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*Research Article*

# Bianchi Type-V Bulk Viscous Cosmic String in $f(R,T)$ Gravity with Time Varying Deceleration Parameter

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We study the Bianchi type-V string cosmological model with bulk viscosity in $f(R,T)$ theory of gravity by considering a special form and linearly varying deceleration parameter. This is an extension of the earlier work of Naidu et al., 2013, where they have constructed the model by considering a constant deceleration parameter. Here we find that the cosmic strings do not survive in both models. In addition we study some physical and kinematical properties of both models. We observe that in one of our models these properties are identical to the model obtained by Naidu et al., 2013, and in the other model the behavior of these parameters is different.

## 1. Introduction

Recent data obtained in observational cosmology [1–5] indicate that currently our universe is accelerating. Researchers are trying to describe this late inflation of the universe in two different ways. Some by modifying Einstein's theory of gravity and others by introducing an exotic type of fluid like a cosmological constant or quintessential type of scalar field. But the use of cosmological constant is surrounded with serious theoretical problems. Therefore various alternatives are used such as K-essence [6], tachyon [7], Phantom [8], and quintom [9]. While all these alternatives have both positive and negative aspects, a large number of researchers used a matter field, namely, chaplygin gas. In the second approach where gravitational theory is modified researchers study the action described by an arbitrary function of the scalar curvature $R$, which is called $f(R)$ theory of gravity [10–12]. Carroll et al. [13] pointed out that the late time acceleration of the universe can be explained by $f(R)$ gravity. Sharif and Yousaf [14] analyzed the role of the electromagnetic field and a viable $f(R)$ model on the range of dynamical instability. Sharif and Kausar [15] obtained that $f(R)$ with constant scalar curvature plays the role of cosmological constant and slows down the collapsing process. Sharif and Yousaf [16] performed stability analysis of an adiabatic anisotropic cylindrical collapsing system in metric $f(R)$ gravity. Sharif and Yousaf [17] studied the effects of polynomial $f(R)$ model on the stability of homogeneous energy density in self-gravitating spherical stellar object and found that shear, pressure, dissipative parameters, and $f(R)$ terms affect the existence of inhomogeneous energy density. Sharif and Yousaf [18] obtained that in addition to other fluid variables higher order $f(R)$ corrections, relaxation processes, and electromagnetic field affect the energy density in homogeneity of spherical stars. Furthermore Sharif and Yousaf [19] explored the effects of the three-parametric $f(R)$ model on the stability of the regular energy density of planar fluid configurations with the Palatini $f(R)$ formalism.

There exist other interesting classes of modified gravity theories such as $f(G)$ gravity, $f(R,G)$ gravity, $f(T)$ gravity, and $f(R,T)$ gravity. In $f(R,T)$ gravity the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar $(R)$ and the trace $(T)$ of the stress energy tensor and also it depends on a source term representing the variation of the matter stress energy tensor with respect to the metric. The source term expression is obtained as a function matter Lagrangian $(L_m)$; as a result for each choice of $L_m$, it would generate a specific set of field equations. The field equation of

$f(R,T)$ gravity is obtained by Harko et al. [20] from Hilbert-Einstein type variational principle. The action for this gravity is given by

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R,T) + L_m\right) d^4x, \quad (1)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar $(R)$ and trace $(T)$ of energy tensor of the matter $T_{ij}$ and $L_m$ represents matter Lagrangian density. They have derived the field equations of $f(R,T)$ gravity by varying the action $S$ of the gravitational field with respect to the metric tensor components $g_{ij}$. For the choice of $f(R,T) = R + 2f(T)$, the field equation takes the form

$$G_{ij} = \left[8\pi + 2f'(T)\right] T_{ij} + \left[2pf'(T) + f(T)\right] g_{ij}, \quad (2)$$

where the overhead prime indicates derivative with respect to the argument and $T_{ij}$ is given by

$$T_{ij} = (\rho + p) u_i u_j + p g_{ij}, \quad (3)$$

where $\rho$ and $p$ stand for energy density and isotropic pressure, respectively.

Studies of string cosmological models are crucial as they play an important role in structure formation of the early stages of evolution of the universe. Cosmic strings are one-dimensional topological defects, which may be formed during symmetry breaking phase transition in the early universe along with other defects like domain walls and monopoles. The density perturbation arising out of them plays an important role in cosmology since they are considered as the route for galaxy formation. After Stachel [21] and Hendi and Momeni [22] many researchers [23–32] have worked on string cosmological models in the context of Einstein's theory or modified theories of gravity. Pradhan et al. [33] studied the LRS Bianchi type-II massive string cosmological model in general relativity. Five-dimensional Bianchi type-I string cosmological models in Lyra manifold are analysed by Samanta and Debata [34]. Rikhvitsky et al. [35] discussed the magnetic Bianchi type-II string cosmological model in loop quantum cosmology.

Bulk viscosity is useful for the study of early stages of evolution of the universe. Bulk viscosity driven inflation is primarily due to the negative bulk viscous pressure giving rise to a total negative effective pressure which may overcome the pressure due to the usual gravity of matter distribution in the universe and provides an impetus for rapid expansion of the universe. Thus many researchers have shown interest to study bulk viscous string cosmological model in the context of Einstein theory or modified theories of gravity. Several authors [36–43] studied bulk viscous cosmological models in general relativity. Anisotropic bulk viscous cosmological models with particle creation have been investigated by Singh and Kale [44]. Rao and Sireesha [45] studied the Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation. Numerous researchers, Samanta and Dhal [46]; Chaubey and Shukla [47]; Adhav [48]; Reddy et al. [49], have studied cosmological model of the universe in $f(R,T)$ theory of gravity in different contexts. Recently Naidu et al. [50], Kiran and Reddy [51], and Reddy et al. [52] discussed the Bianchi type-V, Bianchi type-III, Kaluza-Klein space time with cosmic strings, and bulk viscosity in $f(R,T)$ gravity, respectively. Mahanta [53] investigated the bulk viscous cosmological models in $f(R,T)$ theory of gravity.

Motivated by the above discussion, here we have constructed Bianchi type-V bulk viscous string cosmological model in $f(R,T)$ gravity with special form of deceleration parameter and linearly varying deceleration parameter. The main reason to explore the Bianchi type-V model is that the standard FLRW models are contained as special cases of the Bianchi models. The Bianchi type-V model generalizes the open $(k = -1)$ Friedmann model and represents a model in which the fluid flow is not necessarily orthogonal to the three surfaces of homogeneity. At early stage of evolution, the universe was not so smooth as it looks in present time. Therefore anisotropic cosmological models have taken considerable interest of researchers.

In this work we investigate the role of variable deceleration parameter in Bianchi type-V space time with bulk viscous cosmic string and $f(R,T)$ gravity.

## 2. Metric and Field Equations

The spatially homogeneous and anisotropic Bianchi type-V space time is given by

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x}\left(B^2 dy^2 + C^2 dz^2\right), \quad (4)$$

where $\alpha$ is a constant and $A$, $B$, and $C$ are functions of cosmic time $t$ only.

The energy momentum tensor for a bulk viscous fluid containing one-dimensional cosmic strings is considered as

$$T_{ij} = (\rho + \overline{p}) u_i u_j + \overline{p} g_{ij} - \lambda x_i x_j, \quad (5)$$

$$\overline{p} = p - 3\xi H, \quad (6)$$

where $\rho(t)$ is the rest energy density of the system, $H$ is the Hubble parameter, $p$ is the pressure, $\xi(t)$ is the coefficient of bulk viscosity, $\lambda(t)$ is the string tension density, $u^i = (0,0,0,1)$ is the four-velocity vector of the fluid, and $x^i$ is the direction of the string. Also $u^i$ and $x^i$ satisfy the relation

$$g_{ij} u^i u_j = x^i x_j = -1,$$
$$u^i x_j = 0. \quad (7)$$

The field equation (2) for the metric (4), with the help of (5) along with $f(T) = \mu T$ ($\mu$ is a constant quantity), is given as follows:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -\overline{p}(8\pi + 7\mu) + \lambda(8\pi + 3\mu) + \mu\rho,$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -\overline{p}(8\pi + 7\mu) + (\lambda + \mu)\rho,$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-\overline{p}\left(8\pi+7\mu\right)+\left(\lambda+\mu\right)\rho,$$
$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{3\alpha^2}{A^2}=-\rho\left(8\pi+7\mu\right)-5\overline{p}\mu+\mu\lambda,$$
$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{8}$$

where an overhead dot denotes differentiation with respect to $t$.

## 3. Solution of the Field Equations

The field equations (8) reduce to the following independent equations:

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{A}\dot{C}}{AC}=\lambda\left(8\pi+\mu\right), \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{3\alpha^2}{A^2}=-\rho\left(8\pi+7\mu\right)-5\overline{p}\mu+\mu\lambda, \tag{10}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{A}\dot{C}}{AC}=0, \tag{11}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0. \tag{12}$$

Here there are four equations involving six unknowns. Since the field equations are highly nonlinear for the complete determinacy, we need extra conditions among the variables. We consider these conditions in the form SET-I and SET-II as defined below.

(i) The shear ($\sigma$) is proportional to the expansion ($\theta$) [54]. Collins et al. [55] pointed out that, for spatially homogeneous metric, the normal congruence to the homogeneous hyper surface satisfies the condition [$\sigma/\theta$ = Const.] which yields

$$B=C^m. \tag{13}$$

(ii) The combined effect of the proper pressure and the bulk viscous pressure for barotropic fluid [50] can be written as

$$\overline{p}=p-3\xi H=\left(\epsilon_0-\gamma\right)\rho,\quad 0\le\epsilon_0\le 1,\ \ p=\epsilon_0\rho, \tag{14}$$

where $\epsilon_0$ and $\gamma$ are constants. The SET-I consists of (i), (ii), and a special form of deceleration parameter [56]

$$q=-1+\frac{\beta}{1+R^{\beta}}, \tag{15}$$

where $\beta(>\ 0)$ is a constant and $R$ is scale factor of the metric. The SET-II consists of (i), (ii), and a linearly varying deceleration parameter [57]

$$q=-kt+n-1, \tag{16}$$

where $k(\ge 0)$ and $n(\ge 0)$ are constants.

*3.1. Solution of the Field Equations along with Conditions in SET-I.* In this section, we have discussed the solution of the field equations by considering the extra conditions as in SET-I. The Hubble parameter $H$ is defined as $H=\dot{R}/R$ and from (15) we obtained

$$H=\frac{\dot{R}}{R}=A_1\left(1+R^{-\beta}\right), \tag{17}$$

where $A_1$ is a constant of integration. Integrating (17) and using the initial conditions $R=0$ at $t=0$ we have found

$$R=\left(e^{A_1\beta t}-1\right)^{1/\beta}. \tag{18}$$

The scale factor of metric (4) is defined as

$$R^3=ABC. \tag{19}$$

With the help of (12), (13), and (19) we have found

$$A=\left(e^{A_1\beta t}-1\right)^{1/\beta} \tag{20}$$

$$B=\left(e^{A_1\beta t}-1\right)^{2m/\beta(m+1)} \tag{21}$$

$$C=\left(e^{A_1\beta t}-1\right)^{2/\beta(m+1)}. \tag{22}$$

From (9), with the help of (20)–(22) we obtained the string tension density ($\lambda$) as

$$\lambda=-\frac{A_{11}e^{A_1\beta t}\left(\beta-3e^{A_1\beta t}\right)}{\left(e^{A_1\beta t}-1\right)^2}, \tag{23}$$

where $A_{11}=A_1^2(m-1)/(m+1)(8\pi+2\mu)$. From (10), with the help of (14) and (22), we obtained the rest energy density ($\rho$) as

$$\rho=\frac{1}{8\pi+\mu\left(3-5\epsilon_0+5\gamma\right)}\left[\frac{\left(A_{13}\left(\beta-3e^{A_1\beta t}\right)+A_{12}\right)e^{A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^2}-\frac{3\alpha^2}{\left(e^{A_1\beta t}-1\right)^{2/\beta}}\right], \tag{24}$$

where $A_{12}=2A_1^2(m^2+4m+1)/(m+1)^2$, $A_{13}=\mu A_{11}$. From (14), with the help of (24), we have obtained the total pressure

($\overline{p}$), proper pressure ($p$), and the coefficient of bulk viscosity ($\xi$) as follows:

$$\overline{p}=\frac{\epsilon_0-\gamma}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{\left(A_{13}\left(\beta-3e^{A_1\beta t}\right)+A_{12}\right)e^{A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^2}-\frac{3\alpha^2}{\left(e^{A_1\beta t}-1\right)^{2/\beta}}\right],$$
$$p=\frac{\epsilon_0}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{\left(A_{13}\left(\beta-3e^{A_1\beta t}\right)+A_{12}\right)e^{A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^2}-\frac{3\alpha^2}{\left(e^{A_1\beta t}-1\right)^{2/\beta}}\right],\tag{25}$$
$$\xi=\frac{\gamma}{3A_1\left[8\pi+\mu(3-5\epsilon_0+5\gamma)\right]}\left[\frac{A_{13}\left(\beta-3e^{A_1\beta t}\right)+A_{12}}{\left(e^{A_1\beta t}-1\right)}-\frac{3\alpha^2e^{-A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^{2/\beta-1}}\right].$$

Using (13), (20), and (22) in (11) we get

$$(m-1)\left[\frac{\ddot{C}}{C}+\left(\frac{\dot{C}}{C}\right)^2+\frac{\dot{A}\dot{C}}{AC}\right]=0,\tag{26}$$

which provide $m=1$ as

$$\left[\frac{\ddot{C}}{C}+\left(\frac{\dot{C}}{C}\right)^2+\frac{\dot{A}\dot{C}}{AC}\right]=\frac{2A_1^2e^{A_1\beta t}}{(m+1)^2\left(e^{A_1\beta t}-1\right)^2}\times\left[\beta(m+1)-(m+5)e^{A_1\beta t}\right]\neq0.\tag{27}$$

Thus, for $m=1$, the expressions in (17)–(24) take the form

$$A=B=C=\left(e^{A_1\beta t}-1\right)^{1/\beta},$$
$$\lambda=0,$$
$$\rho=\frac{3}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{A_1^2e^{A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^2}-\frac{\alpha^2}{\left(e^{A_1\beta t}-1\right)^{2/\beta}}\right],$$
$$\overline{p}=\frac{3(\epsilon_0-\gamma)}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{A_1^2e^{A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^2}-\frac{\alpha^2}{\left(e^{A_1\beta t}-1\right)^{2/\beta}}\right],$$
$$p=\frac{3\epsilon_0}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{A_1^2e^{A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^2}-\frac{\alpha^2}{\left(e^{A_1\beta t}-1\right)^{2/\beta}}\right],$$
$$\xi=\frac{3\gamma}{3A_1\left[8\pi+\mu(3-5\epsilon_0+5\gamma)\right]}\left[\frac{A_1^2}{\left(e^{A_1\beta t}-1\right)}-\frac{\alpha^2e^{-A_1\beta t}}{\left(e^{A_1\beta t}-1\right)^{2/\beta-1}}\right].\tag{28}$$

*3.1.1. Some Physical Properties of the Model.* Some physical properties of the model are given below, which are crucial in the discussion of cosmology:

(i) The spatial volume $V=(e^{A_1\beta t}-1)^{1/\beta}$.

(ii) The scalar of expansion $\theta=3A_1e^{A_1\beta t}/(e^{A_1\beta t}-1)$.

(iii) The mean Hubble parameter $H=A_1e^{A_1\beta t}/(e^{A_1\beta t}-1)$.

(iv) The mean anisotropy parameter is defined as $3A_h=\sum_{i=1}^{3}((H_i-H)/H)^2$ and obtained as $A_h=0$.

(v) The shear scalar $\sigma^2=0$.

In this model, we observed that at initial epoch the values of energy density ($\rho$), proper pressure ($p$), total pressure ($\overline{p}$), coefficient of bulk viscosity ($\xi$), and Hubble parameter ($H$) are very high and these values gradually decrease with the evolution of time; that is, $\rho$, $p$, $\overline{p}$, $\xi$, $H$ tend to zero as $t\to\infty$ (see Figures 1–5). Spatial volume ($V$) increases with the evolution of time; that is, $V\to\infty$ as $t\to\infty$ (see Figure 6). This model is isotropic as the average anisotropy parameter is zero and it is also shear free. In this model cosmic string does not survive.

*3.2. Solution of the Field Equations along with Conditions in SET-II.* In this section, we have discussed the solution of the field equations by considering the conditions as in SET-II. From (16) we have Akarsu and Dereli [57]:

$$R=\begin{cases}R_1\exp\left[\dfrac{2}{\sqrt{n^2-2c_1k}}\operatorname{arctanh}\left(\dfrac{kt-n}{\sqrt{n^2-2c_1k}}\right)\right], & \text{for } k>0,\ n\geq0\\ R_2\left(nt+c_2\right)^{1/n}, & \text{for } k=0,\ n>0\\ R_3e^{c_3t}, & \text{for } k=0,\ n=0,\end{cases}\tag{29}$$

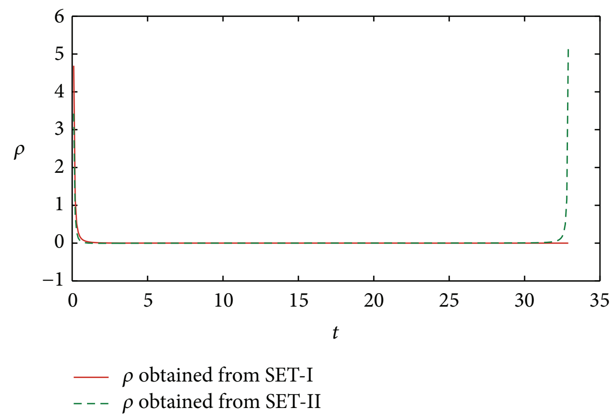


FIGURE 1: Density $\rho$ versus time $t$ for $\mu = 0.5$, $\epsilon_0 = 1/3$, $\gamma = 1$, $\beta = 1.5$, $\alpha = 0.1$, $A_1 = 1$, $R_1 = 1$, $n = 1.6$, and $k = 0.097$.

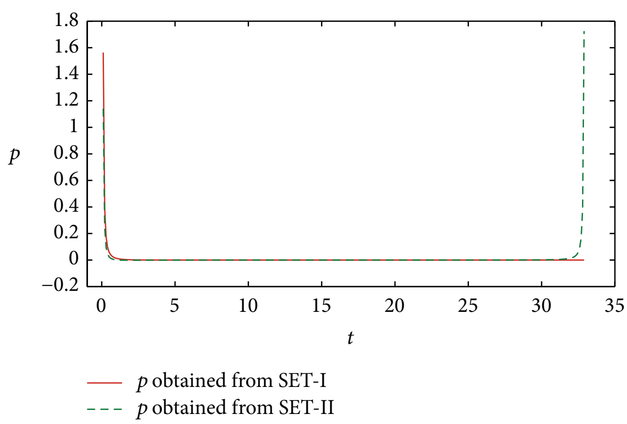


FIGURE 2: Proper pressure $p$ versus time $t$ for $\mu = 0.5$, $\epsilon_0 = 1/3$, $\gamma = 1$, $\beta = 1.5$, $\alpha = 0.1$, $A_1 = 1$, $R_1 = 1$, $n = 1.6$, and $k = 0.097$.

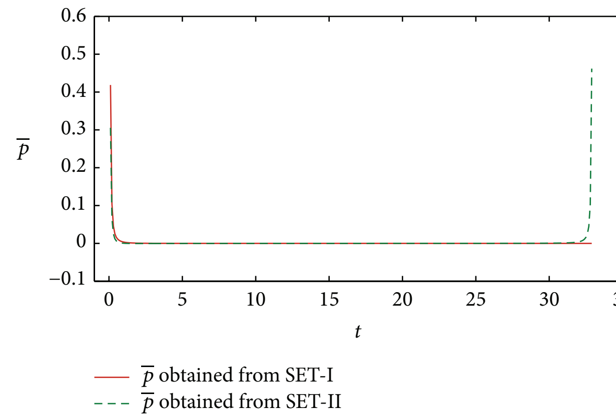


FIGURE 3: Total pressure $\overline{p}$ versus time $t$ for $\mu = 0.5$, $\epsilon_0 = 1/3$, $\gamma = 1/4$, $\beta = 1.5$, $\alpha = 0.1$, $A_1 = 1$, $R_1 = 1$, $n = 1.6$, and $k = 0.097$.

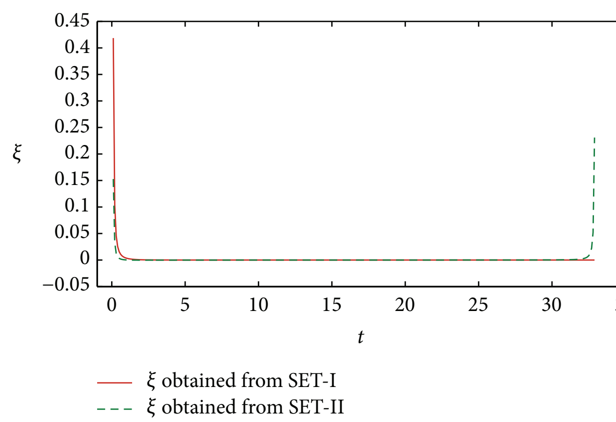


FIGURE 4: Coefficient of bulk viscosity $\xi$ versus time $t$ for $\mu = 0.5$, $\epsilon_0 = 1/3$, $\gamma = 1/4$, $\beta = 1.5$, $\alpha = 0.1$, $A_1 = 1$, $R_1 = 1$, $n = 1.6$, and $k = 0.097$.

where $R_1, R_2, R_3, c_1, c_2$, and $c_3$ are constants of integration. The last two values of $R$ give the constant deceleration parameter. So we neglect these values of $R$ as $q = \text{constant}$ is studied by Naidu et al. [50]. Thus we focus on the first value of scale factor. The scale factor $R$ can also be expressed as follows for $k > 0$ and $n > 1$ as [57]:

$$R = R_1 \exp\left[\frac{2}{n}\operatorname{arctanh}\left(\frac{kt-n}{n}\right)\right]. \tag{30}$$

With the help of (12), (13), and (29) we have obtained

$$\begin{aligned} A &= R_1 \exp\left[\frac{2}{n}\operatorname{arctanh}\left(\frac{kt-n}{n}\right)\right], \\ B &= (R_1)^{2m/(m+1)} \exp\left[\frac{4m}{n(m+1)}\operatorname{arctanh}\left(\frac{kt-n}{n}\right)\right], \\ C &= (R_1)^{2/(m+1)} \exp\left[\frac{4}{n(m+1)}\operatorname{arctanh}\left(\frac{kt-n}{n}\right)\right]. \end{aligned} \tag{31}$$

From (9), with the help of (31), we found the string tension density ($\lambda$) as

$$\lambda = \frac{2(m-1)(kt-n+3)}{(m+1)(4\pi+\mu)t^2(2n-kt)^2}. \tag{32}$$

From (10) with the help of (14) and (32), we found the rest energy density ($\rho$):

$$\begin{aligned} \rho = \frac{2}{8\pi+\mu(3-5\epsilon_0+5\gamma)} &\left[\frac{A_{14}}{t(2n-kt)^2} \right. \\ &+ \frac{A_{15}}{t^2(2n-kt)^2} \\ &\left. - \frac{3\alpha^2}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right], \end{aligned} \tag{33}$$

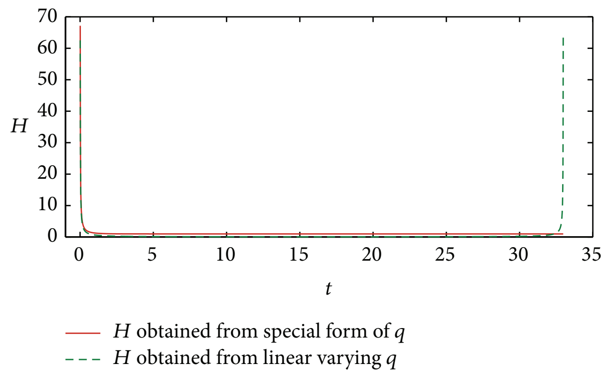


FIGURE 5: Hubble parameter $H$ versus time $t$ for $\beta = 1.5$, $A_1 = 1$, $n = 1.6$, and $k = 0.097$.

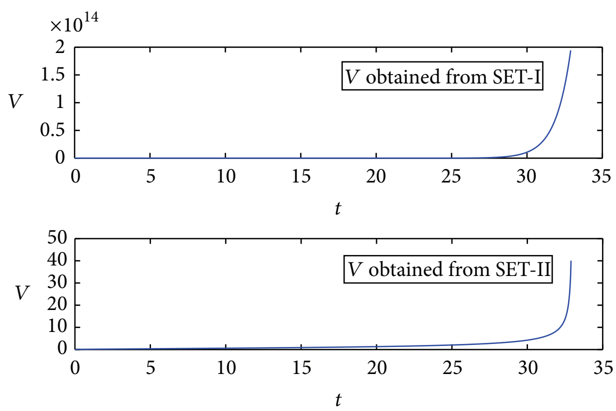


FIGURE 6: Spatial volume $V$ versus time $t$ for $\beta = 1.5$, $A_1 = 1$, $n = 1.6$, $k = 0.097$, and $R_1 = 1$.

where

$$A_{14} = \frac{k\mu(1-m)}{(m+1)(4\pi+\mu)},$$
$$A_{15} = \frac{(16\pi+\mu+n\mu)m^2 + 16(4\pi+\mu)m + 16\pi + 7\mu - n\mu}{(m+1)^2(4\pi+\mu)}. \quad (34)$$

From (14), with the help of (33), we obtained the total pressure ($\overline{p}$), proper pressure ($p$), and the coefficient of bulk viscosity ($\xi$) as follows:

$$\overline{p} = \frac{2(\epsilon_0-\gamma)}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{A_{14}}{t(2n-kt)^2} + \frac{A_{15}}{t^2(2n-kt)^2} - \frac{3\alpha^2}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right], \quad (35)$$

$$p = \frac{2\epsilon_0}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{A_{14}}{t(2n-kt)^2} + \frac{A_{15}}{t^2(2n-kt)^2} - \frac{3\alpha^2}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right], \quad (36)$$

$$\xi = \frac{\gamma}{3[8\pi+\mu(3-5\epsilon_0+5\gamma)]}\left[\frac{A_{14}}{2n-kt} + \frac{A_{15}}{t(2n-kt)} - \frac{3\alpha^2 t(2n-kt)}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right]. \quad (37)$$

Similar to that of (26), here also we get $m = 1$. Thus, for $m = 1$, the expressions in (31)–(37) take the form

$$A = B = C = R_1\exp\left[\frac{2}{n}\operatorname{arctanh}\left(\frac{kt-n}{n}\right)\right],$$
$$\lambda = 0,$$
$$\rho = \frac{3}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{4}{t^2(2n-kt)^2} - \frac{\alpha^2}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right],$$
$$\overline{p} = \frac{3(\epsilon_0-\gamma)}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{4}{t^2(2n-kt)^2} - \frac{\alpha^2}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right], \quad (38)$$
$$p = \frac{3\epsilon_0}{8\pi+\mu(3-5\epsilon_0+5\gamma)}\left[\frac{4}{t^2(2n-kt)^2} - \frac{\alpha^2}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right],$$
$$\xi = \frac{\gamma}{2[8\pi+\mu(3-5\epsilon_0+5\gamma)]}\left[\frac{4}{t(2n-kt)} - \frac{\alpha^2 t(2n-kt)}{2R_1^2\exp[(4/n)\operatorname{arctanh}((kt-n)/n)]}\right].$$

*3.2.1. Some Physical Properties of the Model.* Here some physical properties of the model are given below:

(i) The spatial volume $V = R_1\exp[(2/n)\operatorname{arctanh}((kt-n)/n)]$.
(ii) The scalar of expansion $\theta = 6/t(2n-kt)$.
(iii) The mean Hubble parameter $H = 2/t(2n-kt)$.
(iv) The mean anisotropy parameter $A_h = 0$.
(v) The shear scalar $\sigma^2 = 0$.

In this model, the energy density ($\rho$), proper pressure ($p$), total pressure ($\overline{p}$), coefficient of bulk viscosity ($\xi$), and Hubble parameter ($H$) gradually decrease with the evolution of time but when $t =\sim 33$, all of them diverge (see Figures 1–5). Spatial volume ($V$) increases with the evolution of time up to $t =\sim 33$; after that it also diverges (see Figure 6). This model is isotropic as the average anisotropy parameter is zero and it is also shear-free. In this model also cosmic string does not survive.

## 4. Graphical Results

See Figures 1, 2, 3, 4, 5, and 6.

## 5. Concluding Remarks

In this work we have studied the Bianchi type-V bulk viscous string cosmological model in $f(R,T)$ theory of gravity with variable deceleration parameters. This work is an extension of the earlier work of Naidu et al. [50], in which they solve the field equations with constant deceleration parameter along with two other conditions. Here we found that in one of our models the behavior of the physical and kinematical parameters is same as that of Naidu et al. [50] and these parameters in our other model behave differently. In both the models cosmic strings do not survive. We observed that the type of time variations of deceleration parameter considered here does not affect the nonexistence of cosmic string in this model. Hence the consideration of variable deceleration parameter does not contribute towards the existence of cosmic strings in this theory for Bianchi type-V space time. We have also investigated the stability of our system by the use of speed of sound in viscous fluid. The condition for the stability of the theory is $C_s^2 = d\overline{p}/d\rho \geq 0$. In both of our models we obtained that $C_s^2 = \epsilon_0 - \gamma$. Here the theory is stable if $\epsilon_0 - \gamma \geq 0$; that is, $\epsilon_0 \geq \gamma$.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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