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Research Article

An Optimization Model for Inventory System and the Algorithm for the Optimal Inventory Costs Based on Supply-Demand Balance

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In order to investigate the inventory optimization of circulation enterprises, demand analysis was carried out firstly considering supply-demand balance. Then, it was assumed that the demand process complied with mutually independent compound Poisson process. Based on this assumption, an optimization model for inventory control of circulation enterprises was established with the goal of minimizing the average total costs in unit time of inventory system. In addition, the optimal computing algorithm for inventory costs was presented. Meanwhile, taking the agricultural enterprises in Aksu, Xinjiang, China, for example, the researchers conducted numerical simulation and sensitivity analysis. Through constantly adjusting and modifying the parameters values in model, the optimal stock and the optimal inventory costs were obtained. Therein, the numerical results showed that the uncertainty of lead time greatly influenced the optimal inventory strategy. Besides, it was demonstrated that the research results provided a valuable reference for the agricultural enterprises in terms of optimal management for inventory system.

1. Introduction

Warehousing is an important part of logistics system. Inventory control of warehousing has been widely focused on by circulation enterprises and relevant scholars all the time. If the stock is high, smooth business process can be fully guaranteed to improve service level and customer satisfaction, while if the stock is low, capital backlog of enterprises and the corresponding management costs are able to be reduced (as in [1]), to optimize and control stock matters to the service quality and economic benefits of circulation enterprises. In addition, the sustainable development of circulation enterprises is crucial.

Owing to the significance of inventory optimization and control in circulation enterprises, there are many relevant scholars that begin to pay attention to this. In the last two decades, the issue has attracted much attention from many researchers. Among these researches, economic order quantity (EOQ) model based on stock-dependent demand was established (as in [2]). Besides, the production-inventory model for perishable items with definite productivity and with demand linearly depending on inventory level was considered (as in [3]). Some scholars explored the inventory issue with allowable shortages under inventory-level-dependent demand; at the same time, they also took monetary value as well as the expansion rate caused by external and internal costs into consideration (as in [4]). In addition, EOQ model for perishable items was established, where the perishable items were under the following conditions: the demand rate was related to inventory level and some stock-outs could be supplemented later (as in [5]). Cárdenas-Barrón et al. (as in [6]) studied the optimal solution of multiproduct EOQ model. Moreover, the optimal replenishment strategy for perishable items was investigated aiming at maximizing profits (as in [7]), while the inventory optimization for perishable items under stock-dependent demand was studied as well (as in [8]). Wang et al. (as in [9]) discussed the inventory control model for fresh agricultural products on Weibull distribution under the assumption that the inflation rate is higher than the natural decay rate. Paul and Rajendran (as in [10]) studied the problem of rationing mechanisms and inventory control-policy parameters for a divergent supply chain operating with lost sales and costs of review. Krishnamoorthy and Narayanan (as in [11]) considered the stability and performance analysis of a production-inventory system. Yadavalli et al. (as in [12]) studied the problem of updating service facilities for inventory system to achieve production and service synchronization. From the inventory cost and the cost of order to determine the optimal order point and quantity, Doğru et al. (as in [13]) pointed out enterprises adjust inventory quantity through a large number of buffer stocks and there is a serious bullwhip effect. Hua et al. (as in [14]) studied the carbon emissions in inventory management. The corresponding model is established by the joint of replenishment strategy, and through genetic algorithm, Zhou et al. (as in [15]) gave the solution and simulation of the model. Murray et al. (as in [16]) studied the multiproduct pricing and inventory issues. Choi and Ruszczyński (as in [17]) established a multiproduct risk-averse newsvendor, and they pointed out that the increase of risk aversion does not necessarily lead to the reduction of the order quantity. Schrijver et al. (as in [18]) studied the optimization model and algorithm of multiproduct demand inventory network design for stochastic demand and inventory decision. Based on the theory of nonlinear integer programming, Yang et al. (as in [19]) studied the integrated multiproduct optimization model. Liu et al. (as in [20]) studied the flexible service policies for a Markov inventory system with two demand classes. Zhao and Lian (as in [21]) studied the priority service rule of a queueing-inventory system with two classes of customers. Karimi-Nasab and Konstantaras (as in [22]) studied an inventory control model with stochastic review interval and special sale offer. By determining the level of customer's anchoring effect, Liu and Shum (as in [23]) studied the joint control of pricing and inventory allocation in two periods of retailers based on constructing the customer's disappointment aversion utility function. Mo et al. (as in [24]) researched the inventory issue for the perishable multi-items with just-in-time (JIT) inventory-level-dependent demand. Ji and Jin (as in [25]) established an inventory optimization model meeting the restrained conditions of being controllable in lead time and service level. Li (as in [26]) studied the control and optimization model for multiechelon inventory in supply chain, while the optimization method for two-echelon inventory system based on stochastic lead time was researched by Dai et al. (as in [27]). Zhao (as in [28]) presented an optimization study on multiechelon inventory in supply chain on the basis of time competition, while Wang (as in [29]) studied the optimization model for production-inventory under uncertain environments. Fu and Pan (as in [30]) mainly explored disposing the inventory management problem by using fuzzy theory under uncertainty to derive the fuzzy mathematical model for single inventory management with multiple fuzzy parameters in the case of allowing moderate shortages. Besides, supply chain inventory optimization with controllable lead time under fuzzy environment was investigated by Li and Xu (as in [31]). Wang and Guo (as in [32]) analyzed the inventory risk loss led by the EOQ and order cycles of classical inventory models under fuzzy demand to deduce the economic risk function in fuzzy situation. Kong and Jirimutu (as in [33]) researched the inventory optimization under stochastic demand based on Monte Carlo simulation. Xu et al. (as in [34]) explored the inventory control model during random replenishment interval with inventory-level-dependent demand.

Most of the above researches were conducted on the basis of continuous normal population, which made the researches convenient and operable to some extent. However, on the premise of uncertain supply and demand, there were a lot of uncertain factors for inventory optimization. In fact, most of the demand and supply in reality cannot distribute continually but present in the form of discrete random variables usually. As a result, on the assumption that the demand process of each subwarehouse submitted to the mutually independent compound Poisson process, the authors carried out the researches on some aspects, including the optimization and control of inventory system based on supply-demand balance as well as the algorithm design for the optimal inventory costs. In addition, the related researches have a certain value on theoretical research.

Although there have been quite a few researches on inventory control, it is still necessary to take many factors and variables into account due to its systematicness and complexity of inventory problem. Besides, it is difficult to quantify and define the optimal inventory because the correlation degrees between each factor are fuzzy. In view of the above facts, the optimization and control of inventory can be summarized as a complex dynamic system containing multifactors, while the quantitative model about the optimization and control of inventory is considered as a complex system with multiple variables and multiple parameters. It is very difficult to solve the problem once and for all by using a single model and a unified algorithm for the research work of the optimization and control of inventory. Most of the existing researches were focused on some specific fields of a particular region to only work out the specific issues under a certain environment. In addition, it is inevitable that the research process is influenced by the subjectivity of the researchers themselves, which suggests that the optimization and control of inventory under various situations cannot be solved. Hence, this issue will undoubtedly attract the persistent attention from the relevant experts and scholars. Actually, it still plays a very realistic role in carrying out pertinent researches on optimization and control model for inventory and algorithm with respect to some specific fields in different areas.

2. Inventory System Model

From a practical point of view of research object, a necessary simplification for the research object was conducted during the research combining the actual conditions of regional circulation enterprises. For the underdeveloped regions, the two-echelon inventory system is more common. In order to improve the practical significance of research results and enhance the operability, a typical two-echelon inventory system composed of a central warehouse and several subwarehouses was emphatically studied.

3. Model Assumption and Symbol Description

3.1. Model Assumption

(1) The central warehouse of the two-echelon inventory system mentioned purchases products from material suppliers, while the subwarehouses order goods from the central warehouse.

(2) Both the central warehouse and subwarehouses of the system carry out the (R, Q) ordering strategy of continuous review inventory. In other words, the inventory levels are continuously observed by the subwarehouses and central warehouse. When the inventory level reduces to order point R, the warehouses will purchase with lot-size of Q, where the inventory level refers to the result by subtracting the stock-outs from the total of on-hand inventory and the goods of the orders in transit. In this way, the inventory level is within a range $[R, R + Q]$ after distribution centers and retailers ordering.

(3) The material suppliers can supply materials unlimitedly and the delivery time for central warehouse is a constant, while the transportation time from central warehouse to subwarehouses is a random variable. Then, the lead time of subwarehouses consists of random delay and random transportation time.

(4) The product demand process of subwarehouses is a mutually independent compound Poisson process, that is to say, Poisson arrival of consumers. In addition, the demand of each consumer is a random integer.

(5) $R_i \geq -Q_i$ $(i = 0, 1, 2, \ldots, N)$ are available for all order points.

(6) All stock-outs in the two-echelon inventory system are waiting. Besides, the delayed order-to-delivery follows the principle of "first come first serve."

3.2. Symbol Description. The meaning of the symbols in the research is as follows:

N is the number of retailers.

L_0 is the fixed delivery time from manufacturers to distribution centers, namely, the lead time of distribution centers.

T_i is the random transportation time of goods from distribution centers to retailers i.

τ is the random delay of retailer's orders in distribution centers.

L_i is the lead time of retailers i; $L_i = T_i + \tau$.

Q_0 is the order quantity of distribution centers.

Q_i is the order quantity of retailers i.

R_0 is the order point of distribution centers.

R_i is the order point of retailers i.

h_0 is the storage costs of unit goods in unit time of distribution center.

h_i is the storage costs of unit goods in unit time of retailers i.

P_0 is the stock-out losses of unit goods in unit time of distribution center.

P_i is the stock-out losses of unit goods in unit time of retailers i.

$C_0(R_0, Q_0)$ is the average holding costs and shortage costs of distribution center in unit time.

$C_i(R_i, Q_i)$ is the holding costs and shortage costs of retailers i in unit time.

TC is the expected gross costs of the inventory system.

4. Hypothesis Testing and Demonstration of Poisson Distribution

In order to test whether the order demand is subject to Poisson distribution, we give an empirical research by taking the agricultural warehousing company in Akesu area of Xinjiang as an example. Based on balance theory between supply and demand, the fertilizers supply quantity of the agricultural warehousing company within a week is determined by the demand quantity of farmers, while the supply quantity of the agricultural warehousing company decides its order quantity. Hence, the demand-based order quantity of the agricultural warehousing company within a week depends on the demand quantity of the farmers in this area in this period. In order to simplify the problem, the order quantity of each time with little fluctuation is taken as a constant. By doing so, the order quantity can be determined by controlling the order times within a single period.

In general, agricultural production presents seasonality and periodicity. This epically can be obviously found in agriculture plantation of Aksu area in Xinjiang province. As the demand quantity of fertilizers for the farmers in Aksu area is shown to be stable within some period or a given time in a season, to some extent, the order times of the agricultural warehousing company for an arbitrary time interval within the demand period merely rely on the span of time interval, instead of the end point of time interval. In addition, the order events of the agricultural warehousing company happen independently in the nonoverlapping intervals. Moreover, the probability for the occurrence of two or more than two order times can be nearly neglected when the time interval is enough small. As above mentioned, it is indicated that the order times of the agricultural warehousing company conform to Poisson stream, indicating stability, no aftereffect stream, and ordinary (as in [35]). On this basis, it is supposed that the order demand of the agricultural warehousing company is subject to Poisson distribution.

In order to verify our supposing, we conducted on-site interviews to the large and middle size agricultural warehousing company and the farmers in various corps. for surveying the demands of various fertilizers. With in-depth investigation, we have acquired large volume of information and first-hand data: through the close communication and contact with the managers in each corp., the total arable areas in Aksu and actual demand quantity of various fertilizers of each corp. were acquired. Afterwards, we verified the goodness of fit for Pearson χ^2 of the sample data, to judge whether the sample data are subject to Poisson distribution or not. It is noteworthy that the actual demand quantity of fertilizers is the order demand quantity of the agricultural warehousing company in fact. The method is described as follows.

By randomly selecting a set of sample data with each sample size more than 50, the order quantity of each time is assumed to be a constant when the order quantity of each time shows little fluctuation. In this case, the order demand can be directly presented by controlling order times. The analysis of chosen data indicates that third- or fourth-order times of the agricultural warehousing company are shown to be reasonable.

According to statistical quantity,

$$\chi^2_{i(\alpha)} = \sum_{j=1}^{m} \left[\frac{\left(f_{ij} - p_{ij}\right)^2}{f_i} \right]. \quad (1)$$

m is the number of purchase groups, where $m = 2$.

The empirical and theoretical frequencies of stochastic events are p_i and f_i, respectively, and they indicate significant difference, with a given confidential level α; the distribution of p_i does not agree with that of f_i; if there is no significant difference between p_i and f_i, the random variable distribution is subject to the theoretical scheme. To validate whether or not the empirical scheme of the fertilizers demand in a time interval in some areas is subject to Poisson distribution, a significant difference between the theoretical and empirical frequencies needs to be checked. In the case of $\alpha = 0.05$, $\chi^2_\alpha = 3.841$ refers to the value of χ^2 when the degree of freedom is 1. On this basis, $\chi^2 = 3.5011$ is calculated through R software programming this is obviously obtained that $\chi^2 < \chi^2_\alpha$. As can be seen, the selected sample data have passed the verification of Poisson distribution. This is to say, these data are subject to the Poisson distribution; the detailed processes and methods can be seen in the researches (as in [35, 36]). By frequent sampling on the sample data, various sets of sample data are obtained by repeating abovementioned methods and processes. All chosen sample data have been validated in Poisson distribution. This further confirmed the feasibility of our assumption.

Apart from verifying the goodness of fit for Pearson χ^2, the repeated samplings have been conducted on the sample data. Moreover, the density function was performed on the simulation of different samples. The simulating results indicate that the density function approximates to normal density with increasing quantity of sample size; considering that the order quantity or order number of a warehouse is stochastic and independent, the data obtained are discrete data, as the order number is relatively large. This is because the corps. in Aksu area mainly make a living by planting; the demand quantity of various fertilizers is relatively large: the quantities are usually more than 1,000 tons on average. The average value of real sample data investigated is 1,093 tons. The total average value estimated is the average value of the sample data. Based on these two points, the parameter lambda (total average value) of Poisson distribution is large according to probability knowledge; as its limit distribution is normal, it is supposed that the demand data are subject to Poisson distribution which is assumed to be reasonable according to the abovementioned analysis results.

As shown in Figure 1, $N = 16$ represents the density function image of all data investigated, where $N = 16$ refers to 16 corps. in Aksu. The density functions are obtained by the simulation when three sample sizes are 30, 60, and 300, respectively. The analysis and investigation into the simulation of density function images reveal that, in the average value range of the samples $[800, 1200]$, there is a good similarity between Poisson and normal distributions. The large the sample size, the more favorable the approximation effect.

5. Modeling

On the assumption that the lead time of retailers is a random variable and the demand process is a compound Poisson process, the (R, Q) storage model was established with the aim of minimizing the mean total cost of two-echelon inventory system in unit time (as in [5]).

5.1. The Inventory Model of Central Warehouse. It was assumed that the demand process of subwarehouse i is followed by the compound Poisson process with parameter of λ_i, which is the mean unit time of purchasers arriving at subwarehouse i. If j is the demand of purchasers, $f_{i,j}$ is the probability of demand j $(j > 0)$ in retailer i, μ_i is the average demand of retailer i in unit time, and σ_i^2 is the demand variance of retailer i in unit time, then

$$\mu_i = \lambda_i \sum_{j=1}^{\infty} j^2 f_{i,j},$$
$$\sigma_i^2 = \lambda_i \sum_{j=1}^{\infty} L_0 \sigma_i^2. \quad (2)$$

Thereby, the average demand and demand variance of retailer i in the lead time of distribution centers can be expressed as

$$\mu_i(L_0) = L_0 \mu_i,$$
$$V_i(L_0) = L_0 \sigma_i^2. \quad (3)$$

Because the demand discussed here was Poisson demand, it could be known according to probability knowledge that if the sample capacity was great, Poisson distribution would be approximate to normal distribution. Thus, it was assumed that the demand of subwarehouse in the lead time of central warehouse was close to normal distribution (as in [5, 25]).

If the probability for placing k orders by subwarehouse i to central warehouse in lead time L_0 was $p_{i,k}(L_0)$, the first order placed after demand was up to x, where x obeyed uniform distribution on $(0, Q_i)$, so placing k orders by subwarehouse

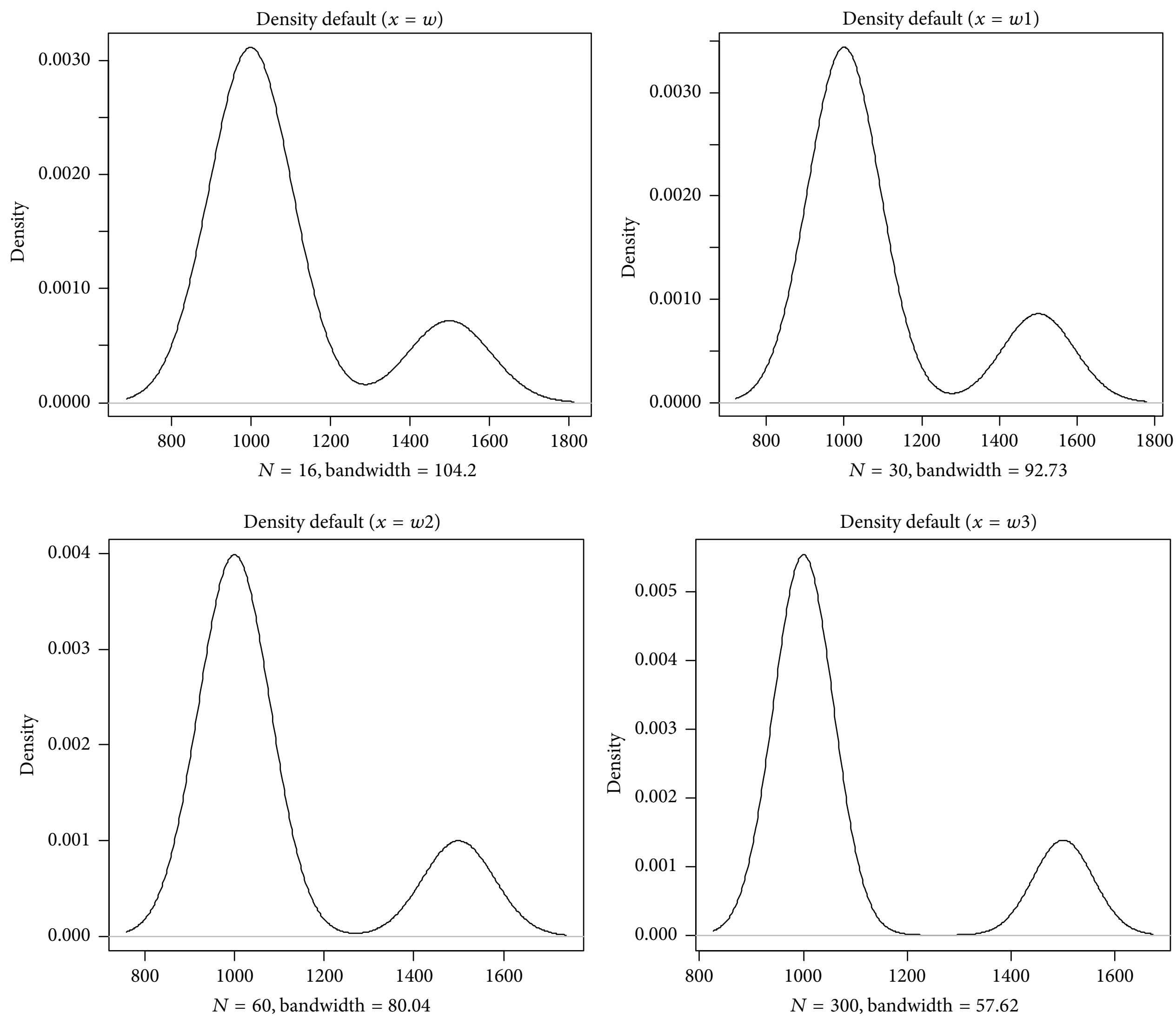

Figure 1: The simulation of density function images.

i within L_0 means that the demand is between $x + (k-1)Q_i$ and $x + kQ_i$; then,

$$\begin{aligned} p_{i,k}(L_0) &= \frac{1}{Q_i}\int_0^{Q_i}\left[\phi\left(\frac{x + kQ_i - \mu_i(L_o)}{\sigma_i(L_0)}\right)\right. \\ &\quad \left. - \phi\left(\frac{x + (k-1)Q_i - \mu_i(L_0)}{\sigma_i(L_0)}\right)\right]dx \\ &= \frac{\sigma_i(L_0)}{Q_i}\left[\phi^{(1)}\left(\frac{(k-1)Q_i - \mu_i(L_0)}{\sigma_i(L_0)}\right)\right. \\ &\quad + \phi^{(1)}\left(\frac{(k+1)Q_i - \mu_i(L_0)}{\sigma_i(L_0)}\right) \\ &\quad \left. - 2\phi^{(1)}\left(\frac{kQ_i - \mu_i(L_0)}{\sigma_i(L_0)}\right)\right], \end{aligned} \tag{4}$$

where $\phi(x)$ is the standard normal density function and $\phi^{(1)}(x)$ is first-order loss function; then, it can be obtained that

$$\phi^{(1)}(x) = \int_x^{\infty}(u - x)\varpi_{(u)}du = \varpi_{(x)} - x\left(1 - \phi_{(x)}\right). \tag{5}$$

Thereby, the mean $\mu_i^0(L_0)$ and variance $V_i^0(L_0)$ of the demand of central warehouse obtained from subwarehouse i in lead time L_0 can be calculated as

$$\begin{aligned} \mu_i^0(L_0) &= \mu(L_0), \\ V_i^0(L_0) &= \sum\left(kQ_i - \mu_i^0(L_0)\right)^2 p_{i,k}(L_0). \end{aligned} \tag{6}$$

To sum up $\mu_i^0(L_0)$ and $V_i^0(L_0)$, the mean $\mu^0(L_0)$ and variance $V^0(L_0)$ of demand of central warehouse in lead time L_0 can be obtained; namely,

$$\mu^0(L_0) = \sum_{i=1}^{N} \mu_i^0(L_0),$$
$$V^0(L_0) = \sum_{i=1}^{N} V_i^0(L_0). \tag{7}$$

I_0 and I_i are set as the stochastic inventory levels of central warehouse and subwarehouse i, respectively ($i = 1, 2, 3, \ldots, N$). Besides, it has been known that the inventory level of central warehouse is independent of the random variable of demand in lead time and the stationary distribution at arbitrary time t in the interval $[R_0 + 1, R_0 + Q_0]$ is uniform distribution (as in [25]).

The total demand of central warehouse is made up of the random demand of each subwarehouse. In addition, based on central limit theorem, when sample capacity N is large enough, it can be considered that the demand of central warehouse in lead time almost obeys normal distribution. Therefore, the average stock-out quantity of central warehouse can be acquired as

$$\begin{aligned} E(I_0)^- &= \frac{1}{Q_0 - 1} \\ &\cdot \int_1^{Q_0} \int_{R_0+y}^{\infty} (x - R_0 - y)\, d\phi\left(\frac{x - \mu^0(L_0)}{\sigma^0(L_0)}\right) dy \\ &= \frac{\left(\sigma^0(L_0)\right)^2}{Q_0 - 1}\left[\phi^{(2)}\left(\frac{R_0 + 1 - \mu^0(L_0)}{\sigma^0(L_0)}\right)\right. \\ &\left. - \phi^{(2)}\left(\frac{R_0 + Q_0 - \mu^0(L_0)}{\sigma^0(L_0)}\right)\right], \end{aligned} \tag{8}$$

where $\phi^{(2)}(x)$ is the quadratic loss function of standard normal distribution, so

$$\begin{aligned} \varphi^{(2)}(x) &= \int_x^{\infty} \varphi^{(1)}(u)\, du \\ &= \frac{1}{2}\left[\left(x^2 + 1\right)(1 - \varphi(x)) - x\varpi(x)\right]. \end{aligned} \tag{9}$$

Then, the average existing inventory of central warehouse is

$$\begin{aligned} E(I_0)^+ &= E(I_0) + E(I_0)^- \\ &= R_0 + \frac{Q_0 + 1}{2} - \mu^0(L_0) + E(I_0)^-. \end{aligned} \tag{10}$$

Finally, according to the practical significance, make h_0 and P_0 the weight value of $E(I_0)^+$ and $E(I_0)^-$, respectively; combine (8) and (10); the average holding cost and shortage cost of distribution center can be expressed as

$$C_0(R_0, Q_0) = h_0 E(I_0)^+ + P_0 E(I_0)^-. \tag{11}$$

5.2. The Inventory Model of Subwarehouse. The randomness of the lead time of subwarehouse is caused by the random delay of central warehouse and the uncertainty of transportation time. Random delay means the random waiting time led by the stock-outs of central warehouse when subwarehouses place orders to central warehouse.

τ was set as the random delay of the orders from subwarehouses in central warehouse. For the sake of simplicity, assuming that it is the same to each subwarehouse, the mean and variance of random delay are $E[\tau]$ and $V[\tau]$, respectively. The time for central warehouse demanding unit goods is t_1, while the arrival time of the goods to central warehouse is t_2. If $t_1 \geq t_2$, there is no delay, while if $t_1 < t_2$, there is a delay, where $\tau = \max(\delta, 0)$ $(\delta = t_2 - t_1)$. According to calculation, the following relationship can be obtained:

$$E[\tau] = \frac{E(I_0)^-}{\sum_{i=1}^{N} \mu_i}. \tag{12}$$

If δ obeys normal distribution, then

$$P(\delta \leq 0) = \left(-\mu_\delta \sigma_\delta\right) = P(\tau = 0), \tag{13}$$

where

$$\begin{aligned} P(\tau = 0) &= \frac{1}{Q_0 - 1}\int_1^{Q_0} \varphi\left(\frac{R_0 + y - \mu^0(L_0)}{\sigma_0(L_0)}\right) dy, \\ E[\tau] &= \int_0^{\infty}\left[1 - \varphi\left(\frac{x - \mu_\delta}{\sigma_\delta}\right)\right] dx \\ &= \sigma_\delta \phi^{(1)}\left(-\mu_\delta \sigma_\delta\right). \end{aligned} \tag{14}$$

So it can be obtained that

$$\begin{aligned} \sigma_\delta &= \frac{E[\tau]}{\phi^{(1)}\left(-\mu_\delta \sigma_\delta\right)}, \\ \mu_\delta &= -\sigma_\delta^{-1}(P(\tau = 0)), \\ V[\tau] &= E\left(\tau^2\right) - (E[\tau])^2 \\ &= \int_0^{\infty} \frac{x^2}{\sigma_\delta}\varpi\left(\frac{x - \mu_\delta}{\sigma_\delta}\right) dx - (E[\tau])^2 \\ &= \sigma_\delta^2(1 - P(\tau = 0)) + E[\tau]\mu_\delta - (E[\tau])^2. \end{aligned} \tag{15}$$

If the transportation time T_i of subwarehouse i obeys the Gamma distribution with α_i and β_i as parameters and the mean and variance of random lead time of subwarehouse i are $E_i(LT)$ and $V_i(LT)$, then

$$\begin{aligned} E_i(LT) &= E\left(T_i\right) + E[\tau] = \frac{\alpha_i}{\beta_i} + E[\tau], \\ V_i(LT) &= V\left[T_i\right] + V[\tau] = \frac{\alpha_i^2}{V[\tau]}. \end{aligned} \tag{16}$$

Through further analysis, it can be known that the mean $E_i(LTD)$ and variance $V_i(LTD)$ of demand of subwarehouse i in random lead time LT are

$$\begin{aligned} E_i(LTD) &= \mu_i E_i(LT), \\ V_i(LTD) &= \sigma_i^2 E_i(LT) + \left(\mu_i\right)^2 V_i(LT). \end{aligned} \tag{17}$$

In general inventory model, if the demand process is Poisson process and the lead time obeys gamma distribution, most of the demand during lead time is approximated by using Poisson distribution. However, if there are many subwarehouses and the mean demand of some subwarehouses in lead time is small, using normal distribution for approximation will produce quite a number of negative values, which is unreasonable. Because its mean and variance values are the same, the change of demand during lead time cannot be well reflected by approximating through Poisson distribution. Besides, the characteristics of negative binomial distribution can preferably conform to the practical situation of the demand of retailers during lead time. In addition, it takes Poisson distribution as limiting distribution. As a result, it is more reasonable to utilize negative binomial distribution for approximating the demand distribution of retailers during lead time (as in [26]).

If the demand of retailer i in lead time can obey the negative binomial distribution with n_i and P_i as parameters, distribution density function and distribution function are, respectively, $g_i(x)$ and $G_i(x)$ $(i = 1, 2, \ldots, N)$; then,

$$\begin{aligned} P_i &= \frac{V_i(LTD) - E_i(LTD)}{V_i(LTD)}, \\ n_i &= \frac{E_i^2(LTD)}{V_i(LTD) - E_i(LTD)}. \end{aligned} \tag{18}$$

Setting $G_i^{(0)}(x) = 1 - G_i(x)$, because n is not an integer, it is impossible to calculate accurately $g_i(x)$ and $G_i(x)$. Thus, the first-order loss function $G^{(1)}(x)$ and quadratic loss function $G^{(2)}(x)$ $(x \geq 0)$ for LTD are as follows:

$$\begin{aligned} G^{(1)}(x) &= E\{[X - x]^+\} = \sum_{y \geq x} G_i^{(0)}(y) = \left[\frac{n_i P_i}{1 - P_i} - x\right] \\ &\quad \cdot G_i^{(0)}(x) + (x + n_i)\frac{P_i}{1 - P_i} g_i(x), \\ G^{(2)}(x) &= \frac{1}{2} E\{[X - x]^+ [X - x - 1]^+\} = \frac{1}{2} \\ &\quad \cdot E\{X[X - 1]\} - \sum_{0 < y \leq x} G^{(1)}(y) \\ &= \frac{1}{2}\left\{n_i(n_i + 1)\left[\frac{P_i}{1 - P_2}\right]^2 - 2\frac{n_i P_i}{1 - P_i} x \right. \\ &\quad + x(x + 1) G^{(1)}(x) \\ &\quad \left. + \left[\frac{(n_i + 1) P_i}{1 - P_i} - x\right](x + n_i)\frac{P_i}{1 - P_i} g_i(x)\right\}. \end{aligned} \tag{19}$$

In the same way of central warehouse, it can be obtained that subwarehouses i are equally distributed on $[R_i+1, R_i+Q_i]$ at arbitrary time. Hence, it can be acquired that the average stock-out quantity of subwarehouse i is as follows:

$$\begin{aligned} E(I_i)^- &= \frac{1}{Q_i - 1}\int_1^Q \int_{R_i + y}^{\infty} (x - R_i - y) g_i(x)\, dx\, dy \\ &= \frac{1}{Q_i - 1}\left[G^{(2)}(R_i + 1) - G^{(2)}(R_i + Q_i)\right]. \end{aligned} \tag{20}$$

Then, corresponding to (10), the average existing inventory of subwarehouse i is

$$\begin{aligned} E(I_i)^+ &= E(I_i) + E(I_i)^- \\ &= R_i + \frac{Q_i + 1}{2} - E_i(LTD) + E(I_i)^-. \end{aligned} \tag{21}$$

Therefore, according to the practical significance, make h_i and P_i the weight value of $E(I_i)^+$ and $E(I_i)^-$, respectively; combine (20) and (21); the average holding cost and shortage cost of subwarehouse i can be obtained as

$$C_i(R_i, Q_i) = h_i E(I_i)^+ + P_i E(I_i)^-. \tag{22}$$

As the total inventory cost is equal to the inventory cost of the central warehouse and the warehouse, combine (11) and (22); through calculation and derivation, the expected total cost function of two-echelon inventory system can be expressed as

$$TC = C_0(R_0, Q_0) + \sum_{i=1}^{N} C_i(R_i, Q_i). \tag{23}$$

5.3. Optimizing and Control Inventory Cost. Through analyzing the cost structure of two-echelon inventory system, the inventory control model based on cost optimization can be obtained as follows:

$$\min TC = \min\left[C_0(R_0, Q_0) + \sum_{i=1}^{N} C_i(R_i, Q_i)\right]. \tag{24}$$

The key to solving the model is to select appropriate order point R_i^* and order quantity Q_i^* $(i = 0, 1, \ldots, N)$ to make the objective function achieve the optimum value.

Firstly, R_0 and Q_0 are given to determine the order point $R_i^*(R_0, Q_0)$ and order quantity $Q_i^*(R_0, Q_0)$ of subwarehouse i. According to the analysis, it can be known that $E(I_i)^-$ is a convex function of R_i and Q_i, and $E(I_i)^+$ is a linear equation of (R_i, Q_i). The nonlinear combination $C_i(R_i, Q_i)$ is still a convex function based on (R_i, Q_i). It can be known that there must be optimal R_i, Q_i to minimize $C_i(R_i, Q_i)$ through the properties of convex function.

After further simplifying the objective function, it can be obtained that

$$TC(R_0, R_i) = C_0(R_0) + \sum_{i=1}^{N} C_i(R_0, R_i). \tag{25}$$

Table 1: Ordering costs of each cycle (10^4 Yuan).

	Actual value	Simulation value					
	$N = 6$	$N = 5$	$N = 7$	$N = 8$	$N = 9$	$N = 10$	$N = 50$
Nitrogen	957.03	1109.0625	848.44	742.8	659.9	633.28	633.28
Phosphate	783.025	783.02	114.6019	112.3126	110.5313	872.9776	872.9776
Potassium	34.829	27.8632	39.80624	43.53625	46.43737	48.7606	48.7606
Other	34.829	27.8632	39.80624	43.53625	46.43737	48.7606	48.7606
Total	1914.0625	2060.1268	1102.778	996.515	913.0875	1696.259	1696.259

5.4. The Analysis and Calculation of the Optimal Inventory Cost. The inventory model established in the research is a two-echelon inventory system model containing a central warehouse and several subwarehouses. If the total cost of the system is the sum of the cost of the central warehouse and all the subwarehouses, then it can be acquired by determining the optimal cost of each warehouse.

The optimal inventory cost can be computed by the following two steps.

First step is to find the optimal order point $R_i^*(R_0)$ of retailer i by fixing R_0 and minimizing $C_i(R_0, R_i)$. Through the above theorem, it can be known that $C_i(R_i, Q_i)$ is a convex function of (R_i, Q_i); then, the necessary condition for obtaining extreme value of $C_i(R_0, R_i)$ is that

$$\frac{\partial C_i(R_0, R_i)}{\partial R_i} = 0. \tag{26}$$

The root of equation $R_i^*(R_0)$ can be solved by using one-dimensional search method.

Second step is to solve R_0^* by minimizing TC. Because TC for R_0 may not be a convex function, a simple search program was designed to find out the locally optimal solution R_0^*.

Step 0. Consider $R_0 = -Q_0$, step $= \lceil \mu^0(L_0) \rceil$, and $C^* = TC[R_0, R_i^*(R_0)]$.

Step 1. Consider $R_0 = R_0 +$ step, marking the corresponding order points as $\widehat{R}_0$.

Step 2. If step = 1, $R_0^* = \widehat{R}_0$; otherwise, let $R_0 = \max(R_0, \widehat{R}_0 -$ step), step = $\lceil$step/10$\rceil$, go back to Step 1.

The complexity of two search algorithms utilized for solving the optimum values of the order points of central warehouse and subwarehouses was added up as $O(\log R_0^* \sum_{i=1}^{N} \log R_i^*)$, which is the linear function of N. According to the solving steps above, the corresponding mathematical model is as follows:

$$\begin{aligned} \min \quad & \left(C_0(R_0) + \sum_{i=1}^{N} C_i(R_0, R_i) \right) \\ \text{s.t.} \quad & \sum_{i=1}^{N} CQ_i \le W \\ & Q_i \in (0, \infty), \quad i = 1, 2, \ldots, N. \end{aligned} \tag{27}$$

6. Model Solving and Main Results

6.1. The Empirical Analysis and Numerical Simulation Test. Through on-site survey to the 16 corps. in Aksu area of South Xinjiang, and the agricultural warehousing company in and surrounding this areas, a large logistics storage company whose business covers whole Aksu area is taken as research object; the company has a developed warehouse system consisting of six bases which are one headquarter warehouse base and five subwarehouse bases. This company supplies fertilizers to the 16 corps. around Aksu areas. We therefore acquired the demand quantities of various fertilizers for each corp. and the order cycles of the logistics warehousing company. Moreover, we constructed an optimization model for the optimal order cost and inventory cost in each circle and provided an optimal inventory cost algorithm. By combining MATLAB programming, the optimal order cost in single circle and the optimal inventory cost for various fertilizers, as well as the corresponding total cost of single circle order and the optimal total inventory cost, are solved. Besides, a statistical simulation method was used to simulate the cases when the warehouse quantities are 5, 7, 8, 9, 10, and 50, respectively. The simulated results of the optimal order cost in single period, and corresponding total order cost in single period, of various fertilizers are compared with empirical results, as shown in Table 1. Moreover, the empirical and simulated results of the optimal inventory cost of various fertilizers and corresponding optimal total inventory cost are compared also, as indicated in Table 2.

Figure 2 indicates the simulated tendency of the order cost and the total order cost in single period for various kinds of fertilizers including nitrogen, phosphate, and potassium fertilizers.

6.2. The Analysis of Empirical and Simulated Results. According to the statistics analysis of the original data obtained, the optimal order cost in single period of nitrogen, phosphate, and potassium fertilizers as well as other fertilizers is obtained when the planting areas of the Aksu in Xinjiang province and 6 warehouses are given. Besides, statistics simulation method was used to conduct simulation when the number of warehouses is 5, 7, 8, 9, 10, and 50, respectively. The simulated results show that as the number of warehouses increases to 9 from 5, the order costs and total order cost of single period, for nitrogen and phosphate fertilizers, decrease obviously. This is because the demands of nitrogen and phosphate fertilizers are large, which occupy higher proportion in

Table 2: The optimal inventory costs (10^4 Yuan).

	Actual value	Simulation value					
	$N = 6$	$N = 5$	$N = 7$	$N = 8$	$N = 9$	$N = 10$	$N = 50$
Nitrogen	5742.1875	5545.3125	5939.08	5942.4	5939.1	6332.8	6332.8
Phosphate	4698.1534	3915.1	802.2133	898.5008	994.7817	8729.776	8729.776
Potassium	208.98	139.316	278.6437	348.29	417.9363	487.606	487.606
Other	208.98	139.316	278.6437	348.29	417.9363	487.606	487.606
Total	11484.375	10300.634	7719.446	7972.12	8217.788	16962.59	16962.59

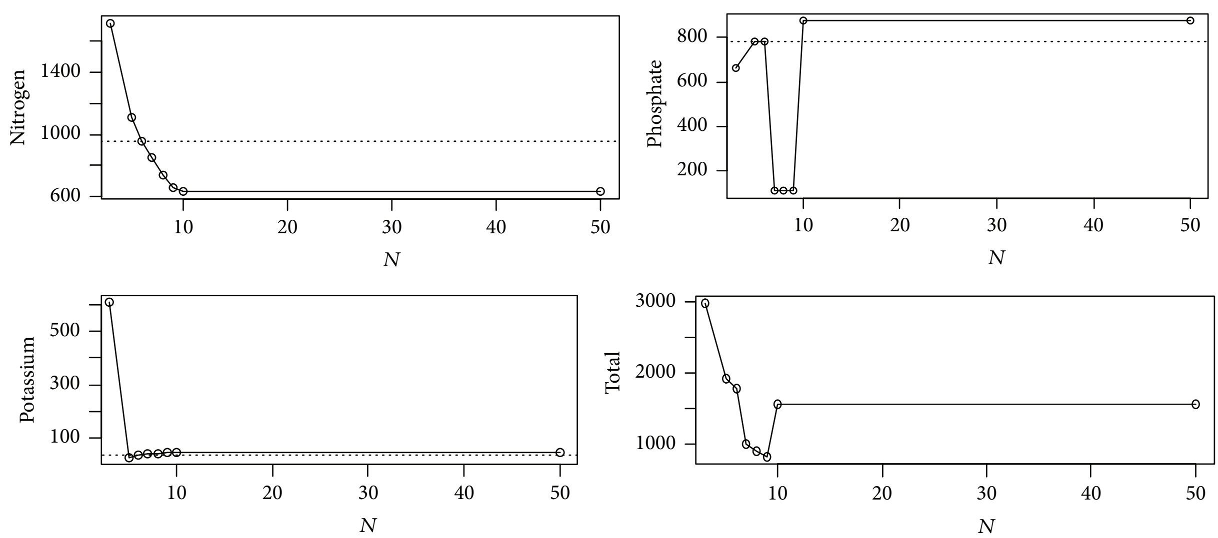

Figure 2: The simulated tendency of the order cost and the total order cost in single period for various kinds of fertilizers.

order cost; the total order cost is mainly determined by nitrogen and phosphate fertilizers. In contrast, as the number of warehouses rises, the order costs of single period for potassium and other fertilizers increase. This is because the demands of potassium fertilizer and other microelement fertilizers are small; the small number of warehouses can sufficiently store minor amount of potassium fertilizer and other fertilizers, while a large number of warehouses tend to cause waste resources and unnecessary cost. As the number of warehouses increases to or greater than 10, the total order cost of single period for various fertilizers tend to be stable, as indicated in Figure 1. From the perspective of the order cost of single period for a fertilizer, the demand of nitrogen fertilizer is large and stable. With increasing number of warehouses, the quantity of nitrogen fertilizers has been well supplied in each warehouse so as to satisfy the real time demands of customers. This makes the order cost of single period for nitrogen fertilizer reduce. Phosphate and potassium fertilizers, as well as other fertilizers, all show an apparent fluctuation when the number of warehouses is equal to or greater than 10, and their order costs increase obviously. This is because the fact that as the number of warehouses increases, the order quantities of these fertilizers are likely to rise greatly so as to fully utilize warehouse. In addition, these fertilizers have high price; consequently, the order costs of single period increase. Regardless of construction and management costs, the optimal order cost of single period is proven to be most ideal when the number of warehouses is 9. Based on the abovementioned results, the empirical results are consistent with simulated results.

Table 2 presents the calculated and simulated results of the optimal inventory cost. When the warehouse number is 7, the optimal total inventory cost of all fertilizers is minimum. For the perspective of nitrogen fertilizer, its optimal inventory cost is minimum when the warehouse number is 5; for phosphate fertilizer, its optimal inventory cost is the smallest when the warehouse number is 7; however, the optimal inventory costs of potassium and other fertilizers are minimum when the warehouse number is 5. When the warehouse number is equal to or greater than 10, the inventory costs and optimal inventory costs of different fertilizers tend to be stable. Since Xinjiang belongs to an arid and alpine area, the corps. demand great quantity of phosphate fertilizer. Considering the phosphate fertilizer is relatively expensive, it is suggested to mainly concern a minimum optimal inventory cost of phosphate fertilizer in our optimization decision with the optimal warehouse number of 7.

7. Conclusion

In the process of numerical simulation, the optimal stock and the optimal inventory costs of various agricultural products were obtained by continuously modifying the values of parameters in model. At the same time, the stock and the ordering costs of all products in each cycle were acquired. Through constantly adjusting the initial values of the variables in model, it was found that all models with reasonable initial values can converge to a stable equilibrium solution, namely, the optimal solution in finite steps, so as to verify the reliability of the model. In order to investigate the stability of parameters, the authors performed sensitivity analysis for the parameters in the model. As indicated by the numerical simulation results, there only appeared some fluctuations by slightly varying and modifying the values of parameters in the model during numerical simulation test. It was verified that the stability of the parameters in model was preferable. In the simulation test mentioned, the authors discovered that the convergence efficiency of algorithm was greatly influenced by selecting different initial values for variables. Moreover, the convergence efficiency of algorithms was high by choosing initial values close to equilibrium solution.

Considering supply-demand balance, the research started from demand analysis. On the assumption that the demand process obeyed mutually independent compound Poisson process, a two-echelon inventory system consisting of a central warehouse and several subwarehouses was constructed through simplifying the research object. Besides, aiming at minimizing the mean total cost of inventory system in unit time, the optimization and control model for inventory of the system was established. In addition, the optimal algorithm for computing the inventory cost was provided. Meanwhile, a numerical simulation experiment was conducted, and the numerical results illustrated that the optimal inventory strategy was much influenced by the uncertainty of lead time. In this paper, the researchers explored the issue about the optimization and control of inventory system and established the corresponding optimization and control model for inventory to optimize the allocation of resources and utilize the limited inventory resources effectively by managing them quantitatively. The related research results offered a certain practical value and reference for the optimal management of agricultural materials inventory of agricultural material enterprises in Xinjiang. The modeling idea and method presented in this paper are excepted to be further developed to the higher-level multiechelon inventory structure.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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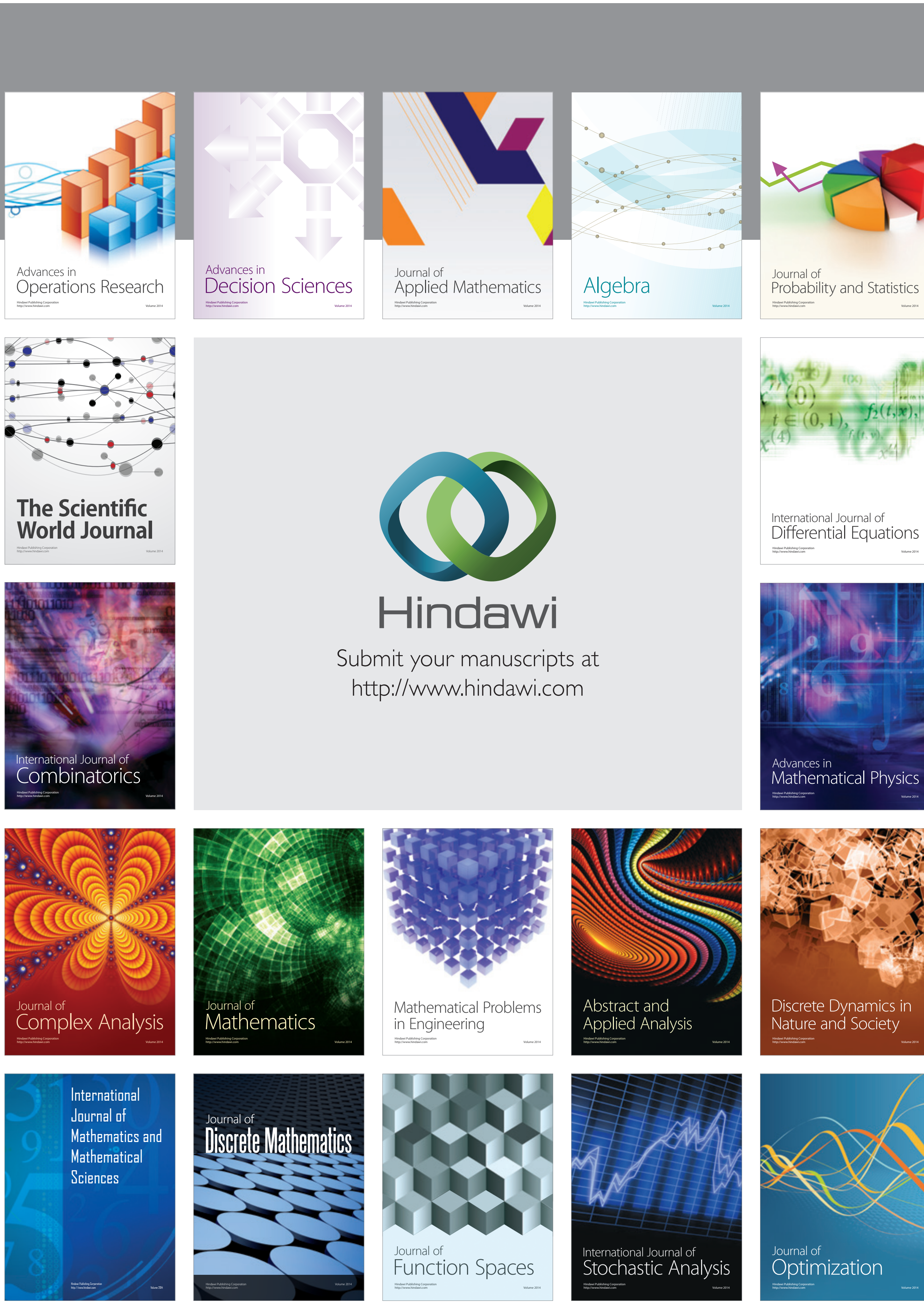
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