# RENAISSANCE ASTRONOMY

# Isaac Newton

Isaac Newton

[1642-1727, England]

**Kepler** discovered the rules that govern planetary orbits, and **Galileo** discovered laws that describe the behavior of falling bodies. Later, **Isaac Newton** unified these and other insights by showing that the force of gravitation that accelerates falling bodies near the Earth is the same force that keeps the Moon in its orbit around the Earth and the planets in their orbits about the Sun.

# Three Laws of Motion

Every body continues in a state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.

(**momentum** $p = mv$)

The change of motion is proportional to the force impressed, and it is made in the direction of the straight line in which that force acts.

(**force** $f = ma$)

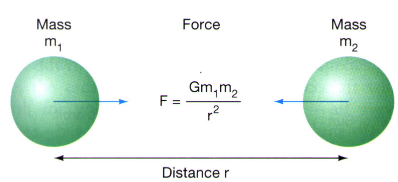
To every action there is always an equal and opposite reaction; or, the mutual actions of two bodies upon each other are always equal and act in opposite directions.

(**reaction**)

# Law of Gravity

$F = G\, m \mathcal{M} / r^2$ ,

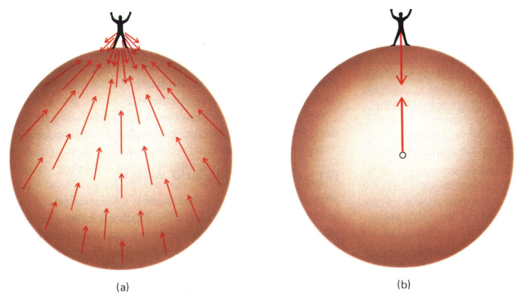
$G = 6.67 \times 10^{-11}\ N\ m^2 / kg^2$



# Acceleration and Weight

$$a = G\,\mathcal{M} / R^2$$

$$W = G\,m\,\mathcal{M} / R^2$$




# Examples

What is the new weight if the planet's mass increases by 4X?

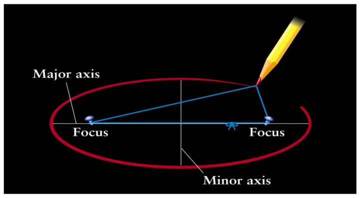
$$W_{new} / W_{old} = (4\mathcal{M} / R^2) / (\mathcal{M} / R^2) = 4$$

What is the new weight if the planet's radius decreases by 4X?

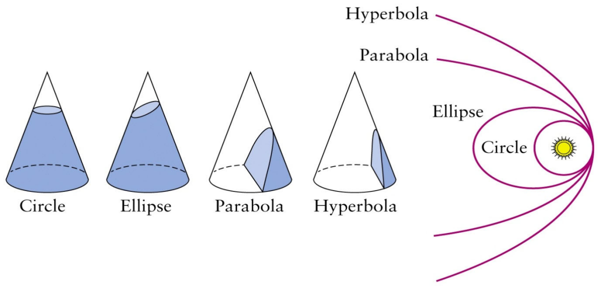
$$W_{new} / W_{old} = (\mathcal{M} / (R/4)^2) / (\mathcal{M} / R^2) = 16$$

# Kepler's Three Laws

1. All planets have elliptical orbits with the Sun at a focus.



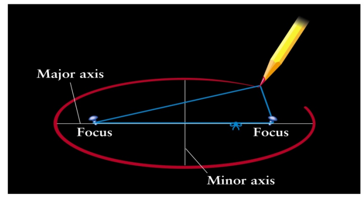

# Modification of Kepler's Laws



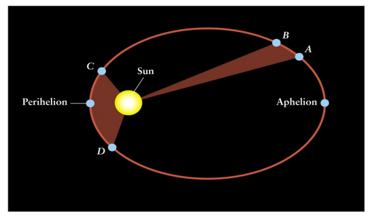

All orbiting bodies have a conic-section orbit, with the massive body (i.e., the Sun) at a focus.

# Kepler’s Three Laws

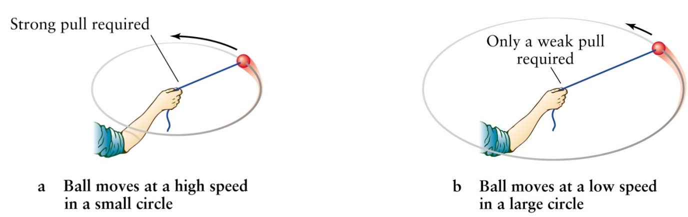
1. All planets have elliptical orbits with the Sun at a focus.




2. Law of Equal Areas: Equal areas are swept out in equal time intervals.




# Modification of Kepler’s Laws




Law of Equal Areas: Equal areas are swept out in equal time intervals.

This is explained by Conservation of Angular Momentum.

$$\mathbf{r_1\,v_1 = r_2\,v_2}$$

Kepler's Second Law Interactive

# Kepler’s Three Laws

3. Harmonic Law (published in *The Harmony of the Worlds)*:

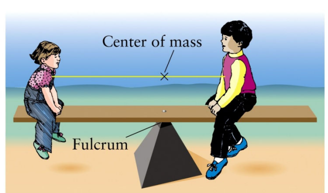
$$\mathbf{P^2 = k\, \mathit{a}^3},$$

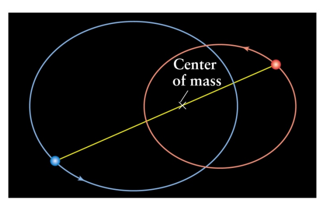
where k = 1 if P is in earth years and *a* is in **AU**s.

**table 4-3** A Demonstration of Kepler's Third Law

| Planet | Sidereal period *P* (years) | Semimajor axis *a* (AU) | $P^2$ | $a^3$ |
|---|---|---|---|---|
| Mercury | 0.24 | 0.39 | 0.06 | 0.06 |
| Venus | 0.61 | 0.72 | 0.37 | 0.37 |
| Earth | 1.00 | 1.00 | 1.00 | 1.00 |
| Mars | 1.88 | 1.52 | 3.53 | 3.51 |
| Jupiter | 11.86 | 5.20 | 140.7 | 140.6 |
| Saturn | 29.46 | 9.54 | 867.9 | 868.3 |
| Uranus | 84.01 | 19.19 | 7,058 | 7,067 |
| Neptune | 64.79 | 30.06 | 27,160 | 27,160 |
| Pluto | 248.54 | 39.53 | 61,770 | 61,770 |

# Modification of Kepler’s Laws




The Third Law needs to have the sum of the masses included.

$$\mathbf{(\mathcal{M} + \mathit{m})\, P^2 = k\, \mathit{a}^3},$$

where k = 1 if P is in earth years, *a* is in AUs, and $(\mathcal{M} + m)$ is in solar masses.

For objects orbiting the Sun, $(\mathcal{M} + m) = 1$.

Kepler's Third Law Interactive

# Example

$P = 4.00$ days (/ 365.25 d/yr) $= 0.01095$ yr

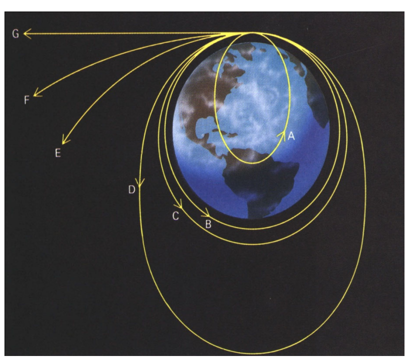
$a = 16.26\ R_{sun}$ (x $6.96 \times 10^5$ km/$R_{sun}$) (/ $1.5 \times 10^8$ km/AU) = 0.07544 AU

$(\mathcal{M}_1 + \mathcal{M}_2)\ P^2 = a^3$

$(\mathcal{M}_1 + \mathcal{M}_2) = (0.07544)^3 \ / \ (0.01095)^2$

$(\mathcal{M}_1 + \mathcal{M}_2) = 3.58\ \mathcal{M}_{sun}$

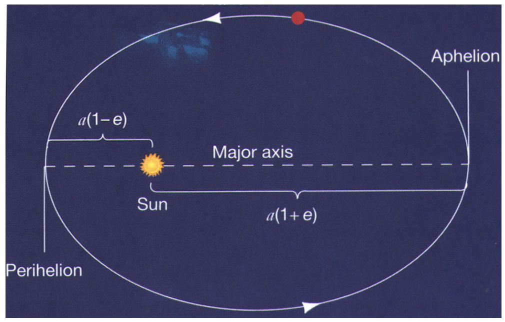
# Orbital Trajectories




Orbits
Function of Velocity

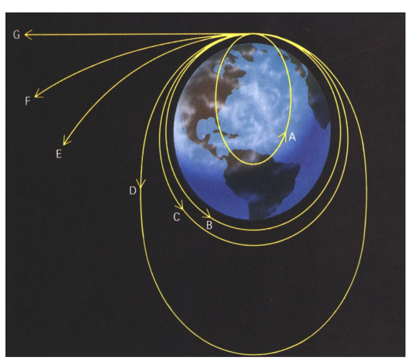
A: Elliptical
B: Elliptical
C: Circular
D: Elliptical
E: Parabolic
F: Hyperbolic

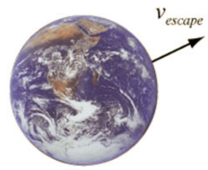
# Orbital Positions




| | | | |
|---|---|---|---|
| Nearest Positions: | Perihelion, | Perigee, | Periastron |
| Farthest Positions: | Aphelion, | Apogee, | Apastron |

# Escape Velocity



$$v_{escape} = 11.2\ km/s$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

Escape Velocity Interactive

# Geosynchronous Orbit

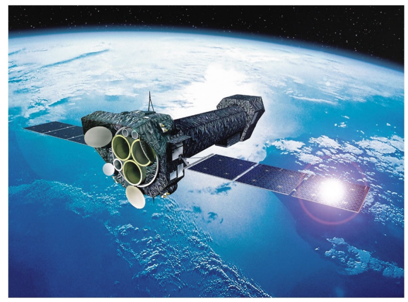
What is the distance from the center of the Earth for a geosynchronous orbit?

$P = ( 1 \text{ day} / 365.25 \text{ day/yr} )$
$= 2.74 \times 10^{-3}$ yr

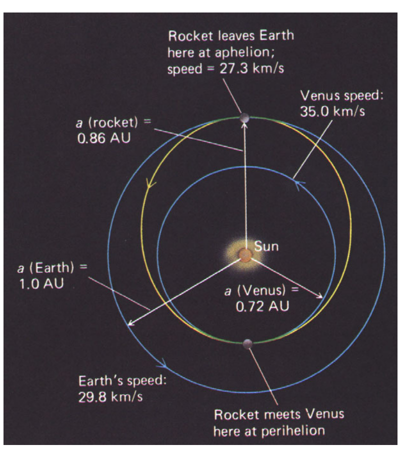
$\mathcal{M} = \mathcal{M}_{earth} / \mathcal{M}_{sun} = 3 \times 10^{-6}$

$(\mathcal{M} + m)\, P^2 = a^3$

$a = [ (3 \times 10^{-6}) (2.74 \times 10^{-3} \text{ yr})^2 ]^{1/3}$
$= 2.82 \times 10^{-4}$ AU $\times (1.5 \times 10^8$ km/AU)
= 42,400 km
= 0.11 Earth-Moon = 6.6 earth radii



# Least Energy Orbit




Trip Time

$(\mathcal{M} + m)\, P^2 = a^3$

$a = (1 \text{ AU} + 0.72 \text{ AU}) / 2$

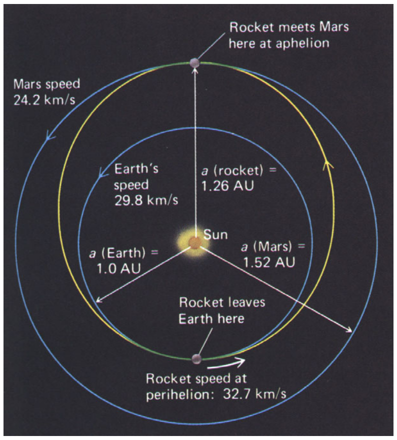
$P^2 = a^3 = (0.86)^3 = 0.636$

$P = 0.798$ yr

$T = P / 2 = 0.399$ yr

= 4.8 months

## Least Energy Orbit




Trip Time

$(\mathcal{M} + m)\, P^2 = a^3$

$a = (1\ \text{AU} + 1.52\ \text{AU}) / 2$

$P^2 = a^3 = (1.26)^3 = 2.000$

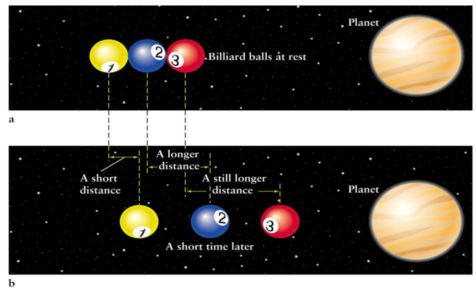
$P = 1.414$ yr

$T = P / 2 = 0.707$ yr

$= 8.5$ months

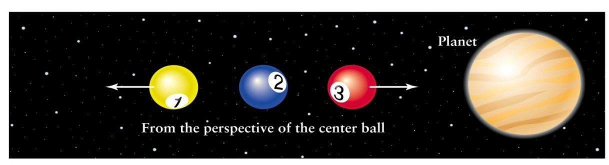
## Tides

# Tidal Forces



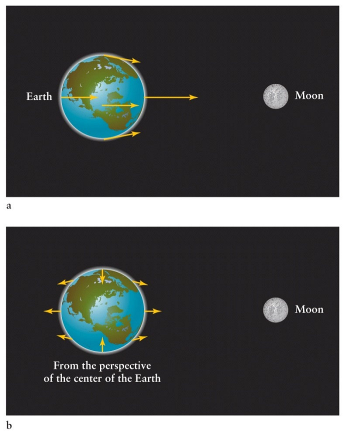

The nearest ball feels the greatest gravitational pull, so it moves the farthest.
The middle ball feels the next greatest pull, but does not move as far.
The farthest ball feels the least pull, but it still moves some toward the planet.

# Tidal Forces




From the perspective of the middle ball, the two outer balls move away by equal amounts of distance.

# Tidal Distortion



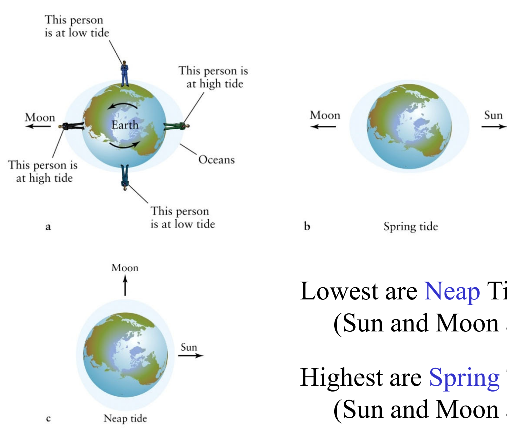

Now let's consider the pull of the Moon on the Earth.

The nearest edge toward the Moon is pulled the most, the center somewhat, and the farthest edge just a little.

From the Earth's perspective, its shape is deformed into two bulges and two low regions.

Consequently, there are 2 high tides and 2 low tides daily.

# Variations in Tides




Lowest are Neap Tides
(Sun and Moon at 90°)

Highest are Spring Tides
(Sun and Moon at 0°)

# The Principia

*The Mathematical Principles of Natural Philosophy*
*(Principia)*