RENSSELAER POLYTECHNIC INSTITUTE
TROY, NY
MAKEUP EXAM NO. 3 INTRODUCTION TO ENGINEERING ANALYSIS
(ENGR-1100) – Fall 13

NAME: **Solution** Section: ___________

RIN: ______________________________

Wednesday, November 20, 2013
5:00 – 6:50

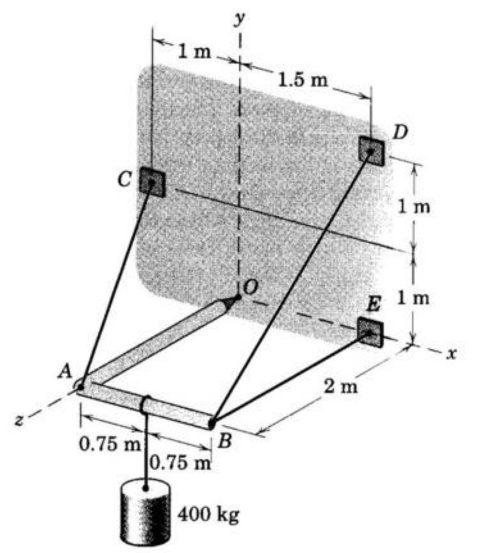
Please state clearly all assumptions made in order for full credit to be given.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Problem #1 (25)

The right-angle boom that supports the 400-kg cylinder is supported by three cables and a ball-and-socket joint at *O* attached to the vertical *x-y* plane. Neglect the weight of the boom.

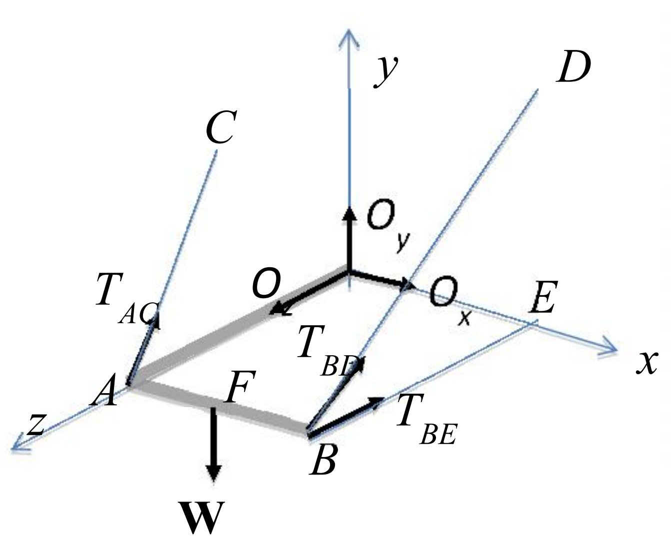
(a) Draw complete and separate free-body-diagram for the boom. (4)
(b) Express all forces in vector form. (6)
(c) Write equilibrium equations for the boom that include all detailed force and moment terms (12)
(d) Determine reactions at *O* and the magnitude of cable tensions (3)



Note: You need to show your work to receive credit.

Solution

(a) Each force 0.5 point, coordinate system 0.5 point, totally 4 points.



(b) $\mathbf{O} = O_x\mathbf{i} + O_y\mathbf{j} + O_z\mathbf{k}$ N (1.5, each component 0.5)

$\mathbf{T}_{AC} = T_{AC}\frac{-\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{(-1)^2+1^2+(-2)^2}} = \frac{T_{AC}}{\sqrt{6}}(-\mathbf{i}+\mathbf{j}-2\mathbf{k})$ N (1.5, each component 0.5)

$\mathbf{T}_{BD} = T_{BD}\frac{2\mathbf{j}-2\mathbf{k}}{\sqrt{2^2+(-2)^2}} = \frac{T_{BD}}{\sqrt{2}}(\mathbf{j}-\mathbf{k})$ N (1.5, each component 0.5)

$\mathbf{T}_{BE} = -T_{BE}\mathbf{k}$ N (1.0)

$\mathbf{W} = -3920\mathbf{j}$ N (0.5)

(c)

$$\sum \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T}_{AC} + \mathbf{r}_{OF} \times \mathbf{W} + \mathbf{r}_{OB} \times (\mathbf{T}_{BD} + \mathbf{T}_{BE}) = 0$$

$$\mathbf{r}_{OA} \times \mathbf{T}_{AC} = \frac{T_{AC}}{\sqrt{6}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ -1 & 1 & -2 \end{vmatrix} = -\frac{T_{AC}}{\sqrt{6}} 2 \ (\mathbf{i} + \mathbf{j}) \quad \text{N-m} \quad \text{(2 points)}$$

$$\mathbf{r}_{OF} \times \mathbf{W} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 0 & 2 \\ 0 & -3920 & 0 \end{vmatrix} = 7840\mathbf{i} - 2940\mathbf{k} \quad \text{N-m} \quad \text{(2 points)}$$

$$\mathbf{r}_{OB} \times (\mathbf{T}_{BD} + \mathbf{T}_{BE}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 0 & 2 \\ 0 & \frac{T_{BD}}{\sqrt{2}} & -\left(T_{BE} + \frac{T_{BD}}{\sqrt{2}}\right) \end{vmatrix}$$

$$= -\sqrt{2}T_{BD}\mathbf{i} + 1.5\left(T_{BE} + \frac{T_{BD}}{\sqrt{2}}\right)\mathbf{j} + 1.5\frac{T_{BD}}{\sqrt{2}}\mathbf{k} \quad \text{N-m} \quad \text{(2 points)}$$

$$\sum M_{Ox} = -2\frac{T_{AC}}{\sqrt{6}} - \sqrt{2}T_{BD} + 7840 = 0 \quad \text{Eq 1} \quad \text{(1 point)}$$

$$\sum M_{Oy} = -2\frac{T_{AC}}{\sqrt{6}} + 1.5\left(T_{BE} + \frac{T_{BD}}{\sqrt{2}}\right) = 0 \quad \text{Eq(2)} \quad \text{(1 point)}$$

$$\sum M_{Oz} = 1.5\frac{T_{BD}}{\sqrt{2}} - 2940 = 0 \quad \text{Eq (3)(1 point)}$$

$$\mathbf{R} = \mathbf{O} + \mathbf{T}_{AC} + \mathbf{T}_{BD} + \mathbf{T}_{BE} + \mathbf{W} = 0$$

$$\sum F_x = O_x - \frac{T_{AC}}{\sqrt{6}} = 0 \quad \text{Eq(4) (1 point)}$$

$$\sum F_y = O_y + \frac{T_{AC}}{\sqrt{6}} + \frac{T_{BD}}{\sqrt{2}} - 3920 = 0 \quad \text{Eq(5) (1 point)}$$

$$\sum F_z = O_z - 2\frac{T_{AC}}{\sqrt{6}} - \frac{T_{BD}}{\sqrt{2}} - T_{BE} = 0 \quad \text{Eq(6)} \quad \text{(1 point)}$$

(a) from Eq(3), T_{BD} = 2771 N
from Eq(1), T_{AC} = 4802 N
from Eq(2), T_{BE} = 654 N
from Eq(4), O_x = 1960 N
from Eq(5), O_y = 0 N
from Eq(6), O_z = 6534 N

each of the above: 0.5 point.

Problem #2 (25)

There is a distributed load across a 10’ long weightless beam supported at the left end by a smooth pin and at the right end by a smooth roller. The weight on the beam per linear foot [lb/ft] is characterized by a function as follows:

$0 < x < 2$	0
$2 < x < 6$	$(x - 2)^3$
$6 < x < 10$	64

Where x = 0 corresponds to the left end of the beam.

What is the centroid of the weight distribution along the x-axis? (12)
What are the support reactions at x=0’ and x=10’? (8)
(Show all work to maximize credit).

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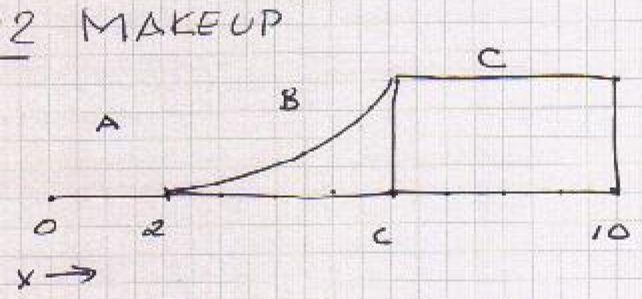
Q2 MAKEUP



1. Convert A, B, C to distributed loads

A: 0

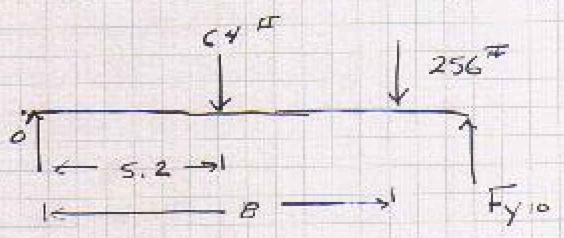
B: $\int_0^4 x^3\,dx = 4^4/4 = 64$

$\int_0^4 x^4\,dx = 4^5/5 = 204.8$

distance to centroid of B: $\frac{204.8}{64} = 3.2$

$x = 3.2 + 2 = 5.2$

C : $4 * 64 = 256^{\#}$ acting at $x = 8$



$\Sigma M_0 = (64 * 5.2) + (256 * 8) - (F_{y10} * 10)$

$F_{y10} = \frac{332.8 + 2048}{10} = \boxed{238.08}$

$F_{0y} + F_{10y} = 64 + 256$

$F_{0y} = 320 - 238.08 = \boxed{81.92}$

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Centroid of Weight Distribution

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$$

Region	A_i	$x_i A_i$
A	0	0 = 0
B	64	64 * 5.2 = 332.8
C	256	256 * 8 = 2048
	320	2380.8

$$\bar{x} = \frac{2380.8}{320} = 7.44$$

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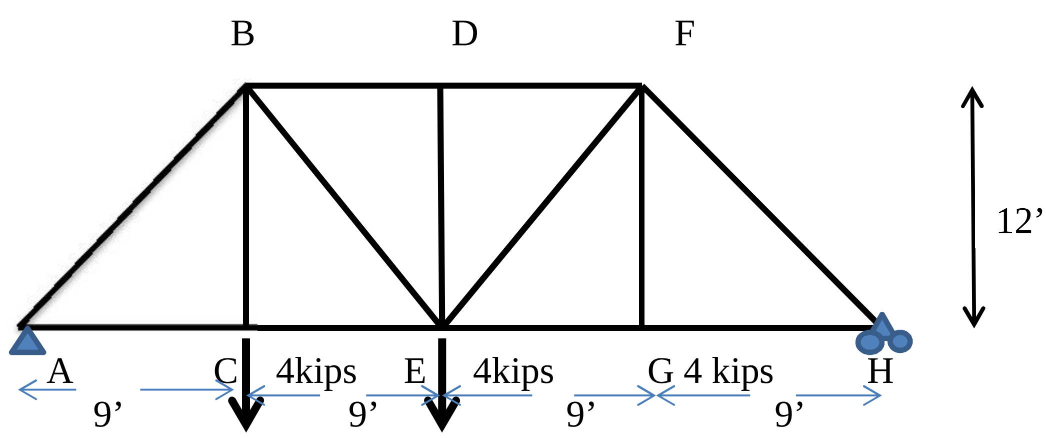
Problem #3 (25)

Determine the force in each of the 6 left-side members (including DE) of the Pratt truss illustrated below. Determine if they are in compression (C) or tension (T)

Support reaction (y-direction) at A	(2)
Support reaction (x-direction) at A	(2)
AB	(3)
AC	(3)
BC	(3)
BE	(3)
BD	(3)
CE	(3)
DE	(3)



You must show all steps and required FBD’s. You may use any method you like (Joints or Sections) in your solution.

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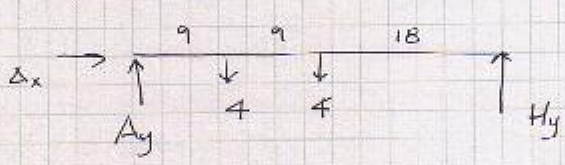
FBD of ENTIRE TRUSS



$\Sigma F_X = 0$ $\quad A_x = 0$

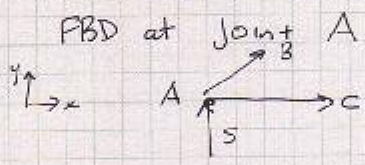
$\Sigma F_y = 0$ $\quad A_y + H_y = 8$

$\Sigma M_A = 0$ $\quad (4 \cdot 9) + (4 \cdot 18) - H_y(36) = 0$

$\frac{36 + 72}{36} = H_y$ $\quad \therefore H_y = 5^k$

$A_y = 3^k$

FBD at Joint A



$\Sigma F_x = 0$ $\quad AC + 9/15\ ~~BC~~\ AB = 0$

$AC = -9/15\ ~~BC~~\ AB$

$\Sigma F_y = 0$ $\quad 5 + \frac{12}{15}\ ~~BC~~\ AB = 0$

$AB\ ~~BC~~ = -6.25K$ $\quad$ ~~sign~~ direction is wrong

[AB 6.25 C]

$AC = -\frac{9}{15}(-6.25) =$ **[3.75 T]**

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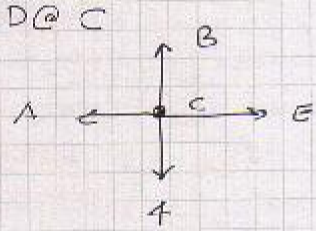
FBD@ C

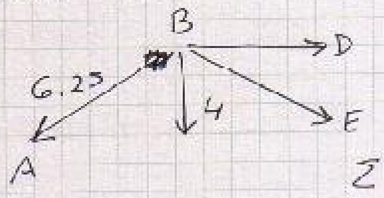


$\Sigma F_x = 0$

$CE = AC =$ **3.75 (T)**

$\Sigma F_y = 0$ **BC = 4 (T)**

FBD@ B



$\Sigma F_x = 0$

$\frac{9}{15} 6.25 + BD + 9/15 (BE) = 0$

$\Sigma F_y = 0$

$4 + \frac{12}{15} 6.25 + \frac{12}{15} BE = 0$

$4 + 5 + .8 BE = 0$

$\frac{9 = -11.25 BE}{.8}$

sign wrong

BE = 11.25 T

$\Sigma F_x = 0 \quad \frac{9}{15}(6.25) + BD + \frac{9}{15}(11.25) = 0$

$3.75 + BD + 6.75 = 0$

$BD = -10.5$

sign wrong

BD 10.5 T

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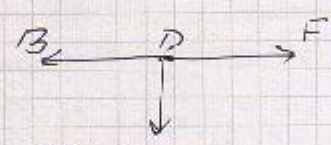
FBD AT D



BD

$\Sigma F_x = 0$

DE IS A ZERO FORCE MEMBER

$\boxed{DE = 0}$

Problem #4 (25)

Consider the three equations

$$x_1 + 2x_2 + x_3 = 2$$
$$3x_1 + 8x_2 + x_3 = 12$$
$$4x_2 + x_3 = 2$$

(a) Write the equation Ax=b in parametric matrix form where A is the associated coefficient matrix and b is the solution matrix to the equations.
(b) Given the associated coefficient matrix A, find its determinant using cofactor expansion along column #1.
(c) Determine A^{-1}, the inverse of A, using only row operations.
(d) Utilizing A^{-1}, solve the system of equations, for the variables x_1, x_2, and x_3.
Show all work.

a)

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix}$$

b) $c_{1,1} = [8x1\text{-}1x4] = 4$

$c_{1,2} = -[2x1\text{-}1x4] = 2$

$c_{1,3} = -[2\,x1\text{-}1\,x\,8] = \text{-}6$

$\det(A) = 1\,x\,4 + 3\,x\,2 + 0x\,6 = 10$

c)

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 8 & 1 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{bmatrix} \text{Subtract } 3R_1 \text{ from } R_2$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & -2 & -3 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{bmatrix} \text{Divide } R_2 \text{ by } 2$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -3/2 & 1/2 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{bmatrix} \text{Subtract 2 R2 from R1, and 4 R2 from R3}$$

$$\begin{bmatrix} 1 & 0 & 3 & 4 & -1 & 0 \\ 0 & 1 & -1 & -3/2 & 1/2 & 0 \\ 0 & 0 & 5 & 6 & -2 & 1 \end{bmatrix} \text{Divide } R_3 \text{ by } 5$$

$$\begin{bmatrix} 1 & 0 & 3 & 4 & -1 & 0 \\ 0 & 1 & -1 & -3/2 & 1/2 & 0 \\ 0 & 0 & 1 & 6/5 & -2/5 & 1/5 \end{bmatrix} \text{Subtract } 3R_3 \text{ from } R_1 \text{ and Add } R_3 \text{ to R2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2/5 & 1/5 & -3/5 \\ 0 & 1 & 0 & -3/10 & 1/10 & 1/5 \\ 0 & 0 & 1 & 6/5 & -2/5 & 1/5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2/5 & 1/5 & -3/5 \\ -3/10 & 1/10 & 1/5 \\ 6/5 & -2/5 & 1/5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2/5 & 1/5 & -3/5 \\ -3/10 & 1/10 & 1/5 \\ 6/5 & -2/5 & 1/5 \end{bmatrix} \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix}$$

$$x = 0.4 \text{ x } 2 + 0.2 \text{ x } 12 - 0.6 \text{ x } 2 = -0.8 + 2.4 - 1.2 = 2$$

$$y = -0.3 \text{ x } 2 + 0.1 \text{ x } 12 + 0.2 \text{ x } 2 = -0.6 + 1.2 + 0.4 = 1$$

$$z = 1.2 \text{ x } 2 - 0.4 \text{ x } 12 + 0.2 \text{ x } 2 = 2.4 - 4.8 + 0.4 = -2$$