

RELATIONSHIP BETWEEN DEFORMATION-RELAXATION AND HEAT-AND-MASS-TRANSFER PROCESSES IN THE DRYING OF CAPILLARY-POROUS BODIES

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In the convection drying of capillary-porous bodies with coupled heat and mass transfer, the bodies undergo deformation because the specific volume of the constituent material depends on temperature and humidity. In the high-rate treatment of such bodies by heat and moisture, the attendant change in their volume due to mechanical stresses can lead to partial or complete failure of the material. In turn, the deformation also has a certain effect on heat and mass transfer. Thus, the optimization of many hydrothermal technologies is closely allied with study of the relationship between deformation-relaxation and heat-and-mass-transfer processes. A general approach was proposed in [7, 10] for describing the effect of heat and mass transfer on the stress-strain state of solids within the framework of the theory of small elastoplastic strains. Coupled equations of thermoviscoelasticity were derived as part of the basic research reported in [4, 5, 6]. Diffusion processes in a viscoelastic body undergoing deformation were examined in [11].

In this investigation, we proceed on the basis of the thermodynamics of irreversible processes to derive general equations describing coupled deformation-relaxation and heat-and-mass-transfer processes with allowance for failure during the drying of capillary-porous bodies.

To do this, we examine free energy [2] as the main thermodynamic potential of a system which undergoes changes in temperature and volume

$$dF = dF_1 + dF_2. \tag{1}$$

Here, dF_1 is the change in free energy as a result of heat and mass transfer

$$dF_1 = -SdT + \rho_0 \mu dU, \tag{2}$$

S is the entropy of a unit volume V ; T is temperature; U is moisture content; μ is chemical potential; ρ_0 is the density of the substance when it is completely dry; dF_2 is the change in free energy during deformation and failure

$$dF_2 = \sigma_{ij} d\varepsilon_{ij} + \varphi_{nm} d\omega_{nm}, \tag{3}$$

σ_{ij} and ε_{ij} are the components of the stress and strain tensors, respectively; ω_{nm} are the components of the damage tensor; φ_{nm} is the corresponding potential.

It follows from Eqs. (1-3) that

$$\begin{aligned} S &= - \left(\frac{\partial F}{\partial T} \right)_{U, \varepsilon_{ij}, \omega_{nm}}; \quad \rho_0 \mu = \left(\frac{\partial F}{\partial U} \right)_{T, \varepsilon_{ij}, \omega_{nm}}; \\ \sigma_{ij} &= \left(\frac{\partial F}{\partial \varepsilon_{ij}} \right)_{T, U, \omega_{nm}}; \quad \varphi_{nm} = \left(\frac{\partial F}{\partial \omega_{nm}} \right)_{T, U, \varepsilon_{ij}}. \end{aligned} \tag{4}$$

We represent the relative change in a unit volume of the material due to heat and mass transfer processes in the form

$$\frac{dV}{V_0} = \frac{V - V_0}{V_0} = 3 \left(\alpha_T (T - T_0) + \beta_U (U - U_0) + \gamma (\Pi - \Pi_0) \right),$$

where

$$\alpha_T = \frac{1}{3V_0} \frac{\partial V}{\partial T}; \quad \beta_U = \frac{1}{3V_0} \frac{\partial V}{\partial U}; \quad \gamma = \frac{1}{3V_0} \frac{\partial V}{\partial \Pi} \quad (5)$$

are the coefficient of linear expansion, expansion factor, and coefficient of structural expansion; Π is the volume of the damages (pores, cracks, etc.); the subscript "0" denotes the corresponding characteristics of the material in the equilibrium state.

In accordance with [1, 9], we describe the damage tensor ω_{nm} by means of a single parameter $l = \Pi/\Pi_0$. Then $\omega_{nm} = l$ when $n = m$ and $\omega_{nm} = 0$ when $n \neq m$.

For relatively small elastic and viscoelastic strains, free energy can be expanded into a series in the neighborhood of the undeformed states $F(T, U, l_0)$ in the invariants I_1, I_2 of the principal values of the strain tensor [4]. Here, l_0 is the damage in the undeformed material. In a special case, we can take $l_0 = 1$ for $\varepsilon_{ij} = 0$.

We thus have

$$F = F_0 + I_1 \frac{\partial F_0}{\partial I_1} + I_2 \frac{\partial F_0}{\partial I_2} + \frac{I_1^2}{2} \frac{\partial^2 F_0}{\partial I_1^2}, \quad (6)$$

where

$$I_1 = \varepsilon_{ii}, \quad I_2 = \varepsilon_{ij} \varepsilon_{ij}.$$

Numerical values of the coefficients of the expansion can be found on the basis of Hooke's empirical laws and the expansion of the bodies that accompanies the heating and moistening of their materials [4, 10]

$$\frac{\partial F_0}{\partial I_1} = -\frac{dV}{V_0} \left(3 \frac{\partial^2 F}{\partial I_1^2} + 2\mu \right); \quad \frac{\partial^2 F_0}{\partial I_1^2} = \lambda; \quad \frac{\partial F_0}{\partial I_2} = \frac{E}{2(1+\nu)},$$

where $\mu = \mu(T, U)$, $E = E(T, U)$, and $\nu = \nu(T, U)$ are respectively the shear modulus, elastic modulus, and Poisson's ratio; $\lambda = E\nu/(1 - 2\nu)(1 + \nu)$ is the Lamé constant. To generalize Eq. (6) to the case of viscoelastic strains, we write it in the form:

$$F = F_0 - \frac{K}{2} \left(\frac{dV}{V_0} \right)^2 - K \frac{dV}{V_0} \varepsilon_{kk} + \mu \left(\varepsilon_{ij} - \frac{1}{3} \delta_{ij} \varepsilon_{kk} \right)^2 + \frac{K}{2} \left(\varepsilon_{kk}^2 + \left(\frac{dV}{V_0} \right)^2 \right), \quad (7)$$

where $K = \lambda + 2\mu/3$ is the compressive bulk modulus; δ_{ij} is the unit tensor; $\delta_{ij} = 1$ when $i = j$; $\delta_{ij} = 0$ when $i \neq j$.

To further generalize the problem, we isolate the deviatoric part of the stress and strain tensors by means of the formulas $\tau_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$, $\varepsilon_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_{kk}/3$ in Hooke's law. This part of the tensors accounts for the damage to the material and heat and mass transfer processes and is obtained from Eqs. (4), (6)

$$\sigma_{ij} = -\delta_{ij} \left(\frac{dV}{V_0} \left(\frac{2}{3}\mu + \lambda \right) - \xi P - \lambda \varepsilon_{kk} \right) + 2\mu \varepsilon_{ij}, \quad (8)$$

where P is the pressure of the liquid.

Thus, by using a differential relation that is linear with respect to time [4], we obtain the following formula to express the relationship between the first invariants of the stress and strain tensors in a rheological model of viscoelastic bodies that is sufficiently general in scope

$$A\sigma_{ij} = B(\varepsilon_{ij} - \delta_{ij}\varepsilon_{kk}/3) + \delta_{ij}AK\left(\varepsilon_{kk} - \frac{dV}{V_0}\right), \quad (9)$$

where A and B are differential operators that are linear with respect to time [4].

Resorting to a rheological description of capillary-porous bodies such as wood [12], we use (9) to obtain relations to determine the stress–strain state with allowance for heat and mass transfer processes

$$\begin{aligned} & \frac{E_2}{\eta_1}\sigma_{ij} + \left(1 + \frac{\eta_2}{\eta_1} + \frac{E_2}{E_1}\right)\frac{\partial\sigma_{ij}}{\partial t} + \frac{\eta_2}{E_1}\frac{\partial^2\sigma_{ij}}{\partial t^2} = \\ & = E_2\frac{\partial}{\partial t}(\varepsilon_{ij} - \delta_{ij}\varepsilon_{kk}/3) + \eta_2\frac{\partial^2}{\partial t^2}(\varepsilon_{ij} - \delta_{ij}\varepsilon_{kk}/3) + \\ & + \delta_{ij}K\left[\frac{E_2}{E_1}\left(\varepsilon_{kk} - \frac{V - V_0}{V_0}\right) + \left(1 + \frac{\eta_2}{\eta_1} + \frac{E_2}{E_1}\right)\frac{\partial}{\partial t}\left(\varepsilon_{kk} - \frac{V - V_0}{V_0}\right) + \right. \\ & \quad \left. + \frac{\eta_2}{E_2}\frac{\partial^2}{\partial t^2}\left(\varepsilon_{kk} - \frac{V - V_0}{V_0}\right)\right], \end{aligned} \quad (10)$$

where E_1 , E_2 , η_1 , and η_2 are the rheological coefficients of wood. The values of these coefficients depend on the temperature-moisture conditions and are found experimentally.

Thus, proceeding on the basis of the thermodynamics of irreversible processes, we have obtained relations to determine the thermohydroviscoelastic stress–strain state of capillary-porous bodies in general and wood in particular.

If convection takes place among the liquid components in a capillary-porous medium, as it typically does in high-rate drying operations (drying with high-frequency currents, contact drying, etc.), additional heat and mass transfer occurs in the material due to the existence of a gradient of the total pressure of the moist air in the medium. The gradient is the result of vaporization of the liquid and the resistance presented by the skeleton of the body to the motion of the vapor. Convection is facilitated by the presence of microcapillaries. Molecular infiltration of air into the body from the surrounding medium takes place through the system of microcapillaries, and diffusion also takes place in the system. There may therefore be a difference between the stresses applied to a unit area of the skeleton of the porous body σ^s_{ij} and a unit area of the liquid σ^l_{ij} that fills the pores of the skeleton. The effective stresses on a unit area are equal to [10]

$$\sigma_{ij} = \sigma^s_{ij}(1 - \xi) + \sigma^l_{ij}\xi,$$

where ξ is the volume fraction of liquid in the body.

In the general case, determination of the stress tensor σ^l_{ij} entails solving a problem of fluid flow. If the velocity of the liquid in the pores of the given capillary-porous body is low and inertial forces can be ignored compared to viscous and pressure forces, then the stress tensor σ^l_{ij} becomes part of the fluid pressure P and the motion of the fluid is described by the corresponding filtration equation. In terms of structure, this equation is analogous to the equations that describe heat conduction and diffusion.

Proceeding on this basis, we can write the following to account for the additional stresses due to the action of excess pressure in the wood

$$\frac{\partial F_0}{\partial I_1} = \xi P - \left(\frac{2}{3}\mu + \lambda\right)\frac{dV}{V_0},$$

where the quantity ξ can be determined from the following relation on the basis of the change in pressure and the corresponding change in specific volume Δv_p at $T = \text{const}$ and $U = \text{const}$

$$\xi = \frac{\Delta v_p}{3V_0 P}(2\mu + 3\lambda).$$

In addition to generalizing relations (8), (10), we use (4) and (6) to determine the values of chemical potential, entropy, and damage potential.

In accordance with the formula that determines heat capacity at constant volume in the absence of strains [10], the free energy function can be represented in the form

$$F_0 = - \int_{T_0}^T dT \int_{T_0}^T \frac{c_v}{T} dT.$$

We should note that $S = 0$ and $F = 0$ when $\varepsilon_{ij} = 0$ and $T = T_0$.

We then have

$$S = \int_{T_0}^T \frac{c_v}{T} dT + \varepsilon_{\mu\kappa} \frac{\partial}{\partial T} \left(\frac{V - V_0}{V_0} \left(\frac{2}{3} \mu + \lambda \right) - \xi P \right) - \varepsilon_{ij} \varepsilon_{ij} \frac{\partial \mu}{\partial T} - \frac{1}{2} \varepsilon_{\mu\kappa}^2 \frac{\partial \lambda}{\partial T}; \quad (11)$$

$$\mu = - \int_{T_0}^T dT \int_{T_0}^T \frac{\partial c_v}{\partial U} dT - \varepsilon_{\mu\kappa} \frac{\partial}{\partial U} \left(\frac{V - V_0}{V_0} \left(\frac{2}{3} \mu + \lambda \right) - \xi P \right) + \varepsilon_{ij} \varepsilon_{ij} \frac{\partial \mu}{\partial U} + \frac{1}{2} \varepsilon_{\mu\kappa}^2 \frac{\partial \lambda}{\partial U}; \quad (12)$$

$$\varphi_{nm} = \varphi_{nm}^0 - \varepsilon_{\mu\kappa} \frac{\partial}{\partial \omega_{nm}} \left(\frac{V - V_0}{V_0} \left(\frac{2}{3} \mu + \lambda \right) - \xi P \right) + \varepsilon_{ij} \varepsilon_{ij} \frac{\partial \mu}{\partial \omega_{nm}} + \frac{1}{2} \varepsilon_{\mu\kappa}^2 \frac{\partial \lambda}{\partial \omega_{nm}}, \quad (13)$$

where the components of the tensor φ_{nm}^0 are represented in a form analogous to ω_{nm}

$$\varphi_{nm}^0 = \left(\frac{\partial F_0}{\partial l} \right)_{v,T} = h, \quad n = m; \quad \varphi_{nm}^0 = 0, \quad n \neq m.$$

Here, h is determined by the intrinsic properties of the body and its structure. Alternative tensor descriptions of capillary-porous bodies were given in [1, 9].

In accordance with the law of thermodynamics of irreversible processes, the product of absolute temperature and the rate of increase in entropy is equal to the sum of the products of the fluxes and the corresponding thermodynamic forces [2]. Thus, the energy and mass transport equations for the high-rate drying of moist capillary-porous bodies can be written in the form of a system of differential equations

$$T \frac{\partial S}{\partial t} = \text{div} (\lambda_{TT} \text{grad } T) + \text{div} (\lambda_{TU} \text{grad } U) + \text{div} (\lambda_{TP} \text{grad } P) + \mu \rho_0 \frac{\partial U}{\partial t}; \quad (14)$$

$$\frac{\partial \rho}{\partial t} = \text{div} (\lambda_{UT} \text{grad } T) + \text{div} (\lambda_{UU} \text{grad } U) + \text{div} (\lambda_{UP} \text{grad } P),$$

where λ_{ij} represents the thermophysical coefficients and thermodynamic characteristics of the mass transport process, values of these quantities having been presented in [8] for specific systems; ρ is density.

If we ignore the terms in Eq. (11) that are of second-order smallness relative to the strain ε_{ij} , then

$$\frac{\partial S}{\partial t} = \frac{c_v}{T} \frac{\partial T}{\partial t} + \frac{\partial}{\partial t} \left(\varepsilon_{\mu\kappa} \frac{\partial}{\partial t} \left(\frac{V - V_0}{V_0} \left(\frac{2}{3} \mu + \lambda \right) - \xi P \right) \right). \quad (15)$$

Since $c_v = \rho c_v$ (where c_v is heat capacity at $V = \text{const}$), then heat-conduction equation (14) will appear as follows with allowance for the deformation-relaxation processes that take place during the drying of wood-based materials

$$\rho c_v \frac{\partial T}{\partial t} = \text{div} (\lambda_{TT} \text{grad } T) + \text{div} (\lambda_{TU} \text{grad } U) + \text{div} (\lambda_{TP} \text{grad } P) +$$

$$+ \mu \rho_0 \frac{\partial U}{\partial t} - T \frac{\partial}{\partial t} \left(\varepsilon_{kk} \frac{\partial}{\partial T} \left(\frac{V - V_0}{V_0} \left(\frac{2}{3} \mu + \lambda \right) - \xi P \right) \right). \quad (16)$$

In connection with the change in mass content and volume due to deformation, the density of the moist capillary-porous body during drying can be written as

$$\frac{\partial \rho}{\partial t} = \rho_0 \frac{\partial U}{\partial t} + (1 + U) \frac{\partial \rho_0}{\partial t}.$$

Allowing for the change in the density of the solid skeleton in the description of the viscoelastic strains leads to quantities of second-order smallness, which allows us to take $\rho = \text{const}$. Given this assumption and with allowance for (12), (14), the mass transport equation for a capillary-porous body undergoing deformation during drying has the form

$$\begin{aligned} \rho_0 \frac{\partial U}{\partial t} = & \text{div} (\lambda_{UT} \text{grad } T) + \text{div} (\lambda_{UU} \text{grad } U) + \text{div} (\lambda_{UP} \text{grad } P) - \\ & - \frac{\partial}{\partial t} \left(\varepsilon_{kk} \frac{\partial}{\partial U} \left(\frac{V - V_0}{V_0} \left(\frac{2}{3} \mu + \lambda \right) - \xi P \right) \right). \end{aligned} \quad (17)$$

Equation (17) accounts for molecular transport of the bound substance under the influence of the gradient of the total pressure of the vapor–air medium in the pores of the body. Convective mass transfer in the medium undergoing deformation is described by means of the transport equation

$$\frac{\partial (\rho_0 U)}{\partial t} = - \text{div } J + J_0, \quad (18)$$

where $J = -(\lambda_{PT} \text{grad } T + \lambda_{PU} \text{grad } U + \lambda_{PP} \text{grad } P)$ is the total filtration flux of the vapor–air mixture and vapor interdiffusion; $J_0 = -\rho_0 \varepsilon (\partial U / \partial t)$ is the volumetric capacity of the vapor sources; ε is the phase transformation criterion, equal to the ratio of the absolute total vapor flow to the sum of the absolute flows of vapor and liquid.

The mass of the capillary-porous body per unit volume will be expressed through total porosity, which is a variable quantity due to the deformation that occurs in the body

$$\frac{1}{c_B} \frac{\partial U}{\partial t} = \frac{\partial P}{\partial t} + \frac{P}{\Pi} \frac{\partial \Pi}{\partial t}, \quad (19)$$

where c_M is the specific content of vapor-gas moisture (the capacity of the capillary body in terms of moist air); $\Pi^{-1}(\partial \Pi / \partial t)$ is the relative rate of change in total porosity, equal to the rate of deformation of a unit volume of the system, i.e.,

$$\frac{1}{\Pi} \left(\frac{\partial \Pi}{\partial t} \right) = \frac{\partial \varepsilon_{kk}}{\partial t}.$$

With allowance for deformation, the equation that describes convective heat and mass transfer can be written in the form

$$\begin{aligned} \frac{\partial P}{\partial t} = & \text{div} (\lambda_{PT} \text{grad } T) + \text{div} (\lambda_{PU} \text{grad } U) + \\ & + \text{div} (\lambda_{PP} \text{grad } P) - \frac{\varepsilon}{c_B} \frac{\partial U}{\partial t} - P \frac{\partial \varepsilon_{kk}}{\partial t}. \end{aligned} \quad (20)$$

In order to completely describe coupled deformation-relaxation and heat-and-mass-transfer processes at the internal points of a capillary-porous body, system (9) (or system (10), (16)–(17), (20), in the case of wood) must be supplemented by the equations of motion

$$\sigma_{ij,j} + X_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (21)$$

and the relations between the strains and the displacements

$$\epsilon_{ij} = \epsilon_{ji} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

where u_i are the components of the displacement vector; X_i are projections of the body force.

Inserting (9) into (21) and taking the last formulas into account, we obtain a system of scalar equations of motion in displacements

$$\frac{1}{2}B(u_{i,kk} + u_{k,ki}/3) - KA\left(u_{k,ki} - \frac{V - V_0}{V_0}\right) + X_i = \rho \frac{\partial^2 u_i}{\partial t^2}. \quad (22)$$

Thus, system of differential equations (9), (16)-(17), (20), and (21) or (22) describes coupled deformation-relaxation and heat-and-mass transfer processes in viscoelastic capillary-porous bodies. Third-order heat-and-mass-transfer boundary conditions and the initial conditions $T(X, t) = T_0(X)$, $U(X, t) = U_0(X)$ (where x is the position vector of a point of the body) for the drying of capillary-porous bodies must be assigned in order to solve the system. The specific form of the conditions depends on the drying methods and regimes that are used [8]. The boundary conditions for deformation can be assigned in displacements or in stresses by the classical method. In the special case when the thermodynamic potentials associated with heat and mass transfer are absent, the resulting system leads to the equations of the classical theory of viscoelasticity [4]. If the components of the stresses and the displacements are equal to zero, we obtain the heat and mass transfer equations proposed by A. V. Lykov [8].

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