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Research Article

Helmholtz and Diffusion Equations Associated with Local Fractional Derivative Operators Involving the Cantorian and Cantor-Type Cylindrical Coordinates

Ya-Juan Hao,[1] H. M. Srivastava,[2] Hossein Jafari,[3] and Xiao-Jun Yang[4]

[1] *College of Science, Yanshan University, Qinhuangdao 066004, China*
[2] *Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia, Canada V8W 3R4*
[3] *Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar 47415-416, Iran*
[4] *Department of Mathematics and Mechanics, China University of Mining and Technology, Jiangsu, Xuzhou 221008, China*

Correspondence should be addressed to Ya-Juan Hao; moonhyj@sina.com.cn

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The main object of this paper is to investigate the Helmholtz and diffusion equations on the Cantor sets involving local fractional derivative operators. The Cantor-type cylindrical-coordinate method is applied to handle the corresponding local fractional differential equations. Two illustrative examples for the Helmholtz and diffusion equations on the Cantor sets are shown by making use of the Cantorian and Cantor-type cylindrical coordinates.

1. Introduction

In the Euclidean space, we observe several interesting physical phenomena by using the differential equations in the different styles of planar, cylindrical, and spherical geometries. There are many models for the anisotropic perfectly matched layers [1], the plasma source ion implantation [2], fractional paradigm and intermediate zones in electromagnetism [3, 4], fusion [5], reflectionless sponge layers [6], time-fractional heat conduction [7], singular boundary value problems [8], and so on (see also the references cited in each of these works).

The Helmholtz equation was applied to deal with problems in such fields as electromagnetic radiation, seismology, transmission, and acoustics. Kreß and Roach [9] discussed the transmission problems for the Helmholtz equation. Kleinman and Roach [10] studied the boundary integral equations for the three-dimensional Helmholtz equation. Karageorghis [11] presented the eigenvalues of the Helmholtz equation. Heikkola et al. [12] considered the parallel fictitious domain method for the three-dimensional Helmholtz equation. Fu and Mura [13] suggested the volume integrals of the inhomogeneous Helmholtz equation. Samuel and Thomas [14] proposed the fractional Helmholtz equation.

Diffusion theory has become increasingly interesting and potentially useful in solids [15, 16]. Some applications of physics, such as superconducting alloys [17], lattice theory [18], and light diffusion in turbid material [19], were considered. Fractional calculus theory (see [20–28]) was applied to model the diffusion problems in engineering, and fractional diffusion equation was discussed (see, e.g., [29–36]).

Recently, the local fractional calculus theory was applied to process the nondifferentiable phenomena in fractal domain (see [37–48] and the references cited therein). There are some local fractional models, such as the local fractional Fokker-Planck equation [37], the local fractional stress-strain relations [38], the local fractional heat conduction equation [45], wave equations on the Cantor sets [47], and the local fractional Laplace equation [48].

The main aim of this paper is present in the mathematical structure of the Helmholtz and diffusion equations within local fractional derivative and to propose their forms in the Cantor-type cylindrical coordinates by using the Cantor-type cylindrical-coordinate method [46].

Our present investigation is structured as follows. In Section 2, the Helmholtz equation on the Cantor sets with local fractional derivative is investigated. The diffusion equation on the Cantor sets based upon the local fractional vector calculus is structured in Section 3. The Helmholtz and diffusion equations in the Cantor-type cylindrical coordinates are studied in Section 4. Finally, the conclusions are presented in Section 5.

2. The Helmholtz Equation on the Cantor Sets

In order to derive the Helmholtz equation on the Cantor sets, if the local fractional derivative is defined through [43–46]

$$f^{(\alpha)}(x_0) = \left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0} = \lim_{x\to x_0}\frac{\Delta^\alpha (f(x)-f(x_0))}{(x-x_0)^\alpha} \quad (1)$$

with

$$\Delta^\alpha (f(x)-f(x_0)) \cong \Gamma(1+\alpha)\,\Delta(f(x)-f(x_0)), \quad (2)$$

then the wave equation on the Cantor sets was suggested in [44] by

$$\nabla^{2\alpha} u(r,t) = \frac{1}{a^{2\alpha}}\frac{\partial^{2\alpha} u(r,t)}{\partial t^{2\alpha}}, \quad (3)$$

where the local fractional Laplace operator is given by [43, 44, 48]

$$\nabla^{2\alpha} = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}, \quad (4)$$

where $1/a^{2\alpha}$ is a constant and $u(r,t)$ is satisfied with local fractional continuous conditions (see [47]).

Using separation of variables in nondifferentiable functions, which begins by assuming that the fractal wave function $u(r,t)$ may be separable, namely,

$$u(r,t) = M(r)\,T(t), \quad (5)$$

we have

$$\frac{\nabla^{2\alpha} M(r)}{M(r)} = \frac{1}{a^{2\alpha}T(t)}\frac{\partial^{2\alpha} T(t)}{\partial t^{2\alpha}}, \quad (6)$$

such that

$$\nabla^{2\alpha} M(r) + \omega^{2\alpha} M(r) = 0, \quad (7)$$

$$\frac{1}{a^{2\alpha}T(t)}\frac{\partial^{2\alpha} T(t)}{\partial t^{2\alpha}} = -\omega^{2\alpha}. \quad (8)$$

In the three-dimensional Cantorian coordinate system, by following (7), we have

$$\frac{\partial^{2\alpha} M(x,y,z)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x,y,z)}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha} M(x,y,z)}{\partial z^{2\alpha}} + \omega^{2\alpha} M(x,y,z) = 0, \quad (9)$$

where the operator is a local fractional derivative operator.

For the two-dimensional Cantorian coordinate system, the local fractional homogeneous Helmholtz equation is given by

$$\frac{\partial^{2\alpha} M(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x,y)}{\partial y^{2\alpha}} + \omega^{2\alpha} M(x,y) = 0. \quad (10)$$

For a fractal dimension $\alpha = 1$, (9) becomes

$$\frac{\partial^{2} M(x,y,z)}{\partial x^{2}} + \frac{\partial^{2} M(x,y,z)}{\partial y^{2}} + \frac{\partial^{2} M(x,y,z)}{\partial z^{2}} + \omega^{2} M(x,y,z) = 0, \quad (11)$$

which is the classical Helmholtz equation [10].

In view of (9), the inhomogeneous Helmholtz equation reads as follows:

$$\frac{\partial^{2\alpha} M(x,y,z)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x,y,z)}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha} M(x,y,z)}{\partial z^{2\alpha}} + \omega^{2\alpha} M(x,y,z) = f(x,y,z), \quad (12)$$

where $f(x,y,z)$ is a local fractional continuous function.

In the two-dimensional Cantorian coordinate system, following (12), the local fractional inhomogeneous Helmholtz equation can be suggested by

$$\frac{\partial^{2\alpha} M(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x,y)}{\partial y^{2\alpha}} + \omega^{2\alpha} M(x,y) = f(x,y), \quad (13)$$

where $f(x,y)$ is a local fractional continuous function.

We notice that the fractional Helmholtz equation was applied to deal with the differentiable wave equations in [14]. However, the Helmholtz equation with local fractional derivative arises in physical problems in such areas as, for example, fractal electromagnetic radiation, seismology, and acoustics, because their wave functions are the local fractional continuous functions (nondifferentiable functions). So, the Helmholtz equation on the Cantor sets can be used to describe the fractal electromagnetic radiation, the fractal seismology, the fractal acoustics, and so on.

3. Diffusion Equation on the Cantor Sets

In this section, we derive the diffusion equation on the Cantor sets with local fractional vector calculus [44].

Let us recall Fick's law within the local fractional derivative, which was presented as

$$\mathbf{J}(r,t) = -D(\varphi)\,\nabla^\alpha \varphi(r,t), \quad (14)$$

where $\varphi(r,t)$ and $\mathbf{J}(r,t)$ are local fractional continuous functions.

It is noticed that the flux of the diffusing material in any part of the fractal system is proportional to the local fractional density gradient. If the diffusion coefficient $D(\varphi) = D$ is constant, the local fractional Fick law was suggested as [44]

$$\mathbf{J}(r,t) = -D\nabla^\alpha \varphi(r,t), \quad (15)$$

which was expressed as [44]

$$\oiint \mathbf{J}(r,t)\cdot d\mathbf{S}^{(\beta)} = -\oiint D(\varphi)\,\nabla^\alpha \varphi(r,t)\cdot d\mathbf{S}^{(\beta)}, \quad (16)$$

where the local fractional vector integral is defined as [44]

$$\iint \mathbf{u}(r_P)\cdot d\mathbf{S}^{(\beta)} = \lim_{N\to\infty}\sum_{P=1}^{N}\mathbf{u}(r_P)\cdot \mathbf{n}_P \Delta S_P^{(\beta)}, \quad (17)$$

with N elements of area with a unit normal local fractional vector $\mathbf{n}_P$, $\Delta S_P^{(\beta)} \to 0$ as $N \to \infty$ for $\beta = 2\alpha$, and $\varphi(r,t)$ is the density of the diffusing material in local fractional field.

The conservation of mass within local fractional vector operator was presented as [44]

$$\frac{d^\alpha}{dt^\alpha} \iiint \varphi(r,t)\, dV^{(\gamma)} = -\oiint \mathbf{J}(r,t) \cdot d\mathbf{S}^{(\beta)}, \quad (18)$$

where local fractional volume integral is given by [44]

$$\iiint \mathbf{u}(r_P)\, dV^{(\gamma)} = \lim_{N\to\infty} \sum_{P=1}^{N} \mathbf{u}(r_P)\, \Delta V_P^{(\gamma)}, \quad (19)$$

with N elements of volume $\Delta V_P^{(\gamma)} \to 0$ as $N \to \infty$ for $\gamma = (3/2)\beta = 3\alpha$.

Following (18), and by using the divergence theorem of local fractional field [44], we have

$$\frac{d^\alpha \varphi(r,t)}{dt^\alpha} + \nabla^\alpha \cdot \mathbf{J}(r,t) = 0, \quad (20)$$

where $\mathbf{J}(r,t)$ is the flux of the diffusing material in local fractional field.

Submitting (14) into (20), we obtain

$$\frac{d^\alpha \varphi(r,t)}{dt^\alpha} + \nabla^\alpha \left[-D(\varphi)\nabla^\alpha \varphi(r,t)\right] = 0, \quad (21)$$

which is the so-called diffusion equation on the Cantor sets. This result differs from the fractional diffusion equation [29–36].

For the diffusion coefficient $D(\varphi) = D$, (21) becomes

$$\frac{d^\alpha \varphi(r,t)}{dt^\alpha} = D\nabla^{2\alpha}\varphi(r,t). \quad (22)$$

In the three-dimensional Cantorian coordinate system, following (22), we have

$$\frac{d^\alpha \varphi(x,y,z,t)}{dt^\alpha} = D\left[\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\varphi(x,y,z,t) + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}}\varphi(x,y,z,t) + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}\varphi(x,y,z,t)\right]. \quad (23)$$

In the two-dimensional Cantorian coordinate system, we get

$$\frac{d^\alpha \varphi(x,y,t)}{dt^\alpha} = D\left[\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\varphi(x,y,t) + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}}\varphi(x,y,t)\right]. \quad (24)$$

In the one-dimensional Cantorian coordinate system, we obtain [48]

$$\frac{d^\alpha \varphi(x,t)}{dt^\alpha} = D\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\varphi(x,t). \quad (25)$$

We notice that when fractal dimension α is equal to 1, we get the classical diffusion equation [15, 16]. However, the diffusion equation on the Cantor sets with local fractional derivative is derived from local fractional field, whose quantities are local fractional continuous functions.

4. The Cantor-Type Cylindrical-Coordinate Method to the Helmholtz and Diffusion Equations on the Cantor Sets

Let us consider the Cantor-type cylindrical coordinates, which read as follows:

$$\begin{aligned} x^\alpha &= R^\alpha \cos_\alpha \theta^\alpha, \\ y^\alpha &= R^\alpha \sin_\alpha \theta^\alpha, \\ z^\alpha &= z^\alpha, \end{aligned} \quad (26)$$

with $R \in (0, +\infty)$, $z \in (-\infty, +\infty)$, $\theta \in (0, \pi]$, and $x^{2\alpha} + y^{2\alpha} = R^{2\alpha}$.

We now have a local fractional vector given by

$$\begin{aligned} \mathbf{r} &= R^\alpha \cos_\alpha \theta^\alpha \mathbf{e}_1^\alpha + R^\alpha \sin_\alpha \theta^\alpha \mathbf{e}_2^\alpha + z^\alpha \mathbf{e}_3^\alpha \\ &= r_R \mathbf{e}_R^\alpha + r_\theta \mathbf{e}_\theta^\alpha + r_z \mathbf{e}_z^\alpha, \end{aligned} \quad (27)$$

such that [46]

$$\nabla^\alpha \phi(R,\theta,z) = \mathbf{e}_R^\alpha \frac{\partial^\alpha}{\partial R^\alpha}\phi + \mathbf{e}_\theta^\alpha \frac{1}{R^\alpha}\frac{\partial^\alpha}{\partial \theta^\alpha}\phi + \mathbf{e}_z^\alpha \frac{\partial^\alpha}{\partial z^\alpha}\phi, \quad (28)$$

$$\nabla^{2\alpha}\phi(R,\theta,z) = \frac{\partial^{2\alpha}}{\partial R^{2\alpha}}\phi + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}}{\partial \theta^{2\alpha}}\phi + \frac{1}{R^\alpha}\frac{\partial^\alpha}{\partial R^\alpha}\phi + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}\phi, \quad (29)$$

$$\nabla^\alpha \cdot \mathbf{r} = \frac{\partial^\alpha r_R}{\partial R^\alpha} + \frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} + \frac{r_R}{R^\alpha} + \frac{\partial^\alpha r_z}{\partial z^\alpha}, \quad (30)$$

$$\begin{aligned} \nabla^\alpha \times \mathbf{r} = {} & \left(\frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha r_\theta}{\partial z^\alpha}\right)\mathbf{e}_R^\alpha + \left(\frac{\partial^\alpha r_R}{\partial z^\alpha} - \frac{\partial^\alpha r_z}{\partial R^\alpha}\right)\mathbf{e}_\theta^\alpha \\ & + \left(\frac{\partial^\alpha r_\theta}{\partial R^\alpha} + \frac{r_R}{R^\alpha} - \frac{1}{R^\alpha}\frac{\partial^\alpha r_R}{\partial \theta^\alpha}\right)\mathbf{e}_z^\alpha, \end{aligned} \quad (31)$$

where

$$\begin{aligned} \mathbf{e}_R^\alpha &= \cos_\alpha \theta^\alpha \mathbf{e}_1^\alpha + \sin_\alpha \theta^\alpha \mathbf{e}_2^\alpha, \\ \mathbf{e}_\theta^\alpha &= -\sin_\alpha \theta^\alpha \mathbf{e}_1^\alpha + \cos_\alpha \theta^\alpha \mathbf{e}_2^\alpha, \\ \mathbf{e}_z^\alpha &= \mathbf{e}_3^\alpha. \end{aligned} \quad (32)$$

Submitting (29) into (9) and (12), it yields

$$\begin{aligned} & \frac{\partial^{2\alpha} M(R,\theta,z)}{\partial R^{2\alpha}} + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha} M(R,\theta,z)}{\partial \theta^{2\alpha}} + \frac{1}{R^\alpha}\frac{\partial^\alpha M(R,\theta,z)}{\partial R^\alpha} \\ & \quad + \frac{\partial^{2\alpha} M(R,\theta,z)}{\partial z^{2\alpha}} + \omega^{2\alpha} M(R,\theta,z) = 0, \\ & \frac{\partial^{2\alpha} M(R,\theta,z)}{\partial R^{2\alpha}} + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha} M(R,\theta,z)}{\partial \theta^{2\alpha}} + \frac{1}{R^\alpha}\frac{\partial^\alpha M(R,\theta,z)}{\partial R^\alpha} \\ & \quad + \frac{\partial^{2\alpha} M(R,\theta,z)}{\partial z^{2\alpha}} + \omega^{2\alpha} M(R,\theta,z) = f(R,\theta,z), \end{aligned} \quad (33)$$

which is the Helmholtz equation in the Cantor-type cylindrical coordinates.

In the like manner, from (23), we get

$$\frac{d^{\alpha}\varphi(R,\theta,z,t)}{dt^{\alpha}} = D\left[\frac{\partial^{2\alpha}\varphi(R,\theta,z,t)}{\partial R^{2\alpha}} + \frac{1}{R^{2\alpha}}\frac{\partial^{2\alpha}\varphi(R,\theta,z,t)}{\partial\theta^{2\alpha}} + \frac{1}{R^{\alpha}}\frac{\partial^{\alpha}\varphi(R,\theta,z,t)}{\partial R^{\alpha}} + \frac{\partial^{2\alpha}\varphi(R,\theta,z,t)}{\partial z^{2\alpha}}\right], \quad (34)$$

which is the diffusion equation in the Cantor-type cylindrical coordinates.

5. Concluding Remarks and Observations

In the present work, we have derived the Helmholtz and diffusion equations on the Cantor sets in the Cantorian coordinates, which are based upon the local fractional derivative operators. By applying the Cantor-type cylindrical-coordinate method, we have also investigated the Helmholtz and diffusion equations on the Cantor sets in the Cantor-type cylindrical coordinates. Furthermore, we have presented two illustrative examples for the corresponding fractional Helmholtz and diffusion equations on the Cantor sets by using the Cantorian and Cantor-type cylindrical coordinates.

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