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INTERMEDIATE-FREQUENCY GAIN STABILIZATION WITH INVERSE FEEDBACK

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ABSTRACT

Increased gain stability and gain-bandwidth product result from the use of inverse feedback in an i-f amplifier. Improvement in gain stability is related to the number of cascaded stages, the stage gain, and the magnitude of the feedback. A circuit is described which uses feedback over a pair of cascaded stages. Generalized selectivity curves for this feedback couple are shown, and the design procedure is outlined. A description of an experimental amplifier concludes the paper.

INTERMEDIATE-FREQUENCY GAIN STABILIZATION WITH INVERSE FEEDBACK

INTRODUCTION

Increased gain stability and gain-bandwidth product result from the use of inverse feedback in an i-f amplifier. In addition, the response curve of the amplifier may be designed to have a flatter top and steeper skirts than the curve for an amplifier of cascaded, synchronous, single-tuned stages without feedback. The improvement in flatness and gain-bandwidth product has been described by previous investigators for the case of feedback over a single stage.[1,2] This paper analyses a method using feedback over pairs of stages and presents experimental confirmation of the design procedure.

Formulas introduced in the text are derived in detail in the appendix, where a complete list of symbol definitions appears.

STABILITY RELATIONS

The voltage gain of an amplifier of n identical cascaded stages with no feedback is

$$G_1 = \mu^n$$

where G_1 is the overall gain, and μ is the gain of a single stage. For good gain stability, it is desirable that the derivative

$$\frac{dG_1}{d\mu} = n\mu^{n-1} \qquad (1)$$

be as small as possible. Ordinarily, μ may be expected to vary with changes in tube transconductance due to power supply variations and tube aging, and with variations in constants of the interstage coupling networks. The derivative given in (1) represents the worst possible case, that of a particular variation in μ occurring simultaneously in all stages.

Now consider an amplifier of a identical cascaded stages in which feedback is applied over groups of *m* stages. Then

$$G_2 = \left[\frac{\mu^m}{1+\beta\mu^m}\right]^{\frac{a}{m}} \tag{2}$$

where G_2 is the overall gain, and β is the gain of the feedback network.[3] In order to compare the feedback amplifier with the zero feedback amplifier, we let $G_2 = G_1$ and find that

$$\frac{dG_2}{d\mu} = h\,\frac{dG_1}{d\mu} \tag{3}$$

where

$$h = \frac{a}{n}\left[\frac{G_2^{\frac{1}{a}}}{\mu}\right]^m \tag{4}$$

Again, for good gain stability, $\frac{dG_2}{d\mu}$ should be small. For maximum stability, *h* should be a minimum. If *a*, and consequently β, is increased to obtain this minimum, it is found that

$$G_2 = \epsilon^{\frac{a}{m}} \tag{5}$$

and

$$h = \frac{m\epsilon}{\mu^m}\log\mu \tag{6}$$

where $\epsilon = 2.718$, and *log* indicates the logarithm to this base. Obviously, maximum stability has been achieved at the cost of stage gain, each *m* group having a gain of ϵ. The maximum stability conditions demonstrate the possibility of increasing feedback beyond the point where increased stability results. A given overall gain may be maintained with increasing feedback and stability by increasing the number of stages, but stability is not improved by increasing feedback beyond the value.

$$\beta = \frac{1}{\epsilon} - \frac{1}{\mu^m} \tag{7}$$

Practically, because of the many stages required, there will be few cases where feedback of this magnitude is desirable.

If G_2 and *a* in equation (4) are both constant, it is more advantageous to increase feedback by increasing m rather then β, since $\frac{G^{\frac{1}{a}}}{\mu} < 1$. In most cases, m will not be greater than three.

The preceding argument does not consider bandwidth; the gain equations refer to center-frequency gains only..

PARTICULAR METHOD

Figure (1) is the schematic diagram of a cascaded pair of radio-frequency stages with inverse feedback. Power supply connections are omitted for simplicity. Symbols necessary to the design of this feedback couple are defined in the following list:

- A = center-frequency voltage gain of couple with zero feedback.
- $1+B$ = feedback factor.
- C_1 = plate load capacitance in farads.
- C_2 = feedback capacitance in farads.
- C_3 = divider capacitance in farads.
- C_4 = divider capacitance in farads.
- f_o = center frequency in cycles per second.
- Δf = bandwidth in cycles per second.
- G = voltage gain of feedback couple.
- G_o = center-frequency voltage gain of feedback couple
- g = normalized gain
- P = design factor.
- Q_1 = Q of tuned circuit consisting of shunt-connected L_1, C_1, R_1.
- Q_2 = Q of tuned circuit consisting of shunt-connected L_2, C_2, R_2.
- R_o = cathode bias resistance in ohms.
- R_1 = effective plate load resistance.
- R_2 = effective feedback resistance.
- S = tube transconductance in ohms.
- ω_o = $2\pi f_o$ radians per second.
- $\Delta\omega$ = $2\pi\Delta f$ radians per second.
- ρ = step-down ratio of tuned output circuit.

Normalized gain curves for several values of feedback are given in Figure

(2), where g is plotted against $Q_1\frac{\Delta f}{f_o}$. The curve labeled B=O is the normalized response of the couple with zero feedback. Note that the curves for appreciable feedback display flatter tops and much greater skirt attenuation than the curve for zero feedback.

The improvement in gain-bandwidth product varies with the degree of feedback. For a single-tuned stage with zero feedback, the gain-bandwidth product is

$$\Pi_o = \frac{S}{2\pi C_1} \tag{8}$$

For the feedback couple, the gain-bandwidth product, per stage, is

$$\Pi_B \simeq 1.7\,\Pi_o \tag{9}$$

in the more useful range of feedback values.

Generally, the feedback factor 1+B should be chosen as large as possible in order to obtain maximum stability. The improvement in center-frequency gain stability over a zero-feedback amplifier can be calculated directly from (4).

For example, if $a = 4$, $n = 2$, $G_o = 10^4$, $\mu = 10^2$, $h = 2(0.1)^2 = .02$, or an amplifier consisting of two feedback couples with the given constants has one-fiftieth the gain variation of a similar two-stage amplifier with no feedback.

In the appendix, it is shown that the design may be proportioned to give peaked responses at the extremes of the pass band. Curves for several normalized peaked responses are given in figure (3).

A feedback couple may be designed using formulas developed in the appendix and information given in the curves. The design procedure follows:

Given Δf, fo, S, and G_o

a. Choose value of 1+B. In Figure (2) or (3), find $Q_1\frac{\Delta f}{f_o}$ for $g = \frac{1}{\sqrt{2}}$.

b. Calculate Q_1. If Q_1 is impractically large, choose a smaller value of 1+B.

c. Calculate $C_1 = \frac{S Q_1}{\omega_o \sqrt{G_o}\,(1+B)}$.

d. Calculate $R_1 = \frac{Q_1}{\omega_o C_1}$.

e. In Figure (4), find P and A/ρ.

f. Calculate $A = (1+B)G_o$, and ρ.

g. Choose R_o from tube data.

h. Calculate $R_2 = (\frac{A}{\rho B} - 1)R_o$.

i. Verify $\rho^2 R_2 >> R_1$. If this is not true, choose a smaller 1+B and re-design.

j. Calculate $Q_2 = \frac{AP}{\rho B} Q_1$.

k. Calculate $C_2 = \frac{Q_2}{\omega_o R_2}$.

l. Calculate $C_3 = \frac{\rho C_1}{\rho - 1}$.

$$C_4 = \rho C_1 .$$

EXPERIMENTAL RESULTS

The electrical arrangement represented by Figure (1) must be duplicated in practice as closely as possible if results are to match the predictions of the design. Several practices, noted as critical during the course of the experimental work, should be followed:

1. Ensure adequate by-passing of the "ground" ends of the tuned plate circuits. This is especially important in the tapped output circuit. The design formulas are based on an output impedance at the tap which is usually a few ohms, and it does not take much reactance in the by-pass capacitor to modify the output impedance considerably.

2. By-pass the screen grid of the first stage directly to the cathode. The formulas for R_o and C_1 will not be correct if the screen is by-passed to ground.

3. Install interstage shielding and power lead decoupling as in an ordinary zero feedback amplifier. Because of the greater gain-bandwidth product, the gain per stage will be even larger than for a zero feedback amplifier. A small amount of regeneration may work mischief with a carefully calculated design.

Tuning the amplifier is greatly facilitated by providing a switch to break the feedback line to the cathode of the first stage. The amplifier plate circuits are peaked in the normal fashion. The feedback is then switched in, and the feedback tuned circuit is adjusted for maximum response at the center frequency. If a slight asymmetry of the response develops, it is usually possible to minimize it by detuning the feedback circuit. A large asymmetry indicates a design error, regeneration, or inadequate by-passing. A useful final adjustment is the value of R_o. A bandwidth which is too large can be decreased by decreasing R_o. If the bandwidth is too small, R_o should be increased.

In Figure (5) is shown the gain characteristic of a feedback couple using two 6SK7 tubes with the following design values: Δf = 30 Kc/s, f_o = 450 Kc/s, S = 2000 micromhos, and G_o = 100. The normalized gain curve for 1+B = 10, taken from Figure (2), is superimposed for comparison.

The overall gain and bandwidth agree well with the given values. The skirts are not quite as narrow as the normalized gain curve predicts. The lack of agreement is due to approximations that were made in the analysis. In particular, it was assumed that $\rho^2 R_2 >> R_1$, and that $|\frac{\bar{A}}{\rho}| >> |SZ_2|$. The latter inequality will not hold as well for frequencies far removed from resonance as for the center frequency. To the extent that the approximations are not achieved in practice, the skirts may be expected to deviate from the calculated values by small amounts.

The normalized gain curves plotted in Figures (2) and (3) are also approximate in that the quantity μ is considered equal to $\frac{\Delta f}{f_o}$, as explained in the appendix. This approximation was chosen because it allows the normalized response curves to be plotted as symmetrical characteristics, facilitating reading of bandwidth values. The approximation fails for bandwidths which are a large fraction of the center frequency, and in such cases it is advisable to plot curves with μ equal to its exact value.

In Figure (6) is shown the variation in gain of the experimental couple with plate supply voltage. A similar curve is shown for the same amplifier with zero feedback.

References

1. S. N. Van Voorhis, "Microwave Receivers", McGraw-Hill, 1948. pp. 88-91, 175-178.

2. E. H. B. Bartelink, J. Kahnke, R. L. Watters, "A Flat-Response Single-Tuned I. F. Amplifier", Proc. I. R. E., Vol. 36, April 1948, p. 474.

3. F. E. Terman, "Radio Engineering", McGraw-Hill, 1937. p. 248.

APPENDIX

Complete list of symbols

A = center frequency voltage gain of zero feedback couple.

$\bar{A}$ = complex voltage gain of zero feedback couple.

a = number of stages in feedback amplifier.

B = design factor.

1+B = feedback factor.

C_1 = plate load capacitance in farads.

C_2 = feedback capacitance in farads.

C_3 = divider capacitance in farads.

C_4 = divider capacitance in farads.

D,E,F, = design parameters.

f = frequency in cycles per second.

f_o = center frequency in cycles per second.

Δf = bandwidth in cycles per second.

G = voltage gain of feedback couple.

G_o = center frequency voltage gain of feedback couple.

$\bar{G}$ = complex voltage gain of feedback couple.

G_1 = voltage gain of zero feedback amplifier.

G_2 = voltage gain of feedback amplifier.

g = normalized gain.

h = instability reduction factor.

L_1 = plate load inductance in henries.

L_2 = feedback inductance in henries.

m = number of stages in each feedback loop.

n = number of stages in zero feedback amplifier.

P = design factor

Q_1 = Q of tuned circuit consisting of shunt-connected L_1, C_1, R_1.

Q_2 = Q of tuned circuit consisting of shunt-connected L_2, C_2, R_2.

R_o = cathode-bias resistance in ohms.

R_1 = effective plate load resistance in ohms.

R_2 = effective feedback resistance in ohms.

S = tube transconductance in mhos.

u = $\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}$

x = Q_1 u.

x_c = value of x corresponding to g = ½.

Z = parallel-tuned circuit impedance in ohms.

Z_2 = feedback tuned circuit impedance in ohms.

β = voltage gain of feedback network

ϵ = natural logarithmic base, 2.718.

μ = voltage gain of single stage.

ω = $2\pi f$ radians per second.

ω_o = $2\pi f_o$.

$\Delta\omega$ = $2\pi\Delta f$.

Π_o = zero feedback gain-bandwidth product.

Π_s = feedback gain-bandwidth product.

ρ = step-down ratio of tuned output circuit.

Stability Equations

From the formula for gain in a feedback amplifier[3], we find

$$G_2 = \left[\frac{\mu^m}{1+\beta\mu^m}\right]^{\frac{a}{m}} \tag{2}$$

and

$$\frac{dG_2}{d\mu} = h\,\frac{dG_1}{d\mu} \tag{3}$$

where

$$h = \frac{a}{n}\left[\frac{G_2^{\frac{1}{a}}}{\mu}\right]^m \tag{4}$$

Now

$$\frac{dh}{da} = \frac{1}{n}\left[1 - \frac{m}{a}\log G_2\right]\left[\frac{G_2^{\frac{1}{a}}}{\mu}\right]^m$$

If

$\frac{dh}{da} = 0$, then

$$\log G_2 = \frac{a}{m}$$

$$G_2 = \epsilon^{\frac{a}{m}} \tag{5}$$

and

$$h = \frac{a\epsilon}{n\mu^m}$$

But, since we have allowed $G_2 = G_1 = \mu^n$,

$$\log G_2 = n \log \mu = \frac{a}{m}$$

$$\frac{a}{n} = m \log \mu$$

and

$$h = \frac{m\epsilon}{\mu^m}\log \mu \tag{6}$$

In addition,

$$\frac{\mu^m}{1+\beta\mu^m} = \epsilon$$

so that

$$\beta = \frac{1}{\epsilon} - \frac{1}{\mu^m} \tag{7}$$

Feedback couple analysis

The complex impedance of a two-pole formed by shunt connecting inductance L, capacitance C, and resistance R is

$$Z = \frac{R}{1+jQu}$$

where $Q = \frac{R}{\omega_o L}$, and $u = \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \simeq \frac{\Delta\omega}{\omega_o}$

In the circuit of Figure (1),

$$\bar{G} = \frac{\bar{A}}{1 + \left(\frac{\bar{A}}{\rho} + SZ_2\right)\left(\frac{R_o}{R_o + Z_2}\right)}$$

where it is assumed that $\rho^2 R_2 >> R_1$ and

$$\bar{A} = \frac{A}{(1+jQ_1u)^2}$$

If it is also assumed that $\left|\frac{\bar{A}}{\rho}\right| >> |SZ_2|$, then

$$\bar{G} = \frac{\bar{A}}{1 + \frac{\bar{A}}{\rho}\left(\frac{R_o}{R_o + Z_2}\right)}$$

Substition of the expression for $\bar{A}$ in $\bar{G}$, and expansion,

gives $$\bar{G} = \frac{1 + jPx}{1+B - (1+2P)x^2 + j\left[2 + \frac{AP}{\rho} + P(1 - x^2)\right] x}$$

where $$B = \frac{A}{\rho\left(1 + \frac{R_2}{R_o}\right)}$$

$$P = \frac{\rho B Q_2}{A Q_1}$$

and $$Q_1 u = x$$

Now
$$g = G\left(\frac{1+B}{A}\right) = \left[\frac{1+P^2x^2}{\left(1-\frac{1+2P}{1+B}x^2\right)^2 + \left(\frac{2+\frac{AP}{\rho}+P}{1+B} - \frac{F}{1+B}x^2\right)^2 x^2}\right]^{1/2}$$

Let
$$D = \frac{1+2P}{1+B},\quad E = \frac{2+\frac{AP}{\rho}+P}{1+B},\quad F = \frac{P}{1+B}.$$
Then

$$g^2 = \frac{1+P^2x^2}{1+(E^2-2D)x^2+(D^2-2EF)x^4+F^2x^6}$$

The relative values of the coefficients in the above equation determine the shape of the generalized response.

It can be seen by inspection that, for maximum flatness with no inflection,

$$E^2 - 2D = P^2,\quad D^2 - 2EF = 0$$

These two conditions give

$$\frac{A}{\rho} = \frac{1}{2P^2} + 1$$

and
$$1+B = \frac{1+2P}{P^2}\left[\sqrt{1+(1/2+P)^2} - 1\right]$$

Plots of these two functions are given in Figure (4). The equation for g now becomes

$$g = \left[\frac{1}{1+\frac{P^2x^6}{(1+B)^2\,(1+P^2x^2)}}\right]^{1/2}$$

Plots of this function are given in Figure (2). For a slightly peaked response with one inflection,

$$E^2 - 2D = P^2 \;,\; D^2 - 2\,EF = \frac{1}{5(1+B)^2}$$

These two conditions give

$$\frac{A}{\rho} = \frac{3}{5P^2} + 1$$

and

$$1 + B = \frac{1 + 2P}{P^2}\left[\sqrt{1.1 + (\tfrac{1}{2} + P)^2 + \frac{0.0025}{(\tfrac{1}{2} + P)^2}} - 1\right]$$

Plots of these two functions are given in Figure (4). The equation for slightly peaked g now becomes

$$g = \left[\frac{1}{1 + \dfrac{P^2x^6 - \dfrac{x^4}{5}}{(1+B)^2(1+P^2x^2)}}\right]^{1/2}$$

Plots of this function are given in Figure (3).

It should be noted that responses with almost any degree of peaking are available with a different choice of the coefficients D,E,F. For any particular set of these coefficients, it is necessary to derive new expressions for $\frac{A}{\rho}$, $1 + B$, and g in terms of P. Responses may be obtained which give three peaks to the usual i-f amplifier selectivity curve, or two inflections in the expression for g.

Design formulas

$$A = G_o\,(1 + B) = (SR_1)^2$$

$$G_o\,(1 + B) = \frac{S^2Q_1^{\,2}}{\omega_o^{\,2}C_1^{\,2}}$$

$$C_1 = \frac{SQ_1}{\omega_o \sqrt{G_o (1 + B)}}$$

By definition,

$$B = \frac{\frac{A}{\rho}}{1 + \frac{R_2}{R_o}} \qquad \text{(c)}$$

$$R_2 = \left(\frac{A}{\rho B} - 1\right) R_o \qquad \text{(h)}$$

$$\rho = \frac{C_4}{C_1}$$

$$\frac{1}{C_1} = \frac{1}{C_3} + \frac{1}{C_4}$$

$$C_3 = \frac{\rho C_1}{\rho - 1} \qquad \text{(I)}$$

$$C_4 = \rho C_1$$

Gain-bandwidth product

For a single-tuned stage with zero feedback,

$$\mu = \frac{SQ_1}{\omega_o C_1} = \frac{S}{\Delta\omega C_1}$$

$$\Pi_o = \mu \Delta f = \frac{S}{2\pi C_1} \qquad \text{(8)}$$

For the smooth response feedback couple,

$$G_o = \frac{A}{1 + B} = \frac{(SQ_1)^2}{(\omega_o C_1)^2 \ (1 + B)}$$

$$G_o = \frac{(Sx_c)^2}{(\Delta\omega C_1)^2\ (1+B)}$$

where x_c is the value of x at $g = ½$

$$\Pi_B = \sqrt{G_o}\,\Delta f = \frac{Sx_c}{2\pi C_1 \sqrt{1+B}}$$

then $$\Pi_B = \Pi_o \frac{x_c}{\sqrt{1+B}}$$

Reference to Figure (2) results in the following table:

$1 + B$	$\frac{x_c}{\sqrt{1+B}}$
10	1.58
100	1.75
1000	1.77

For most useful values of $1 + B$, $\frac{x_c}{\sqrt{1+B}}$ is thus approximately 1.7 , or

$$\Pi_B \simeq 1.7\ \Pi_o \qquad (9)$$

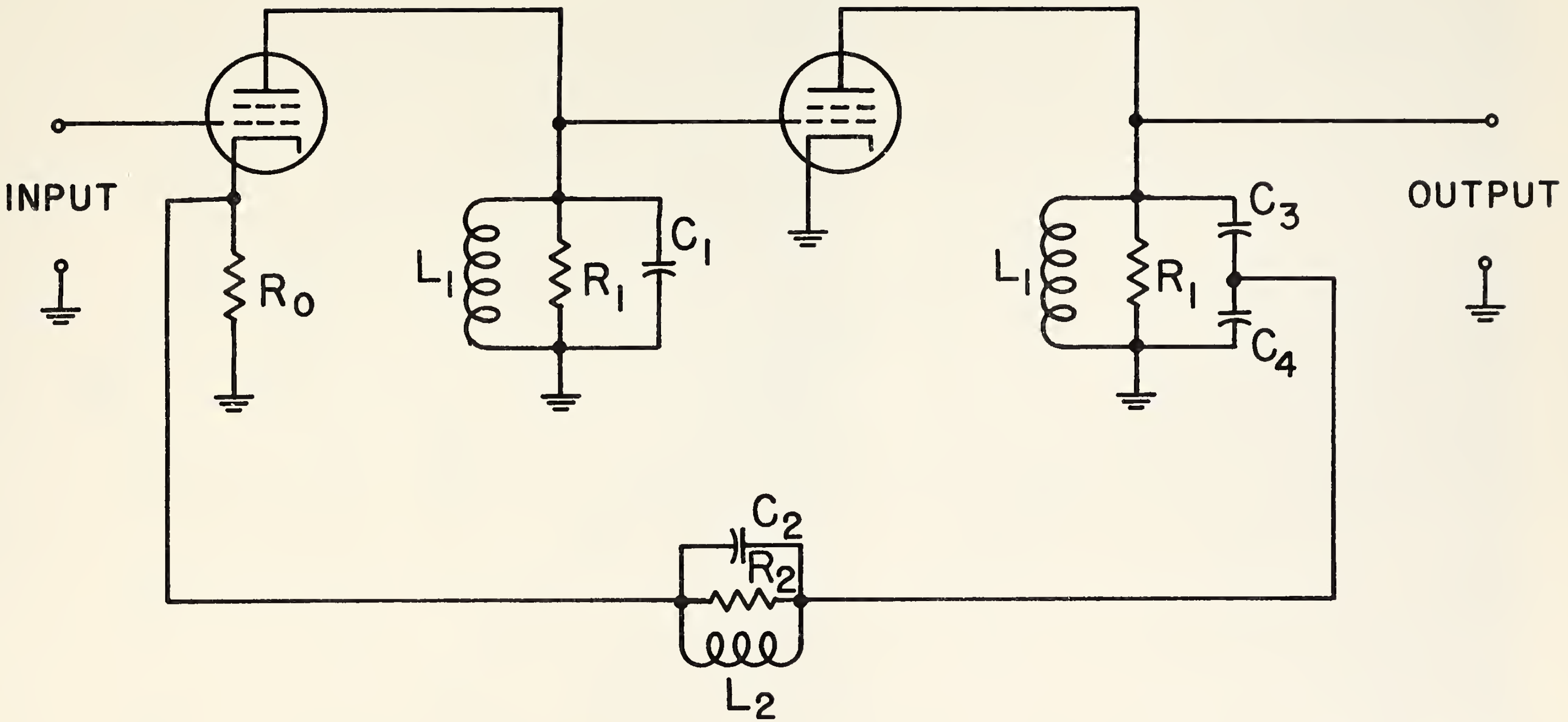

FIG. 1. SCHEMATIC DIAGRAM OF FEEDBACK COUPLE.

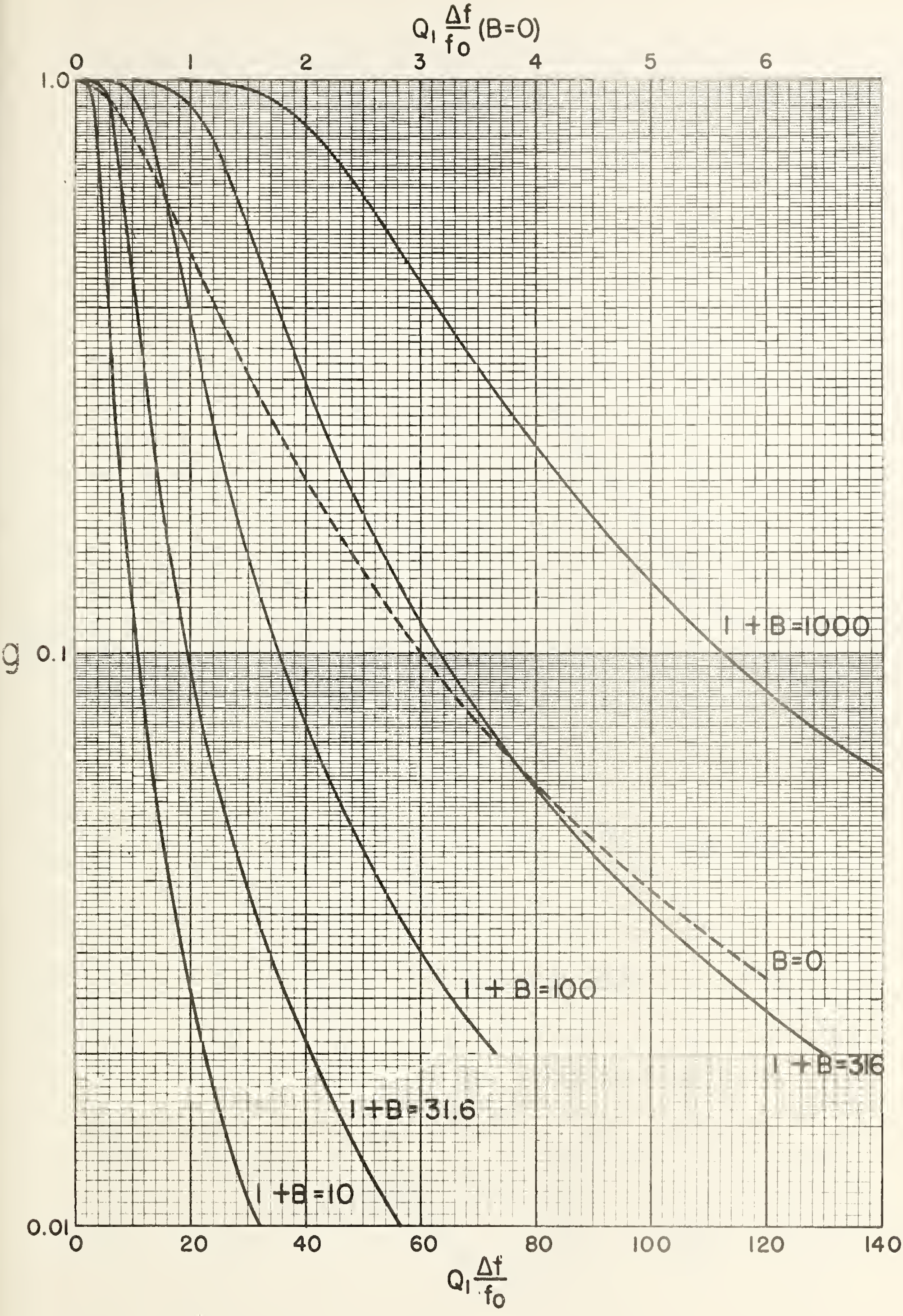

FIG. 2. NORMALIZED GAIN FOR SMOOTH RESPONSE

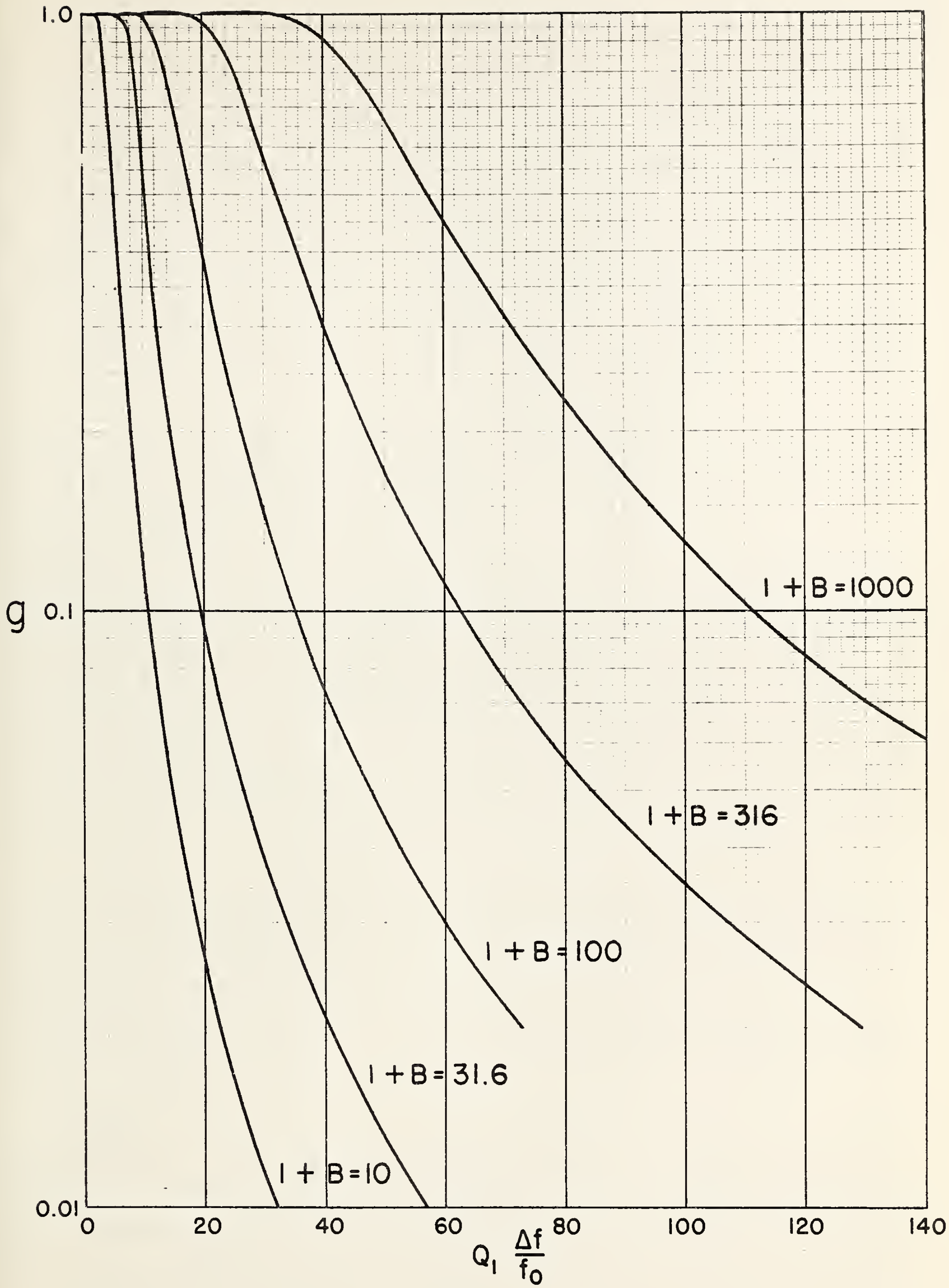

FIG. 3. NORMALIZED GAIN FOR PEAKED RESPONSE

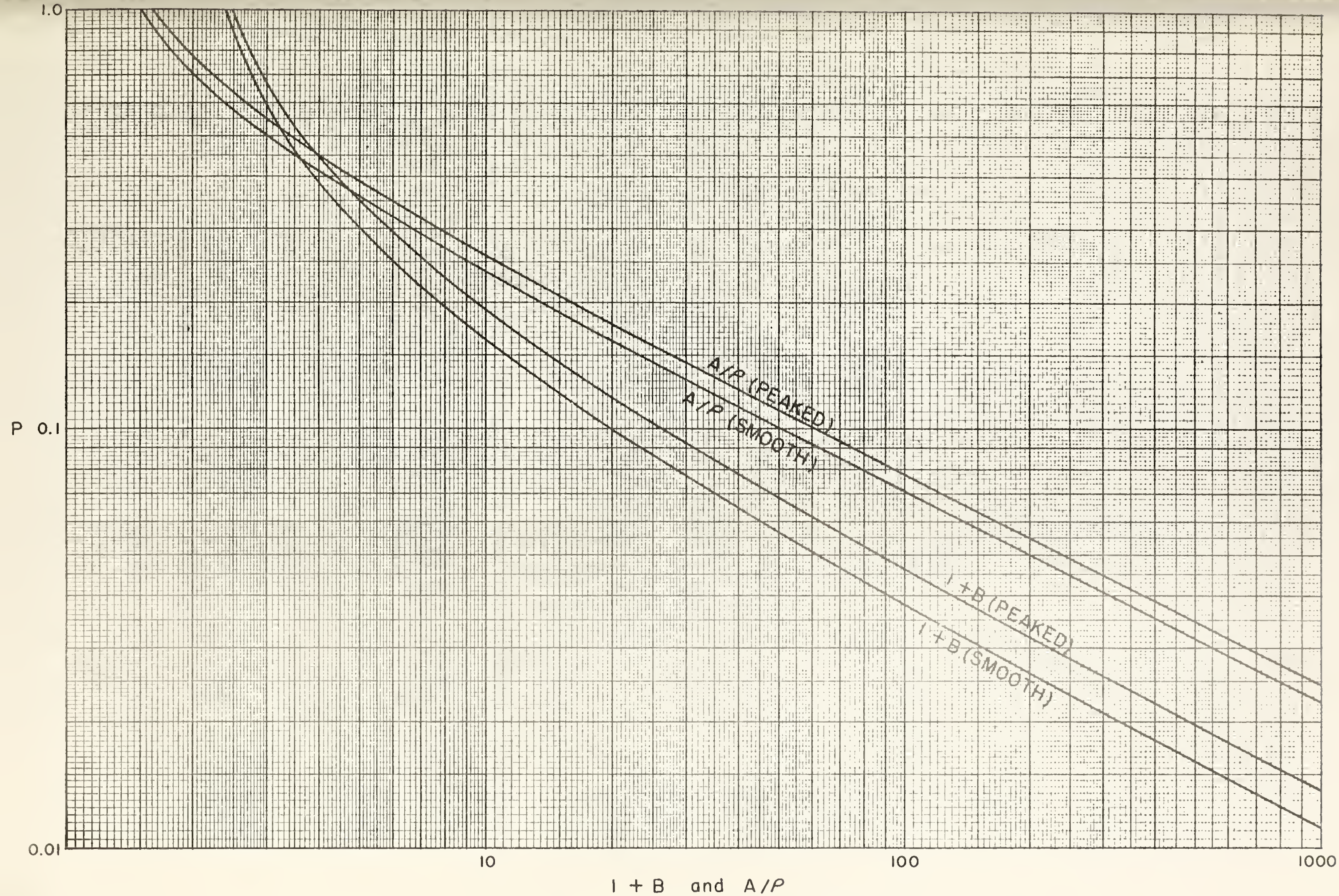

FIG. 4. DESIGN FACTOR CHART

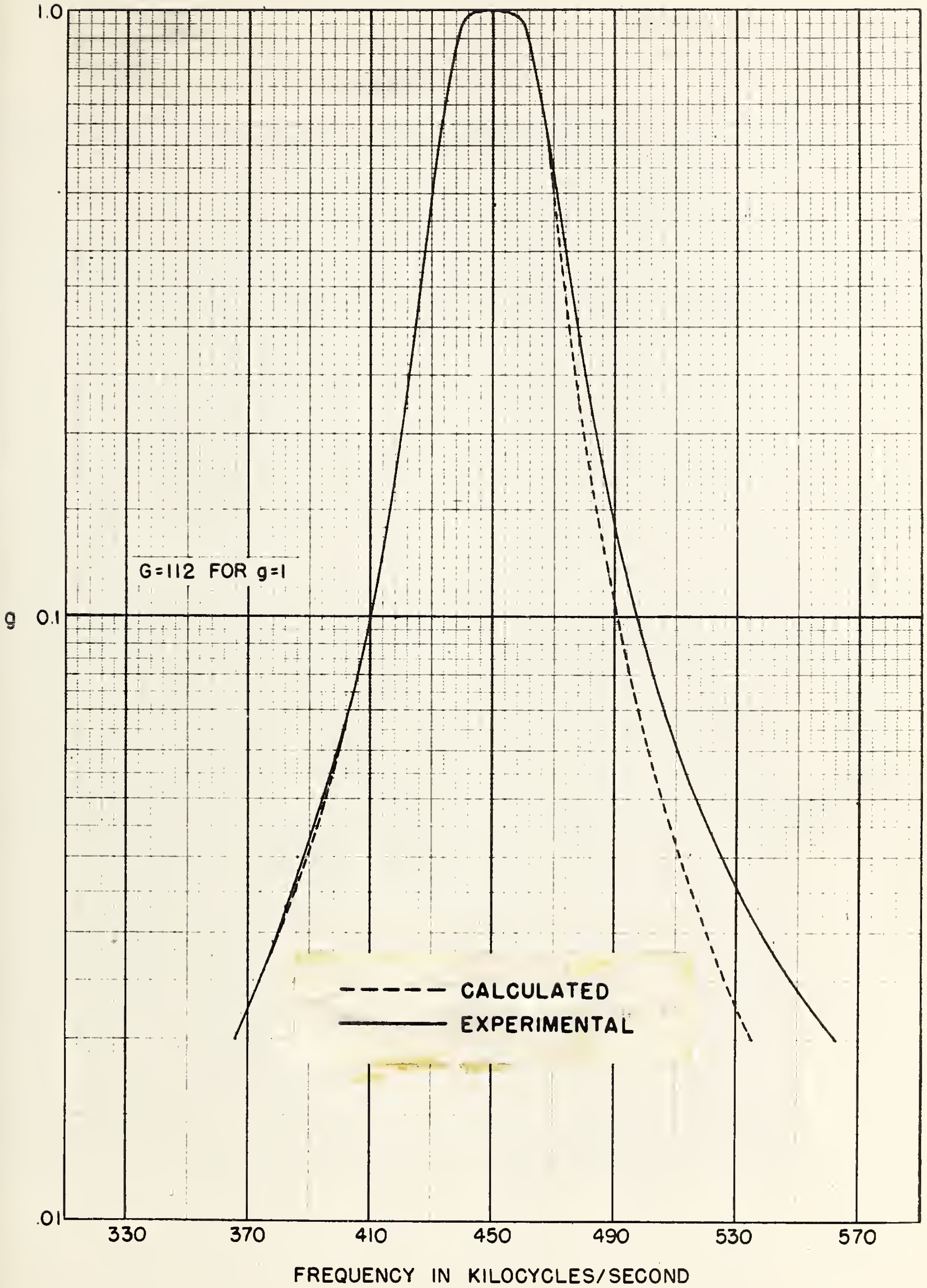

Fig.5. RESPONSE OF EXPERIMENTAL AMPLIFIER

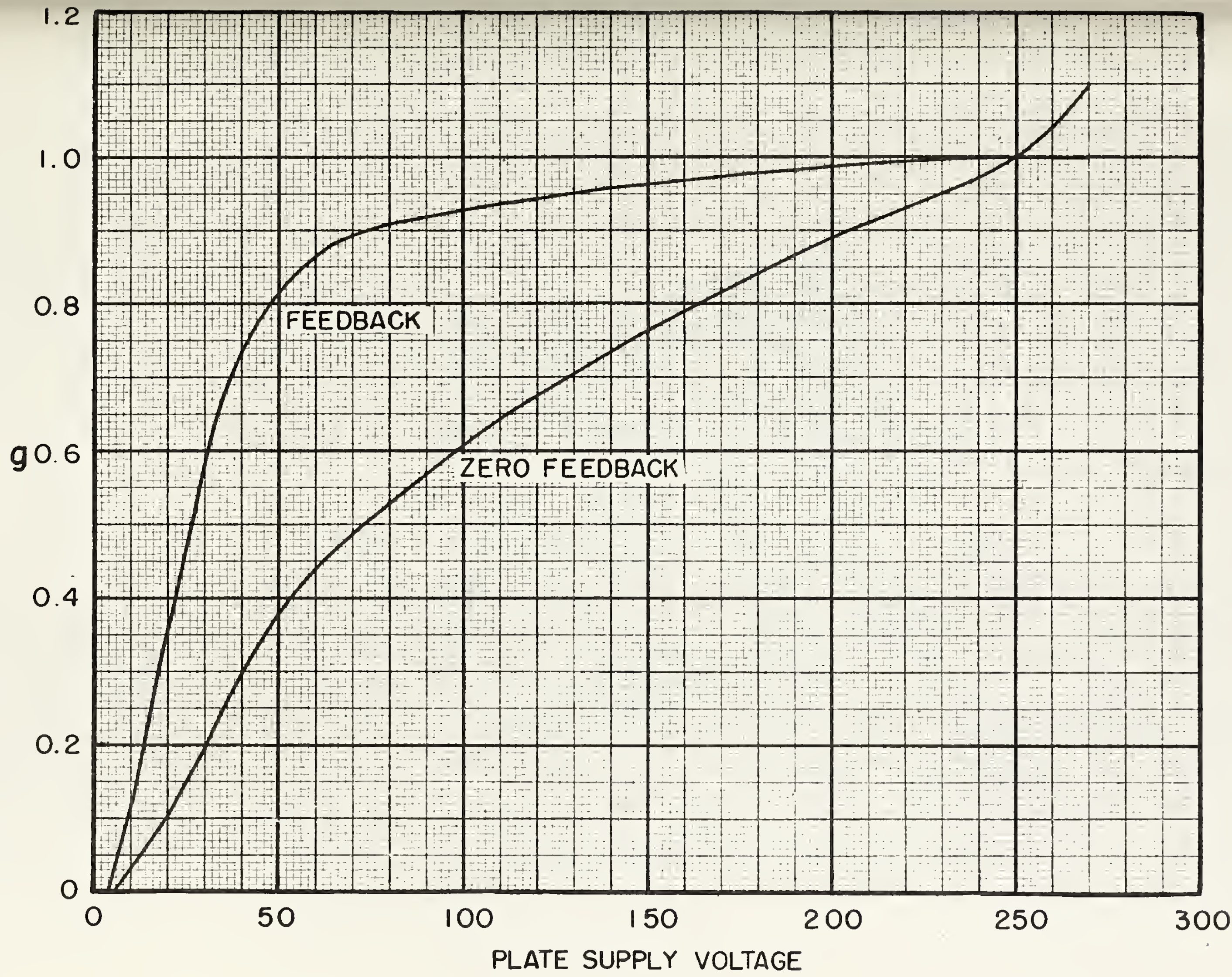

FIG. 6. GAIN VARIATION WITH SUPPLY VOLTAGE