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...RACT Research on this project has centered on the Phi-transform and wavelets and their application to various problems in image/signal processing, differential equations, and computer aided design. Construction of Wavelets - While the Phi-transform and wavelets have been developed in a quite general setting, several applications, most notably in image processing and numerical methods for differential equations, have pointed to several deficiencies in the traditional wavelet constructions. This has led the researchers to consider alternate constructions of wavelets and related questions Partial Differential Equations - Concentration has been on two of the most prominent problems in partial differential equations: nonlinear equations such as conservation laws which arise in the study of wave propagation and elliptic problems on nonsmooth domains which arise in scattering theory and many aspects of computer aided design. Image Processing - The researchers have investigated algorithms for image and surface compression based on wavelet decompositions. The algorithms developed have been shown by the researchers to be optimal methods for compression in the sense of width or optimal recovery. Computer Aided Design - They have continued their development of the second generation of the CAD system SLIP They now have a prototype running which allows them to reconstruct a 3D CAD description from photographic images of a manufactured part. Collaboration With Industry - They continue their collaboration with various industries, government laboratories, and university centers. Industrial Mathematics Initiative - The goal of IMI is to enact the rapid transfer of advanced mathematics and numerical analysis into the industrial sector.

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Final Report

Applications of the Phi–transform

Principal investigators: R. DeVore/B. Jawerth

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1. Research progress.

Research on this project has centered on the Phi-transform and wavelets and their application to various problems in image/signal processing, differential equations, and computer aided design.

Construction of wavelets. While the Phi-transform and wavelets have been developed in a quite general setting, several applications, most notably in image processing and numerical methods for differential equations, have pointed to several deficiencies in the traditional wavelet constructions. This has led us to consider alternate constructions of wavlelets and related questions.

In a series of papers [6,7,8], C. de Boor, R. DeVore and A. Ron study the structure of shift invariant spaces which play a prominent role in wavelet constructions, multigrid methods, applications of radial basis functions, and various aspects of approximation (spaces of univariate and multivariate splines often have this property). A space $\mathcal{S}$ of functions on $\mathbb{R}^d$ is shift-invariant if $f \in \mathcal{S}$ implies $f(\cdot + j)$ is in $\mathcal{S}$ for every multi-integer $j \in \mathbb{Z}^d$. These works characterize the structure of shift-invariant spaces and utilize this structure to settle several questions in approximation and the construction of wavelets.

It is shown for example how the closed shift-invariant subspaces of $L_2(\mathbb{R}^d)$ can be characterized by their Fourier transforms and how to describe the $L_2(\mathbb{R}^d)$ projectors onto finitely generated shift-invariant spaces. This characterization can then be used to solve several other problems. For example, for an arbitrary closed shift invariant subspace $\mathcal{S}$ of $L_2(\mathbb{R}^d)$, it is characterized when the dilates $\mathcal{S}_h$ of $\mathcal{S}$ provide $L_2(\mathbb{R}^d)$-approximation order $O(h^k)$ for all functions functions f in the Sobolev space $W^k(L_2(\mathbb{R}^d))$. This extends and completes earlier investigations of this type of approximation inititiated by Strang and Fix, in that no assumptions are made on $\mathcal{S}$. Applications to wavelets extend and complete earlier investigations of Micchelli, Chui and Wang, Jia and Micchelli, Riemenschneider and Shen, Lorentz and Madych, Chui, Stöckler, and Ward, and others. The main contribution is that all a priori conditions of growth and smoothness on the generating functions in wavelet constructions can be eliminated.

There are several obstacles to the application of wavelets for problems on a finite domain caused by the spill over (outside the domain) of the supports of these wavelets. This is one of the obstacles in applying wavelets to partial differential equations and to the use of wavelets in segmentation procedures for image processing. Recent work of the PI's and their collaborators have given wavelet construction which eliminate some of these difficulties. In [28] A. Cohen, I. Daubechies, B. Jawerth, and P. Vial give a construction of wavelets on the unit interval [0,1]. The construction is very simple and robust, and leads to fast algorithms. In [2] L. Andersson, N. Hall, B. Jawerth, and G. Peters extend these results in different ways. We consider arbitrary intervals, which is important for image analysis and PDEs, and recursive generation of wavelets through general orthonormal sets. We also show how these constructions lead to smoothness preserving extension theorems, important for PDE's and CAD, and how they can be used to construct wavelets satisfying boundary conditions. The latter has been an open problem for some time. In [5], Andersson, Dahlberg, and Jawerth give similar constructions for general closed sets (including sets with Lipschitz boundary). The closed set wavelets and related ideas are ideal for parallel processing and this is exploited by Huntsberger, Jawerth, Metlapalli and Peters in [22].

Our study of nonlinear multivariate wave propogation [12] described below has pointed to several deficiencies in the usual construction and application of wavelets in several space dimensions. In particular we observed that the usual wavelet bases are deficient in being able to resolve multivariate shocks because their dyadic scaling is the same in each coordinate direction. To circumvent these drawbacks, we have introduced hyperbolic wavelets in [26] which allow for variable scaling in coordinate directions. The theoretical properties of these nonisotropic wavelet bases were developed in [26] including their use in linear approximation and nonlinear compression. We are currently investigating application of these bases to image processing.

While the usual wavelet basis and the hyperbolic basis are two contrasting alternatives for wavelet decompositions, there is actually a plethora of wavelet bases which mitigate between these two extremes. In this direction, we have investigated in [27], the adaptive selection of bases with variable coordinate scaling for applications to image processing described below.

Partial Differential Equations. We have concentrated on two of the most prominent problems in partial differential equations: nonlinear equations such as conservation laws which arise in the study of wave propogation and elliptic problems on nonsmooth domains which arise in scattering theory and many aspects of computer aided design.

Ron DeVore and Brad Lucier have studied regularity theorems and numerical methods for solving nonlinear conservation laws. The solution to such equations develop discontinuities (so called "shocks") even for C^∞ initial conditions. Their earlier results showed that the Besov spaces $B^\alpha_\tau(L_\tau)$, $\tau := (\alpha+1)^{-1}$ are regularity spaces for scalar conservation laws in one space dimension. In [12], they characterize all Besov spaces which are regularity spaces for such conservation laws. They consist of the above Besov spaces and certain other Besov spaces with $\alpha < 1$. These results can also be appliedto multivariate scalar conservation laws. They show that none of the Besov spaces, with $\alpha > 1$, can be regularity spaces for conservation laws in several space dimensions. The importance of these results is that it indicates that conservation laws in several space dimensions must involve non-classical smoothness spaces. An indication of the structure of these non-classical spaces is also given.

Dahlberg and Jawerth and their collaborators have also studied numerical methods for the fast solution of elliptic PDE's on domains. In [29] we have established that the Galerkin method can be used for solving integral equations corresponding to classical elliptic PDE's on nonsmooth domains. In fact, we have shown that the mesh size can be chosen in an adaptive way controlled by local curvature properties of certain approximate domains. We have also constructed finite element spaces based on adaptive constructions of wavelets. These constructions were precursors to those that lead to wavelets on closed sets and can now be improved.

Image processing. Ron DeVore, Björn Jawerth, and Brad Lucier [13], [14] have investigated algorithms for image and surface compression based on wavelet decompositions. The algorithms developed in these papers have been shown by R. DeVore, G. Kyriazis, D. Leviatan, and V. Tikhomirov [16] to be optimal methods for compression in the sense of widths or optimal recovery. R. DeVore and B. Lucier [17] have derived new algorithms for noise removal based on wavelet decompositions. The algorithms for noise removal can be shown to be optimal in the sense of nondeterministic optimal recovery. These results compliment parallel investigations of Donoho and Johnstone on wavelet shrinkage.

In [23], L. Hsiao, B. Jawerth, B. Lucier, and X. Yu have shown optimal compression algorithms for spaces with a Littlewood-Paley description, such as L^p, Sobolev spaces, BMO, certain Besov spaces, etc. These results improve and extend some in [14]. In this paper, optimal algorithms for compression of matrices are also established. These results have been implemented numerically by two graduate students and a systematic comparison with the results of Beylkin-Coifman-Rokhlin, *The fast wavelet transform*, has been carried out (the results are now part of the students' theses).

In [25] B. Jawerth, A. Laine, and A. Tirumalai have developed a fast technique for magnifying and reducing images based on wavelet methods. In contrast to the standard method, so called pixel replication, this method does not introduce edge artifacts. In addition, it commutes with wavelet compression in the sense that the magnification can be done first and then the compression or vice versa.

We are developing fast algorithms in image processing for their application to digital mammography and the early detection of breast cancer. These include fast algorithms for compression of mammographic images [27] which do not lose any of the essential features of the mammogram; fast methods for noise removal; and adaptive algorithms for feature classification including microcalcification clusters, marginated masses, and architectural distortions.

Computer aided design We have continued our development of the second generation of the CAD system SLIP. We now have a prototype running which allows us to reconstruct a 3D CAD description from photographic images of a manufactured part.

2. Collaboration with industry. We are continuing our collaboration with various industries, government laboratories, and university centers.

We are continuing our close collaboration with Volvo, Sweden. This collaboration involves CAD, optimal design and tool design. We are also working with Metimur and Volvo concerning computer representation, and manufacturing, of industrial objects from their photographic images. One of the goals with this project is to allow the use of photographic images for quality control and verification of specifications of parts supplied by subcontractors.

A collaboration with several of the companies within McDonnell-Douglas, St. Louis, has also started. The collaboration involves PDE solvers, image compression, ATR, and transient analysis. The companies involved are McAir, MD Missile SC , and MD Research

Laboratories.

Collaraboration is also ongoing with Magnavox on video compression and corresponding hardware design.

We have also had initial discussions with United Technologies, concerning PDE solvers, manufacturing and CAD, and with KSR concerning video compression.

We collaborate with Hercules Corporation on the develoment of wavelet based algorithms for Automated Target Detection. This research has centered around preprocessing using wavelet decompositions to be followed by a nonlinear neural network for the detection and classification.

With the University of South Florida Cancer Institute we collaborate on the applications of our fast image processing algorithms in medical imagery including the early detection of breast cancer through digitized mammography.

Our close collaboration with SMTC is also continuing. SMTC is a national center at USC. Its objective is to provide small and midsize manufacturing companies in the Southeastern region with access to start of the art CAD/CAM technology. Currently, about 400 companies are associated with SMTC. Our work with SMTC concerns wavelet compression methods, fracture mechanics, CAD, and quality control through contact free measuring techniques.

The activities which were initiated by this AFOSR grant are also continuing as part of a project funded by a new DARPA grant for Surface modelling and Fast PDE solvers (PIs B. Dahlberg and B. Jawerth). This new project involves SMTC as well as the Computer Science and Mechanical Engineering Departments.

The Industrial Mathematics Initiative. Within the Department of Mathematics at USC, we have established a semi-autonomous unit, "The Industrial Mathematics Initiative" (IMI). The goal of IMI is to enact the rapid transfer of advanced mathematics and numerical analysis into the industrial sector. We have recently also been awarded an NSF EPSCoR grant in support of IMI for projects related to the work on this AFOSR contract.

RESULTING PUBLICATIONS ACKNOWLEDGING SUPPORT

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2. L. Andersson, N. Hall, B. Jawerth, and G. Peters, *Wavelets on closed subsets of the real line*, submitted.
3. B. Dahlberg, and B. Jawerth, and G. Peters, *Wavelets on closed sets*, in preparation.
4. R. Andersson and B. Dahlberg, *Interactive techniques for visual design*, to appear.
5. R. Andersson, B. Dahlberg, and B. Jawerth, *Algorithms for surface generation*, in preparation.
6. C. de Boor, R. DeVore, and A. Ron, *Approximation from shift-invariant subspaces of* $L_2(\mathbf{R}^d)$, preprint.
7. C. de Boor, R. DeVore, and A. Ron, *Wavelets and prewavelets*, preprint.
8. C. de Boor, R. DeVore, and A. Ron, *The structure of finitely generated shift-invariant spaces in* $L_2(\mathbf{R}^d)$, preprint.
9. B. Dahlberg and B. Jawerth, *The collocation method for the boundary integral method in nonsmooth domains*, in preparation.
10. W. Dahmen, R. DeVore, and C. Micchelli, *Monotonicity preserving extensions*, submitted.
11. A. Deliu and B. Jawerth, *Geometrical dimension versus smoothness*, Constructive Approximation Theory 1992, 211-222.
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13. R. DeVore, B. Jawerth, and B. Lucier, *Surface compression*, Computer Aided Geometric Design **9** (1992), 219-239.
14. R. DeVore, B. Jawerth, and B. Lucier, *Image compression through transform coding*, IEEE Proceedings on Information Theory **36** (1992), 719-746.
15. R. DeVore, P. Petrushev, and X. Yu, *Wavelet approximation in C*, submitted.
16. R. DeVore, G. Kyriazis, D. Leviatan, and V. Tikhomirov, *Wavelet compression and widths of Besov spaces*, in preparation.
17. R. DeVore and B. Lucier, *Optimal methods for compression and noise removal*, in preparation.
18. M. Frazier, B. Jawerth, and G. Weiss, *Littlewood-Paley theory and the study of function spaces*, monograph in the CBMS-AMS Regional Conference Series (1991).

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21. M. Hilton, T. Huntsberger and B. Jawerth, *Local image enhancement of compressed images*, to appear.
22. T. Huntsberger, B. Jawerth, K. Metlapalli and G. Peters, *Wavelet probing and parallel processing*, in preparation.
23. B. Jawerth, L. Hsiao, B. Lucier and X. Yu, *Near optimal compression of orthonormal wavelet expansions*, to appear.
24. B. Jawerth and M. Milman, *Wavelets and best approximation in Besov spaces*, Israel MCP **5** (1992), 107-112.
25. B. Jawerth, A. Laine, and A. Tirumalai, *A multiscale approach to magnification*, preprint.
25. R. DeVore, S. Konjagin, P.Petrushev, and V. Temlyakov, *Hyperbolic waqvelets*, preprint.
26. R. DeVore, B. Lucier, E. Moskona, and P. Petrushev, *Adaptive wavelet bases with variable coordinate scaling*, preprint.
27. (with B. Lucier, M. Kallergi, Wei Qian, R. Clark, E. Saff, and L.P. Clarke), *Wavelet compression and segmentation of mammographic images*, submitted to Journal of Digital Imaging.
28. A. Cohen, I. Daubechies, B. Jawerth, P. Vial, *Wavelets on [0,1] and Fast algorithms*, Comptes Rendus **312** (1992), 32-42.

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