# Reliability Block Diagram Extensions for Non-Parametric Probabilistic Analysis

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*Abstract*—**Multi-Valued Reliability Block Diagrams (MVRBD) are introduced as a generalization of classical reliability block diagrams (RBD) commonly used in system analysis. MVRBD offer the advantage of allowing systems and subsystem components to be modeled with arbitrary hazard failure rate relationships. MVRBD are based upon a multiple-valued discrete switching algebra that is functionally complete with constants thereby affording a corresponding model to be formulated for any system that is capable of being modeled in a reliability block diagram form. The utility of this new model is that system failure and reliability analysis can be performed without restricting component hazard rate relationships to the binary case of either "failure" or "fully operational." The incorporation of any desired number of "degraded" states into the MVRBD model allows for non-parametric probability mass functions (*pmf*) to be used. Any arbitrary number of intermediate system states of "degraded" may be incorporated into the MVRBD model without significant increase in the complexity of the analyses methodologies.**

*Keywords—system reliability, block diagram, hazard rate, reliability, degraded system analysis, multi-valued algebra, non-parametric pmf*

## I. INTRODUCTION

Reliability Block Diagrams (RBD) are commonly used for analysis of system reliability [1, 2]. These diagrams model how the combinations of system components contribute to the success or failure of systems. A series combination of subsystems means that all such subsystems must be operational for overall reliability to hold. Similarly, a parallel combination of subsystems infers that only one of the components needs to be operational for overall reliability. The series combinations can be modeled by the Boolean algebraic AND operation, while the required parallel combinations can be modeled by the OR operation for reliability calculations.

Modeling different operational modes other than just the binary case of failure or normal operation are critical in analyzing system reliability. As an example, in the 2003 blackout of the US power grid, many complex interactions caused a blackout to occur in a large portion of the northeastern US [3]; however, it would be incorrect to state that the entire US power grid failed. To address these issues, we propose the application of Multiple-Valued Logic (MVL) to RBDs.

## II. PREVIOUS WORK

Other past work has claimed the use of MVL in block diagrams, but fell short in several key areas. Some approaches only used combinations of multiple binary variables in a single block diagram element to attempt to simulate MVL [4]. This allows subcomponents to be grouped together and help abstract lower level systems in the block diagram for a higher level view of the system. Since the elements themselves were not truly multiple-valued, MVL algebras are not used, which limits the information that can be gained from the block diagrams. In this case, the limitations of Boolean algebra are still present although the radix is $2^n$.

Other approaches use multistate systems, which are similar to MVL systems in that the system input and intermediate values have more than two states [5, 6, 7, 8]. However, the system output values are still binary, so these systems are not true MVL systems.

## III. BACKGROUND CONCEPTS

### A. Multiple-Valued Logic

Multiple-valued logic is related to Boolean or binary logic and may be viewed as a generalization of binary switching theory from radix-2 to an arbitrary radix. The binary logic functions AND, OR, and NOT correspond with the multiple-valued functions MIN, MAX and literal selection, $\{J_i\}$. The MIN function accepts two or more inputs and outputs the value of its lowest (or minimum) input. The MAX function accepts two or more inputs and outputs the value of its highest (or maximum) input. The literal selection gate accepts one input and acts as a logic level detector. In an MVL system with a radix value $r$, there are $r$ different unary literal selection functions denoted by their subscripts. This gate outputs the highest radix value of the system for the selected input value. For example, in a ternary ($r=3$ or radix-3) system, a $J_1$ (select-1) gate outputs zero (0) for inputs of zero (0) or two (2), and outputs the maximum value of two (2) for an input of one (1). The ternary system supports literal selection gates $J_0$, $J_1$, and $J_2$. In general, the set of supported literal selection gates $\{J_i\}$ are $\{J_0, J_1, \ldots, J_{r-1}\}$.

In order to represent all possible functions, the set of gates used in the logic system must be functionally complete. In binary logic, AND, OR and NOT form a functionally complete set with constants $\{0, 1\}$. Likewise, in MVL, MIN, MAX and $\{J_i\}$ form a functionally complete set with constants $\{0, 1, \ldots,$

*r*-1} [9]. Without a functionally complete set, it is not possible to model all systems, limiting the applications of the algebraic system (binary or MVL). Table 1 provides a definition of the MIN, MAX and $J_i$ operations for the case of a ternary, $r=3$, or radix-3, system.

**Table 1: Truth Table for Ternary MVL Operators**

| Variables | | MIN | MAX | Literal Selection | | |
|---|---|---|---|---|---|---|
| $x$ | $y$ | $xy$ | $x+y$ | $J_0(x)$ | $J_1(x)$ | $J_2(x)$ |
| 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| 0 | 1 | 0 | 1 | 2 | 0 | 0 |
| 0 | 2 | 0 | 2 | 2 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 2 | 0 |
| 1 | 1 | 1 | 1 | 0 | 2 | 0 |
| 1 | 2 | 1 | 2 | 0 | 2 | 0 |
| 2 | 0 | 0 | 2 | 0 | 0 | 2 |
| 2 | 1 | 1 | 2 | 0 | 0 | 2 |
| 2 | 2 | 2 | 2 | 0 | 0 | 2 |

*B. Shannon's Decomposition Theorem*

An important theorem in a discrete switching algebra such as the binary Boolean algebra or its extension to a multi-valued algebra is the Shannon decomposition theorem. This theorem allows for any discrete switching function to be expressed as a disjoint set of conjunctive terms. In the binary case, each conjunctive term is formed using the multiplicative AND operation with a chosen variable of decomposition and a function known as a co-factor.

**Definition 1**: (*Binary Function Co-factor*)

Consider a binary switching function, $f(x_1, x_2, \ldots, x_i, \ldots, x_n)$ where each of the dependent variables $x_i$ are also binary-valued. Because the radix is binary, there are exactly two co-factor functions about each potential variable of decomposition. The two co-factors about variable $x_i$ are denoted as $f_{\overline{x}_i}$ and $f_{x_i}$. The zero co-factor with respect to $x_i$ is

$$f_{\overline{x}_i} = f(x_1, x_2, \ldots, x_i, \ldots, x_n)\big|_{x_i=0} = f(x_1, x_2, \ldots, 0, \ldots, x_n)$$

and the one co-factor with respect to $x_i$ is

$$f_{x_i} = f(x_1, x_2, \ldots, x_i, \ldots, x_n)\big|_{x_i=1} = f(x_1, x_2, \ldots, 1, \ldots, x_n). \ \square$$

Shannon's decomposition Theorem is stated for the binary-radix case without proof. A proof is available in [9].

**Theorem 1**: (*Shannon's Decomposition*)

Consider a binary switching function, $f(x_1, x_2, \ldots, x_i, \ldots, x_n)$ and a chosen variable of decomposition $x_i$. The function may always be expressed as

$$f(x_1, x_2, \ldots, x_i, \ldots, x_n) = \overline{x}_i f_{\overline{x}_i} + x_i f_{x_i}. \quad \square$$

An important Corollary of Shannon's decomposition Theorem is that the multiplicative or conjunctive terms are disjoint among one another meaning that it is impossible for two or more of the terms to simultaneously evaluate to values greater than zero. For any given valuation of the dependent variables of *f*, there may be at most one conjunctive term that can result in a value greater than zero due to the fact that the conjunctive terms are disjoint.

In terms of the algebraic operations described previously, the conjunctive terms for the binary case are formed with the AND operator and the disjunction of all the terms is formed

with the OR operator. The Shannon decomposition can be generalized and holds for higher-valued radices with the operators MIN, MAX, and $\{J_i\}$. The generalized Shannon decomposition theorem is stated as follows.

**Theorem 2**: (*Generalized Shannon's Decomposition Theorem*)

Consider a discrete function, $f(x_1, x_2, \ldots, x_i, \ldots, x_n)$ and a chosen variable of decomposition $x_i$. where the dependent variable set $\{x_i\}$ are radix-*r* variables. The function may always be expressed as

$$f(x_1, x_2, \ldots, x_i, \ldots, x_n)$$
$$= x_i^{\{0\}} f_{x_i^{\{0\}}} + x_i^{\{1\}} f_{x_i^{\{1\}}} + \cdots + x_i^{\{j\}} f_{x_i^{\{j\}}} + \cdots + x_i^{\{r-1\}} f_{x_i^{\{r-1\}}} \quad \square$$

The variable of decomposition is expressed as a literal where the $j^{\text{th}}$ literal is denoted as $x^{\{j\}}$. This provided by the literal selection operator $J_i(x) = x^{\{j\}}$. This notation is consistent with binary notation for the literals $\overline{x}$ and $x$ and is simply a generalization of that notation to allow for conveniently expressing literals over radices greater than two (i.e., non-binary or MVL radices). That is, $\overline{x} = x^{\{0\}}$ and $x = x^{\{1\}}$. Likewise, the conjunctive or multiplicative operations are provided by the MIN operation and the disjunctive or additive operations are provided by the MAX operation.

*C. System Block Diagrams with Binary States*

Block diagrams are commonly used to model systems and analyze their reliability. From a block diagram, one can create a reliability function or a failure function. In general, the symbolic function that represents some aspect of the represented system in block diagram form is referred to as the *structure function*. System structure functions may represent system reliability, failure, availability, or any of a variety of other characteristics.

One may view the block diagram as a set of elements connected in series or parallel. To extract the structure function that models reliability, we can treat the series connections as logical ANDs between the elements in series. That is, both the elements in series are required to be functional for this overall portion of the system to be functional. Likewise, for those elements in parallel, either one or the other must be functional for the overall parallel structure to be functional. Thus, parallel structures are modeled with a logical OR operation.

Conversely, to create the structure function that models failure, we treat the series connections as logical ORs and treat the parallel connections as logical ANDs. Thus if any of the elements in series fail, the combination of those elements fail, and if all of the elements in parallel fail, the combination of those elements fail. Mathematically, the reliability and failure structure functions are duals of one another.

For example, in the block diagram in Fig. 1, elements *B* and *C* are in parallel and element *A* is in series with the parallel combination of *B* and *C*. Using the techniques described above, we can create the reliability and failure structure functions.

Reliability: $\quad F_{rel} = A(B+C)$

Failure: $\quad F_{fail} = \overline{A} + (\overline{B}\overline{C})$
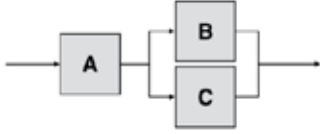
**Fig. 1: Small Example System**

These functions are mathematical duals, and we can use DeMorgan's Theorem to show: $F_{rel} = \overline{F}_{fail}$ .

Starting with $F_{rel}$,

$$F_{rel} = A(B+C)$$

next, we negate both sides,

$$\overline{F}_{rel} = \overline{A(B+C)}$$

and repeatedly apply DeMorgan's law

$$\overline{F}_{rel} = \overline{A} + (\overline{B+C})$$

$$\overline{F}_{rel} = \overline{A} + (\overline{B}\,\overline{C}).$$

Recall definition of $F_{fail}$,

$$F_{fail} = \overline{A} + (\overline{B}\,\overline{C}).$$

Thus,

$$F_{rel} = \overline{F}_{fail} .$$

### D. Probabilitistic Models for System Failure

System fault tolerance analysis methods using RBD are well established and accepted in the community as described in [10] for example. Central to this theory is the concept of the hazard failure rate function that is often used to model components, particularly electrical components, where their lifetime is characterized in three phases; the "burn-in" period, the useful lifetime period, and the "burn-out" period. Due to the shape of the hazard failure rate function, $h(t)$, it is colloquially referred to as the "bathtub" curve. An illustration of the bathtub failure rate curve is given in Fig. 2.
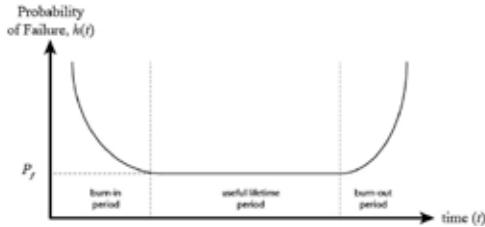


**Fig. 2: Bathtub Curve Failure Rate Function, $h(t)$**

The useful lifetime period of the hazard failure rate is a simple function, $h(t)=P_f$ where $P_f$ is a constant. Due to this simple relationship, subsystems can be assigned one of two binary states, either a state of failure or a state of being fully operational with corresponding constant discrete point probabilities of $P_f$ or $1-P_f$. Because reliability is defined as the probability that a system is operational, the dual theory of reliability analysis is also conveniently simplified due to the bathtub failure curve since the reliability of a system can be modeled as $P_r=1-P_f$. This well-known theory allows for a random variable $(RV)$ $X$ to be used based on two probabilistic events; when a system suffers the event of failure, $X=0$, and when the system is in the operational state, $X=1$. The probability mass function ($pmf$) for $X$ is shown in Fig. 3.
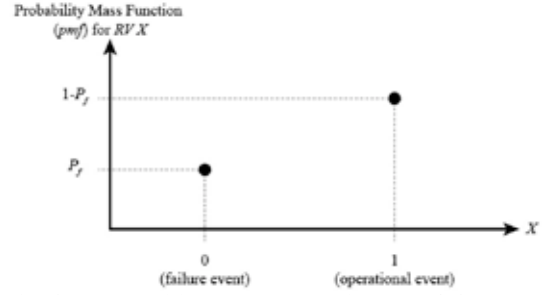


**Fig. 3: Probability Mass Function ($pmf$) for $RV$ $X$**

Fig. 3 is a well-known $pmf$ known as the Bernoulli distribution which is a form of the discrete binomial distribution with the binomial $pmf$ parameter $n$ set to unity, $n=1$. Symbolically, the Bernoulli $pmf$ for $RV$ $X$ as defined previously is

$$F_{BERN}(X;P_f) = \begin{cases} P_f, & X = 0 \\ P_r = P_f - 1, & X = 1 \end{cases}.$$

### IV. MVRBD ALGEBRAIC MODEL AND ANALYSIS

The contribution described here is twofold. First, we extend RBD to a non-binary case with a theoretically rigorous MVL switching algebraic model. Our model is rigorous in the sense that the underlying algebra denoted as $\langle \{d_i\},$ MIN, MAX, $\{J_i\}$, 0, $d_{r-1} \rangle$ is a functionally complete algebra with constants. Functional completeness with constants ensures that any possible system has a corresponding algebraic model. $\{d_i\}$ represents a set of values where $d_i \in \mathbb{Z}$, $r$ represents the radix or number of discrete valuations, $\{J_i\}$ represents the set of $r$-1 unary operators previously defined, MIN represents the multiplicative operator whose identity element is $d_{r-1}$, and MAX represents the additive operator whose corresponding identity element is 0, and it is always the case that $0 \in \{d_i\}$ and $d_{r-1} \in \{d_i\}$.

The second contribution is that the new algebraic model allows for a probabilistic model to be formulated in systems reliability, failure, or other applications that is not restricted to a binary case. Classical RBD theory is typically based upon the binary concept of subsystem failure or fully operational behavior. In many cases, such as systems composed of electronic devices, the binary model is sufficient since the well-known "bathtub" curve failure rate relationship is accepted as being applicable. In other systems, such a relationship may not be as accurate and thus we are motivated to utilize a non-binary $pmf$ to model subsystem components.

### A. Multi-Valued Block Diagram Model (MVRBD)

Thus, to supplement and extend the existing binary-based block diagrams, we propose extending block diagrams to create Multi-Valued Reliability Block Diagrams (MVRBDs). These MVRBDs, in general, have the same form and appearance as binary-based block diagrams, making them instantly familiar to anyone that has used traditional block diagrams. When considering the reliability of system represented by a MVRBD, the elements in series are related by the MIN of the elements and the elements in parallel are related by the MAX of the

elements. That is, the relationships between block diagram elements represented by AND and OR in a binary block diagram are represented by MIN and MAX respectively.

However, to maintain the ability to represent any function, providing the ability to model any system or process, it is necessary to maintain the functional completeness of binary block diagrams. Binary block diagrams are able to represent logical ANDs and ORs as previously described with series and parallel connections and logical NOT is handled internally in individual blocks. Logical NOT of larger expressions can be modeled through applications of DeMorgan's Theorem.

In simple cases, even systems that only appear to use MIN and MAX do contain literal select although this is not initially obvious. We can show that literal select is in fact present through the use of Shannon's decomposition, expanding the function into a more complex equivalent. For example, if we apply Shannon's decomposition to MVRBD version of Fig. 1, the literal select gates become apparent.

$$F_{rel} = A(B+C)$$
$$F_{rel} = A^{\{0\}}F_{rel\,A=0} + A^{\{1\}}F_{rel\,A=1} + A^{\{2\}}F_{rel\,A=2}$$
$$F_{rel} = A^{\{0\}}(A|_{A=0}(B+C)) + A^{\{1\}}(A|_{A=1}(B+C)) + A^{\{2\}}(A|_{A=2}(B+C))$$
$$F_{rel} = A^{\{0\}}(0(B+C)) + A^{\{1\}}(1(B+C)) + A^{\{2\}}(2(B+C))$$
$$F_{rel} = 0 + A^{\{1\}}1(B+C) + A^{\{2\}}(B+C)$$
$$F_{rel} = A^{\{1\}}1(B+C) + A^{\{2\}}(B+C)$$

Clearly, with the existing capabilities of block diagrams, there is no direct way to represent this form of the equation. We propose the use of literal select as a switch or multiplexing structure in the block diagram. The inputs to the multiplexer are the different components that are conjunctively combined with the literal select operation in Shannon's decomposition. For example, for the reliability structure function that was previously derived, $F_{rel}$ may be represented with a switching structure as shown in Fig. 4.

This construct can be very useful for where certain subsystems are "switched in" under conditions. For example, modern datacenter power distribution typically has several layers of redundancy. An illustrative and simplified example may be a datacenter composed of three power sources that can be switched into operation in the event of a primary power failure. Those sources being the primary utility grid power, short-term and rapidly available battery power, and longer-term secondary power supplied by a petroleum-fueled generator. These sources are modeled with variables $U$, $B$, and $G$ in Fig. 5. Also, the power switch $S$ and the condition indicator subsystem $C$ is shown in Fig. 5.

In the datacenter MVRBD, the resulting system structure function utilizes the literal selection operator in a natural way since the power sources $U$, $B$, or $G$ are only utilized one at a time. The datacenter reliability structure function, $DC_{rel}$, can be directly extracted from this diagram as

$$DC_{rel} = S \bullet P \bullet \left(C^{\{0\}}U + C^{\{1\}}B + C^{\{2\}}G\right).$$

In the case of the datacenter, the reliability structure function has two single points of failure due to the switch $S$ that selects among the power sources $U$, $B$, and $G$ and the power condition circuit $P$ that produces the condition $C$. For this reason, $S$ and $P$ are conjunctively related to the expression

that selects among the sources. Another interesting characteristic of $DC_{rel}$, is the fact that the power condition subsystem $P$ is related to the variables $U$, $B$, and $G$ since this subsystem is monitoring those resources. For this reason, it is inaccurate to model $C$ with an independent Bernoulli *pmf*. Rather, this three-state subsystem should be modeled with a $RV$ that is dependent upon the $RV$s that model the $U$, $B$, and $G$ subsystems. These observations indicate the limitations of traditional RBD and the resulting reliability and failure structure functions they represent. To simplify the example, the power condition indicator $P$ is assumed to be perfectly reliable, Pr(*power condition subsystem fails*)=0.
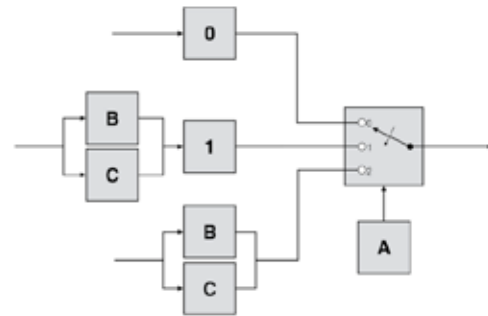


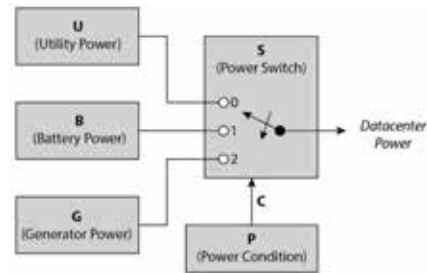**Fig. 4: Reliability Block Diagram with Switch**



**Fig. 5: Datacenter Power Generation Example**

*B. Probabilistic Analysis of MVRBD*

As an example of the use of intermediate system states that represent a degraded but neither fully operational nor full failure state, consider a random variable $Y$ representing three states; failure, degraded, and operational. These states cause $RV$ $Y$ to take on values {0, 1, 2} respectively and the *pmf* of $Y$ is depicted in Fig. 6 where point probabilities $P_f$, $P_d$, and $P_o$ correspond to each of the possible system states. It should be noted that more than one intermediate degraded state is possible within the MVRBD framework, this example includes a single degraded state only for the purpose of conciseness.

Assume that the datacenter components have the states and probabilities shown in Table 2. The utility source has three possible states: 2↔fully operational, 1↔degraded operation (brownout), and 0↔non-operational (blackout). Both the battery and generator are either fully operational or non-operational, 2↔fully operational and 0↔ non-operational.

The power condition $C$ depends on the states of $U$, $B$, and $G$ and is provided by the Power Condition subsystem $P$. If $U=2$, then the datacenter has full power from the utility, so the condition is $C=0$. However, if $U=1$, there is a brownout condition that is assumed to be temporary, so the datacenter

receives power from the battery $B$, thus $C$=1. If $U$=0, there is a blackout condition, so the datacenter will receive power from the generator $G$, thus $C$=2. If none of the units are operational, then $C$ defaults to 0. The truth table for the switch operation $C$ is shown in Table 3. Table 3 describes the functionality of the power condition subsystem.
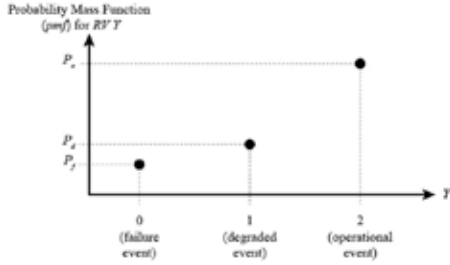


**Fig 6: *pmf* of Random Variable *Y***

**Table 2: Datacenter power source state probabilities**

| State | U | B | G |
|---|---|---|---|
| 2 | 0.80 | 0.90 | 0.95 |
| 1 | 0.15 | - | - |
| 0 | 0.05 | 0.10 | 0.05 |

**Table 3: Truth Table for Power Condition Subsystem**

| U | B | G | C |
|---|---|---|---|
| 2 | X | X | 0 |
| 1 | 2 | X | 1 |
| 1 | 0 | 2 | 2 |
| 1 | 0 | 0 | 0 |
| 0 | X | 2 | 2 |
| 0 | 2 | 0 | 1 |
| 0 | 0 | 0 | 0 |

We can determine the probabilities that the datacenter is receiving power from $U$, $B$, or $G$ by using the truth table and probability values for the power source states and other components. The probability of $U$↔state $i$, $B$↔state $j$, and $G$↔state $k$ is denoted by Pr($U$, $B$, $G$). The operational states of the power sources are assumed to be independent, so P($U$, $B$, $G$)=Pr($U$)×Pr($B$)×Pr($G$). If we expand the truth table of Table 3 and add the corresponding probability values, we get Table 4.

We can determine the probability that datacenter power condition is in state $j$ by adding the corresponding probability values Pr($U$,$B$,$G$) for state $j$. The probability that the datacenter is powered by the utility, is obtained by summing P($U$,$B$,$G$) values where $C$=0 and where the values of $U$, $B$, and $G$ are **not** 0 (recall the default condition above).

Pr($C$=0)=0.684+0.036+0.076+0.004+0.00075
   =0.80075

Similarly, we can determine the probabilities that the datacenter is powered by the battery or by the generator with the following two calculations.

Pr($C$=1)=0.12825+0.00675+0.00225=0.13725
Pr($C$=2)=0.01425+0.04275+0.00475=0.06175

The previous calculations focused on the datacenter power condition or, which power source the datacenter is powered by based on the condition. To determine the probability that the datacenter is actually powered by the utility, battery, or

generator, the reliabilities of the power condition subsystem and the power switch must be accounted for using the structure function $DC_{rel}$. To simplify the example, we assume that the power condition circuit is highly reliable and model the probability of failure as zero. To illustrate the process of adding more detail, we assume that the power switch has reliability of 0.98 and is independent of the other subsystems in Fig. 5. Thus the overall probabilities that the datacenter is receiving power from $U$, $B$, or $G$ can be calculated as

Pr(*utility power*) = 0.98×Pr($C$=0)=0.784735,
Pr(*battery power*) = 0.98×Pr($C$=1)=0.134505,
Pr(*generator power*) = 0.98×Pr($C$=2)=0.060515,

Pr(*no power*) = 1 - [Pr(*utility power*) + Pr(*battery power*) + Pr(*generator power*)] = 1 – (0.784735 + 0.134505 + 0.060515) = 0.020245.

**Table 4: Operation with Power Source Probabilities**

| U | B | G | C | P(U) | P(B) | P(G) | P(U,B,G) |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 0 | 0.80 | 0.90 | 0.95 | 0.684 |
| 2 | 2 | 0 | 0 | 0.80 | 0.90 | 0.05 | 0.036 |
| 2 | 0 | 2 | 0 | 0.80 | 0.10 | 0.95 | 0.076 |
| 2 | 0 | 0 | 0 | 0.80 | 0.10 | 0.05 | 0.004 |
| 1 | 2 | 2 | 1 | 0.15 | 0.90 | 0.95 | 0.128 |
| 1 | 2 | 0 | 1 | 0.15 | 0.90 | 0.05 | 0.00675 |
| 1 | 0 | 2 | 2 | 0.15 | 0.10 | 0.95 | 0.01425 |
| 1 | 0 | 0 | 0 | 0.15 | 0.10 | 0.05 | 0.00075 |
| 0 | 2 | 2 | 2 | 0.05 | 0.90 | 0.95 | 0.04275 |
| 0 | 2 | 0 | 1 | 0.05 | 0.90 | 0.05 | 0.00225 |
| 0 | 0 | 2 | 2 | 0.05 | 0.10 | 0.95 | 0.00475 |
| 0 | 0 | 0 | 0 | 0.05 | 0.10 | 0.05 | 0.00025 |

## V. EXAMPLE APPLICATION

To demonstrate the utility of the MVRBD, we will show an example using the features of the MVRBD by modeling the reliability of the power generation for a city. Modeling the power in a city has clear benefits to using a MVRBD over a traditional binary RBD. The state of the power available to the city as a whole has more than two states, non-operational and fully operational. Instead, in addition to the states present in binary, the power grid can be in a state of partial operation, such brownouts or rolling blackouts. This example could be extended to have more intermediate states, representing various states of power grid degradation; however, we will limit the number of states to 3 for simplicity. Thus, each element in our power system can have one of the flowing functional states:

- 0 – non-operational
- 1 – partially operation (degraded operation)
- 2 – fully operational

Clearly, 0 and 2 are the states that are possible in a binary RBD, and 1 is the intermediate state, possible only in the MVRBD.

The power is provided by a variety of sources: two coal plants, three wind farms, a nuclear plant, a hydroelectric plant and a solar plant. Due to limitations in the grid layout, the power is supplied in one of the following ways: (1) using at least one coal plant and one wind farm, (2) using a nuclear plant, or (3) using a hydroelectric plant and solar plan. From these relationships, we determine the structure of the MVBD.

We know that the coal plants will be in parallel with each other. Similarly, the wind farms will be in parallel together. Since both a coal plant and a wind farm is required to be a possible power delivery method, the parallel combinations of the coal plants and wind farms will in series. Likewise, the hydroelectric and solar plants will be in series. Finally, all three groups will be in parallel overall since any of the three can provide power.

Next, we assign the power system elements to variables that appear in the corresponding MVRBD as provided in Table 5. These variables are also used in the equation representing the MVRBD, which is useful in analysis of the system.

**Table 5: Variables for City Power Generation Model**

| Variable | Power System Element |
|----------|----------------------|
| A | Coal Plant 1 |
| B | Coal Plant 2 |
| C | Wind Farm 1 |
| D | Wind Farm 2 |
| E | Wind Farm 3 |
| F | Nuclear Plant |
| G | Hydroelectric Plant |
| H | Solar Plant |

After assigning variables, we create the MVRBD in Fig. 7 using the relationships we determined.
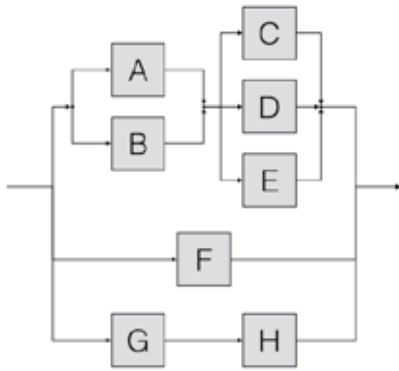


**Fig. 7: MVRBD for Power Generation Model**

We can observe in Fig. 7 that the MVRBD has the properties we described and using the MVRBD we can create the following equation.

$$Z = [(A+B)(C+D+F)] + E + (GH)$$

For analysis automated tools can quickly create the equation representing a system and calculate the output for a given set of inputs. The analysis on the systems can be used in a variety of manners.

## VI. CONCLUSION

A generalization of RBD are developed based upon a multi-valued switching algebra. Because the multi-valued switching algebra is functionally complete with constants from a mathematical point of view, MVRBDs are a robust model that allow for any system to be modeled since all possible functions can be represented with the algebra $\langle \{d_i\}, \text{MIN}, \text{MAX}, \{J_i\}, 0, d_{r-1} \rangle$. The utility of the MVRBD model is that subsystem components may be modeled with any arbitrary hazard rate relationship since corresponding random variables may be formulated that represent states of degraded behavior in addition to those of complete failure or fully operational. The multi-state $RV$s may be characterized with any general non-parametric $pmf$ and the total number of chosen states defines the radix value $r$ used in the multi-valued discrete algebra.

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