Chapter 13

Tidal power

13.1 Introduction

The level of water in the large oceans of the Earth rises and falls according to predictable patterns. The main periods τ of these tides are diurnal at about 24 h and semidiurnal at about 12 h 25 min. The change in height between successive high and low tides is the range, R. This varies between about 0.5 m in general and about 10 m at particular sites near continental land masses. The movement of the water produces tidal currents, which may reach speeds of $\sim 5\,\mathrm{m\,s^{-1}}$ in coastal and inter-island channels.

The seawater can be trapped at high tide in an estuarine basin of area A behind a dam or barrier to produce tidal *range* power. If the water of density ρ runs out through turbines at low tide, it is shown in Section 13.5 that the average power produced is

$$\bar{P} = \rho A R^2 g/(2\tau).$$

For example, if $A = 10\,\mathrm{km}^2$, $R = 4\,\mathrm{m}$, $\tau = 12\,\mathrm{h}\ 25\ \mathrm{min}$, then $\overline{P} = 17\,\mathrm{MW}$. Obviously sites of large range give the greatest potential for tidal power, but other vital factors are the need for the power, and the costs and secondary benefits of the construction. The civil engineering costs charged to a tidal range power scheme could be reduced if other benefits are included. Examples are the construction of roads on dams, flood control, irrigation improvement, pumped water catchments for energy storage and navigation or shipping benefits. Thus the development of tidal power is very site-specific.

The power of tidal *currents* may be harnessed in a manner similar to wind power; this is also called '*tidal stream power*'. It is shown in Section 13.4 that the average power per unit area $\overline{q}$ in a current of maximum speed u_0 is $\overline{q} \sim 0.1\rho u_0^3$. For $u_0 = 3\,\mathrm{m\,s^{-1}}$, $\overline{q} \sim 14\,\mathrm{kW\,m^{-2}}$. In practice, tidal current is likely to be attractive for power generation only where it is enhanced in speed by water movement in straights between islands and mainland, or between relatively large islands. Therefore the opportunities for viable commercial sites are unusual. Where it is possible however, much of the

discussion concerning the use of the power is in common with tidal range power. We may also note that the flow power in a river has similar characteristics, but without the temporal variation. Harnessing river-stream power in the same manner as tidal stream power is certainly possible, but seldom considered.

The harnessing of *tidal range power* (henceforward called 'tidal power' as distinct from 'tidal current/stream power') has been used for small mechanical power devices, e.g. in medieval England and in China. The best-known large-scale electricity generating system is the 240 MW_e 'La Rance' system at an estuary into the Gulf of St Malo in Brittany, France, which has operated reliably since 1967, thereby proving the technical feasibility of this technology at large scale. However, economic and environmental constraints have meant that very few similar systems have been constructed since; see Section 13.6. Other sites with large tidal range, such as the Severn estuary in England and the Bay of Fundy on the eastern boundary between Canada and the United States, have been the subject of numerous feasibility studies over the past hundred years.

The range, flow and periodic behaviour of tides at most coastal regions are well documented and analysed because of the demands of navigation and oceanography. The behaviour may be predicted accurately, within an uncertainty of less than ±4%, and so tidal power presents a very reliable and assured form of renewable power. The major drawbacks are:

1 The mismatch of the principal lunar driven periods of 12 h 25 min and 24 h 50 min with the human (solar) period of 24 h, so that optimum tidal power generation is not in phase with demand.
2 The changing tidal range and flow over a two-week period, producing changing power production.
3 The requirement for large water volume flow at low head, necessitating many specially constructed turbines set in parallel.
4 The very large capital costs of most potential installations.
5 The location of sites with large range may be distant from the demand for power.
6 Potential ecological harm and disruption to extensive estuaries or marine regions.

For optimum electrical power generation from tides, the turbines should be operated in a regular and repeatable manner. The mode of operation will depend on the scale of the power plant, the demand and the availability of other sources. Very many variations are possible, but certain generalisations apply:

a If the tidal-generated electricity is for local use, then other assured power supplies must exist when the tidal power is unavailable. However,

the tidal basin provides energy storage, so extending power generation times and being available for storage from other power sources.

b If the generated electricity can feed into a large grid and so form a proportionately minor source within a national system, then the predictable tidal power variations can be submerged into the national demand.

c If the immediate demand is not fixed to the human (solar) period of 24 h, then the tidal power can be used whenever available. For example, if the electrical power is for transport by charging batteries or by electrolysing water for hydrogen, then such a decoupling of supply and use can occur.

The following sections outline the physical background to tides and tidal power. Readers interested only in power-generating installations should turn directly to Sections 13.4 and 13.5. Section 13.6 briefly reviews the social and environmental aspects of the technology.

13.2 The cause of tides

The analysis of tidal behaviour has been developed by many notable mathematicians and applied physicists, including Newton, Airy, Laplace, George Darwin (son of Charles Darwin) and Kelvin. We shall use Newton's physical theory to explain the phenomena of tides. However, present day analysis and prediction depends on the mathematical method of harmonic analysis developed by Lord Kelvin in Glasgow. A complete physical understanding of tidal dynamics has not yet been attained owing to the topological complexity of the ocean basins.

The seas are liquids held on the solid surface of the rotating Earth by gravity. The gravitational attraction of the Earth with the Moon and the Sun perturbs these forces and motions so that tides are produced. Tidal power is derived from turbines set in this liquid, so harnessing the kinetic energy of the rotating Earth. Even if all the world's major tidal power sites were utilised, this would lead to an extra slowing of the Earth's rotation by no more than one day in 2000 years; this is not a significant extra effect.

13.2.1 The lunar induced tide

The Moon and Earth revolve about each other in space (Figure 13.1), but since the mass of the Earth is nearly hundred times greater than the Moon's mass, the Moon's motion is more apparent. The centre of revolution is at O, such that

$$ML = M'L'$$

$$L' = MD/(M' + M) \tag{13.1}$$

$$L' = 4670\,\text{km}.$$

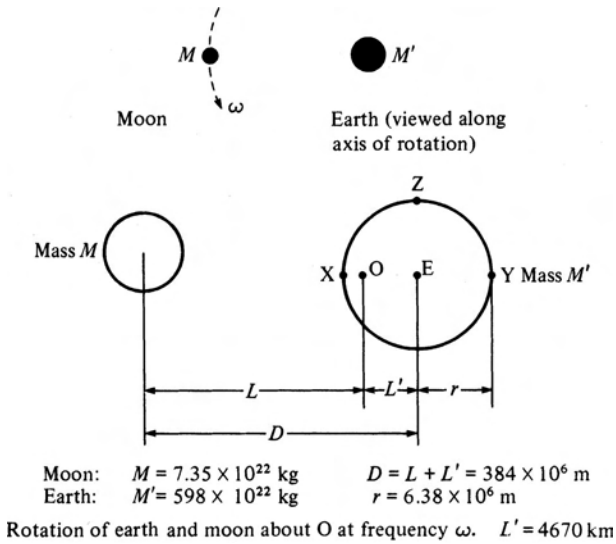

Figure 13.1 Motion of the Moon and the Earth.

The Earth's mean radius is 6371 km, so the point of revolution O is *inside* the surface of the Earth.

A balance of gravitational attraction and centrifugal force maintains the Earth–Moon separation. If the gravitational constant is G,

$$\frac{GMM'}{D^2} = ML\omega^2 = M'L'\omega^2 \tag{13.2}$$

If all the mass of the Earth could be located at the centre of the Earth E, then each element of mass would be at the equilibrium position with respect to the Moon. However, the mass of the Earth is not all at one point, and so is not all in this equilibrium. Material furthest from the Moon at Y (see Figure 13.1), experiences an increased outward centrifugal force with distance of rotation $(r+L')$ and a decreased gravitational force from the Moon. Material nearest the Moon at X has an increased gravitational force towards the Moon, plus the centrifugal force, also towards the Moon but reduced, because of the reduced rotation distance $(r-L')$. The solid material of the Earth experiences these changing forces as the Moon revolves, but is held with only small deformation by the structural forces of the solid state. Liquid on the surface is, however, free to move, and it is this movement relative to the Earth's surface that causes the tides. If the Moon is in the equatorial plane of the Earth, the water of the open seas attempts to heap together to form peaks at points X and Y, closest to and furthest from the

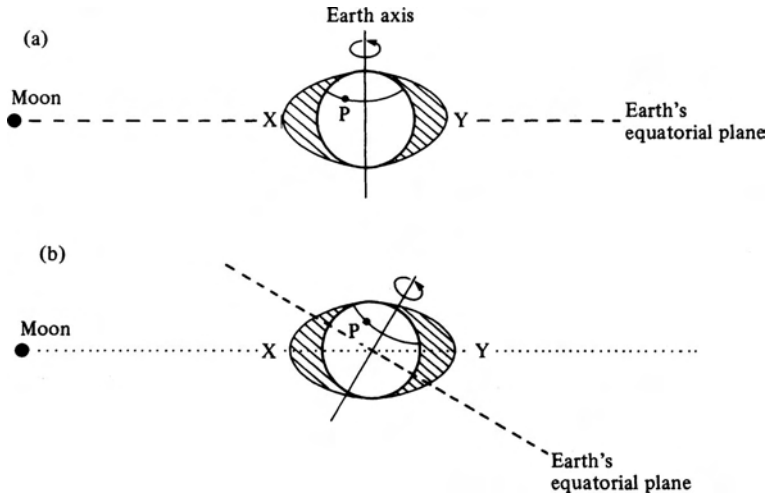

Figure 13.2 Basic physical explanation of the semidiurnal and diurnal tide. (a) Simple theory of equilibrium tide with the Moon in the plane of the Earth's equator, P experiences two equal tides each day (semidiurnal tide). (b) Normally the Moon is not in the Earth's equatorial plane, and so P may experience only one tide each day (diurnal tide).

Moon. The solid Earth would rotate with a period of one day underneath these two peaks (Figure 13.2(a)). Thus with no other effect occurring, each sea-covered position of the Earth would experience two rises and two falls of the water level as the Earth turns through the two peaks. This is the semidiurnal (half daily) tide. Note that the daily rotation of the Earth on its own axis has no first order effect, as such, in producing tidal range.

We may estimate the resultant force causing the tides from (i) the centrifugal force about O at the lunar frequency ω, and (ii) the force of lunar gravitational attraction (see Figure 13.1). For a mass m of water, furthest from the Moon at Y,

$$F_Y = m(L' + r)\omega^2 - \frac{GMm}{(D+r)^2} \tag{13.3}$$

Nearest to the Moon at X

$$F_x = \frac{GMm}{(D-r)^2} + m(r - L')\omega^2 \tag{13.4}$$

At position E (see Figure 13.1), by definition of L',

$$\frac{GMm}{D^2} = mL'\omega^2 \tag{13.5}$$

However, since $r \ll D$,

$$\frac{1}{(D \pm r)^2} = \frac{1}{D^2}\left(1 \pm \frac{2r}{D}\right)$$

So by substituting (13.5) in (13.3) and (13.4),

$$F_X = F_Y = mr\omega^2\left(1 + \frac{2L'}{D}\right) \tag{13.6}$$

We therefore expect two lunar tidal ranges each day of equal amplitude. This does indeed happen in the large oceans when the Moon is in the equatorial plane of the Earth.

At low tide on this equilibrium tide model the lunar-related force is $mr\omega^2$, and so the tide-raising force within (13.6) is $mr\omega^2 2L'/D$. It can be shown, see Problem 13.1, that this would produce a maximum equilibrium tidal range of 0.36 m.

There are three principal reasons why actual tidal behaviour is different from this simplistic 'equilibrium tide' explanation:

1 The explanation of the forcing function producing the tides is physically correct, but the theory has failed to determine if the peaks of water can move at about 1600 km h^{-1} to keep up with the Earth's rotation. In practice, the tidal wave cannot move fast enough to remain in the meridian of the Moon (see Problem 13.3). Thus as the Moon passes over an ocean, tidal motion is induced, which propagates at a speed of about 500 km/h and lags behind the Moon's position. The time of this lag is the 'age of the tide'. Each ocean basin tends to have its own system of moon-induced tides that are not in phase from one ocean to another.
2 The Moon is not usually in the equatorial plane of the Earth (Figure 13.2(b)), and so a diurnal component of the tide occurs. Other minor frequency components of the tidal motion occur. For instance, the Moon–Earth distance oscillates slightly, from a maximum of 4.06×10^8 m at apogee to a minimum of 3.63×10^8 m at perigee in a period of 27.55 solar days (the anomalistic month). Also the Moon's plane of motion moves about 2° in and out of the Earth–Sun ecliptic plane. (Unrelated to tidal forces, it is interesting to realise that the Moon rotates on its own axis in a period of 27.396 Earth days, and is locked to show one 'face' to the Earth. No place on the Moon is permanently dark, so solar systems would function everywhere, but, for the same production, would require more storage capacity than on Earth!)
3 A great many other complications occur, mostly associated with particular ocean basins. For instance, resonances occur across oceans and

especially near continental shelves, which produce distinct enhancements of the tidal range. We will show in Section 13.3 that these resonant enhancements are of great importance for tidal power installations.

13.2.2 Period of the lunar tides

To calculate the period of the tides more precisely, we have to be more precise about what we mean by a 'day' (Figure 13.3). At a point A on the Earth, a solar day is the interval between when the Sun crosses the meridional plane at A on a specified day and when it does so the subsequent day. This period actually varies through the year because of the irregularities in the Earth's orbit, and so the common unit of time, the *mean solar day* t_S, is defined to be the interval averaged over a whole year. Its value is defined as exactly 24 h, i.e.

$$t_S = 86\,400\,\text{s} \tag{13.7}$$

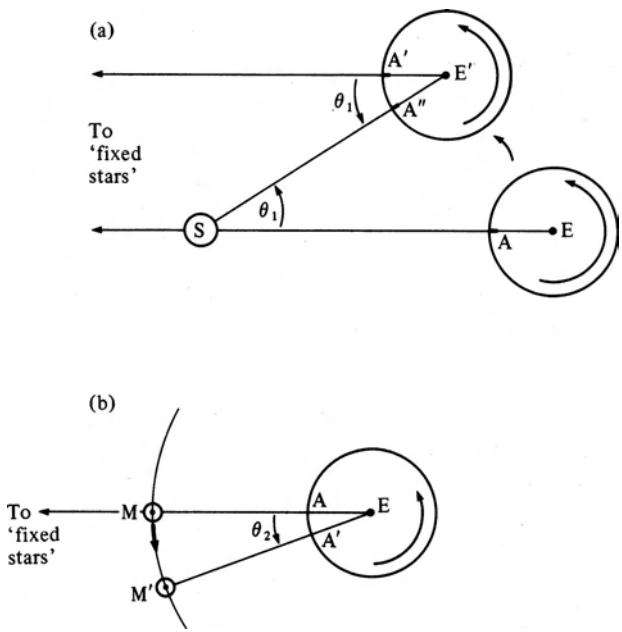

Figure 13.3 Comparison of three different 'days' that can be observed from Earth. (a) Sidereal and solar day. (b) Sidereal and lunar. The solar day is 24 hours exactly by definition, the sidereal is slightly shorter and the lunar slightly longer. The diagrams are not to scale. See also Figure 4.4 for meridional plane.

The *sidereal day* t^* is similarly defined to be the average interval between successive transits of a 'fixed star', i.e. one so distant that its apparent motion relative to the Earth is negligible. The sidereal day is therefore the 'true' period of rotation of the Earth, as seen by a distant observer.

Figure 13.3(a) shows how the difference between t_S and t^* is related to the revolution of the Earth around the Sun (period $T_S = 365.256 t_S$). Suppose that at midday on a certain day the centre of the Earth E, point A on the Earth's surface, the Sun S and some fixed star are all aligned. One solar day later, the Sun, A and the Earth's centre are again aligned. In this time E has moved through an angle θ_1 around the Sun to E′. Since t_S is the mean solar day, for time-keeping purposes we can regard the Earth as moving uniformly around a circular orbit so that

$$\frac{\theta_1}{2\pi} = \frac{t_S}{T_S} \tag{13.8}$$

In this time A has rotated around E through an angle $(2\pi + \theta_1)$ to A''. Its time to rotate through an angle 2π (as seen by a distant observer) is just t^*, so that

$$\frac{\theta_1}{2\pi} = \frac{t_S - t^*}{t^*} \tag{13.9}$$

Equating (13.8) and (13.9) gives

$$\begin{aligned} t^* &= \frac{t_S}{1 + (t_S/T_S)} \\ &= 86\,164\,\text{s} \\ &= 23\,\text{h}\ 56\ \text{min}\ 4\,\text{s} \end{aligned} \tag{13.10}$$

Similarly the *mean lunar day* t_M is defined to be the mean interval between successive alignments of E, A and the Moon's centre. Figure 13.3(b) shows the fictitious mean Moon M moving uniformly in a circular orbit around the Earth. In a time t_M, the Moon moves through an angle θ_2 from M to M′, while A on the Earth rotates through $2\pi + \theta_2$. Thus as seen by a distant observer

$$\frac{\theta_2}{2\pi} = \frac{t_M}{T^*} = \frac{T_M - t^*}{t^*} \tag{13.11}$$

where $T^* = 27.32 t_S$ (called the sidereal month, the 'true' lunar month) is the period of revolution of the Moon about the Earth's position as seen by a distant observer. This is shorter than the lunar month as recorded by an

observer on Earth ($T_M = 29.53$ days) owing to the Earth moving around the Sun. Equation (13.11) implies that

$$t_M = \frac{t^*}{1-(t^*/T^*)} = 89\,428\,\text{s} = 24\,\text{h}\ 50\,\text{min}\ 28\ \text{s} \tag{13.12}$$

Such a period is called 'diurnal' because it is near to 24 h.

13.2.3 The solar induced tide and combined effects

The same Newtonian theory that explains the major aspects of the twice daily lunar tide can be applied to the Sun/Earth system. A further twice daily tide is induced with a period of exactly half the solar day of 24 h. Other aspects being equal, the range of the solar tide will be 2.2 times less than the range of the lunar tide, which therefore predominates. This follows from considering that the tidal range is proportional to the *difference* of the gravitational forces from the Moon and the Sun across the diameter d of the Earth. If M_M and M_S are the masses of the Moon and the Sun at distances from the Earth of D_M and D_S, then for either system:

$$\text{gravitational force} \propto \frac{M}{D^2}$$
$$\text{difference in force} \propto \frac{\partial F}{\partial D} d = \frac{-2Md}{D^3} \tag{13.13}$$

The range of the lunar tide R_M and solar tide R_S are proportional to the difference, so

$$\frac{R_M}{R_S} = \frac{(M_M/D_M^3)}{(M_S/D_S^3)} = \left(\frac{D_S}{D_M}\right)^3 \frac{M_M}{M_S} = \left(\frac{1.50\times10^{11}\,\text{m}}{3.84\times10^{8}\,\text{m}}\right)^3 \left(\frac{7.35\times10^{22}\,\text{kg}}{1.99\times10^{30}\,\text{kg}}\right) = 2.2 \tag{13.14}$$

The solar tide moves in and out of phase with the lunar tide. When the Sun, Earth and Moon are aligned in conjunction, the lunar and solar tides are in phase, so producing tides of maximum range. These are named '*spring tides*' of maximum range occurring twice per lunar (synodic) month at times of both full and new Moons (Figure 13.8(c)).

When the Sun/Earth and Moon/Earth directions are perpendicular (in quadrature) the ranges of the tides are least. These are named '*neap tides*' that again occur twice per synodic month. If the spring tide is considered to result from the sum of the lunar and solar tides, and the neap tide from

their difference, then the ratio of spring to neap ranges might be expected to be

$$\frac{R_s(\text{spring})}{R_n(\text{neap})} = \frac{1+(1/2.2)}{1-(1/2.2)} = 3 \tag{13.15}$$

In practice, dynamical and local effects alter this rather naive model, and the ratio of spring to neap range is more frequently about 2. Spring tides at the Moon's perigee have greater range than spring tides at apogee, and a combination of effects including wind can occur to cause unusually high tides.

13.3 Enhancement of tides

The normal mid-ocean tidal range is less than one metre and of little use for power generation. However, near many estuaries and some other natural features, enhancement of the tidal range may occur by (i) funnelling of the tides (as with sound waves in an old-fashioned trumpet-shaped hearing aid), and (ii) by resonant coupling to natural frequencies of water movement in coastal contours and estuaries. This *local enhancement is essential* for tidal power potential; we stress this point most strongly.

The ordinary tidal movement of the sea has the form of a particular type of moving wave called a 'tidal wave'. The whole column of water from surface to sea bed moves at the same velocity in a tidal wave, and the wavelength is very long compared with the sea depth (Figure 13.4). (This is the same relative proportion as the so-called 'shallow' water waves, which may be a correct, but totally misleading, name given to tidal waves.) Motion of a continuously propagating natural tidal wave has a velocity c related to the acceleration of gravity g and the sea depth h such that $c = \sqrt{(gh)}$, i.e. $\sim 750\,\text{km}\,\text{h}^{-1}$ across major oceans, which have depth $\sim 4000\,\text{m}$.

Underwater volcanic or earthquake activity can induce a freely propagating 'seismic sea wave' in deep oceans correctly called a *tsunami*, but

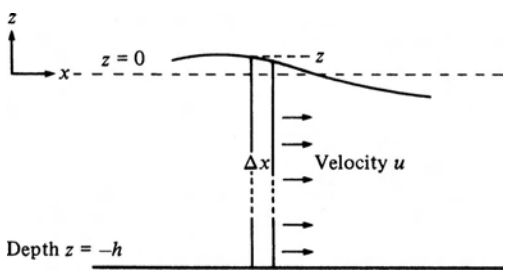

Figure 13.4 Motion of water in a tidal wave; the elemental section of sea has thickness Δx, depth h and width b (along the y axis).

sometimes incorrectly called a 'tidal wave' since there is no relationship to tides. A tsunami is initiated by a relatively localised, but extreme, sudden change in the height of the sea bottom, which injects an immense pulse of energy over a short relatively horizontal distance on the sea bed. The resulting 'shock' creates the physical equivalent of a 'shallow' depth wave (with $\lambda/4 >$ depth), where 'shallow' has to be interpreted as the $\sim$4000 m sea depth, and wave movement encompasses the whole depth. The wave spreads rapidly at speed $c = \sqrt{(gh)}$ and wavelength $\lambda \approx 150$ km. When the tsunami reaches the decreasing sea depth near shore, friction at the sea bed slows the wave and so shortens the wavelength, with the consequence of rapidly increased surface amplitude to perhaps 30 m. This amplitude will be apparent at the coast as perhaps an exceptional outflow of sea water followed quickly by huge and damaging breaking waves.

Considering the solar and the lunar forces, neither is in the form of a pulse, so no 'tsunami-like' behaviour occurs. The only possibility for enhanced motion is for the natural tidal motion to be in resonance with the solar and the lunar forces. But, as seen from Earth, the Sun moves overhead at $\sim$2000 km h^{-1} and the Moon at $\sim$60 km h^{-1}, therefore, the tidal forcing motions for the lunar- and the solar-induced tides do not, in general, coincide with the requirements for a freely propagating tidal wave in the deep ocean, and so resonant enhancement of the forced motion does not occur in the open oceans.

In certain estuaries and bays, resonance can occur, however, and most noticeable changes in tidal motion therefore appear. We consider a slab of water of depth h, width b, thickness Δx and surface level above the mean position z. The change in surface level over the thickness Δx is $(\partial z/\partial x)\Delta x$, and this is small compared with z. The side area of the slab is $A = hb$.

The form of the wave is obtained by considering Newton's equation of motion for the slab, and the requirement for conservation of water mass. The pressure difference across each side of the slab arises from the small change in height of the surface:

$$\Delta p = -\rho g \left(\frac{\partial z}{\partial x}\right) \Delta x \tag{13.16}$$

So the equation of motion of the slab of velocity u is

$$\text{force} = \text{mass} \times \text{acceleration}$$

$$\left[-\rho g \left(\frac{\partial z}{\partial x}\right) \Delta x\right] hb = \rho hb \Delta x \frac{\partial u}{\partial t} \tag{13.17}$$

$$\frac{\partial u}{\partial t} = -g \frac{\partial z}{\partial x} \tag{13.18}$$

The difference between the flow of water into and out of the slab must be accounted for by a change in volume V of the slab with time, and this conservation of water mass leads to the equation of continuity

$$-\left[\frac{\partial}{\partial x}(A+bz)u\right]\Delta x = \frac{\partial V}{\partial t} = \frac{\partial[(A+bz)\Delta x]}{\partial t} \tag{13.19}$$

Since $A = bh$ is constant and much larger than bz,

$$-A\frac{\partial u}{\partial x} = b\frac{\partial z}{\partial t}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{h}\frac{\partial z}{\partial t} \tag{13.20}$$

From (13.18) and (13.20),

$$\frac{\partial^2 u}{\partial t \partial x} = -g\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 u}{\partial x \partial t} = -\frac{1}{h}\frac{\partial^2 z}{\partial t^2} \tag{13.21}$$

So

$$\frac{\partial^2 z}{\partial t^2} = gh\frac{\partial^2 z}{\partial x^2} = c^2\frac{\partial^2 z}{\partial x^2} \tag{13.22}$$

This is the equation of a wave of speed c, with

$$c = \sqrt{(gh)} \tag{13.23}$$

Resonant enhancement of the tides in estuaries and bays occurs in the same manner as the resonance of sound waves in open and closed pipes, for example as in Figure 13.5. Resonance with the open sea tide occurs when

$$L = \frac{j\lambda}{4}, j \text{ an odd integer} \tag{13.24}$$

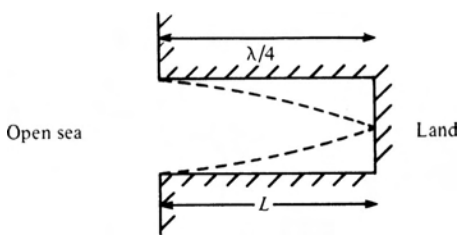

Figure 13.5 Resonant enhancement of a tidal wave in an estuary, plan view. Idealised bay of constant depth h. Amplitude of tidal range indicated for a quarter wavelength resonance.

The natural frequency of the resonance f_r and the period T_r is given by

$$f_r = \frac{1}{T_r} = \frac{c}{\lambda} \tag{13.25}$$

So

$$T_r = \frac{\lambda}{c} = \frac{4L}{jc} = \frac{4L}{j\sqrt{(gh)}} \tag{13.26}$$

Resonance occurs when this natural period equals the forced period of the tides in the open sea T_f, in which case

$$T_f = \frac{4L}{j\sqrt{(gh)}}; \frac{L}{\sqrt{h}} = \frac{j}{4}\sqrt{(g)}T_f \tag{13.27}$$

The semidiurnal tidal period is about 12 h 25 min (45 000 s), so resonance for $j = 1$ occurs when

$$\frac{L}{\sqrt{h}} = \frac{45\,000\,\mathrm{s}}{4}\sqrt{(9.8\,\mathrm{m\,s^{-2}})} = 36\,000\,\mathrm{m}^{1/2} \tag{13.28}$$

Usually, if it occurs at all, such enhancement occurs in river estuaries and ocean bays, as in the Severn Estuary (Example 13.1). However, there is a small general enhancement for the whole Atlantic Ocean.

Example 13.1 Resonance in the Severn Estuary
The River Severn Estuary between Wales and England has a length of about ~200 km and depth of about 30 m, so

$$\frac{L}{\sqrt{h}} \approx \frac{200 \times 10^3\,\mathrm{m}}{\sqrt{(30\,\mathrm{m})}} \approx 36\,400\,\mathrm{m}^{1/2} \tag{13.29}$$

As a result there is close matching of the estuary's resonance frequency with the normal tidal frequency, and large amplitude tidal motions of 10–14 m range occur.

In practice, estuaries and bays do not have the uniform dimensions implied in our calculations, and analysis is extremely complicated. It becomes necessary to model the conditions (i) in laboratory wave tanks using careful scaling techniques, and (ii) by theoretical analysis. One dominant consideration for tidal power installations is to discover how barriers and dams will affect the resonance enhancement. For the Severn estuary, some studies have concluded the barriers would reduce the tidal range and hence the power available: yet other studies have concluded the range will be increased! The

construction of tidal power schemes is too expensive to allow for mistakes to occur in understanding these effects.

13.4 Tidal current/stream power

Near coastlines and between islands, tides may produce strong water currents that can be considered for generating power. This may be called tidal-current, tidal-stream or tidal-flow power. The total power produced may not be large, but generation at competitive prices for export to a utility grid or for local consumption may be possible.

The theory of tidal stream power is similar to wind power, see Chapter 9. The advantages are (a) predictable velocities of the fluid and hence predictable power generation, and (b) water density 1000 times greater than air and hence smaller scale turbines. The main disadvantages are (a) small fluid velocity and (b) the intrinsically difficult marine environment.
The power density in the water current is, from (9.2),

$$q = \frac{\rho u^3}{2} \tag{13.30}$$

For a tidal or river current of velocity, for example, $3\,\mathrm{m\,s^{-1}}$,

$$q = \frac{(1025\,\mathrm{kg\,m^{-3}})(27\,\mathrm{m^3\,s^{-3}})}{2} = 13.8\,\mathrm{kW\,m^{-2}}$$

Only a fraction η of the power in the water current can be transferred to useful power and, as for wind, η will not exceed about 60%. In practice, η may approach a maximum of 40%.

Tidal current velocities vary with time approximately as

$$u = u_0 \sin(2\pi t/\tau) \tag{13.31}$$

where τ is the period of the natural tide, 12 h 25 min for a semidiurnal tide, and u_0 is the maximum speed of the current.

Generation of electrical power per unit cross section may therefore be on average (assuming 40% efficiency of tidal current power to electricity),

$$\bar{q} \approx \frac{0.4}{2}\rho u_0^3 \frac{\int_{t=0}^{t=\tau/4} \sin^3(2\pi t/\tau)\mathrm{d}t}{\int_{t=0}^{t/4} \mathrm{d}t} \tag{13.32}$$

$$= 0.2\rho u_0^3(\tau/3\pi)(4/\tau)$$

$$\approx 0.1\rho u_0^3 \tag{13.33}$$

for a device that could generate power in the *ebb* (out) and *flow* (in) tidal currents, and with a maximum current of $3\,\mathrm{m\,s^{-1}}$, $\bar{q} \sim 2.8\,\mathrm{kW\,m^{-2}}$. With a maximum current of $5\,\mathrm{m\,s^{-1}}$, which occurs in a very few inter-island

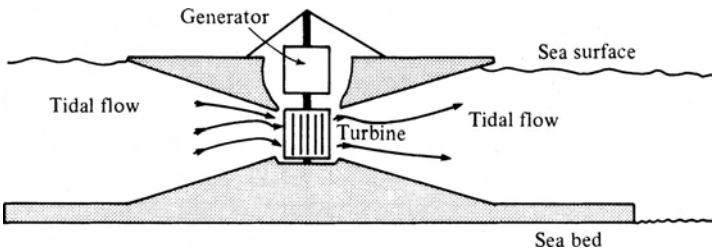

Figure 13.6 Tidal current power device.

channels, $\bar{q} \sim 14\,\mathrm{kW\,m^{-2}}$; if the intercepted area is a circle of area $100\,\mathrm{m}^2$ (radius 5.6 m), then the total average power generation would be 1.4 MW. (We may note that in most cases a 4 MW capacity wind turbine would be expected to have a similar average power production, but the rotor radius would be ~60 m, see Table 9.2.)

The periodic nature of the power generation would lead to complications, but we note that tidal flow power lags about $\pi/2$ behind range power from a single basin, so the two systems could be complementary.

Few modern tidal flow power devices have been developed, but Figure 13.6 shows the design of one device. Some prototype devices have very similar form to wind turbines; visit <www.itpower.co.uk/OceanEnergy.htm> and <www.marineturbines.com/projects.htm>. The capital cost per unit capacity depends on eventual commercial series manufacture. If tidal flow devices are to be used at all, the best opportunities are obviously where unusually fast tidal stream flows occur, where alternative sources are expensive and where ships can be excluded. The predictable nature of the generated power is an advantage, despite the tidal rhythms.

13.5 Tidal range power

13.5.1 Basic theory

The basic theory of tidal power, as distinct from the tides themselves, is quite simple. Consider water trapped at high tide in a basin, and allowed to run out through a turbine at low tide (Figure 13.7). The basin has a constant surface area A that remains covered in water at low tide. The trapped water, having a mass ρAR at a centre of gravity $R/2$ above the low tide level, is all assumed to run out at low tide. The potential maximum energy available per tide if all the water falls through $R/2$ is therefore (neglecting small changes in density from the sea water value, usually $\rho = 1025\,\mathrm{kg\,m^{-3}}$)

$$\text{energy per tide} = (\rho AR)\, g \frac{R}{2} \tag{13.34}$$

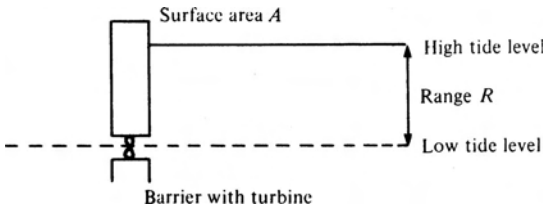

Figure 13.7 Power generation from tides.

If this energy is averaged over the tidal period τ, the average potential power for one tidal period becomes

$$\overline{P} = \frac{\rho A R^2 g}{2\tau} \tag{13.35}$$

The range varies through the month from a maximum R_s for the *spring* tides, to a minimum R_n for the *neap* tides. The envelope of this variation is sinusoidal, according to Figure 13.8, with a period of half the lunar month.

At any time t after a mean high tide within the lunar month of period T(=29.53 days), the range is given by

$$\frac{R}{2} = \left(\frac{R_s + R_n}{4}\right) + \left(\frac{R_s - R_n}{4}\right)\sin(4\pi t/T) \tag{13.36}$$

If

$$R_n = \alpha R_s \tag{13.37}$$

then the range is given by

$$R = \frac{R_s}{2}[(1+\alpha) + (1-\alpha)\sin(4\pi t/T)] \tag{13.38}$$

The power is obtained from the mean square range:

$$\overline{R^2} = \frac{R_s^2}{4}\frac{\int_{t=0}^{T}[(1+\alpha)+(1-\alpha)\sin(4\pi t/T)]^2 \mathrm{d}t}{\int_{t=0}^{T}\mathrm{d}t} \tag{13.39}$$

Hence

$$\overline{R^2} = \frac{R_s^2}{8}(3 + 2\alpha + 3\alpha^2) \tag{13.40}$$

The mean power produced over the month is

$$\overline{P}_{\text{month}} = \frac{\rho A g}{2\tau}\frac{R_s^2}{8}(3 + 2\alpha + 3\alpha^2) \tag{13.41}$$

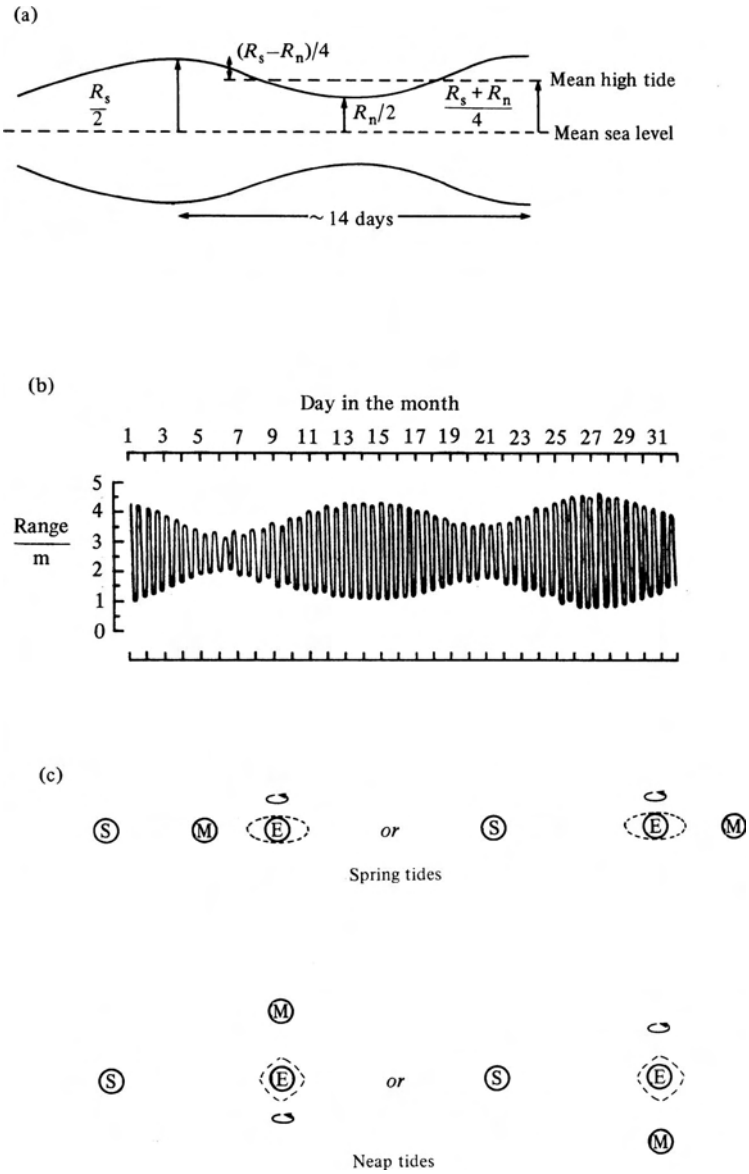

Figure 13.8 (a) Sinusoidal variation of tidal range. (b) Tidal range variation for one month (from Bernshtein 1965) for a regular semidiurnal tide. Large range at spring tides, small range at neap tides. (c) Positions of the Sun (S), Moon (M) and Earth (E) that produce spring and neap tides twice per month.

where $R_n = \alpha R_s$ and τ is the intertidal period.

Since $\alpha \sim 0.5$, (13.41) differs little from the approximations often used in the literature, i.e.

$$\overline{P} \approx \frac{\rho A g}{2\tau}(\overline{R})^2 \tag{13.42}$$

where $\overline{R}$ is the mean range of all tides, and

$$\overline{P} \approx \frac{\rho A g}{2\tau}\frac{(R_{max}^2 + R_{min}^2)}{2} \tag{13.43}$$

where R_{max} and R_{min} are the maximum and minimum ranges.

Example 13.2 Typical values of mean tidal power
If $R_s = 5\,\text{m}, R_n = 2.5\,\text{m}, \alpha = 0.5, \overline{R} = 3.7\,\text{m}, R_{max} = 5\,\text{m}, R_{min} = 2.5\,\text{m}, A = 10\,\text{km}^2, \rho = 1.03 \times 10^3\,\text{kg m}^{-3}$ and $\tau = 12$ h 25 min $= 4.47 \times 10^4$ s,

then

$$\begin{aligned} &(13.41)\text{yields} \quad \overline{P} = 16.6\,\text{MW} \\ &(13.42)\text{yields} \quad \overline{P} = 15.4\,\text{MW} \end{aligned} \tag{13.44}$$

and

$$(13.43)\text{yields} \quad \overline{P} = 16.1\,\text{MW}$$

13.5.2 Application

The maximum potential power of a tidal range system cannot be obtained in practice, although high efficiencies are possible. The complications are:

1 Power generation cannot be maintained near to low tide conditions and so some potential energy is not harnessed.
2 The turbines must operate at low head with large flow rates – a condition that is uncommon in conventional hydropower practice, but similar to 'run-of-the-river' hydropower. The French have most experience of such turbines, having developed low head, large flow bulb turbines for generation from rivers and the Rance tidal scheme. The turbines are least efficient at lowest head.

3 The electrical power is usually needed at a near constant rate, and so there is a constraint to generate at times of less than maximum head.

Efficiency can be improved if the turbines are operated as pumps at high tide to increase the head. Consider a system where the range is 5 m. Water lifted 1 m at high tide can be let out for generation at low tide when the head becomes 6 m. Even if the pumps and generators are 50% efficient there will be a net energy gain of ~200%; see Problem 13.5.

In Figure 13.7, note that power can be produced as water flows with both the incoming ('flow') and the outgoing ('ebb') tide. Thus a carefully optimised tidal power system that uses reversible turbines to generate at both ebb and flow, and where the turbine can operate as pumps to increase the head, can produce energy of 90% of the potential given by (13.41).

13.6 World range power sites

The total dissipation of energy by water tides in the Earth is estimated to be 3000 GW, including no more than about 1000 GW dissipated in shallow sea areas accessible for large civil engineering works. The sites of greatest potential throughout the world are indicated in Figure 13.9 and detailed in Table 13.1. They have a combined total potential of about 120 GW. This is about 12% of near-shoreline potential and 10% of the total world

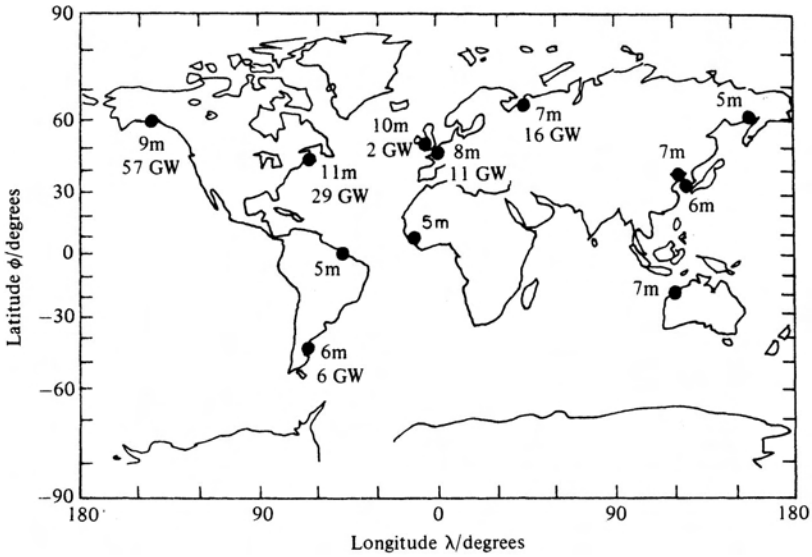

Figure 13.9 Location of major world tidal power sites, showing the average tidal power range and power potential. From Sørensen (2000) with permission of Elsevier.

Table 13.1 Major world tidal sites (authors' tabulation from various sources, including the classic tabulation of Hubbert, 1971)

Location	*Mean range*	*Basin area*	*Potential Mean power*	*Potential Annual prodn*	*Actual Installed capacity*	*Date commissioned*
	(*m*)	(km^2)	(*MW*)	($GWh\,y^{-1}$)	(*MW*)	
North America						
Passamaquoddy	5.5	262	1800	15 800	–	
Cobscook	5.5	106	722	6330	–	
Bay of Fundy	6.4	83	765	6710	17.8	1985
Minas-Cobequid	10.7	777	19 900	175 000	–	
Amherst Point	10.7	10	256	2250	–	
Shepody	9.8	117	520	22 100	–	
Cumberland	10.1	73	1680	14 700	–	
Petitcodiac	10.7	31	794	6960	–	
Memramcook	10.7	23	590	5170	–	
South America						
San Jose Argentina	5.9	750	5870	51 500	–	
UK						
Severn	9.8	70	1680	15 000	–	
Mersey	6.5	~7	130	1300		
Solway Firth	5.5	~60	1200	10 000		
Thames	4.2	~40	230	1400		
France						
Aber-Benoit	5.2	2.9	18	158	–	
Aber-Wrac'h	5	1.1	6	53	–	
Arguenon	8.4	28	446	3910	–	
Frenaye	7.4	12	148	1300	–	
La Rance	8.4	22	349	3060	240	1966
Rotheneuf	8	1.1	16	140	–	
Mont St Michel	8.4	610	9700	85 100	–	
Somme	6.5	49	466	4090	–	
Ireland						
Strangford Lough	3.6	125	350	3070	–	
Former Soviet Union						
Kislaya	2.4	2	2	22	0.4	1966
Lumbouskii Bay	4.2	70	277	2430	–	
WhiteSea	5.65	2000	14 400	126 000	–	
Mezen Estuary	6.6	140	370	12 000	–	
Australia						
Kimberley	6.4	600	630	5600	–	
China						
Baishakou	2.4				0.64	1978
Jiangxia	7.1	2			3.2	1980
Xinfuyang	4.5				1.3	1989
About five other small sites	~5				0.53	1961–76
total			~63 000	~570 000	~1000	

Note

* If no commissioning date indicated then only studies have been made at the site but no installation.

hydropower (river) potential. This is a significant power potential and of great potential importance for certain countries, e.g. the UK, where, in principle, about 25% of annual electricity could be generated by tidal power from known estuaries with enhanced tidal range. Unfortunately, as also indicated in Table 13.1, few tidal power stations have been constructed, mostly due to the large capital cost and small short-term financial gains.

13.7 Social and environmental aspects of tidal range power

Sites for tidal range power are chosen for their large tidal range; a characteristic that is associated with estuaries having large areas of mud flats exposed at lower tides. Tidal range power depends on the placing of a barrier for a height difference in water level across the turbines. In operation, (i) the level of water in the basin is always above the unperturbed low tide and always below the unperturbed high tide, (ii) the rates of flow of both the incoming and the outgoing tides are reduced in the basin, and (iii) sea waves are stopped at the barrier. These mechanical factors are the driving functions likely to cause the following effects:

1 The areas of exposed mud flats are reduced, so significantly reducing the food available for birds; usually including migratory birds habitually passing such special habitats. The change in flow, depth and sea waves can be expected to change many other ecological characteristics, many of which may be unique to particular sites.
2 Visual impact is changed, but with a barrier the only necessary construction.
3 River flow can be controlled to reduce flooding.
4 Access for boats to harbours in the basin is increased.
5 Controlled depth and flow of the basin allows leisure activities such as sailing.
6 The barrier can be used as a viaduct for transport and for placing other constructions, e.g. wind turbines.

Tidal barriers are large and expensive structures that may require years to construct. No power can be produced, and hence no income generated, until the last section of the barrier is complete. Difficulties in finance may lead to lack of environmental care. Although the installation at La Rance now features a flourishing natural ecosystem, it is noticeably different from that which was there before the dam, and took some years to establish itself. Therefore it has been observed that La Rance may not have been constructed if it had had to face today's environmental impact procedures.

A developer's main criterion for the success of a tidal power plant is the cost per unit kWh of the power produced. As with other capital-intensive

energy technologies, the economic cost per kWh generated can be reduced (i) if other advantages can be costed as benefit to the project, including carbon abatement, (ii) if interest rates of money borrowed to finance the high capital cost are small, and (iii) if the output power can be used to decrease consumption of expensive fuels such as oil. (See Chapter 17 for a more general discussion of these issues.) With such economic and environmental complexity, large-scale (~1000 MW) tidal power plants may not be best. Smaller schemes may perhaps be more economic.

Problems

13.1 a In Figure 13.1, consider the lunar-related force F_Z on a mass m of seawater along the Earth's radius EZ. Since $D \gg r$, show that $F_Z = mr\omega^2$.

b Hence show that the difference in the lunar-related force on this mass between high and low tide is the tide raising force

$$F_t = F_X - F_Z = 2MmGr/D^3$$

c The tide raising force must equal the difference in the Earth's gravitational attraction on m between low and high tide. Hence show that the tidal range R is 0.36 m and is given by

$$R = \frac{Mr^4}{M'D^3}$$

13.2 The sidereal month T^* is defined after (13.11). The synodic month T_m is defined as the average period between two new Moons as seen by an observer on Earth. T_m is greater than T^* because of the motion of the Earth and the Moon together about the Sun that effectively 'delays' the appearance of the new Moon. What is the relation between T^* and T_m?

13.3 a The Earth's oceans have an average depth of 4400 m. Show that the speed of a naturally propagating tidal wave is about 200 m s^{-1}(750 km h^{-1}).

b Compare this speed with the speed of the lunar tidal force passing round the Earth's equator.

c What is the effect of the difference between these two speeds?

13.4 A typical ocean on the Earth's surface has a depth of 4400 m.

a What is the speed of a freely travelling tidal wave for this depth?

b At this speed, how long would it take the tidal wave to circle the Earth?

c If the tidal wave is started by the influence of the Moon, can its motion be reinforced continually as the Earth rotates?

13.5 Water is pumped rapidly from the ocean at high tide to give an increased water level in a tidal power basin of 1.0 m. If the tidal range is 5.0 m and if the pump/generator system is only 50% efficient, show that the extra energy gained can be nearly twice the energy needed for pumping.

13.6 The *capacity factor* of a power system is the ratio of energy produced to the energy that would have been produced if the system ran continuously at its rated (maximum) power. By integrating over a day (i.e. two full tides), calculate the capacity factor of a tidal range station. (Assume that the range $R = \beta R_{\max}$ with $0 < \beta < 1$ and that the rated power is the maximum output on a day when $\beta = 1$.)

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