Chapter 5

Forecasting

To accompany
Quantitative Analysis for Management, *Eleventh Edition*,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Forecasting Models

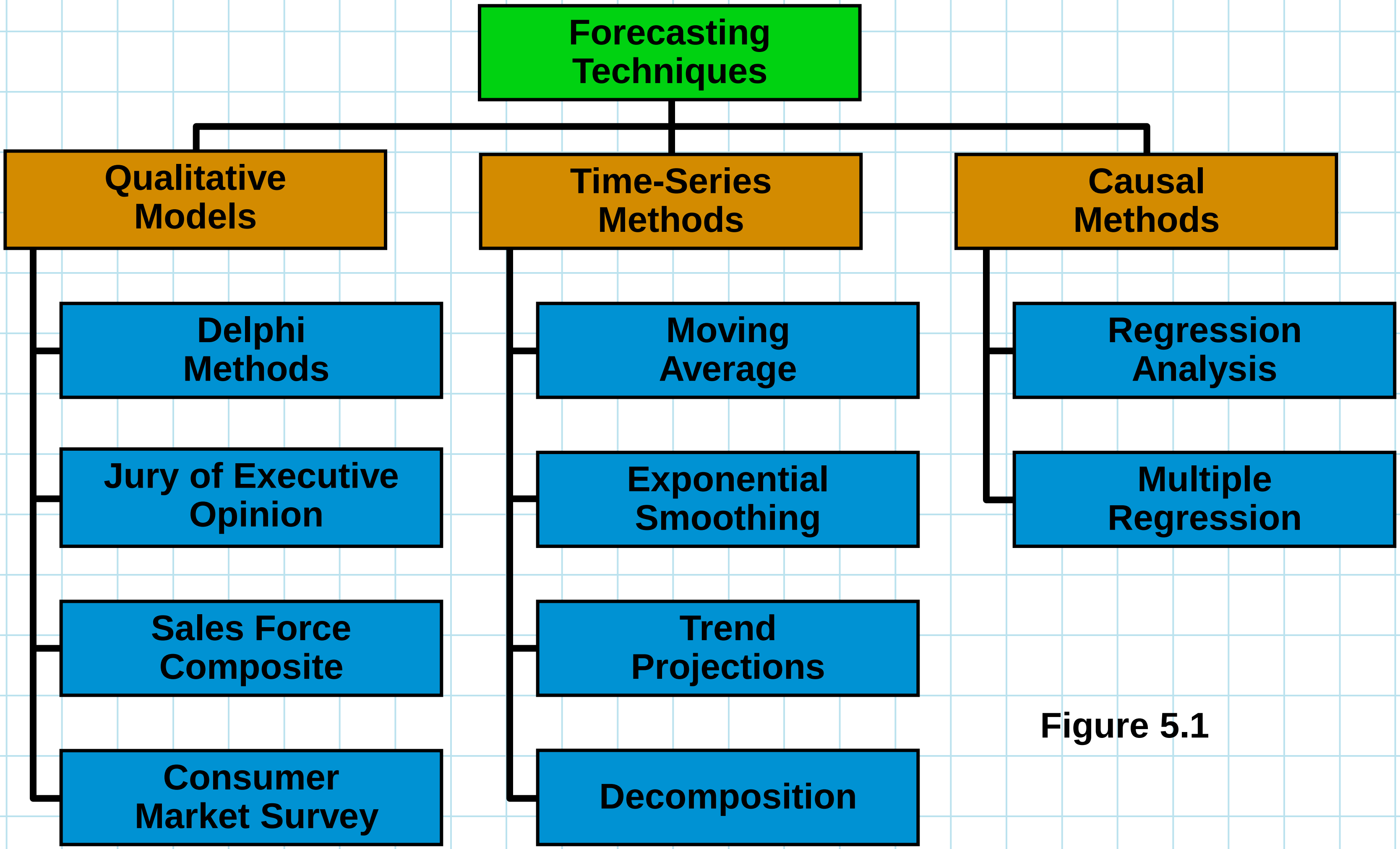

Figure 5.1

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Qualitative Models

- ***Qualitative models*** **incorporate judgmental or subjective factors.**
- **These are useful when subjective factors are thought to be important or when accurate quantitative data is difficult to obtain.**
- **Common qualitative techniques are:**
  - **Delphi method.**
  - **Jury of executive opinion.**
  - **Sales force composite.**
  - **Consumer market surveys.**

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Qualitative Models

- ***Delphi Method*** **– This is an iterative group process where (possibly geographically dispersed) *respondents* provide input to *decision makers.***
- ***Jury of Executive Opinion*** **– This method collects opinions of a small group of high-level managers, possibly using statistical models for analysis.**
- ***Sales Force Composite*** **– This allows individual salespersons estimate the sales in their region and the data is compiled at a district or national level.**
- ***Consumer Market Survey*** **– Input is solicited from customers or potential customers regarding their purchasing plans.**

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Time-Series Models

- ***Time-series models*** **attempt to predict the future based on the past.**
- **Common time-series models are:**
  - **Moving average.**
  - **Exponential smoothing.**
  - **Trend projections.**
  - **Decomposition.**
- **Regression analysis is used in trend projections and one type of decomposition model.**

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Scatter Diagrams

Wacker Distributors wants to forecast sales for three different products (annual sales in the table, in units):

YEAR	TELEVISION SETS	RADIOS	COMPACT DISC PLAYERS
1	250	300	110
2	250	310	100
3	250	320	120
4	250	330	140
5	250	340	170
6	250	350	150
7	250	360	160
8	250	370	190
9	250	380	200
10	250	390	190

Table 5.1

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Scatter Diagram for TVs

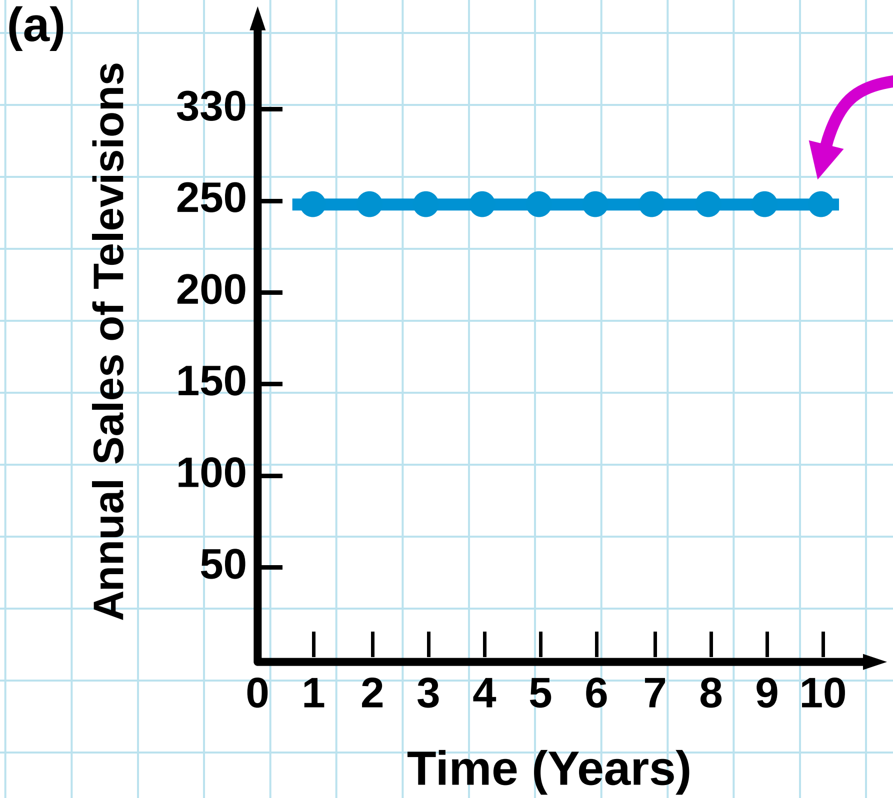

Figure 5.2a

- Sales appear to be constant over time

  Sales = 250

- A good estimate of sales in year 11 is 250 televisions

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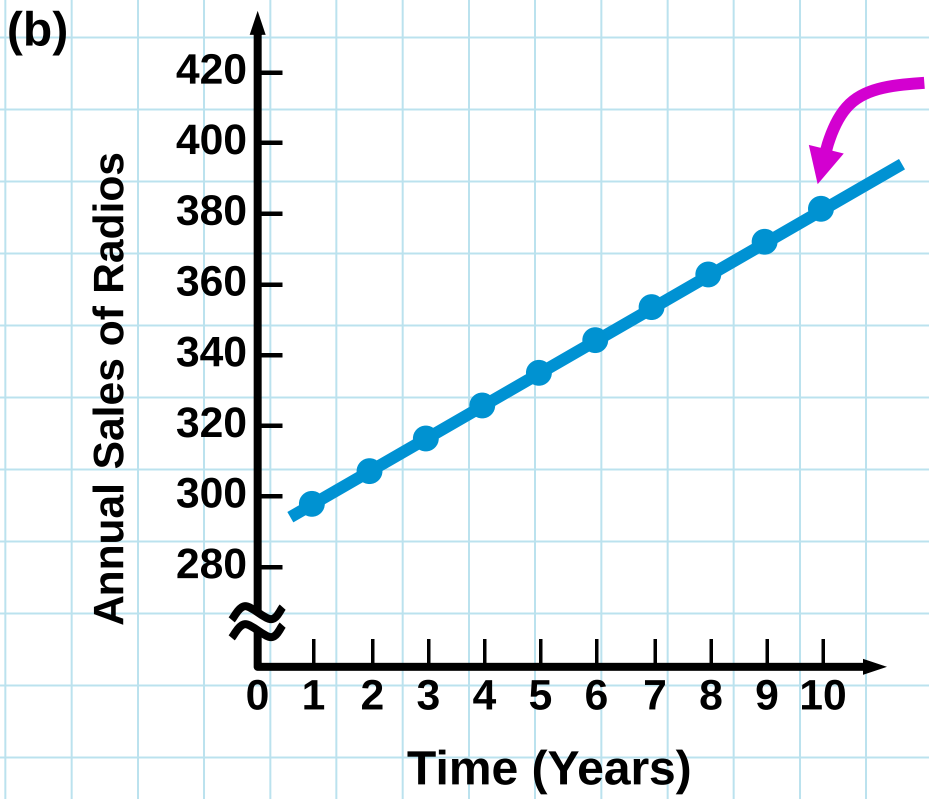
Scatter Diagram for Radios



- Sales appear to be increasing at a constant rate of 10 radios per year

Sales = 290 + 10(Year)

- A reasonable estimate of sales in year 11 is 400 radios.

Figure 5.2b

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Scatter Diagram for CD Players

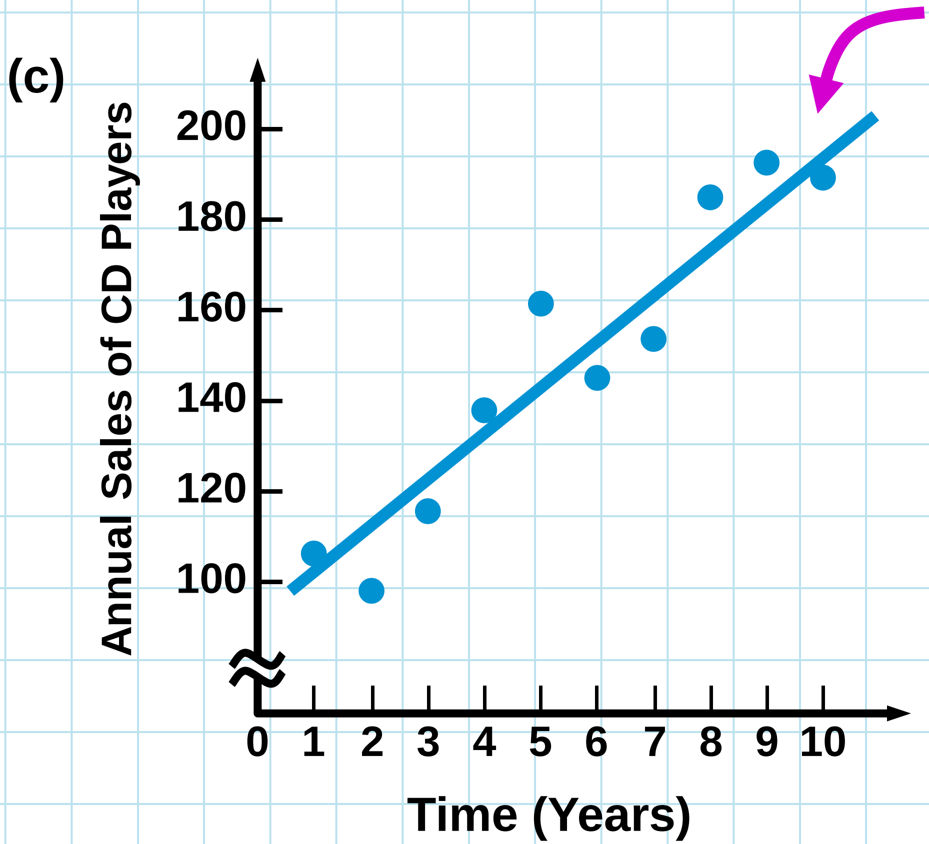

Figure 5.2c

- This trend line may not be perfectly accurate because of variation from year to year
- Sales appear to be increasing
- A forecast would probably be a larger figure each year

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Measures of Forecast Accuracy

- **We compare forecasted values with actual values to see how well one model works or to compare models.**

Forecast error = Actual value – Forecast value

- **One measure of accuracy is the *mean absolute deviation* (*MAD*):**

$$\text{MAD} = \frac{\sum \left|\text{forecast error}\right|}{n}$$

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Measures of Forecast Accuracy

Using a *naïve* forecasting model we can compute the MAD:

YEAR	ACTUAL SALES OF CD PLAYERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
2	100	110	|100 – 110| = 10
3	120	100	|120 – 110| = 20
4	140	120	|140 – 120| = 20
5	170	140	|170 – 140| = 30
6	150	170	|150 – 170| = 20
7	160	150	|160 – 150| = 10
8	190	160	|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

Table 5.2

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Measures of Forecast Accuracy

Using a *naïve* forecasting model we can compute the MAD:

YEAR	ACTUAL SALES OF CD PLAYERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
[illegible]	[illegible]	[illegible]	– 110| = 10
[illegible]	[illegible]	[illegible]	– 110| = 20
[illegible]	[illegible]	[illegible]	– 120| = 20
[illegible]	[illegible]	[illegible]	– 140| = 30
[illegible]	[illegible]	[illegible]	– 170| = 20
[illegible]	[illegible]	[illegible]	– 150| = 10
8	190	160	|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{160}{9} = 17.8$$

Table 5.2

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Measures of Forecast Accuracy

- There are other popular measures of forecast accuracy.
- The ***mean squared error:***

$$\text{MSE} = \frac{\sum (\text{error})^2}{n}$$

- The ***mean absolute percent error:***

$$\text{MAPE} = \frac{\sum \left| \frac{\text{error}}{\text{actual}} \right|}{n} 100\%$$

- And ***bias*** is the average error.

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Time-Series Forecasting Models

- **A time series is a sequence of evenly spaced events.**
- **Time-series forecasts predict the future based solely on the past values of the variable, and other variables are ignored.**

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Components of a Time-Series

A time series typically has four components:

1. ***Trend*** **(*T*) is the gradual upward or downward movement of the data over time.**
2. ***Seasonality*** **(*S*) is a pattern of demand fluctuations above or below the trend line that repeats at regular intervals.**
3. ***Cycles*** **(*C*) are patterns in annual data that occur every several years.**
4. ***Random variations*** **(*R*) are "blips" in the data caused by chance or unusual situations, and follow no discernible pattern.**

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Decomposition of a Time-Series

Product Demand Charted over 4 Years, with Trend and Seasonality Indicated

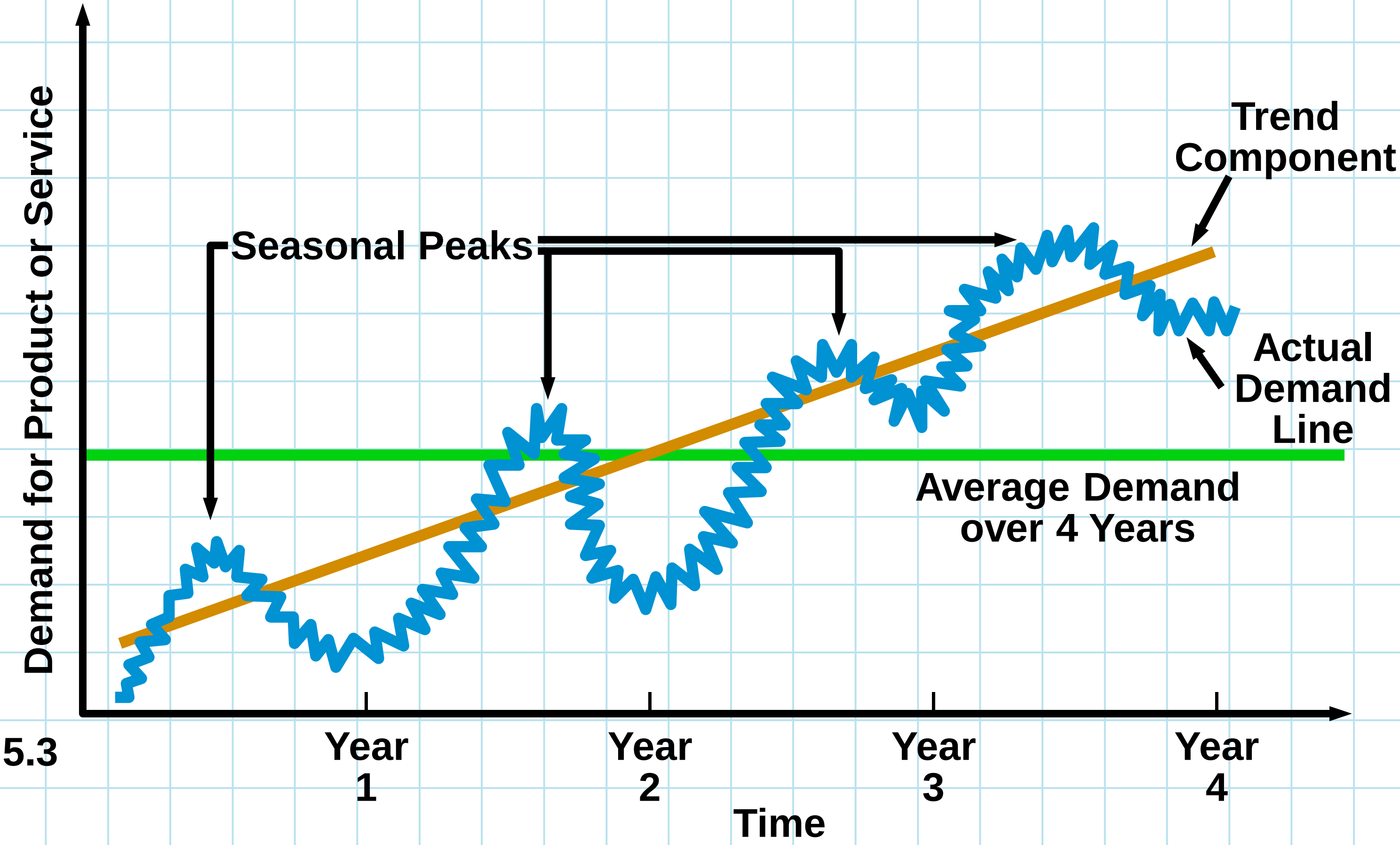

Figure 5.3

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Decomposition of a Time-Series

- There are two general forms of time-series models:
  - The multiplicative model:

$$\text{Demand} = T \times S \times C \times R$$

  - The additive model:

$$\text{Demand} = T + S + C + R$$

- Models may be combinations of these two forms.
- Forecasters often assume errors are normally distributed with a mean of zero.

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Moving Averages

- ***Moving averages*** can be used when demand is relatively steady over time.
- The next forecast is the average of the most recent n data values from the time series.
- This methods tends to smooth out short-term irregularities in the data series.

$$\text{Moving average forecast} = \frac{\text{Sum of demands in previous } n \text{ periods}}{n}$$

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Moving Averages

- **Mathematically:**

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

Where:

F_{t+1} = forecast for time period $t + 1$

Y_t = actual value in time period t

n = number of periods to average

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Wallace Garden Supply

- Wallace Garden Supply wants to forecast demand for its Storage Shed.
- They have collected data for the past year.
- They are using a three-month moving average to forecast demand (n = 3).

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Wallace Garden Supply

MONTH	ACTUAL SHED SALES	THREE-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	(10 + 12 + 13)/3 = 11.67
May	19	(12 + 13 + 16)/3 = 13.67
June	23	(13 + 16 + 19)/3 = 16.00
July	26	(16 + 19 + 23)/3 = 19.33
August	30	(19 + 23 + 26)/3 = 22.67
September	28	(23 + 26 + 30)/3 = 26.33
October	18	(26 + 30 + 28)/3 = 28.00
November	16	(30 + 28 + 18)/3 = 25.33
December	14	(28 + 18 + 16)/3 = 20.67
January	—	(18 + 16 + 14)/3 = 16.00

Table 5.3

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Weighted Moving Averages

- ***Weighted moving averages*** use weights to put more emphasis on previous periods.
- This is often used when a trend or other pattern is emerging.

$$F_{t+1} = \frac{\sum (\text{Weight in period } i)(\text{Actual value in period})}{\sum (\text{Weights})}$$

- Mathematically:

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$$

where

w_i = weight for the i^{th} observation

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Wallace Garden Supply

- Wallace Garden Supply decides to try a weighted moving average model to forecast demand for its Storage Shed.
- They decide on the following weighting scheme:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago

3 x Sales last month + 2 x Sales two months ago + 1 X Sales three months ago

6 — Sum of the weights

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Wallace Garden Supply

MONTH	ACTUAL SHED SALES	THREE-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 X 13) + (2 X 12) + (10)]/6 = 12.17
May	19	[(3 X 16) + (2 X 13) + (12)]/6 = 14.33
June	23	[(3 X 19) + (2 X 16) + (13)]/6 = 17.00
July	26	[(3 X 23) + (2 X 19) + (16)]/6 = 20.50
August	30	[(3 X 26) + (2 X 23) + (19)]/6 = 23.83
September	28	[(3 X 30) + (2 X 26) + (23)]/6 = 27.50
October	18	[(3 X 28) + (2 X 30) + (26)]/6 = 28.33
November	16	[(3 X 18) + (2 X 28) + (30)]/6 = 23.33
December	14	[(3 X 16) + (2 X 18) + (28)]/6 = 18.67
January	—	[(3 X 14) + (2 X 16) + (18)]/6 = 15.33

Table 5.4

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Exponential Smoothing

- ***Exponential smoothing*** **is a type of moving average that is easy to use and requires little record keeping of data.**

New forecast = Last period's forecast
+ α(Last period's actual demand
– Last period's forecast)

Here α is a weight (or *smoothing constant*) in which $0 \leq \alpha \leq 1$.

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Exponential Smoothing

Mathematically:

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

Where:

F_{t+1} = new forecast (for time period t + 1)

F_t = pervious forecast (for time period t)

α = smoothing constant ($0 \leq \alpha \leq 1$)

Y_t = pervious period's actual demand

The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period.

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Exponential Smoothing Example

- In January, February's demand for a certain car model was predicted to be 142.
- Actual February demand was 153 autos
- Using a smoothing constant of $\alpha = 0.20$, what is the forecast for March?

New forecast (for March demand) $= 142 + 0.2(153 - 142)$
$= 144.2$ or 144 autos

- If actual demand in March was 136 autos, the April forecast would be:

New forecast (for April demand) $= 144.2 + 0.2(136 - 144.2)$
$= 142.6$ or 143 autos

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Selecting the Smoothing Constant

- **Selecting the appropriate value for α is key to obtaining a good forecast.**
- **The objective is always to generate an accurate forecast.**
- **The general approach is to develop trial forecasts with different values of α and select the α that results in the lowest *MAD.***

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Exponential Smoothing

Port of Baltimore Exponential Smoothing Forecast for *α=0.1 and α=0.5.*

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST USING α =0.10	FORECAST USING α =0.50
1	180	175	175
2	168	175.5 = 175.00 + 0.10(180 – 175)	177.5
3	159	174.75 = 175.50 + 0.10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + 0.10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + 0.10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + 0.10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + 0.10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + 0.10(180 – 178.02)	186.30
9	?	178.60 = 178.22 + 0.10(182 – 178.22)	184.15

Table 5.5

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Exponential Smoothing

Absolute Deviations and MADs for the Port of Baltimore Example

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = 0.10$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.10$	FORECAST WITH $\alpha = 0.50$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.50$
1	180	175	5	175	5
2	168	175.5	7.5	177.5	9.5
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.3
Sum of absolute deviations			82.45		98.63
		$MAD = \frac{\Sigma\|deviations\|}{n}$ =	10.31	$MAD =$	12.33

Table 5.6

Best choice

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Trend Projections

- **Trend projection fits a trend line to a series of historical data points.**
- **The line is projected into the future for medium- to long-range forecasts.**
- **Several trend equations can be developed based on exponential or quadratic models.**
- **The simplest is a linear model developed using regression analysis.**

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Trend Projection

The mathematical form is

$$\hat{Y} = b_0 + b_1 X$$

Where

$\hat{Y}$ = predicted value
b_0 = intercept
b_1 = slope of the line
X = time period (i.e., X = 1, 2, 3, …, n)

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Midwestern Manufacturing

- Midwest Manufacturing has a demand for electrical generators from 2004 – 2010 as given in the table below.

YEAR	ELECTRICAL GENERATORS SOLD
2004	74
2005	79
2006	80
2007	90
2008	105
2009	142
2010	122

Table 5.7

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Midwestern Manufacturing Company Example

- The forecast equation is

$$\hat{Y} = 56.71 + 10.54X$$

- To project demand for 2011, we use the coding system to define $X = 8$

(sales in 2011) = 56.71 + 10.54(8)
= 141.03, or 141 generators

- Likewise for $X = 9$

(sales in 2012) = 56.71 + 10.54(9)
= 151.57, or 152 generators

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Midwestern Manufacturing

Electrical Generators and the Computed Trend Line

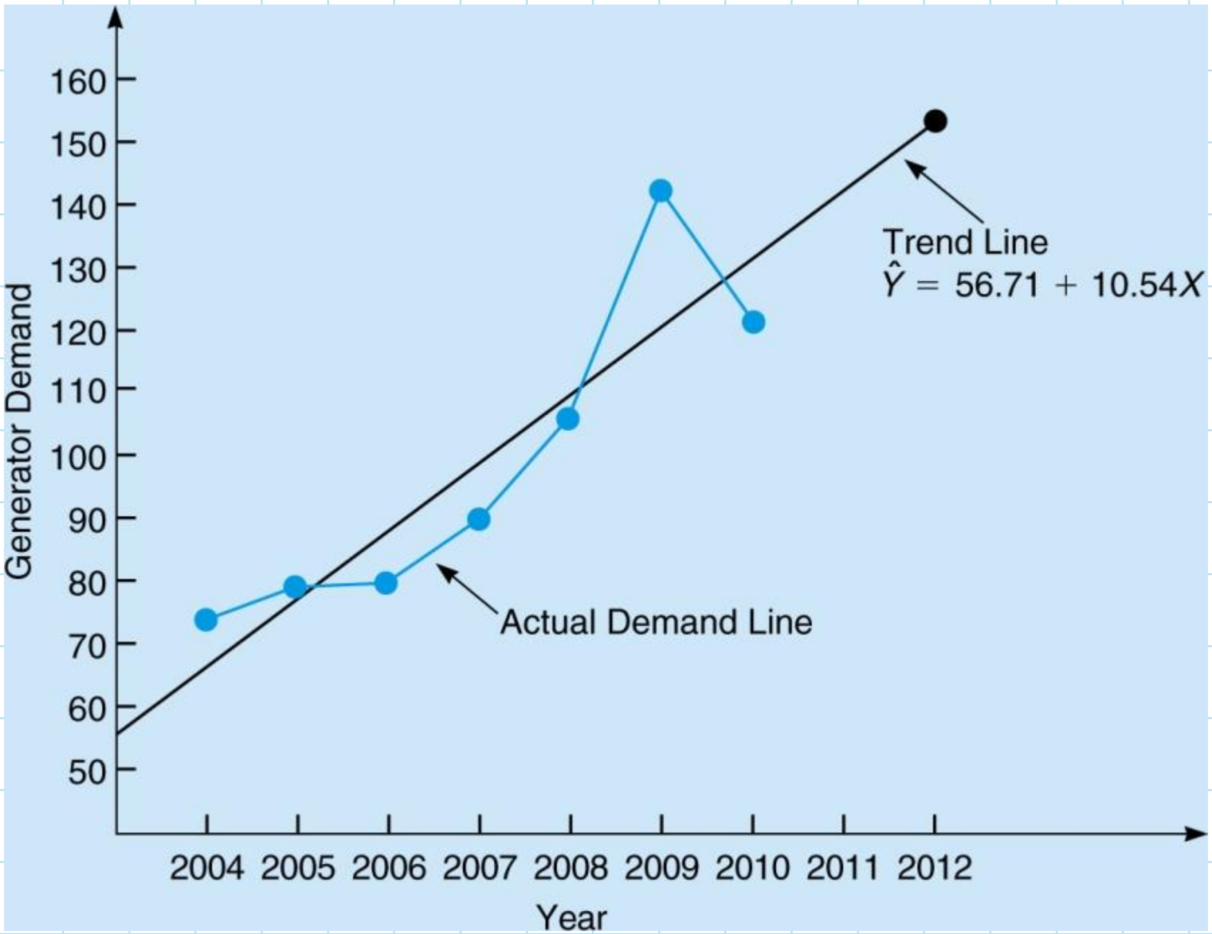

Figure 5.4

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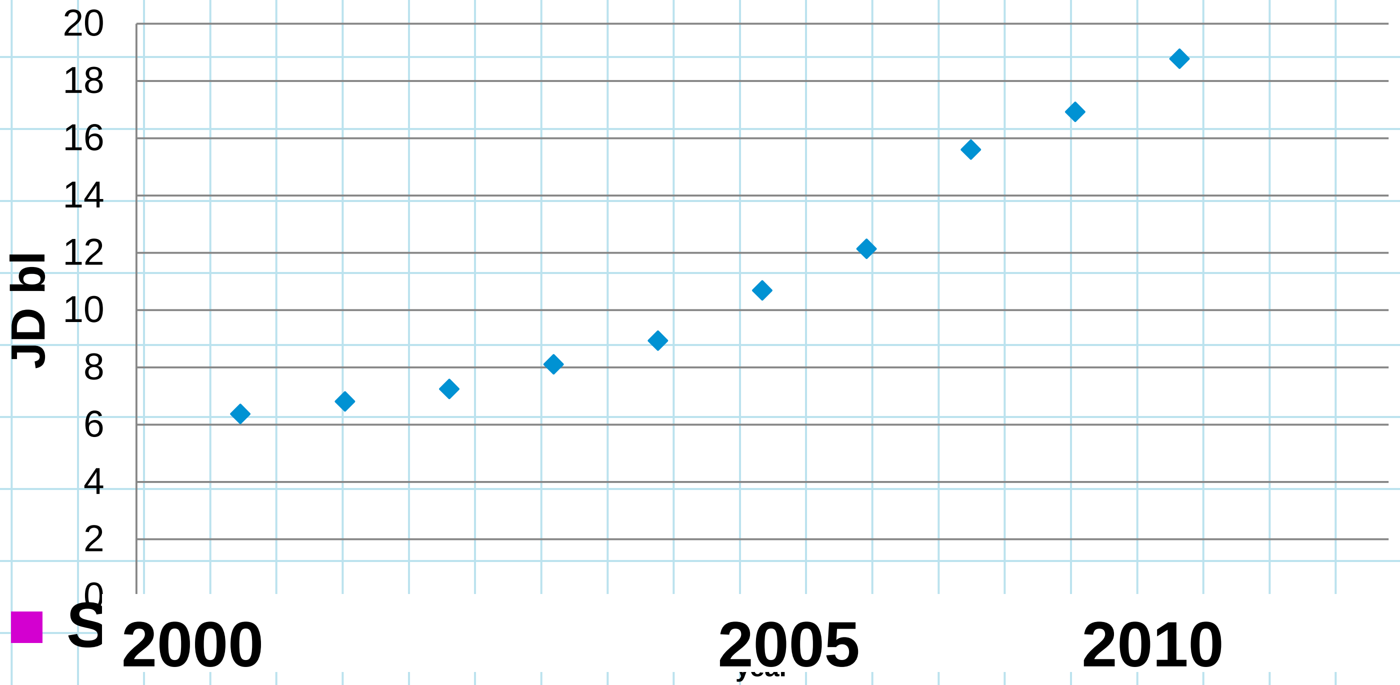
Trend projections

- Observing a time series for the GDP of Jordan from 2001 to 2011, annual data



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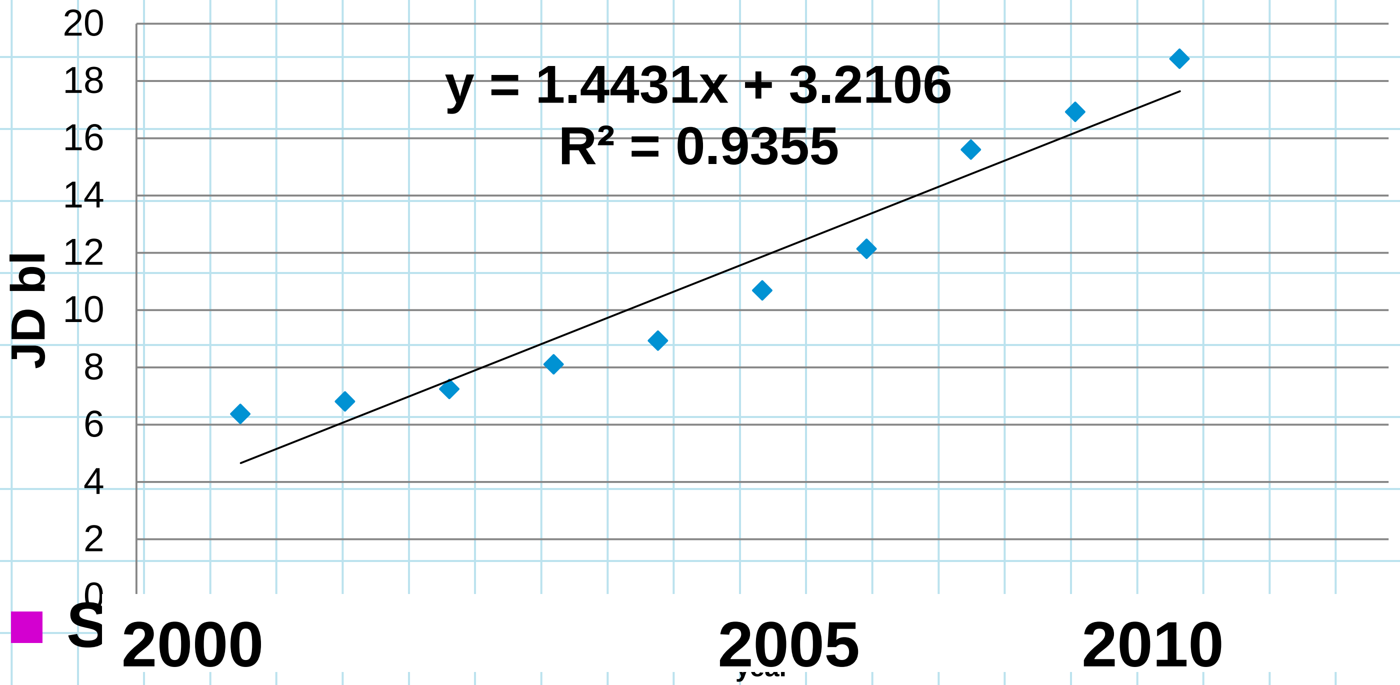
Trend projections

- Observing a time series for the GDP of Jordan from 2001 to 2011, annual data



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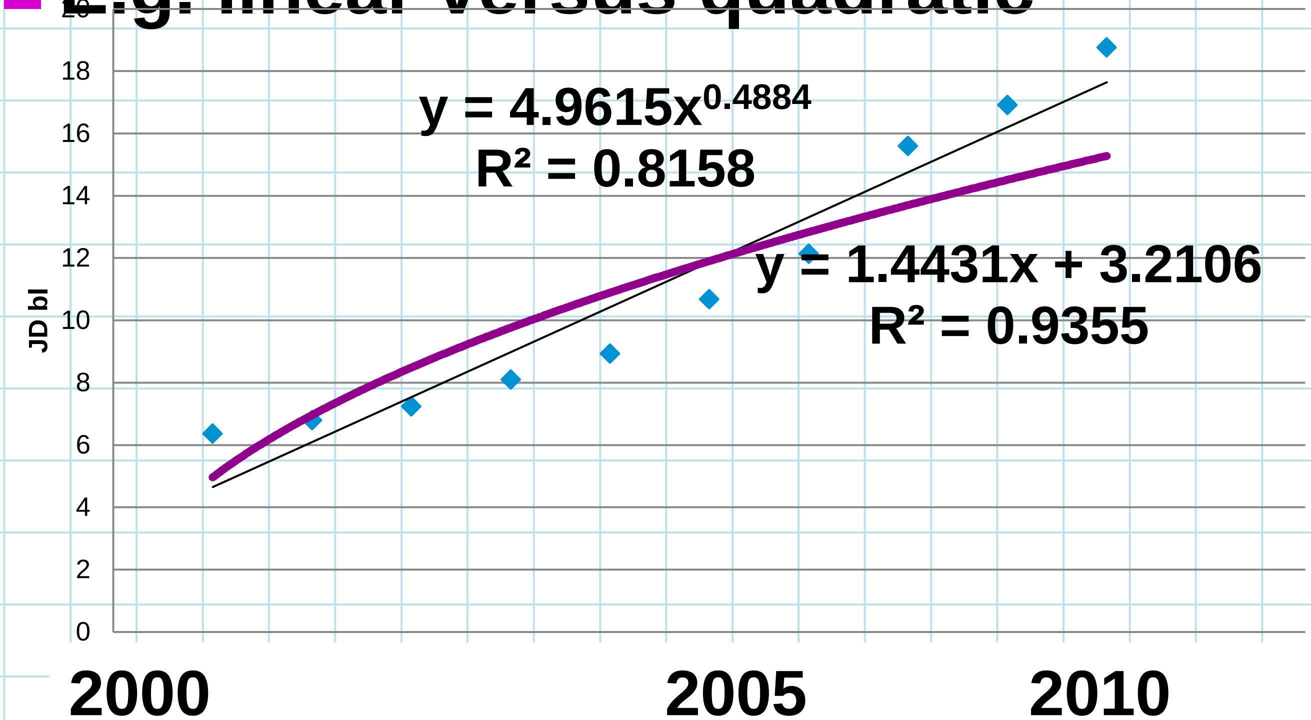
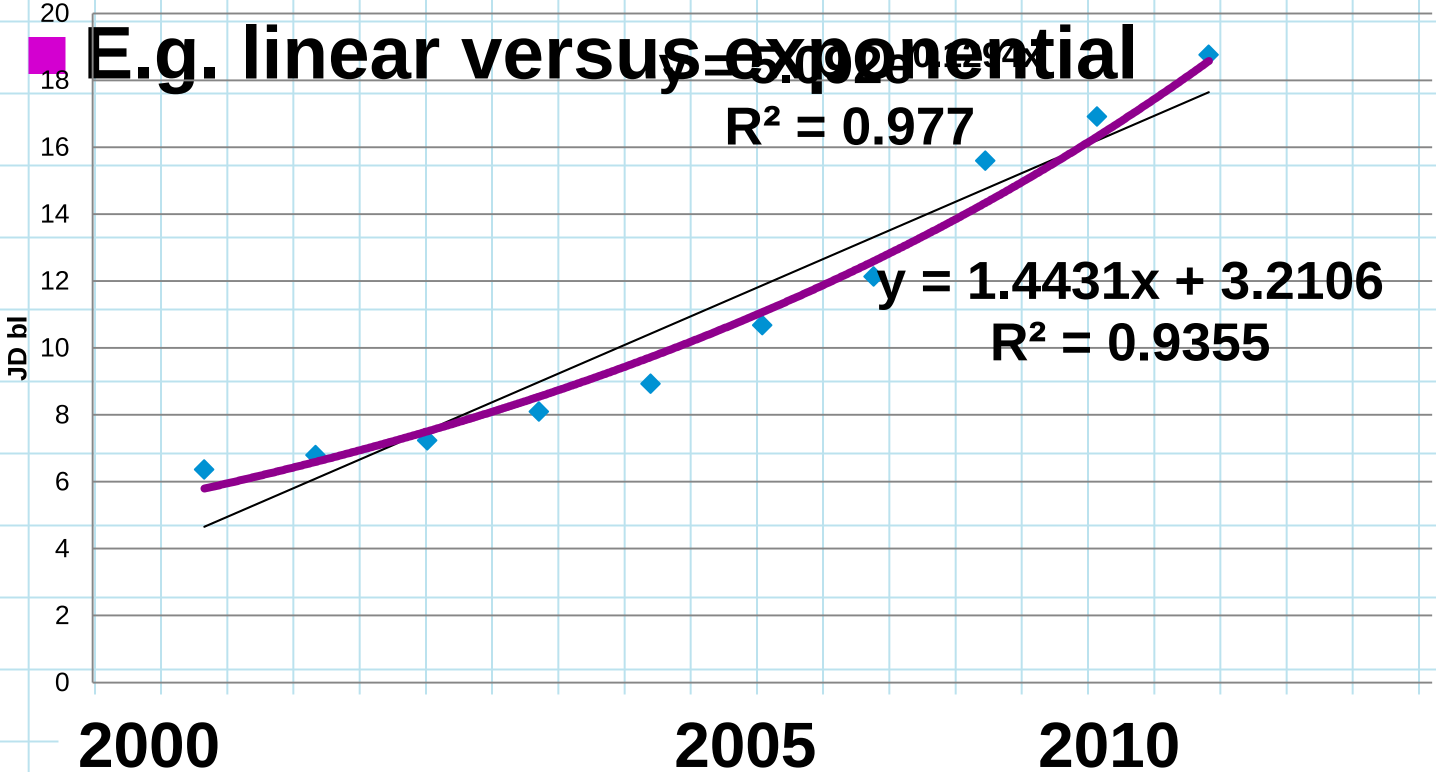
Trend projections

- **Different trend models can be compared**
  - **E.g. linear versus quadratic**



Trend projections

- **Different trend models can be compared**
  - **E.g. linear versus exponential**



Seasonal Variations

- Recurring variations over time may indicate the need for seasonal adjustments in the trend line.
- A seasonal index indicates how a particular season compares with an average season.
- When no trend is present, the seasonal index can be found by dividing the average value for a particular season by the average of all the data.

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Eichler Supplies

- Eichler Supplies sells telephone answering machines.
- Sales data for the past two years has been collected for one particular model.
- The firm wants to create a forecast that includes seasonality.

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Eichler Supplies Answering Machine Sales and Seasonal Indices

MONTH	SALES DEMAND YEAR 1	SALES DEMAND YEAR 2	AVERAGE TWO-YEAR DEMAND	MONTHLY DEMAND	AVERAGE SEASONAL INDEX
January	80	100	90	94	0.957
February	85	75	80	94	0.851
March	80	90	85	94	0.904
April	110	90	100	94	1.064
May	115	131	123	94	1.309
June	120	110	115	94	1.223
July	100	110	105	94	1.117
August	110	90	100	94	1.064
September	85	95	90	94	0.957
October	75	85	80	94	0.851
November	85	75	80	94	0.851
December	80	80	80	94	0.851
		Total average demand =	1,128		

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

$$\text{Seasonal index} = \frac{\text{Average two-year demand}}{\text{Average monthly demand}}$$

Table 5.9

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Seasonal Variations

- The calculations for the seasonal indices are

Jan.	$\frac{1{,}200}{12} \times 0.957 = 96$	July	$\frac{1{,}200}{12} \times 1.117 = 112$
Feb.	$\frac{1{,}200}{12} \times 0.851 = 85$	Aug.	$\frac{1{,}200}{12} \times 1.064 = 106$
Mar.	$\frac{1{,}200}{12} \times 0.904 = 90$	Sept.	$\frac{1{,}200}{12} \times 0.957 = 96$
Apr.	$\frac{1{,}200}{12} \times 1.064 = 106$	Oct.	$\frac{1{,}200}{12} \times 0.851 = 85$
May	$\frac{1{,}200}{12} \times 1.309 = 131$	Nov.	$\frac{1{,}200}{12} \times 0.851 = 85$
June	$\frac{1{,}200}{12} \times 1.223 = 122$	Dec.	$\frac{1{,}200}{12} \times 0.851 = 85$

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