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978-0-521-76726-2 - Relativistic Quantum Physics: From Advanced Quantum Mechanics to Introductory Quantum Field Theory

Tommy Ohlsson

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RELATIVISTIC QUANTUM PHYSICS
From Advanced Quantum Mechanics to Introductory
Quantum Field Theory

Quantum physics and special relativity theory were two of the greatest breakthroughs in physics during the twentieth century and contributed to paradigm shifts in physics. This book combines these two discoveries to provide a complete description of the fundamentals of relativistic quantum physics, guiding the reader effortlessly from relativistic quantum mechanics to basic quantum field theory.

The book gives a thorough and detailed treatment of the subject, beginning with the classification of particles, the Klein–Gordon equation and the Dirac equation. It then moves on to the canonical quantization procedure of the Klein–Gordon, Dirac, and electromagnetic fields. Classical Yang–Mills theory, the LSZ formalism, perturbation theory and elementary processes in QED are introduced, and regularization, renormalization, and radiative corrections are explored. With exercises scattered through the text and problems at the end of most chapters, the book is ideal for advanced undergraduate and graduate students in theoretical physics.

TOMMY OHLSSON is Professor of Theoretical Physics at the Royal Institute of Technology (KTH), Sweden. His main research field is theoretical particle physics, especially neutrino physics and physics beyond the Standard Model.

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In memory of my father Dick

Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction to relativistic quantum mechanics	1
1.1 Tensor notation	1
1.2 The Lorentz group	3
1.3 The Poincaré group	9
1.4 Casimir operators	11
1.5 General description of relativistic states	12
1.6 Irreducible representations of the Poincaré group	13
1.7 One-particle relativistic states	16
Problems	21
Guide to additional recommended reading	21
2 The Klein–Gordon equation	22
2.1 Transformation properties	24
2.2 The current	25
2.3 Solutions to the Klein–Gordon equation	26
2.4 Charged particles	28
2.5 The Klein paradox	30
2.6 The pionic atom	34
Problems	38
Guide to additional recommended reading	39
3 The Dirac equation	40
3.1 Free particle solutions to the Dirac equation	45
3.2 Problems with the Dirac equation: the hole theory and the Dirac sea	50
3.3 Some gamma gymnastics and trace technology	52
3.4 Spin operators	57

3.5	Orthogonality conditions and energy projection operators	61
3.6	Relativistic invariance of the Dirac equation	63
3.7	Bilinear covariants	66
3.8	Electromagnetic structure of Dirac particles and charge conjugation	68
3.9	Constants of motion	72
3.10	Central potentials	74
3.11	The hydrogenic atom	77
3.12	The Weyl equation	86
3.13	Helicity and chirality	89
	Problems	90
	Guide to additional recommended reading	92
4	Quantization of the non-relativistic string	94
4.1	Equation of motion for the non-relativistic string	94
4.2	Solutions to the wave equation: normal modes	97
4.3	Generalized positions and momenta	98
4.4	Quantization	99
4.5	Quanta as particles	101
	Problem	103
	Guide to additional recommended reading	104
5	Introduction to relativistic quantum field theory: propagators, interactions, and all that	105
5.1	Propagators	106
5.2	Lagrangians	109
5.3	Gauge interactions	111
5.4	Scattering theory and Møller wave operators	113
5.5	The S operator	115
	Guide to additional recommended reading	121
6	Quantization of the Klein–Gordon field	122
6.1	Canonical quantization	122
6.2	Field operators and commutators	126
6.3	Green's functions and propagators	129
6.4	The energy–momentum tensor	132
6.5	Classical external sources	134
6.6	The charged Klein–Gordon field	135
	Problems	135
	Guide to additional recommended reading	137

Contents

ix

7	Quantization of the Dirac field	138
	7.1 The free Dirac field	138
	7.2 Quantization	140
	7.3 Positive energy	141
	7.4 The charge operator	144
	7.5 Parity, time reversal, and charge conjugation	145
	7.6 The Majorana field	148
	7.7 Green's functions and propagators	150
	7.8 Perturbation of electromagnetic interaction	152
	7.9 Expansion of the S operator	153
	Problems	154
	Guide to additional recommended reading	154
8	Maxwell's equations and quantization of the electromagnetic field	155
	8.1 Maxwell's equations	155
	8.2 Quantization of the electromagnetic field	157
	8.3 The Casimir effect	163
	8.4 Covariant quantization of the electromagnetic field	167
	Problems	174
	Guide to additional recommended reading	174
9	The electromagnetic Lagrangian and introduction to Yang–Mills theory	176
	9.1 The electromagnetic Lagrangian	176
	9.2 Massive vector fields	180
	9.3 Gauge transformations and the covariant derivative	182
	9.4 The Yang–Mills Lagrangian	183
	Problems	186
	Guide to additional recommended reading	187
10	Asymptotic fields and the LSZ formalism	188
	10.1 Asymptotic fields and the S operator	188
	10.2 The LSZ formalism for real scalar fields	192
	10.3 Proton–meson scattering	195
	Guide to additional recommended reading	196
11	Perturbation theory	197
	11.1 Three different pictures	198
	11.2 The unitary time-evolution operator	199
	11.3 Perturbation of VEVs for T -ordered products	202
	11.4 The relation between the physical vacuum $ \Omega\rangle$ and the free theory ground state $ 0\rangle$	205

x	<i>Contents</i>	
	11.5 Specific correlation functions	207
	11.6 Wick's theorem	211
	11.7 Feynman rules and diagrams	215
	11.8 Kinematics for binary reactions	222
	11.9 The S matrix, the T matrix, cross-sections, and decay rates	225
	Problems	232
	Guide to additional recommended reading	234
12	Elementary processes of quantum electrodynamics	235
	12.1 $e^+ + e^- \rightarrow \mu^+ + \mu^-$	236
	12.2 $e^- + \mu^- \rightarrow e^- + \mu^-$	240
	12.3 $e^+ + e^- \rightarrow e^+ + e^-$	242
	12.4 $e^- + e^- \rightarrow e^- + e^-$	246
	12.5 $e^- + \gamma \rightarrow e^- + \gamma$ and $e^+ + e^- \rightarrow 2\gamma$	250
	Problems	253
	Guide to additional recommended reading	255
13	Introduction to regularization, renormalization, and radiative corrections	257
	13.1 The electron vertex correction	260
	13.2 The electron self-energy	265
	13.3 The photon self-energy	268
	13.4 The renormalized electron charge	272
	Problems	275
	Guide to additional recommended reading	276
Appendix A	A brief survey of group theory and its notation	278
	A.1 Groups	278
	A.2 Lie groups	279
	A.3 Lie algebras	281
	A.4 Lie algebras of Lie groups	282
	A.5 The angular momentum algebra	283
	<i>Bibliography</i>	286
	<i>Index</i>	288

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[More information](#)

Preface

This book is based on my lectures in the course ‘Relativistic Quantum Physics’ at the Royal Institute of Technology (KTH) in Stockholm, Sweden. These lectures have been given four times during the academic years 2006–2007, 2007–2008, 2008–2009, and 2009–2010. The main sources of inspiration for the lectures were the books A. Z. Capri, *Relativistic Quantum Mechanics and Introduction to Quantum Field Theory*, World Scientific (2002) and M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995), and indeed, this book serves as a textbook for relativistic quantum mechanics with continuation to basic quantum field theory. The book is mainly intended for final-year undergraduate students in physics or first-year graduate students in physics and/or theoretical physics, who want to learn relativistic quantum mechanics, the basics of quantum field theory, and the techniques of calculating cross-sections for elementary reactions in quantum electrodynamics. Thus, the book should be suitable for any course on relativistic quantum mechanics as well as it might be suitable for a beginners’ course on quantum field theory. In summary, the book is a self-contained technical treatment on relativistic quantum mechanics, introductory quantum field theory, and the step in between, i.e. it should fill the gap between advanced quantum mechanics and quantum field theory, which I have called *relativistic quantum physics*. It contains a thorough and detailed mathematical treatment of the subject with smaller exercises throughout the whole text and larger problems at the end of most chapters.

I am deeply grateful to Johannes Bergström, Jonas de Woul, and Dr Jens Wirstam for careful proof-reading of earlier versions of the manuscript of this book and for useful comments, discussions, and suggestions how to improve the book. I am indebted to my former Ph.D. supervisor Professor emeritus Håkan Snellman for teaching me that physics is a descriptive science, which indeed does not explain anything. I would also like to thank my two friends Björn Sjödin and Jens Wirstam, who left science for ‘industry’, but never lost interest in it, and with whom I

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Frontmatter

[More information](#)

xii

Preface

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The author gratefully acknowledges financial support from the degree program ‘Engineering Physics’ (especially, Professor Leif Kari) at KTH for the development of this book.

Finally, last but not least, I would like to thank my family and friends for always being there for me. This applies particularly to my wife Linda, but also to my mother Inga-Lill and my sister Ther ese.