.3.1.1 (see *Slabs*). The span lengths for continuous beams or slabs is taken as the clear span,  $l_n$ .

**Table 9.3.1.1—Minimum depth of nonprestressed beams**

Support condition	Minimum $h^{(1)}$
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

<sup>(1)</sup>Expressions applicable for normalweight concrete and  $f_y = 60,000$  psi. For other cases, minimum  $h$  shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

**Criteria for Flat Slab & Plate System Design**

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

**9.3.1.1.1** For  $f_y$  other than 60,000 psi, the expressions in Table 9.3.1.1 shall be multiplied by  **$(0.4 + f_y/100,000)$** .

**9.3.1.1.2** For nonprestressed beams made of lightweight concrete having  $w_c$  in the range of 90 to 115 lb/ft<sup>3</sup>, the expressions in Table 9.3.1.1 shall be multiplied by the greater of (a) and (b):

- (a)  **$1.65 - 0.005w_c$**
- (b) **1.09**

**Criteria for Column Design**

(American Concrete Institute) ACI 318-14 Code and Commentary:

$$P_u \leq \phi_c P_n \quad \text{where}$$

- $P_u$  is a factored load
- $\phi$  is a resistance factor
- $P_n$  is the nominal load capacity (strength)

Load combinations, ex:

- 1.4D (D is dead load)
- 1.2D + 1.6L (L is live load)

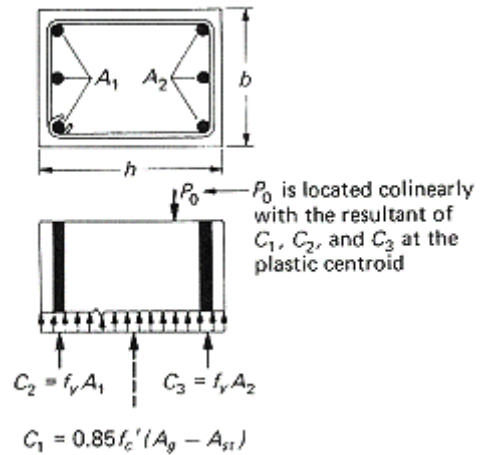
For compression,  $\phi_c = 0.75$  and  $P_n = 0.85P_o$  for spirally reinforced,  $\phi_c = 0.65$  and  $P_n = 0.8P_o$  for tied columns where  $P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st}$  and  $P_o$  is the name of the maximum axial force with no concurrent bending moment.

Columns which have reinforcement ratios,  $\rho_g = \frac{A_{st}}{A_g}$ ,

in the range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

Spiral ties are harder to construct.



Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The **interaction** diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

*Rigid Frames*

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating k for non-sway and sway frames can be found in the ACI code.

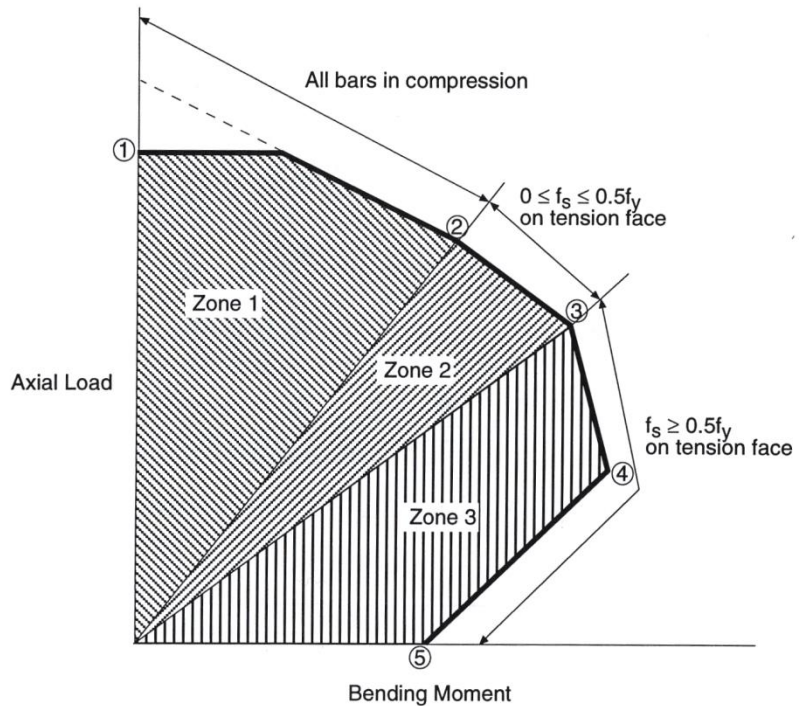
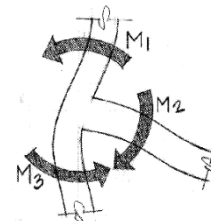


Figure 5-3 Transition Stages on Interaction Diagram

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or  $\Psi$ , must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced frames.

$$G = \Psi = \frac{\sum EI/l_c}{\sum EI/l_b}$$



where

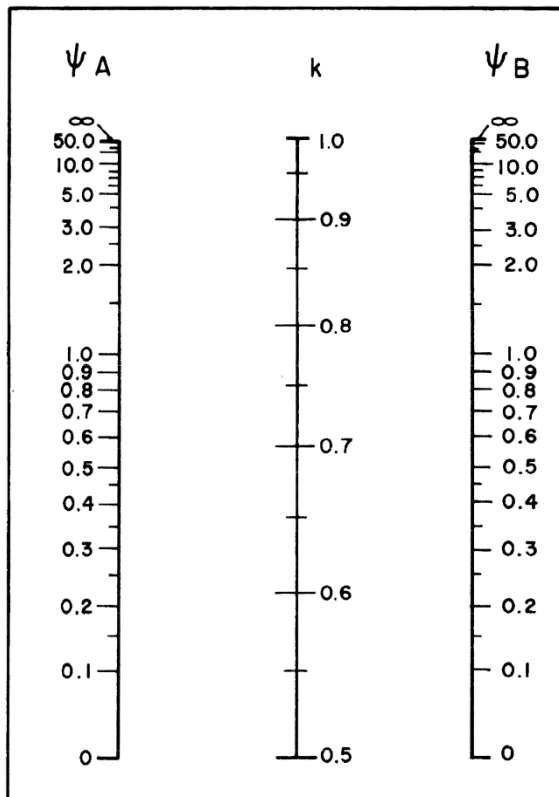
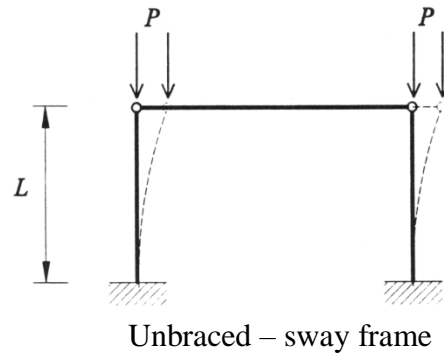
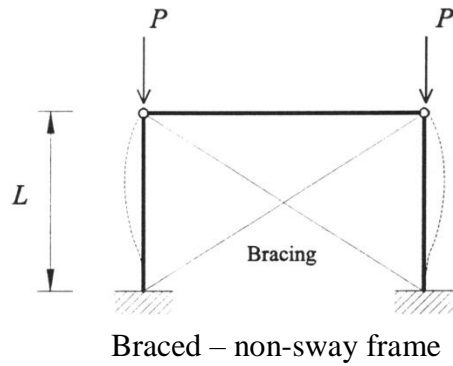
$E$  = modulus of elasticity for a member

$I$  = moment of inertia of for a member

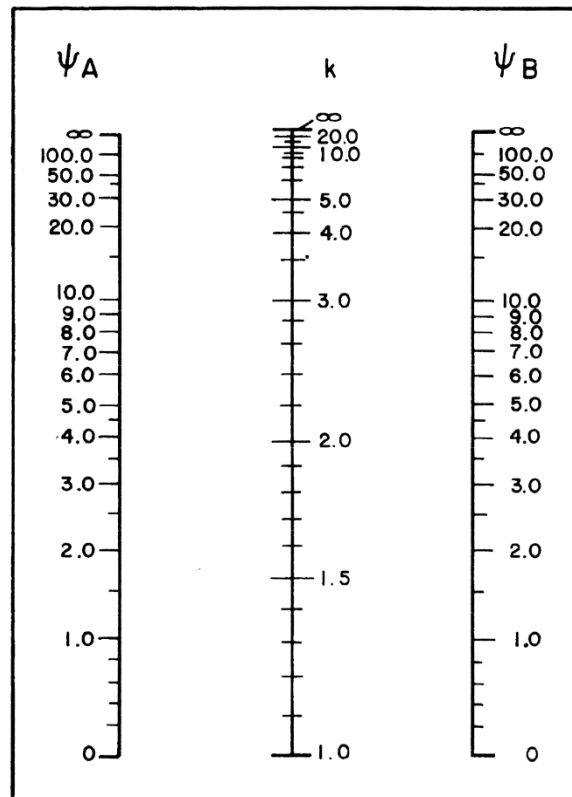
$l_c$  = length of the column from center to center

$l_b$  = length of the beam from center to center

- For pinned connections we typically use a value of 10 for  $\Psi$ .
- For fixed connections we typically use a value of 1 for  $\Psi$ .



(a)  
Nonsway Frames



(b)  
Sway Frames



### Slenderness

Slenderness effects can be neglected if  $kl/r \leq 22$  for columns not braced against side sway,  $kl/r \leq 34 + 12(M_1/M_2)$  and less than 40 for columns braced against sidesway where  $M_1/M_2$  is negative if the column is bent in single curvature, and positive for double curvature.

**Example 1** (a) Determine the ultimate moment capacity of a beam with dimensions  $b = 10$  in. and  $d_{\text{effective}} = 15$  in. and that has three No. 9 bars ( $3.0 \text{ in.}^2$ ) of tension-reinforcing steel. Assume that  $h = 18$  in.,  $F_y = 40$  ksi, and  $f'_c = 5$  ksi. (b) Assume also that the section is used as a cantilever beam 10 ft long, where the service loads are dead load = 400 lb/ft and live load = 300 lb/ft. Is the beam adequate in bending? Calculate the ultimate moment capacity of the beam first.

**Solution:**

$$(a) \quad a = A_s F_y / 0.85 f'_c b = (3)(40,000) / (0.85)(5000)(10) = 2.82 \text{ in.}$$

$$\phi M_n = \phi A_s F_y [d - a/2] = 0.9(3)(40,000)[15 - (2.82)/(2)] = 1,466,640 \text{ in.-lb}$$

Check for overreinforcement,  $c = 0.375 \cdot 15 = 5.625$ . Depth of stress block  $a = 0.80 \cdot 5.625 \text{ in.} = 4.5 \text{ in.}$   $A_{s,\text{max}} = (0.85)(5\text{ksi})(4.5\text{in.})(10\text{in.}) / (40\text{ksi}) = 4.78 \text{ in.}^2$  The beam is not over reinforced. Check for minimum steel:  $A_{s,\text{min}} = \frac{3\sqrt{f'_c}}{F_y} bd = 0.16 \text{ in.}^2$ , so beam is sufficiently reinforced.

$$(b) \quad U = 1.2D + 1.6L = 1.2(400) + 1.6(300) = 960 \text{ lb/ft}$$

$$M_u = w_u L^2 / 2 = (960)(10^2) / 2 = 48,000 \text{ ft-lb} = 576,000 \text{ in.-lb}$$

Since  $M_u = 576,000 < \phi M_n = 1,466,640$ , the beam is adequate in bending.

### EXAMPLE

Determine the ultimate moment capacity of a beam of dimensions  $b = 250$  mm and  $d = 350$  mm and that has  $300 \text{ mm}^2$  of reinforcing steel. Assume that  $F_y = 400$  MPa and  $f'_c = 25$  MPa.

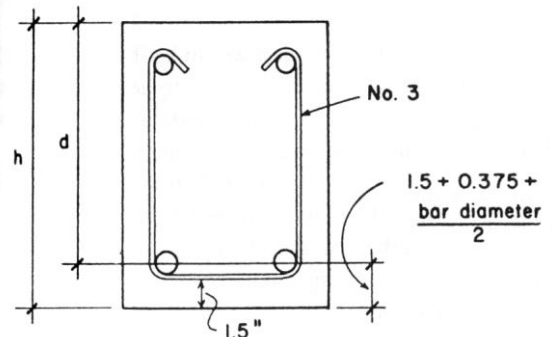
**Solution:**

$$a = \frac{A_s F_y}{0.85 f'_c b} = \frac{(300)(400)}{(0.85)(25)(250)} = 22.6$$

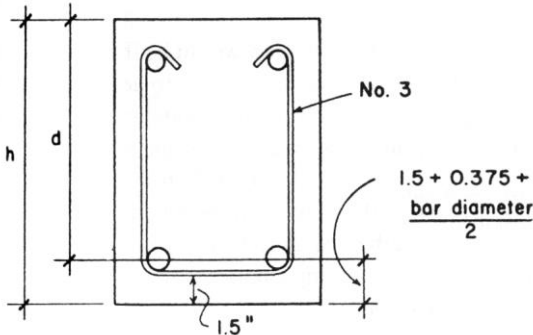
$$\phi M_n = \phi A_s F_y \left( d - \frac{a}{2} \right) = 0.9(300)(400) \left( 350 - \frac{22.6}{2} \right) = 36.5 \text{ kN} \cdot \text{m}$$

### Example 2 (pg 407)

**Example 1.** The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. [254 mm] wide,  $f'_c$  is 3000 psi [27.6 MPa], and  $f_y$  is 60 ksi [414 MPa]. Determine the depth of the beam and the tensile reinforcing required.



Example 2 (continued)



Example 3

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment,  $M_u$ , from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft<sup>3</sup> multiplied by the cross section area: self weight =  $150 \frac{\text{lb}}{\text{ft}^3} (10\text{in})(12\text{in}) \cdot \left(\frac{1\text{ft}}{12\text{in}}\right)^2 = 125 \text{ lb/ft}$

$$w_u = 1.2(300 \text{ lb/ft} + 125 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft}$$

$$\text{The maximum moment for a simply supported beam is } \frac{wl^2}{8} : \quad M_u = \frac{w_u l^2}{8} = \frac{1310 \frac{\text{lb}}{\text{ft}} (20\text{ft})^2}{8} = 65,500 \text{ lb-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{65,500 \text{ lb-ft}}{0.9} = 72,778 \text{ lb-ft}$$

To use the design chart aid, find  $R_n = \frac{M_n}{bd^2}$ , estimating that d is about 1.75 inches less than h:

$$d = 12\text{in} - 1.75\text{in} - (0.375) = 10.25\text{in} \quad (\text{NOTE: If there are stirrups, you must also subtract the diameter of the stirrup bar.})$$

$$R_n = \frac{72,778 \text{ lb-ft}}{(10\text{in})(10.25\text{in})^2} \cdot (12 \frac{\text{in}}{\text{ft}}) = 831 \text{ psi}$$

$\rho$  corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319), so the estimated area required,  $A_s$ , can be found:

$$A_s = \rho bd = (0.023)(10\text{in})(10.25\text{in}) = 2.36 \text{ in}^2$$

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at  $\rho_{\max}$  do not choose an area bigger than the maximum!)

$$\text{Try } A_s = 2.37 \text{ in}^2 \text{ from } 3\#8 \text{ bars}$$

$$d = 12 \text{ in} - 1.5 \text{ in (cover)} - \frac{1}{2} (8/8 \text{ in diameter bar}) = 10 \text{ in}$$

$$\text{Check } \rho = 2.37 \text{ in}^2 / (10 \text{ in})(10 \text{ in}) = 0.0237 \text{ which is less than } \rho_{\max-0.005} = 0.0319 \text{ OK (We cannot have an over reinforced beam!!)}$$

Find the moment capacity of the beam as designed,  $\phi M_n$

$$a = A_s f_y / 0.85 f_c b = 2.37 \text{ in}^2 (40 \text{ ksi}) / [0.85 (5 \text{ ksi}) 10 \text{ in}] = 2.23 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9 (2.37 \text{ in}^2) (40 \text{ ksi}) \left(10 \text{ in} - \frac{2.23 \text{ in}}{2}\right) \cdot \left(\frac{1}{12 \frac{\text{in}}{\text{ft}}}\right) = 63.2 \text{ k-ft} < 65.5 \text{ k-ft needed (not OK)}$$

So, we can increase d to 13 in, and  $\phi M_n = 70.3 \text{ k-ft}$  (OK). Or increase  $A_s$  to 2 # 10's (2.54 in<sup>2</sup>), for a = 2.39 in and  $\phi M_n$  of 67.1 k-ft (OK). Don't exceed  $\rho_{\max}$  Or  $\rho_{\max-0.005}$  if you want to use  $\phi=0.9$

Example 4

A simply supported beam 20 ft long carries a service dead load of 425 lb/ft (including self weight) and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment,  $M_u$ , from the factored load combination of 1.2D + 1.6L. *If self weight is not included in the service loads, you need to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.*

$$w_u = 1.2(425 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft}$$

The maximum moment for a simply supported beam is  $\frac{wl^2}{8}$  :

$$M_u = \frac{w_u l^2}{8} = \frac{1310 \text{ lb/ft} (20 \text{ ft})^2}{8} = 65,500 \text{ lb-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{65,500 \text{ lb-ft}}{0.9} = 72,778 \text{ lb-ft}$$

To use the design chart aid, we can find  $R_n = \frac{M_n}{bd^2}$ , and estimate that h is roughly 1.5-2 times the size of b, and  $h = 1.1d$  (rule of thumb):  $d = h/1.1 = (2b)/1.1$ , so  $d \approx 1.8b$  or  $b \approx 0.55d$ .

We can find  $R_n$  at the maximum reinforcement ratio for our materials, keeping in mind  $\rho_{\max}$  at a strain = 0.005 is 0.0319 off of the chart at about 1070 psi, with  $\rho_{\max} = 0.037$ . Let's substitute b for a function of d:

$$R_n = 1070 \text{ psi} = \frac{72,778 \text{ lb-ft}}{(0.55d)(d)^2} \cdot (12 \text{ in/ft}) \quad \text{Rearranging and solving for } d = 11.4 \text{ inches}$$

That would make b a little over 6 inches, which is impractical. 10 in is commonly the smallest width.

So if h is commonly 1.5 to 2 times the width, b, h ranges from 14 to 20 inches. ( $10 \times 1.5 = 15$  and  $10 \times 2 = 20$ )

Choosing a depth of 14 inches,  $d \cong 14 - 1.5$  (clear cover) -  $\frac{1}{2}(1'' \text{ diameter bar guess}) - \frac{3}{8}$  in (stirrup diameter) = 11.625 in.

$$\text{Now calculating an updated } R_n = \frac{72,778 \text{ lb-ft}}{(10 \text{ in})(11.625 \text{ in})^2} \cdot (12 \text{ in/ft}) = 646.2 \text{ psi}$$

$\rho$  now is 0.020 (under the limit at 0.005 strain of 0.0319), so the estimated area required,  $A_s$ , can be found:

$$A_s = \rho b d = (0.020)(10 \text{ in})(11.625 \text{ in}) = 1.98 \text{ in}^2$$

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at  $\rho_{\max-0.005}$  do not choose an area bigger than the maximum!)

Try  $A_s = 2.37 \text{ in}^2$  from 3#8 bars. (or  $2.0 \text{ in}^2$  from 2 #9 bars. 4#7 bars don't fit...)

$d(\text{actually}) = 14 \text{ in.} - 1.5 \text{ in (cover)} - \frac{1}{2}(8/8 \text{ in bar diameter}) - \frac{3}{8} \text{ in. (stirrup diameter)} = 11.625 \text{ in.}$

Check  $\rho = 2.37 \text{ in}^2 / (10 \text{ in})(11.625 \text{ in}) = 0.0203$  which is less than  $\rho_{\max-0.005} = 0.0319$  OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed,  $\phi M_n$

$$a = A_s f_y / 0.85 f_c b = 2.37 \text{ in}^2 (40 \text{ ksi}) / [0.85(5 \text{ ksi})10 \text{ in}] = 2.23 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9(2.37 \text{ in}^2)(40 \text{ ksi})(11.625 \text{ in} - \frac{2.23 \text{ in}}{2}) \cdot (\frac{1}{12 \text{ in/ft}}) = 74.7 \text{ k-ft} > 65.5 \text{ k-ft needed}$$

**OK!** Note: If the section doesn't work, you need to increase d or  $A_s$  as long as you don't exceed  $\rho_{\max-0.005}$

Example 5

A simply supported beam 25 ft long carries a service dead load of 2 k/ft, an estimated self weight of 500 lb/ft and a live load of 3 k/ft. Design an appropriate beam (for flexure only). Use grade 60 steel and concrete strength of 3000 psi.

SOLUTION:

Find the design moment,  $M_u$ , from the factored load combination of  $1.2D + 1.6L$ . If self weight is estimated, and the selected size has a larger self weight, the design moment must be adjusted for the extra load.

$$w_u = 1.2(2 \text{ k/ft} + 0.5 \text{ k/ft}) + 1.6(3 \text{ k/ft}) = 7.8 \text{ k/ft} \quad \text{So, } M_u = \frac{w_u L^2}{8} = \frac{7.8 \text{ k/ft} (25 \text{ ft})^2}{8} = 609.4 \text{ k-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{609.4 \text{ k-ft}}{0.9} = 677.1 \text{ k-ft}$$

To use the design chart aid, we can find  $R_n = \frac{M_n}{bd^2}$ , and estimate that  $h$  is roughly 1.5-2 times the size of  $b$ , and  $h = 1.1d$  (rule of thumb):  $d = h/1.1 = (2b)/1.1$ , so  $d \approx 1.8b$  or  $b \approx 0.55d$ .

We can find  $R_n$  at the maximum reinforcement ratio for our materials off of the chart at about 700 psi with  $\rho_{\max-0.005} = 0.0135$ . Let's substitute  $b$  for a function of  $d$ :

$$R_n = 700 \text{ psi} = \frac{677.1 \text{ k-ft} (1000 \text{ lb/k})}{(0.55d)(d)^2} \cdot (12 \text{ in/ft}) \quad \text{Rearranging and solving for } d = 27.6 \text{ inches}$$

That would make  $b$  15.2 in. (from 0.55d). Let's try 15. So,

$$h \cong d + 1.5 \text{ (clear cover)} + \frac{1}{2}(1" \text{ diameter bar guess}) + \frac{3}{8} \text{ in (stirrup diameter)} = 27.6 + 2.375 = 29.975 \text{ in.}$$

Choosing a depth of 30 inches,  $d \cong 30 - 1.5 \text{ (clear cover)} - \frac{1}{2}(1" \text{ diameter bar guess}) - \frac{3}{8} \text{ in (stirrup diameter)} = 27.625 \text{ in.}$

$$\text{Now calculating an updated } R_n = \frac{677,100 \text{ lb-ft}}{(15 \text{ in})(27.625 \text{ in})^2} \cdot (12 \text{ in/ft}) = 710 \text{ psi} \quad \text{This is larger than } R_n \text{ for the } 0.005 \text{ strain limit!}$$

We can't just use  $\rho_{\max-0.005}$ . The way to reduce  $R_n$  is to increase  $b$  or  $d$  or both. Let's try increasing  $h$  to 31 in., then  $R_n = 661 \text{ psi}$  with  $d = 28.625 \text{ in.}$  That puts us under  $\rho_{\max-0.005}$ . We'd have to remember to keep UNDER the area of steel calculated, which is hard to do.

From the chart,  $\rho \approx 0.013$ , less than the  $\rho_{\max-0.005}$  of 0.0135, so the estimated area required,  $A_s$ , can be found:

$$A_s = \rho b d = (0.013)(15 \text{ in})(29.625 \text{ in}) = 5.8 \text{ in}^2$$

The number of bars for this area can be found from handy charts. Our charts say there can be 3 – 6 bars that fit when  $\frac{3}{4}$ " aggregate is used. We'll assume 1 inch spacing between bars. The actual limit is the maximum of 1 in, the bar diameter or 1.33 times the maximum aggregate size.

$$\text{Try } A_s = 6.0 \text{ in}^2 \text{ from 6\#9 bars. Check the width: } 15 - 3 (1.5 \text{ in cover each side}) - 0.75 \text{ (two \#3 stirrup legs)} - 6 * 1.128 - 5 * 1.128 \text{ in.} = -1.16 \text{ in NOT OK.}$$

$$\text{Try } A_s = 5.08 \text{ in}^2 \text{ from 4\#10 bars. Check the width: } 15 - 3 (1.5 \text{ in cover each side}) - 0.75 \text{ (two \#3 stirrup legs)} - 4 * 1.27 - 3 * 1.27 \text{ in.} = 2.36 \text{ OK.}$$

$$d(\text{actually}) = 31 \text{ in.} - 1.5 \text{ in (cover)} - \frac{1}{2} (1.27 \text{ in bar diameter}) - \frac{3}{8} \text{ in. (stirrup diameter)} = 28.49 \text{ in.}$$

Find the moment capacity of the beam as designed,  $\phi M_n$

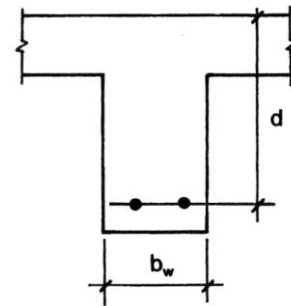
$$a = A_s f_y / 0.85 f_c b = 5.08 \text{ in}^2 (60 \text{ ksi}) / [0.85(3 \text{ ksi})15 \text{ in}] = 8.0 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9(5.08 \text{ in}^2)(60 \text{ ksi})(28.49 \text{ in} - \frac{8.0 \text{ in}}{2}) \cdot (\frac{1}{12 \text{ in/ft}}) = 559.8 \text{ k-ft} < 609 \text{ k-ft needed!! (NO GOOD)}$$

More steel isn't likely to increase the capacity much unless we are close. It looks like we need more steel **and** lever arm. Try  $h = 32 \text{ in.}$  AND  $b = 16 \text{ in.}$ , then  $M_u^*$  (with the added self weight of 33.3 lb/ft) = 680.2 k-ft,  $\rho \approx 0.012$ ,  $A_s = 0.012(16 \text{ in})(29.42 \text{ in}) = 5.66 \text{ in}^2$ . 6#9's won't fit, but 4#11's will:  $\rho = 0.0132$  ✓,  $a = 9.18 \text{ in}$ , and  $\phi M_n = 697.2 \text{ k-ft}$  which is finally larger than 680.2 k-ft **OK**

Example 6 (pg 420)

**Example 4.** A T-section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft [5.49 m], beams are 9 ft [2.74 m] center to center, slab thickness is 4 in. [0.102 m], beam stem dimensions are  $b_w = 15$  in. [0.381 m] and  $d = 22$  in. [0.559 m],  $f'_c = 4$  ksi [27.6 MPa],  $f_y = 60$  ksi [414 MPa]. Find the required area of steel and select the reinforcing bars for a dead load moment of 125 kip-ft [170 kN-m] plus a live load moment of 100 kip-ft [136 kN-m].



Example 7

Design a T-beam for a floor with a 4 in slab supported by 22-ft-span-length beams cast monolithically with the slab. The beams are 8 ft on center and have a web width of 12 in. and a total depth of 22 in.;  $f'_c = 3000$  psi and  $f_y = 60$  ksi. Service loads are 125 psf and 200 psf dead load which does not include the weight of the floor system

## SOLUTION:

1. Establish the design moment:

$$\text{slab weight} = \frac{96(4)}{144}(0.150) = 0.400 \text{ kip/ft}$$

$$\text{stem weight} = \frac{12(18)}{144}(0.150) = 0.225$$

$$\text{total} = 0.625 \text{ kip/ft}$$

$$\text{service DL} = 8(0.200) = 1.60 \text{ kips/ft}$$

$$\text{service LL} = 8(0.125) = 1.00 \text{ kip/ft}$$

Calculate the factored load and moment:

$$w_u = 1.2(0.625 + 1.60) + 1.6(1.00) = 4.27 \text{ kip/ft}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{4.27(22)^2}{8} = 258 \text{ ft-kips}$$

2. Assume an effective depth
- $d = h - 3$
- in.:

$$d = 22 - 3 = 19 \text{ in.}$$

3. Determine the effective flange width:

$$\frac{1}{4} \text{ span length} = 0.25(22)(12) = 66 \text{ in.}$$

$$b_w + 16h_f = 12 + 16(4) = 76 \text{ in.}$$

$$\text{beam spacing} = 96 \text{ in.}$$

Use an effective flange width  $b = 66$  in.

4. Determine whether the beam behaves as a true T-beam or as a rectangular beam by computing the practical moment strength
- $\phi M_{nf}$
- with the full effective flange assumed to be in compression. This assumes that the bottom of the compressive stress block coincides with the bottom of the flange, as shown in Figure 3-10. Thus

$$\begin{aligned} \phi M_{nf} &= \phi(0.85f'_c)bh_f \left( d - \frac{h_f}{2} \right) \\ &= 0.9(0.85)(3)(66) \frac{4(19 - 4/2)}{12} = 858 \text{ ft-kips} \end{aligned}$$

5. Since  $858 \text{ ft-kips} > 258 \text{ ft-kips}$ , the total effective flange need not be completely utilized in compression (i.e.,  $a < h_f$ ), and the T-beam behaves as a wide rectangular beam with a width  $b$  of 66 in.
6. Design as a rectangular beam with  $b$  and  $d$  as known values (see Section 2-15):

$$\text{required } R_n = \frac{M_u}{\phi b d^2} = \frac{258(12)}{0.9(66)(19)^2} = 0.1444 \text{ ksi}$$

7. From Table A-8, select the required steel ratio to provide a
- $R_n$
- of 0.1444 ksi:

$$\text{required } \rho = 0.0024$$

8. Calculate the required steel area:

$$\begin{aligned} \text{required } A_s &= \rho b d \\ &= 0.0024(66)(19) = 3.01 \text{ in.}^2 \end{aligned}$$

9. Select the steel bars. Use 3#9 (
- $A_s = 3.00 \text{ in.}^2$
- )

$$\text{minimum } b_w = 7.125 \text{ in.} \quad (\text{O.K.})$$

Check the effective depth  $d$ :

$$d = 22 - 1.5 - 0.38 \frac{1.129}{2} = 19.56 \text{ in.}$$

$$19.49 \text{ in.} > 19 \text{ in.}$$

(O.K.)

10. Check
- $A_{s,\min}$
- . From Table A-5:

$$\begin{aligned} A_{s,\min} &= 0.0033b_w d \\ &= 0.0033(12)(19) = 0.75 \text{ in.}^2 \\ 0.75 \text{ in.}^2 &< 3.00 \text{ in.}^2 \end{aligned}$$

11. Check
- $A_{s,\max}$
- :

$$\begin{aligned} A_{s,\max} &= 0.0135(66)(19) \\ &= 16.93 \text{ in.}^2 > 3.00 \text{ in.}^2 \quad (\text{O.K.}) \end{aligned}$$

12. Verify the moment capacity:

$$(\text{Is } M_u \leq \phi M_n \text{ ?})$$

$$a = (3.00)(60)/[0.85(3)(66)] = 1.07 \text{ in.}$$

$$\begin{aligned} \phi M_n &= 0.9(3.00)(60)(19.56 - \frac{1.07}{2}) \frac{1}{12} \\ &= 256.9.1 \text{ ft-kips} \quad (\text{Not O.K.}) \end{aligned}$$

Choose more steel,  $A_s = 3.16 \text{ in.}^2$  from 4-#8's

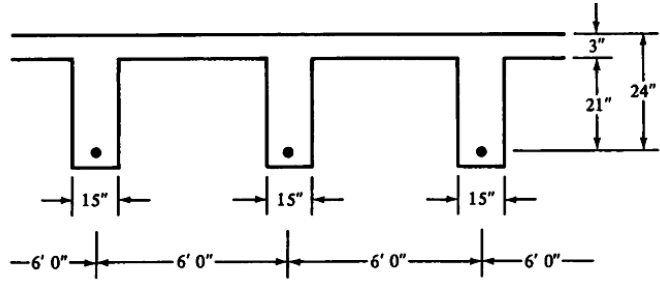
$$d = 19.62 \text{ in.}, a = 1.13 \text{ in.}$$

$$\phi M_n = 271.0 \text{ ft-kips, which is OK}$$

13. Sketch the design

**Example 8**

Design a T-beam for the floor system shown for which  $b_w$  and  $d$  are given.  $M_D = 200$  ft-k,  $M_L = 425$  ft-k,  $f'_c = 3000$  psi and  $f_y = 60$  ksi, and simple span = 18 ft.



**SOLUTION**

**Effective Flange Width**

- (a)  $\frac{1}{4} \times 18' = 4'6'' = \underline{54''}$
- (b)  $15'' + (2)(8)(3) = 63''$
- (c)  $6'0'' = 72''$

**Moments Assuming  $\phi = 0.90$**

$$M_u = (1.2)(200) + (1.6)(425) = 920 \text{ ft-k}$$

$$M_n = \frac{M_u}{0.90} = \frac{920}{0.90} = 1022 \text{ ft-k}$$

First assume  $a \leq h_f$  (which is very often the case. Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T beam flange.

$$\frac{M_u}{\phi b d^2} = \frac{920(12,000)}{(0.9)(54)(24)^2} = 394.4 \text{ psi}$$

from Table A.12,  $\rho = 0.0072$

$$a = \frac{\rho f_y d}{0.85 f'_c} = \frac{0.0072(60)(24)}{(0.85)(3)} = 4.06 \text{ in.} > h_f = 3 \text{ in.}$$

The beam acts like a T beam, not a rectangular beam, and if the values for  $\rho$  and  $a$  above are not correct. If the value of  $a$  had been  $\leq h_f$ , the value of  $A_s$  would have been simply  $\rho b d = 0.0072(54)(24) = 9.33 \text{ in}^2$ . Now break the beam up into two parts (Figure 5.7) and design it as a T beam.

Assuming  $\phi = 0.90$

$$A_{sf} = \frac{(0.85)(3)(54 - 15)(3)}{60} = 4.97 \text{ in.}^2$$

$$M_{uf} = (0.9)(4.97)(60)(24 - \frac{3}{2}) = 6039 \text{ in.-k} = 503 \text{ ft-k}$$

$$M_{uw} = 920 - 503 = 417 \text{ ft-k}$$

Designing a rectangular beam with  $b_w = 15$  in. and  $d = 24$  in. to resist 417 k-ft

$$\frac{M_{uw}}{\phi b_w d^2} = \frac{(12)(417)(1000)}{(0.9)(15)(24)^2} = 643.5$$

$\rho_w = 0.0126$  from Appendix Table A.12

$$A_{sw} = (0.0126)(15)(24) = 4.54 \text{ in.}^2$$

$$A_s = 4.97 + 4.54 = \underline{9.51 \text{ in.}^2}$$

Check minimum reinforcing:

$$A_{s \text{ min}} = \frac{3\sqrt{f'_c}}{f_y} b_w d = \frac{3\sqrt{3000}(15)(24)}{60,000} = 0.986 \text{ in}^2$$

but not less than

$$A_{s \text{ min}} = \frac{200 b_w d}{f_y} = \frac{200(15)(24)}{60,000} = \underline{1.2 \text{ in}^2}$$

Only 2 rows fit, so try 8-#10 bars,  $A_s = 10.16 \text{ in}^2$  for equilibrium:  $T = C_w + C_f$

$$T = A_s f_y = (10.16)(60) = 609.6 \text{ k}$$

$$C_f = 0.85 f'_c (b - b_w) h_f \text{ and } C_w = 0.85 f'_c a b_w$$

$$C_w = T - C_f = 609.6 \text{ k} - (0.85)(3)(54 - 15)3 = 311.25 \text{ k}$$

$$a = 311.25 / (0.85 * 3 * 15) = 8.14 \text{ in}$$

Check strain ( $\epsilon_s$ ) and  $\phi$ :

$$c = a / \beta_1 = 8.14 \text{ in} / 0.85 = 9.58$$

$$\epsilon_s = \left( \frac{d - c}{c} \right) (0.003) = \left( \frac{24 - 9.58}{9.58} \right) (0.003) = 0.0045 > 0.005!$$

We could try 10-#9 bars at  $10 \text{ in}^2$ ,  $T = 600 \text{ k}$ ,  $C_w = 301.65 \text{ k}$ ,  $a = 7.89$ ,  $\epsilon_s = 0.0061$ ;  $\phi = 0.9!$

Finally check the capacity:

$$\begin{aligned} M_n &= C_w \left( d - \frac{a}{2} \right) + C_f \left( d - \frac{h_f}{2} \right) \\ &= [301.65(24 - 7.89/2) + 298.35(24 - 3/2)] 1 \text{ ft} / 12 \text{ in} \\ &= 1063.5 \text{ k-ft} \end{aligned}$$

So:  $\phi M_n = 0.9(1063.5) = 957.2 \text{ k-ft} \geq \underline{920 \text{ k-ft}}$  (OK)

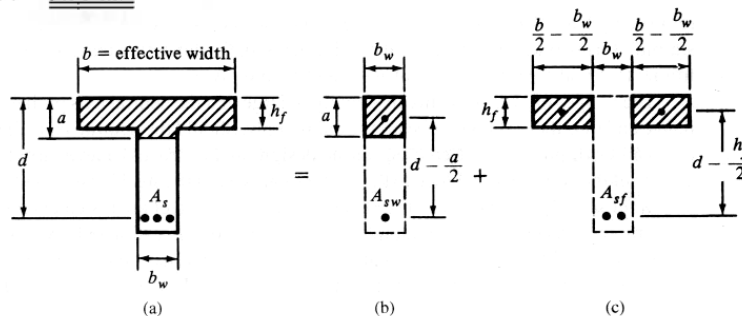
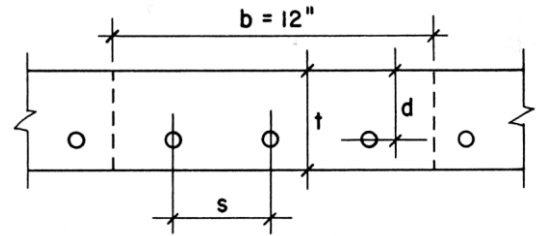


Figure 5.7 Separation of T beam into rectangular parts.



Example 9 (pg 432)

**Example 7.** A one-way solid concrete slab is to be used for a simple span of 14 ft [4.27 m]. In addition to its own weight, the slab carries a superimposed dead load of 30 psf [1.44 kPa] plus a live load of 100 psf [4.79 kPa]. Using  $f'_c = 3$  ksi [20.7 MPa] and  $f_y = 40$  ksi [276 MPa], design the slab for minimum overall thickness.



**TABLE 13.7 Areas Provided By Spaced Reinforcement**

Bar Spacing (in.)	Area Provided (in. <sup>2</sup> /ft width)									
	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	
3	0.44	0.80	1.24	1.76	2.40	3.16	4.00			
3.5	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35		
4	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68	
4.5	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16	
5	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74	
5.5	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40	
6	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12	
7	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67	
8	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34	
9	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08	
10	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87	
11	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70	
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	
13	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44	
14	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34	
15	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25	
16	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17	
18	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04	
24	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78	

## Example 10

### Example 2-9

Design a simple-span one-way slab to carry a uniformly distributed live load of 400 psf. The span is 10 ft (center to center of supports). Use  $f'_c = 4000$  psi and  $f_y = 60,000$  psi. Select the thickness to be not less than the ACI minimum thickness requirement.

#### Solution:

Determine the required minimum  $h$  and use this to estimate the slab dead weight.

1. From ACI Table 7.3.1.1, for a simply supported, solid, one-way slab,

$$\text{minimum } h = \frac{\ell}{20} = \frac{10(12)}{20} = 6.0 \text{ in.}$$

Try  $h = 6$  in. and design a 12-in.-wide segment.

2. Determine the slab weight dead load:

$$\frac{6(12)}{144}(0.150) = 0.075 \text{ kip/ft}$$

The total design load is

$$\begin{aligned} w_u &= 1.2w_{DL} + 1.6w_{LL} \\ &= 1.2(0.075) + 1.6(0.400) \\ &= 0.730 \text{ kip/ft} \end{aligned}$$

3. Determine the design moment:

$$M_u = \frac{w_u \ell^2}{8} = \frac{0.73(10)^2}{8} = 9.125 \text{ ft-kips}$$

4. Establish the approximate  $d$ . Assuming No. 6 bars and minimum concrete cover on the bars of  $\frac{3}{8}$  in.,

$$\text{assumed } d = 6.0 - 0.75 - 0.375 = 4.88 \text{ in.}$$

5. Determine the required  $R_n$ :

$$\begin{aligned} \text{required } R_n &= \frac{M_u}{\phi b d^2} \\ &= \frac{9.125(12)}{0.9(12)(4.88)^2} = 0.4257 \text{ ksi} \end{aligned}$$

6. From Table A-10, for a required  $R_n = 0.4257$ , the required  $\rho = 0.0077$ . (Note that the required  $\rho$  selected is the next higher value from Table A-10.) Thus

$$\rho_{\max} = 0.0181 > 0.0077 \quad (\text{O.K.})$$

Use  $\rho = 0.0077$ .

7. required  $A_s = \rho b d = 0.0077(12)(4.88) = 0.45 \text{ in.}^2/\text{ft}$

8. Select the main steel (from Table A-4). Select No. 5 bars at  $7\frac{1}{2}$  in. o.c. ( $A_s = 0.50 \text{ in.}^2$ ). The assumption on bar size was satisfactory. The code requirements for maximum spacing have been discussed in Section 2-13. Minimum spacing of bars in slabs, practically, should not be less than 4 in., although the ACI Code allows bars to be placed closer together, as discussed in Example 2-7. Check the maximum spacing (ACI Code, Section 7.6.5):

$$\text{maximum spacing} = 3h \text{ or } 18 \text{ in.}$$

$$3h = 3(6) = 18 \text{ in.}$$

$$7\frac{1}{2} \text{ in.} < 18 \text{ in.} \quad (\text{O.K.})$$

Therefore use No. 5 bars at  $7\frac{1}{2}$  in. o.c.

9. Select shrinkage and temperature reinforcement (ACI Code, Section 7.12):

$$\begin{aligned} \text{required } A_s &= 0.0018bh \\ &= 0.0018(12)(6) = 0.13 \text{ in.}^2/\text{ft} \end{aligned}$$

Select No. 3 bars at 10 in. o.c. ( $A_s = 0.13 \text{ in.}^2$ ) or No. 4 bars at 18 in. o.c. ( $A_s = 0.13 \text{ in.}^2$ ):

$$\text{maximum spacing} = 5h \text{ or } 18 \text{ in.}$$

Use No. 3 bars at 10 in. o.c.

10. The main steel area must exceed the area required for shrinkage and temperature steel (ACI Code, Section 10.5.4):

$$0.50 \text{ in.}^2 > 0.13 \text{ in.}^2 \quad (\text{O.K.})$$

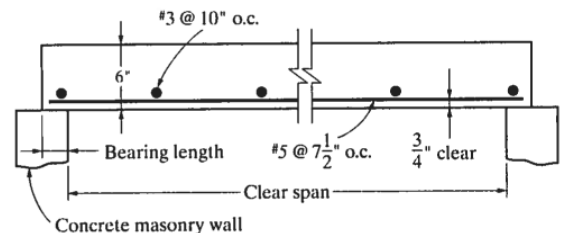
11. Verify the moment capacity:

$$(\text{Is } M_u \leq \phi M_n)$$

$$a = \frac{(0.50)(60)}{0.85(4)(12)} = 0.74 \text{ in.}$$

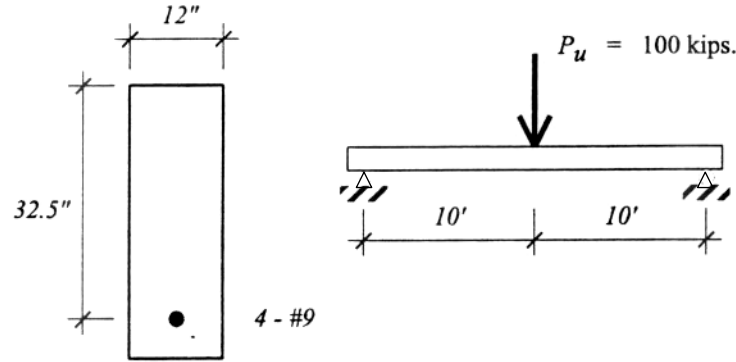
$$\begin{aligned} \phi M_n &= 0.9(0.50)(60)(5.0625 - 0.74/2) / 12 \\ &= 10.6 \text{ ft-kips} \quad (\text{OK}) \end{aligned}$$

12. A design sketch is drawn:



**Example 11**

For the simply supported concrete beam shown in Figure 5-61, determine the stirrup spacing (if required) using No. 3 U stirrups of Grade 60 ( $f_y = 60$  ksi). Assume  $f'_c = 3000$  psi.



**Figure 5-61:** Simply supported concrete beam for Example 5-15.

$$f'_c = 3000 \text{ psi.} \quad \text{For \#3 bars, } A_s = 0.11 \text{ in.}^2,$$

$$F_y = 60 \text{ ksi.} \quad \text{with 2 legs, then } A_v = 0.22 \text{ in.}^2$$

Solution:

$$V_u = 50 \text{ kips (neglecting weight of the beam)}$$

$$\phi V_c = \phi \lambda 2 \sqrt{f'_c} b_w d$$

$$= \frac{(0.75)(1)2\sqrt{3000}(12)(32.5)}{1000} = 32.0 \text{ kips} < V_u \therefore \text{Need Stirrups}$$

Note: If  $V_u = \frac{1}{2} \phi V_c$ , minimum stirrups would still be required.

$$V_u \leq \phi V_c + \phi V_s$$

$$\therefore \phi V_s = V_u - \phi V_c = 50 - 32.0 = 18.0 \text{ kips} \quad (< \phi 4 \sqrt{f'_c} b_w d = 64.1 \text{ kips})$$

$$s_{req'd} \leq \frac{\phi A_v f_y d}{\phi V_s} = \frac{(0.75)(0.22 \text{ in}^2)(60 \text{ ksi})(32.5 \text{ in})}{18.0 \text{ k}}$$

$$= 17.875 \text{ in.}$$

$$s_{max} = \frac{d}{2} = \frac{32.5}{2} = 16.2 \text{ in.} \quad \text{controls}$$

$$= 24 \text{ in.}$$

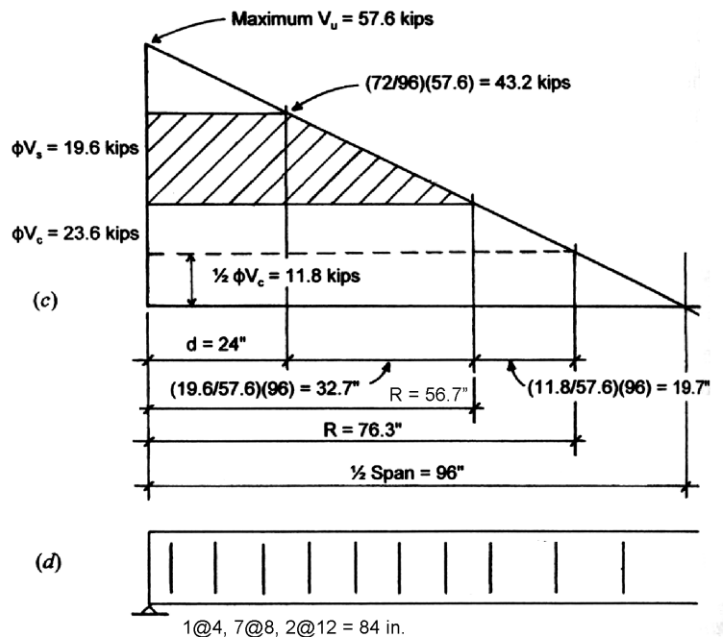
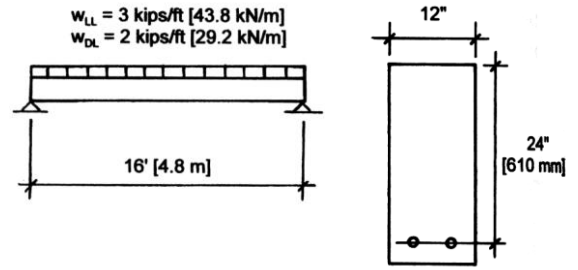
$$\text{Only IF } \phi V_c > V_u > \frac{\phi V_c}{2} \quad s_{req'd} \leq \frac{A_v f_y}{50 b_w} = \frac{(0.22)(60,000)}{50(12)} = 22.0 \text{ in.} \text{ but } 16'' (d/2) \text{ would be the maximum as well.}$$

$$\text{and } \frac{A_v f_y}{0.75 \sqrt{f'_c} b_w} = \frac{(0.22)(60,000)}{0.75 \sqrt{3000}(12)} = 26.8 \text{ in. use } 22, \text{ which is smaller}$$

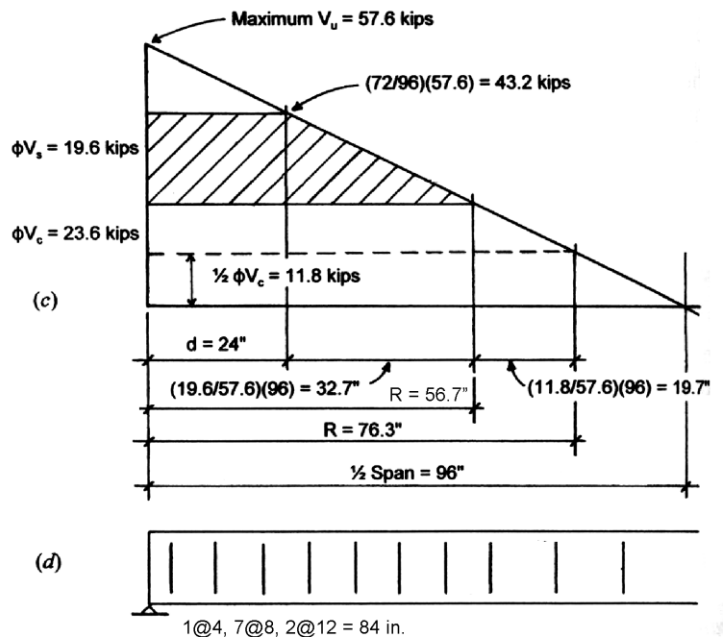
$\therefore$  Use #3 U @ 16'' max spacing

Example 12 (pg 444)

**Example 8.** Design the required shear reinforcement for the simple beam shown in Figure 13.18. Use  $f'_c = 3$  ksi [20.7 MPa] and  $f_y = 40$  ksi [276 MPa] and single U-shaped stirrups.

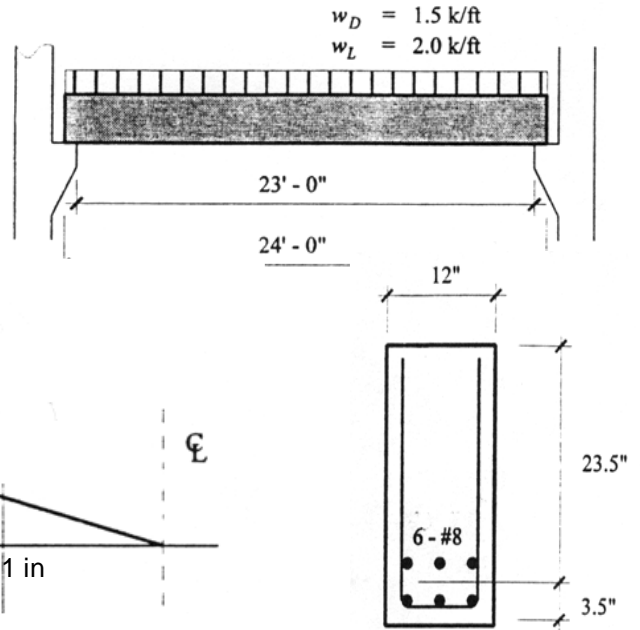


Example 12 (continued)

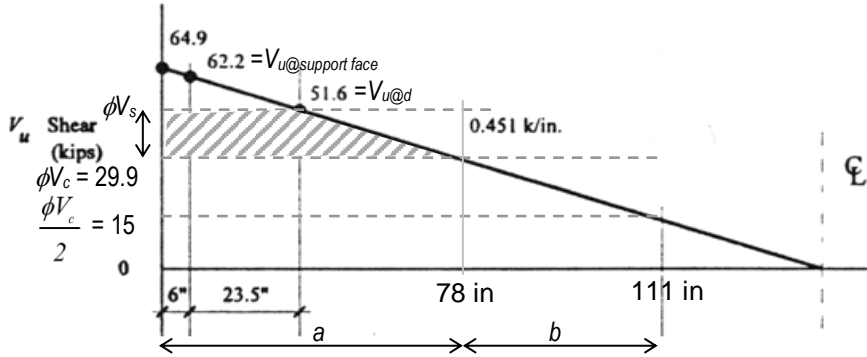


**Example 13**

Design the shear reinforcement for the simply supported reinforced concrete beam shown with a dead load of 1.5 k/ft and a live load of 2.0 k/ft. Use 5000 psi concrete and Grade 60 steel. Assume that the point of reaction is at the end of the beam.



SOLUTION:



**Shear diagram:**

Find self weight =  $\gamma A = 150 \text{ lb/ft}^3 \times 1 \text{ ft} \times (27/12 \text{ ft}) = 338 \text{ lb/ft} = 0.338 \text{ k/ft}$

$w_u = 1.2 (1.5 \text{ k/ft} + 0.338 \text{ k/ft}) + 1.6 (2 \text{ k/ft}) = 5.41 \text{ k/ft} (= 0.451 \text{ k/in})$

$V_{u(\text{max})}$  is at the ends =  $w_u L / 2 = 5.41 \text{ k/ft} (24 \text{ ft}) / 2 = 64.9 \text{ k}$

$V_{u(\text{support})} = V_{u(\text{max})} - w_u(\text{distance}) = 64.9 \text{ k} - 5.41 \text{ k/ft} (6/12 \text{ ft}) = 62.2 \text{ k}$

$V_u$  for design is  $d$  away from the support =  $V_{u(\text{support})} - w_u(d) = 62.2 \text{ k} - 5.41 \text{ k/ft} (23.5/12 \text{ ft}) = 51.6 \text{ k}$

**Concrete capacity:** ( $\lambda = 1$  for normal weight concrete)

We need to see if the concrete needs stirrups for strength or by requirement because  $V_u \leq \phi V_c + \phi V_s$  (design requirement)

$\phi V_c = \phi 2\lambda \sqrt{f'_c} b_w d = 0.75 (2)(1.0) \sqrt{5000} \text{ psi} (12 \text{ in}) (23.5 \text{ in}) = 299106 \text{ lb} = 29.9 \text{ kips} (< 51.6 \text{ k!})$

**Stirrup design and spacing**

We need stirrups:  $A_v$  for 2 # 3 legs is  $0.22 \text{ in}^2$ . ( $A_v = V_s / f_y d$  if you know the spacing.)

$\phi V_s \geq V_u - \phi V_c = 51.6 \text{ k} - 29.9 \text{ k} = 21.7 \text{ k}$

Spacing requirements are in Table 3-8 and depend on  $\phi V_c / 2 = 15.0 \text{ k}$  and  $\phi 4 \sqrt{f'_c} b_w d = 59.8 \text{ k}$

$s_{req'd} \leq \phi A_v f_y d / \phi V_s = 0.75 (0.22 \text{ in}^2) (60 \text{ ksi}) (23.5 \text{ in}) / 21.7 \text{ k} = 10.72 \text{ in}$   $\phi V_s (21.7 \text{ k})$  is smaller than  $59.8 \text{ k}$ , so our maximum is  $d/2$ , and not larger than  $24''$ , so  $d/2$  governs with  $11.75 \text{ in}$  The minimum is  $4 \text{ in}$ . Use  $10''$  (less than  $10.72$  &  $11.75$ ).

This spacing is valid until  $V_u = \phi V_c$  and that happens at  $(64.9 \text{ k} - 29.9 \text{ k}) / 0.451 \text{ k/in} = 78 \text{ in}$

We can put the first stirrup at a minimum of  $2 \text{ in}$  from the support face, so we need  $10''$  spacing for  $(78 - 2 - 6 \text{ in}) / 10$  in which is 7 even spaces (8 stirrups altogether ending at  $78 \text{ in}$ ).

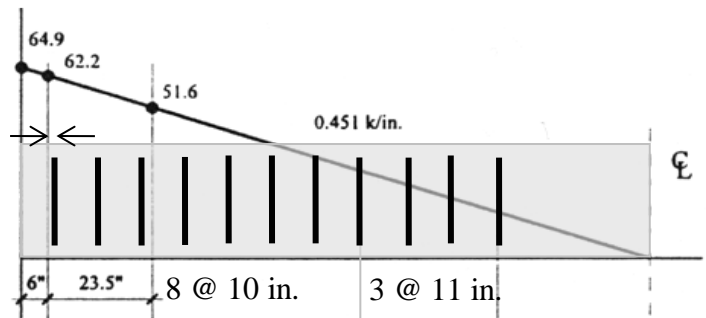
After  $78''$  we can change the spacing to the required where we NEED them (but not more than the maximum of  $d/2 \leq 24 \text{ in}$ );

$$s = A_v f_y / 50 b_w = 0.22 \text{ in}^2 (60,000 \text{ psi}) / 50 (12 \text{ in}) =$$

$$22 \text{ in} \leq A_v f_y / 0.75 \sqrt{f'_c} b_w =$$

$$0.22 \text{ in}^2 (60,000 \text{ psi}) / [0.75 \sqrt{5000} \text{ psi} (12 \text{ in})] = 20.74 \text{ in}$$

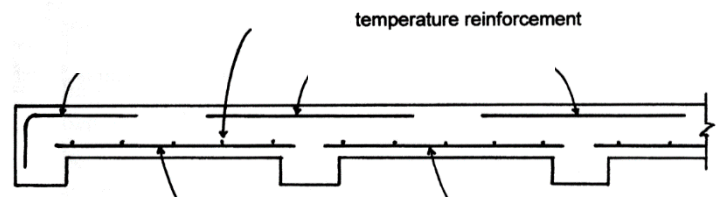
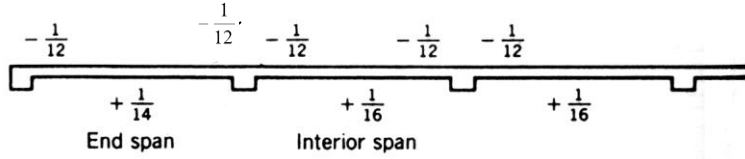
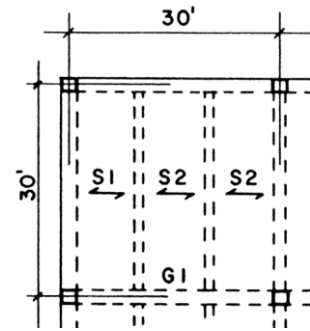
which is larger than the maximum of  $11.75$ , so use  $11 \text{ in}$ . We need to continue to  $111 \text{ in}$ , so  $(111 - 78 \text{ in}) / 11 \text{ in} = 3$  even spaces.

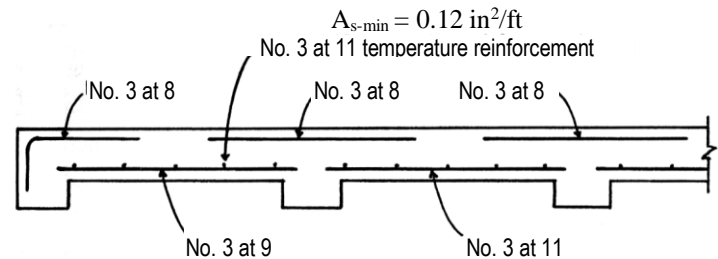


**Locating end points:**  
 $29.9 \text{ k} = 64.9 \text{ k} - 0.451 \text{ k/in} \times (a)$   
 $a = 78 \text{ in}$   
 $15 \text{ k} = 64.9 \text{ k} - 0.451 \text{ k/in} \times (b)$   
 $a + b = 111 \text{ in}$ .

Example 14 (pg 466)

**Example 1.** A solid one-way slab is to be used for a framing system similar to that shown in Figure 14.1. Column spacing is 30 ft. with evenly spaced beams occurring at 10 ft. center to center. Superimposed loads on the structure (floor live load plus other construction dead load) are a dead load of 38 psf [1.82 kPa] and a live load of 100 psf [4.79 kPa]. Use  $f'_c = 3$  ksi [20.7 MPa] and  $f_y = 40$  ksi [275 MPa]. Determine the thickness for the slab and select its reinforcement.





### Example 15

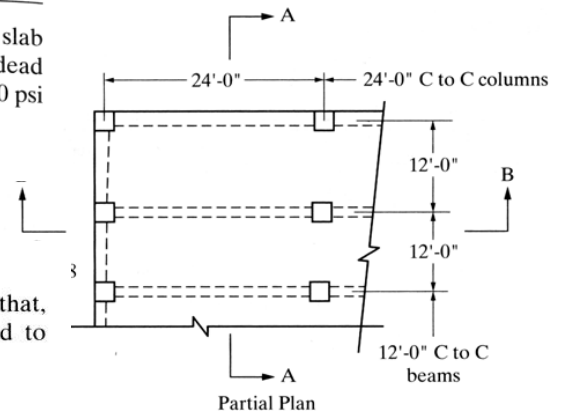
#### Example 6-1

The floor system shown in Figure 6-4 consists of a continuous one-way slab supported by continuous beams. The service loads on the floor are 25 psf dead load (does not include weight of slab) and 250 psf live load. Use  $f'_c = 3000$  psi (normal-weight concrete) and  $f_y = 60,000$  psi. The bars are uncoated.

Design the continuous one-way floor slab.

#### Solution:

The primary difference in this design from previous flexural designs is that, because of continuity, the ACI coefficients and equations will be used to determine design shears and moments.



#### A. Continuous one-way floor slab

1. Determine the slab thickness. The slab will be designed to satisfy the ACI minimum thickness requirements from Table 9.5(a) of the code and this thickness will be used to estimate slab weight.

With both ends continuous,

$$\text{minimum } h = \frac{1}{28} \ell_n = \frac{1}{28} (11)(12) = 4.71 \text{ in.}$$

With one end continuous,

$$\text{minimum } h = \frac{1}{24} \ell_n = \frac{1}{24} (11)(12) = 5.5 \text{ in.}$$

Try a 5½-in.-thick slab. Design a 12-in.-wide segment ( $b = 12$  in.).

2. Determine the load:

$$\text{slab dead load} = \frac{5.5}{12} (150) = 68.8 \text{ psf}$$

$$\text{total dead load} = 25.0 + 68.8 = 93.8 \text{ psf}$$

$$\begin{aligned} w_u &= 1.2 w_{DL} + 1.6 w_{LL} \\ &= 1.2(93.8) + 1.6(250) \\ &= 112.6 + 400.0 \\ &= 512.6 \text{ psf} \quad (\text{design load}) \end{aligned}$$

Because we are designing a slab segment that is 12 in. wide, the foregoing loading is the same as 512.6 lb/ft or 0.513 kip/ft.



Example 15 (continued)

3. Determine the moments and shears. Moments are determined using the ACI moment equations. Refer to Figures 6-1 and 6-4. Thus

$$+M_u = \frac{1}{14} w_u \ell_n^2 = \frac{1}{14} (0.513)(11)^2 = 4.43 \text{ ft-kips} \quad (\text{end span})$$

$$+M_u = \frac{1}{16} w_u \ell_n^2 = \frac{1}{16} (0.513)(11)^2 = 3.88 \text{ ft-kips} \quad (\text{interior span})$$

$$-M_u = \frac{1}{10} w_u \ell_n^2 = \frac{1}{10} (0.513)(11)^2 = 6.20 \text{ ft-kips} \quad (\text{end span - first interior support})$$

$$-M_u = \frac{1}{11} w_u \ell_n^2 = \frac{1}{11} (0.513)(11)^2 = 5.64 \text{ ft-kips} \quad (\text{interior span - both supports})$$

$$-M_u = \frac{1}{24} w_u \ell_n^2 = \frac{1}{24} (0.513)(11)^2 = 2.58 \text{ ft-kips} \quad (\text{end span - exterior support})$$

Similarly, the shears are determined using the ACI shear equations. In the end span at the face of the first interior support,

$$V_u = 1.15 \frac{w_u \ell_n}{2} = 1.15(0.513) \left( \frac{11}{2} \right) = 3.24 \text{ kips} \quad (\text{end span - first interior support})$$

whereas at all other supports,

$$V_u = \frac{w_u \ell_n}{2} = (0.513) \left( \frac{11}{2} \right) = 2.82 \text{ kips}$$

4. Design the slab. Assume #4 bars for main steel with  $\frac{3}{4}$  in. cover:  $d = 5.5 - 0.75 - \frac{1}{2}(0.5) = 4.5$  in.  
 5. Design the steel. (All moments must be considered.) For example, the negative moment in the end span at the first interior support:

$$R_n = \frac{M_u}{\phi b d^2} = \frac{6.20(12)(1000)}{0.9(12)(4.5)^2} = 340 \text{ ft-kips} \quad \text{so } \rho \cong 0.006$$

$$A_s = \rho b d = 0.006(12)(4.5) = 0.325 \text{ in}^2 \text{ per ft. width of slab} \quad \therefore \text{Use \#4 at 7 in. (16.5 in. max. spacing)}$$

The minimum reinforcement required for flexure is the same as the shrinkage and temperature steel.

(Verify the moment capacity is achieved:  $a = 0.67$  in. and  $\phi M_n = 6.38$  ft-kips  $>$  6.20 ft-kips)

For grade 60 the minimum for shrinkage and temperature steel is:

$$A_{s-min} = 0.0018 b t = 0.0018 (12)(5.5) = 0.12 \text{ in}^2 \text{ per ft. width of slab} \quad \therefore \text{Use \#3 at 11 in. (18 in. max spacing)}$$

6. Check the shear strength. ( $\lambda = 1$  for normal weight material)

$$\phi V_c = \phi 2 \lambda \sqrt{f'_c} b d = 0.75(2)(1)\sqrt{3000}(12)(4.5) = 4436.6 \text{ lb} = 4.44 \text{ kips}$$

$$V_u \leq \phi V_c \quad \text{Therefore the thickness is O.K.}$$

7. Development length for the flexure reinforcement is required. (Hooks are required at the spandrel beam.) For example, #6 bars:

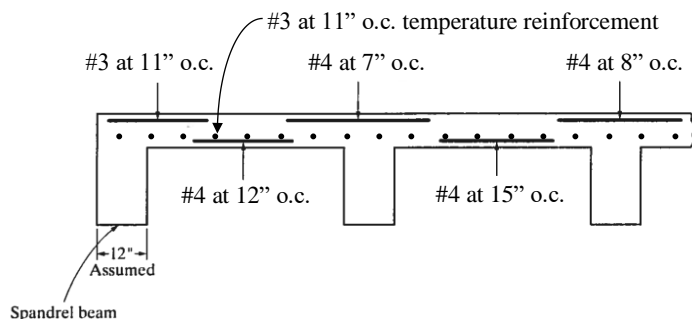
$$l_d = \frac{d_b f_y}{25 \lambda \sqrt{f'_c}} \quad \text{or 12 in. minimum}$$

With grade 40 steel and 3000 psi concrete:

$$l_d = \frac{\frac{5}{8} \text{ in} (40,000 \text{ psi})}{25(1)\sqrt{3000 \text{ psi}}} = 21.9 \text{ in}$$

(which is larger than 12 in.)

8. Sketch:



**Example 16**

A building is supported on a grid of columns that is spaced at 30 ft on center in both the north-south and east-west directions. Hollow core planks with a 2 in. topping span 30 ft in the east-west direction and are supported on precast L and inverted T beams. Size the hollow core planks assuming a live load of 100 lb/ft<sup>2</sup>. Choose the shallowest plank with the least reinforcement that will span the 30 ft while supporting the live load.

**SOLUTION:**

The shallowest that works is an 8 in. deep hollow core plank.

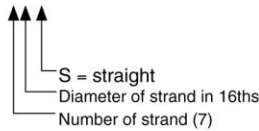
The one with the least reinforcing has a strand pattern of 68-S, which contains 6 strands of diameter 8/16 in. = 1/2 in. The S indicates that the strands are straight. The plank supports a superimposed service load of 124 lb/ft<sup>2</sup> at a span of 30 ft with an estimated camber at erection of 0.8 in. and an estimated long-time camber of 0.2 in.

The weight of the plank is 81 lb/ft<sup>2</sup>.

**3.6 Hollow-Core Load Tables (cont.)**

**Strand Pattern Designation**

76-S



Safe loads shown include dead load of 10 lb/ft<sup>2</sup> for untopped members and 15 lb/ft<sup>2</sup> for topped members. Remainder is live load. Long-time cambers include superimposed dead load but do not include live load.

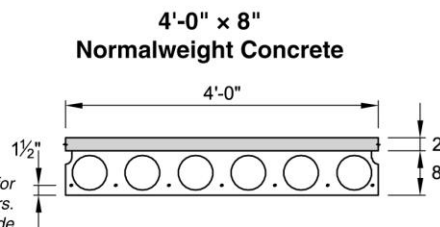
Capacity of sections of other configurations are similar. For precise values, see local hollow-core manufacturer.

**Key**

385- Safe superimposed service load, lb/ft<sup>2</sup>

0.1 - Estimated camber at erection, in.

0.2 - Estimated long-time camber, in.



$f'_c = 5000 \text{ psi}$

$f_{pu} = 270,000 \text{ psi}$

**Section Properties**

	No Topping	2 in. topping
A	= 215 in. <sup>2</sup>	-
I	= 1666 in. <sup>4</sup>	3071 in. <sup>4</sup>
y <sub>b</sub>	= 4.00 in.	5.29 in.
y <sub>t</sub>	= 4.00 in.	4.71 in.
S <sub>b</sub>	= 417 in. <sup>3</sup>	581 in. <sup>3</sup>
S <sub>t</sub>	= 417 in. <sup>3</sup>	652 in. <sup>3</sup>
wt	= 224 lb/ft	324 lb/ft
DL	= 56 lb/ft <sup>2</sup>	81 lb/ft <sup>2</sup>
V/S	= 1.92 in.	

**4HC8 + 2**

**Table of safe superimposed service load, lb/ft<sup>2</sup>, and cambers, in.**

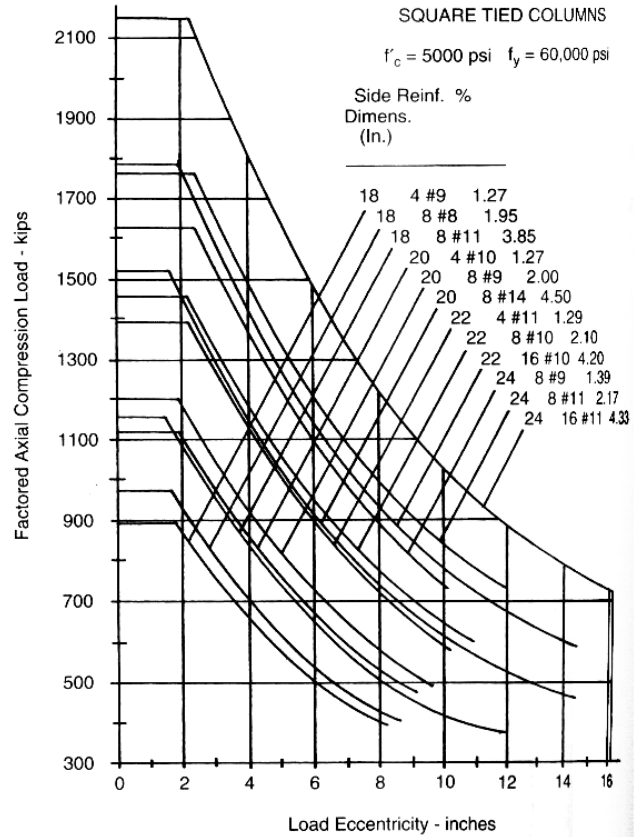
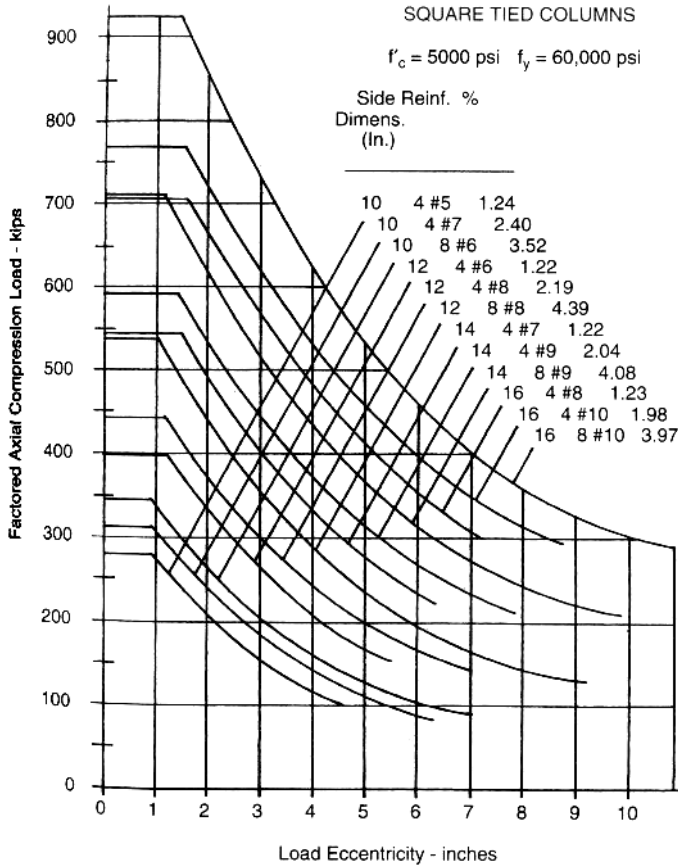
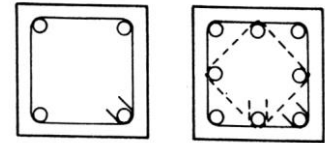
**2 in. Normalweight Topping**

Strand designation code	Span, ft																																									
	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40														
<b>66-S</b>	400	365	333	308	282	256	224	197	173	153	135	119	105	93	82	68	56	45	36	26																						
<b>76-S</b>	474	435	396	366	340	304	267	235	208	184	164	146	130	116	103	88	74	62	51	41	31																					
<b>58-S</b>	445	405	374	342	318	298	275	260	243	228	217	196	177	159	143	126	110	95	82	70	59	49	40	32																		
<b>68-S</b>	463	426	393	366	342	319	299	282	267	251	239	216	195	177	158	140	124	110	97	84	73	62	53	44	36	28																
<b>78-S</b>	472	435	402	375	348	325	305	288	273	257	245	232	220	207	186	167	149	133	119	106	94	83	73	64	55	46	38															

Strength is based on strain compatibility; bottom tension is limited to 7.5√f<sub>c</sub>; see pages 3-8 through 3-11 for explanation. See item 3, note 4, Section 3.3.2 for explanation of vertical line.

Example 17 (pg 490)

**Example 1.** A square tied column with  $f'_c = 5$  ksi and steel with  $f_y = 60$  ksi sustains an axial compression load of 150 kips dead load and 250 kips live load with no computed bending moment. Find the minimum practical column size if reinforcing is a maximum of 4% and the maximum size if reinforcing is a minimum of 1%. Also, design for  $e = 6$  in.



**Example 18**

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.

SOLUTION:

Find  $\phi P_n$ , with  $\phi=0.65$  and  $P_n = 0.80P_o$  for tied columns and

$$P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

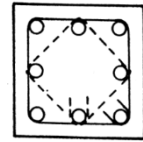
$$A_{st} = 8 \text{ bars} \times (1.27 \text{ in}^2) = 10.16 \text{ in}^2$$

Concrete area (gross):

$$A_g = 16 \text{ in} \times 16 \text{ in} = 256 \text{ in}^2$$

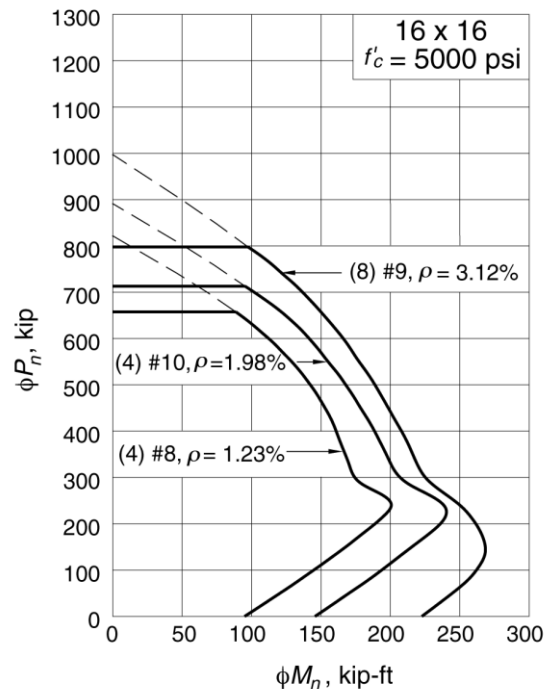
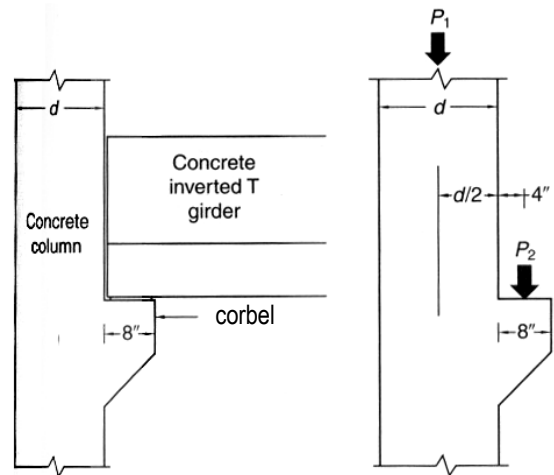
Grade 40 reinforcement has  $f_y = 40,000 \text{ psi}$  and  $f'_c = 4000 \text{ psi}$

$$\phi P_n = (0.65)(0.80)[0.85(4000 \text{ psi})(256 \text{ in}^2 - 10.16 \text{ in}^2) + (40,000 \text{ psi})(10.16 \text{ in}^2)] = 646,026 \text{ lb} = 646 \text{ kips}$$



**Example 19**

16" x 16" precast reinforced columns support inverted T girders on corbels as shown. The unfactored loads on the corbel are 81 k dead, and 72 k live. The unfactored loads on the column are 170 k dead and 150 k live. Determine the reinforcement required using the interaction diagram provided. Assume that half the moment is resisted by the column above the corbel and the other half is resisted by the column below. Use grade 60 steel and 5000 psi concrete.



## Example 20

**EXAMPLE 5-4**

Design a short square tied column to carry an axial dead load of 300 kip and a live load of 200 kip. Assume that the applied moments on the column are negligible. Use  $f'_c = 4,000$  psi and  $f_y = 60,000$  psi.

**Solution**

Step 1 The factored load,  $P_u$ , is:

$$\begin{aligned} P_u &= 1.2P_D + 1.6P_L \\ P_u &= 1.2(300) + 1.6(200) \\ P_u &= 680 \text{ kip} \end{aligned}$$

Assume  $\rho_g = 0.03$ .

Step 2 The required area of the column,  $A_g$ , is:

$$\begin{aligned} A_g &= \frac{P_u}{0.8\phi[0.85f'_c(1 - \rho_g) + f_y\rho_g]} \\ A_g &= \frac{680}{0.80(0.65)[0.85(4)(1 - 0.03) + 60(0.03)]} \\ A_g &= 257 \text{ in}^2 \end{aligned}$$

Step 3 For a square column, the size,  $h$ , is:

$$\begin{aligned} h &= \sqrt{A_g} = \sqrt{257} \\ \therefore h &= 16.0 \text{ in.} \end{aligned}$$

Try a 16 in.  $\times$  16 in. column:

$$A_g = (16)(16) = 256 \text{ in}^2$$

Step 4 The required amount of steel,  $A_{st}$ , is:

$$\begin{aligned} A_{st} &= \frac{P_u - 0.8\phi(0.85f'_c A_g)}{0.8\phi(f_y - 0.85f'_c)} \\ A_{st} &= \frac{680 - 0.8 \times 0.65(0.85 \times 4 \times 256)}{0.8 \times 0.65(60 - 0.85 \times 4)} = 7.73 \text{ in}^2 \end{aligned}$$

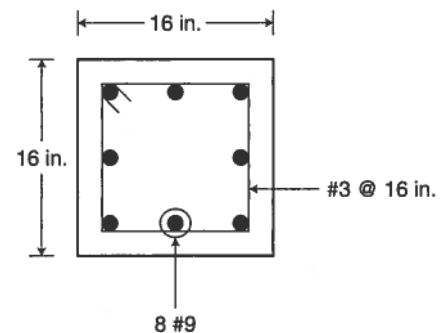
Step 5 Select the size and number of bars. For a square column with bars uniformly distributed along the edges, we keep the number of bars as multiples of four. Using Table A2-9, 8 #9 bars ( $A_s = 8 \text{ in}^2$ ) are selected.

From Table A5-1  $\rightarrow$  Maximum of 12 #9 bars  $\therefore$  ok

Step 6 Because the longitudinal bars are #9, select #3 bars for the ties. The maximum spacing of the ties ( $s_{\max}$ ) is:

$$\begin{aligned} s_{\max} &= \min\{16d_b, 48d_t, b_{\min}\} \\ s_{\max} &= \min\{16(1.128), 48(\frac{3}{8}), 16\} \\ s_{\max} &= \min\{18.0, 18.0, 16.0\} \\ \therefore s_{\max} &= 16 \text{ in.} \end{aligned}$$

The selected ties are #3 @ 16 in.



Example 21

Design a 10 ft long circular spiral column for a braced system to support the service dead and live loads of 300k and 460k, respectively, and the service dead and live moments of 100 ft-k each. The moment at one end is zero. Use  $f'_c = 4,000$  psi and  $f_y = 60,000$  psi.

**Solution**

- $P_u = 1.2(300) + 1.6(460) = 1096$  k  
 $M_u = 1.2(100) + 1.6(100) = 280$  ft-k
- Assume  $\rho_g = 0.01$ , from Equation 16.10:

$$A_g = \frac{P_u}{0.60[0.85f'_c(1-\rho_g) + f_y\rho_g]}$$

$$= \frac{1096}{0.60[0.85(4)(1-0.01) + 60(0.01)]}$$

$$= 460.58 \text{ in.}^2$$

$$\frac{\pi h^2}{4} = 460.58$$

or  $h = 24.22$  in.

Use  $h = 24$  in.,  $A_g = 452$  in.<sup>2</sup>

- Assume #9 size of bar and 3/8 in. spiral center-to-center distance  
 $= 24 - 2(\text{cover}) - 2(\text{spiral diameter}) - 1$  (bar diameter)  
 $= 24 - 2(1.5) - 2(3/8) - 1.128 = 19.12$  in.

ACI 20.6: Concrete exposed to earth or weather:  
No. 6 through No. 18 bars..... 2 in. minimum

$$\gamma = \frac{19.12}{24} = 0.8$$

Use the interaction diagram Appendix D.21

$$4. K_n = \frac{P_u}{\phi f'_c A_g} = \frac{1096}{(0.75)(4)(452)} = 0.808$$

$$R_n = \frac{M_u}{\phi f'_c A_g h} = \frac{3360}{(0.75)(4)(452)(24)} = 0.103$$

- At the intersection point of  $K_n$  and  $R_n$ ,  $\rho_g = 0.02$
- The point is above the strain line = 1, hence  $\phi = 0.7$  **OK**
- $A_{st} = (0.02)(452) = 9.04$   
From Appendix D.2, select 12 bars of #8,  $A_{st} = 9.48$  in.<sup>2</sup>  
From Appendix D.14 for a core diameter of  $24 - 3 = 21$  in. 17 bars of #8 can be arranged a row
- Selection of spirals  
From Appendix D.13, size = 3/8 in.  
pitch = 2 1/4 in.  
Clear distance =  $2.25 - 3/8 = 1.875 > 1$  in. **OK**
- $K = 1$ ,  $l = 10 \times 12 = 120$  in.,  $r = 0.25(24) = 6$  in.

$$\frac{Kl}{r} = \frac{1(120)}{6} = 20$$

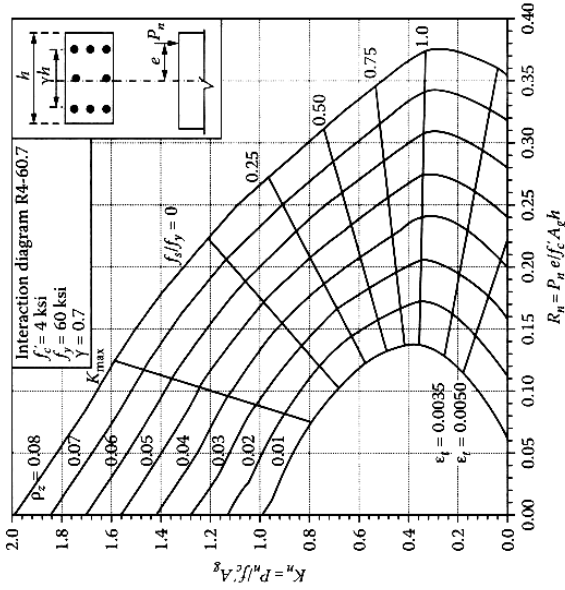
$$\left( \frac{M_1}{M_2} \right) = 0$$

$$34 - 12 \left( \frac{M_1}{M_2} \right) = 34$$

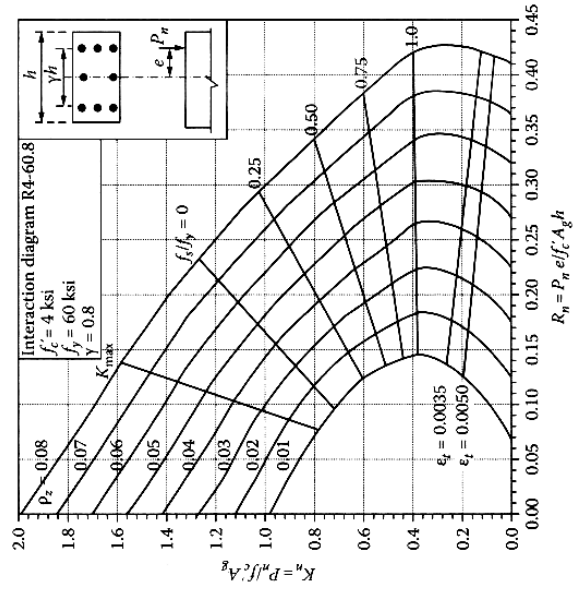
ACI 6.2: In nonsway frames it shall be permitted to ignore slenderness effects for compression members that satisfy:  $\frac{kl_u}{r} \leq 34 - 12 \left( \frac{M_1}{M_2} \right)$

since  $(Kl/r) < 34$ , short column.

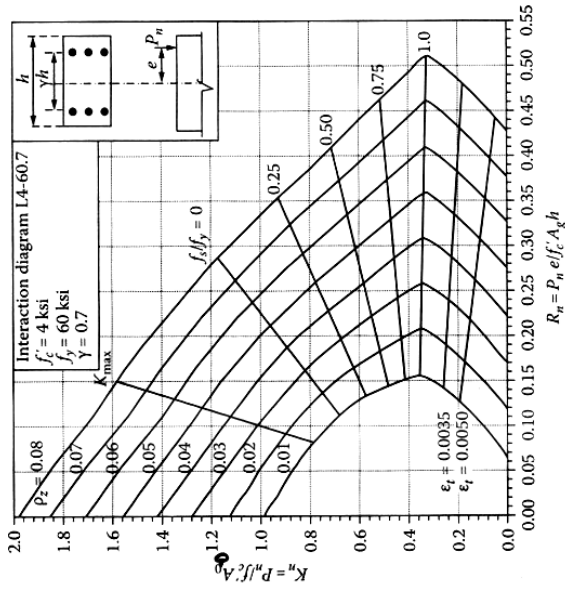
# Column Interaction Diagrams



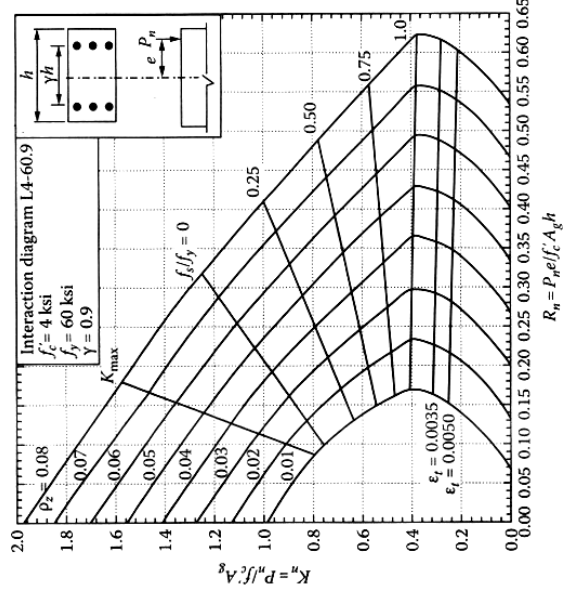
**FIGURE D.17** Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



**FIGURE D.18** Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

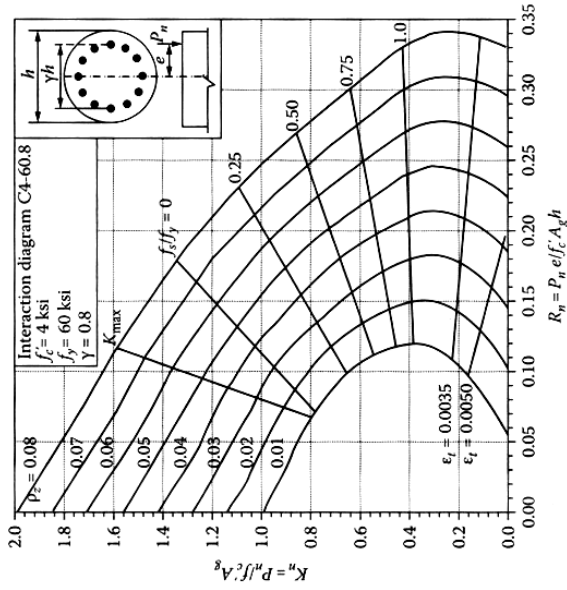


**FIGURE D.15** Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

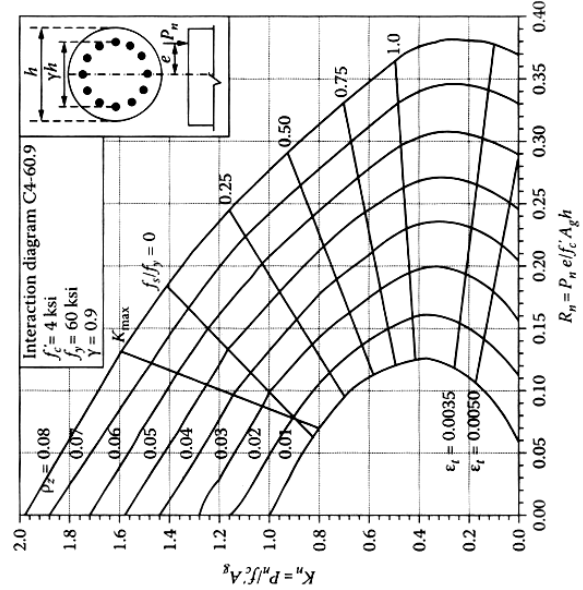


**FIGURE D.16** Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

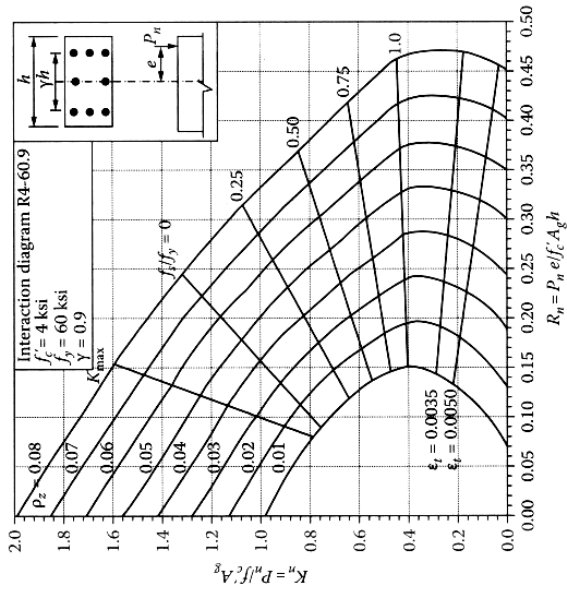
# Column Interaction Diagrams



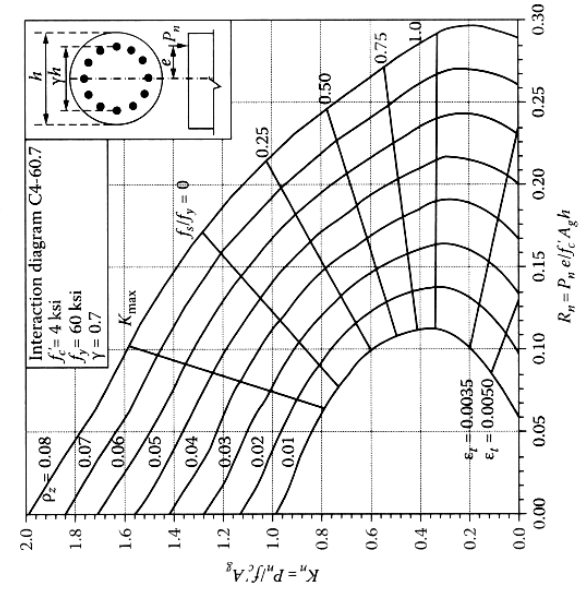
**FIGURE D.21** Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



**FIGURE D.22** Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



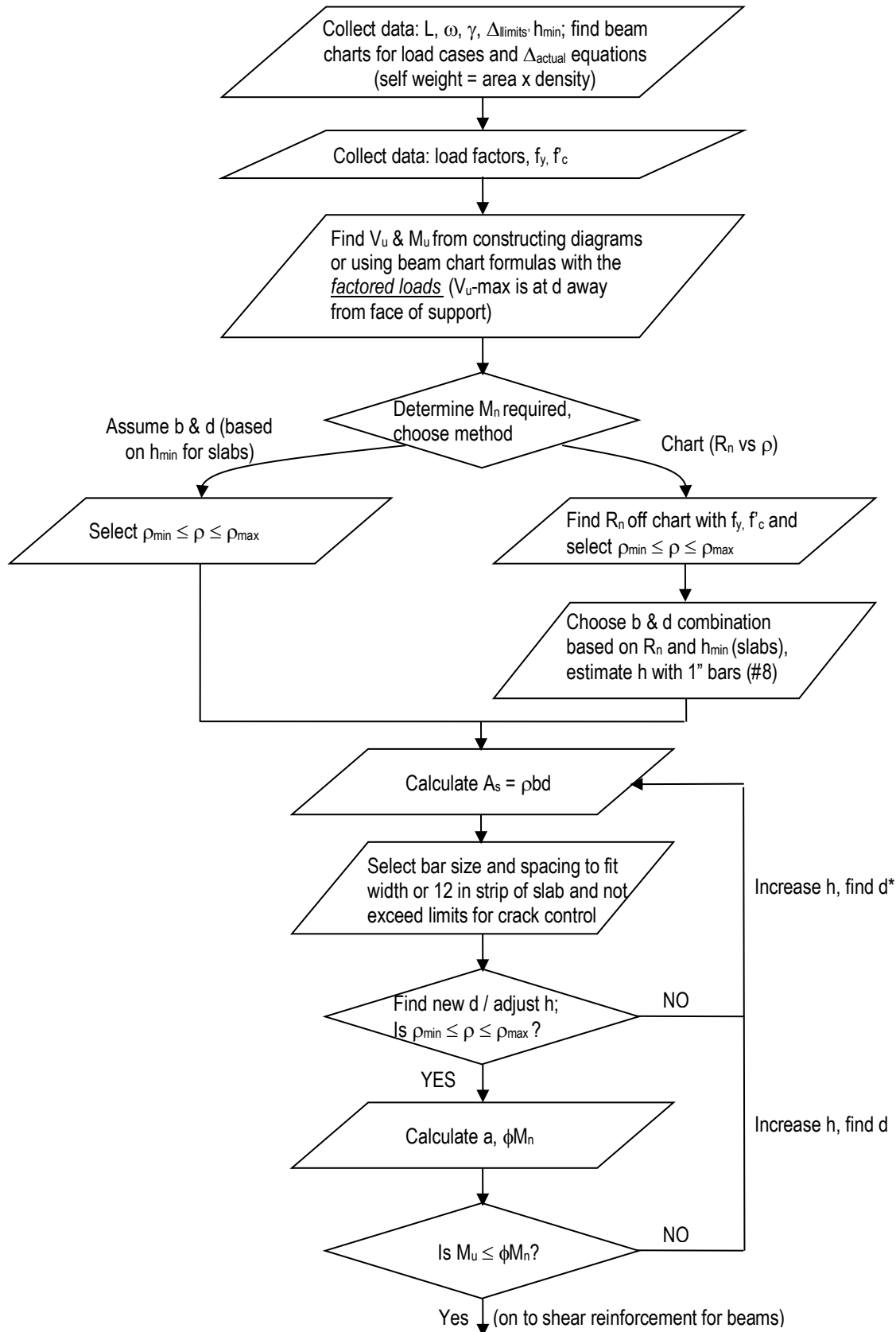
**FIGURE D.19** Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



**FIGURE D.20** Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)



### Beam / One-Way Slab Design Flow Chart



### Beam / One-Way Slab Design Flow Chart - continued

