RELATIVE-MOTION ANALYSIS OF TWO PARTICLES USING TRANSLATING AXES

Today's Objectives:

Students will be able to:

1. Understand translating frames of reference.
2. Use translating frames of reference to analyze relative motion.

In-Class Activities:

- Check Homework,
- Reading Quiz
- Applications
- Relative Position, Velocity and Acceleration
- Vector & Graphical Methods
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The velocity of B relative to A is defined as

A) $\boldsymbol{v}_B - \boldsymbol{v}_A$.

B) $\boldsymbol{v}_A - \boldsymbol{v}_B$.

C) $\boldsymbol{v}_B + \boldsymbol{v}_A$.

D) $\boldsymbol{v}_A + \boldsymbol{v}_B$.

2. Since vector addition forms a triangle, there can be at most __________ unknowns (either magnitudes and/or directions of the vectors).

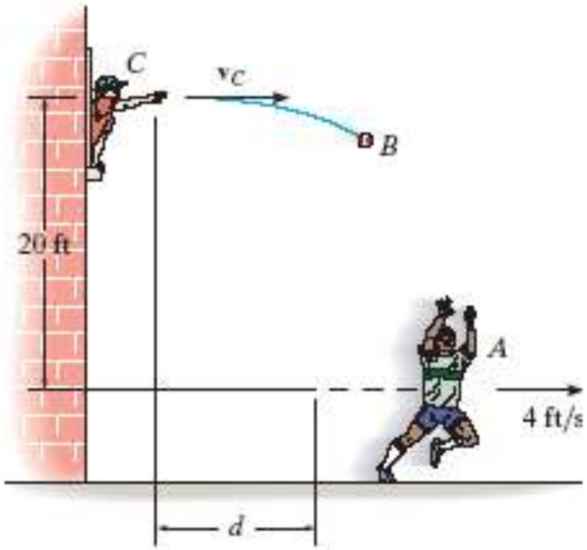
A) one

B) two

C) three

D) four

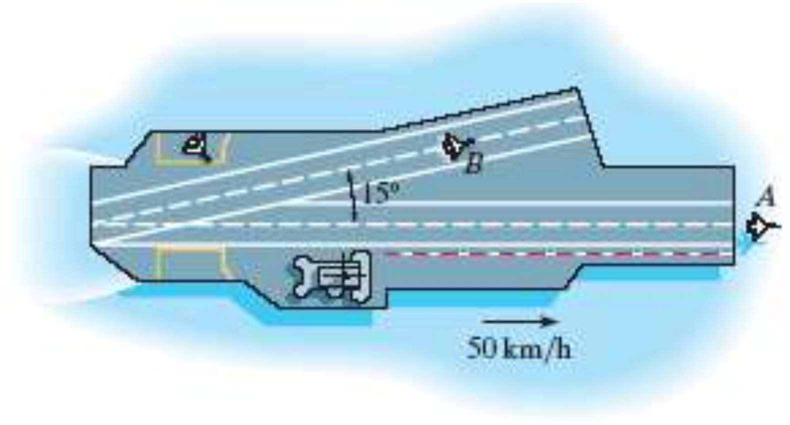
APPLICATIONS



When you try to hit a moving object, the position, velocity, and acceleration of the object must be known. Here, the boy on the ground is at d = 10 ft when the girl in the window throws the ball to him.

If the boy on the ground is running at a constant speed of 4 ft/s, how fast should the ball be thrown?

APPLICATIONS
(continued)



When fighter jets take off or land on an aircraft carrier, the velocity of the carrier becomes an issue.

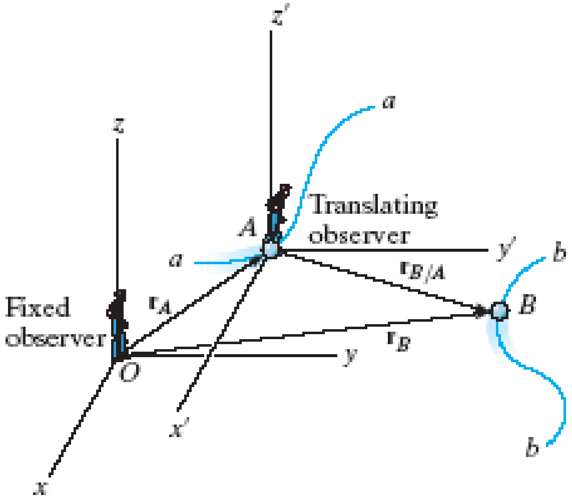
If the aircraft carrier travels at a forward velocity of 50 km/hr and plane A takes off at a horizontal air speed of 200 km/hr (measured by someone on the water), how do we find the velocity of the plane relative to the carrier?

How would you find the same thing for airplane B?

How does the wind impact this sort of situation?

RELATIVE POSITION

(Section 12.10)



The absolute position of two particles A and B with respect to the fixed x, y, z reference frame are given by $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. The position of B relative to A is represented by

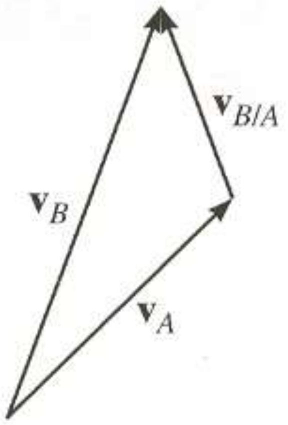
$$\boldsymbol{r}_{B/A} = \boldsymbol{r}_B - \boldsymbol{r}_A$$

Therefore, if $\boldsymbol{r}_B = (10\,\boldsymbol{i} + 2\,\boldsymbol{j})$ m

and $\boldsymbol{r}_A = (4\,\boldsymbol{i} + 5\,\boldsymbol{j})$ m,

then $\boldsymbol{r}_{B/A} = (6\,\boldsymbol{i} - 3\,\boldsymbol{j})$ m.

RELATIVE VELOCITY



To determine the relative velocity of B with respect to A, the time derivative of the relative position equation is taken.

$$v_{B/A} = v_B - v_A$$

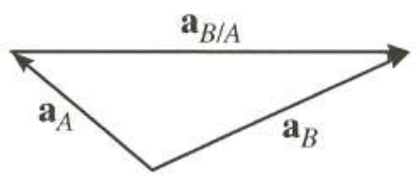
or

$$v_B = v_A + v_{B/A}$$

In these equations, v_B and v_A are called absolute velocities and $v_{B/A}$ is the relative velocity of B with respect to A.

Note that $v_{B/A} = - v_{A/B}$.

RELATIVE ACCELERATION



The time derivative of the relative velocity equation yields a similar vector relationship between the absolute and relative accelerations of particles A and B.

$$a_{B/A} = a_B - a_A$$

or

$$a_B = a_A + a_{B/A}$$

SOLVING PROBLEMS

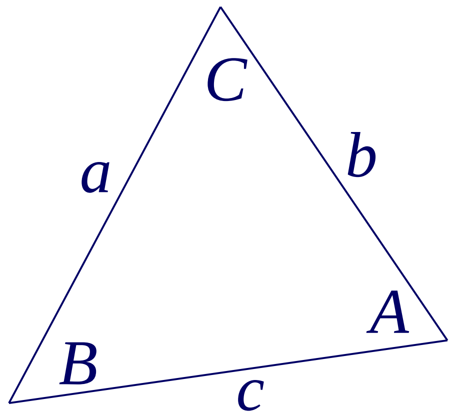
Since the relative motion equations are vector equations, problems involving them may be solved in one of two ways.

For instance, the velocity vectors in $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$ could be written as Cartesian vectors and the resulting scalar equations solved for up to two unknowns.

Alternatively, vector problems can be solved "graphically" by use of trigonometry. This approach usually makes use of the law of sines or the law of cosines.

Could a CAD system be used to solve these types of problems?

LAWS OF SINES AND COSINES



Since vector addition or subtraction forms a triangle, sine and cosine laws can be applied to solve for relative or absolute velocities and accelerations. As review, their formulations are provided below.

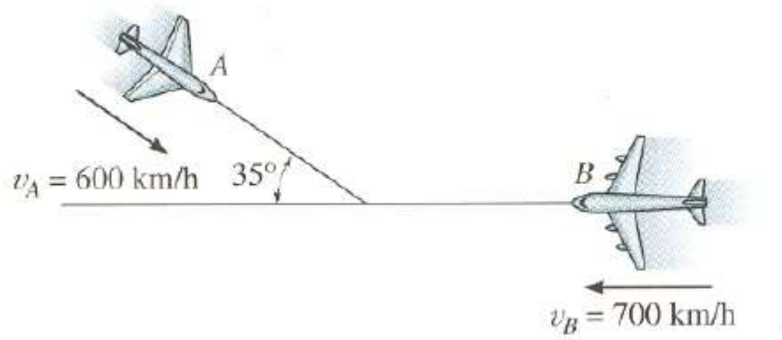
Law of Sines: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines: $$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

EXAMPLE



Given: $v_A = 600$ km/hr
$v_B = 700$ km/hr

Find: $\boldsymbol{v}_{B/A}$

Plan:

a) Vector Method: Write vectors $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ in Cartesian form, then determine $\boldsymbol{v}_B - \boldsymbol{v}_A$

b) Graphical Method: Draw vectors $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ from a common point. Apply the laws of sines and cosines to determine $\boldsymbol{v}_{B/A}$.

EXAMPLE
(continued)

Solution:

a) Vector Method:

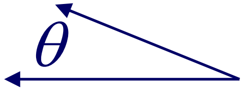
$$\mathbf{v}_A = 600 \cos 35\, \mathbf{i} - 600 \sin 35\, \mathbf{j}$$
$$= (491.5\, \mathbf{i} - 344.1\, \mathbf{j}) \text{ km/hr}$$

$$\mathbf{v}_B = -700\, \mathbf{i} \text{ km/hr}$$

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = (-1191.5\, \mathbf{i} + 344.1\, \mathbf{j}) \text{ km/hr}$$

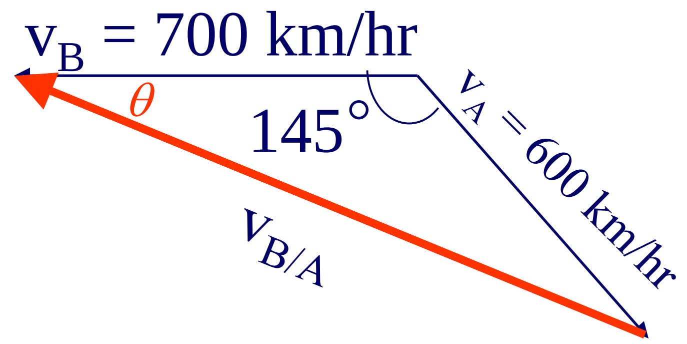
$$v_{B/A} = \sqrt{(1191.5)^2 + (344.1)^2} = 1240.2 \frac{\text{km}}{\text{hr}}$$

$$\text{where} \quad \theta = \tan^{-1}\left(\frac{344.1}{1191.5}\right) = 16.1°$$



EXAMPLE
(continued)

b) Graphical Method:
Note that the vector that measures the tip of B relative to A is $\boldsymbol{v_{B/A}}$.



Law of Cosines:

$$v_{B/A}^2 = (700)^2 + (600)^2 - 2(700)(600)\cos 145^\circ$$

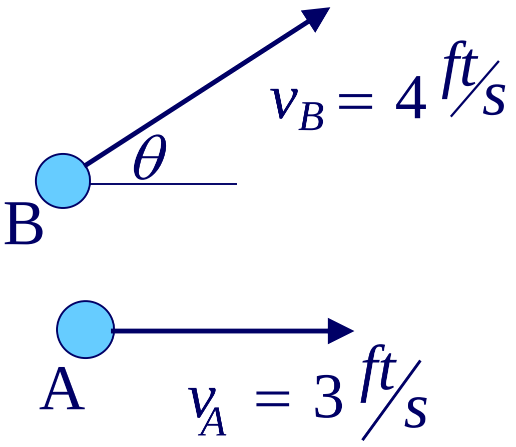
$$v_{B/A} = 1240.2\ \frac{\text{km}}{\text{hr}}$$

Law of Sines:

$$\frac{v_{B/A}}{\sin(145^\circ)} = \frac{v_A}{\sin\theta} \quad \text{or} \quad \theta = 16.1^\circ$$

CONCEPT QUIZ

1. Two particles, A and B, are moving in the directions shown. What should be the angle θ so that $v_{B/A}$ is minimum?



A) 0°   B) 180°

C) 90°   D) 270°

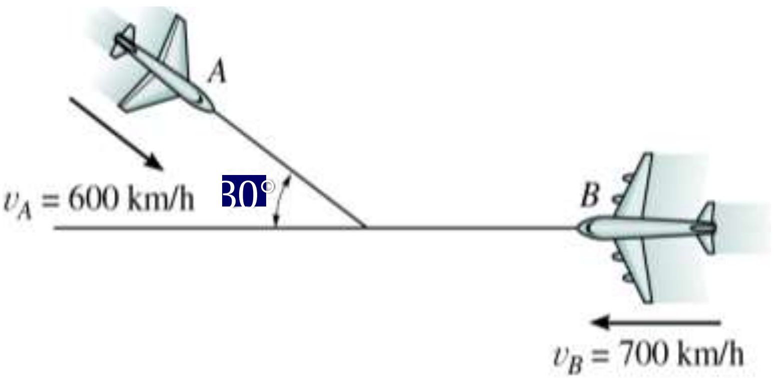
2. Determine the velocity of plane A with respect to plane B.

A) (400 ***i*** + 520 ***j***) km/hr

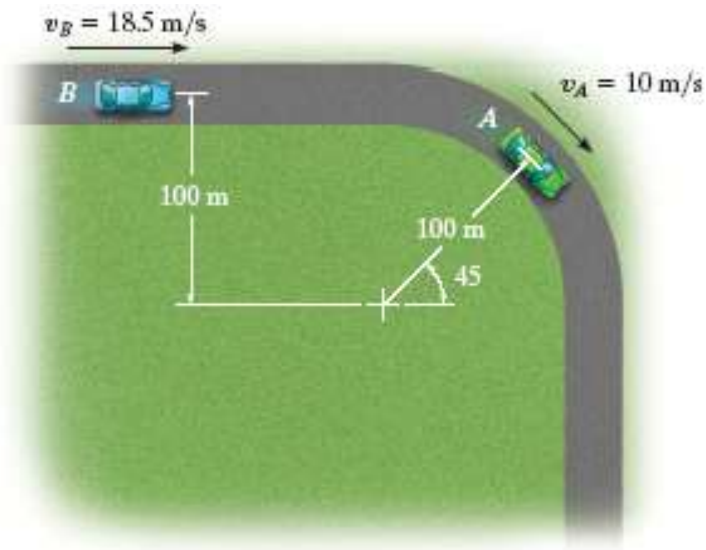
B) (1220 ***i*** - 300 ***j***) km/hr

C) (-181 ***i*** - 300 ***j***) km/hr

D) (-1220 ***i*** + 300 ***j***) km/hr



GROUP PROBLEM SOLVING



Given: $v_A = 10$ m/s
$v_B = 18.5$ m/s
$(a_t)_A = 5$ m/s^2
$a_B = 2$ m/s^2

Find: $\boldsymbol{v}_{A/B}$
$\boldsymbol{a}_{A/B}$

Plan: Write the velocity and acceleration vectors for A and B and determine $\boldsymbol{v}_{A/B}$ and $\boldsymbol{a}_{A/B}$ by using vector equations.

Solution:

The velocity of A is:

$$\boldsymbol{v}_A = 10\cos(45)\boldsymbol{i} - 10\sin(45)\boldsymbol{j} = (7.07\boldsymbol{i} - 7.07\boldsymbol{j})\text{ m/s}$$

GROUP PROBLEM SOLVING
(continued)

The velocity of B is:

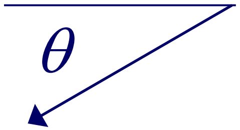
$$\boldsymbol{v_B} = 18.5\boldsymbol{i} \text{ (m/s)}$$

The relative velocity of A with respect to B is ($\boldsymbol{v_{A/B}}$):

$$\boldsymbol{v_{A/B}} = \boldsymbol{v_A} - \boldsymbol{v_B} = (7.07\boldsymbol{i} - 7.07\boldsymbol{j}) - (18.5\boldsymbol{i}) = -11.43\boldsymbol{i} - 7.07\boldsymbol{j}$$

or $$v_{B/A} = \sqrt{(11.43)^2 + (7.07)^2} = 13.4 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{7.07}{11.43}\right) = 31.73^\circ$$



GROUP PROBLEM SOLVING
(continued)

The acceleration of A is:

$$\mathbf{a}_A = (\mathbf{a}_t)_A + (\mathbf{a}_n)_A = [5\cos(45)\mathbf{i} - 5\sin(45)\mathbf{j}]$$

$$+ [-(\frac{10^2}{100})\sin(45)\mathbf{i} - (\frac{10^2}{100})\cos(45)\mathbf{j}]$$

$$\mathbf{a}_A = 2.83\mathbf{i} - 4.24\mathbf{j}\ (\mathrm{m/s^2})$$

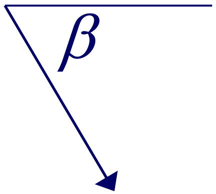
The acceleration of B is:

$$\mathbf{a}_B = 2\mathbf{i}\ (\mathrm{m/s^2})$$

The relative acceleration of A with respect to B is:

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = (2.83\mathbf{i} - 4.24\mathbf{j}) - (2\mathbf{i}) = 0.83\mathbf{i} - 4.24\mathbf{j}$$

$$a_{A/B} = \sqrt{(0.83)^2 + (4.24)^2} = 4.32\ \mathrm{m/s^2}$$



$$\beta = \tan^{-1}(\frac{4.24}{0.83}) = 78.9°$$

ATTENTION QUIZ

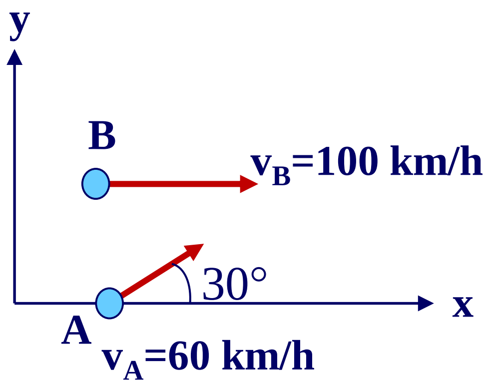
1. Determine the relative velocity of particle B with respect to particle A.

   A) (48***i*** + 30***j***) km/h

   B) (- 48***i*** + 30***j***) km/h

   C) (48***i*** - 30***j***) km/h

   D) (- 48***i*** - 30***j***) km/h



2. If theta equals 90° and A and B start moving from the same point, what is the magnitude of $\mathbf{r}_{B/A}$ at t = 5 s?

   A) 20 ft

   B) 15 ft

   C) 18 ft

   D) 25 ft