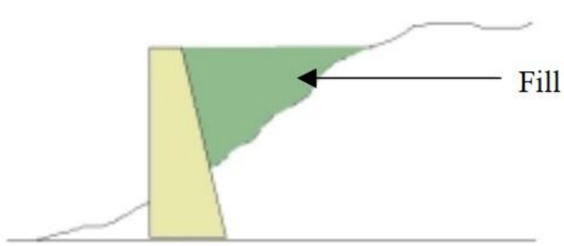
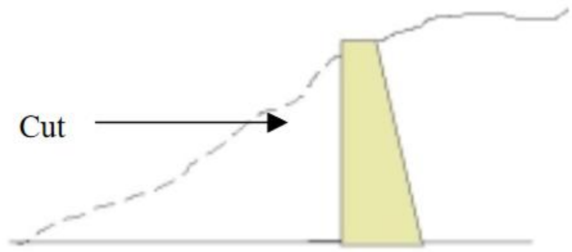
Retaining Wall

-Lecture Note by Dr. E. R. Latifee

Retaining walls are usually built to support or hold back (retain) soil mass. However, retaining walls can also be constructed for aesthetic landscaping purposes. Retaining walls are structures that support backfill and allow for a change of grade. For instance a retaining wall can be used to retain fill along a slope or it can be used to support a cut into a slope



Retaining Wall to Support a Fill.

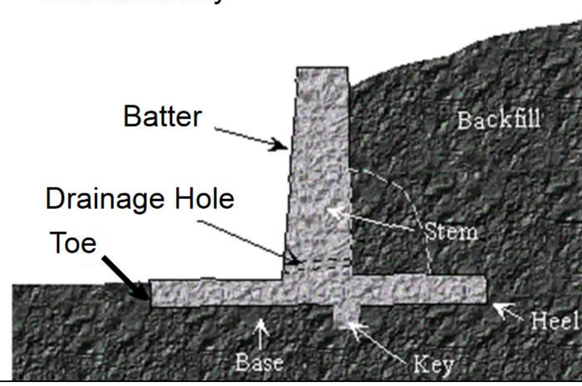


Retaining Wall to Support a Cut.

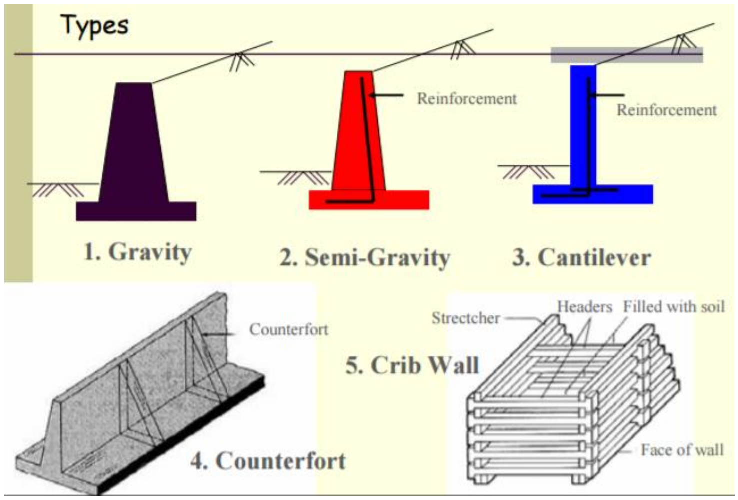
Classification of
Retaining walls

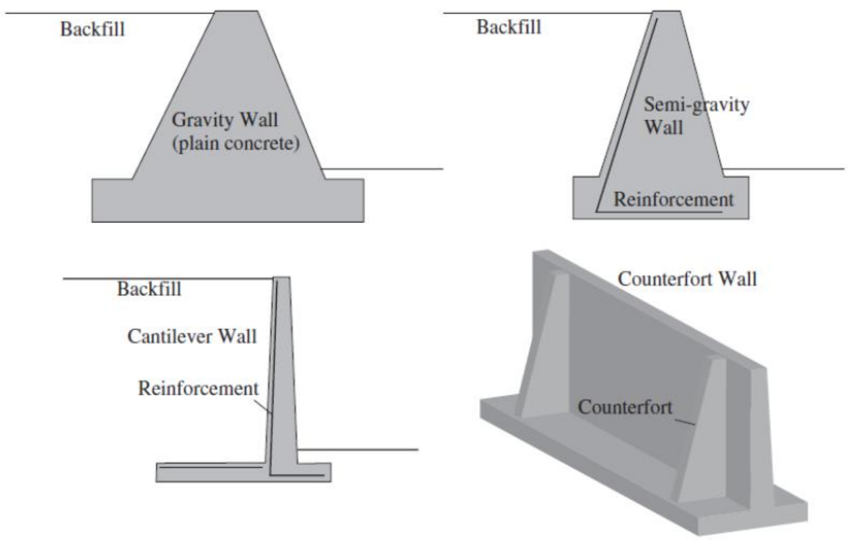
- Gravity wall-Masonry or Plain concrete
- Cantilever retaining wall-RCC (Inverted T and L)
- Counterfort retaining wall-RCC
- Buttress wall-RCC

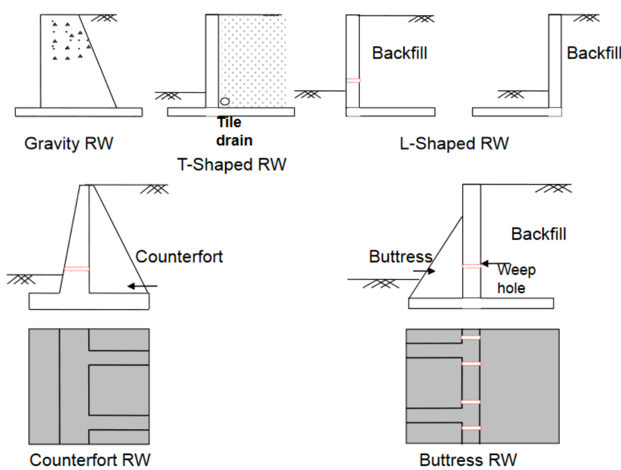
Cantilever Retaining wall
with shear key



Photos of Retaining walls







The gravity retaining wall is used for walls of up to about 10 ft to 12 ft in height. It is usually constructed with plain concrete and depends completely on its own weight for stability against sliding and overturning. It is usually so massive that it is unreinforced.

Semigravity retaining walls fall between the gravity and cantilever types (to be discussed in the next paragraph). They depend on their own weights plus the weight of some soil behind the wall to provide stability. Semigravity walls are used for approximately the same range of heights as the gravity walls and usually have some light reinforcement.

The cantilever retaining wall or one of its variations is the most common type of retaining wall. Such walls are generally used for heights from about 10 ft to 25 ft. In discussing retaining walls, the vertical wall is referred to as the stem. The outside part of the footing that is pressed down into the soil is called the toe, while the part that tends to be lifted is called the heel.

When it is necessary to construct retaining walls of greater heights than approximately 20 ft to 25 ft, the bending moments at the junction of the stem and footing become so large that the designer will, from economic necessity, have to consider other types of walls to handle the moments. This can be done by introducing vertical cross walls on the front or back of the stem. If the cross walls are behind the stem (i.e., inside the soil) and not visible, the retaining walls are called counterfort walls. Should the cross walls be visible (i.e., on the toe side), the walls are called buttress walls.

The stems for these walls are continuous members supported at intervals by the buttresses or counterforts. Counterforts or buttresses are usually spaced at distances approximately equal to one-half (or a little more) of the retaining wall heights.

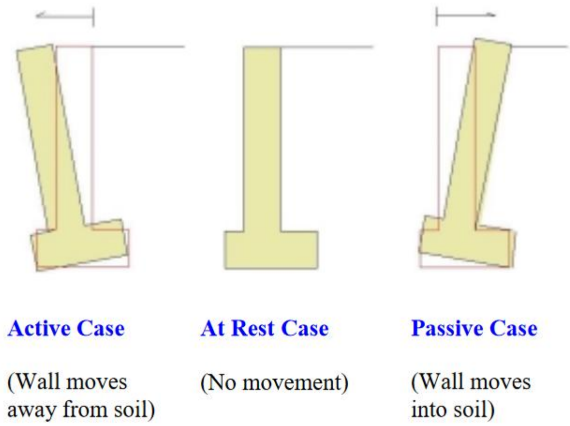
Categories of Lateral Earth Pressure

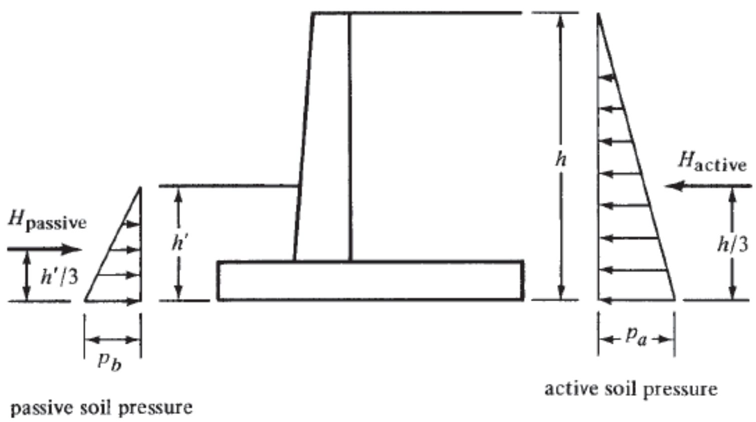
There are three categories of lateral earth pressure and each depends upon the movement experienced by the vertical wall on which the pressure is acting

The three categories are:

- **At rest-earth pressure**
- **Active earth pressure**
- **Passive earth pressure**

The at rest pressure develops when the wall experiences no lateral movement. This typically occurs when the wall is restrained from movement such as along a basement wall that is restrained at the bottom by a slab and at the top by a floor framing system prior to placing soil backfill against the wall. The active pressure develops when the wall is free to move outward such as a typical retaining wall and the soil mass stretches sufficiently to mobilize its shear strength. On the other hand, if the wall moves into the soil, then the soil mass is compressed, which also mobilizes its shear strength and the passive pressure develops. This situation might occur along the section of wall that is below grade and on the opposite side of the retained section of fill.





Lateral earth pressure is related to the vertical earth pressure by a coefficient termed the:

- At Rest Earth Pressure Coefficient (Ko)
- Active Earth Pressure Coefficient (Ka)
- Passive Earth Pressure Coefficient (Kp)

If a linear pressure variation is assumed, the active pressure at any depth can be determined as

$$p_a = k_a w h$$

or, for passive pressure,

$$p_p = k_p w h'$$

In these expressions, *ka* and *kp* are the approximate coefficients of active and passive pressures, respectively. These coefficients can be calculated by theoretical equations such as those of Rankine or Coulomb.2 For a granular material, typical values of *ka* and *kp* are 0.3 and 3.3. The Rankine equation (published in 1857) neglects the friction of the soil on the wall, whereas the Coulomb formula (published in 1776) takes it into consideration. These two equations were developed for cohesionless soils. For cohesive soils containing clays and/or silts, it is necessary to use empirical values determined from field measurements.

It has been estimated that the cost of constructing retaining walls varies directly with the square of their heights. Thus, as retaining walls become higher, the accuracy of the computed lateral pressures becomes more and more important in providing economical designs. Since the Coulomb equation does take into account friction on the wall, it is thought to be the more accurate one and is often used for walls of over 20 ft. The Rankine equation is commonly used for ordinary retaining walls of 20 ft or less in height. It is interesting to note that the two methods give identical results if the friction of the soil on the wall is neglected.

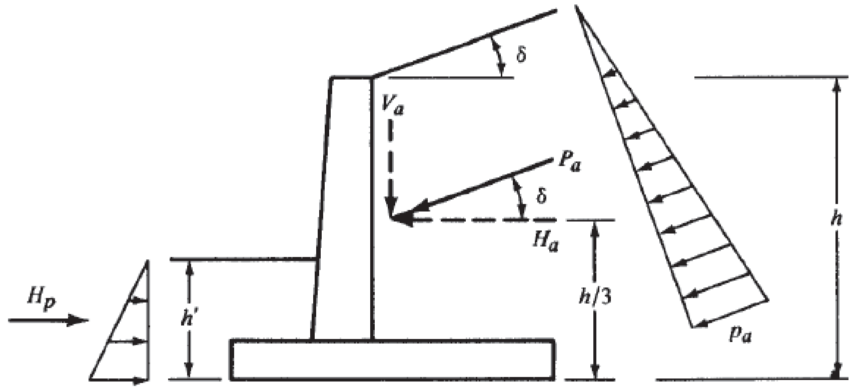


Figure: Active and passive soil pressures with sloping backfill.

The Rankine expressions for the active and passive pressure coefficients are given below . In these expressions, δ is the angle the backfill makes with the horizontal, while φ is the angle of internal friction of the soil. For well-drained sand or gravel backfills, the angle of internal friction is often taken as the angle of repose of the slope. One common slope used is 1 vertically to 1.5 horizontally (33 degree 40 minute).

$$k_a = \cos\delta\left(\frac{\cos\delta - \sqrt{\cos^2\delta - \cos^2\phi}}{\cos\delta + \sqrt{\cos^2\delta - \cos^2\phi}}\right)$$

$$k_p = \cos\delta\left(\frac{\cos\delta + \sqrt{\cos^2\delta - \cos^2\phi}}{\cos\delta - \sqrt{\cos^2\delta - \cos^2\phi}}\right)$$

Should the backfill be horizontal—that is, should δ be equal to zero—the expressions become

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$k_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$H_a = \left(\frac{1}{2}\right)(p_a)(h) = \left(\frac{1}{2}\right)(k_a wh)(h)$ $H_a = \frac{k_a wh^2}{2}$	Passive Pressure $H_p = \frac{k_p wh'^2}{2}$

Footing Soil Pressure

Because of lateral forces, the resultant of the horizontal and vertical forces, R, intersects the soil underneath the footing as an eccentric load, causing greater pressure at the toe. This toe pressure should be less than the permissible value, q_a, of the particular soil. It is also desirable to keep the resultant force within the kern or the middle third of the footing base.

If the resultant force intersects the soil within the middle third of the footing, the soil pressure at any point can be calculated with the formula to follow exactly as the stresses are determined in an eccentrically loaded column.

$$q = -\frac{R_v}{A} \pm \frac{R_v ec}{I}$$

In this expression, R_v is the vertical component of R or the total vertical load, e is the eccentricity of the load from the center of the footing, A is the area of a 1-ft-wide strip of soil of a length equal to the width of the footing base, and I is the moment of inertia of the same area about its centroid. This expression is correct only if R_v falls within the kern.

This expression can be reduced to the following expression, in which L is the width of the footing from heel to toe.

$$q = -\frac{R_v}{L} \pm \frac{R_v e(L/2)}{L^3/12} = -\frac{R_v}{L}\left(1 \pm \frac{6e}{L}\right)$$

If the resultant force falls outside of the middle third of the footing, the preceding expressions are not applicable because they indicate a tensile stress on one side of the footing—a Such case should not be permitted.

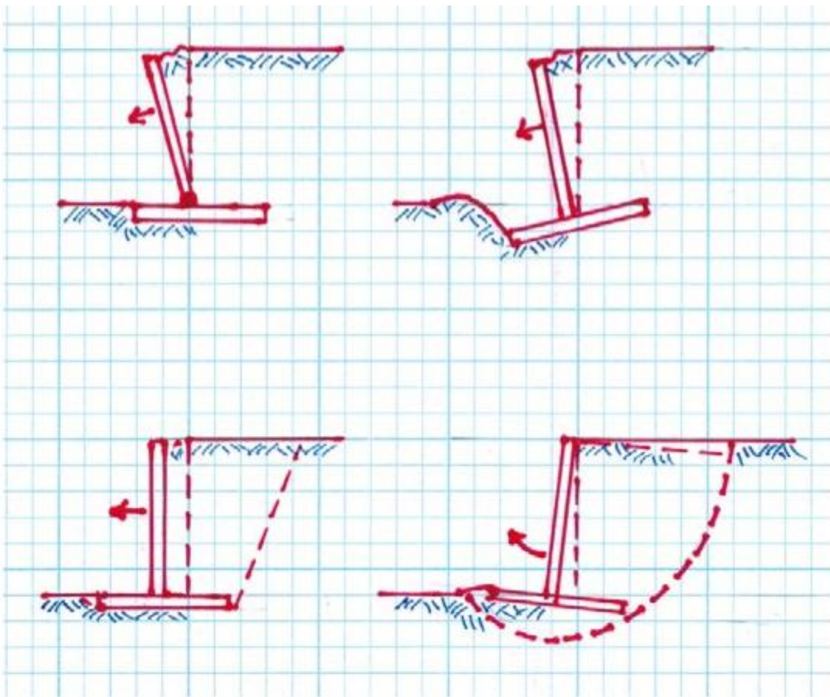
Possible modes of failure for free -standing concrete cantilever retaining walls

a) **Wall stem structural failure:** The wall stem fails in bending. Most likely location is at the base of the wall where the stem connects to the foundation.

b) **Foundation bearing failure:** A bearing failure of the soil under the toe of the foundation and a forwards rotation of the wall.

c) **Sliding failure of wall:** Possible mode for non-cohesive soils. Wall moves outwards with passive failure of soil in front of foundation and active failure of soil behind wall. Often a key is required beneath the foundation to prevent sliding.

d) **Deep seated rotational failure:** Possible mode for cohesive soils. Factor of safety controlled by increasing length of heel or depth of key. Factor of safety calculated using limiting equilibrium "Bishop" analysis or similar. Unlikely to govern design unless wall is embedded into sloping ground with sloping backfill or there is a weak layer at the toe of the wall.



Possible modes of failure for free-standing concrete cantilever retaining walls.

- Basic soil parameters;
  - Unit weight of soil
  - Angle of friction
  - Cohesion
- Then the lateral pressure distribution will be known.
- There are 2 phases in the design of a retaining wall;
  - The retaining wall is checked for stability: overturning, sliding and bearing capacity failures.
  - Each component of the retaining wall is checked for adequate strength and the steel reinforcement.

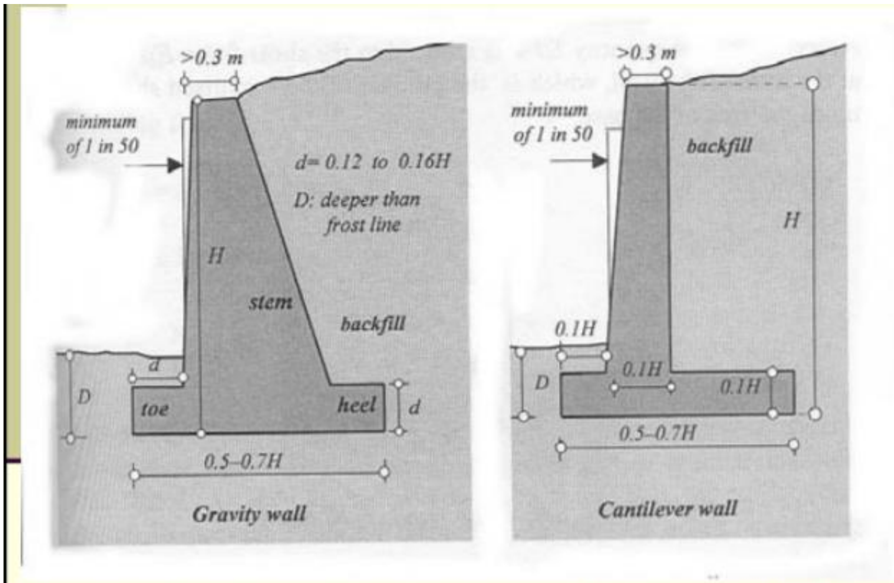
Factors affecting earth pressure:

Earth pressure depends on type of backfill, the height of wall and the soil conditions

Soil conditions: The different soil conditions are-•Dry leveled back fill

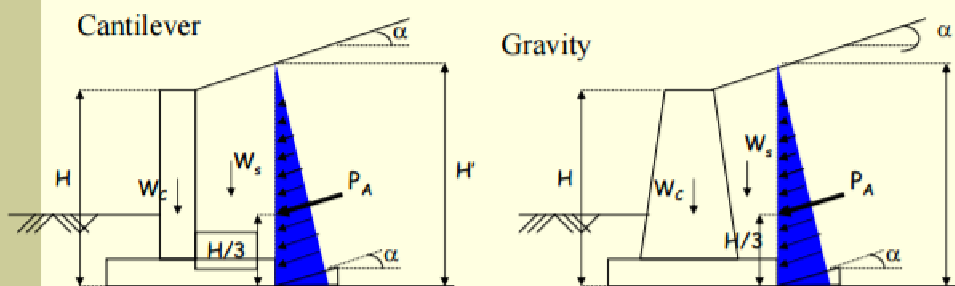
•Moist leveled backfill, •Submerged leveled backfill

•Leveled backfill with uniform surcharge •Backfill with sloping surface



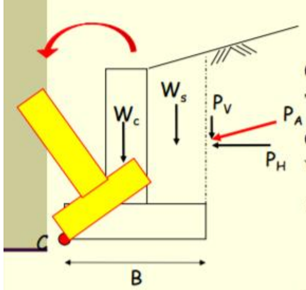
Empirical relationships related to the design of walls (Azizi, 2000)

Application of Lateral Earth Pressure Theories to Design



Retaining Wall Stability

1) Safety Against Overturning (Rotational stability) :



Consider forces W_C, W_S, P_V, P_H
Take moment w.r.t '**C**' (TOE)
clockwise : resisting (M_R) (W_C, W_S, P_V) a.clockwise :overturning (M_O) (P_H)

if not increase the base 'B' ;use piles ;increase wall dimensions.

- F_s=2 (for cohesive backfill) and 1.5 (for granular backfill)

Analysis for dry back fills

Maximum pressure at any height, $p = k_a \gamma h$

Total pressure at any height from top,

$p_a = 1/2[k_a \gamma h]h = [k_a \gamma h^2]/2$

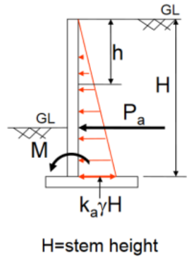
Bending moment at any height

$M = p_a x h/3 = [k_a \gamma h^3]/6$

$\therefore$ Total pressure, $P_a = [k_a \gamma H^2]/2$

$\therefore$ Total Bending moment at bottom,

$M = [k_a \gamma H^3]/6$



Where, k_a= Coefficient of active earth pressure

$= (1-\sin\phi)/(1+\sin\phi) = \tan^2\phi$

$= 1/k_p$, coefficient of passive earth pressure

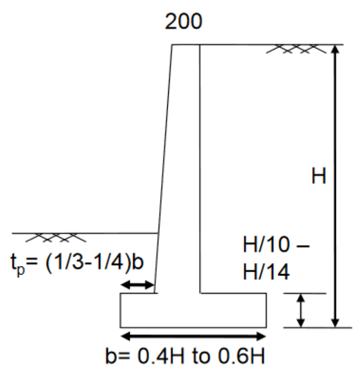
ϕ= Angle of internal friction or angle of repose

γ=Unit weigh or density of backfill

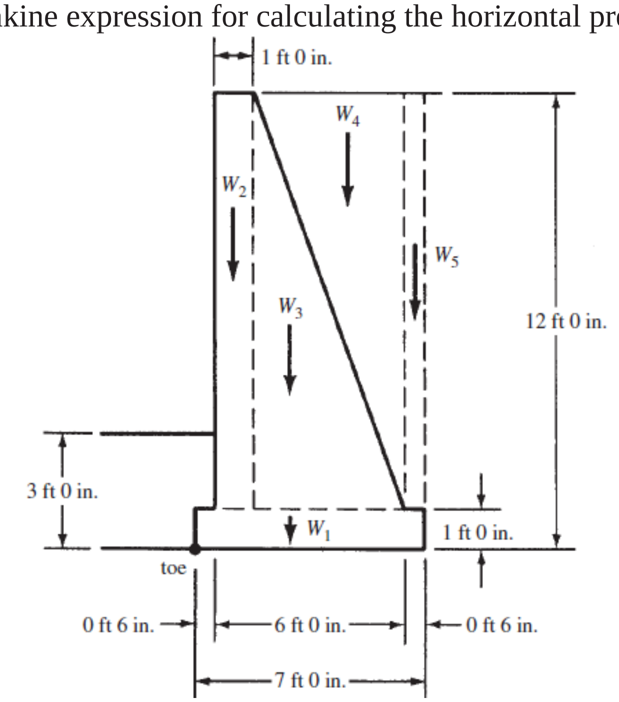
If $\phi = 30°$, $k_a = 1/3$ and $k_p = 3$.

Preliminary Proportioning (T shaped wall)

- Stem: Top width 200 mm to 400 mm
- Base slab width b= 0.4H to 0.6H, 0.6H to 0.75H for surcharged wall
- Base slab thickness= H/10 to H/14
- Toe projection= (1/3-1/4) Base width



Example 1: A semigravity retaining wall consisting of plain concrete (weight = 145 lb/ft3) is shown in the Figure below. The bank of supported earth is assumed to weigh 110 lb/ft3, to have a φ of 30◦, and to have a coefficient of friction against sliding on soil of 0.5. Determine the safety factors against overturning and sliding and determine the bearing pressure underneath the toe of the footing. Use the Rankine expression for calculating the horizontal pressures.



Solution:

Computing the Soil Pressure Coefficients

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - 0.5}{1 + 0.5} = 0.333$$

$$k_p = \frac{1 + \sin\phi}{1 - \sin\phi} = \frac{1 + 0.5}{1 - 0.5} = 3.00$$

Value of H_a

$$H_a = \frac{k_a w h^2}{2} = \frac{(0.333)(110 \text{ pcf})(12 \text{ ft})^2}{2} = 2637 \text{ lb/ft}$$

Overturning Moment

$$\text{O.T.M.} = (2637 \text{ lb/ft})\left(\frac{12 \text{ ft}}{3}\right) = 10{,}548 \text{ ft-lb/ft}$$

Resisting Moment:

Force		Moment Arm		Moment
$W_1 = (7)(1)(145 \text{ pcf})$	=	1,015 lb × 3.5 ft	=	3,552 ft-lb
$W_2 = (1)(11)(145 \text{ pcf})$	=	1,595 lb × 1.0 ft	=	1,595 ft-lb
$W_3 = (\frac{1}{2})(5)(11)(145 \text{ pcf})$	=	3,988 lb × 3.17 ft	=	12,642 ft-lb
$W_4 = (\frac{1}{2})(5)(11)(110 \text{ pcf})$	=	3,025 lb × 4.83 ft	=	14,611 ft-lb
$W_5 = (0.5)(11)(110 \text{ pcf})$	=	605 lb × 6.75 ft	=	4,084 ft-lb
R_v	=	10,228 lb M	=	36,484 ft-lb

Factor of Safety against Overturning,

$$\text{Safety factor} = \frac{36{,}484 \text{ ft-lb}}{10{,}548 \text{ ft-lb}} = 3.46 > 2.00 \quad \underline{\underline{\text{OK}}}$$

Factor of Safety against Sliding:

Assuming soil above the footing toe has eroded, and thus the passive pressure is due only to soil of a depth equal to footing thickness,

$$H_p = \frac{k_p w h'^2}{2} = \frac{(3.0)(110 \text{ pcf})(1 \text{ ft})^2}{2} = 165 \text{ lb}$$

$$\text{Safety factor against sliding} = \frac{(0.5)(10{,}228 \text{ lb}) + 165 \text{ lb}}{2637 \text{ lb}} = 2.00 > 1.50 \quad \underline{\underline{\text{OK}}}$$

Distance of Resultant from Toe

$$\text{Distance} = \frac{36{,}484 \text{ ft-lb} - 10{,}548 \text{ ft-lb}}{10{,}228 \text{ lb}} = 2.54 \text{ ft} > 2.33 \text{ ft} \qquad \underline{\underline{\therefore \text{ Inside middle third}}}$$

Soil Pressure Under Heel and Toe

$$A = (1\ \text{ft})(7.0\ \text{ft}) = 7.0\ \text{ft}^2$$

$$I = \left(\frac{1}{12}\right)(1\ \text{ft})(7\ \text{ft})^3 = 28.58\ \text{ft}^4$$

$$f_{\text{toe}} = -\frac{R_v}{A} - \frac{R_v ec}{I} = -\frac{10{,}228\ \text{lb}}{7.0\ \text{ft}^2} - \frac{(10{,}228\ \text{lb})(3.50\ \text{ft} - 2.54\ \text{ft})(3.50\ \text{ft})}{28.58\ \text{ft}^4}$$

$$= -1461\ \text{psf} - 1202\ \text{psf} = -2663\ \text{psf}$$

$$f_{\text{heel}} = -\frac{R_v}{A} + \frac{R_v ec}{I} = -1461\ \text{psf} + 1202\ \text{psf} = -259\ \text{psf}$$

Effect of Surcharge

Should there be earth or other loads on the surface of the backfill, as shown in Figure below, the horizontal pressure applied to the wall will be increased. If the surcharge is uniform over the sliding area behind the wall, the resulting pressure is assumed to equal the pressure that would be caused by an increased backfill height having the same total weight as the surcharge. It is usually easy to handle this situation by adding a uniform pressure to the triangular soil pressure for a wall without surcharge, as shown in the figure. If the surcharge does not cover the area entirely behind the wall, some rather complex soil theories are available to consider the resulting horizontal pressures developed. As a consequence, the designer usually uses a rule of thumb to cover the case, a procedure that works reasonably well.

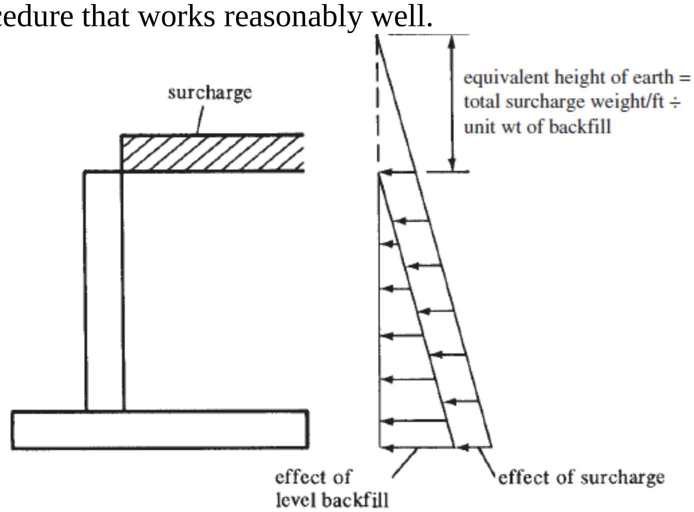
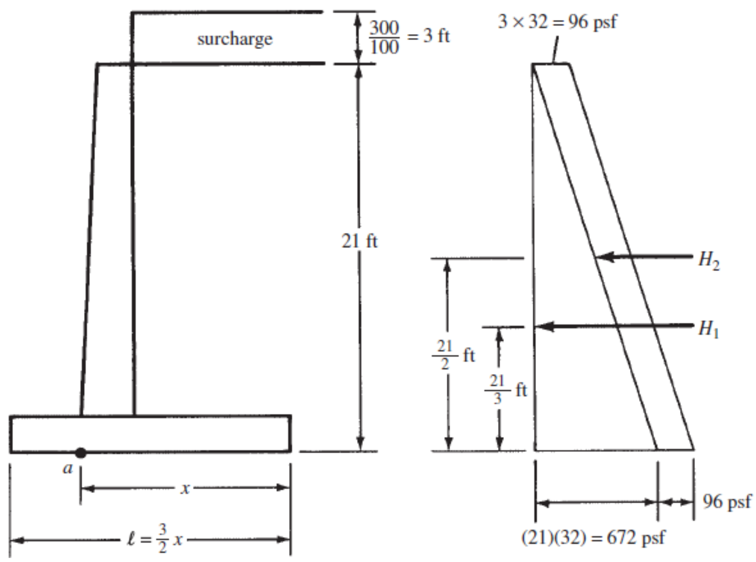
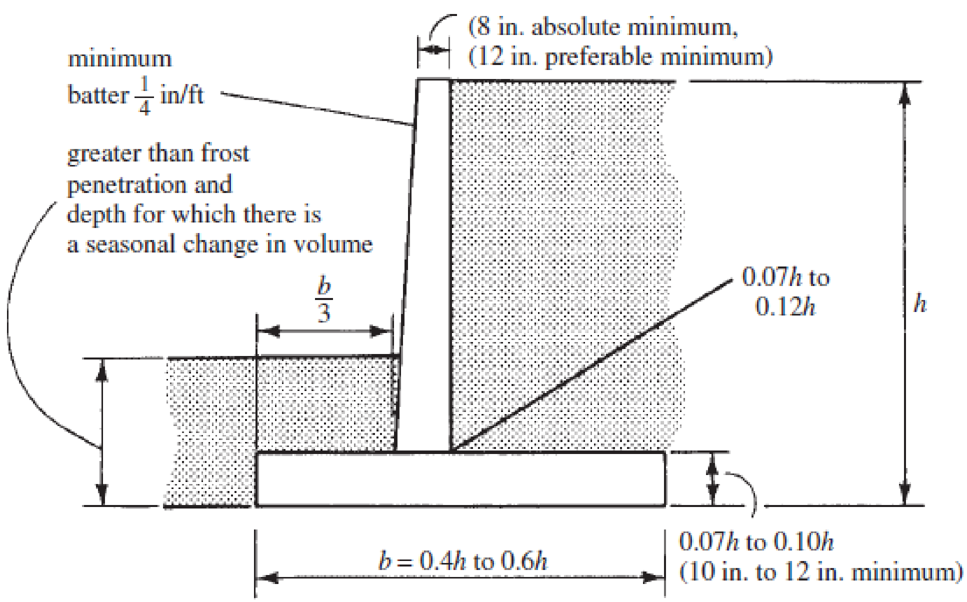


Figure: Equivalent Weight of Surcharge


surcharge
300/100 = 3 ft
3 × 32 = 96 psf
21 ft
H2
H1
21/2 ft
21/3 ft
a
x
ℓ = 3/2 x
96 psf
(21)(32) = 672 psf


(8 in. absolute minimum,
(12 in. preferable minimum)
minimum
batter 1/4 in/ft
greater than frost
penetration and
depth for which there is
a seasonal change in volume
b/3
0.07h to
0.12h
h
b = 0.4h to 0.6h
0.07h to 0.10h
(10 in. to 12 in. minimum)

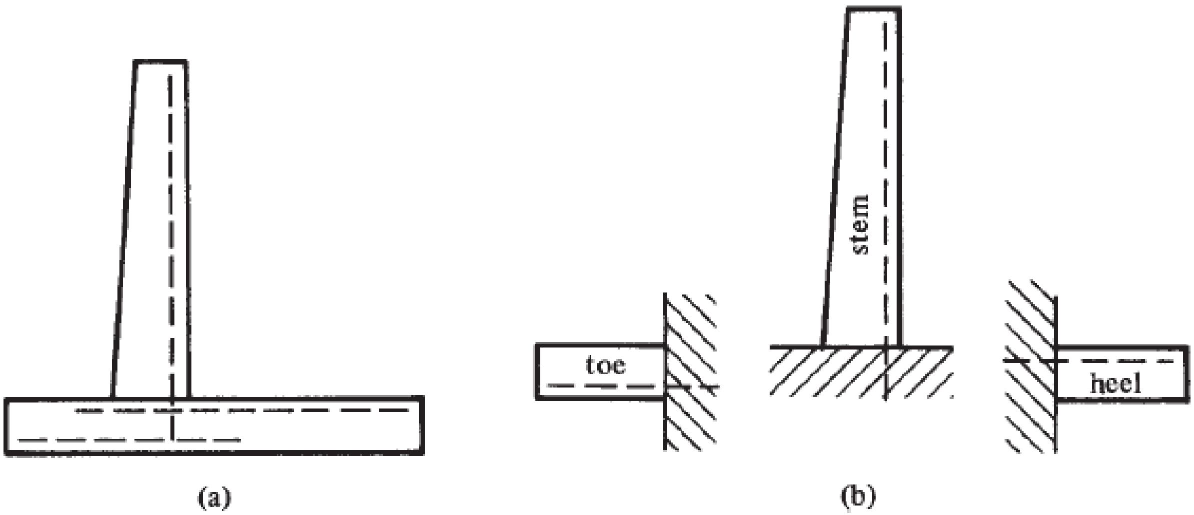


Figure: Cantilever beam model used to design retaining wall stem, heel, and toe.

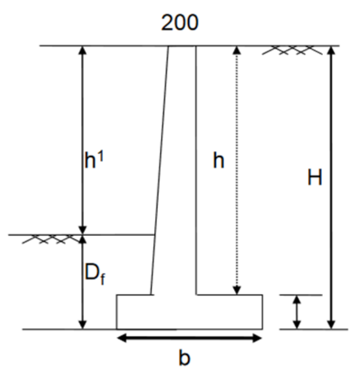
Example:

Design a cantilever retaining wall (T type) to retain earth for a height of 4m. The backfill is horizontal. The density of soil is $18kN/m^3$. Safe bearing capacity (SBC) of soil is 200 kN/m^2. Take the co-efficient of friction between concrete and soil as 0.6. The angle of repose is 30°.

Solution

Data: h' = 4m, SBC= 200 kN/m^2, γ= 18 kN/m^3, μ=0.6, φ=30°

Depth of foundation



- To fix the height of retaining wall [H]
- $H= h' + D_f$
- Depth of foundation
- $D_f = \frac{SBC}{\gamma}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$
- = 1.23m say 1.2m ,
- Therefore H= 5.2m

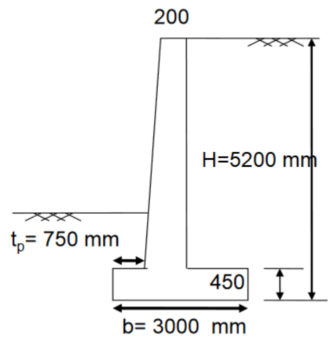
Proportioning of Wall

- Thickness of base slab=(1/10 to1/14)H
- 0.52m to 0.43m, say 450 mm

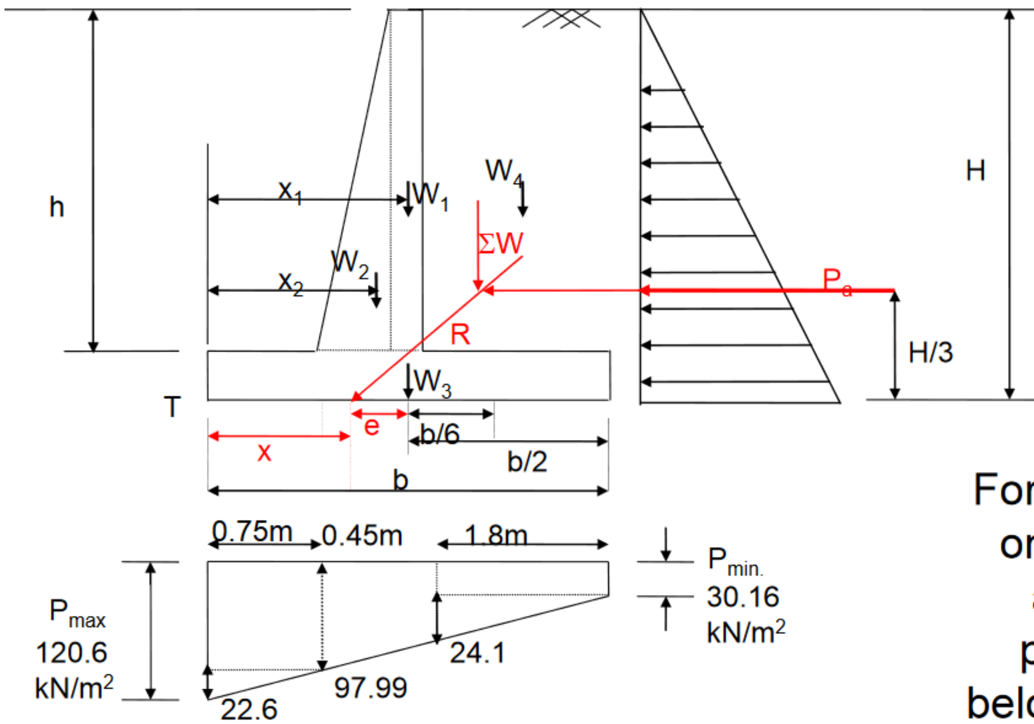
- Width of base slab=b = (0.5 to 0.6) H
- 2.6m to 3.12m say 3m

- Toe projection= pj= (1/3 to ¼)H
- 1m to 0.75m say 0.75m

- Provide 450 mm thickness for the stem at the base and 200 mm at the top



Stability Analysis



Pressure below the Retaining Wall

Forces acting on the wall and the pressure below the wall

Load	Magnitude, kN	Distance from A, m	BM about A kN-m
Stem W1	0.2x4.75x1x25 = 23.75	1.1	26.13
Stem W2	½ x0.25x4.75x1x25 = 14.84	0.75 + 2/3x0.25 =0.316	13.60
B. slab W3	3.0x0.45x1x25=33.75	1.5	50.63
Back fill, W4	1.8x4.75x1x18 = 153.9	2.1	323.20
Total	**ΣW= 226.24**		**ΣM_R=413.55**
Earth Pre. =P_H	P_H =0.333x18x5.2²/2	H/3 =5.2/3	**M_O=140.05**

***Backfill height = (5.2, H - 0.45, bottom slab)m = 4.75m**

For Earth pressure, Ka for fi, when angle of repose is 30°, is 1/3 or 0.333; density of soil is 18kN/m^3. Total height = 5.2 m $Pa = \frac{1}{2}\ Ka\ \gamma\ H^2$

Stability Check

Check for overturning

FOS = $\Sigma M_R / M_O$= 2.94 >1.55 safe

Check for Sliding FOS = $\mu\ \Sigma W / P_h$	P_h= ½ x 1/3 x 18 x 4.75²=67.68 kN **= 0.6 * 226.24/67.68 =2 > A minimum factor of safety of 1.5 is desirable.**

Bibliography

- Design of Reinforced Concrete by Jack C. McCormac and Russell Brown
- Design of Concrete by Arthur Nilson (Author), David Darwin (Author), Charles Dolan (Author)
- Various Websites

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