Reteaching Masters

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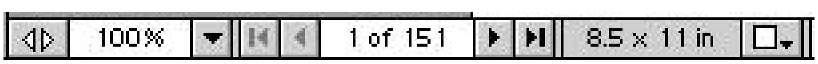

Menu

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NAME ______________________ CLASS ____________ DATE ____________

Reteaching

1.1 The Building Blocks of Geometry

◆ Skill A Identifying and naming points, lines, and planes

Recall The undefined terms *point*, *line*, and *plane* represent geometric figures. Such figures do not exist in the real world, but may be represented by real-world objects or by drawings.

A **point** has no size. It may be represented by a dot and named with a single capital letter.

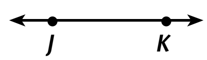
A **line** extends forever in both directions, but has no thickness. A line is named using either the names of two points on the line or a single lower-case letter. Points that lie on the same line are called *collinear*. Any two points are collinear.

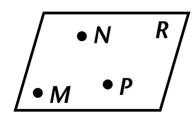
A **plane** is like a flat surface that extends forever in all directions but has no thickness. A plane is named using either the names of three points that lie in the plane and are not collinear, or by a single script capital letter. Points that lie in the same plane are *coplanar*. Any three points are coplanar.

◆ **Example**

Name each figure.

a. 

b. 

c. 

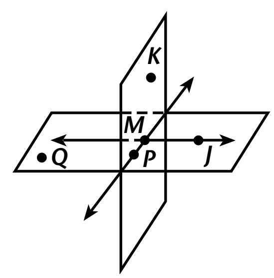
◆ **Solution**

a. point *P*

b. $\overleftrightarrow{JK}$, $\overleftrightarrow{KJ}$, or line *m*

c. plane *MNP* or plane

Name the indicated figures in the drawing at the right.



1. two lines ______________________

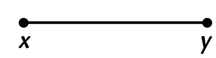
2. two planes ______________________

3. three noncollinear points ______________________

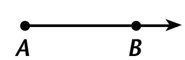
4. four noncoplanar points ______________________

◆ Skill B Identifying and naming segments, rays, and angles

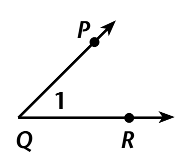
Recall A **segment** is a part of a line that consists of two points, called the endpoints, and all the points between them. The segment with endpoints *X* and *Y* is denoted $\overline{XY}$.



A **ray** is a part of a line that has one endpoint and extends without end in one direction. The ray that has endpoint *A* and contains point *B* is denoted $\overrightarrow{AB}$.



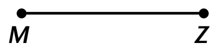
An **angle** is a figure formed by two rays that do not lie on the same line, but have the same endpoint. The common endpoint is the **vertex** of the angle. The angle in the figure at the right may be referred to as $\angle PQR$, $\angle RQP$, $\angle 1$, or $\angle Q$. If two angles have the same vertex, they must be named using numbers or three letters.

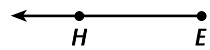


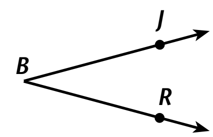
CLASS ______________ DATE ______________

◆ **Example**

Name each figure.

a. 

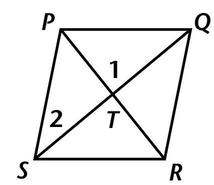
b. 

c. 

◆ **Solution**

a. $\overline{MZ}$ or $\overline{ZM}$ **b.** $\overrightarrow{EH}$ **c.** $\angle JBR$, $\angle RBJ$, or $\angle B$

Refer to the figure at the right.



5. Name all the segments. ______________________________

6. Name four rays with endpoint T. ______________________________

7. Give two other names for $\angle 1$. ______________________________

8. Name the rays that form the sides of $\angle 2$. ______________________________

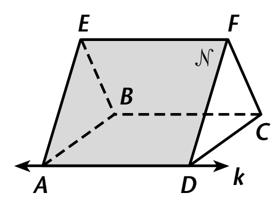
◆ **Skill C** Classifying and identifying intersections of geometric figures

Recall The postulates that follow are fundamental geometric ideas.
The intersection of two lines is a point.
The intersection of two planes is a line.
Through any two points, there is exactly one line.
Through any three noncollinear points, there is exactly one plane.
If two points are in a plane, then the line containing them is in the plane.

◆ **Example**

Name each of the following.

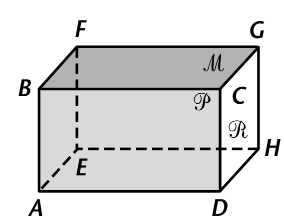
a. the intersection of $\overleftrightarrow{AB}$ and line k
b. the intersection of planes $\mathcal{N}$ and CDF
c. the line containing points A and D
d. all the planes shown that contain $\overleftrightarrow{BE}$



◆ **Solution**

a. point A **b.** $\overleftrightarrow{DF}$
c. $\overleftrightarrow{AD}$ (line k) **d.** plane ABE and plane BEF

Refer to the figure at the right. Name each of the following.



9. the intersection of $\overleftrightarrow{BC}$ and $\overleftrightarrow{CG}$ ______________________________

10. a plane containing points D and G ______________________________

11. the intersection of planes P and M ______________________________

12. the line containing points F and E ______________________________

ANSWERS

Reteaching — Chapter 1

Lesson 1.1

1. $\overleftrightarrow{PN}$ and $\overleftrightarrow{JN}$
2. Sample: KNP and PNJ
3. Sample: P, N, Q
4. Sample: K, N, P, Q
5. $\overline{PQ}$, $\overline{PS}$, $\overline{SR}$, $\overline{QR}$, $\overline{PR}$, $\overline{QS}$, $\overline{TP}$, $\overline{TQ}$, $\overline{TR}$, $\overline{TS}$
6. $\overrightarrow{TP}$, $\overrightarrow{TQ}$, $\overrightarrow{TR}$, $\overrightarrow{TS}$ **7.** $\angle PTQ$, $\angle QTP$
8. $\overrightarrow{SP}$ and $\overrightarrow{ST}$ or $\overrightarrow{SQ}$
9. point C **10.** R **11.** $\overleftrightarrow{BC}$ **12.** $\overleftrightarrow{FE}$

Lesson 1.2

1. 3 **2.** 5 **3.** 3 **4.** 0.5 **5.** 6.5 **6.** 4.5

7. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$; $\overline{PA}$ and $\overline{PC}$; $\overline{PD}$ and $\overline{PB}$

8. $\overline{QP}$ and $\overline{RS}$; $\overline{PU}$ and $\overline{TS}$; $\overline{QU}$ and $\overline{RT}$; $\overline{QR}$ and $\overline{UT}$

9. $\overline{NK}$ and $\overline{NM}$; $\overline{JK}$ and $\overline{JM}$; $\overline{LK}$ and $\overline{LM}$

10. $JM = 27$ **11.** 8 **12.** 1.75 **13.** 29

14. Axel to Patrice: 205 feet; Patrice to Zaleika: 95 feet

Lesson 1.3

1. 115° **2.** 42° **3.** 78° **4.** 102° **5.** 57°

6. $\angle FDC$ **7.** $\angle B$ **8.** $\angle FDE$

Lesson 1.4

1. k and n appear to be perpendicular.
2. . . . it is perpendicular to the other line.
3. Check students' drawings; the perpendicular bisector of $\overline{CD}$ contains P.
4. . . . on the perpendicular bisector of the segment.
5. Check students' drawings; m appears to bisect $\angle CBD$.
6. . . . bisects the other angle in the linear pair.

Lesson 1.5

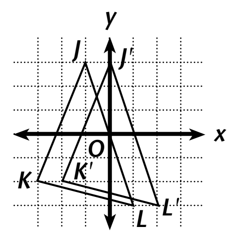
1–4. Check students' drawings.

Lesson 1.6

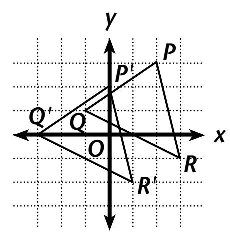
1–6. Check students' drawings.

Lesson 1.7

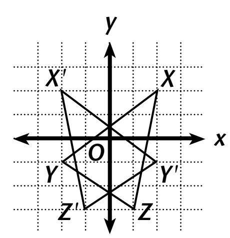
1. horizontal move right 1 unit



2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the y-axis

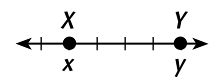


NAME ______________________ CLASS ____________ DATE ____________

Reteaching
1.2 Measuring Length

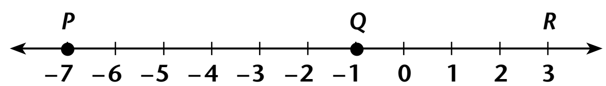
◆ Skill A Finding the length of a segment on a number line

Recall If x and y are the *coordinates* of points X and Y on a number line, then XY, the length of $\overline{XY}$, is given by $|x - y|$ or $|y - x|$. Notice that the length of $\overline{XY}$ is also the distance between X and Y.



◆ **Example**

Find the length of each segment on the number line.

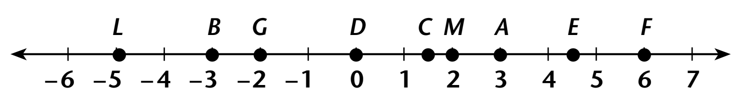


a. $\overline{PR}$ **b.** $\overline{PQ}$ **c.** $\overline{QR}$

◆ **Solution**

a. The coordinates of P and R are -7 and 3. Thus, $PR = |3 - (-7)| = |10| = 10$.
b. The coordinates of P and Q are -7 and -1. Thus, $PQ = |-1 - (-7)| = |6| = 6$.
c. The coordinates of Q and R are -1 and 3. Thus, $QR = |3 - (-1)| = |4| = 4$.

In Exercises 1–6, use the number line below. Find the length of each line segment.



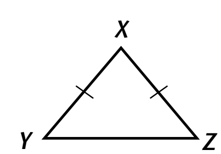
1. $\overline{DA}$ ____________ **2.** $\overline{MB}$ ____________ **3.** $\overline{CE}$ ____________

4. $\overline{CM}$ ____________ **5.** $\overline{LC}$ ____________ **6.** $\overline{CF}$ ____________

◆ Skill B Using the Segment Congruence Postulate to identify and measure congruent segments

Recall The **Segment Congruence Postulate** states that if two segments have the same length, they are congruent. If two segments are congruent, they have the same length.

In illustrations, congruent segments are indicated by equal numbers of "tick marks." In the figure at the right, for example, $\overline{XY}$ and $\overline{XZ}$ are congruent. You write this as, $\overline{XY} \cong \overline{XZ}$.



◆ **Example**

Use the Segment Congruence Postulate to complete each sentence.

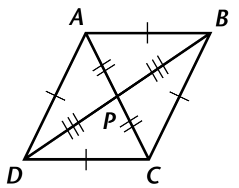
a. If $DE = MN$, then ____________ . **b.** If $\overline{DE} \cong \overline{MN}$, then ____________ .

◆ **Solution**

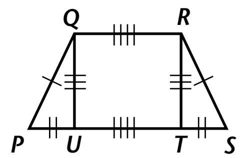
a. If $DE = MN$, then $\overline{DE} \cong \overline{MN}$. **b.** If $\overline{DE} \cong \overline{MN}$, then $DE = MN$.

Name all congruent segments.

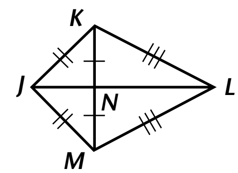
7.



8.



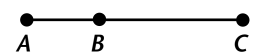
9.



10. In Exercise 9, if $JK = 27$, what else can you determine? ______

◆ **Skill C** Using the Segment Addition Postulate to solve problems

Recall The Segment Addition Postulate states that if A, B, and C are collinear with B between A and C, then $AB + BC = AC$



◆ **Example**

Acadia, Brentwood, and Cedarville lie along a straight stretch of interstate highway. Brentwood lies between Acadia and Cedarville. The distance from Acadia to Cedarville is 200 miles. If Cedarville is three times as far from Brentwood as Brentwood is from Acadia, find each distance.

◆ **Solution**

Let points A, B, and C represent Acadia, Brentwood, and Cedarville.
Let x = the distance in miles from Acadia to Brentwood.
Then $3x$ = the distance in miles from Brentwood to Cedarville.
Point B is between points A and C, and $AC = 200$.

$$AB + BC = AC \quad \rightarrow \quad x + 3x = 200$$
$$4x = 200$$
$$x = 50$$

distance from Acadia to Brentwood $= x = 50$ miles
distance from Brentwood to Cedarville $= 3x = 150$ miles
Check: $50 + 150 = 200$, which is the distance from Acadia to Cedarville.

In Exercises 11–14, point *B* is between points *A* and *C*. Find the indicated value.

11. If $AC = 22$, $AB = x$, and $BC = x + 6$, find x. ______

12. If $AB = 6x - 2$, $BC = 2x + 1$, and $AC = 4$, find AB. ______

13. Point P is on $\overline{DE}$, between D and E. If $DP = 12$ and $PE = 17$, find DE. ______

14. The members of a drill team are lined up along the sideline of a football field. Axel is at one end and Zaleika is at the other, 300 feet away. Patrice is between Axel and Zaleika. Her distance from Axel is 15 feet more than twice her distance from Zaleika. Find the distance from Axel to Patrice and from Patrice to Zaleika. ______

ANSWERS

Reteaching — Chapter 1

Lesson 1.1

1. $\overleftrightarrow{PN}$ and $\overleftrightarrow{JN}$
2. Sample: *KNP* and *PNJ*
3. Sample: *P*, *N*, *Q*
4. Sample: *K*, *N*, *P*, *Q*
5. $\overline{PQ}$, $\overline{PS}$, $\overline{SR}$, $\overline{QR}$, $\overline{PR}$, $\overline{QS}$, $\overline{TP}$, $\overline{TQ}$, $\overline{TR}$, $\overline{TS}$
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8. $\overrightarrow{SP}$ and $\overrightarrow{ST}$ or $\overrightarrow{SQ}$
9. point C **10.** *R* **11.** $\overleftrightarrow{BC}$ **12.** $\overleftrightarrow{FE}$

Lesson 1.2

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10. $JM = 27$ **11.** 8 **12.** 1.75 **13.** 29

14. Axel to Patrice: 205 feet;
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Lesson 1.3

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Lesson 1.4

1. *k* and *n* appear to be perpendicular.
2. . . . it is perpendicular to the other line.
3. Check students' drawings; the perpendicular bisector of $\overline{CD}$ contains *P*.
4. . . . on the perpendicular bisector of the segment.
5. Check students' drawings; *m* appears to bisect $\angle CBD$.
6. . . . bisects the other angle in the linear pair.

Lesson 1.5

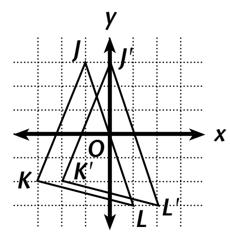
1–4. Check students' drawings.

Lesson 1.6

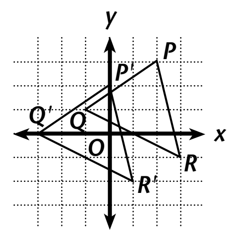
1–6. Check students' drawings.

Lesson 1.7

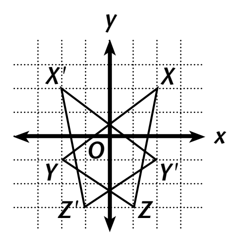
1. horizontal move right 1 unit



2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the *y*-axis

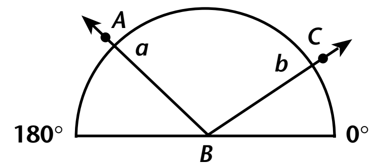


NAME ______________________ CLASS __________ DATE __________

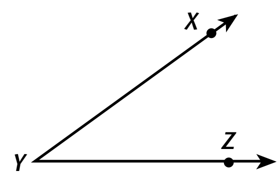
Reteaching
1.3 Measuring Angles

◆ Skill A Measuring angles with a protractor

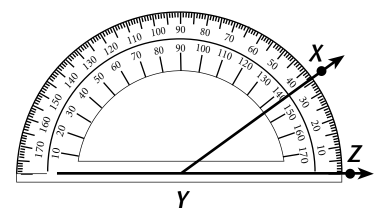
Recall A protractor is a type of geometry ruler used to find the measure of an angle. If the vertex of $\angle ABC$ is placed at the center point of a protractor and $\overrightarrow{BA}$ and $\overrightarrow{BC}$ intersect the protractor at a and b, respectively, then the measure of $\angle ABC$, written $m\angle ABC$, is $|a - b|$ or $|b - a|$.



◆ Example
Use a protractor to find the measure of $\angle XYZ$.

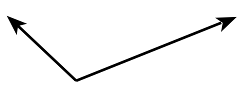


◆ Solution
1. Put the center of the protractor at Y, the vertex of the angle.
2. Position the protractor so that $\overrightarrow{YZ}$ passes through point 0.
3. Find the intersection point for $\overrightarrow{YX}$,36.
$m\angle XYZ = |36 - 0| = 36°$.



Use a protractor to find the measure of each angle. You may trace the figures or use a piece of paper to extend the rays if necessary.

1.



____

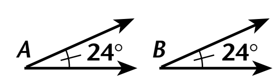
2.

____

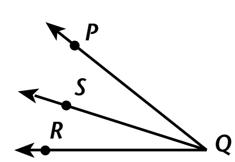
◆ Skill B Using the Angle Addition Postulate

Recall **The Angle Congruence Postulate**: If two angles have the same measure, then they are congruent. If two angles are congruent, then they have the same measure.

In the figure at the right, $m\angle A = m\angle B$, so $\angle A$ and $\angle B$ are congruent. This may be written $\angle A \cong \angle B$. The “tick marks” indicate that the angles are congruent.

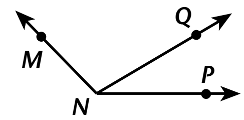


The Angle Addition Postulate: If a point S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.



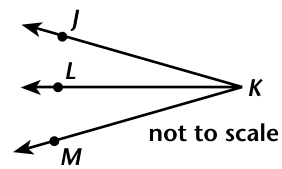
NAME ______________________________ CLASS ____________ DATE ____________

◆ **Example**
In the figure, m∠*MNQ* = 104° and m∠*QNP* = 31°.
Find m∠*MNP*.



◆ **Solution**
By the Angle Addition Postulate,
m∠*MNP* = m∠*MNQ* + m∠*QNP* = 104° + 31° = 135°.

Find the missing angle measures.



3. m∠*JKL* = 41°, m∠*LKM* = 37°, m∠*JKM* = ______________________

4. m∠*JKL* = 73°, m∠*LKM* = 29°, m∠*JKM* = ______________________

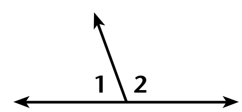
5. m∠*JKM* = 83°, m∠*JKL* = 26°, m∠*LKM* = ______________________

◆ **Skill C** Identifying and using special pairs of angles

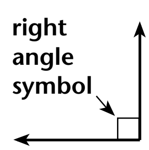
Recall Two angles are **complementary** if the sum of their measures is 90°. Each is called a **complement** of the other.

Two angles are **supplementary** if the sum of their measures is 180°. Each is called a **supplement** of the other.

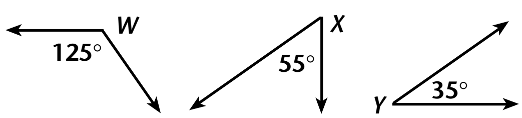
If the endpoint of a ray is on a line so that two angles are formed, the angles are called a **linear pair**. In the figure, ∠1 and ∠2 form a linear pair. By the **Linear Pair Property**, the angles in a linear pair are supplementary.



Angles may be classified according to their measures.
A **right angle** has measure 90°.
An **acute angle** has measure between 0° and 90°.
An **obtuse angle** has measure between 90° and 180°.

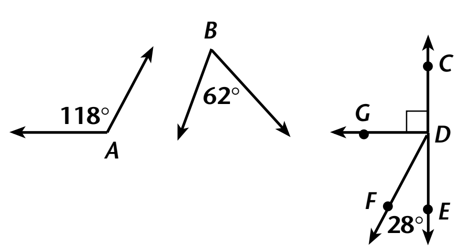


◆ **Example**
Identify any pairs of complementary angles or supplementary angles.



◆ **Solution**
m∠*X* + m∠*Y* = 55° + 35° = 90°, so ∠*X* and ∠*Y* are complementary angles.
m∠*W* + m∠*X* = 125° + 55° = 180°, so ∠*W* and ∠*X* are supplementary angles.

Refer to the figure at the right. Complete each statement.



6. ∠*FDE* and ∠____________ form a linear pair.

7. ∠____________ and ∠*A* are supplementary angles.

8. ∠____________ and ∠*B* are complementary angles.

ANSWERS

Reteaching — Chapter 1

Lesson 1.1

1. $\overleftrightarrow{PN}$ and $\overleftrightarrow{JN}$

2. Sample: *KNP* and *PNJ*

3. Sample: *P*, *N*, *Q*

4. Sample: *K*, *N*, *P*, *Q*

5. $\overline{PQ}$, $\overline{PS}$, $\overline{SR}$, $\overline{QR}$, $\overline{PR}$, $\overline{QS}$, $\overline{TP}$, $\overline{TQ}$, $\overline{TR}$, $\overline{TS}$

6. $\overrightarrow{TP}$, $\overrightarrow{TQ}$, $\overrightarrow{TR}$, $\overrightarrow{TS}$ **7.** $\angle PTQ$, $\angle QTP$

8. $\overrightarrow{SP}$ and $\overrightarrow{ST}$ or $\overrightarrow{SQ}$

9. point C **10.** *R* **11.** $\overleftrightarrow{BC}$ **12.** $\overleftrightarrow{FE}$

Lesson 1.2

1. 3 **2.** 5 **3.** 3 **4.** 0.5 **5.** 6.5 **6.** 4.5

7. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$; $\overline{PA}$ and $\overline{PC}$; $\overline{PD}$ and $\overline{PB}$

8. $\overline{QP}$ and $\overline{RS}$; $\overline{PU}$ and $\overline{TS}$; $\overline{QU}$ and $\overline{RT}$; $\overline{QR}$ and $\overline{UT}$

9. $\overline{NK}$ and $\overline{NM}$; $\overline{JK}$ and $\overline{JM}$; $\overline{LK}$ and $\overline{LM}$

10. $JM = 27$ **11.** 8 **12.** 1.75 **13.** 29

14. Axel to Patrice: 205 feet;
Patrice to Zaleika: 95 feet

Lesson 1.3

1. 115° **2.** 42° **3.** 78° **4.** 102° **5.** 57°

6. $\angle FDC$ **7.** $\angle B$ **8.** $\angle FDE$

Lesson 1.4

1. *k* and *n* appear to be perpendicular.

2. . . . it is perpendicular to the other line.

3. Check students' drawings; the perpendicular bisector of $\overline{CD}$ contains *P*.

4. . . . on the perpendicular bisector of the segment.

5. Check students' drawings; *m* appears to bisect $\angle CBD$.

6. . . . bisects the other angle in the linear pair.

Lesson 1.5

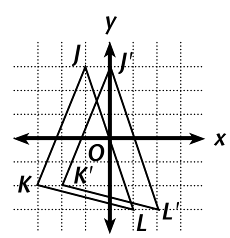
1–4. Check students' drawings.

Lesson 1.6

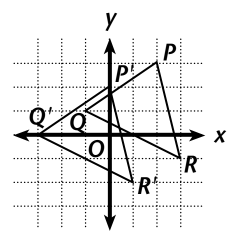
1–6. Check students' drawings.

Lesson 1.7

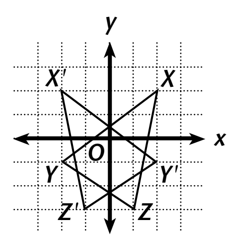
1. horizontal move right 1 unit



2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the *y*-axis



NAME ______________________ CLASS ____________ DATE ____________

Reteaching
1.4 Geometry Using Paper Folding

Throughout this lesson, you will need folding paper and a marker or pencil that will write on the paper. For Skill B, you will need a compass as well.

◆ **Skill A** Using paper folding to construct and study perpendicular and parallel lines

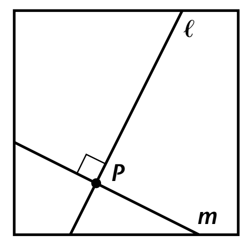
Recall **Perpendicular lines** are lines that intersect to form a right angle.

Parallel lines are coplanar lines that do not intersect.
You can use paper folding to produce a line perpendicular to, or parallel to a given line.

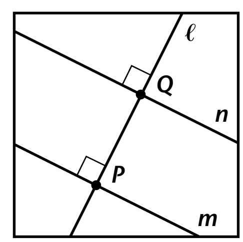
◆ **Example**
Use paper folding to construct two parallel lines and a line perpendicular to one of the lines.

◆ **Solution**

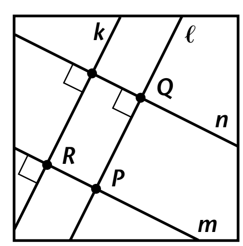
1. Fold the paper once to make a line. Label it ℓ. Draw a point P on ℓ. Fold the paper through P so that ℓ matches up with itself. Label the new line you folded m.



2. Draw a second point Q on ℓ. Fold the paper through Q so that ℓ matches up with itself. Label the new line you folded n. Then lines ℓ and m are parallel.



3. Draw a point R on m. Fold the paper through R so that m matches up with itself. Label the new line k. Line k is perpendicular to m.



Refer to lines *m*, *n*, and *k* in the example above.

1. What appears to be true of lines k and n? ______________________

2. Complete the following conjecture about parallel and perpendicular lines.

If a line is perpendicular to one of two parallel lines, then ______________________.

CLASS ______________ DATE ______________

◆ **Skill B** Using paper folding to construct and study segment bisectors and angle bisectors

Recall An **angle bisector** is a line or a ray that divides an angle into two congruent angles.

◆ **Example**

Use paper folding to construct each line.

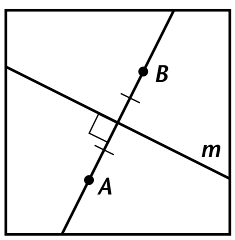
a. the bisector of a segment

b. the bisector of an angle.

◆ **Solution**

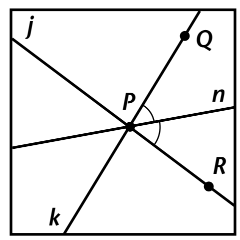
a. First fold line ℓ. Choose points A and B, the endpoints of a segment.

Then fold the paper so that A matches up with B. Label the resulting line m. m is the perpendicular bisector of $\overline{AB}$.



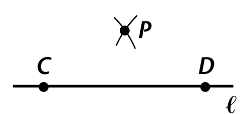
b. First fold two intersecting lines j and k. Label the point of intersection P. Label a point Q on line k and a point R on line j.

Then fold the paper through P so that line j matches up with line k. Label the new line n. n is a bisector of $\angle PQR$.



Refer to the constructions in the example above.

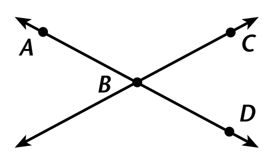
3. Use a new piece of paper. Fold line ℓ and choose points C and D. Using a compass, draw two congruent arcs with centers C and D. Label the intersection of the arcs P. Repeat part **a** of the example to construct the bisector of $\overline{CD}$. What do you notice about point P?



____

4. Complete this conjecture: If a point is equally distant from the endpoints of a segment, then the point lies ______________________________.

5. Use a new piece of paper. Fold two intersecting lines, labeled as shown in the figure. Construct the bisector of $\angle ABC$ and label it ℓ. Fold the paper through B so that ℓ matches up with itself. Label the new line m. What appears to be true of m and $\angle CBD$?



____.

6. Complete this conjecture: If a line that passes through the vertex of a linear pair is perpendicular to a bisector of one of the angles, then the line ______________________________.

Reteaching — Chapter 1

Lesson 1.1

1. $\overleftrightarrow{PN}$ and $\overleftrightarrow{JN}$
2. Sample: *KNP* and *PNJ*
3. Sample: *P*, *N*, *Q*
4. Sample: *K*, *N*, *P*, *Q*
5. $\overline{PQ}$, $\overline{PS}$, $\overline{SR}$, $\overline{QR}$, $\overline{PR}$, $\overline{QS}$, $\overline{TP}$, $\overline{TQ}$, $\overline{TR}$, $\overline{TS}$
6. $\overrightarrow{TP}$, $\overrightarrow{TQ}$, $\overrightarrow{TR}$, $\overrightarrow{TS}$ **7.** $\angle PTQ$, $\angle QTP$
8. $\overrightarrow{SP}$ and $\overrightarrow{ST}$ or $\overrightarrow{SQ}$
9. point C **10.** *R* **11.** $\overleftrightarrow{BC}$ **12.** $\overleftrightarrow{FE}$

Lesson 1.2

1. 3 **2.** 5 **3.** 3 **4.** 0.5 **5.** 6.5 **6.** 4.5
7. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$; $\overline{PA}$ and $\overline{PC}$; $\overline{PD}$ and $\overline{PB}$
8. $\overline{QP}$ and $\overline{RS}$; $\overline{PU}$ and $\overline{TS}$; $\overline{QU}$ and $\overline{RT}$; $\overline{QR}$ and $\overline{UT}$
9. $\overline{NK}$ and $\overline{NM}$; $\overline{JK}$ and $\overline{JM}$; $\overline{LK}$ and $\overline{LM}$
10. $JM = 27$ **11.** 8 **12.** 1.75 **13.** 29
14. Axel to Patrice: 205 feet; Patrice to Zaleika: 95 feet

Lesson 1.3

1. 115° **2.** 42° **3.** 78° **4.** 102° **5.** 57°
6. $\angle FDC$ **7.** $\angle B$ **8.** $\angle FDE$

Lesson 1.4

1. *k* and *n* appear to be perpendicular.
2. . . . it is perpendicular to the other line.
3. Check students' drawings; the perpendicular bisector of $\overline{CD}$ contains *P*.
4. . . . on the perpendicular bisector of the segment.
5. Check students' drawings; *m* appears to bisect $\angle CBD$.
6. . . . bisects the other angle in the linear pair.

Lesson 1.5

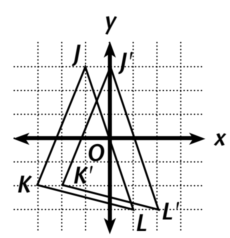
1–4. Check students' drawings.

Lesson 1.6

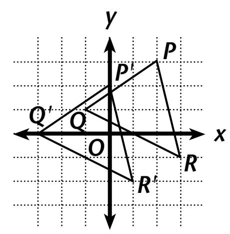
1–6. Check students' drawings.

Lesson 1.7

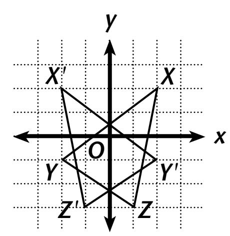
1. horizontal move right 1 unit



2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the *y*-axis



Menu

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NAME ______________________ CLASS __________ DATE __________

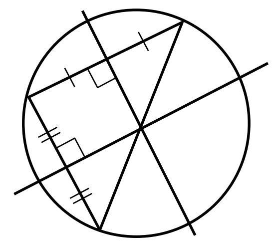
Reteaching
1.5 Special Points in Triangles

In this lesson, you will be required to do constructions. You will need folding paper and an appropriate marker or geometry software. You will also need a compass.

◆ **Skill A** Using the perpendicular bisectors of a triangle to construct the circumscribed circle of the triangle

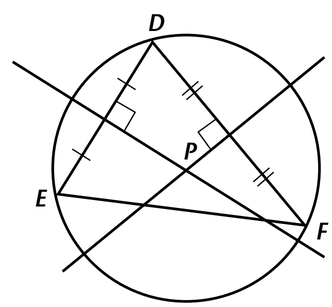
Recall A triangle is named using the symbol △ and the vertices.

The perpendicular bisectors of a triangle intersect in a point. The point is equidistant from each of the vertices and is the center of the **circumscribed circle** of the triangle. This circle is outside the triangle and contains the three vertices of the triangle. Given a triangle, you can locate the center of the circumscribed circle by constructing two of the perpendicular bisectors. You may then use the third perpendicular bisector as a check.



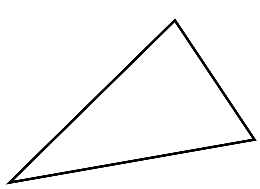
◆ **Example**
Construct the circumscribed circle of $\triangle DEF$.

◆ **Solution**
1. Choose two sides of $\triangle DEF$, for example, $\overline{DE}$ and $\overline{DF}$ as shown in the figure. Construct the perpendicular bisectors of $\overline{DE}$ and $\overline{DF}$. Label the point where the perpendicular bisectors intersect P.
2. Place the point of a compass at P. Choose any vertex of the triangle, say E, and open the compass to the width PE.
3. Draw the circle with center P and radius PE.

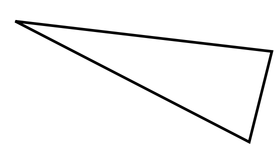


Construct the circumscribed circle of each triangle.

1.

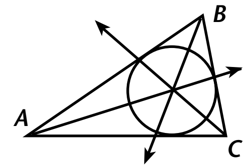


2.



◆ **Skill B** Using the angle bisectors of a triangle to construct the inscribed circle of the triangle

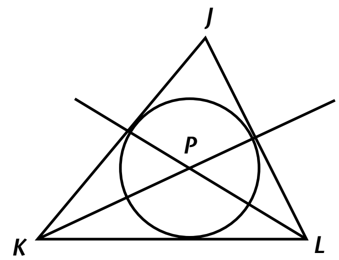
Recall The angle bisectors of a triangle intersect in a point. The point is equidistant from the sides of the triangle and is the center of the **inscribed circle** of the triangle. This circle is inside the triangle and just touches the three sides. Given a triangle, you can locate the center of the inscribed circle by constructing the bisectors of two of the angles. You may then use the bisector of the third angle as a check.



◆ **Example**
Construct the inscribed circle of $\triangle JKL$.

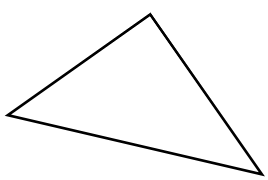
◆ **Solution**

1. Choose two angles of $\triangle JKL$, for example, $\angle K$ and $\angle L$ as shown in the figure. Construct the bisector of $\angle K$ and the bisector of $\angle L$. Label the point where the angle bisectors intersect P.
2. Find the distance from P to one side of the triangle, $\overline{KL}$ for instance. Use the distance from P to $\overline{KL}$ as the setting for your compass.
3. Draw the circle with center P and radius equal to the distance you found in Step 2.

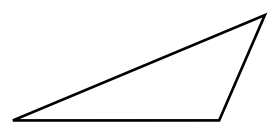


Construct the inscribed circle of each triangle.

3.



4.



ANSWERS

Reteaching — Chapter 1

Lesson 1.1

1. $\overleftrightarrow{PN}$ and $\overleftrightarrow{JN}$

2. Sample: *KNP* and *PNJ*

3. Sample: *P*, *N*, *Q*

4. Sample: *K*, *N*, *P*, *Q*

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10. $JM = 27$ **11.** 8 **12.** 1.75 **13.** 29

14. Axel to Patrice: 205 feet;
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Lesson 1.3

1. 115° **2.** 42° **3.** 78° **4.** 102° **5.** 57°

6. $\angle FDC$ **7.** $\angle B$ **8.** $\angle FDE$

Lesson 1.4

1. *k* and *n* appear to be perpendicular.

2. . . . it is perpendicular to the other line.

3. Check students' drawings; the perpendicular bisector of $\overline{CD}$ contains *P*.

4. . . . on the perpendicular bisector of the segment.

5. Check students' drawings; *m* appears to bisect $\angle CBD$.

6. . . . bisects the other angle in the linear pair.

Lesson 1.5

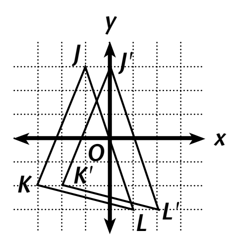
1–4. Check students' drawings.

Lesson 1.6

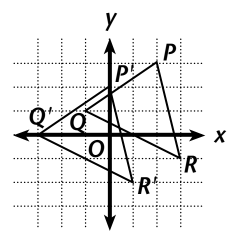
1–6. Check students' drawings.

Lesson 1.7

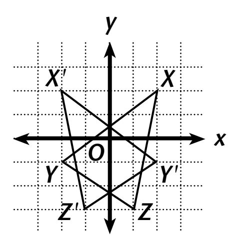
1. horizontal move right 1 unit



2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the *y*-axis



NAME ______________________ CLASS ______________ DATE ______________

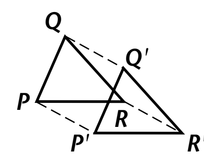
Reteaching
1.6 Motions in Geometry

For this lesson, you will need patty paper or tracing paper.

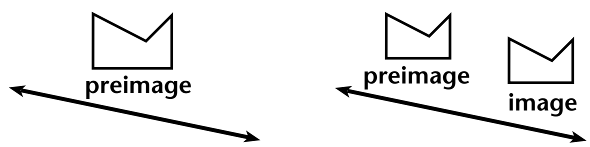
◆ Skill A Drawing a translation

Recall A rigid transformation of a plane figure does not change the size or shape of the figure. The original figure, or *preimage*, and the result, or *image*, are congruent.

A **translation** is a rigid transformation in which every point of the figure moves in a straight line with no turns. All points move the same distance and in the same direction. In the figure, $\overline{PP'}$, $\overline{QQ'}$, and $\overline{RR'}$ are all congruent and parallel.



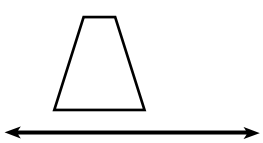
◆ **Example**
Translate the preimage along the given line.



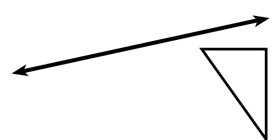
◆ **Solution**
1. Trace the figure and the line onto a piece of patty paper.
2. Start with your drawing directly over the diagram. Slide the patty paper so the image of the line stays directly over the preimage. Trace the figure a second time.
3. Label the first figure you drew *preimage* and the second *image*.

Trace the diagram. Then translate the figure along the given line.

1.

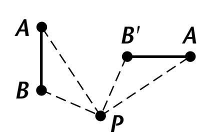


2.



◆ Skill B Drawing a rotation

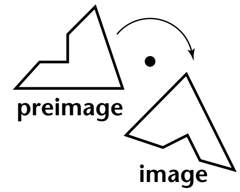
Recall A **rotation** is a rigid transformation in which every point of a figure moves around a given point, called the center of rotation. All points rotate through the same angle measure. In the figure, P is the center of rotation and $\angle APA'$ and $\angle BPB'$ are congruent.



NAME ______________________ CLASS ____________ DATE ____________

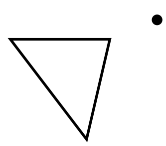
◆ **Example**
Rotate the preimage about the given point.

◆ **Solution**
1. Trace the preimage and the point onto a piece of patty paper.
2. Start with your drawing directly over the diagram. Press the point of your pencil onto the point to hold the patty paper in place. Then turn the patty paper around the point as far as you like. Trace the figure a second time.
3. Label the first figure you drew *preimage* and the second *image*.

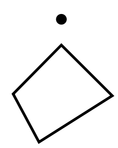


Trace the diagram. Then rotate the figure around the given point.

3.

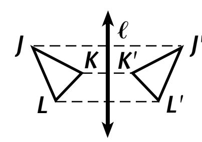


4.



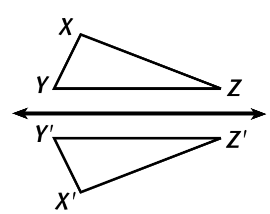
◆ **Skill C** Drawing a reflection

Recall A **reflection** is a rigid transformation in which every point of a figure is reflected across a line. The line is the perpendicular bisector of each segment formed by a point and its image. In the figure, ℓ is the perpendicular bisector of $\overline{JJ'}$, $\overline{KK'}$ and $\overline{LL'}$.



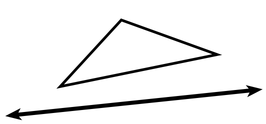
◆ **Example**
Reflect $\triangle XYZ$ across the given line.

◆ **Solution**
1. Draw $\triangle XYZ$ and line.
2. Fold the paper along ℓ. Punch holes through points X, Y, and Z to locate the image points X', Y', and Z'.
3. Connect the points to draw the image triangle, $\triangle X'Y'Z'$.



Trace the diagram. Then reflect the figure across the given line.

5.



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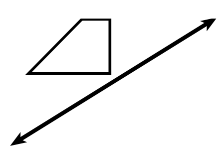


ANSWERS

Reteaching — Chapter 1

Lesson 1.1

1. $\overleftrightarrow{PN}$ and $\overleftrightarrow{JN}$

2. Sample: *KNP* and *PNJ*

3. Sample: *P*, *N*, *Q*

4. Sample: *K*, *N*, *P*, *Q*

5. $\overline{PQ}$, $\overline{PS}$, $\overline{SR}$, $\overline{QR}$, $\overline{PR}$, $\overline{QS}$, $\overline{TP}$, $\overline{TQ}$, $\overline{TR}$, $\overline{TS}$

6. $\overrightarrow{TP}$, $\overrightarrow{TQ}$, $\overrightarrow{TR}$, $\overrightarrow{TS}$ **7.** $\angle PTQ$, $\angle QTP$

8. $\overrightarrow{SP}$ and $\overrightarrow{ST}$ or $\overrightarrow{SQ}$

9. point C **10.** *R* **11.** $\overleftrightarrow{BC}$ **12.** $\overleftrightarrow{FE}$

Lesson 1.2

1. 3 **2.** 5 **3.** 3 **4.** 0.5 **5.** 6.5 **6.** 4.5

7. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$; $\overline{PA}$ and $\overline{PC}$; $\overline{PD}$ and $\overline{PB}$

8. $\overline{QP}$ and $\overline{RS}$; $\overline{PU}$ and $\overline{TS}$; $\overline{QU}$ and $\overline{RT}$; $\overline{QR}$ and $\overline{UT}$

9. $\overline{NK}$ and $\overline{NM}$; $\overline{JK}$ and $\overline{JM}$; $\overline{LK}$ and $\overline{LM}$

10. $JM = 27$ **11.** 8 **12.** 1.75 **13.** 29

14. Axel to Patrice: 205 feet;
Patrice to Zaleika: 95 feet

Lesson 1.3

1. 115° **2.** 42° **3.** 78° **4.** 102° **5.** 57°

6. $\angle FDC$ **7.** $\angle B$ **8.** $\angle FDE$

Lesson 1.4

1. *k* and *n* appear to be perpendicular.

2. . . . it is perpendicular to the other line.

3. Check students' drawings; the perpendicular bisector of $\overline{CD}$ contains *P*.

4. . . . on the perpendicular bisector of the segment.

5. Check students' drawings; *m* appears to bisect $\angle CBD$.

6. . . . bisects the other angle in the linear pair.

Lesson 1.5

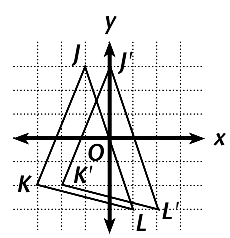
1–4. Check students' drawings.

Lesson 1.6

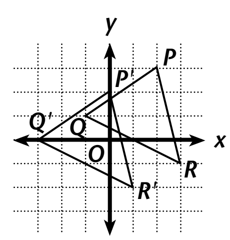
1–6. Check students' drawings.

Lesson 1.7

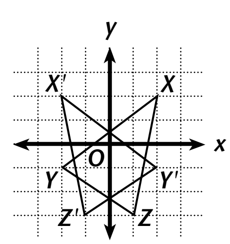
1. horizontal move right 1 unit



2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the *y*-axis



NAME ______________________ CLASS ____________ DATE ____________

Reteaching
1.7 Motion in the Coordinate Plane

◆ Skill A Using algebraic operations on the coordinates of a point to describe a translation

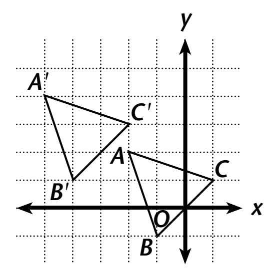
Recall A **coordinate plane** has a horizontal axis, called the ***x*-axis**, and a vertical axis called the ***y*-axis**. The **origin** is the point where the axes intersect. Each point in the plane is identified with an ordered pair. The first number is the x-coordinate. The second is the y-coordinate.

You can apply algebraic operations to the coordinates (x, y) of a point to describe a translation.
horizontal translation by h units: $H((x, y)) = (x + h, y)$
vertical translation by v units: $V((x, y)) = (x, y + v)$
translation by h units horizontally and v units vertically: $T((x, y)) = (x + h, y + v)$

◆ Example
Describe the effect of the transformation $T((x, y)) = (x - 3, y + 2)$. Then draw the triangle with vertices $A(-2, 2)$, $B(-1,-1)$, and $C(1, 1)$ on a coordinate plane and use the rule to transform the figure.

◆ Solution
$T((x, y)) = (x - 3, y + 2) = (x + (-3), y + 2)$
The horizontal change is -3, or 3 units to the left.
The vertical change is 2, or up 2 units.

$T((-2, 2)) = (-2 + (-3), 2 + 2) = (-5, 4)$
$T((-1, -1)) = (-1 + (-3), -1 + 2) = (-4, 1)$
$T((1, 1)) = (1 + (-3), 1 + 2) = (-2, 3)$



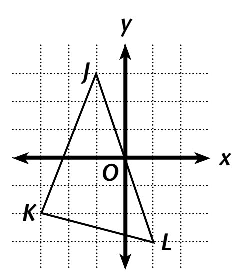
Describe the effect of each transformation. Use the given rule to transform the figure.

1. $P((x, y)) = (x + 1 ,y)$

____

____

____



2. $M((x, y)) = (x - 2, y - 1)$

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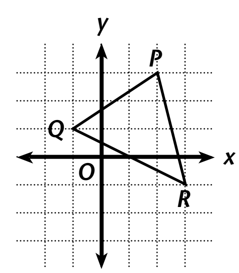


◆ Skill B Using algebraic operations on the coordinates of a point to describe a reflection across the x-axis or the y-axis

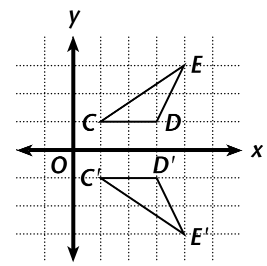
Recall You can apply algebraic operations to the coordinates (x, y) of a point to describe a reflection across the x-axis or the y-axis.
reflection across the x-axis: $F((x, y)) = (x, -y)$
reflection across the y-axis: $G((x, y)) = (-x, y)$

◆ **Example**

Describe the effect of the transformation $J((x, y)) = (x, -y)$. Then draw the triangle with vertices $C(1, 1)$, $D(3, 1)$, and $E(4, 3)$ on a coordinate plane and use the rule to transform the figure.

◆ **Solution**

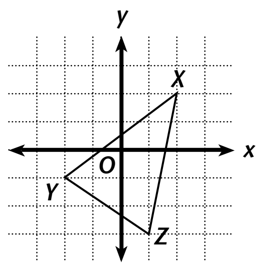
$J((x, y)) = (x, -y)$ is a reflection across the x-axis.
$J((1, 1)) = (1, -1)$
$J((3, 1)) = (3, -1)$
$J((4, 3)) = (4, -3)$



Describe the effect of each transformation. Use the given rule to transform the figure.

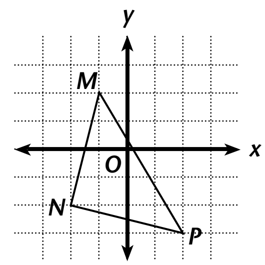
3. $B((x, y)) = (-x, y)$

____
____
____



4. $K((x, y)) = (x, -y)$

____
____
____



◆ **Skill C** Using algebraic operations on the coordinates of a point to describe a 180° rotation about the origin

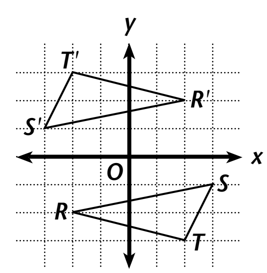
Recall You can apply algebraic operations to the coordinates (x, y) of a point to describe a 180° rotation about the origin: $R((x, y)) = (-x, -y)$

◆ **Example**

Draw the triangle with vertices $R(-2, -2)$, $S(3, -1)$, and $T(2, -3)$ on a coordinate plane. Then draw its image after a 180° rotation about the origin.

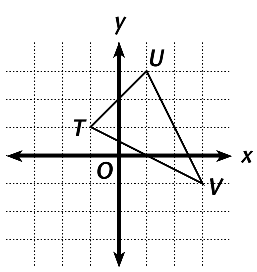
◆ **Solution**

$J((-2, -2)) = (2, 2)$
$J((3, -1)) = (-3, 1)$
$J((2, -3)) = (-2, 3)$



Use the transformation $J(x, y) = (-x, -y)$ to rotate each figure 180° about the origin.

5.



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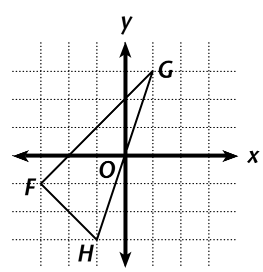

Reteaching — Chapter 1

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3. Sample: *P*, *N*, *Q*

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Lesson 1.5

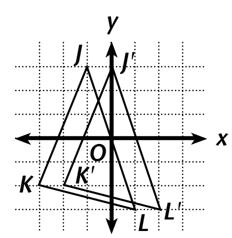
1–4. Check students' drawings.

Lesson 1.6

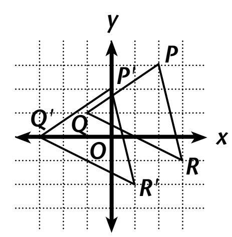
1–6. Check students' drawings.

Lesson 1.7

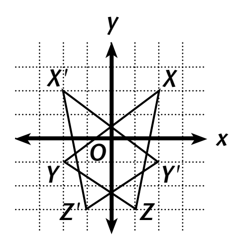
1. horizontal move right 1 unit



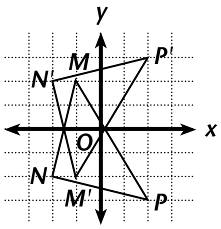
2. horizontal move 2 units left, vertical move 1 unit down



3. reflection across the *y*-axis



4. reflection across the x-axis



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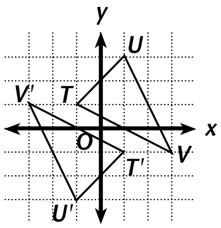

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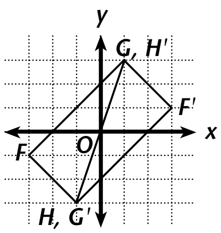


Reteaching — Chapter 2

Lesson 2.1

1. Yes; the proof would be identical. If n is a positive odd number, then the next positive odd number is $n + 2$.

2. If you take the bottom row of dots and make it a column at the right, the diagram would be nearly square. It would be missing one dot. For the specific case in the diagram, there are six dots and each row and eight dots in each column. If the bottom row is made into a column, the diagram would be a 7×7 square with one dot missing. That is, $6 \times 8 = 7^2 - 1$.

3. Since x and y are even numbers, let $x = 2n$ and $y = 2m$, where n and m are whole numbers. Then $x + y = 2n + 2m = 2(n + m)$, which is an even number, since $n + m$ is a whole number.

4.

n	$n^2 - n + 41$	Prime?
20	421	Yes
30	911	Yes
40	1601	Yes

Yes.

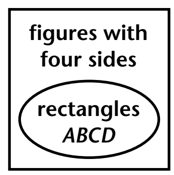
5. 1681; $1681 = 41^2$

6. No. When $n = 41$, the value of the expression is 41×41, which is not prime.

7. only one case

Lesson 2.2

1.



ABCD is a figure with four congruent sides.

2. a line intersects a segment at its midpoint

3. the line bisects the segment

4. If a line bisects a segment, then the line intersects the segment at its midpoint. True.

5. If cows have wings, then they can fly. If an animal can fly, then it is a bird. If I am a bird, then I can sing. If I can sing, then I will be a star. If cows have wings, then I will be a star.

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
2.1 An Introduction to Proofs

◆ Skill A Investigating informal proofs

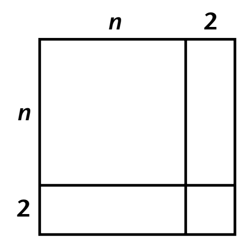
Recall Positive even numbers are 2, 4, 6, . . . and positive odd numbers are 1, 3, 5, . . .

◆ Example

If you know the square of a positive even number, you can easily find the square of the next even number by adding 4 plus 4 times the original number. In algebraic terms, this can be expressed as:
For any positive even number n, $(n + 2)^2 = n^2 + 4n + 4$.
Draw a diagram and prove informally that the statement is true.

◆ Solution

The diagram shows a rectangle whose dimensions are $n + 2$ and $n + 2$.
The areas of the four regions are:
$n \times n = n^2$ $\quad 2 \times n = 2n$ $\quad n \times 2 = 2n$ $\quad 2 \times 2 = 4$
The sum of the areas is $n^2 + 2n + 2n + 4$.
Then $(n + 2)^2 = n^2 + 4n + 4$.

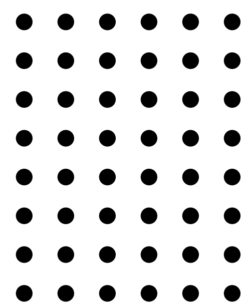


Write informal proofs.

1. Could you use the method in the solution to prove that for any positive *odd* number n, $(n + 2)^2 = n^2 + 4n + 4$? Explain.

____

2. The diagram at right represents the product $(n + 1)(n - 1)$. Explain how you could use the diagram shown to prove that for every positive integer n, $(n + 1)(n - 1) = n^2 - 1$.



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3. Prove that the sum of two even numbers is even. (Use the fact that an even number is a number that can be written as $2n$, where n is a whole number. Show that if x and y are both even, then $x + y$ is even.)

____

____

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NAME ______________________________ CLASS ____________ DATE ____________

◆ **Skill B** Investigating conjecture and proof

Recall A prime number is a number that is divisible by exactly two numbers, itself and 1. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

◆ **Example**
Consider the numbers in the table. Given a whole number n, what appears to be true of the expression $n^2 - n + 41$?

n	$n^2 - n + 41$
1	$1^2 - 1 + 41 = 41$
2	$2^2 - 2 + 41 = 43$
3	$3^2 - 3 + 41 = 47$
4	$4^2 - 4 + 41 = 53$
5	$5^2 - 5 + 41 = 61$

n	$n^2 - n + 41$
6	$6^2 - 6 + 41 = 71$
7	$7^2 - 7 + 41 = 83$
8	$8^2 - 8 + 41 = 97$
9	$9^2 - 9 + 41 = 113$
10	$10^2 - 10 + 41 = 131$

◆ **Solution**
For each whole number n from 1 to 10, the expression $n^2 - n + 41$ gives a prime number.

Refer to the example above.

4. Complete the table.

n	$n^2 - n + 41$	Prime?
20		
30		
40		

Do the results of your table prove that $n^2 - n + 41$ gives a prime number? ____________

5. Find the value of $n^2 - n + 41$ for $n = 41$. Show that $n^2 - n + 41$ is *not* prime.

____

6. Is the conjecture true that $n^2 - n + 41$ gives a prime number? Explain your response.

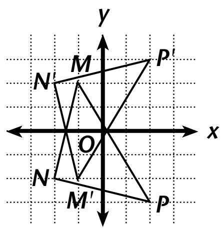
____

7. A conjecture cannot be proven true by demonstrating that it is true for specific cases, no matter how many are presented. How many cases must be presented to show that a conjecture is *not* true?

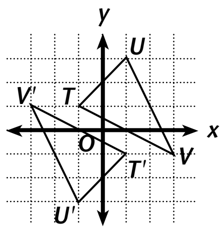
____

____

4. reflection across the x-axis



5.



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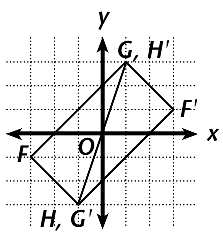

Reteaching — Chapter 2

Lesson 2.1

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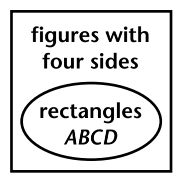
Lesson 2.2

1.



ABCD is a figure with four congruent sides.

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NAME ______________________ CLASS ____________ DATE ____________

Reteaching
2.2 An Introduction to Logic

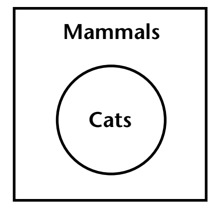
◆ **Skill A** Drawing conclusions from conditionals

Recall A conditional is a statement that can be written in "If-then" form. The part following *if* is called the **hypothesis**. The part following *then* is called the **conclusion**.

The statement "All cats are mammals" is a conditional statement.

If-then form: If an animal is a cat, then it is a mammal.

↓ hypothesis ↓ conclusion



The *Euler* (or *Venn*) *diagram* at the right shows that all cats are mammals. This demonstrates that the given conditional statement is true.

The process of **deductive reasoning**, or **deduction**, involves making logical arguments to reach logically certain conclusions.

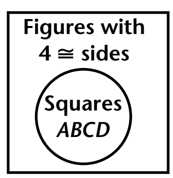
◆ **Example**

a. Draw an Euler diagram that conveys the following information.
If a figure is a square, then it has four congruent sides. *ABCD* is a square.

b. What conclusion can be drawn about *ABCD*?

◆ **Solution**

a.



b. *ABCD* has four congruent sides.

Use the statements "If a figure is a rectangle, then it has four sides" and "*PQRS* is a rectangle" in Exercises 1 and 2.

1. Draw an Euler diagram to represent the information.
What conclusion can be drawn from the diagram?

____

◆ **Skill B** Forming the converse of a conditional

Recall In logical notation, letters are used to represent the hypothesis and the conclusion of a conditional statement. The conditional "If p then q" is written "$p \to q$." This may also be read "p implies q." The **converse** of a conditional is formed by interchanging the hypothesis and the conclusion.

Conditional: $p \to q$ Converse: $q \to p$

The converse of a true conditional statement may be true or false. A conditional statement is false if you can find one example, called a **counterexample**, for which the hypothesis is true and the conclusion is false.

◆ **Example**
The statement "If a person lives in San Antonio, then he or she lives in Texas" is true. Write the hypothesis, the conclusion, and the converse of the statement. If the converse is false, give a counterexample.

◆ **Solution**
hypothesis: A person lives in San Antonio conclusion: he or she lives in Texas
converse: If a person lives in Texas, then he or she lives in San Antonio.
The converse is false. For example, the person may live in Dallas.

The statement "If a line intersects a segment at its midpoint, then the line bisects the segment" is true.

2. Write the hypothesis. ______

3. Write the conclusion. ______

4. Write the converse. Is it true? ______

◆ Skill C Writing and using logical chains

Recall Conditional statements can sometimes be linked together in a logical chain. A conditional can be derived from the logical chain using the *If-Then* Transitive Property:

When you are given "If A, then B" and "If B, then C," you can conclude "If A, then C." This can also be written "If $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$."

◆ **Example**
Arrange the statements in a logical chain. Write the conditional that follows.
If it is cold, I can go ice skating.
If it is March, then the wind is blowing.
If the wind is blowing, then it is cold.

◆ **Solution**
Use letters to represent the hypothesis and conclusion of each statement.
A: it is cold B: I can go ice skating C: it is March D: the wind is blowing
Write the statements in logical notation and look for a chain.

$$A \rightarrow B \qquad C \rightarrow D \qquad D \rightarrow A$$

The chain is $C \rightarrow D$, $D \rightarrow A$, and $A \rightarrow B$. The conditional that follows is $C \rightarrow B$, that is, if it is March, then I can go ice skating.

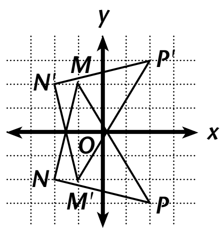
Arrange the statements in a logical chain and write the conditional that follows.

5. If I can sing, then I will be a star. If I am a bird, then I can sing. If cows have wings, then they can fly. If an animal can fly, then it is a bird.

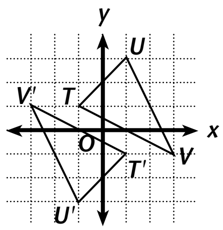
______

______

4. reflection across the x-axis



5.



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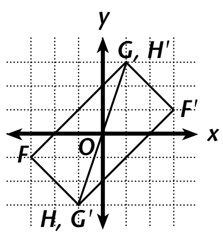


Reteaching — Chapter 2

Lesson 2.1

1. Yes; the proof would be identical. If n is a positive odd number, then the next positive odd number is $n + 2$.

2. If you take the bottom row of dots and make it a column at the right, the diagram would be nearly square. It would be missing one dot. For the specific case in the diagram, there are six dots and each row and eight dots in each column. If the bottom row is made into a column, the diagram would be a 7×7 square with one dot missing. That is, $6 \times 8 = 7^2 - 1$.

3. Since x and y are even numbers, let $x = 2n$ and $y = 2m$, where n and m are whole numbers. Then $x + y = 2n + 2m = 2(n + m)$, which is an even number, since $n + m$ is a whole number.

4.

n	$n^2 - n + 41$	Prime?
20	421	Yes
30	911	Yes
40	1601	Yes

Yes.

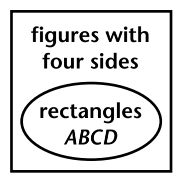
5. 1681; $1681 = 41^2$

6. No. When $n = 41$, the value of the expression is 41×41, which is not prime.

7. only one case

Lesson 2.2

1.



ABCD is a figure with four congruent sides.

2. a line intersects a segment at its midpoint

3. the line bisects the segment

4. If a line bisects a segment, then the line intersects the segment at its midpoint. True.

5. If cows have wings, then they can fly. If an animal can fly, then it is a bird. If I am a bird, then I can sing. If I can sing, then I will be a star. If cows have wings, then I will be a star.

NAME ______________________ CLASS __________ DATE __________

Reteaching
2.3 Definitions

◆ **Skill A** Using Euler diagrams to study definitions

Recall When a conditional statement and its converse are combined, the result is a conditional statement known as a **biconditional**, or an "if-and-only-if" statement. In logical notation, if you are given $p \rightarrow q$ and $q \rightarrow p$, you can combine them to form the biconditional statement $p \leftrightarrow q$, which is read "p if and only q." A biconditional can be represented by a pair of Euler diagrams, one representing one conditional statement and one representing its converse.

Every definition can be written as a biconditional. For example, the definition of adjacent angles can be written as follows.

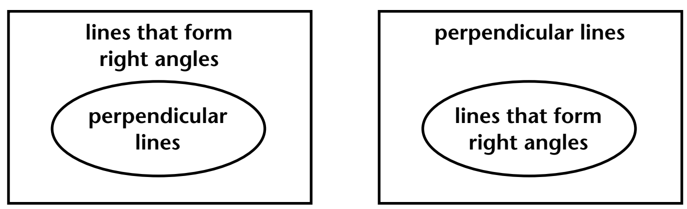
Two angles in the same plane are **adjacent angles** if and only if they share a vertex and a common side, but no interior points.

◆ **Example**
Determine whether the following sentence is a definition.

Perpendicular lines are lines that meet to form right angles.

If the sentence is a definition, write it as a biconditional and represent it with a pair of Euler diagrams.

◆ **Solution**
If two lines are perpendicular, then they meet to form right angles. If two lines meet to form right angles, then they are perpendicular. The sentence is a definition. Two lines are **perpendicular** if and only if they meet to form right angles.



Determine whether the sentence is a definition. If so, write it as a biconditional and represent it with a pair of Euler diagrams. If not, explain why not.

1. A square is a rectangle with four congruent sides. ______________________

____

____

2. An acute angle is an angle that is not obtuse. ______________________

____

____

NAME ______________________________ CLASS ______________ DATE ______________

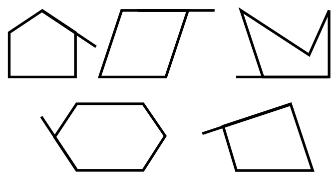
◆ Skill B Using the principles of logic to formulate definitions

Recall Every definition can be written as a biconditional.

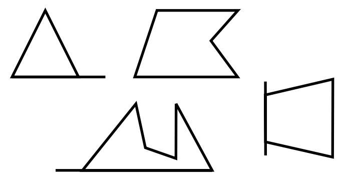
◆ Example

Suppose, with no information other than that given in the figures below, you are asked to define the word *frisbend*.

These figures are frisbends.



These figures are not frisbends.



Use the figures above to write a definition of the word *frisbend*.

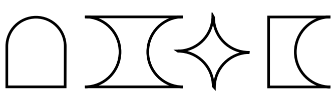
◆ Solution

You need to determine what frisbends have in common that is not true of objects that are *not* frisbends. Each of the objects in the first group is a closed figure with at least four sides, including one that extends beyond a vertex. Some objects in the second group have fewer than four sides. Some have no extended sides, or more than one.

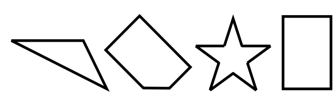
Definition: A figure is a *frisbend* if and only if it is a closed figure with at least four sides, exactly one of which extends beyond a vertex.

Use the given information to determine which figures are greems.

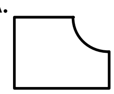
3. These figures are greems.

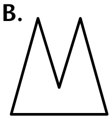


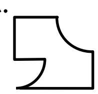
These figures are not greems.



Tell whether each figure is a greem.

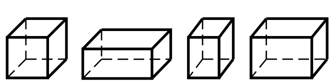
A.  ______________

B.  ______________

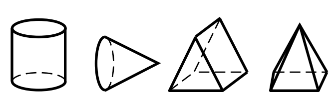
C.  ______________

Use the given information to write a definition of the phrase *rectangular solid*.

4. These figures are rectangular solids.



These figures are not rectangular solids.



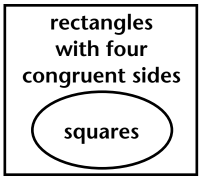
A space figure is a *rectangular solid* if and only if ____

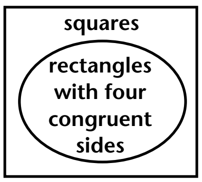
____.

ANSWERS

Lesson 2.3

1. Yes; a figure is a square if and only if it is a rectangle with four congruent sides.





2. No; if an angle is acute, then it is not obtuse. However an angle that is not obtuse may be a right angle or a straight angle.

3. **a.** Yes; **b.** no; **c.** yes.

4. Sample: all of its surfaces are rectangles

Lesson 2.4

1. $3x - 7 = 29$ (Given); $3x = 36$ (Addition Property of Equality); $x = 12$ (Division Property of Equality)

2. Addition Property of Equality

3. Segment Addition Postulate

4. $PR = QS$

5. Given

6. Addition Property of Equality

7. Angle Addition Postulate

8. $m\angle ADC$

9. $m\angle ABC = m\angle ADC$

Lesson 2.5

1. 11 **2.** 12 **3.** 7 **4.** 9 **5.** 10

6. 8 **7.** 33° **8.** 139° **9.** 110°

10. 28 **11.** 11 **12.** 12

13. Definition of perpendicular lines

14. Transitive (or Substitution) Property of Equality

15. Angle Addition Postulate

16. Transitive (or Substitution) Property of Equality

17. Subtraction Property of Equality

18. $\angle 1 \cong \angle 2$

19. Transitive **20.** $\angle 4$ **21.** $\angle 3 \cong \angle 4$

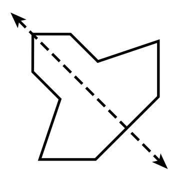
22. $\angle 1 \cong \angle 4$

Reteaching — Chapter 3

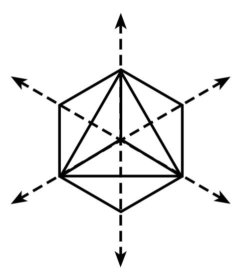
Lesson 3.1

1. none

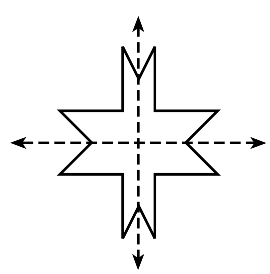
2.



3.



4.



Menu

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Reteaching

2.4 Building a System of Geometry Knowledge

◆ **Skill A** Using the properties of equality and congruence

Recall The Algebraic and Equivalence Properties of Equality which you are already familiar are often used in geometry. Since the measures of segments and angles are real numbers, the algebraic properties can be applied to such measures. The equivalence properties also extend to geometric figures. The properties are summarized below.

The Algebraic Properties of Equality

For all real numbers a, b, and c, if $a = b$, then:

$a + c = b + c$	(Addition)	$a - c = b - c$	(Subtraction)
$ac = bc$	(Multiplication)	$\frac{a}{c} = \frac{b}{c} (c \neq 0)$	(Division)

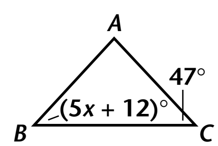
a and b may be substituted for each other in any equation or inequality. (Substitution)

The Equivalence Properties

	Equality	Congruence
	For all real numbers a, b, and c:	For geometric figures A, B, and C:
Reflexive	$a = a$	$A \cong A$
Symmetric	If $a = b$, then $b = a$.	If $A \cong B$, then $B \cong A$.
Transitive	If $a = b$ and $b = c$, then $a = c$.	If $A \cong B$ and $B \cong C$, then $A \cong C$.

◆ **Example**

In the figure, $\angle B \cong \angle C$. Write and solve an equation to find the value of x. Write the steps of the solution at the left. Write the reason that justifies each step to the right of the step.

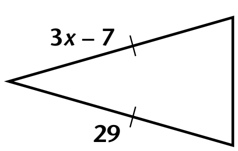


◆ **Solution**

$\angle B \cong \angle C$	Given
$m\angle B = m\angle C$	Definition of congruence
$5x + 12 = 47$	Substitution Property of Equality
$5x + 12 - 12 = 47 - 12$	Subtraction Property of Equality
$5x = 35$	Subtraction
$x = 7$	Division Property of Equality

In each triangle congruent sides are indicated. Write and solve an equation to find the value of *x*. Write the steps of the solution at the left side of the blanks and the reasons at the right.

1.



____

____

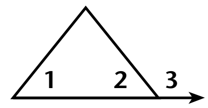
____

____

◆ Skill B Providing justifications in proofs

Recall In the proof of a theorem, each step (unless it is given) must be justified by a definition, a property, a postulate, or a theorem.

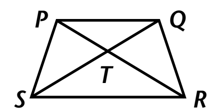
◆ **Example**
Given: $m\angle 1 + m\angle 3 = 180°$
Prove: $\angle 1 \cong \angle 2$



◆ **Solution**

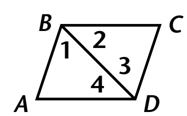
Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180°$	1. Given
2. $m\angle 2 + m\angle 3 = 180°$	2. Linear Pair Property
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	3. Substitution Property of Equality
4. $m\angle 1 = m\angle 2$	4. Subtraction Property of Equality
5. $\angle 1 \cong \angle 2$	5. Definition of congruence

Complete each proof.



Given: $PT = QT, TR = TS$
Prove: $PR = QS$

Statements	Reasons
$PT = QT$, $TR = TS$	Given
$PT + TR = QT + TS$	**2.** ____________
$PT + TR = PR$, $QT + TS = QS$	**3.** ____________
4. ____________	Substitution Property of Equality



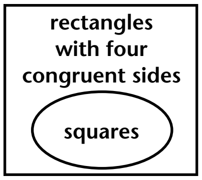
Given: $m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 4$
Prove: $m\angle ABC = m\angle ADC$

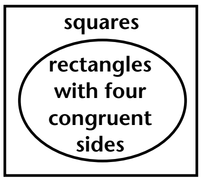
Statements	Reasons
$m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 4$	**5.** ____________
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**6.** ____________
$m\angle 1 + m\angle 2 = m\angle ABC$	**7.** ____________
8. $m\angle 3 + m\angle 4 = m\angle$ ____________	Angle Addition Postulate
9. ____________	Substitution Property of Equality

ANSWERS

Lesson 2.3

1. Yes; a figure is a square if and only if it is a rectangle with four congruent sides.





2. No; if an angle is acute, then it is not obtuse. However an angle that is not obtuse may be a right angle or a straight angle.

3. **a.** Yes; **b.** no; **c.** yes.

4. Sample: all of its surfaces are rectangles

Lesson 2.4

1. $3x - 7 = 29$ (Given); $3x = 36$ (Addition Property of Equality); $x = 12$ (Division Property of Equality)

2. Addition Property of Equality

3. Segment Addition Postulate

4. $PR = QS$

5. Given

6. Addition Property of Equality

7. Angle Addition Postulate

8. $m\angle ADC$

9. $m\angle ABC = m\angle ADC$

Lesson 2.5

1. 11 **2.** 12 **3.** 7 **4.** 9 **5.** 10

6. 8 **7.** 33° **8.** 139° **9.** 110°

10. 28 **11.** 11 **12.** 12

13. Definition of perpendicular lines

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15. Angle Addition Postulate

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17. Subtraction Property of Equality

18. $\angle 1 \cong \angle 2$

19. Transitive **20.** $\angle 4$ **21.** $\angle 3 \cong \angle 4$

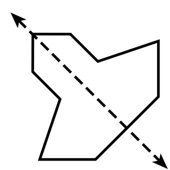
22. $\angle 1 \cong \angle 4$

Reteaching — Chapter 3

Lesson 3.1

1. none

2.



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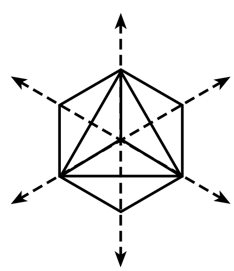

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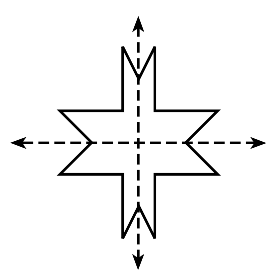
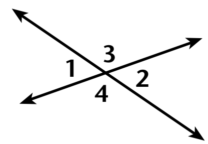


NAME ______________________ CLASS __________ DATE __________

Reteaching
2.5 Conjectures That Lead to Theorems

◆ Skill A Using the Vertical Angles Theorem

Recall When two lines intersect, they form two pairs of **vertical angles**. According to the **Vertical Angles Theorem**, if two angles form a pair of vertical angles, then they are congruent. In the figure at the right, $\angle 1$ and $\angle 2$ are vertical angles, so $\angle 1 \cong \angle 2$. Also, $\angle 3$ and $\angle 4$ are vertical angles, so $\angle 3 \cong \angle 4$.



◆ Example

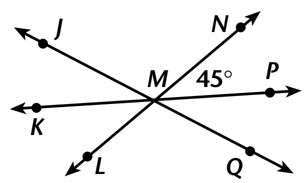
Complete:

a. $\angle KMN$ and $\angle$ ____?____ are vertical angles.

b. If $m\angle JML = 77°$, then $m\angle NMQ =$ ____?____.

c. If $m\angle NMP = 45°$, and $m\angle KML = (4x - 3)°$,

then $x =$ ____?____.



◆ Solution

a. $\angle KMN$ is formed by intersecting lines $\overleftrightarrow{KP}$ and $\overleftrightarrow{LN}$, so $\angle KMN$ and $\angle LMP$ are vertical angles.

b. $\angle JML$ and $\angle NMQ$ are vertical angles, so $m\angle NMQ = m\angle JML = 77°$.

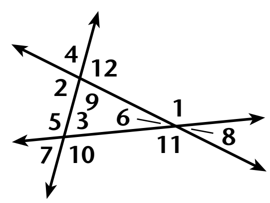
c. $\angle NMP$ and $\angle KML$ are vertical angles. Then their measures are equal and $4x - 3 = 45$, so $4x = 48$, and $x = 12$.

Identify the vertical angles in the figure.

1. $\angle 1$ and $\angle$ __________ **2.** $\angle 2$ and $\angle$ __________

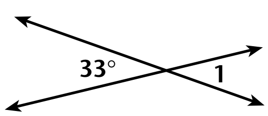
3. $\angle 3$ and $\angle$ __________ **4.** $\angle 4$ and $\angle$ __________

5. $\angle 5$ and $\angle$ __________ **6.** $\angle 6$ and $\angle$ __________



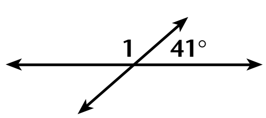
Find the measure of $\angle 1$.

7.



____

8.



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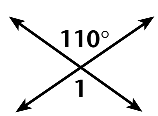
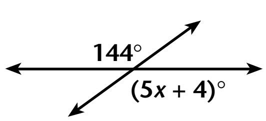
9.



____

Find the value of *x*.

10.



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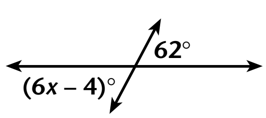
11.



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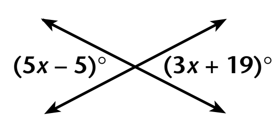
12.



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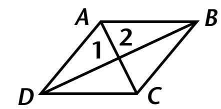
◆ Skill B Using two-column and paragraph forms of proof

◆ Example

Given: m∠1 = m∠2

Prove: $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BD}$.

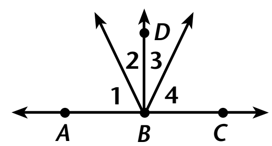


◆ Solution

Statements	Reasons
1. m∠1 = m∠2	1. Given
2. m∠1 + m∠2 = 180°	2. Linear Pair Property
3. m∠1 + m∠1 = 180°; 2m∠1 = 180°	3. Substitution Property of Equality
4. m∠1 = 90°	4. Division Property of Equality
5. $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BD}$.	5. Definition of perpendicular lines

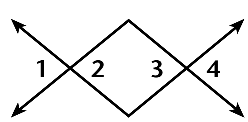
Complete each proof.

Given: $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BD}$
∠1 ≅ ∠4
Prove: ∠2 ≅ ∠3



Statements	Reasons
$\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BD}$.	Given
m∠*ABD* = 90°; m∠*CBD* = 90°	**13.** ______________
m∠*ABD* = m∠*CBD*	**14.** ______________
m∠1 + m∠2 = m∠*ABD*; m∠3 + m∠4 = m∠*CBD*	**15.** ______________
m∠1 + m∠2 = m∠3 + m∠4	**16.** ______________
m∠2 = m∠3 or ∠2 ≅ ∠3	**17.** ______________

Given: ∠2 ≅ ∠3
Prove: ∠1 ≅ ∠4

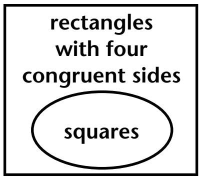


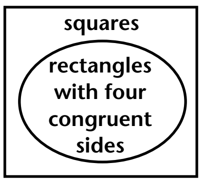
Proof: ∠1 and ∠2 are vertical angles, so **(18.)** ______________ by the Vertical Angles Theorem. It is given that ∠2 ≅ ∠3, so ∠1 ≅ ∠3 by the **(19.)** ______________ Property of Congruence. ∠3 and **(20.)** ______________ are also vertical angles, so **(21.)** ______________ as well. Then, by the Transitive Property of Congruence, **(22.)** ______________.

ANSWERS

Lesson 2.3

1. Yes; a figure is a square if and only if it is a rectangle with four congruent sides.





2. No; if an angle is acute, then it is not obtuse. However an angle that is not obtuse may be a right angle or a straight angle.

3. **a.** Yes; **b.** no; **c.** yes.

4. Sample: all of its surfaces are rectangles

Lesson 2.4

1. $3x - 7 = 29$ (Given); $3x = 36$ (Addition Property of Equality); $x = 12$ (Division Property of Equality)

2. Addition Property of Equality

3. Segment Addition Postulate

4. $PR = QS$

5. Given

6. Addition Property of Equality

7. Angle Addition Postulate

8. $m\angle ADC$

9. $m\angle ABC = m\angle ADC$

Lesson 2.5

1. 11 **2.** 12 **3.** 7 **4.** 9 **5.** 10

6. 8 **7.** 33° **8.** 139° **9.** 110°

10. 28 **11.** 11 **12.** 12

13. Definition of perpendicular lines

14. Transitive (or Substitution) Property of Equality

15. Angle Addition Postulate

16. Transitive (or Substitution) Property of Equality

17. Subtraction Property of Equality

18. $\angle 1 \cong \angle 2$

19. Transitive **20.** $\angle 4$ **21.** $\angle 3 \cong \angle 4$

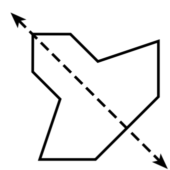
22. $\angle 1 \cong \angle 4$

Reteaching — Chapter 3

Lesson 3.1

1. none

2.



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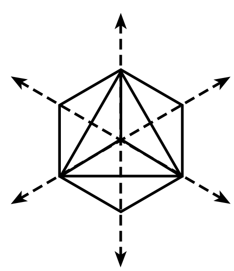

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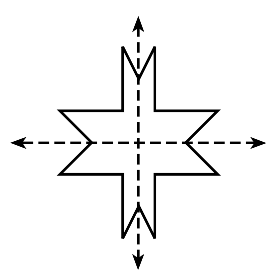

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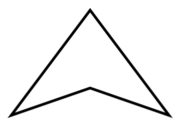
NAME ______________________ CLASS __________ DATE __________

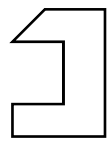
Reteaching
3.1 Symmetry in Polygons

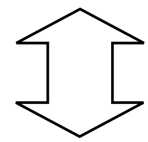
◆ **Skill A** Identifying and drawing figures with reflectional symmetry

Recall A figure has **reflectional symmetry** if there is a line, called an **axis of symmetry**, that divides the figure into two parts that are mirror images of each other. A figure may have no axis of symmetry, or it may have one or more. If you fold a figure along an axis of symmetry, the two parts of the figure will coincide.

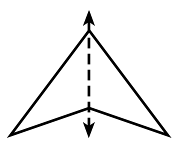
◆ **Example 1**
Tell how many axes of symmetry the figure has. Draw them all.

a.


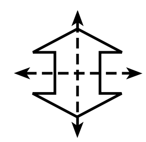
b.


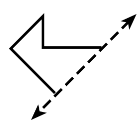
c.


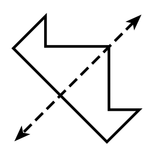
◆ **Solution**

a. There is one.


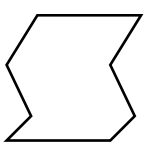
b. There are none.

c. There are two.


◆ **Example 2**
Complete the figure so that the dashed line is a line of symmetry.


◆ **Solution**


Draw all the axes of symmetry of each figure. If there are none, write *none*.

1.

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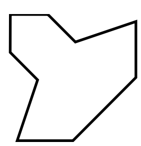
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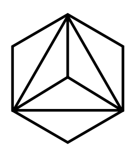
3.

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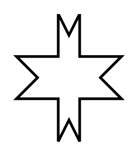
4.

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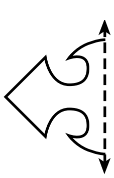
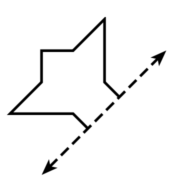
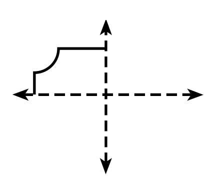
Complete the figure so that each dashed line is an axis of symmetry.

5.


6.


7.


NAME ______ CLASS ______ DATE ______

◆ Skill B Identifying and drawing figures with rotational symmetry

Recall A figure has rotational symmetry of $x°$ about a point P if, when the figure is rotated $x°$ about P, the figure and its image coincide.

A **regular polygon** is a polygon with all sides congruent and all angles congruent. Its center is the point that is equidistant from the sides.

The measure of the central angle of a regular polygon with n sides has measure $\frac{360°}{n}$. The polygon has n rotational symmetries about its center.

◆ **Example 1**

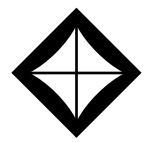
Describe the rotational symmetries of a regular hexagon.

◆ **Solution**

A regular hexagon has six sides, so it has six rotational symmetries about its center. The measure of a central angle is $\frac{360°}{6} = 60°$, so the measures of the angles of rotation for the symmetries are multiples of 60° less than or equal to 360°: 60°, 120°, 180°, 240°, 300°, and 360°.

◆ **Example 2**

Describe the rotational symmetries of the figure.



◆ **Solution**

The points of the figure are the vertices of a square. The figure has rotational symmetries of 90°, 180°, 270°, and 360° about the center.

Describe the rotational symmetries of the figure.

8. an equilateral triangle with center P

______

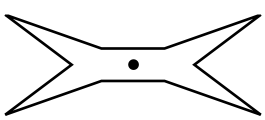
______

9. a regular pentagon with center P

______

______

10.



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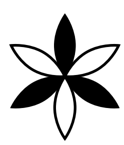
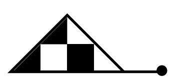
11.



______

______

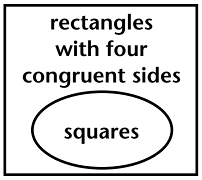
12. Complete the drawing so that the figure has 180° rotational symmetry about the indicated point.

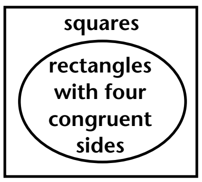


ANSWERS

Lesson 2.3

1. Yes; a figure is a square if and only if it is a rectangle with four congruent sides.





2. No; if an angle is acute, then it is not obtuse. However an angle that is not obtuse may be a right angle or a straight angle.

3. **a.** Yes; **b.** no; **c.** yes.

4. Sample: all of its surfaces are rectangles

Lesson 2.4

1. $3x - 7 = 29$ (Given); $3x = 36$ (Addition Property of Equality); $x = 12$ (Division Property of Equality)

2. Addition Property of Equality

3. Segment Addition Postulate

4. $PR = QS$

5. Given

6. Addition Property of Equality

7. Angle Addition Postulate

8. $m\angle ADC$

9. $m\angle ABC = m\angle ADC$

Lesson 2.5

1. 11 **2.** 12 **3.** 7 **4.** 9 **5.** 10

6. 8 **7.** 33° **8.** 139° **9.** 110°

10. 28 **11.** 11 **12.** 12

13. Definition of perpendicular lines

14. Transitive (or Substitution) Property of Equality

15. Angle Addition Postulate

16. Transitive (or Substitution) Property of Equality

17. Subtraction Property of Equality

18. $\angle 1 \cong \angle 2$

19. Transitive **20.** $\angle 4$ **21.** $\angle 3 \cong \angle 4$

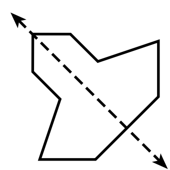
22. $\angle 1 \cong \angle 4$

Reteaching — Chapter 3

Lesson 3.1

1. none

2.



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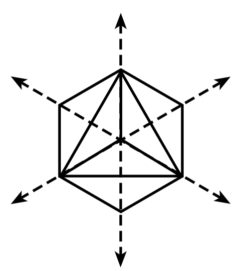

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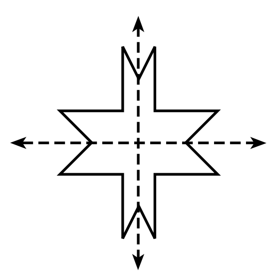

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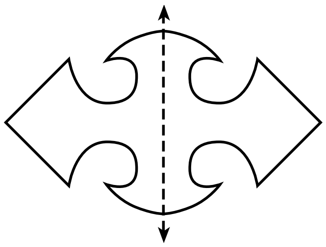

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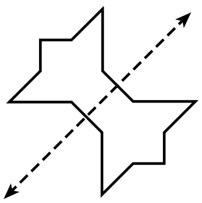
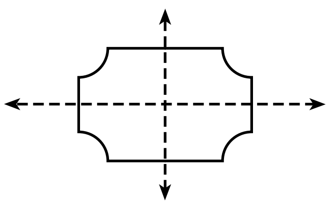


7.



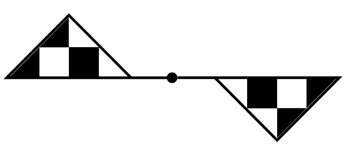
8. rotational symmetries of 120°, 240°, and 360° about the center

9. rotational symmetries of 72°, 144°, 216°, 288°, and 360°

10. rotational symmetres of 180° and 360° about the center

11. rotational symmetries of 120°, 240°, and 360° about the center

12.



Lesson 3.2

1. rhombus, parallelogram

2. square, rhombus, rectangle, parallelogram

3. trapezoid **4.** parallelogram **5.** 18

6. 24 **7.** 30 **8.** 15 **9.** 15 **10.** 90°

11. 20 **12.** 20 **13.** 7.5 **14.** 90°

15. 135° **16.** 45° **17.** 34 **18.** 24

19. 48 **20.** 48 **21.** 90° **22.** 90°

Lesson 3.3

1. $\angle 1$ and $\angle 3$; $\angle 2$ and $\angle 4$; $\angle 5$ and $\angle 7$; $\angle 6$ and $\angle 8$

2. $\angle 2$ and $\angle 6$; $\angle 3$ and $\angle 7$

3. $\angle 1$ and $\angle 5$; $\angle 4$ and $\angle 8$

4. $\angle 2$ and $\angle 3$; $\angle 6$ and $\angle 7$

5. Justifications may vary. $\angle 4$ (Vertical angles are congruent.); $\angle 8$ (If 2 $\parallel$ lines are int. by a trans., corr. $\angle$s are $\cong$.); $\angle 2$ (If 2 $\parallel$ lines are int. by a trans., alt. ext. $\angle$s are $\cong$.)

6. **a.** 143° **b.** 37° **c.** 143° **d.** 37° **e.** 143° **f.** 37°

7. **a.** 121° **b.** 51° **c.** 59° **d.** 70°

8. **a.** 24 **b.** 55° **c.** 125°

Lesson 3.4

1. $j \parallel k$; if 2 lines are int. by a trans. and alt. int. $\angle$s are $\cong$, the lines are $\parallel$.

2. $m \parallel n$; if 2 lines are int. by a trans. and alt. ext. $\angle$s are $\cong$, the lines are.

3. none

4. $m \parallel n$; if 2 lints are int. by a trans. and same-side int. $\angle$s are supp., the lines are $\parallel$.

5. Given **6.** $\overline{MQ}$ **7.** $\overline{NP}$ **8.** Given

9. $\overline{MN}$ **10.** $\overline{QP}$

11. If 2 lines are int. by a trans. and alt. int. $\angle$s are $\cong$, the lines are $\parallel$.

12. *MNPQ* is a parallelogram.

Lesson 3.5

1. 23° **2.** 80° **3.** 54° **4.** 68°

5. 7; 70°; 71°; 39° **6.** 8; 64°; 46°; 70°

NAME ______________________ CLASS __________ DATE __________

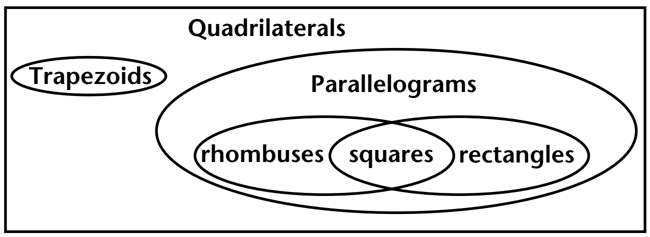
Reteaching
3.2 *Properties of Quadrilaterals*

◆ **Skill A** Identifying special quadrilaterals

Recall The table summarizes the definitions of special quadrilaterals.

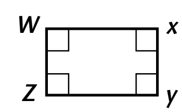
Trapezoid	Exactly one pair of opposite sides are parallel.
Parallelogram	Both pairs of opposite sides are parallel.
Rhombus	All four sides are congruent.
Rectangle	All four angles are right angles. (All four angles are congruent.)
Square	All four sides are congruent; all four angles are right angles.

The Euler diagram shows how special quadrilaterals are related.



◆ **Example**

In quadrilateral *WXYZ*, $\overleftrightarrow{WX}$ is perpendicular to both $\overleftrightarrow{WZ}$ and $\overleftrightarrow{XY}$. $\overleftrightarrow{YZ}$ is also perpendicular to both $\overleftrightarrow{WZ}$ and $\overleftrightarrow{XY}$. Identify the special type(s) of quadrilateral that *WXYZ* must be.



◆ **Solution**

All four angles of *WXYZ* are right angles, so *WXYZ* is a rectangle. According to the Euler diagram, all rectangles are parallelograms, so *WXYZ* is also a parallelogram.

Identify the special type(s) of quadrilateral that WXYZ must be.

1. $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, and $\overline{WZ}$ are congruent.

____

____

2. *WXYZ* is both equilateral and equiangular.

____

____

3. $\overline{WX}$ and $\overline{ZY}$ are parallel; $\overline{WZ}$ and $\overline{XY}$ are not parallel.

____

____

4. $\overline{WX}$ and $\overline{ZY}$ are parallel; $\overline{WZ}$ and $\overline{XY}$ are parallel.

____

____

◆ Skill B Using the properties of special quadrilaterals

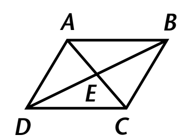
Recall Since rhombuses, rectangles, and squares are all parallelograms, they all have the properties of parallelograms. However, they have special properties of their own as well. The properties are summarized in the table.

All parallelograms	Opposite sides are congruent. Opposite angles are congruent. The diagonals bisect each other. Consecutive angles are supplementary.
Rhombuses	The diagonals are perpendicular.
Rectangles	The diagonals are congruent.
Squares	The diagonals are perpendicular and congruent.

The diagonals of a square are both perpendicular and congruent because a square is both a rhombus and a rectangle.

◆ Example

In parallelogram $ABCD$, $AB = 4$, $AD = 3$, $BD = 6$ and $m\angle DAB = 120°$. Find DC, BC, DE, and $m\angle ADC$.



◆ Solution

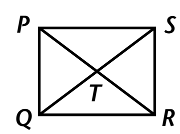
$ABCD$ is a parallelogram.

Opposite sides are congruent, so $DC = AB = 4$ and $BC = AD = 3$.

The diagonals of $ABCD$ bisect each other, so $DE = \frac{1}{2} \cdot DB = 3$

$\angle DAB$ and $\angle ADC$ are supplementary, so $m\angle ADC = 180° - 120° = 60°$.

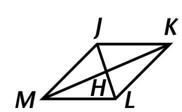
In rectangle *PQRS*, *PS* = 24, *PQ* = 18, and *QS* = 30. Complete.



5. $SR =$ ____________ **6.** $QR =$ ____________ **7.** $PR =$ ____________

8. $PT =$ ____________ **9.** $QT =$ ____________ **10.** $m\angle QPS =$ ________

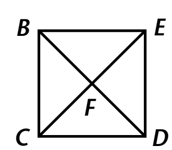
In rhombus *JKLM*, m∠*JML* = 45°, *ML* = 20, *JL* = 15, and *MN* = 18.5. Complete.



11. $KL =$ ____________ **12.** $JK =$ ____________ **13.** $JN =$ ____________

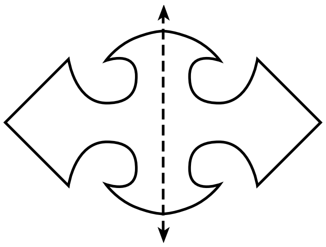
14. $m\angle JNM =$ ________ **15.** $m\angle MJK =$ ________ **16.** $m\angle JKL =$ ________

In square *BCDE*, *CD* = 34 and *BF* = 24. Complete.



17. $BE =$ ____________ **18.** $FD =$ ____________ **19.** $BD =$ ____________

20. $CE =$ ____________ **21.** $m\angle CBE =$ ________ **22.** $m\angle BFC =$ ________

5.


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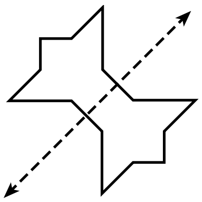

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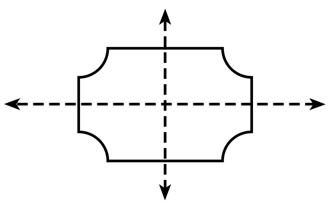

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9. rotational symmetries of 72°, 144°, 216°, 288°, and 360°

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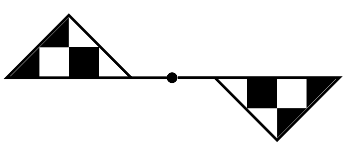

Lesson 3.2

1. rhombus, parallelogram

2. square, rhombus, rectangle, parallelogram

3. trapezoid **4.** parallelogram **5.** 18

6. 24 **7.** 30 **8.** 15 **9.** 15 **10.** 90°

11. 20 **12.** 20 **13.** 7.5 **14.** 90°

15. 135° **16.** 45° **17.** 34 **18.** 24

19. 48 **20.** 48 **21.** 90° **22.** 90°

Lesson 3.3

1. ∠1 and ∠3; ∠2 and ∠4; ∠5 and ∠7; ∠6 and ∠8

2. ∠2 and ∠6; ∠3 and ∠7

3. ∠1 and ∠5; ∠4 and ∠8

4. ∠2 and ∠3; ∠6 and ∠7

5. Justifications may vary. ∠4 (Vertical angles are congruent.); ∠8 (If 2 ∥ lines are int. by a trans., corr. ∠s are ≅.); ∠2 (If 2 ∥ lines are int. by a trans., alt. ext. ∠s are ≅.)

6. **a.** 143° **b.** 37° **c.** 143° **d.** 37° **e.** 143° **f.** 37°

7. **a.** 121° **b.** 51° **c.** 59° **d.** 70°

8. **a.** 24 **b.** 55° **c.** 125°

Lesson 3.4

1. $j \parallel k$; if 2 lines are int. by a trans. and alt. int. ∠s are ≅, the lines are ∥.

2. $m \parallel n$; if 2 lines are int. by a trans. and alt. ext. ∠s are ≅, the lines are.

3. none

4. $m \parallel n$; if 2 lints are int. by a trans. and same-side int. ∠s are supp., the lines are ∥.

5. Given **6.** $\overline{MQ}$ **7.** $\overline{NP}$ **8.** Given

9. $\overline{MN}$ **10.** $\overline{QP}$

11. If 2 lines are int. by a trans. and alt. int. ∠s are ≅, the lines are ∥.

12. *MNPQ* is a parallelogram.

Lesson 3.5

1. 23° **2.** 80° **3.** 54° **4.** 68°

5. 7; 70°; 71°; 39° **6.** 8; 64°; 46°; 70°

Menu
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NAME ______________________ CLASS __________ DATE __________

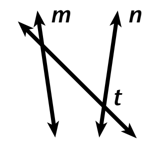
Reteaching
3.3 Parallel Lines and Transversals

◆ **Skill A** Identifying related pairs of angles

Recall A transversal of two lines is a line that intersects the given lines in two different points. When a transversal *t* intersects two lines *m* and *n*, the pairs of angles formed have special names.

Alternate exterior angles: a pair of angles outside *m* and *n* on opposite sides of *t*

Corresponding angles: a pair of angles in corresponding positions relative to *m* and *n*

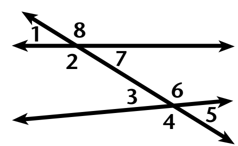


Alternate interior angles: a pair of angles between *m* and *n* on opposite sides of *t*

Same-side interior angles: a pair of consecutive angles between *m* and *n* on the same side of *t*

◆ **Example**

Indicate whether the pairs of angles are corresponding angles, alternate interior angles, alternate exterior angles, or same-side interior angles.



a. $\angle 3$ and $\angle 7$ **b.** $\angle 8$ and $\angle 6$

c. $\angle 4$ and $\angle 8$ **d.** $\angle 2$ and $\angle 3$

◆ **Solution**

a. $\angle 3$ and $\angle 7$ are alternate interior angles.
b. $\angle 8$ and $\angle 6$ are corresponding angles.
c. $\angle 4$ and $\angle 8$ are alternate exterior angles.
d. $\angle 2$ and $\angle 3$ are same-side interior angles.

Using the figure in the above example, identify all pairs of angles of the type indicated.

1. corresponding angles ______________________

2. alternate interior angles ______________________

3. alternate exterior angles ______________________

4. same-side interior angles ______________________

◆ **Skill B** Using the postulates and properties of parallel lines

Recall When two parallel lines are intersected by a transversal:

- Corresponding angles are congruent.
- Alternate interior angles are congruent.
- Alternate exterior angles are congruent.
- Same-side interior angles are supplementary.

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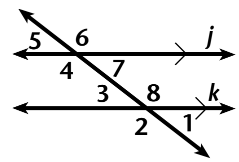
◆ **Example 1**

Lines *j* and *k* are parallel. Name all the angles that are congruent to $\angle 1$. Justify your answer.



◆ **Solution**

$\angle 7$; when two parallel lines are intersected by a transversal, corresponding angles are congruent.
$\angle 5$; when two parallel lines are intersected by a transversal, alternate exterior angles are congruent.
$\angle 3$; vertical angles are congruent.

◆ **Example 2**

Refer to the figure in Example 1 above. If $m\angle 4 = 142°$ find $m\angle 3$.

◆ **Solution**

When two parallel lines are intersected by a transversal, same-side interior angles are supplementary, so $m\angle 3 = 180° - m\angle 4 = 38°$.

Refer to the figure in Example 1 above.

5. Name all the angles that are congruent to $\angle 6$. Justify your answer.

____

____

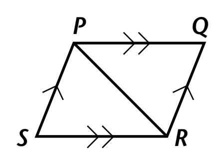
____

6. If $m\angle 1 = 37°$, find the measure of each numbered angle.

a. $m\angle 2 =$ ____________ **b.** $m\angle 3 =$ ____________ **c.** $m\angle 4 =$ ____________

d. $m\angle 5 =$ ____________ **e.** $m\angle 6 =$ ____________ **f.** $m\angle 7 =$ ____________

For Exercises 7 and 8, refer to the figure at the right, in which $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{SR}$ and $\overleftrightarrow{PS}$ is parallel to $\overleftrightarrow{QR}$.



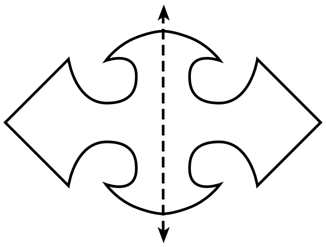
7. If $m\angle S = 59°$ and $m\angle PRS = 51°$, find the measure of each angle.

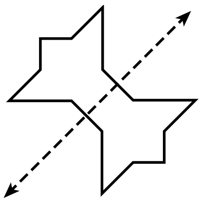
a. $m\angle SPQ =$ ____________ **b.** $m\angle QPR =$ ____________

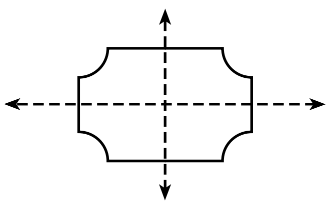
c. $m\angle Q =$ ____________ **d.** $m\angle PRQ =$ ____________

8. If $m\angle S = (2x + 7)°$ and $m\angle SPQ = 5(x + 1)°$, find each value.

a. $x =$ ____________ **b.** $m\angle S =$ ____________ **c.** $m\angle SPQ =$ ____________

5. 

6. 

7. 

8. rotational symmetries of 120°, 240°, and 360° about the center

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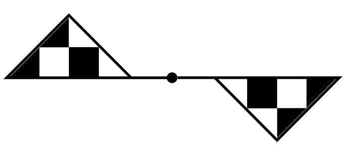
12. 

Lesson 3.2

1. rhombus, parallelogram

2. square, rhombus, rectangle, parallelogram

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6. 24 **7.** 30 **8.** 15 **9.** 15 **10.** 90°

11. 20 **12.** 20 **13.** 7.5 **14.** 90°

15. 135° **16.** 45° **17.** 34 **18.** 24

19. 48 **20.** 48 **21.** 90° **22.** 90°

Lesson 3.3

1. ∠1 and ∠3; ∠2 and ∠4; ∠5 and ∠7; ∠6 and ∠8

2. ∠2 and ∠6; ∠3 and ∠7

3. ∠1 and ∠5; ∠4 and ∠8

4. ∠2 and ∠3; ∠6 and ∠7

5. Justifications may vary. ∠4 (Vertical angles are congruent.); ∠8 (If 2 ∥ lines are int. by a trans., corr. ∠s are ≅.); ∠2 (If 2 ∥ lines are int. by a trans., alt. ext. ∠s are ≅.)

6. **a.** 143° **b.** 37° **c.** 143° **d.** 37° **e.** 143° **f.** 37°

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8. **a.** 24 **b.** 55° **c.** 125°

Lesson 3.4

1. $j \parallel k$; if 2 lines are int. by a trans. and alt. int. ∠s are ≅, the lines are ∥.

2. $m \parallel n$; if 2 lines are int. by a trans. and alt. ext. ∠s are ≅, the lines are.

3. none

4. $m \parallel n$; if 2 lints are int. by a trans. and same-side int. ∠s are supp., the lines are ∥.

5. Given **6.** $\overline{MQ}$ **7.** $\overline{NP}$ **8.** Given

9. $\overline{MN}$ **10.** $\overline{QP}$

11. If 2 lines are int. by a trans. and alt. int. ∠s are ≅, the lines are ∥.

12. *MNPQ* is a parallelogram.

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1. 23° **2.** 80° **3.** 54° **4.** 68°

5. 7; 70°; 71°; 39° **6.** 8; 64°; 46°; 70°

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Reteaching
3.4 Proving That Lines are Parallel

◆ **Skill A** Identifying parallel lines

Recall You have now learned six ways to prove that two lines are parallel.

Two lines are parallel if, when they are intersected by a transversal:
- corresponding angles are congruent.
- alternate interior angles are congruent.
- alternate exterior angles are congruent.
- same-side interior angles are supplementary.

Two coplanar lines that are perpendicular to the same line are parallel.
Two lines that are parallel to the same line are parallel to each other.

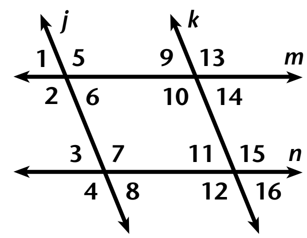
◆ **Example**

Based on the given information, tell which lines, if any, must be parallel. Explain why the lines are parallel.

a. $\angle 4 \cong \angle 5$

b. $\angle 7 \cong \angle 16$

c. $\angle 7$ and $\angle 11$ are supplementary.



◆ **Solution**

a. *m* and *n*; if two lines are intersected by a transversal and alternate exterior angles are congruent, then the lines are parallel.

b. None.

c. *j* and *k*; if two lines are intersected by a transversal and same-side interior angles are supplementary, the lines are parallel.

Refer to the figure in the example above. Determine which lines, if any must be parallel. Explain why the lines are parallel.

1. $\angle 6 \cong \angle 9$ ______________________________

____

2. $\angle 12 \cong \angle 13$ ______________________________

____

3. $\angle 3$ and $\angle 13$ are supplementary. ______________________________

____

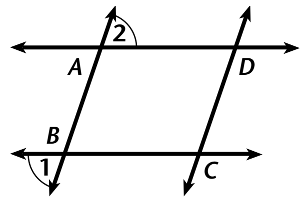
4. $\angle 14$ and $\angle 15$ are supplementary. ______________________________

____

◆ Skill B Using theorems about parallel lines to complete proofs

◆ **Example**

Given: $\angle 1 \cong \angle 2$;
$\angle B$ and $\angle BCD$ are supplementary.
Prove: *ABCD* is a parallelogram.



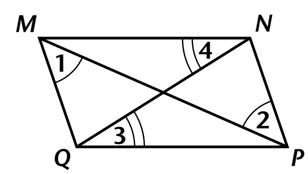
◆ **Solution**

Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	Given
2. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$	If two lines are intersected by a transversal and alt. ext. $\angle$s are $\cong$, the lines are $\parallel$.
3. $\angle CBA$ and $\angle BCD$ are supplementary.	Given
4. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$	If two lines are intersected by a transversal and same-side int. $\angle$s are supp., the lines are $\parallel$.
5. *ABCD* is a parallelogram.	Definition

Complete the two-column proof.

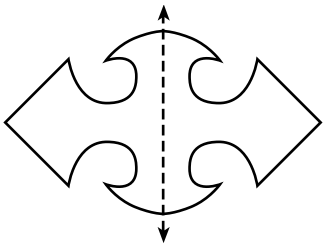
Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$
Prove: *MNPQ* is a parallelogram.



Proof:

Statements	Reasons
$\angle 1 \cong \angle 2$	**5.** ______________________
6. __________ and **(7.)** __________ are $\parallel$.	If two lines are intersected by a transversal and alt. int. $\angle$s are $\cong$, the lines are $\parallel$.
$\angle 3 \cong \angle 4$	**8.** ______________________
9. ________ and **(10.)** _________ are $\parallel$.	**11.** ______________________ ______________________
12. ______________________	Definition

5.



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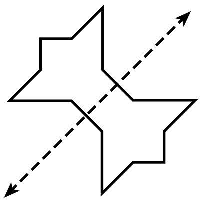

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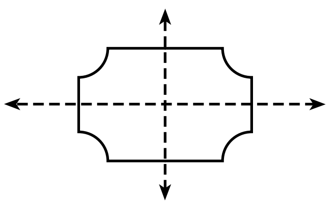

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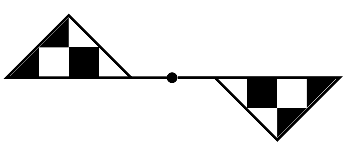

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2. ∠2 and ∠6; ∠3 and ∠7

3. ∠1 and ∠5; ∠4 and ∠8

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5. Given **6.** $\overline{MQ}$ **7.** $\overline{NP}$ **8.** Given

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5. 7; 70°; 71°; 39° **6.** 8; 64°; 46°; 70°

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Print

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
3.5 The Triangle Sum Theorem

◆ **Skill A** Using the Triangle Sum Theorem to find angle measures

Recall **The Triangle Sum Theorem:** The sum of the measures of the angles of a triangle is 180°.

◆ **Example 1**

In $\triangle XYZ$, $m\angle X = 41°$ and $m\angle Y = 66°$. Find $m\angle Z$.

◆ **Solution**

By the Triangle Sum Theorem, $m\angle X + m\angle Y + m\angle Z = 180°$.

$41° + 66° + m\angle Z = 180°$; $m\angle Z = 73°$

◆ **Example 2**

In $\triangle JKL$, $m\angle J = (x^2 - 10)°$, $m\angle K = (4x + 10)°$, and $m\angle L = (5x - 10)°$. Find the value of x and the measure of each angle of $\triangle JKL$.

◆ **Solution**

By the Triangle Sum Theorem, $m\angle J + m\angle K + m\angle L = 180°$.

$$x^2 - 10 + 4x + 10 + 5x - 10 = 180$$
$$x^2 + 9x - 190 = 0$$
$$(x - 10)(x + 19) = 0$$
$$x = 10 \text{ or } x = -19$$

If $x = -19$, $4x + 10$ and $5x - 10$ are negative, so $x = 10$.

$m\angle J = (x^2 - 10)° = (100 - 10)° = 90°$

$m\angle K = (4x + 10)° = (40 + 10) = 50°$

$m\angle L = (5x - 10)° = (50 - 10)° = 40°$

The measures of two angles of a triangle are given. Find the measure of the third angle.

1. $m\angle W = 92°$ and $m\angle X = 65°$; $m\angle Y =$ ______

2. $m\angle R = 50°$ and $m\angle S = 50°$; $m\angle T =$ ______

3. $m\angle D = 22°$ and $m\angle F = 104°$; $m\angle E =$ ______

4. $m\angle B = 78°$ and $m\angle D = 34°$; $m\angle C =$ ______

The measures of the angles of a triangle are given. Find the value of x and the measure of each angle.

5. $m\angle P = (x^2 + 3x)°$, $m\angle Q = (10x + 1)°$, $m\angle R = (5x + 4)°$

$x =$ __________ $m\angle P =$ __________ $m\angle Q =$ __________ $m\angle R =$ __________

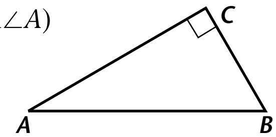
6. $m\angle K = (x^2)°$, $m\angle L = (5x + 6)°$, $m\angle M = (7x + 14)°$

$x =$ __________ $m\angle K =$ __________ $m\angle L =$ __________ $m\angle M =$ __________

◆ Skill B Using the Triangle Sum Theorem to complete proofs

◆ **Example**

Given: $\triangle ABC$ with $m\angle B = 2 \times (m\angle A)$ and $m\angle C = 3 \times (m\angle A)$
Prove: $\triangle ABC$ is a right triangle.



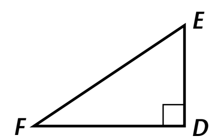
◆ **Solution**

Proof:

Statements	Reasons
1. $m\angle B = 2 \times (m\angle A)$, $m\angle C = 3 \times (m\angle A)$	Given
2. $m\angle A + m\angle B + m\angle C = 180°$	Triangle Sum Theorem
3. $m\angle A + 2 \times (m\angle A) + 3 \times (m\angle A) = 180°$	Substitution Property of Equality
4. $6 \times (m\angle A) = 180°$; $m\angle A = 30°$	Division Property of Equity
5. $m\angle C = 90°$	Substitution Property of Equality
6. $\angle C$ is a right angle; $\triangle ABC$ is a right triangle.	Definition

Complete the two-column proof.

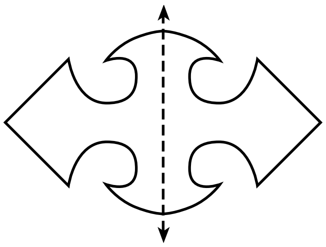
Given: $\angle D$ is a right angle.
Prove: $\angle E$ and $\angle F$ are complementary.



Proof:

Statements	Reasons
$\angle D$ is a right angle.	Given
$m\angle D = 90°$	**7.** ______
$m\angle D + m\angle E + m\angle F = 180°$	**8.** ______ ______
$90° + m\angle E + m\angle F = 180°$	**9.** ______
$m\angle E + m\angle F = 90°$	**10.** ______
$\angle E$ and $\angle F$ are complementary.	**11.** ______

5.



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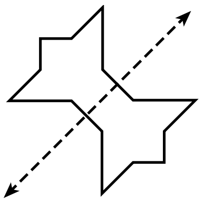
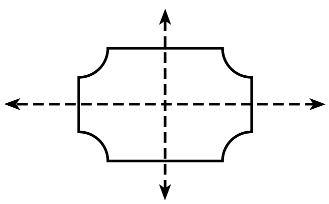


7.



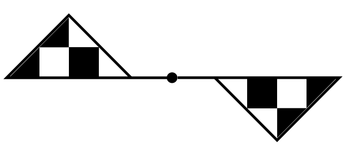
8. rotational symmetries of 120°, 240°, and 360° about the center

9. rotational symmetries of 72°, 144°, 216°, 288°, and 360°

10. rotational symmetres of 180° and 360° about the center

11. rotational symmetries of 120°, 240°, and 360° about the center

12.



Lesson 3.2

1. rhombus, parallelogram

2. square, rhombus, rectangle, parallelogram

3. trapezoid 4. parallelogram 5. 18

6. 24 7. 30 8. 15 9. 15 10. 90°

11. 20 12. 20 13. 7.5 14. 90°

15. 135° 16. 45° 17. 34 18. 24

19. 48 20. 48 21. 90° 22. 90°

Lesson 3.3

1. ∠1 and ∠3; ∠2 and ∠4; ∠5 and ∠7; ∠6 and ∠8

2. ∠2 and ∠6; ∠3 and ∠7

3. ∠1 and ∠5; ∠4 and ∠8

4. ∠2 and ∠3; ∠6 and ∠7

5. Justifications may vary. ∠4 (Vertical angles are congruent.); ∠8 (If 2 ∥ lines are int. by a trans., corr. ∠s are ≅.); ∠2 (If 2 ∥ lines are int. by a trans., alt. ext. ∠s are ≅.)

6. **a.** 143° **b.** 37° **c.** 143° **d.** 37° **e.** 143° **f.** 37°

7. **a.** 121° **b.** 51° **c.** 59° **d.** 70°

8. **a.** 24 **b.** 55° **c.** 125°

Lesson 3.4

1. $j \parallel k$; if 2 lines are int. by a trans. and alt. int. ∠s are ≅, the lines are ∥.

2. $m \parallel n$; if 2 lines are int. by a trans. and alt. ext. ∠s are ≅, the lines are.

3. none

4. $m \parallel n$; if 2 lints are int. by a trans. and same-side int. ∠s are supp., the lines are ∥.

5. Given 6. $\overline{MQ}$ 7. $\overline{NP}$ 8. Given

9. $\overline{MN}$ 10. $\overline{QP}$

11. If 2 lines are int. by a trans. and alt. int. ∠s are ≅, the lines are ∥.

12. $MNPQ$ is a parallelogram.

Lesson 3.5

1. 23° 2. 80° 3. 54° 4. 68°

5. 7; 70°; 71°; 39° 6. 8; 64°; 46°; 70°

7. Def.

8. Triangle Sum Theorem: The sum of the measures of the angles of a triangle is 180°.

9. Substitution Prop. of =

10. Subtraction. Prop. of = **11.** Def.

Lesson 3.6

1. 120° **2.** 130° **3.** 55° **4.** 113° **5.** 34°

6. 150° **7.** 2160° **8.** 360° **9.** 140°; 40°

10. 156°; 24° **11.** 168°; 12°

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17. 32 sides **18.** 50 sides **19.** 48 sides

20. 27 sides

Lesson 3.7

1. 39 **2.** 46.5 **3.** 58

4. Use the method of Example 2. Draw $\overline{JL}$. Midsegments $\overline{WX}$ of $\triangle JKL$ and $\overline{ZY}$ of $\triangle JLM$ are both parallel to $\overline{JL}$, so $\overline{WX} \parallel \overline{ZY}$. Since Ex. 2 showed that $\overline{WZ} \parallel \overline{XY}$, $WXYZ$ is a parallelogram.

5. 39 **6.** 19.5 **7.** 11 **8.** 16 **9.** 32

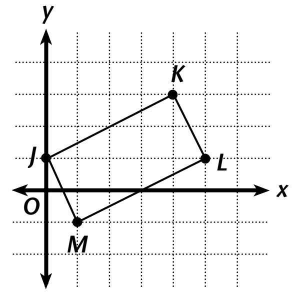
10. 13

Lesson 3.8

1. -3 **2.** $\frac{5}{4}$ **3.** $\frac{1}{8}$

4. parallel **5.** perpendicular

6.



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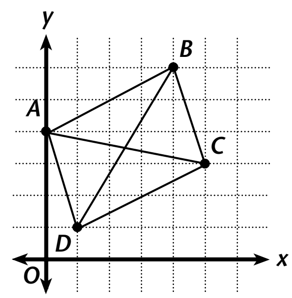

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Reteaching — Chapter 4

Lesson 4.1

1. $\angle C$ **2.** $\angle A$ **3.** $\overline{UA}$ **4.** $\overline{GN}$ **5.** $\overline{QU}$

6. Yes; $\triangle MNP \cong \triangle RTS$.

7. Yes; $JKLMN \cong BCDEF$.

8. Yes; $CDEF \cong LKJM$.

9. Given **10.** Polygon Congruence Postulate

11. Given **12.** Polygon Congruence Postulate

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
3.6 *Angles in Polygons*

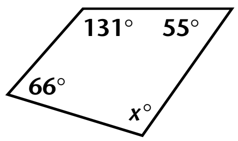
◆ **Skill A** Determining the measures of interior angles of polygons

Recall The sum, S, of the measures of the interior angles of a polygon with n sides is given by $S = (n - 2)180°$.

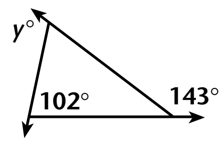
For any polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

◆ **Example**

a. Find the value of x.



b. Find the value of y.



◆ **Solution**

a. The sum of the measures of the interior angles of a quadrilateral is 360°.

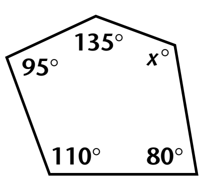
$x + 66 + 131 + 55 = 360$
$x + 252 = 360$
$x = 108$

b. The sum of the measures of the exterior angles of a polygon is 360°.

$y + 143 + 102 = 360$
$y + 245 = 360$
$y = 115$

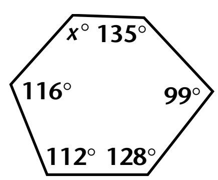
Find the value of *x*.

1.



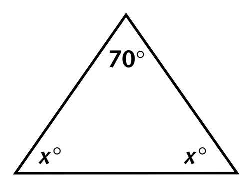
$x =$ ____________

2.



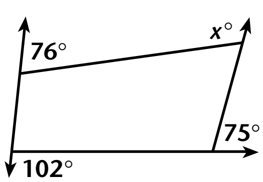
$x =$ ____________

3.



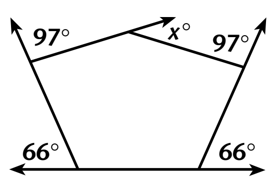
$x =$ ____________

4.



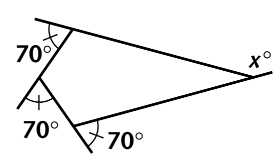
$x =$ ____________

5.



$x =$ ____________

6.



$x =$ ____________

7. The sum of the measures of the interior angles of a 14-sided polygon is

______________________________________.

8. The sum of the measures of the exterior angles of a 7-sided polygon is

______________________________________.

CLASS ______________ DATE ______________

◆ Skill B Determining the measures of interior and exterior angles of a regular polygon

Recall The measure, m, of an interior angle of a regular polygon with n sides is given by $m = \frac{(n - 2)180°}{n}$.

The measure, m, of an exterior angle of a regular polygon with n sides is given by $m = \frac{360°}{n}$. It is sometimes useful to solve this equation for n and use the equivalent expression $n = \frac{360°}{n}$.

◆ **Example 1**

Find the measure of each angle.

a. an interior angle of a regular polygon with 20 sides

b. an exterior angle of a regular polygon with 16 sides

◆ **Solution**

a. $n = 20; m = \frac{(n - 2)180°}{n} = \frac{18 \cdot 180°}{20} = 162°$

The measure of an interior angle of a regular 20-sided polygon is 162°.

b. $n = 16; m = \frac{360°}{16} = 22.5°$

The measure of an exterior angle of a regular 16-sided polygon is 22.5°.

◆ **Example 2**

How many sides does the regular polygon described have?

a. The measure of an interior angle is 160°.

b. The measure of an exterior angle is 15°.

◆ **Solution**

a. If the measure of an interior angle is 160°, the measure of an exterior angle is 20°.

$n = \frac{360°}{m}; n = \frac{360°}{20°} = 18$

The regular polygon has 18 sides.

b. $n = \frac{360°}{m}; n = \frac{360°}{15°} = 24$

The regular polygon has 24 sides.

Find the measure of an interior angle and an exterior angle of a regular polygon with the given number of sides.

9. 9 ____________ **10.** 15 ____________ **11.** 30 ____________ **12.** 25 ____________

The measure of an interior angle of a regular polygon is given. How many sides does the polygon have?

13. 174° ____________ **14.** 172° ____________ **15.** 171° ____________ **16.** 157.5° ____________

The measure of an exterior angle of a regular polygon is given. How many sides does the polygon have?

17. 11.25° ____________ **18.** 7.2° ____________ **19.** 7.5° ____________ **20.** $13\frac{1}{3}°$ ____________

ANSWERS

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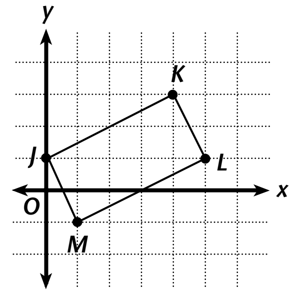

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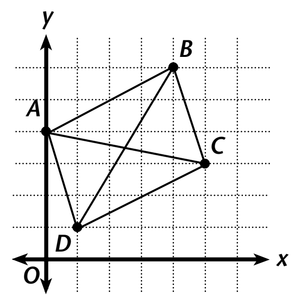

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Reteaching — Chapter 4

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9. Given **10.** Polygon Congruence Postulate

11. Given **12.** Polygon Congruence Postulate

Menu

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Reteaching

3.7 Midsegments of Triangles and Trapezoids

◆ **Skill A** Using midsegments of triangles

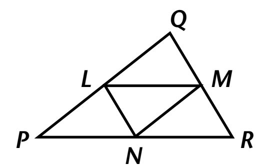
Recall A **midsegment of a triangle** is a segment whose endpoints are midpoints of two sides of the triangle. A midsegment is parallel to the third side of the triangle and half as long as that side.

◆ **Example 1**

In $\triangle PQR$, L, M, and N are the midpoints of $\overline{PQ}$, $\overline{QR}$, and $\overline{PR}$, respectively.

a. Name each midsegment of $\triangle PQR$ and the side of $\triangle PQR$ to which the midsegment is parallel.

b. If $PQ = 10$, $QR = 8$, and $PR = 12$, find the length of each midsegment.



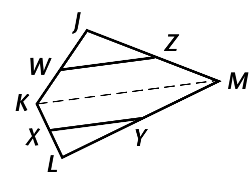
◆ **Solution**

a. $\overline{LM} \parallel \overline{PR}$, $\overline{MN} \parallel \overline{PQ}$, and $\overline{LN} \parallel \overline{QR}$.

b. $MN = \frac{1}{2}PQ = 5$, $LN = \frac{1}{2}QR = 4$, and $LM = \frac{1}{2}PR = 6$.

◆ **Example 2**

In quadrilateral $JKLM$, W, X, Y, and Z are the midpoints of $\overline{JK}$, $\overline{KL}$, $\overline{LM}$, and $\overline{JM}$, respectively. Show that $\overline{WZ}$ is parallel to $\overline{XY}$.

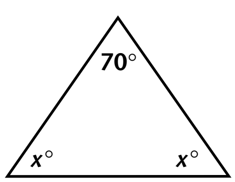


◆ **Solution**

Draw $\overline{KM}$. (Through any two points there is one and only line.) W and Z are the midpoints of $\overline{JK}$ and $\overline{JM}$ so $\overline{WZ}$ is a midsegment of $\triangle JKM$ and is parallel to $\overline{KM}$. Similarly, $\overline{XY}$ is a midsegment of $\triangle KML$ and is parallel to $\overline{KM}$. Two lines parallel to a third line are parallel to each other, so $\overline{WZ} \parallel \overline{XY}$.

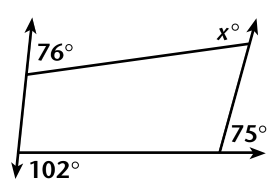
Find the indicated measure.

1.



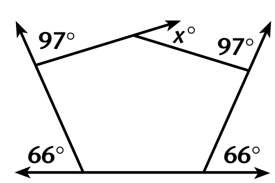
$AB =$ ______________

2.



$UV =$ ______________

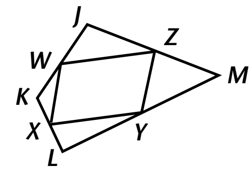
3.



$AB =$ ______________

4. W, X, Y, and Z are the midpoints of the sides of $JKLM$. Example 2 showed that $\overline{WZ} \parallel \overline{XY}$. Explain how you would show that $WXYZ$ is a parallelogram. ______________________________

____

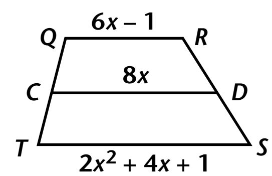


◆ Skill B Using the midsegment of a trapezoid

Recall The **midsegment of a trapezoid** is the segment joining the endpoints of the nonparallel sides. The midsegment of a trapezoid is parallel to the bases. Its length is half the sum of the lengths of the bases.

◆ **Example**

C and D are the midpoints of $\overline{QT}$ and $\overline{RS}$, respectively. Find the value of x and the lengths of $\overline{QR}$, $\overline{TS}$, and $\overline{CD}$.



◆ **Solution**

$\overline{CD}$ is the midsegment of $QRST$, so $CD = \frac{1}{2}(QR + ST)$.

$8x = \frac{1}{2}(2x^2 + 4x + 1 + 6x - 1)$

$8x = x^2 + 5x$

$x^2 - 3x = 0$

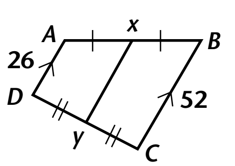
$x(x - 3) = 0$

$x = 0$ or $x = 3$ ← If $x = 0$, $CD = 0$ and QR is negative.

Since x cannot be 0, $x = 3$. Then $QR = 6(3) - 1 = 17$, $ST = 2(3)^2 + 4(3) + 1 = 31$, and $CD = 8(3) = 24$. As a check, notice that $CD = \frac{1}{2}(17 + 31) = 24$.

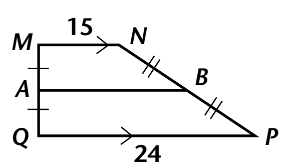
Find the indicated measure.

5.



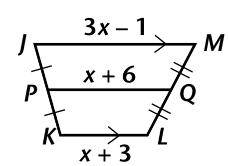
$XY =$ ______________

6.



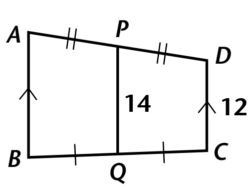
$AB =$ ______________

7.



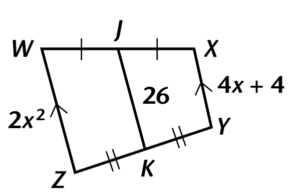
$PQ =$ ______________

8.



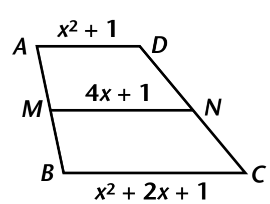
$AB =$ ______________

9.



$WZ =$ ______________

10.



$MN =$ ______________

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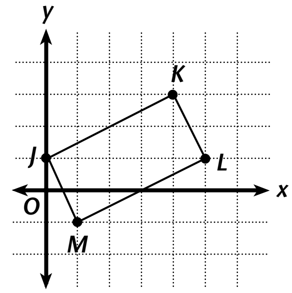

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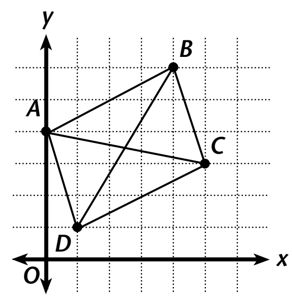

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1. $\angle C$ **2.** $\angle A$ **3.** $\overline{UA}$ **4.** $\overline{GN}$ **5.** $\overline{QU}$

6. Yes; $\triangle MNP \cong \triangle RTS$.

7. Yes; $JKLMN \cong BCDEF$.

8. Yes; $CDEF \cong LKJM$.

9. Given **10.** Polygon Congruence Postulate

11. Given **12.** Polygon Congruence Postulate

NAME ______________________ CLASS __________ DATE __________

Reteaching
3.8 *Analyzing Polygons Using Coordinates*

◆ Skill A Finding the slope of a line in the coordinate plane

Recall The slope, m, of a nonvertical line that contains the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of a horizontal line is 0. The slope of a vertical line is undefined.

◆ Example
Find the slope of the line that contains the points (5, 2) and (12, −1).

◆ Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{12 - 5} = -\frac{3}{7}$$

Determine the slope, *m*, of the line that contains the given points.

1. (5, −3) and (2, 6)

$m =$ ______________

2. (5, 8) and (−3, −2)

$m =$ ______________

3. (14, 8) and (−2, 6)

$m =$ ______________

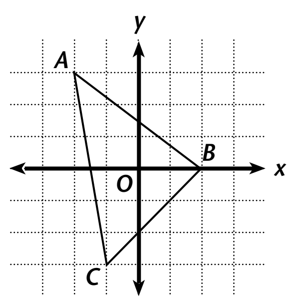
◆ Skill B Using slope to determine whether lines are parallel or perpendicular

Recall Two nonvertical lines are parallel if and only if their slopes are equal. All vertical lines in the coordinate plane are parallel.

Two nonvertical lines are perpendicular if and only if the product of their slopes is −1. Every vertical line in the coordinate plane is perpendicular to every horizontal line in the plane.

◆ Example
Draw the triangle with vertices $A(-2, 3)$, $B(2, 0)$, and $C(-1, -4)$.
Determine whether ABC is a right triangle.

◆ Solution



Since $\overline{AB}$ and $\overline{BC}$ appear to be perpendicular, check their slopes.

slope of $\overline{AB} = \frac{0 - 3}{2 - (-2)} = -\frac{3}{4}$

slope of $\overline{BC} = \frac{-4 - 0}{-1 - 2} = \frac{4}{3}$

Since $-\frac{3}{4} \cdot \frac{4}{3} = -1$, $\overline{AB} \perp \overline{BC}$. Then $\angle ABC$ is a right angle and $\triangle ABC$ is a right triangle.

_______________________ CLASS __________ DATE __________

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither.

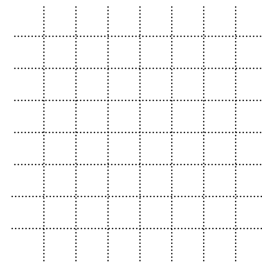
4. $A(-3, -1)$, $B(4, 2)$, $C(0, 2)$, $D(7, 5)$

____

5. $A(-2, -2)$, $B(3, 2)$, $C(0, 2)$, $D(4, -3)$

____

6. Sketch the quadrilateral with vertices $J(0, 1)$, $K(4, 3)$, $L(5, 1)$, and $M(1, -1)$. Show that the quadrilateral is a rectangle.



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◆ Skill C Using the Midpoint Formula

Recall The midpoint of the segment with endpoints (x_1, y_1) and (x_2, y_2) has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

◆ **Example**

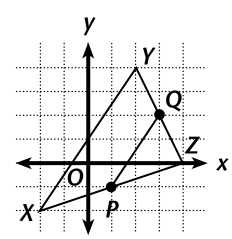
A triangle has vertices $X(-2, -2)$, $Y(2, 4)$, and $Z(4, 0)$. Let $\overline{PQ}$ be the midsegment of $\triangle XYZ$ that is parallel to $\overline{XY}$. Determine the coordinates of P and Q.

◆ **Solution**

The endpoints of $\overline{PQ}$ are the midpoints of $\overline{XZ}$ and $\overline{YZ}$.
Let P be the midpoint of $\overline{XZ}$ and Q the midpoint of $\overline{YZ}$.

P has coordinates $\left(\frac{-2 + 4}{2}, \frac{-2 + 0}{2}\right) = (1, -1)$.

Q has coordinates $\left(\frac{2 + 4}{2}, \frac{4 + 0}{2}\right) = (3, 2)$



Determine the coordinates of M, the midpoint of $\overline{AB}$.

7. $A(-13, 8)$; $B(7, 0)$

$M =$ _______________

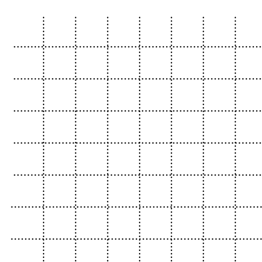
8. $A(5, -9)$; $B(11, -1)$

$M =$ _______________

9. $A(7, -3)$; $B(9, -2)$

$M =$ _______________

10. Draw the quadrilateral with vertices $A(0, 4)$, $B(4, 6)$, $C(5, 3)$, and $D(1, 1)$. Show that $ABCD$ is a parallelogram and that the diagonals bisect each other, that is, have the same midpoint.



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Menu

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7. Def.

8. Triangle Sum Theorem: The sum of the measures of the angles of a triangle is 180°.

9. Substitution Prop. of =

10. Subtraction. Prop. of = **11.** Def.

Lesson 3.6

1. 120° **2.** 130° **3.** 55° **4.** 113° **5.** 34°

6. 150° **7.** 2160° **8.** 360° **9.** 140°; 40°

10. 156°; 24° **11.** 168°; 12°

12. 165.6°; 14.4° **13.** 60 sides

14. 45 sides **15.** 40 sides **16.** 16 sides

17. 32 sides **18.** 50 sides **19.** 48 sides

20. 27 sides

Lesson 3.7

1. 39 **2.** 46.5 **3.** 58

4. Use the method of Example 2. Draw $\overline{JL}$. Midsegments $\overline{WX}$ of $\triangle JKL$ and $\overline{ZY}$ of $\triangle JLM$ are both parallel to $\overline{JL}$, so $\overline{WX} \parallel \overline{ZY}$. Since Ex. 2 showed that $\overline{WZ} \parallel \overline{XY}$, $WXYZ$ is a parallelogram.

5. 39 **6.** 19.5 **7.** 11 **8.** 16 **9.** 32

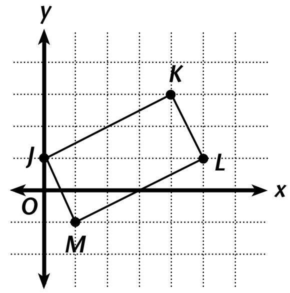
10. 13

Lesson 3.8

1. -3 **2.** $\frac{5}{4}$ **3.** $\frac{1}{8}$

4. parallel **5.** perpendicular

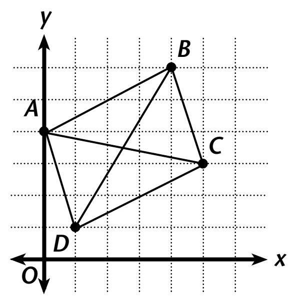
6.



slope $\overline{JK}$ = slope $\overline{ML} = \frac{1}{2}$, so $\overline{JK} \parallel \overline{ML}$; slope $\overline{JM}$ = slope $\overline{KL} = -2$, so $\overline{JM} \parallel \overline{KL}$. Then $JKLM$ is a parallelogram. Moreover, $\overline{JM}$ is perpendicular to both $\overline{JK}$ and $\overline{ML}$ (The product of their slopes is -1) and $\overline{KL}$ is perpendicular to both $\overline{JK}$ and $\overline{ML}$. Then $JKLM$ is a rectangle.

7. $(-3, 4)$ **8.** $(8, -5)$ **9.** $(8, -2.5)$

10.



slope $\overline{AB}$ = slope $\overline{CD} = \frac{1}{2}$, so $\overline{AB} \parallel \overline{CD}$; slope $\overline{AD}$ = slope $\overline{BC} = -3$, so $\overline{AD} \parallel \overline{BC}$; $ABCD$ is a parallelogram; midpoint of $\overline{AC}$ = (2.5, 3.5) = midpoint of $\overline{BD}$; so the diagonals of $ABCD$ bisect each other.

Reteaching — Chapter 4

Lesson 4.1

1. $\angle C$ **2.** $\angle A$ **3.** $\overline{UA}$ **4.** $\overline{GN}$ **5.** $\overline{QU}$

6. Yes; $\triangle MNP \cong \triangle RTS$.

7. Yes; $JKLMN \cong BCDEF$.

8. Yes; $CDEF \cong LKJM$.

9. Given **10.** Polygon Congruence Postulate

11. Given **12.** Polygon Congruence Postulate

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
4.1 Congruent Polygons

◆ Skill A Identifying congruent polygons and congruent parts of congruent polygons

Recall A polygon is named using consecutive vertices in order, either clockwise or counterclockwise around the polgyon.

Two polygons are **congruent** if and only if there is a correspondence between their sides and angles so that:

1. Each pair of corresponding angles is congruent.
2. Each pair of corresponding sides is congruent.

In a *congruence statement*, a statement identifying congruent polygons, the corresponding vertices of the two polygons are named in the same order.

◆ Example

Determine whether the quadrilaterals are congruent. If so, write a congruence statement.

◆ Solution

$\angle P \cong \angle J$, $\angle Q \cong \angle M$, $\angle R \cong \angle L$, $\angle S \cong \angle K$

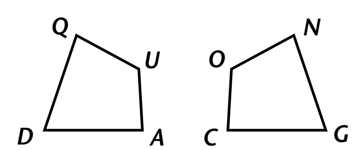
$\overline{PQ} \cong \overline{JM}$, $\overline{QR} \cong \overline{ML}$, $\overline{RS} \cong \overline{LK}$, $\overline{PS} \cong \overline{JK}$

Since the quadrilaterals have four pairs of congruent angles and four corresponding pairs of congruent sides, the quadrilaterals are congruent.

Three of the many possible congruence statements are:

$PQRS \cong JMLK$ $\quad$ $PSRQ \cong JKLM$ $\quad$ $SPQR \cong KJML$

***QUAD* and *CONG* are quadrilaterals and *QUAD* ≅ *CONG*. Complete each statement.**



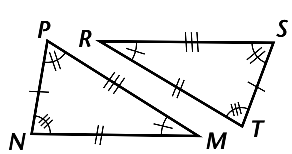
1. $\angle Q \cong$ ______________ **2.** $\angle N \cong$ ______________

3. $\overline{ON} \cong$ ______________ **4.** $\overline{DA} \cong$ ______________

5. $\overline{CO} \cong$ ______________

Determine whether the polygons are congruent. If so, write a congruence statement.

6.



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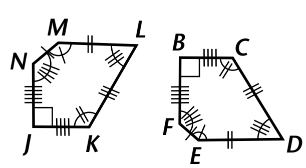
7.



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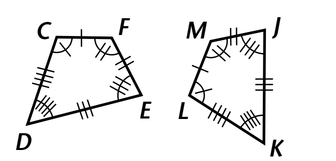
8.



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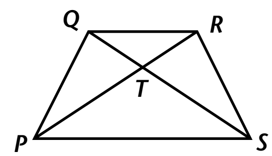
◆ Skill B Using the Polygon Congruence Postulate

Recall The Polygon Congruence Postulate can be written as a a pair of conditional statements: If all corresponding parts of two polygons are congruent, then the polygons are congruent. If two polygons are congruent, then all corresponding parts are congruent. Both statements are useful in proofs.

◆ **Example**

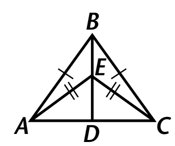
Given that $\triangle PQR \cong \triangle SRQ$, $\overline{QT} \cong \overline{RT}$, and $\overline{TP} \cong \overline{TS}$, write a paragraph proof that $\triangle QTP \cong \triangle RTS$.



◆ **Solution**

Since $\triangle PQR \cong \triangle SRQ$, $\overline{QP} \cong \overline{RS}$ and $\angle QPR \cong \angle RSQ$ by the Polygon Congruence Postulate. $\angle QTP \cong \angle RTS$ by the Vertical Angles Theorem. It is given that $\overline{QT} \cong \overline{RT}$ and $\overline{PT} \cong \overline{ST}$. Since each pair of corresponding angles of the two triangles is congruent and each pair of corresponding sides is congruent, $\triangle QTP \cong \triangle RTS$ by the Polygon Congruence Postulate.

Given: $\triangle ABE \cong \triangle CBE$ and $\triangle AED \cong \triangle CED$
Prove: $\triangle ABD \cong \triangle CBD$



Proof:

Statements	Reasons
$\triangle ABE \cong \triangle CBE$	**9.** ______________
$\overline{AB} \cong \overline{CB}$, $\angle ABE \cong \angle CBE$, $\angle BAE \cong \angle BCE$	**10.** ______________
$\triangle AED \cong \triangle CED$	**11.** ______________
$\overline{AD} \cong \overline{DC}$, $\angle ADE \cong \angle CDE$, $\angle DAE \cong \angle DCE$	**12.** ______________
$m\angle BAE = m\angle BCE$, $m\angle DAE = m\angle DCE$	**13.** ______________
$m\angle BAE + m\angle DAE = m\angle BCE + m\angle DCE$	**14.** ______________
$m\angle BAD = m\angle CBD$ or $\angle BAD \cong \angle CBD$	**15.** ______________
$\overline{BD} \cong \overline{BD}$	**16.** ______________
$\triangle ABD \cong \triangle CBD$	**17.** ______________

7. Def.

8. Triangle Sum Theorem: The sum of the measures of the angles of a triangle is 180°.

9. Substitution Prop. of =

10. Subtraction. Prop. of = **11.** Def.

Lesson 3.6

1. 120° **2.** 130° **3.** 55° **4.** 113° **5.** 34°

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Lesson 3.7

1. 39 **2.** 46.5 **3.** 58

4. Use the method of Example 2. Draw $\overline{JL}$. Midsegments $\overline{WX}$ of $\triangle JKL$ and $\overline{ZY}$ of $\triangle JLM$ are both parallel to $\overline{JL}$, so $\overline{WX} \parallel \overline{ZY}$. Since Ex. 2 showed that $\overline{WZ} \parallel \overline{XY}$, $WXYZ$ is a parallelogram.

5. 39 **6.** 19.5 **7.** 11 **8.** 16 **9.** 32

10. 13

Lesson 3.8

1. −3 **2.** $\frac{5}{4}$ **3.** $\frac{1}{8}$

4. parallel **5.** perpendicular

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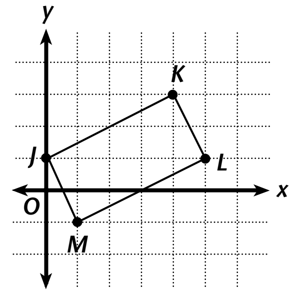
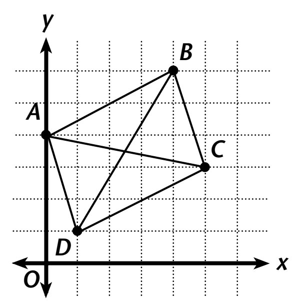


slope $\overline{JK}$ = slope $\overline{ML} = \frac{1}{2}$, so $\overline{JK} \parallel \overline{ML}$; slope $\overline{JM}$ = slope $\overline{KL} = -2$, so $\overline{JM} \parallel \overline{KL}$. Then $JKLM$ is a parallelogram. Moreover, $\overline{JM}$ is perpendicular to both $\overline{JK}$ and $\overline{ML}$ (The product of their slopes is −1) and $\overline{KL}$ is perpendicular to both $\overline{JK}$ and $\overline{ML}$. Then $JKLM$ is a rectangle.

7. (−3, 4) **8.** (8, −5) **9.** (8, −2.5)

10.



slope $\overline{AB}$ = slope $\overline{CD} = \frac{1}{2}$, so $\overline{AB} \parallel \overline{CD}$; slope $\overline{AD}$ = slope $\overline{BC} = -3$, so $\overline{AD} \parallel \overline{BC}$; $ABCD$ is a parallelogram; midpoint of $\overline{AC}$ = (2.5, 3.5) = midpoint of $\overline{BD}$; so the diagonals of $ABCD$ bisect each other.

Reteaching — Chapter 4

Lesson 4.1

1. $\angle C$ **2.** $\angle A$ **3.** $\overline{UA}$ **4.** $\overline{GN}$ **5.** $\overline{QU}$

6. Yes; $\triangle MNP \cong \triangle RTS$.

7. Yes; $JKLMN \cong BCDEF$.

8. Yes; $CDEF \cong LKJM$.

9. Given **10.** Polygon Congruence Postulate

11. Given **12.** Polygon Congruence Postulate

13. Angle Congruence Postulate

14. Add. Prop. of Equality

15. Substitution Prop., Angle Congruence Postulate

16. Reflexive Prop. of Congruence

17. Polygon Congruence Postulate

Lesson 4.2

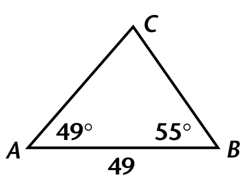
1. Yes; $\triangle PQR \cong \triangle XZY$; ASA.

2. Yes; $\triangle RST \cong \triangle FED$; SSS.

3. no

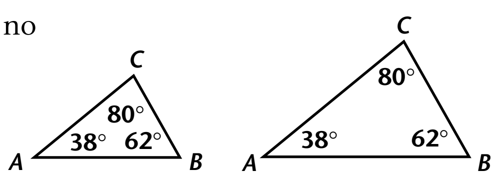
4. Yes; $\triangle GHI \cong \triangle RTS$; SAS.

5. yes

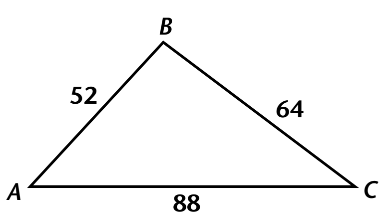


ASA

6. no

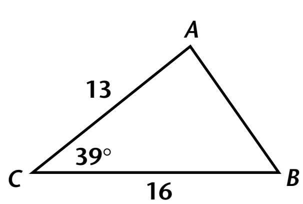


7. yes



SSS

8. yes



SAS

Lesson 4.3

1. Yes; $\triangle QRP \cong \triangle XWY$; SAS.

2. Yes; $\triangle NQM \cong \triangle NQP$; HL.

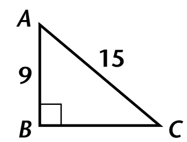
3. Yes; $\triangle HJK \cong \triangle HJI$; SSS.

4. Yes; $\triangle TUV \cong \triangle PMN$; AAS.

5. Yes; $\triangle DEG \cong \triangle FEG$; ASA.

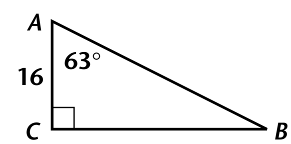
6. no

7. yes



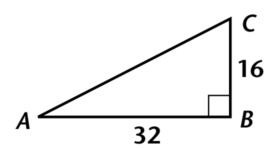
HL

8. yes



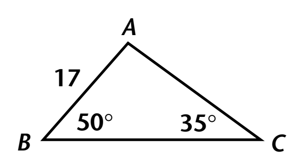
ASA

9. yes



SAS

10. yes



AAS

Lesson 4.4

1. $\triangle AED \cong \triangle\triangle BEC$ (Given); $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{BE}$ or $AE = BE$ (CPCTC); Segment Congruence Postulate); $\overline{ED} \cong \overline{EC}$ or $ED = EC$ (CPCTC); Segment Congruence Postulate); $AE + EC = BE + ED$ (Segment Addition Postulate); $AC = BD$ or $\overline{AC} \cong \overline{BD}$ (Substitution Prop.; Segment Congruence Postulate); $\overline{DC} \cong \overline{DC}$ (Reflexive Prop. of Congruence); $\triangle ADC \cong \triangle BCD$ (SSS Postulate)

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
4.2 Triangle Congruence

◆ **Skill A** Using the SSS, SAS, and ASA Congruence Postulates for triangles

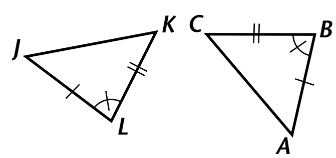
Recall If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. (SSS Postulate)

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. (SAS Postulate)

If two angles and the included side of one triangle are congruent to two sides of another triangle, then the triangles are congruent. (ASA Postulate)

◆ **Example**

Determine whether the triangles can be shown to be congruent using the SSS, SAS, or ASA Postulate. If so, write a congruence statement and identify the postulate.



◆ **Solution**

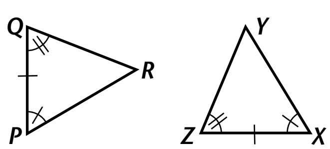
According to the markings in the figure:

$\overline{AB} \cong \overline{JL}$ $\quad \overline{BC} \cong \overline{LK}$ $\quad \angle B \cong \angle L$

Therefore, $\triangle ABC \cong \triangle JLK$ by the SAS Postulate.

Determine whether the triangles can be shown to be congruent using the SSS, SAS, or ASA Postulate. If so, write a congruence statement and identify the postulate.

1.



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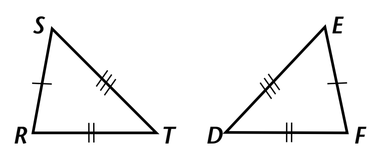
2.



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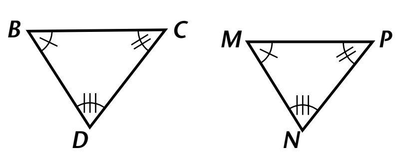
3.



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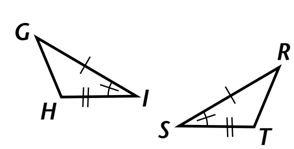
4.



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◆ Skill B Determining triangle uniqueness based on the SSS, SAS, and ASA congruence postulates

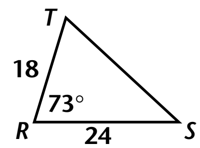
Recall You can use congruence postulates to determine whether given conditions determine a unique triangle.

◆ **Example 1**

Determine whether there is a unique triangle *RST* with $m\angle R = 73°$, $RS = 24$, and $RT = 18$.

◆ **Solution**

The lengths of two sides and the measure of the included angle are given. By the SAS Postulate, any two triangles with the given measures are congruent. Then there is exactly one triangle *RST*; that is, $\triangle RST$ is unique.

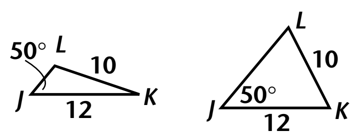


◆ **Example 2**

Determine whether there is a unique triangle *JKL* with $m\angle J = 50°$, $JK = 12$, and $KL = 10$.

◆ **Solution**

The lengths of two sides and the measure of an angle are given. The angle is *not* included between the given sides. None of the three postulates guarantee a unique triangle. The sketches show two such triangles.



Determine whether any of the SSS, SAS, or ASA postulates guarantee that $\triangle ABC$ with the given measures is unique. If so, use a protractor and a ruler to sketch the triangle and identify the postulate that guarantees uniqueness. If not, sketch two such triangles.

5. $AB = 2.8$ cm, $m\angle A = 49°$, $m\angle B = 55°$

______

6. $m\angle A = 38°$, $m\angle B = 62°$, $m\angle C = 80°$

______

7. $AB = 2.7$ cm, $BC = 3.1$ cm, $AC = 4.4$ cm

______

8. $m\angle C = 39°$, $CB = 2.5$ cm, $CA = 3.1$ cm

______

13. Angle Congruence Postulate

14. Add. Prop. of Equality

15. Substitution Prop., Angle Congruence Postulate

16. Reflexive Prop. of Congruence

17. Polygon Congruence Postulate

Lesson 4.2

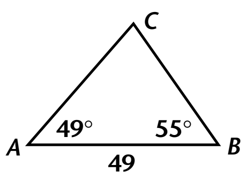
1. Yes; $\triangle PQR \cong \triangle XZY$; ASA.

2. Yes; $\triangle RST \cong \triangle FED$; SSS.

3. no

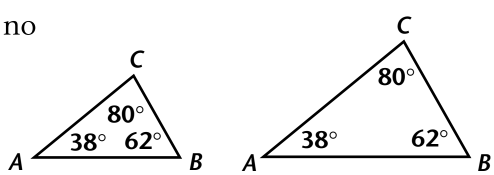
4. Yes; $\triangle GHI \cong \triangle RTS$; SAS.

5. yes

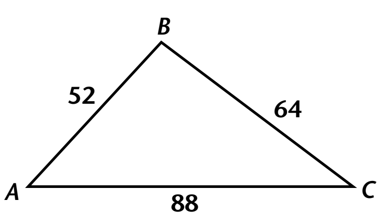


ASA

6. no

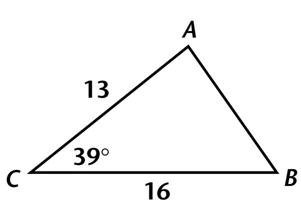


7. yes



SSS

8. yes



SAS

Lesson 4.3

1. Yes; $\triangle QRP \cong \triangle XWY$; SAS.

2. Yes; $\triangle NQM \cong \triangle NQP$; HL.

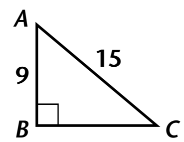
3. Yes; $\triangle HJK \cong \triangle HJI$; SSS.

4. Yes; $\triangle TUV \cong \triangle PMN$; AAS.

5. Yes; $\triangle DEG \cong \triangle FEG$; ASA.

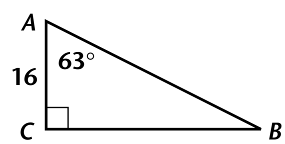
6. no

7. yes



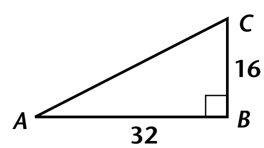
HL

8. yes



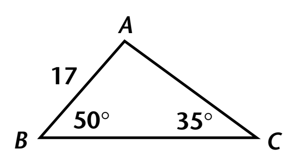
ASA

9. yes



SAS

10. yes



AAS

Lesson 4.4

1. $\triangle AED \cong \triangle\triangle BEC$ (Given); $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{BE}$ or $AE = BE$ (CPCTC); Segment Congruence Postulate); $\overline{ED} \cong \overline{EC}$ or $ED = EC$ (CPCTC); Segment Congruence Postulate); $AE + EC = BE + ED$ (Segment Addition Postulate); $AC = BD$ or $\overline{AC} \cong \overline{BD}$ (Substitution Prop.; Segment Congruence Postulate); $\overline{DC} \cong \overline{DC}$ (Reflexive Prop. of Congruence); $\triangle ADC \cong \triangle BCD$ (SSS Postulate)

NAME ______________________ CLASS ____________ DATE ____________

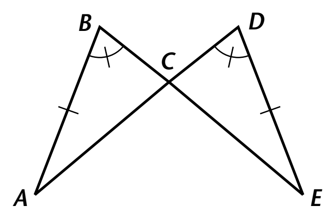
Reteaching
4.3 Analyzing Triangle Congruence

◆ **Skill A** Using postulates and theorems to determine whether triangles are congruent

Recall If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and sides of another triangle, then the triangles are congruent. (AAS Theorem)

If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the triangles are congruent. (HL Theorem)

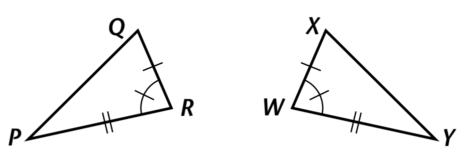
◆ **Example**
Determine whether you could prove that the triangles are congruent. If so, write a congruence statement and identify the postulate or theorem that justifies the statement.



◆ **Solution**
Notice that ∠ACB and ∠ECD are vertical angles, so ∠ACB > ∠ECD.
Then a correspondence can be set up between $\triangle ABC$ and $\triangle EDC$, with $\overline{AB} \cong \overline{ED}$, $\angle B \cong \angle D$, and $\angle ACB \cong \angle ECD$. Then $\triangle ABC \cong \triangle EDC$ by the AAS Theorem.

Determine whether you could prove that the triangles are congruent. If so, write a congruence statement and identify the postulate or theorem you could use.

1.



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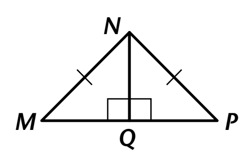
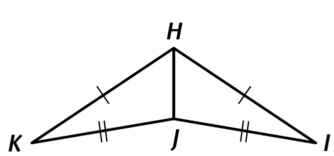
2.



____

3.



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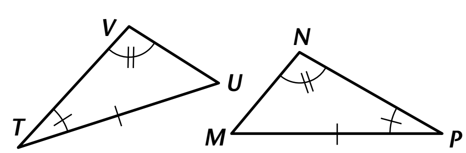
4.



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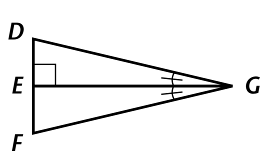
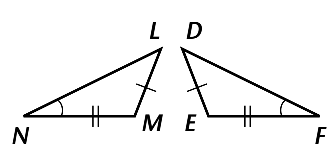
5.



____

6.



____

◆ **Skill B** Determining triangle uniqueness based on congruence postulates and theorems

Recall You have learned six ways of proving that two triangles are congruent or that a given set of conditions determines a unique triangle. They are the Polygon Congruence Postulate, the SSS, SAS, and ASA Postulates, and the AAS and HL Theorems.

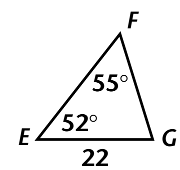
The HL Theorem is a special case of SSA, that is, two sides and the nonincluded angle. Except for right triangles, SSA cannot be used to prove congruence or uniqueness.

◆ **Example**

Determine whether there is a unique triangle *EFG* with m∠*E* = 52°, m∠*F* = 55°, and *EG* = 22.

◆ **Solution**

The measures of two angles and the length of a side are given. The side is not included by the given angles. By the AAS Theorem, any two triangles with the given measures are congruent. Then there is exactly one triangle *EFG*; that is, △*EFG* is unique.



Determine whether any of the congruence postulates or theorems guarantee that △*PQR* with the given measures is unique. If so, use a protractor and a ruler to sketch the triangle and identify the postulate that guarantees uniqueness. If not, sketch two such triangles.

7. *AB* = 1.5 cm, AC = 2 cm, m∠*B* = 90°

____

8. m∠*A* = 63°, m∠*C* = 90°, AC = 1.5 cm

____

9. *AB* = 3 cm, *BC* = 1.5 cm, m∠*B* = 90°

____

10. m∠*B* = 50°, m∠*C* = 35°, *AB* = 1.5 cm

____

13. Angle Congruence Postulate

14. Add. Prop. of Equality

15. Substitution Prop., Angle Congruence Postulate

16. Reflexive Prop. of Congruence

17. Polygon Congruence Postulate

Lesson 4.2

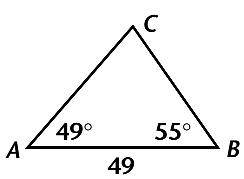
1. Yes; $\triangle PQR \cong \triangle XZY$; ASA.

2. Yes; $\triangle RST \cong \triangle FED$; SSS.

3. no

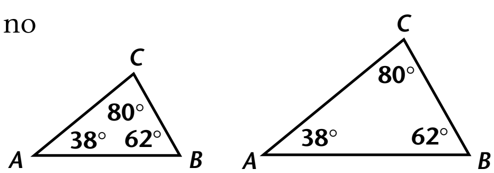
4. Yes; $\triangle GHI \cong \triangle RTS$; SAS.

5. yes

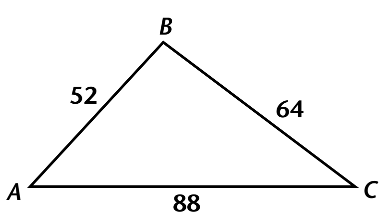


ASA

6. no

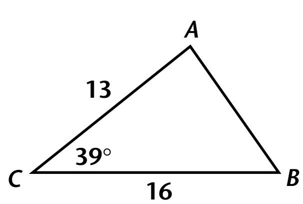


7. yes



SSS

8. yes



SAS

Lesson 4.3

1. Yes; $\triangle QRP \cong \triangle XWY$; SAS.

2. Yes; $\triangle NQM \cong \triangle NQP$; HL.

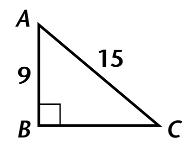
3. Yes; $\triangle HJK \cong \triangle HJI$; SSS.

4. Yes; $\triangle TUV \cong \triangle PMN$; AAS.

5. Yes; $\triangle DEG \cong \triangle FEG$; ASA.

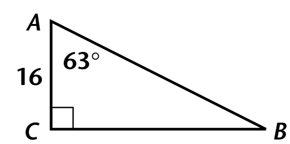
6. no

7. yes



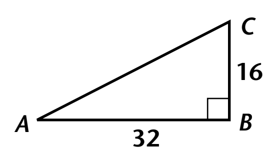
HL

8. yes



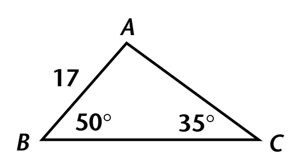
ASA

9. yes



SAS

10. yes



AAS

Lesson 4.4

1. $\triangle AED \cong \triangle\triangle BEC$ (Given); $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{BE}$ or $AE = BE$ (CPCTC); Segment Congruence Postulate); $\overline{ED} \cong \overline{EC}$ or $ED = EC$ (CPCTC); Segment Congruence Postulate); $AE + EC = BE + ED$ (Segment Addition Postulate); $AC = BD$ or $\overline{AC} \cong \overline{BD}$ (Substitution Prop.; Segment Congruence Postulate); $\overline{DC} \cong \overline{DC}$ (Reflexive Prop. of Congruence); $\triangle ADC \cong \triangle BCD$ (SSS Postulate)

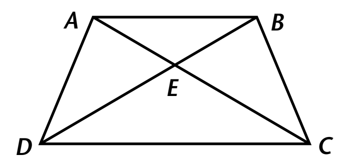
NAME ______________________ CLASS __________ DATE __________

Reteaching
4.4 Using Triangle Congruence

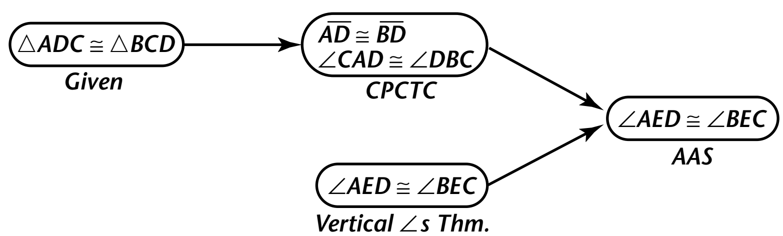
◆ **Skill A** Using congruence of triangles to determine congruence of corresponding parts.

Recall By the Polygon Congruence Postulate, corresponding parts of congruent triangles are congruent. This statement, abbreviated CPCTC, is often used to justify statements in proofs such as the flowchart proof in the example below.

◆ **Example**
Given: $\triangle ADC \cong \triangle BCD$
Prove: $\triangle AED \cong \triangle BEC$



◆ **Solution**
Plan: Use CPCTC to show that $\overline{AD} \cong \overline{BC}$ and $\angle CAD \cong \angle DBA$. Then use the Vertical Angles Theorem to show that $\angle AED \cong \angle BEC$, and so $\triangle AED \cong \triangle BEC$ by the AAS Theorem.



Refer to polygon *ABCD* in the figure above. Write a flowchart proof.

1. Given: $\triangle AED \cong \triangle BEC$
Prove: $\triangle ADC \cong \triangle BCD$

Plan: Use CPCTC to show that $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{BE}$, and $\overline{ED} \cong \overline{EC}$. Then use the Segment Addition Postulate to show that $AC = BD$, or $\overline{AC} \cong \overline{BD}$. Finally, show that, since $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence, $\triangle ADC \cong \triangle BCD$ by the SSS Postulate.

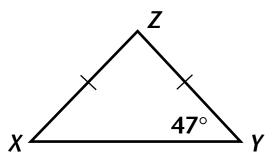
◆ Skill B Using the Isosceles Triangle Theorem and its converse

Recall The Isosceles Triangle Theorem states that if two sides of a triangle are congruent, then the angles opposite those sides are congruent. The converse of the Isosceles Triangle Theorem is true. That is, if two angles of a triangle are congruent, then the sides opposite those angles are congruent. The Isosceles Triangle Theorem has two easily proven corollaries:

The measure of each angle of an equilateral triangle is 60°.
The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

◆ **Example**

Find the measure of $\angle Z$.



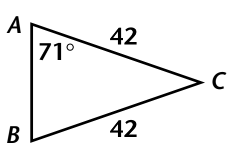
◆ **Solution**

$\triangle XYZ$ is isosceles, with $\overline{XY} \cong \overline{YZ}$. By the Isosceles Triangle Theorem, $\angle X \cong \angle Y$, or, $m\angle X = m\angle Y = 47°$.

$$m\angle X + m\angle Y + m\angle Z = 180°$$
$$47° + 47° + m\angle Z = 180°$$
$$m\angle Z = 86°$$

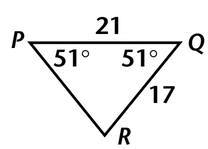
Find the indicated value.

2.



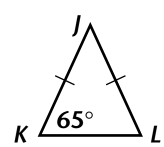
$m\angle B$ = ______________

3.



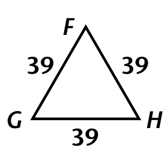
PR = ______________

4.



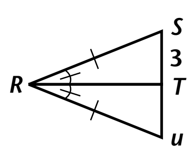
$m\angle J$ = ______________

5.



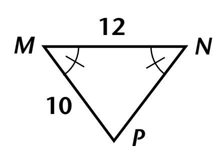
$m\angle F$ = ______________

6.



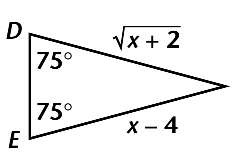
SU = ______________

7.



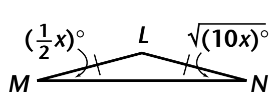
NP = ______________

8.



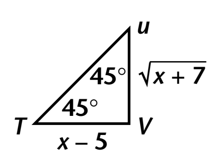
x = ______________

9.



x = ______________

10.



x = ______________

13. Angle Congruence Postulate

14. Add. Prop. of Equality

15. Substitution Prop., Angle Congruence Postulate

16. Reflexive Prop. of Congruence

17. Polygon Congruence Postulate

Lesson 4.2

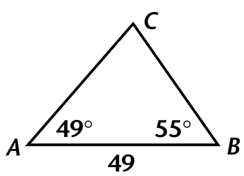
1. Yes; $\triangle PQR \cong \triangle XZY$; ASA.

2. Yes; $\triangle RST \cong \triangle FED$; SSS.

3. no

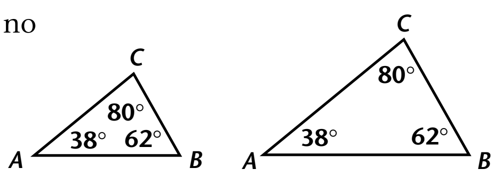
4. Yes; $\triangle GHI \cong \triangle RTS$; SAS.

5. yes

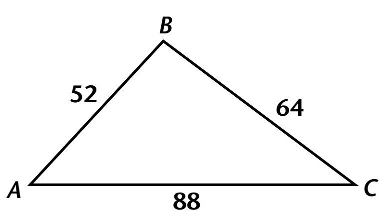


ASA

6. no

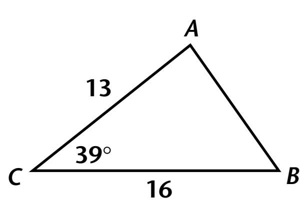


7. yes



SSS

8. yes



SAS

Lesson 4.3

1. Yes; $\triangle QRP \cong \triangle XWY$; SAS.

2. Yes; $\triangle NQM \cong \triangle NQP$; HL.

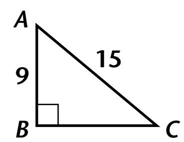
3. Yes; $\triangle HJK \cong \triangle HJI$; SSS.

4. Yes; $\triangle TUV \cong \triangle PMN$; AAS.

5. Yes; $\triangle DEG \cong \triangle FEG$; ASA.

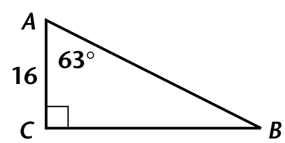
6. no

7. yes



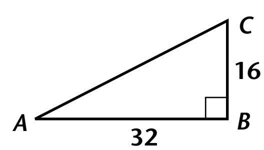
HL

8. yes



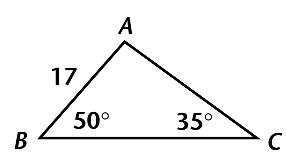
ASA

9. yes



SAS

10. yes



AAS

Lesson 4.4

1. $\triangle AED \cong \triangle\triangle BEC$ (Given); $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{BE}$ or $AE = BE$ (CPCTC); Segment Congruence Postulate); $\overline{ED} \cong \overline{EC}$ or $ED = EC$ (CPCTC); Segment Congruence Postulate); $AE + EC = BE + ED$ (Segment Addition Postulate); $AC = BD$ or $\overline{AC} \cong \overline{BD}$ (Substitution Prop.; Segment Congruence Postulate); $\overline{DC} \cong \overline{DC}$ (Reflexive Prop. of Congruence); $\triangle ADC \cong \triangle BCD$ (SSS Postulate)

ANSWERS

2. 71° **3.** 17 **4.** 50° **5.** 60°

6. 6 **7.** 10 **8.** 7 **9.** 40 **10.** 9

Lesson 4.5

1. 24 **2.** 3 **3.** $1\frac{1}{2}$ **4.** 5

5. Answers may vary. Sample answer: Since the diagonals of a parallelogram bisect each other, $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$. Also, vertical angles PTQ and RTS are congruent, so $\triangle PTQ \cong \triangle RTS$ by the SAS Postulate.

6. 3 **7.** 0.5 **8.** 4 **9.** 90° **10.** 45°

11. $\triangle LMP$, $\triangle LMN$, $\triangle LPN$, $\triangle MNP$, $\triangle LQP$, $\triangle LQM$, $\triangle MQN$, $\triangle PQN$

Lesson 4.6

1. Yes; if one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

2. no

3. Yes; if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. Yes; if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5. no

6. Rhombus; if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

7. Rectangle; if one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

8. Rhombus; if a diagonal of a rhombus bisects a pair of opposite angles, then the parallelogram is a rhombus.

9. Square; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle; if two adjacent sides of a parallelogram are congruent, then the parallelogram is a rhombus.

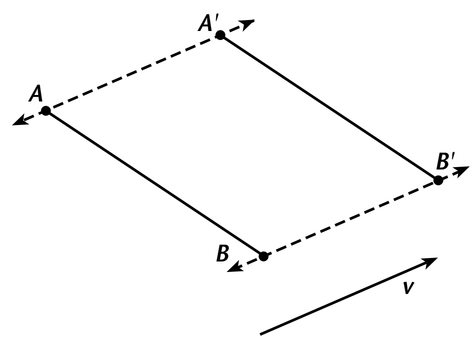
10. Rectangle; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Lesson 4.7

1–6. Check students' drawings.

Lesson 4.8

1.



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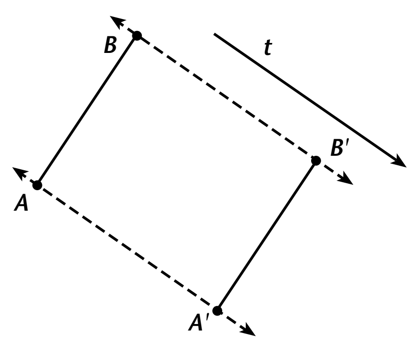

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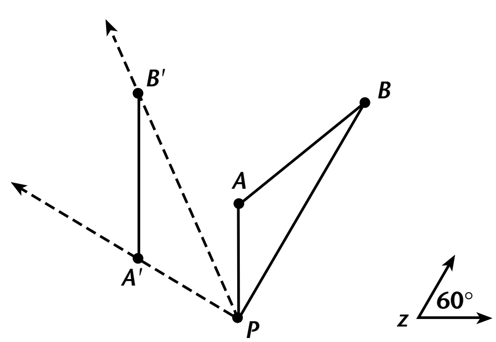


NAME ______________________ CLASS __________ DATE __________

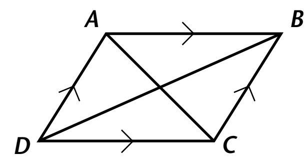
Reteaching

4.5 Proving Quadrilateral Properties

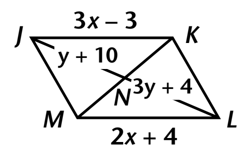
◆ **Skill A** Using the properties of parallelograms

Recall In the figure at the right, $ABCD$ is a parallelogram. $ABCD$ has the following properties:

$\overline{AB}$ and $\overline{CD}$ are congruent, as are $\overline{AD}$ and $\overline{BC}$.
$\angle A$ and $\angle C$ are congruent, as are $\angle B$ and $\angle D$.
The following angles are supplementary:
$\angle A$ and $\angle B$ $\angle A$ and $\angle D$ $\angle B$ and $\angle C$ $\angle C$ and $\angle D$
$\overline{AC}$ and $\overline{BD}$ bisect each other.
$\overline{AC}$ and $\overline{BD}$ each divide $ABCD$ into two congruent triangles.

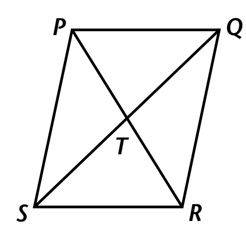


◆ **Example**
In parallelogram $JKLM$, $m\angle MLK = 60°$.
Find each indicated value.
a. x **b.** $m\angle MJK$ **c.** $m\angle JKL$ **d.** y



◆ **Solution**
a. Opposite sides of a parallelogram are congruent, so $JK = ML$ means that $3x - 3 = 2x + 4$, and $x = 7$.
b. Opposite angles of a parallelogram are congruent, so $m\angle MJK = m\angle MLK = 60°$.
c. Consecutive angles of a parallelogram are supplementary, so $m\angle JKL = 180° - m\angle MLK = 180° - 60° = 120°$
d. The diagonals of a parallelogram bisect each other, so $JM = KL$. Then $y + 10 = 3y + 4$ and $y = 3$.

***PQRS* is a parallelogram. Complete each statement.**



1. If $m\angle QPS = (4x + 6)°$ and $m\angle PQR = (3x + 6)°$, then $x =$ ______________.

2. If $PS = 5y - 3$ and $QR = 2y + 6$, then $y =$ ______________.

3. If $QT = 5z + 3$ and $TS = 3z + 6$, then $z =$ ______________.

4. If $m\angle PQR = (3w^2 + 3)°$ and $m\angle PSR = (14w + 8)°$, then $w =$ ______________.

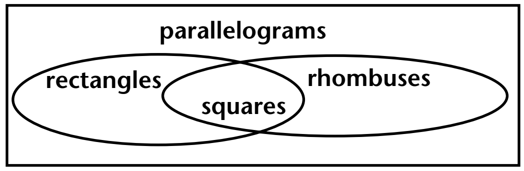
5. Write a plan for proving that $\triangle PTQ \cong \triangle RTS$. ______________________________

____

____

◆ **Skill B** Using the properties of special parallelograms.

Recall Rectangles, rhombuses, and squares are special types of parallelograms. The Euler diagram depicts how the figures are related.

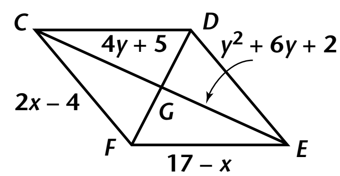


Rectangles, rhombuses, and squares have all the properties of parallelograms and special properties as well.
The diagonals of a rectangle are congruent.
The diagonals of a rhombus are perpendicular.
A square is a rectangle, so its diagonals are congruent. A square is a rhombus, so its diagonals are perpendicular bisectors of each other.

◆ **Example**

CDEF is a rhombus. Find each indicated value.

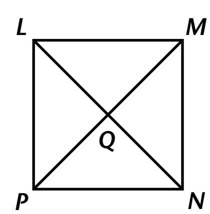
a. x **b.** y
c. $m\angle DGE$ **d.** If $m\angle EDG = 65°$, find $m\angle DEG$.



◆ **Solution**

a. All four sides of a rhombus are congruent, so $CF = FE$. That is, $2x - 4 = 17 - x$ and $x = 7$.
b. A rhombus is a parallelogram, so the diagonals of a rhombus bisect each other. Then $y^2 + 6y + 2 = 4y + 5$, and $y = 1$.
c. The diagonals of a rhombus are perpendicular, so $m\angle DGE = 90°$.
d. $m\angle DGE = 90°$ and $m\angle EDG = 65°$, so $m\angle DEG = 180° - (90° + 65°) = 25°$.

Exercises 6–11 refer to square *LMNP*. Complete each statement.



6. If $LP = x^2 - 4$ and $PN = x + 2$, then $x =$ ______.

7. If $LQ = 3y + 2$ and $NQ = 6y + 0.5$, then $y =$ ______.

8. If $LN = z^2 - 9$ and $PM = z + 3$, then $z =$ ______.

9. $m\angle LQP =$ ______.

10. $m\angle QLP =$ ______.

11. Name all the isosceles right triangles shown in the figure. ______

______

ANSWERS

2. 71° **3.** 17 **4.** 50° **5.** 60°

6. 6 **7.** 10 **8.** 7 **9.** 40 **10.** 9

Lesson 4.5

1. 24 **2.** 3 **3.** $1\frac{1}{2}$ **4.** 5

5. Answers may vary. Sample answer: Since the diagonals of a parallelogram bisect each other, $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$. Also, vertical angles PTQ and RTS are congruent, so $\triangle PTQ \cong \triangle RTS$ by the SAS Postulate.

6. 3 **7.** 0.5 **8.** 4 **9.** 90° **10.** 45°

11. $\triangle LMP$, $\triangle LMN$, $\triangle LPN$, $\triangle MNP$, $\triangle LQP$, $\triangle LQM$, $\triangle MQN$, $\triangle PQN$

Lesson 4.6

1. Yes; if one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

2. no

3. Yes; if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. Yes; if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5. no

6. Rhombus; if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

7. Rectangle; if one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

8. Rhombus; if a diagonal of a rhombus bisects a pair of opposite angles, then the parallelogram is a rhombus.

9. Square; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle; if two adjacent sides of a parallelogram are congruent, then the parallelogram is a rhombus.

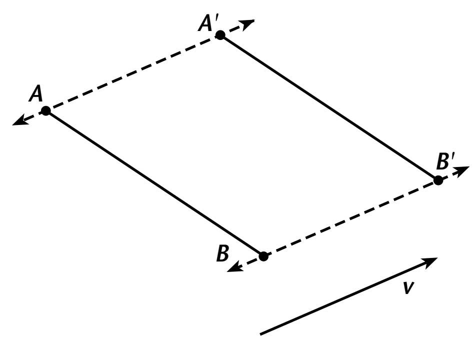
10. Rectangle; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Lesson 4.7

1–6. Check students' drawings.

Lesson 4.8

1.



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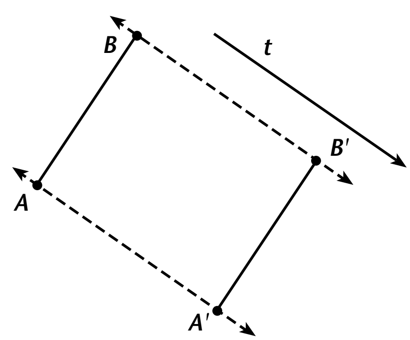

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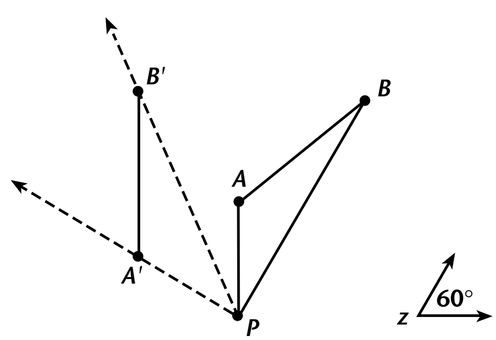


NAME ______________________ CLASS ____________ DATE ____________

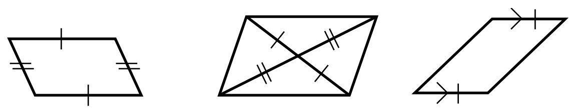
Reteaching
4.6 Conditions for Special Quadrilaterals

◆ **Skill A** Recognizing conditions that determine that a quadrilateral is a parallelogram

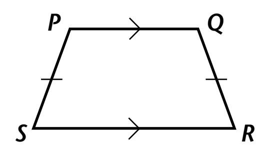
Recall Given a parallelogram, you know that both pairs of opposite sides of the parallelogram are congruent and that the diagonals bisect each other. It is also true that each of these conditions *determine* a parallelogram.

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
A third condition determines a parallelogram as well: If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Each quadrilateral below is a parallelogram.



◆ **Example**
PQRS is a quadrilateral with $\overline{PQ} \parallel \overline{SR}$ and $\overline{PS} \cong \overline{QR}$. Is *PQRS* necessarily a parallelogram? If so, identify the theorem that justifies your answer.



◆ **Solution**
PQRS has one pair of parallel sides and one pair of congruent sides. The given information is insufficient to prove that *PQRS* is a parallelogram. For example, consider the sketch at the right. If it were given that $\overline{PQ} \cong \overline{RS}$ or that $\overline{PS} \cong \overline{QR}$, then *PQRS* would necessarily be a parallelogram.

***VWXY* is a quadrilateral with diagonals intersecting at *Z*. Tell whether *VWXY* is necessarily a parallelogram. If so, state the theorem that justifies your answer.**

1. $\overline{VY} \cong \overline{WX}$ and $\overline{VY} \cong \overline{WX}$ ____________________

2. $\overline{VY} \cong \overline{VW}$ and $\overline{XY} \cong \overline{XW}$ ____________________

3. $\overline{WX} \cong \overline{VY}$ and $\overline{XY} \cong \overline{WV}$ ____________________

4. $\overline{VZ} \cong \overline{XZ}$ and $\overline{WZ} \cong \overline{YZ}$ ____________________

5. $\overline{VZ} \cong \overline{WZ}$ and $\overline{YZ} \cong \overline{XZ}$ ____________________

CLASS ______________ DATE ______________

◆ **Skill B** Recognizing conditions that determine that a parallelogram is a rectangle or a rhombus

Recall Given a parallelogram *ABCD*, the following theorems help you decide whether *ABCD* is a rectangle or a rhombus.

If one angle of *ABCD* is a right angle, then *ABCD* is a rectangle.
If the diagonals of *ABCD* are congruent, then *ABCD* is a rectangle.
If two adjacent sides of *ABCD* are congruent, then *ABCD* is a rhombus.
If the diagonals, $\overline{AC}$ and $\overline{BD}$, of *ABCD* are perpendicular, then *ABCD* is a rhombus.
If a diagonal of *ABCD* bisects a pair of opposite angles, then *ABCD* is a rhombus.

◆ **Example**
BCDE is a parallelogram. $\overline{BD} \perp \overline{CE}$ and $\overline{BD} \cong \overline{CE}$. Tell whether *BCDE* is necessarily a rhombus, a rectangle, both, or neither. Explain.

◆ **Solution**
$\overline{BD}$ and $\overline{CE}$ are the diagonals of *BCDE*. Since $\overline{BD} \perp \overline{CE}$, *BCDE* is a rhombus. Since $\overline{BD} \cong \overline{CE}$, *BCDE* is a rectangle. Since *BCDE* is both a rhombus and a rectangle, *BCDE* is a square.

***MNPQ* is a parallelogram with diagonals intersecting at *R*. Tell whether *MNPQ* is necessarily a rhombus, a rectangle, both, or neither. If *MNPQ* is one or both, justify your answer by stating a theorem or theorems.**

6. $m\angle MRQ = 90°$ ______________________________

____

7. $\overline{MQ} \perp \overline{MN}$ ______________________________

____

8. $\angle QMP \cong \angle NMP$ and $\angle QPM \cong \angle NPM$ ______________________________

____

9. $\overline{NP} \perp \overline{QP}$ and $\overline{MQ} \cong \overline{QP}$ ______________________________

____

10. $\overline{MP} \cong \overline{NQ}$ ______________________________

____

2. 71° **3.** 17 **4.** 50° **5.** 60°

6. 6 **7.** 10 **8.** 7 **9.** 40 **10.** 9

Lesson 4.5

1. 24 **2.** 3 **3.** $1\frac{1}{2}$ **4.** 5

5. Answers may vary. Sample answer: Since the diagonals of a parallelogram bisect each other, $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$. Also, vertical angles PTQ and RTS are congruent, so $\triangle PTQ \cong \triangle RTS$ by the SAS Postulate.

6. 3 **7.** 0.5 **8.** 4 **9.** 90° **10.** 45°

11. $\triangle LMP$, $\triangle LMN$, $\triangle LPN$, $\triangle MNP$, $\triangle LQP$, $\triangle LQM$, $\triangle MQN$, $\triangle PQN$

Lesson 4.6

1. Yes; if one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

2. no

3. Yes; if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. Yes; if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5. no

6. Rhombus; if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

7. Rectangle; if one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

8. Rhombus; if a diagonal of a rhombus bisects a pair of opposite angles, then the parallelogram is a rhombus.

9. Square; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle; if two adjacent sides of a parallelogram are congruent, then the parallelogram is a rhombus.

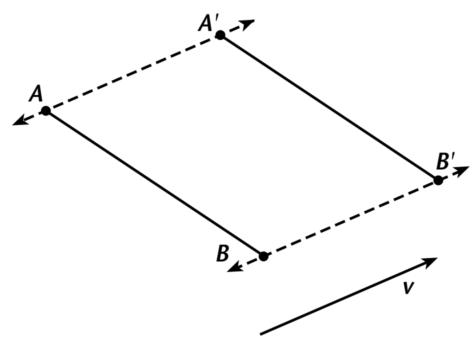
10. Rectangle; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Lesson 4.7

1–6. Check students' drawings.

Lesson 4.8

1.



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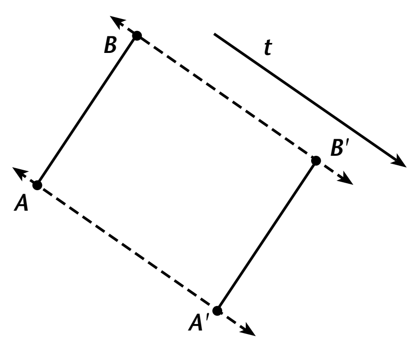

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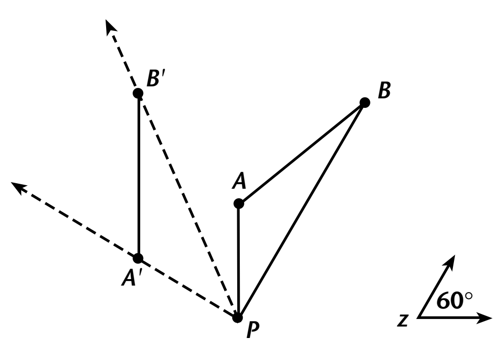


NAME ______________________ CLASS ____________ DATE ____________

Reteaching
4.7 Compass and Straightedge Constructions

◆ **Skill A** Use a compass and straightedge to duplicate segments and angles and construct figures

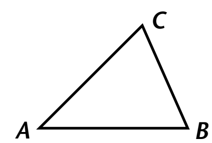
Recall In compass and straightedge constructions, a compass is used to copy distances. A straightedge is used to draw rays and lines. When you draw an arc with a compass, the center of the arc is indicated by the compass point. The radius is the distance between the points of the pencil and the compass.

◆ **Example**

Given $\overline{AB}$ and $\angle C$, use a compass and straightedge to construct each of the following.

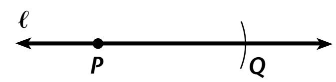
a. a segment congruent to $\overline{AC}$

b. an angle congruent to $\angle A$.

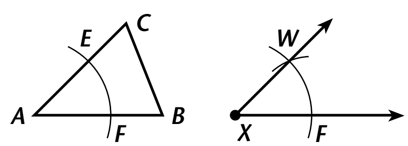


◆ **Solution**

a. Draw line ℓ and choose a point P on ℓ. Draw an arc with radius AB and center P intersecting ℓ at Q; $\overline{PQ} \cong \overline{AB}$.



b. Draw a ray with endpoint X. Draw an arc with center A intersecting the sides of $\angle A$ at points E and F. Using the same radius, draw an arc with center X intersecting the ray at point Y. Next draw an arc with center Y and radius EF intersecting the first arc at point W. Draw $\overrightarrow{XW}$; $\angle WXY \cong \angle A$.



Refer to $\triangle ABC$ in the example above. Choose segments and angles as indicated and construct a triangle congruent to $\triangle ABC$ using the indicated postulate.

1. two sides and the included angle (SAS)

2. two angles and the included side (ASA)

◆ Skill B Constructing lines parallel to or perpendicular to a given line

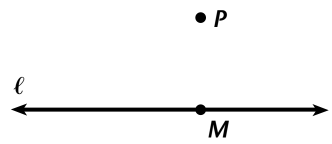
◆ Example

Given line ℓ, point M on ℓ and point P not on ℓ, construct each of the following.

a. a line through P parallel to ℓ

b. a line through P perpendicular to ℓ

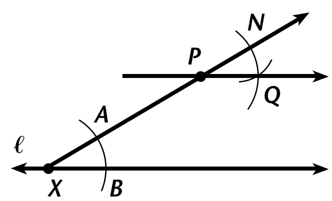
c. a line through M perpendicular to ℓ



◆ Solution

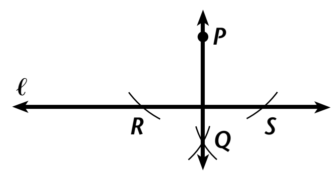
a. Choose a point X on ℓ and draw $\overrightarrow{XP}$, forming $\angle X$ with sides $\overrightarrow{XP}$ and ℓ. Construct an angle with vertex P that is congruent to $\angle X$ as shown. Draw $\overleftrightarrow{PQ}$;

$\overleftrightarrow{PQ} \parallel \ell$.



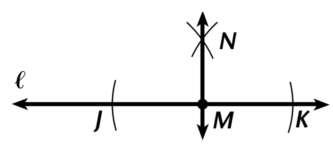
b. Draw an arc with center P intersecting line ℓ at points R and S. Using any reasonable radius, draw arcs with centers R and S that intersect at a point Q on the opposite side of ℓ from P. Draw $\overleftrightarrow{PQ}$;

$\overleftrightarrow{PQ} \perp \ell$.



c. Draw two arcs with center M and the same radius intersecting ℓ at points J and K. Using a radius greater than MJ, draw two arcs with centers J and K intersecting at point N. Draw $\overleftrightarrow{MN}$;

$\overleftrightarrow{MN} \perp \ell$.



Use a compass and a straightedge to construct each figure .

3. rectangle

4. rhombus

5. parallelogram

6. square

ANSWERS

2. 71° **3.** 17 **4.** 50° **5.** 60°

6. 6 **7.** 10 **8.** 7 **9.** 40 **10.** 9

Lesson 4.5

1. 24 **2.** 3 **3.** $1\frac{1}{2}$ **4.** 5

5. Answers may vary. Sample answer: Since the diagonals of a parallelogram bisect each other, $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$. Also, vertical angles PTQ and RTS are congruent, so $\triangle PTQ \cong \triangle RTS$ by the SAS Postulate.

6. 3 **7.** 0.5 **8.** 4 **9.** 90° **10.** 45°

11. $\triangle LMP$, $\triangle LMN$, $\triangle LPN$, $\triangle MNP$, $\triangle LQP$, $\triangle LQM$, $\triangle MQN$, $\triangle PQN$

Lesson 4.6

1. Yes; if one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

2. no

3. Yes; if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. Yes; if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5. no

6. Rhombus; if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

7. Rectangle; if one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

8. Rhombus; if a diagonal of a rhombus bisects a pair of opposite angles, then the parallelogram is a rhombus.

9. Square; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle; if two adjacent sides of a parallelogram are congruent, then the parallelogram is a rhombus.

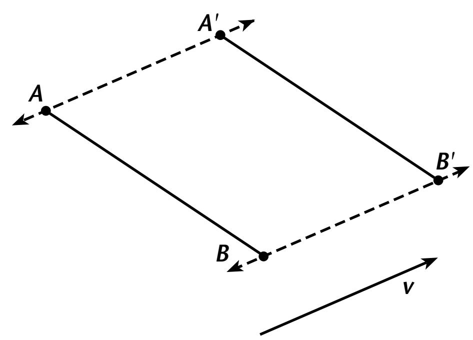
10. Rectangle; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Lesson 4.7

1–6. Check students' drawings.

Lesson 4.8

1.



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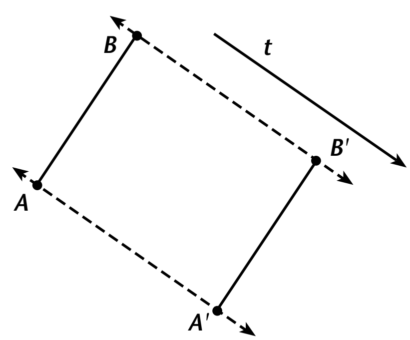

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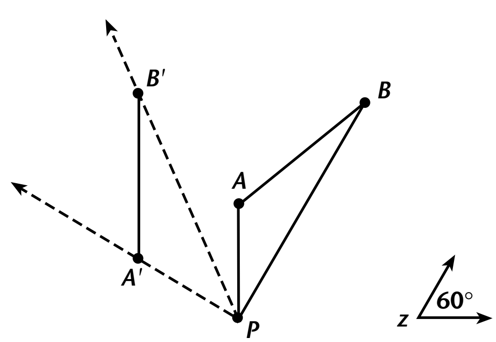

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NAME ______________________ CLASS ____________ DATE ____________

Reteaching

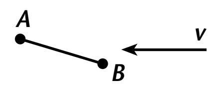
4.8 Constructing Transformations

◆ **Skill A** Translating a segment using a compass and straightedge

Recall A translation vector shows the direction and distance of a translation. To translate a segment, translate its endpoints and connect their images.

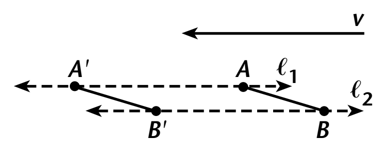
◆ **Example**

Use a compass and straightedge to translate the segment as indicated by the translation vector.



◆ **Solution**

Let x be the length of the translation vector v. Construct a line l_1 through A parallel to v and a line l_2 through B parallel to v. Using a compass opening of x, construct $\overline{AA'}$ on l_1 and $\overline{BB'}$ on l_2. Draw $\overline{A'B'}$. Then $\overline{A'B'}$ is the image of $\overline{AB}$.

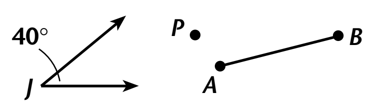


◆ **Skill B** Rotating a segment using a compass and straightedge

Recall To rotate a segment, rotate its endpoints and connect their images.

◆ **Example**

Rotate $\overline{AB}$ about P by m$\angle J$.

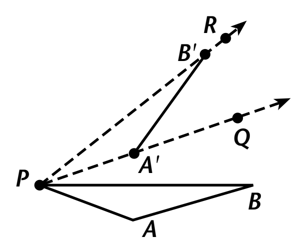


◆ **Solution**

Draw $\overline{PA}$ and $\overline{PB}$. Construct $\angle APQ$ and $\angle BPR$, both congruent to $\angle J$.

Construct $\overline{PA'} \cong \overline{PA}$ on $\overrightarrow{PQ}$ and $\overline{PB'} \cong \overline{PB}$ on $\overrightarrow{PR}$.

Draw $\overline{A'B'}$. Then $\overline{A'B'}$ is the rotation image of $\overline{AB}$.

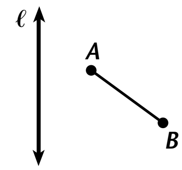


◆ **Skill C** Reflecting a segment using a compass and straightedge

Recall To reflect a segment, reflect its endpoints and connect their images.

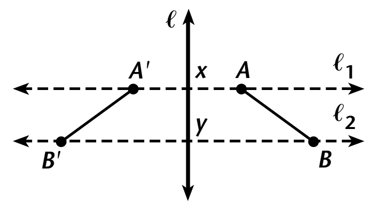
◆ **Example**

Reflect $\overline{AB}$ across ℓ.



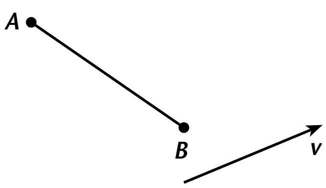
◆ **Solution**

Construct a perpendicular, ℓ_1, to ℓ through A intersecting ℓ at X. Construct a perpendicular, ℓ_2, to ℓ through B intersecting ℓ at Y. Construct $\overline{XA'} \cong \overline{XA}$ on ℓ_1. Construct $\overline{YB'} \cong \overline{YB}$ on ℓ_2. Draw $\overline{A'B'}$. Then $\overline{A'B'}$ is the reflection of $\overline{AB}$.

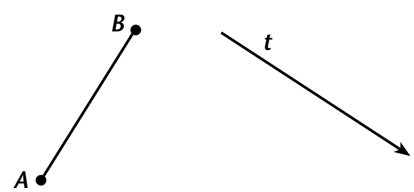


Use a compass and straightedge to construct each transformation of the given segment, $\overline{AB}$.

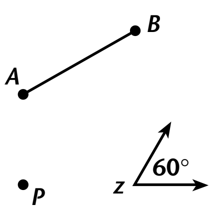
1. the translation indicated by vector v



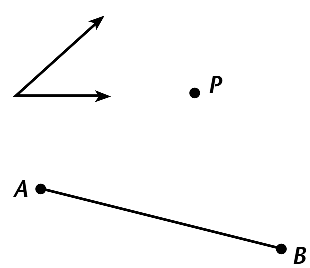
2. the translation indicated by vector t



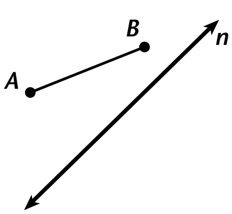
3. rotation about P by $m\angle Z$



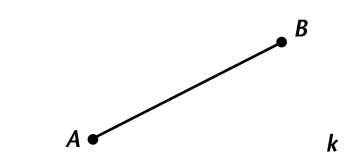
4. rotation about M by $m\angle W$



5. reflection across n



6. reflection across n



2. 71° **3.** 17 **4.** 50° **5.** 60°

6. 6 **7.** 10 **8.** 7 **9.** 40 **10.** 9

Lesson 4.5

1. 24 **2.** 3 **3.** $1\frac{1}{2}$ **4.** 5

5. Answers may vary. Sample answer: Since the diagonals of a parallelogram bisect each other, $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$. Also, vertical angles PTQ and RTS are congruent, so $\triangle PTQ \cong \triangle RTS$ by the SAS Postulate.

6. 3 **7.** 0.5 **8.** 4 **9.** 90° **10.** 45°

11. $\triangle LMP$, $\triangle LMN$, $\triangle LPN$, $\triangle MNP$, $\triangle LQP$, $\triangle LQM$, $\triangle MQN$, $\triangle PQN$

Lesson 4.6

1. Yes; if one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

2. no

3. Yes; if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. Yes; if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5. no

6. Rhombus; if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

7. Rectangle; if one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

8. Rhombus; if a diagonal of a rhombus bisects a pair of opposite angles, then the parallelogram is a rhombus.

9. Square; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle; if two adjacent sides of a parallelogram are congruent, then the parallelogram is a rhombus.

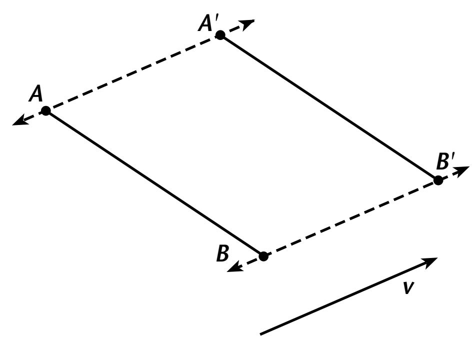
10. Rectangle; if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Lesson 4.7

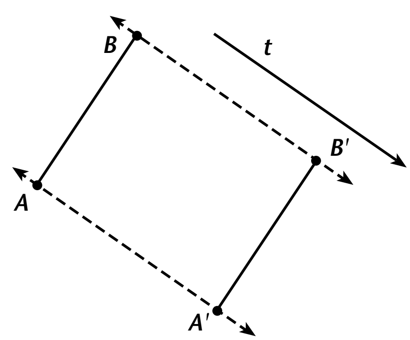
1–6. Check students' drawings.

Lesson 4.8

1.



2.



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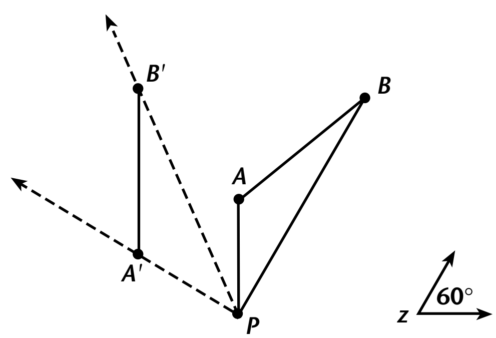

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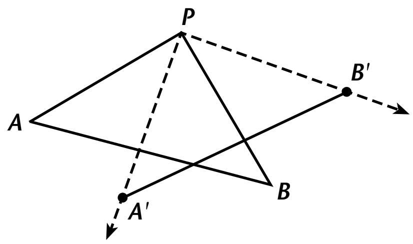

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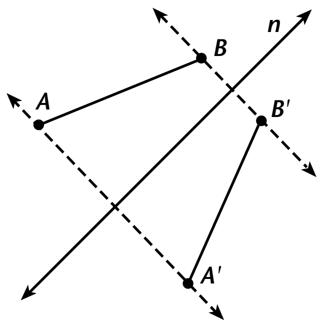

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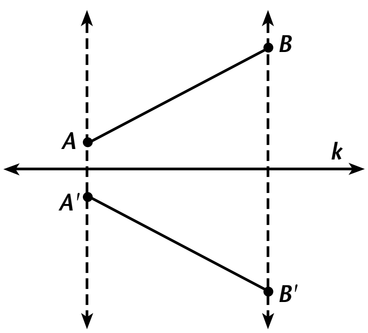


Reteaching — Chapter 5

Lesson 5.1

1. 44 units **2.** 60 units **3.** 88 units

4. 328 units **5.** 45 units **6.** 34 units

7. 38 units **8.** $32\sqrt{3}$ units **9.** 12 units

10. 8 units **11.** 400 square units

12. 272 square units

13. about 8 square units

14. $h = 21$ inches, $b = 14$ inches, $A = 294$ square inches

15. $b = 7$ meters, $h = 15$ meters, $P = 44$ meters

Lesson 5.2

1. 168 square meters

2. 560 square feet

3. 57.6 square centimeters

4. 77 square inches

5. 210 square meters

6. 342.25 square units

7. 336 square millimeters

8. 1120 square feet

9. 115.2 square centimeters

10. 84 square units

11. 25 square meters

12. 126 square inches

13. 84 square centimeters

14. 50 square yards

15. 225 square units

16. 66 square millimeters

17. 17.5 square units

18. 68 square feet

19. 720 square millimeters

Lesson 5.3

1–18. Answers were obtained using 3.14 for π unless otherwise noted.

1. 40π units; 125.6 units

2. 11π inches; 34.54 inches

3. 8π meters; 25.12 meters

4. 48 feet; 48 feet

5. $\frac{7.5}{\pi}$ meters; 2.4 meters

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
5.1 Perimeter and Area

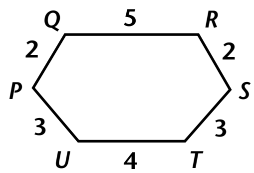
◆ **Skill A** Finding the perimeter of a polygon

Recall The perimeter of a polygon is the sum of the length of its sides. The perimeter of a rectangle with base b and height h is $P = 2b + 2h$.

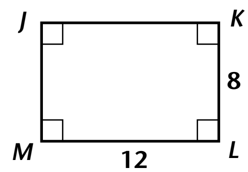
◆ **Example**

Find the perimeter of each figure.

a.



b.



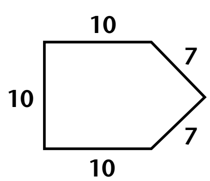
◆ **Solution**

a. $P = 2 + 5 + 2 + 3 + 4 + 3 = 19$ units

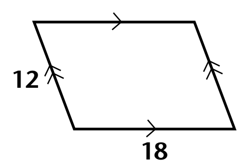
b. $JKLM$ is a rectangle; $P = 2b + 2h = 2(12) + 2(8) = 40$ units.

Find the perimeter of each figure.

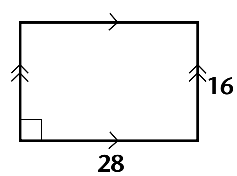
1.



2.



3.



______________ ______________ ______________

4. a rhombus 82 in. on a side ______________

5. a regular pentagon with sides 9 m long ______________

6. a rectangle with base 12 ft and height 5 ft ______________

7. a rectangle with base 11 in. and height 8 in. ______________

8. a regular octagon with sides $4\sqrt{3}$ cm long ______________

9. a decagon with each side $\frac{6}{5}$ units long ______________

10. a dodecagon with each side $\frac{2}{3}$ units long ______________

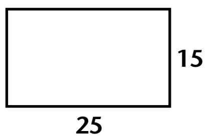
◆ Skill B Finding areas using the formula for the area of a rectangle

Recall To find the area of a rectangle with base b and height h, use the formula $A = bh$. To find the area of region that is not rectangular, you may be able to divide it into nonoverlapping rectangular regions or approximate its shape with rectangles.

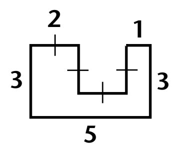
◆ **Example 1**

Find each area. Angles that appear to be right angles are right angles.

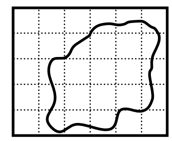
a.



b.



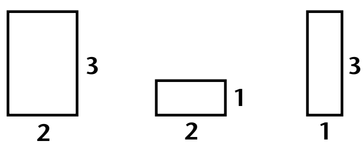
c.



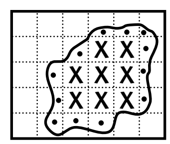
◆ **Solution**

a. Use the formula $A = bh = 15 \cdot 25 = 375$ square units

b. Divide the figure into rectangles One method is shown.
$A = 6 + 2 + 3 = 11$ square units



c. Sample: There are 8 squares completely inside the figure (×) and 12 partly inside the figure (·).
$A \approx 8 + \frac{1}{2} \cdot 12 = 14$ square units



◆ **Example 2**

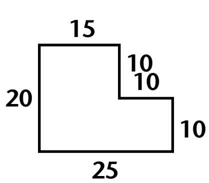
The height of a rectangle is 3 times its base. The perimeter is 48 inches. Find the area of the rectangle.

◆ **Solution**

Use $P = 48$ and $h = 3b$ to write an equation: $P = 2b + 2h$
$48 = 2b + 2(3b)$; $48 = 8b$; $b = 6$
Then $h = 18$ and $A = bh = 6 \cdot 18 = 108$. The area is 108 square inches.

Find the area. Estimate if necessary. Angles that appear to be right angles are right angles.

11.



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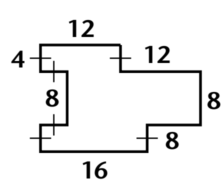
12.



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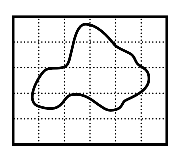
13.



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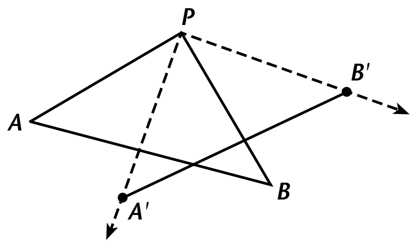
14. The perimeter of a rectangle is 70 inches. The base is $\frac{2}{3}$ the height. Find the base, the height, and the area of the rectangle.

______

15. The area of a rectangle is 105 square meters. The height is 1 more than twice the base. Find the base, the height, and the perimeter of the rectangle.

______

4.



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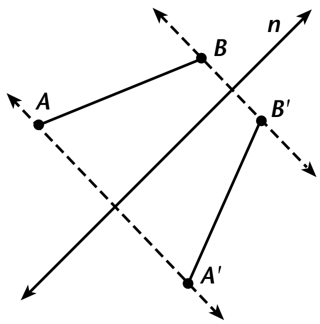

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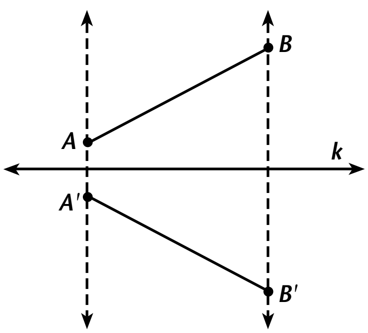

Reteaching — Chapter 5

Lesson 5.1

1. 44 units **2.** 60 units **3.** 88 units

4. 328 units **5.** 45 units **6.** 34 units

7. 38 units **8.** $32\sqrt{3}$ units **9.** 12 units

10. 8 units **11.** 400 square units

12. 272 square units

13. about 8 square units

14. $h = 21$ inches, $b = 14$ inches, $A = 294$ square inches

15. $b = 7$ meters, $h = 15$ meters, $P = 44$ meters

Lesson 5.2

1. 168 square meters

2. 560 square feet

3. 57.6 square centimeters

4. 77 square inches

5. 210 square meters

6. 342.25 square units

7. 336 square millimeters

8. 1120 square feet

9. 115.2 square centimeters

10. 84 square units

11. 25 square meters

12. 126 square inches

13. 84 square centimeters

14. 50 square yards

15. 225 square units

16. 66 square millimeters

17. 17.5 square units

18. 68 square feet

19. 720 square millimeters

Lesson 5.3

1–18. Answers were obtained using 3.14 for π unless otherwise noted.

1. 40π units; 125.6 units

2. 11π inches; 34.54 inches

3. 8π meters; 25.12 meters

4. 48 feet; 48 feet

5. $\frac{7.5}{\pi}$ meters; 2.4 meters

Menu
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NAME ______________________ CLASS ____________ DATE ____________

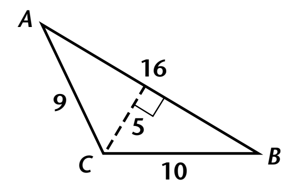
Reteaching

5.2 Areas of Triangles, Parallelograms, and Trapezoids

◆ **Skill A** Finding the area of a triangle

Recall The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.

◆ **Example**
Find the area of $\triangle ABC$.



◆ **Solution**
Since you know the length of the altitude to $\overline{AB}$, let $\overline{AB}$ be the base. Then $b = 16$ and $h = 5$, so $A = \frac{1}{2}(1 \cdot 16) = 40$ square units.

Find the area of the triangle with the given base and height.

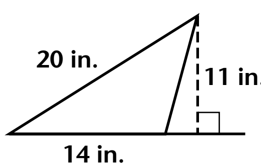
1. $b = 42$ m, $h = 8$ m ____________

2. $b = 16$ ft, $h = 0$ ft ____________

3. $b = 12.8$ cm, $h = 9$ cm ____________

Find the area of each triangle.

4.



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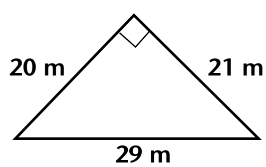
5.



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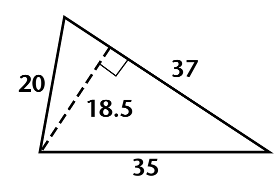
6.



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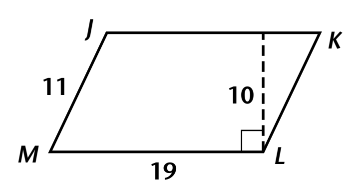
◆ **Skill B** Finding the area of a parallelogram

Recall The area of a parallelogram with base b and height h is $A = bh$.

◆ **Example**
Find the area of parallelogram $JKLM$



◆ **Solution**
Since you know the length of the altitude to $\overline{JK}$ and $\overline{LM}$, let $\overline{JK}$ be the base. Then $b = 19$ and $h = 10$, so $A = (19 \cdot 10) = 190$ square units.

CLASS ________ DATE ________

Find the area of the parallelogram with the given base and height.

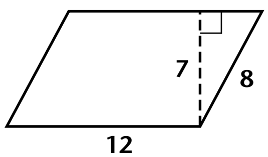
7. $b = 42$ m, $h = 8$ m ________

8. $b = 16$ ft, $h = 70$ ft ________

9. $b = 12.8$ cm, $h = 9$ cm ________

Find the area of each parallelogram.

10.



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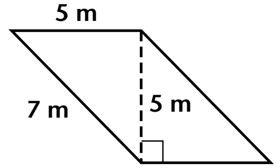

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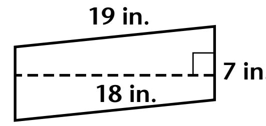

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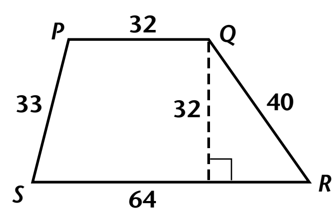
◆ **Skill C** Finding the area of a trapezoid

Recall The area of a trapezoid with bases b_1 and b_2 and height h is $A = \frac{1}{2}(b_1 + b_2)$.

◆ **Example**
Find the area of trapezoid *PQRS*.



◆ **Solution**
$\overline{PQ}$ and $\overline{RS}$ are the bases of *PQRS*, so the height is 32.

Then $A = \frac{1}{2}(32)(32 + 64) = 1536$ square units.

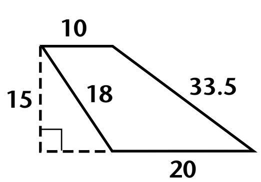
Find the area of the trapezoid with the given bases and height.

13. $b_1 = 15$ cm, $b_2 = 9$ cm, $h = 7$ cm ________

14. $b_1 = 13$ yd, $b_2 = 7$ yd, $h = 5$ yd ________

Find the area of each trapezoid.

15.



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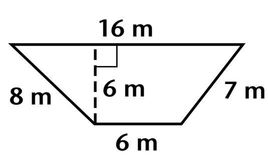

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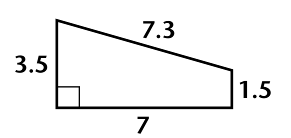
17.



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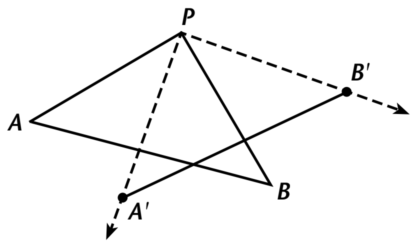
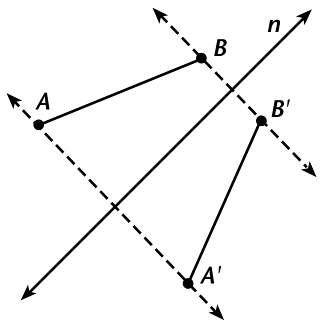
18. $b_1 = 7$ ft, $b_2 = 10$ ft, $h = 8$ ft ________

19. $b_1 = 40$ m, $b_2 = 32$ m, $h = 20$ m ________

4.



5.



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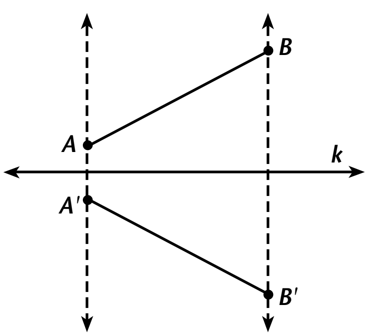


Reteaching — Chapter 5

Lesson 5.1

1. 44 units **2.** 60 units **3.** 88 units

4. 328 units **5.** 45 units **6.** 34 units

7. 38 units **8.** $32\sqrt{3}$ units **9.** 12 units

10. 8 units **11.** 400 square units

12. 272 square units

13. about 8 square units

14. $h = 21$ inches, $b = 14$ inches, $A = 294$ square inches

15. $b = 7$ meters, $h = 15$ meters, $P = 44$ meters

Lesson 5.2

1. 168 square meters

2. 560 square feet

3. 57.6 square centimeters

4. 77 square inches

5. 210 square meters

6. 342.25 square units

7. 336 square millimeters

8. 1120 square feet

9. 115.2 square centimeters

10. 84 square units

11. 25 square meters

12. 126 square inches

13. 84 square centimeters

14. 50 square yards

15. 225 square units

16. 66 square millimeters

17. 17.5 square units

18. 68 square feet

19. 720 square millimeters

Lesson 5.3

1–18. Answers were obtained using 3.14 for π unless otherwise noted.

1. 40π units; 125.6 units

2. 11π inches; 34.54 inches

3. 8π meters; 25.12 meters

4. 48 feet; 48 feet

5. $\frac{7.5}{\pi}$ meters; 2.4 meters

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
5.3 Circumferences and Areas of Circles

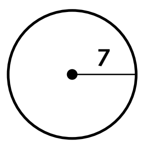
◆ **Skill A** Using the formulas for the circumference of a circle

Recall The circumference of a circle with radius r and diameter d is $C = 2\pi r$ or $C = \pi d$. An approximate value of the circumference can be obtained by substituting one of the values 3.14 or $\frac{22}{7}$ for π.

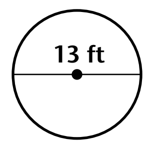
◆ **Example 1**

Find the area of each circle. Give an exact answer and an approximation.

a.



b.



◆ **Solution**

a. $r = 7$, so $C = 2\pi r = 14\pi$; substituting $\frac{22}{7}$ for π, $C \approx \frac{22}{7}(14) = 44$ units.

b. $d = 13$ ft, so $C = 13\pi$ ft; substituting 3.14 for π, $C \approx 13(3.14) = 40.82$ feet.

◆ **Example 2**

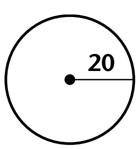
A circle has circumference 72 centimeters. Find the exact radius and an approximation.

◆ **Solution**

$C = 2\pi r$; $72 = 2\pi r$; $r = \frac{72}{2\pi} = \frac{36}{\pi}$ cm and $r \approx \frac{36}{3.14} \approx 11.15$ cm

For each circle, find the exact circumference and an approximation. If necessary, round to the nearest hundredth.

1.



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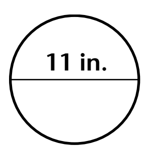
2.



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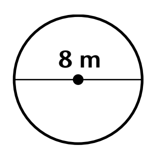
3.



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For each circle, find the exact radius and an approximation to the nearest tenth.

4. 96π ft

____

____

5. $C = 15$ m

____

____

6. $C = 112$ in.

____

____

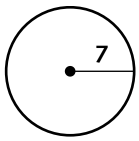
◆ **Skill B** Using the formula for the area of a circle

Recall The area of a circle with radius r is $A = \pi r^2$. An approximate value of the area can be obtained by substituting one of the values 3.14 or $\frac{22}{7}$ for π.

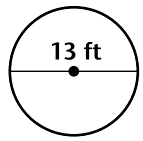
◆ **Example 1**

Find the area of each circle. (In **c** find the area of the shaded region.) Give an exact answer and an approximation.

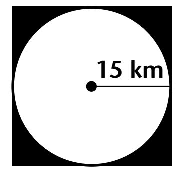
a.



b.



c.



◆ **Solution**

a. $r = 7$, so $A = \pi r^2 = 49\pi$; substituting $\frac{22}{7}$ for π, $A \approx \frac{22}{7}(49) = 154$ square units

b. $d = 13$ ft, so $r = \frac{13}{2} = 6.5$ ft and $A = 42.25\pi$ ft^2; substituting 3.14 for π and rounding to the nearest hundredth, $A \approx 42.25(3.14) \approx 132.67$ ft^2.

c. area of shaded region = area of square − area of circle = $900 - 225\pi$; substituting 3.14 for π, area ≈ 193.5 km^2.

◆ **Example 2**

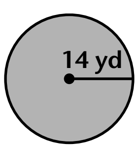
A circle has an area of 56 square units. Find the exact radius of the circle and an approximation to the nearest tenth.

◆ **Solution**

$A = \pi r^2$; $56 = \pi r^2$; $r^2 = \frac{56}{\pi}$, so $r = \sqrt{\frac{56}{\pi}}$ and $r \approx \sqrt{\frac{56}{3.14}} \approx \sqrt{17.83} \approx 4.2$ units

For each shaded region, find the exact area and an approximation. If necessary, round to the nearest hundredth.

7.



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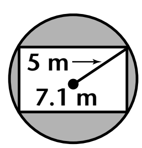

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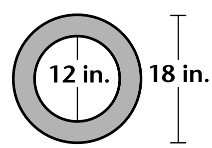
9.



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____

For each circle, find the exact radius and an approximation to the nearest tenth.

10. $A = 53\pi$ km^2

____

____

11. $A = 285$ m^2

____

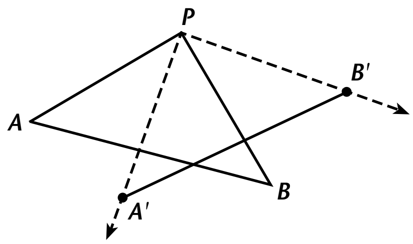
____

12. $A = 498$ in.^{2}

____

____

4.



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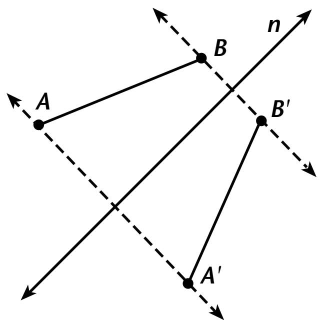

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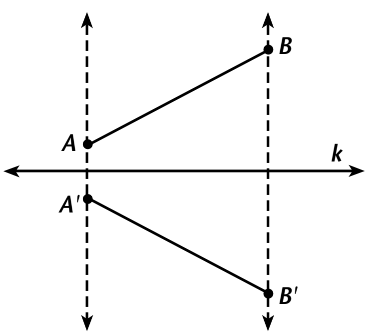

Reteaching — Chapter 5

Lesson 5.1

1. 44 units **2.** 60 units **3.** 88 units

4. 328 units **5.** 45 units **6.** 34 units

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3. 57.6 square centimeters

4. 77 square inches

5. 210 square meters

6. 342.25 square units

7. 336 square millimeters

8. 1120 square feet

9. 115.2 square centimeters

10. 84 square units

11. 25 square meters

12. 126 square inches

13. 84 square centimeters

14. 50 square yards

15. 225 square units

16. 66 square millimeters

17. 17.5 square units

18. 68 square feet

19. 720 square millimeters

Lesson 5.3

1–18. Answers were obtained using 3.14 for π unless otherwise noted.

1. 40π units; 125.6 units

2. 11π inches; 34.54 inches

3. 8π meters; 25.12 meters

4. 48 feet; 48 feet

5. $\frac{7.5}{\pi}$ meters; 2.4 meters

ANSWERS

6. $\frac{56}{\pi}$ inches; 17.8 inches

7. 196π square yards; 616 square yards $\left(\pi \approx \frac{22}{7}\right)$

8. $25\pi - 50.41$ square meters; 28.09 square meters

9. 45π square inches; 141.3 square inches

10. $\sqrt{53}$ kilometers; 7.28 kilometers

11. $\sqrt{\frac{285}{\pi}}$ meters; 9.5 meters

12. $\sqrt{\frac{498}{\pi}}$ inches; 12.6 inches

Lesson 5.4

1. 29 **2.** 17 **3.** $\sqrt{39}$ **4.** $\sqrt{217}$

5. $10\sqrt{3}$ **6.** $4\sqrt{6}$ **7.** 186.8 feet **8.** yes

9. yes **10.** yes **11.** no **12.** yes

13. yes **14.** no **15.** yes **16.** yes

17. acute **18.** obtuse **19.** obtuse

20. acute **21.** obtuse **22.** right

23. acute **24.** acute **25.** obtuse

Lesson 5.5

1. 13; $13\sqrt{2}$ **2.** $4\sqrt{2}$; 4 **3.** $\frac{9\sqrt{2}}{2}$; 4.5

4. 9; 9 **5.** $4\sqrt{3}$; 8 **6.** $6\sqrt{3}$; 9 **7.** 7; 14

8. $12\sqrt{3}$; $6\sqrt{3}$ **9.** 1728 square units

10. 41.6 square inches

11. 25 square meters

12. $600\sqrt{3}$ square feet

13. $36\sqrt{3}$ square units

14. 33.6 square meters

15. 1086 square millimeters

16. 770 square inches

Lesson 5.6

1. 13 **2.** 5 **3.** $\sqrt{34}$; 5.83

4. $10\sqrt{2}$; 14.14 **5.** $\sqrt{181}$; 13.45

6. $3\sqrt{5}$; 6.71

7–10. Answers may vary due to when rounding is done.

7. 31.3 **8.** 44.3 **9.** 31.33 **10.** 35.46

11. 30.56 **12.** 24.56

13. 27.56; Sample: Yes; the value calculated using the formula is 28.27.

Lesson 5.7

1. $A(-a, -a)$, $B(-a, a)$, $D(a, -a)$

2. $M(0, 0)$, $N(0, a)$, $P(a, a)$

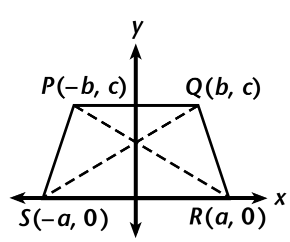
3. $J(-a, 0)$

4. $S(-b, c)$

5. $G(0, 0)$, $E(a - b, c)$

6. $D(0, 0)$, $F(2a, 0)$

7. Sample proof: Let P, Q, R, and S be as shown in the figure.



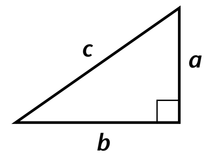
$PR = \sqrt{(a - (-b))^2 + (0 - c)^2} = \sqrt{(a + b)^2 + c^2}$ and $QS = \sqrt{(b - (-a))^2 + (c - 0)^2} = \sqrt{(a + b)^2 + c^2}$, so $\overline{PR} \cong \overline{QS}$.

NAME ______________________ CLASS __________ DATE __________

Reteaching
5.4 The Pythagorean Theorem

◆ **Skill A** Using the Pythagorean Theorem

Recall In any right triangle, the square the length of the hypotenuse is equal to the sum of the squares of the other two sides. For the right triangle shown, $c^2 = a^2 + b^2$.



◆ **Example**
The hypotenuse of a right triangle is 9 inches long and one leg is 5 inches long. Find the length of the other leg.

◆ **Solution**
Let $c = 9$, $a = 5$, and $b =$ the length of the other leg. By the Pythagorean Theorem, $a^2 + b^2 = c^2$, so $b^2 = 9^2 - 5^2 = 81 - 25 = 56$, and $b = \sqrt{56} = 2\sqrt{14}$.

The lengths of two sides of a right triangle are given. Find the length of the third side. If necessary, write your answer in simplest radical form.

1. $a = 20, b = 21, c =$ ______________

2. $a = 8, b = 15, c =$ ______________

3. $a = 5, b = 8, c =$ ______________

4. $b = 12, c = 19, a =$ ______________

5. $a = 10, c = 20, b =$ ______________

6. $a = 5, c = 11, b =$ ______________

7. A rectangular field is 180 feet long and 50 feet wide. Find the length of a diagonal of the field. Give your answer in simplest radical form and as a decimal rounded to the nearest tenth of a foot. ______________

◆ **Skill B** Using the converse of the Pythagorean Theorem

Recall If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

◆ **Example**
The sides of a triangle are 9 cm, 40 cm, and 41 cm long. Is the triangle a right triangle?

◆ **Solution**
Square the length of each side: $9^2 = 81$ $\quad 40^2 = 1600$ $\quad 41^2 = 1681$
Add the squares of the lengths of the two shorter sides: $81 + 1600 = 1681$
Compare: $1681 = 81 + 1600$
By the converse of the Pythagorean Theorem, the triangle is a right triangle.

The lengths of the sides of a triangle are given. Is the triangle a right triangle?

8. 12, 16, 20 ______________

9. 5 in., 12 in., 13 in. ______________

10. 10 m, 24 m, 26 m ______________

11. 14 km, 15 km, 16 km ______________

12. 7, 7, $7\sqrt{2}$ ______________

13. 3.5 cm, 12 cm, 12.5 cm ______________

14. 5 ft, 6 ft, $2\sqrt{15}$ ft ______________

15. 9 mm, $9\sqrt{3}$ mm, 18 mm ______________

16. 15, 20, 25 ______________

◆ Skill C Using the Pythagorean Inequalities

Recall Given any two real numbers x and y, exactly one of the following is true.

$$x = y \qquad x > y \qquad x < y$$

Let $\triangle ABC$ be a triangle with longest side of length c and shorter sides of lengths a and b. The converse of the Pythagorean Theorem deals with the case in which $c^2 = a^2 = \ + b^2$. The Pythagorean Inequalities deal with the other two possibilities.
If $c^2 > a^2 + b^2$, then $\triangle ABC$ is an obtuse triangle.
If $c^2 < a^2 + b^2$, then $\triangle ABC$ is an acute triangle.

◆ **Example**
The sides of a triangle are 23 feet, 15 feet , and 12 feet long. Is the triangle right, obtuse, or acute?

◆ **Solution**
Square the length of each side: $23^2 = 529 \qquad 15^2 = 225 \qquad 12^2 = 144$
Add the squares of the lengths of the two shorter sides: $225 + 144 = 369$
Compare: $529 > 225 + 144$
The triangle is an obtuse triangle.

The lengths of the sides of a triangle are given. Determine whether the triangle is right, acute, or obtuse.

17. 17, 20, 25 ______________

18. 8 ft, 9 ft, 14 ft ______________

19. 4 in., 4 in., 7 in. ______________

20. 15 m, 18 m, 22 m ______________

21. 16, 8, 20 ______________

22. 11 mm, 60 mm, 61 mm ______________

23. 8, 8, 9 ______________

24. 12 ft, 15 ft, 15 ft ______________

25. 30, 40, 48 ______________

Menu

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ANSWERS

6. $\frac{56}{\pi}$ inches; 17.8 inches

7. 196π square yards; 616 square yards $\left(\pi \approx \frac{22}{7}\right)$

8. $25\pi - 50.41$ square meters; 28.09 square meters

9. 45π square inches; 141.3 square inches

10. $\sqrt{53}$ kilometers; 7.28 kilometers

11. $\sqrt{\frac{285}{\pi}}$ meters; 9.5 meters

12. $\sqrt{\frac{498}{\pi}}$ inches; 12.6 inches

Lesson 5.4

1. 29 **2.** 17 **3.** $\sqrt{39}$ **4.** $\sqrt{217}$

5. $10\sqrt{3}$ **6.** $4\sqrt{6}$ **7.** 186.8 feet **8.** yes

9. yes **10.** yes **11.** no **12.** yes

13. yes **14.** no **15.** yes **16.** yes

17. acute **18.** obtuse **19.** obtuse

20. acute **21.** obtuse **22.** right

23. acute **24.** acute **25.** obtuse

Lesson 5.5

1. 13; $13\sqrt{2}$ **2.** $4\sqrt{2}$; 4 **3.** $\frac{9\sqrt{2}}{2}$; 4.5

4. 9; 9 **5.** $4\sqrt{3}$; 8 **6.** $6\sqrt{3}$; 9 **7.** 7; 14

8. $12\sqrt{3}$; $6\sqrt{3}$ **9.** 1728 square units

10. 41.6 square inches

11. 25 square meters

12. $600\sqrt{3}$ square feet

13. $36\sqrt{3}$ square units

14. 33.6 square meters

15. 1086 square millimeters

16. 770 square inches

Lesson 5.6

1. 13 **2.** 5 **3.** $\sqrt{34}$; 5.83

4. $10\sqrt{2}$; 14.14 **5.** $\sqrt{181}$; 13.45

6. $3\sqrt{5}$; 6.71

7–10. Answers may vary due to when rounding is done.

7. 31.3 **8.** 44.3 **9.** 31.33 **10.** 35.46

11. 30.56 **12.** 24.56

13. 27.56; Sample: Yes; the value calculated using the formula is 28.27.

Lesson 5.7

1. $A(-a, -a)$, $B(-a, a)$, $D(a, -a)$

2. $M(0, 0)$, $N(0, a)$, $P(a, a)$

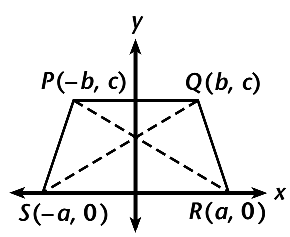
3. $J(-a, 0)$

4. $S(-b, c)$

5. $G(0, 0)$, $E(a - b, c)$

6. $D(0, 0)$, $F(2a, 0)$

7. Sample proof: Let P, Q, R, and S be as shown in the figure.



$PR = \sqrt{(a - (-b))^2 + (0 - c)^2} = \sqrt{(a + b)^2 + c^2}$ and $QS = \sqrt{(b - (-a))^2 + (c - 0)^2} = \sqrt{(a + b)^2 + c^2}$, so $\overline{PR} \cong \overline{QS}$.

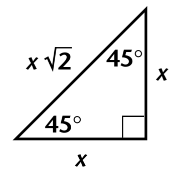
NAME ______________________ CLASS __________ DATE __________

Reteaching

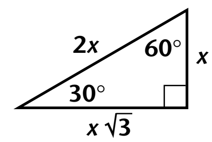
5.5 Special Triangles and Areas of Regular Polygons

◆ **Skill A** Using the 45-45-90 Triangle Theorem and the 30-60-90 Triangle Theorem

Recall A 45-45-90 triangle is a right triangle in which both acute angles have measure 45°. In a 45-45-90 triangle, if the length of each leg is x, the length of the hypotenuse is $x\sqrt{2}$. If the length of the hypotenuse is x, then the length of each leg is $\frac{x}{\sqrt{2}}$ or $\frac{x\sqrt{2}}{2}$.



A 30-60-90 triangle is a right triangle in which the acute angles have measures of 30° and 60°. The shorter leg of the triangle is opposite the 30° angle and the longer leg is opposite the 60° angle. In a 30-60-90 triangle, if the length of the shorter leg is x, then the length of the longer leg is $x\sqrt{3}$, and the length of the hypotenuse is $2x$.



◆ **Example**

a. The legs of a 45-45-90 triangle are each 15 centimeters long. Find the length, h, of the hypotenuse to the nearest tenth of a centimeter.

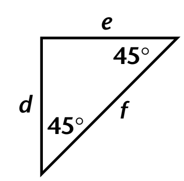
b. The longer leg of a 30-60-90 triangle is 5 inches long. Find the length, h, of the hypotenuse in simplest radical form.

◆ **Solution**

a. Refer to the 45-45-90 triangle in the figure above. Since $x = 15$, $h = x\sqrt{2} = 15\sqrt{2} \approx 21.2$ centimeters.

b. Refer to the 30-6-90 triangle in the figure above. Since $x\sqrt{3} = 5$, $x = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ and $h = 2x = \frac{10\sqrt{3}}{3}$ inches.

The length of one side of a 45-45-90 triangle is given. Find the lengths of the other two sides in simplest radical form.



1. $d = 13$, $e =$ ______________

$f =$ ______________

2. $d = 4$, $e =$ ______________

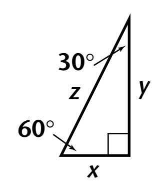
$f =$ ______________

3. $e = 4.5$, $f =$ ______________

$d =$ ______________

4. $f = 9\sqrt{2}$, $d =$ ______________

$e =$ ______________



The length of one side of a 30-60-90 triangle is given. Find the lengths of the other two sides in simplest radical form.

5. $x = 4$, $y =$ ______________

$z =$ ______________

6. $x = 3\sqrt{3}$, $z =$ ______________

$y =$ ______________

7. $y = 7\sqrt{3}$, $x =$ ______________

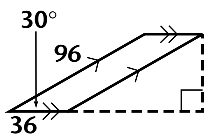
$z =$ ______________

8. $y = 18$, $z =$ ______________

$x =$ ______________

Find the area of each figure. Round your answers to the nearest tenth if necessary.

9.



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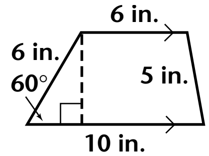

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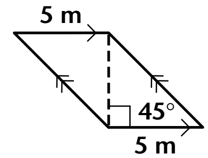

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◆ Skill B Finding the area of a regular polygon

Recall The apothem of a regular polygon is the length of a perpendicular segment from the center of the polygon to a side. The apothem can be used to determine the area of the polygon.

The area of a regular polygon with apothem a and perimeter p is $A = \frac{1}{2}ap$.

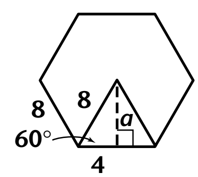
◆ **Example**

Find the area of a regular hexagon with sides that are 8 centimeters long.

◆ **Solution**

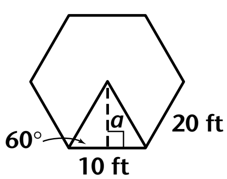
Use the 30-60-90 Triangle Theorem to determine the apothem. In the 30-60-90 triangle shown, the length of the shorter leg is 4, so the apothem is $4\sqrt{3}$.

Since the perimeter is $6 \cdot 8 = 48$, $A = \frac{1}{2} \cdot 4\sqrt{3} \cdot 48 = 96\sqrt{3}$.



Find the area of each regular polygon. Give your answer in simplest radical form.

12.



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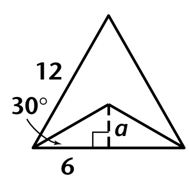
13.



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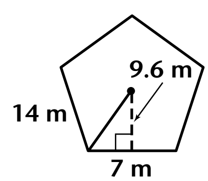
14.



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15. a regular octagon with sides that are 15 mm long and an apothem of 18.1 mm ______________

16. a regular decagon with sides that are 10 in. long and an apothem of 15.4 in. ______________

Menu

Print

6. $\frac{56}{\pi}$ inches; 17.8 inches

7. 196π square yards; 616 square yards $\left(\pi \approx \frac{22}{7}\right)$

8. $25\pi - 50.41$ square meters; 28.09 square meters

9. 45π square inches; 141.3 square inches

10. $\sqrt{53}$ kilometers; 7.28 kilometers

11. $\sqrt{\frac{285}{\pi}}$ meters; 9.5 meters

12. $\sqrt{\frac{498}{\pi}}$ inches; 12.6 inches

Lesson 5.4

1. 29 **2.** 17 **3.** $\sqrt{39}$ **4.** $\sqrt{217}$

5. $10\sqrt{3}$ **6.** $4\sqrt{6}$ **7.** 186.8 feet **8.** yes

9. yes **10.** yes **11.** no **12.** yes

13. yes **14.** no **15.** yes **16.** yes

17. acute **18.** obtuse **19.** obtuse

20. acute **21.** obtuse **22.** right

23. acute **24.** acute **25.** obtuse

Lesson 5.5

1. 13; $13\sqrt{2}$ **2.** $4\sqrt{2}$; 4 **3.** $\frac{9\sqrt{2}}{2}$; 4.5

4. 9; 9 **5.** $4\sqrt{3}$; 8 **6.** $6\sqrt{3}$; 9 **7.** 7; 14

8. $12\sqrt{3}$; $6\sqrt{3}$ **9.** 1728 square units

10. 41.6 square inches

11. 25 square meters

12. $600\sqrt{3}$ square feet

13. $36\sqrt{3}$ square units

14. 33.6 square meters

15. 1086 square millimeters

16. 770 square inches

Lesson 5.6

1. 13 **2.** 5 **3.** $\sqrt{34}$; 5.83

4. $10\sqrt{2}$; 14.14 **5.** $\sqrt{181}$; 13.45

6. $3\sqrt{5}$; 6.71

7–10. Answers may vary due to when rounding is done.

7. 31.3 **8.** 44.3 **9.** 31.33 **10.** 35.46

11. 30.56 **12.** 24.56

13. 27.56; Sample: Yes; the value calculated using the formula is 28.27.

Lesson 5.7

1. $A(-a, -a)$, $B(-a, a)$, $D(a, -a)$

2. $M(0, 0)$, $N(0, a)$, $P(a, a)$

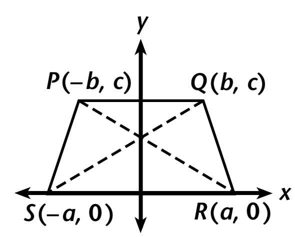
3. $J(-a, 0)$

4. $S(-b, c)$

5. $G(0, 0)$, $E(a - b, c)$

6. $D(0, 0)$, $F(2a, 0)$

7. Sample proof: Let P, Q, R, and S be as shown in the figure.



$PR = \sqrt{(a - (-b))^2 + (0 - c)^2} = \sqrt{(a + b)^2 + c^2}$ and $QS = \sqrt{(b - (-a))^2 + (c - 0)^2} = \sqrt{(a + b)^2 + c^2}$, so $\overline{PR} \cong \overline{QS}$.

NAME ______________________ CLASS ____________ DATE ____________

Reteaching

5.6 The Distance Formula and the Method of Quadrature

◆**Skill A** Using the distance formula

Recall The Pythagorean Theorem can be used to determine the distance between two points in a coordinate plane. The distance between two points (x_1, y_1) and (x_2, y_2) in a coordinate plane is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

To determine the length of a segment in the plane, use the distance formula to determine the distance between the endpoints.

◆ **Example 1**

Find the distance between the points (–4, 7) and (3, 8). Give the answer in simplest radical form and as a decimal rounded to the nearest hundredth.

◆ **Solution**

Let $(x_1, y_1) = (-4, 7)$ and $(x_2, y_2) = (3, 8)$.

$d = \sqrt{(3 - (-4))^2 + (8 - 7)^2} = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2}$

$d \approx 7.07$

◆ **Example 2**

A triangle has vertices $P(-3, -3)$, $Q(-1, 5)$, and $R(6, 0)$. Find the perimeter of $\triangle PQR$ to the nearest tenth.

◆ **Solution**

$PQ = \sqrt{(5 - (-3))^2 + (-1 - (-3))^2} = \sqrt{8^2 + 2^2} = \sqrt{68} = 2\sqrt{17} \approx 8.2$

$QR = \sqrt{(6 - (-1))^2 + (0 - 5)^2} = \sqrt{7^2 + (-5)^2} = \sqrt{74} \approx 8.6$

$PR = \sqrt{(6 - (-3))^2 + (0 - (-3))^2} = \sqrt{9^2 + 3^2} = \sqrt{90} = 3\sqrt{10} \approx 9.5$

perimeter of $\triangle PQR \approx 8.2 + 8.6 + 9.5 \approx 26.3$

Find the distance between the points. Give your answer in simplest radical form and as a decimal rounded to the nearest hundredth.

1. (8, −5) and (3, 7) ____________

2. (−10, −8) and (−6, −5) ____________

3. (4, 5) and (7, 10) ____________

4. (−2, 8) and (8, −2) ____________

5. (7, 7) and (−3, −2) ____________

6. (12, 5) and (18, 8) ____________

The vertices of $\triangle ABC$ are given. Find the perimeter of $\triangle ABC$ to the nearest tenth.

7. $A(0, 0)$, $B(5, 10)$, and $C(-2, -4)$ ____________

8. $A(5, 8)$, $B(-6, -6)$, and $C(1, -10)$ ____________

9. $A = (-4, 4)$, $B = (3, 7)$ and $C = (8, -2)$

10. $A = (1, 1)$, $B = (5, -12)$, and $C = (6, 4)$

◆ Skill B Using quadrature to estimate the area under a curve

Recall Quadrature is a technique involving approximating the area of a region of a coordinate plane using the sum of the areas of rectangles.

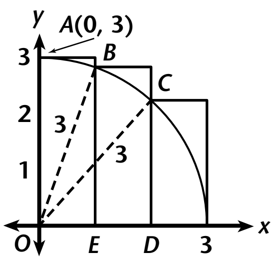
◆ **Example**

Estimate the area of a quarter circle with radius 3 and center $O(0, 0)$.

◆ **Solution**

We will use two different methods and find the average. We will determine BE, CD, AD, and BC by using the Pythagorean Theorem.

1.



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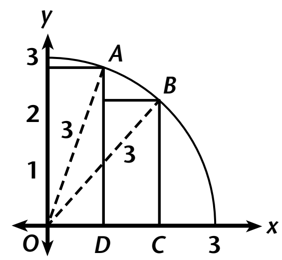
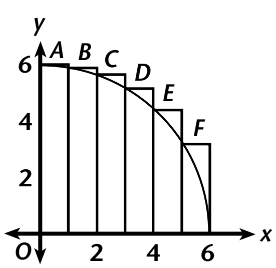


In Figure 1, $\overline{BE}$ is a leg of a right triangle with hypotenuse 3 and base 1, so $BE = \sqrt{3^2 - 1^2} = 2\sqrt{2}$. Similarly, $CD = \sqrt{5}$. In Figure 2, $AD = 2\sqrt{2}$ and $BC = \sqrt{5}$. Then the area from Figure 1 is $3 + 2\sqrt{2} + \sqrt{5}$, or about 8.06. The area from Figure 2 is $2\sqrt{2} + \sqrt{5}$, or about 5.06. The average is about 6.56.

Using the formula for the area of a circle, the area is $\frac{1}{4}\pi \cdot 9 \approx 7.07$, so this is a fairly good estimate.

Use the given rectangles to estimate the area of the region to the nearest hundredth.

11.



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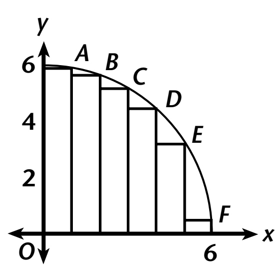

13. Average your estimates from Exercises 11 and 12. Is it a reasonable estimate?

ANSWERS

6. $\frac{56}{\pi}$ inches; 17.8 inches

7. 196π square yards; 616 square yards $\left(\pi \approx \frac{22}{7}\right)$

8. $25\pi - 50.41$ square meters; 28.09 square meters

9. 45π square inches; 141.3 square inches

10. $\sqrt{53}$ kilometers; 7.28 kilometers

11. $\sqrt{\frac{285}{\pi}}$ meters; 9.5 meters

12. $\sqrt{\frac{498}{\pi}}$ inches; 12.6 inches

Lesson 5.4

1. 29 2. 17 3. $\sqrt{39}$ 4. $\sqrt{217}$

5. $10\sqrt{3}$ 6. $4\sqrt{6}$ 7. 186.8 feet 8. yes

9. yes 10. yes 11. no 12. yes

13. yes 14. no 15. yes 16. yes

17. acute 18. obtuse 19. obtuse

20. acute 21. obtuse 22. right

23. acute 24. acute 25. obtuse

Lesson 5.5

1. 13; $13\sqrt{2}$ 2. $4\sqrt{2}$; 4 3. $\frac{9\sqrt{2}}{2}$; 4.5

4. 9; 9 5. $4\sqrt{3}$; 8 6. $6\sqrt{3}$; 9 7. 7; 14

8. $12\sqrt{3}$; $6\sqrt{3}$ 9. 1728 square units

10. 41.6 square inches

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12. $600\sqrt{3}$ square feet

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1. 13 2. 5 3. $\sqrt{34}$; 5.83

4. $10\sqrt{2}$; 14.14 5. $\sqrt{181}$; 13.45

6. $3\sqrt{5}$; 6.71

7–10. Answers may vary due to when rounding is done.

7. 31.3 8. 44.3 9. 31.33 10. 35.46

11. 30.56 12. 24.56

13. 27.56; Sample: Yes; the value calculated using the formula is 28.27.

Lesson 5.7

1. $A(-a, -a)$, $B(-a, a)$, $D(a, -a)$

2. $M(0, 0)$, $N(0, a)$, $P(a, a)$

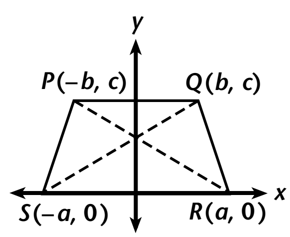
3. $J(-a, 0)$

4. $S(-b, c)$

5. $G(0, 0)$, $E(a - b, c)$

6. $D(0, 0)$, $F(2a, 0)$

7. Sample proof: Let P, Q, R, and S be as shown in the figure.



$PR = \sqrt{(a - (-b))^2 + (0 - c)^2} = \sqrt{(a + b)^2 + c^2}$ and $QS = \sqrt{(b - (-a))^2 + (c - 0)^2} = \sqrt{(a + b)^2 + c^2}$, so $\overline{PR} \cong \overline{QS}$.

NAME ______________________ CLASS ____________ DATE ____________

Reteaching

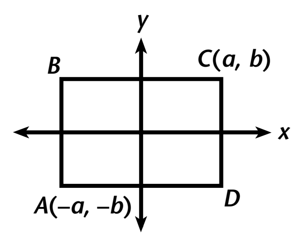
5.7 Proofs Using Coordinate Geometry

◆ **Skill A** Using coordinate geometry to represent geometric properties of figures

Recall Coordinate proofs are particularly convenient for proving that segments are congruent, that two lines intersect or are parallel or perpendicular, and that a line or segment bisects another segment

In writing a coordinate proof, it is helpful to position the axes and assign coordinates in such a way that the calculations are as simple as possible. If a figure is symmetric, position the axes so that one of them is the line of symmetry. If a figure is not symmetric, position the axes so that as many vertices as possible lie on an axis (and have at least one 0 coordinate). When working with midpoints, assign even coordinates such as $2a$ and $2b$. Always be careful not to assign coordinates that give the figure properties it does not actually have.

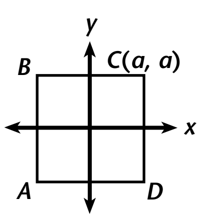
◆ **Example**
ABCD is a rectangle. Give coordinates of the unknown vertices using only the variables shown.



◆ **Solution**
ABCD is a rectangle, so point *B* has the same x-coordinate as point *A* and the same y-coordinate as point *C*, so $B = (-a, b)$. Point *D* has the same x-coordinate as point *C* and the same y-coordinate as point *A*, so $D = (a, -b)$.

If possible, give the coordinates of the unknown vertex or vertices without introducing any new variables. If not, use another variable.

1. square *ABCD*



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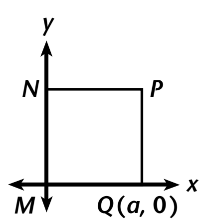
2. square *MNPQ*



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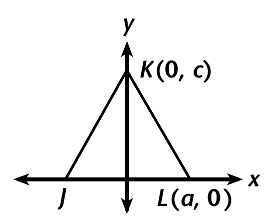
3. isosceles triangle *JKL*



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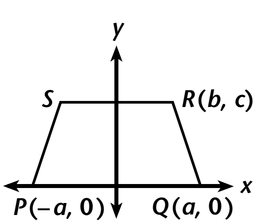
4. isosceles trapezoid *PQRS*



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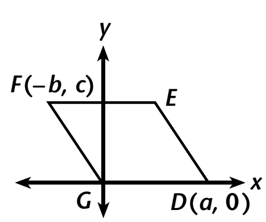
5. parallelogram *DEFG*



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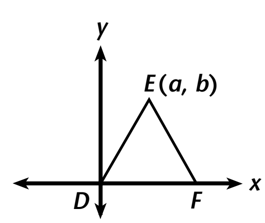
6. equilateral triangle *XYZ*



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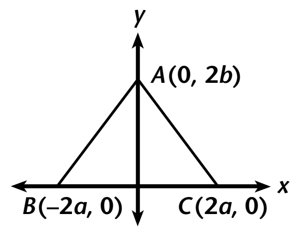
◆ Skill B Using coordinate geometry in proofs

Recall The Distance Formula and the Midpoint Formula are frequently used in coordinate proofs, as are the definition of slope and the equation of a line.

◆ **Example**

Given: isosceles triangle *ABC*; $\overline{AB} \cong \overline{AC}$

Prove: The medians to the legs of $\triangle ABC$ are congruent.



◆ **Solution**

Proof: $\triangle ABC$ is isosceles so let the *y*-axis be an axis of symmetry for $\triangle ABC$. Since we want to determine the coordinates of midpoints, make all coordinates multiples of 2, as shown in the figure.

By the Midpoint Formula, the midpoint, *M*, of $\overline{AB}$ has coordinates $\left(\frac{-2a+0}{2}, \frac{0+2b}{2}\right), = (-a, b)$. Similarly, the midpoint of $\overline{AC}$ is $N(a, b)$.

$MC = \sqrt{(2a-(-a))^2 + (0-b)^2} = \sqrt{9a^2 + b^2}$.

$BN = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{9a^2 + b^2}$.

Make a sketch and write a coordinate proof.

7. The diagonals of an isosceles trapezoid are congruent.
Given: isosceles trapezoid *PQRS*, $\overline{PS} \cong \overline{QR}$
Prove: $\overline{PR} \cong \overline{QS}$

Proof: ____

____

____

8. The segment joining the midpoints of consecutive sides of a rectangle is a rhombus.
Given: rectangle *WXYZ*, with midpoints *M*, *N*, *P*, and *Q*
Prove: *MNPQ* is a rhombus.

Proof: ____

____

____

Menu

Print

ANSWERS

6. $\frac{56}{\pi}$ inches; 17.8 inches

7. 196π square yards; 616 square yards $\left(\pi \approx \frac{22}{7}\right)$

8. $25\pi - 50.41$ square meters; 28.09 square meters

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5. $10\sqrt{3}$ **6.** $4\sqrt{6}$ **7.** 186.8 feet **8.** yes

9. yes **10.** yes **11.** no **12.** yes

13. yes **14.** no **15.** yes **16.** yes

17. acute **18.** obtuse **19.** obtuse

20. acute **21.** obtuse **22.** right

23. acute **24.** acute **25.** obtuse

Lesson 5.5

1. 13; $13\sqrt{2}$ **2.** $4\sqrt{2}$; 4 **3.** $\frac{9\sqrt{2}}{2}$; 4.5

4. 9; 9 **5.** $4\sqrt{3}$; 8 **6.** $6\sqrt{3}$; 9 **7.** 7; 14

8. $12\sqrt{3}$; $6\sqrt{3}$ **9.** 1728 square units

10. 41.6 square inches

11. 25 square meters

12. $600\sqrt{3}$ square feet

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6. $3\sqrt{5}$; 6.71

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3. $J(-a, 0)$

4. $S(-b, c)$

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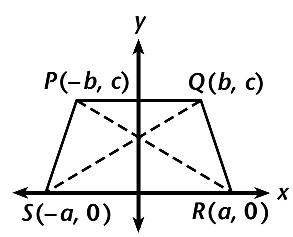
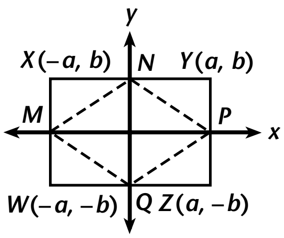
6. $D(0, 0)$, $F(2a, 0)$

7. Sample proof: Let P, Q, R, and S be as shown in the figure.



$PR = \sqrt{(a - (-b))^2 + (0 - c)^2} = \sqrt{(a + b)^2 + c^2}$ and $QS = \sqrt{(b - (-a))^2 + (c - 0)^2} = \sqrt{(a + b)^2 + c^2}$, so $\overline{PR} \cong \overline{QS}$.

8. Sample proof: Let W, X, Y, and Z be as shown.



$MNPQ$ is a parallelogram since the midpoint of $\overline{NQ}$ and MP is (0,0). M, N, P, and Q lie on the axes, so the diagonals, $\overline{MP}$ and $\overline{NQ}$, of $MNPQ$ are perpendicular and $MNPQ$ is a rhombus (Theorem 4.6.8: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.)

Lesson 5.8

1. $\frac{3}{8}$; 0.375; 37.5%

2. $\frac{1}{2}$; 0.5; 50%

3. $\frac{1}{8}$; 0.125; 12.5%

4. $\frac{1}{4}$; 0.25; 25%

5. $\frac{1}{5}$; 0.2; 20%

6. $\frac{1}{9}$; $0.\overline{1}$; $11\frac{1}{9}$%

7. $\frac{1}{6}$; $0.1\overline{6}$; $16\frac{2}{3}$%

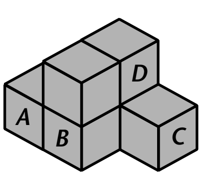
8. $\frac{5}{6}$; $0.8\overline{3}$; $83\frac{1}{3}$%

9. 0.56 **10.** 0.79 **11.** 0.56

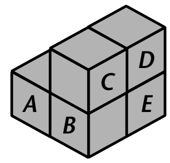
Reteaching — Chapter 6

Lesson 6.1

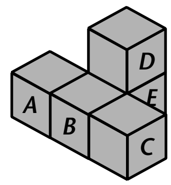
1.



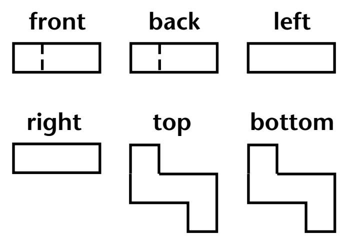
2.



3.



4.



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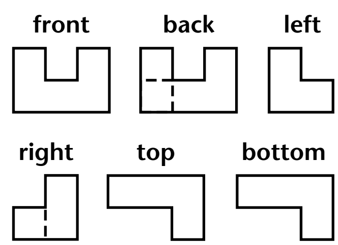


6. 32 sq. units; 8 cu. units

7. 30 sq. units; 8 cu. units

8. 38 sq. units; 11 cu. units

Lesson 6.2

1. $JKLM$, $JMRN$

2. $JKPN$, $MLQR$

3. $\overleftrightarrow{JK}$, $\overleftrightarrow{ML}$, $\overleftrightarrow{NP}$, $\overleftrightarrow{RQ}$

4. $\overleftrightarrow{ML}$, $\overleftrightarrow{RQ}$, $\overleftrightarrow{MR}$, $\overleftrightarrow{LQ}$

5. a. $\overleftrightarrow{FE}$, $\overleftrightarrow{CE}$
b. $\overleftrightarrow{BC}$

6. a. $\overleftrightarrow{BF}$, $\overleftrightarrow{CE}$
b. $\overleftrightarrow{AF}$, $\overleftrightarrow{DE}$

7. $MLQR$ **8.** $JMRN$ **9.** perpendicular

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
5.8 Geometric Probability

◆ **Skill A** Determining the theoretical probability of an event

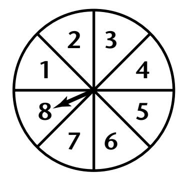
Recall The probability of an event is a number that indicates how likely it is that the event will occur. The probability of an event is always between 0 and 1. If the probability of an event is 0, the event cannot occur. If the probability of an event is 1, the event is sure to happen. A probability can be expressed as a fraction, a decimal, or a percent.

If all possible outcomes are equally likely, the theoretical probability of an event is $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$. A favorable outcome is one for which the event occurs.

◆ **Example**
A bag of 20 marbles contains 7 red marbles, 5 green marbles, and 8 yellow marbles. You remove one marble from the bag without looking. Find the probability that the marble is yellow.

◆ **Solution**
All of the twenty outcomes are equally likely, and eight are favorable. Then $P = \frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$.

The spinner at the right is spun once. Find the probability that the spinner lands on the number indicated. Give each probability as a fraction in lowest terms, as a decimal, and as a percent.



1. a number greater than 5 ____________

2. an even number ____________

3. a number less than 2 ____________

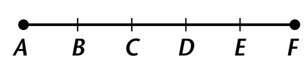
◆ **Skill B** Determining the geometric probability of an event

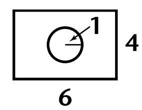
Recall Geometric probabilities are based on length or on area.

◆ **Example**

a. A point on $\overline{AF}$ is chosen at random. Find the probability, P, that the point is on $\overline{AC}$.

b. A point in rectangle $ABCD$ is chosen at random. Find the probability, P, that the point is inside the circle.





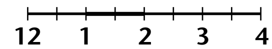
◆ **Solution**

a. $P = \frac{\text{length of } \overline{AC}}{\text{length of } \overline{AF}} = \frac{2}{5} = 0.4 = 40\%$

b. $P = \frac{\text{area of circle}}{\text{area of rectangle}} = \frac{\pi}{24}$ $0.13 = 13\%$

In Exercises 4–5, use the given segment to find the indicated probability.

4. You are expecting a delivery of a package between 12 noon and 4:00 P.M. You are called away at 1:00 P.M. and return half an hour later. Find the probability that you miss the delivery.



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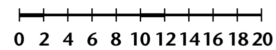
5. A bus arrives at a stop every 10 minutes and waits 2 minutes before leaving. Find the probability that if you arrive at a random time, there is a bus waiting.



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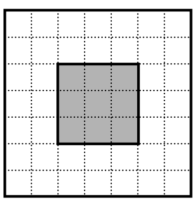
In Exercises 6–8, find the probability that a point chosen at random in the figure is inside the shaded region.

6. 

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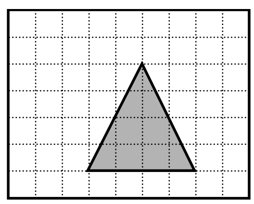
7. 

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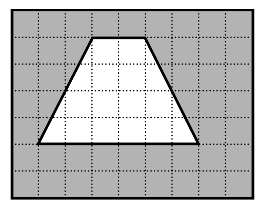
8. 

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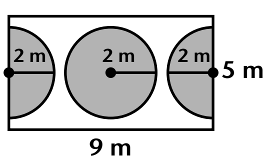
In Exercises 9–11, a dart is tossed at random onto the board. Find the probability that a dart that hits the board hits the shaded region. Give your answer as a decimal rounded to the nearest hundredth. (Use $\pi \approx 3.14$.)

9. 

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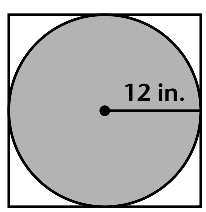
10. 

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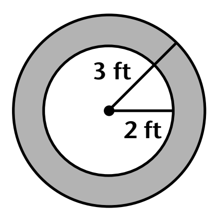
11. 

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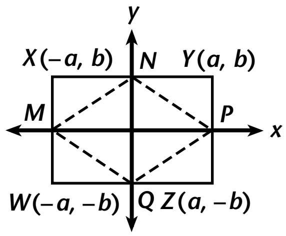
8. Sample proof: Let W, X, Y, and Z be as shown.



$MNPQ$ is a parallelogram since the midpoint of $\overline{NQ}$ and MP is (0,0). M, N, P, and Q lie on the axes, so the diagonals, $\overline{MP}$ and $\overline{NQ}$, of $MNPQ$ are perpendicular and $MNPQ$ is a rhombus (Theorem 4.6.8: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.)

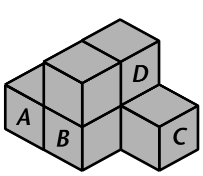
Lesson 5.8

1. $\frac{3}{8}$; 0.375; 37.5%
2. $\frac{1}{2}$; 0.5; 50%
3. $\frac{1}{8}$; 0.125; 12.5%
4. $\frac{1}{4}$; 0.25; 25%
5. $\frac{1}{5}$; 0.2; 20%
6. $\frac{1}{9}$; $0.\overline{1}$; $11\frac{1}{9}$%
7. $\frac{1}{6}$; $0.1\overline{6}$; $16\frac{2}{3}$%
8. $\frac{5}{6}$; $0.8\overline{3}$; $83\frac{1}{3}$%
9. 0.56 **10.** 0.79 **11.** 0.56

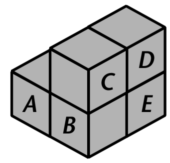
Reteaching — Chapter 6

Lesson 6.1

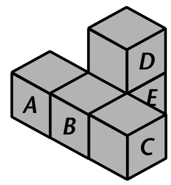
1.



2.



3.



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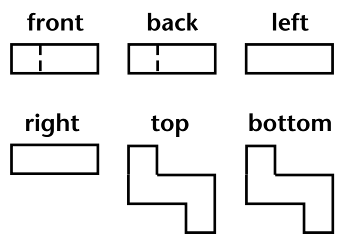
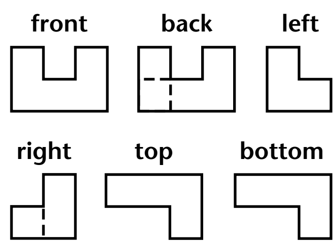


5.



6. 32 sq. units; 8 cu. units
7. 30 sq. units; 8 cu. units
8. 38 sq. units; 11 cu. units

Lesson 6.2

1. $JKLM$, $JMRN$
2. $JKPN$, $MLQR$
3. $\overleftrightarrow{JK}$, $\overleftrightarrow{ML}$, $\overleftrightarrow{NP}$, $\overleftrightarrow{RQ}$
4. $\overleftrightarrow{ML}$, $\overleftrightarrow{RQ}$, $\overleftrightarrow{MR}$, $\overleftrightarrow{LQ}$
5. **a.** $\overleftrightarrow{FE}$, $\overleftrightarrow{CE}$
   b. $\overleftrightarrow{BC}$
6. **a.** $\overleftrightarrow{BF}$, $\overleftrightarrow{CE}$
   b. $\overleftrightarrow{AF}$, $\overleftrightarrow{DE}$
7. $MLQR$ **8.** $JMRN$ **9.** perpendicular

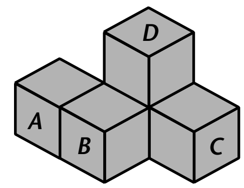
NAME ________________________ CLASS ____________ DATE ____________

Reteaching
6.1 Solid Shapes

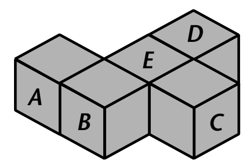
◆ **Skill A** Using isometric dot paper to draw three-dimensional figures

Recall All segments drawn on isometric dot paper by connecting two consecutive dots on a vertical or a diagonal line are the same length, or *isometric*.

◆ **Example**
The figure consists of five unit cubes. Cube E, which is not visible, is underneath cube D. Redraw the figure, placing cube D next to E, so that cubes B, D, and E are in a row.

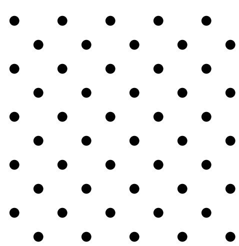


◆ **Solution**
Copy the figure and make erasures to remove cube D. Redraw lines to restore cube E. Then draw the visible edges of cube D in its new position.

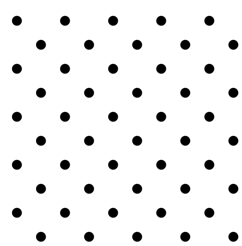


Refer to the top figure in the example. Redraw the figure with the indicated change.

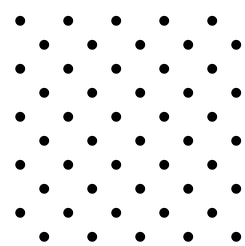
1. Add a cube on top of B.



2. Place C on top of B.



3. Place C in front of B.



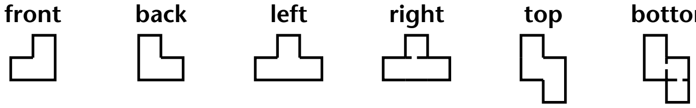
◆ **Skill B** Drawing orthographic projections

Recall An isometric drawing appears to be three-dimensional. Orthographic projection, however, produces an image in which a three-dimensional figure appears to be two-dimensional.

◆ **Example**
In the top figure in the example above, let the face lettered C be the front and the faces lettered A and B be the left side. Draw orthographic projections showing the front, the back, the left, the right, the top, and the bottom.

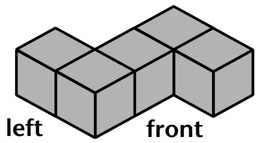
◆ **Solution**

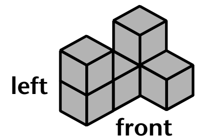
front back left right top bottom



CLASS ______________ DATE ______________

Draw six orthographic projections of each figure. The front and left faces of each figure are indicated.

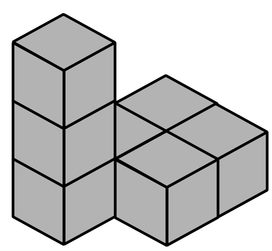
4. 

5. 

◆ **Skill C** Determining surface area and volume using unit cubes

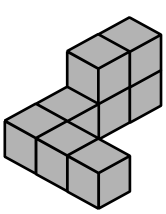
Recall Each face of a unit cube has area 1 square unit. The volume of a unit cube is 1 cubic unit. To determine the surface area of a figure formed from unit cubes, count all exposed cube faces, that is, faces not covered by another cube. To determine the volume, count the number of unit cubes in the figure.

◆ **Example**
Determine the surface area and volume of the figure.



◆ **Solution**
One method of determining surface is area is to consider each unit cube and determine how many of its faces are exposed. For example, the topmost cube has five exposed faces, while the cube at the right front corner has four . The total number of exposed faces is 28, so the surface area of the figure is 28 square units. There are 7 cubes so the volume is 7 cubic units.

Determine the surface area and volume of the figure.

6. 

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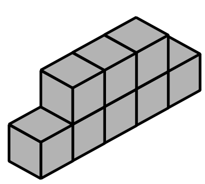
7. 

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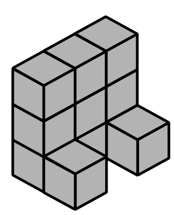
8. 

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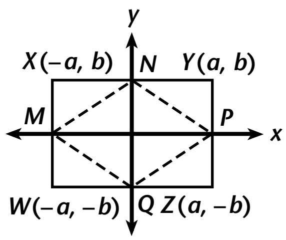
8. Sample proof: Let *W*, *X*, *Y*, and *Z* be as shown.



MNPQ is a parallelogram since the midpoint of $\overline{NQ}$ and $\overline{MP}$ is (0,0). *M*, *N*, *P*, and *Q* lie on the axes, so the diagonals, $\overline{MP}$ and $\overline{NQ}$, of *MNPQ* are perpendicular and *MNPQ* is a rhombus (Theorem 4.6.8: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.)

Lesson 5.8

1. $\frac{3}{8}$; 0.375; 37.5%

2. $\frac{1}{2}$; 0.5; 50%

3. $\frac{1}{8}$; 0.125; 12.5%

4. $\frac{1}{4}$; 0.25; 25%

5. $\frac{1}{5}$; 0.2; 20%

6. $\frac{1}{9}$; $0.\overline{1}$; $11\frac{1}{9}$%

7. $\frac{1}{6}$; $0.1\overline{6}$; $16\frac{2}{3}$%

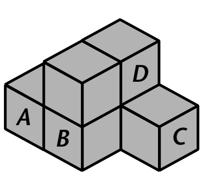
8. $\frac{5}{6}$; $0.8\overline{3}$; $83\frac{1}{3}$%

9. 0.56 **10.** 0.79 **11.** 0.56

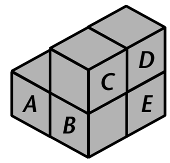
Reteaching — Chapter 6

Lesson 6.1

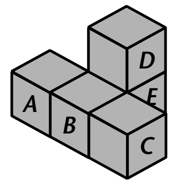
1.



2.



3.



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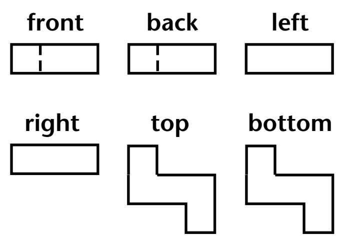

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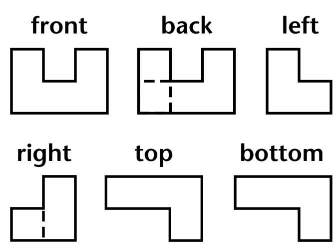

6. 32 sq. units; 8 cu. units

7. 30 sq. units; 8 cu. units

8. 38 sq. units; 11 cu. units

Lesson 6.2

1. *JKLM*, *JMRN*

2. *JKPN*, *MLQR*

3. $\overleftrightarrow{JK}$, $\overleftrightarrow{ML}$, $\overleftrightarrow{NP}$, $\overleftrightarrow{RQ}$

4. $\overleftrightarrow{ML}$, $\overleftrightarrow{RQ}$, $\overleftrightarrow{MR}$, $\overleftrightarrow{LQ}$

5. a. $\overleftrightarrow{FE}$, $\overleftrightarrow{CE}$
b. $\overleftrightarrow{BC}$

6. a. $\overleftrightarrow{BF}$, $\overleftrightarrow{CE}$
b. $\overleftrightarrow{AF}$, $\overleftrightarrow{DE}$

7. *MLQR* **8.** *JMRN* **9.** perpendicular

Menu

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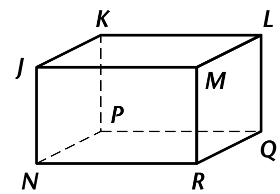
NAME ______________________ CLASS ____________ DATE ____________

Reteaching

6.2 Spatial Relationships

◆ **Skill A** Identifying relationships between lines and planes in space

Recall The space figure shown is a called a rectangular solid, since each face is rectangular. Such a figure can be used to illustrate relationships between lines and planes in space.



A line is perpendicular to a plane at point P if the line is perpendicular to every line in the plane that passes through P.
A line is parallel to a plane if the line and the plane do not intersect.

◆ **Example**
Refer to the figure above. Name each of the following.

a. a plane perpendicular to $\overleftrightarrow{JK}$

b. a plane parallel to $\overleftrightarrow{JK}$

◆ **Solution**

a. Plane *JMRN* is perpendicular to $\overleftrightarrow{JK}$ at J and plane *KLQP* is perpendicular to $\overleftrightarrow{JK}$ at K.

b. Planes *NPQR* and *MLQR* are both parallel to $\overleftrightarrow{JK}$.

Refer to the figure in the example above. Name each of the following.

1. all planes shown that are parallel to $\overleftrightarrow{PQ}$ ______________________

2. all planes shown that are perpendicular to $\overleftrightarrow{KL}$ ______________________

3. all lines shown that are perpendicular to plane *KLQP* ______________________

4. all lines shown that are parallel to plane *JKPN* ______________________

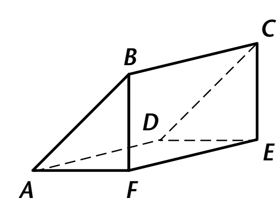
Exercises 5 and 6 refer to the figure at the right. Two surfaces, *ABC* and *DCE*, of the space figure are right triangles. Name all lines shown that are:

5. a. parallel to plane *ABCD* ______________________

b. parallel to plane *ADEF* ______________________

6. a. perpendicular to plane *ADEF* ______________________

b. perpendicular to plane *BCEF* ______________________



CLASS ______________ DATE ______________

◆ Skill B Identifying relationships between planes

Recall Two lines in a plane may be parallel or may intersect. The same is true of two planes in space. Two planes that do not intersect are parallel. Two planes that intersect form *dihedral angles*.

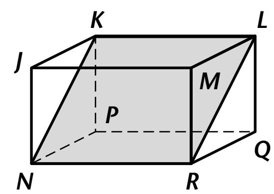
In a plane, the sides of an angle are rays and their intersection is the vertex of the angle. In space, the sides of a dihedral angle are half-planes, called the *faces* of the angle, and their intersection is a line, called the *edge* of the angle.

The measure of a dihedral angle is the measure of an angle formed by rays on the faces that are perpendicular to the edge.

◆ **Example**

a. Name a plane parallel to plane *JKLM*.

b. Identify an angle whose measure you could use to determine the measure of the dihedral angle formed by the intersection of planes *JKLM* and *MLQR*. *Then* give the measure of the dihedral angle.



◆ **Solution**

a. Plane *NPQR* is parallel to plane *JKLM*.

b. $\angle JMR$ or $\angle KLQ$; 90°

Refer to the figure above. Complete each statement.

7. Plane ______________ is parallel to plane *JKPN*.

8. Plane *KLQP* is parallel to plane ______________.

9. Since $m\angle JNP = 90°$, planes *JKPN* and *NPQR* are ______________.

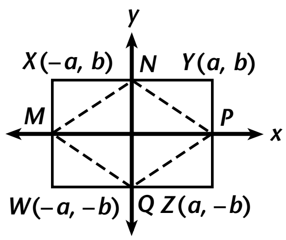
10. Planes ______________, ______________, ______________,

and ______________ are perpendicular to plane *MLQR*.

11. The measure of the dihedral angle formed by the intersection of planes *KLRN* and *NPQR* is equal to m∠ ______________ or m∠ ______________.

12. If *JKPN* and *MLQR* are squares, then the measure of the dihedral angle formed by the intersection of planes *KLRN* and *NPQR* is ______________.

8. Sample proof: Let *W*, *X*, *Y*, and *Z* be as shown.



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Lesson 5.8

1. $\frac{3}{8}$; 0.375; 37.5%

2. $\frac{1}{2}$; 0.5; 50%

3. $\frac{1}{8}$; 0.125; 12.5%

4. $\frac{1}{4}$; 0.25; 25%

5. $\frac{1}{5}$; 0.2; 20%

6. $\frac{1}{9}$; $0.\overline{1}$; $11\frac{1}{9}$%

7. $\frac{1}{6}$; $0.1\overline{6}$; $16\frac{2}{3}$%

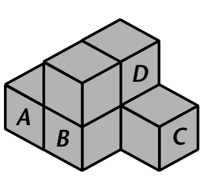
8. $\frac{5}{6}$; $0.8\overline{3}$; $83\frac{1}{3}$%

9. 0.56 **10.** 0.79 **11.** 0.56

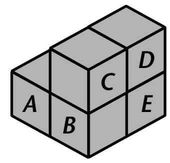
Reteaching — Chapter 6

Lesson 6.1

1.



2.



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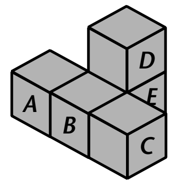

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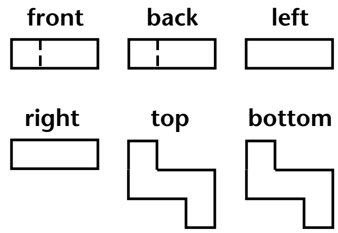

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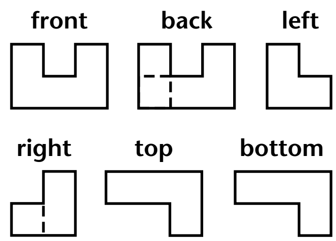


6. 32 sq. units; 8 cu. units

7. 30 sq. units; 8 cu. units

8. 38 sq. units; 11 cu. units

Lesson 6.2

1. *JKLM*, *JMRN*

2. *JKPN*, *MLQR*

3. $\overleftrightarrow{JK}$, $\overleftrightarrow{ML}$, $\overleftrightarrow{NP}$, $\overleftrightarrow{RQ}$

4. $\overleftrightarrow{ML}$, $\overleftrightarrow{RQ}$, $\overleftrightarrow{MR}$, $\overleftrightarrow{LQ}$

5. a. $\overleftrightarrow{FE}$, $\overleftrightarrow{CE}$
b. $\overleftrightarrow{BC}$

6. a. $\overleftrightarrow{BF}$, $\overleftrightarrow{CE}$
b. $\overleftrightarrow{AF}$, $\overleftrightarrow{DE}$

7. *MLQR* **8.** *JMRN* **9.** perpendicular

ANSWERS

10. *JKLM*; *JMRN*; *NPQR*; *KLQP*

11. *LRQ*; *KNP** **12.** 45°

Lesson 6.3

1. No; the figure does not have two parallel faces and only one face is a parallelogram.

2. Yes; right rectangular prism.

3. Yes; oblique pentagonal prism.

4. right trapezoidal prism

5. *ABCD* and *EFGH*

6. $\overline{CD}$, $\overline{EF}$, $\overline{GH}$

7. $\overline{AE}$, $\overline{BF}$, $\overline{CG}$

8. *ABFE*, *DCGH*

9. angles *BAE*, *AEF*, *ABF*, *BFE*, *CBF*, *BFG*, *BCG*, *CGF*, *DCG*, *CGH*, *GHD*, *CDH*, *DAE*, *AEH*, *EHD*, *ADH*

10. $\sqrt{122}$; 11.05 **11.** $2\sqrt{6}$; 4.90

12. $\sqrt{62}$; 7.87 **13.** $\sqrt{22}$; 4.69

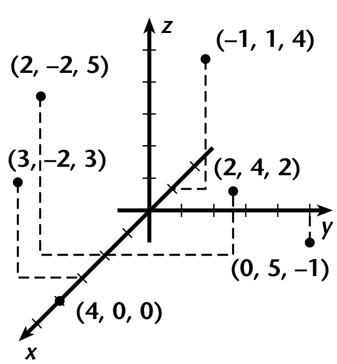
14. $3\sqrt{38}$; 18.49 **15.** $2\sqrt{70}$; 16.73

16. $2\sqrt{57}$; 15.10 **17.** $6\sqrt{5}$; 13.42

18. 8 **19.** $\sqrt{55}$; 7.42

Lesson 6.4

1–6.



7. y-axis

8. back-right-bottom

9. front-right-bottom

10. (2, 0, 2) **11.** (0, 0, 2) **12.** (2, 4, 2)

13. (0, 4, 0) **14** (2, 4, 0) **15.** $5\sqrt{2}$; 7.07

16. $\sqrt{2}$; 1.41 **17.** $\sqrt{66}$; 8.12

18. $\sqrt{185}$; 13.60 **19.** 15

20. 2 **21.** $2\sqrt{5}$; 4.47 **22.** $2\sqrt{2}$; 2.83

23. $2\sqrt{6}$; 4.90

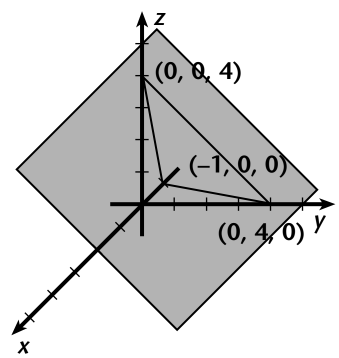
Lesson 6.5

1. (5, 0, 0); (0, −7.5, 0); (0, 0, 3)

2. (8, 0, 0); (0, 2, 0); (0, 0, −4)

3. (3, 0, 0); (0, −6, 0); (0, 0, −9)

4.



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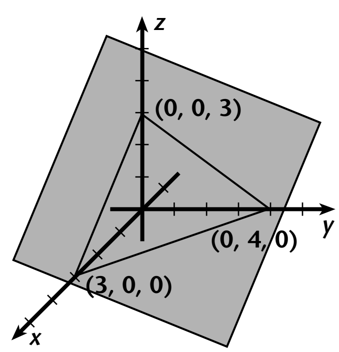


Menu

Print

Reteaching
6.3 Prisms

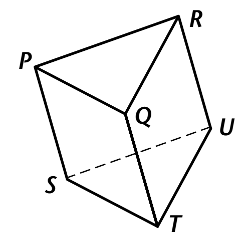
◆ Skill A Identifying prisms and properties of their bases, faces, and edges

Recall The *bases* of a prism are congruent polygons that lie in parallel planes. A prism is named for the shape of its bases. For example, if the bases are pentagons, the prism is called a *pentagonal prism*. The faces that are not bases are called *lateral faces*. The segments joining corresponding vertices in the bases are called *lateral edges*.

All lateral faces of a prism are parallelograms and all lateral edges are congruent. If the lateral faces are rectangles, the prism is called a *right prism*. If the faces are not rectangles, the prism is *oblique*.

◆ Example

The figure at the right is an oblique triangular prism. $\triangle PQR$ and $\triangle STU$ are equilateral triangles.



a. Name the bases of the prism.

b. Name all segments congruent to $\overline{PQ}$.

c. Name all segments congruent to $\overline{PS}$.

d. Are any of the lateral faces congruent?

e. Are any angles of the prism right angles?

◆ Solution

a. $\triangle PQR$ and $\triangle STU$

b. $\triangle PQR$ and $\triangle STU$ are congruent equilateral triangles so, $\overline{QR}$, $\overline{PR}$, $\overline{ST}$, $\overline{TU}$, and $\overline{SU}$ are all congruent to $\overline{PQ}$.

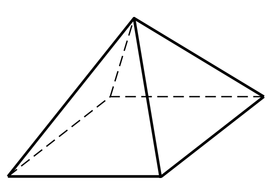
c. All lateral edges of a prism are congruent, so $\overline{QT}$ and $\overline{RU}$ are congruent to $\overline{PS}$.

d. Yes; because the bases are equilateral triangles, all three lateral faces are congruent.

e. No; the angles of the bases are acute angles. Since the lateral faces are parallelograms that are not rectangles, none of their angles are right angles.

Tell whether the figure appears to be a prism. If so, name the type of prism. If not, tell why not.

1.



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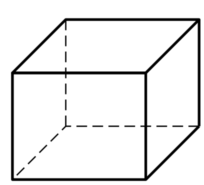
2.



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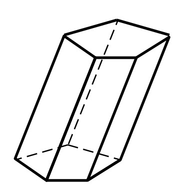
3.



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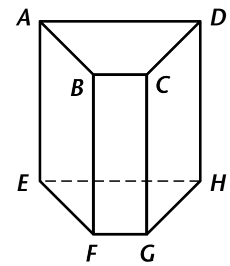
CLASS ______________ DATE ______________

The figure at the right is a right prism. The bases are isosceles trapezoids.



4. What type of prism is the figure? ______________________________

5. Name the bases of the prism. ______________________________

6. Name all segments congruent to $\overline{AB}$. ______________________________

7. Name all segments congruent to $\overline{DH}$. ______________________________

8. Name any congruent lateral faces. ______________________________

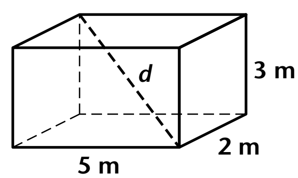
9. Name any right angles in the prism. ______________________________

◆ **Skill B** Using the formula for the length of a diagonal of a right rectangular prism

Recall A diagonal of a polyhedron is a segment that joins two points that are vertices of different faces of the polyhedron. The Pythagorean Theorem can be used to derive a formula for the length of a diagonal of a right rectangular prism.

If a right rectangular prism has length ℓ, width w, and height h, then the length, d, of a diagonal is given by $d = \sqrt{\ell^2 + w^2 + h^2}$.

◆ **Example**
Find the length of a diagonal of the right rectangular prism.



◆ **Solution**

$d = \sqrt{\ell^2 + w^2 + h^2} = \sqrt{5^2 + 2^3 + 3^2} = \sqrt{38} \approx 6.16$ m

Find the length of a diagonal of a right rectangular prism with the given dimensions. Give your answer as a radical in simplest form and as a decimal rounded to the nearest hundredth.

10. $\ell = 9$, $w = 4$, $h = 5$ ______________

11. $\ell = 2$, $w = 2$, $h = 4$ ______________

12. $\ell = 3$, $w = 7$, $h = 2$ ______________

13. $\ell = 3$, $w = 2$, $h = 3$ ______________

14. $\ell = 15$, $w = 6$, $h = 9$ ______________

15. $\ell = 6$, $w = 12$, $h = 10$ ______________

16. $\ell = 8$, $w = 10$, $h = 8$ ______________

17. $\ell = 10$, $w = 8$, $h = 4$ ______________

Find the missing dimension of the right rectangular prism. Give your answer as a radical in simplest form and as a decimal rounded to the nearest hundredth.

18. $d = 17$, $w = 9$, $\ell = 12$, $h =$ ______________

19. $d = 12$, $w = 8$, $h = 5$, $\ell =$ ______________

ANSWERS

10. *JKLM*; *JMRN*; *NPQR*; *KLQP*

11. *LRQ*; *KNP** **12.** 45°

Lesson 6.3

1. No; the figure does not have two parallel faces and only one face is a parallelogram.

2. Yes; right rectangular prism.

3. Yes; oblique pentagonal prism.

4. right trapezoidal prism

5. *ABCD* and *EFGH*

6. $\overline{CD}$, $\overline{EF}$, $\overline{GH}$

7. $\overline{AE}$, $\overline{BF}$, $\overline{CG}$

8. *ABFE*, *DCGH*

9. angles *BAE*, *AEF*, *ABF*, *BFE*, *CBF*, *BFG*, *BCG*, *CGF*, *DCG*, *CGH*, *GHD*, *CDH*, *DAE*, *AEH*, *EHD*, *ADH*

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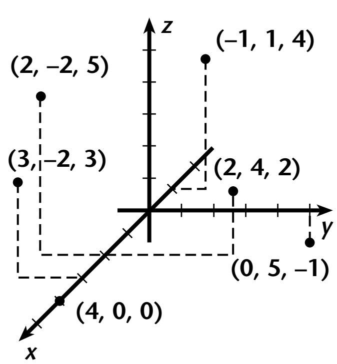
18. 8 **19.** $\sqrt{55}$; 7.42

Lesson 6.4

1–6.



7. *y*-axis

8. back-right-bottom

9. front-right-bottom

10. (2, 0, 2) **11.** (0, 0, 2) **12.** (2, 4, 2)

13. (0, 4, 0) **14** (2, 4, 0) **15.** $5\sqrt{2}$; 7.07

16. $\sqrt{2}$; 1.41 **17.** $\sqrt{66}$; 8.12

18. $\sqrt{185}$; 13.60 **19.** 15

20. 2 **21.** $2\sqrt{5}$; 4.47 **22.** $2\sqrt{2}$; 2.83

23. $2\sqrt{6}$; 4.90

Lesson 6.5

1. (5, 0, 0); (0, −7.5, 0); (0, 0, 3)

2. (8, 0, 0); (0, 2, 0); (0, 0, −4)

3. (3, 0, 0); (0, −6, 0); (0, 0, −9)

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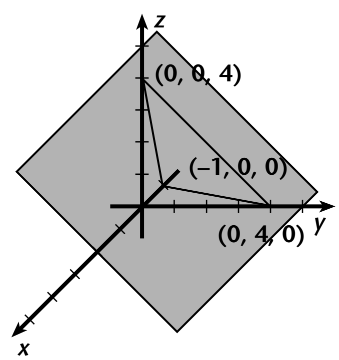

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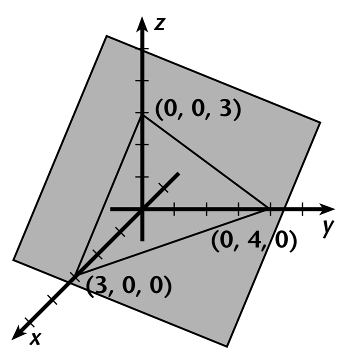

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Reteaching
6.4 Coordinates in Three Dimensions

◆ Skill A Locating points in three dimensions

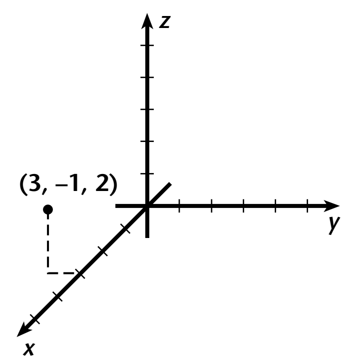
Recall The position of a point in a plane is determined by an ordered pair (x, y). The position of a point is space is determined by an ordered triple (x, y, z)

Each pair of axes determine a plane and the three planes (the *xy*-plane, the *yz*-plane, and the *xz*-plane) divide space into eight octants. The first octant is the octant in which all coordinates of every point are positive. The phrases *front or back*, *left or right*, and *top or bottom* describe the position of an octant with respect to the *yz*-plane, the *xz*-plane, and the *xy*-plane, respectively. The first octant could also be referred to as the front-right-top octant.

◆ Example
Plot the point $(3, -1, 2)$. Determine the octant in which the point is located.

◆ Solution
Begin at the origin and move three units in the positive direction (forward) on the *x*-axis. Mark the *x*-axis at that point.

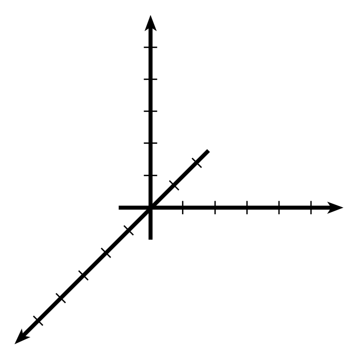
Move one unit in the negative direction (to the left) parallel to the *y*-axis. Draw a line showing the path. Finally, move two units in the positive direction (up) parallel to the *z*-axis. Again, draw a line representing your path. Label the point where you finish $(3, -1, 2)$. Since the point is in front of the *yz*-plane,to the left of the *xz*-plane, and above the *xy*-plane , it is in the front-left-top octant.



Plot each point on the coordinate axes provided.

1. $(4, 0, 0)$ ______ **2.** $(2, 4, 2)$ ______ **3.** $(-1, 1, 4)$ ______

4. $(3, -2, 3)$ ______ **5.** $(0, 5, -1)$ ______ **6.** $(2, -2, 5)$ ______



Determine the octant in which the point is located.
If the point is on an axis, identify the axis.

7. $(0, 12, 0)$ ____

8. $(-2, 2, -2)$ ____

9. $(3, 4, -5)$ ____

CLASS ______________ DATE ______________

Use the given coordinates of the vertices of a right rectangular solid to find the coordinates of the following vertices:

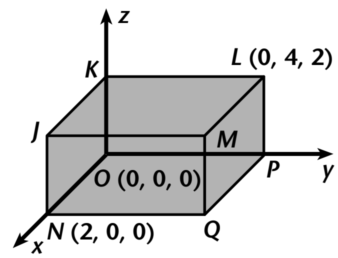
10. J ____________________

11. K ____________________

12. M ____________________

13. P ____________________

14. Q ____________________



◆ **Skill B** Using the distance formula for three dimensions

Recall The distance between two points in space, (x_1, y_1, z_1) and (x_2, y_2, z_2), is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

◆ **Example**
Find the distance between the points (4, 3, 2) and (5, 0, 1).

◆ **Solution**

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(5 - 4)^2 + (0 - 3)^2 + (1 - 2)^2} \\ &= \sqrt{1 + 9 + 1} \\ &= \sqrt{11} \\ &\approx 3.32 \end{aligned}$$

Find the distance between the given points. Give your answer in simplest radical form and as a decimal rounded to the nearest hundredth.

15. (3, 2, 0) and (0, 7, 4) ____________________

16. (2, 4, 2) and (3, 5, 2) ____________________

17. (−3, −5, 2) and (4, −1, 1) ____________________

18. (5, 5, 5) and (−2, −1, −5) ____________________

19. (8, −10, 2) and (−2, 0, 7) ____________________

In the figure above, give the length of each segment in simplest radical form and as a decimal rounded to the nearest hundredth.

20. $\overline{JK}$ ____________________ **21.** $\overline{KM}$ ____________________

22. $\overline{MP}$ ____________________ **23.** $\overline{KQ}$ ____________________

ANSWERS

10. *JKLM*; *JMRN*; *NPQR*; *KLQP*

11. *LRQ*; *KNP* **12.** 45°

Lesson 6.3

1. No; the figure does not have two parallel faces and only one face is a parallelogram.

2. Yes; right rectangular prism.

3. Yes; oblique pentagonal prism.

4. right trapezoidal prism

5. *ABCD* and *EFGH*

6. $\overline{CD}$, $\overline{EF}$, $\overline{GH}$

7. $\overline{AE}$, $\overline{BF}$, $\overline{CG}$

8. *ABFE*, *DCGH*

9. angles *BAE*, *AEF*, *ABF*, *BFE*, *CBF*, *BFG*, *BCG*, *CGF*, *DCG*, *CGH*, *GHD*, *CDH*, *DAE*, *AEH*, *EHD*, *ADH*

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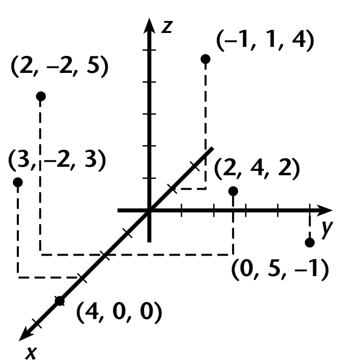
18. 8 **19.** $\sqrt{55}$; 7.42

Lesson 6.4

1–6.



7. y-axis

8. back-right-bottom

9. front-right-bottom

10. (2, 0, 2) **11.** (0, 0, 2) **12.** (2, 4, 2)

13. (0, 4, 0) **14** (2, 4, 0) **15.** $5\sqrt{2}$; 7.07

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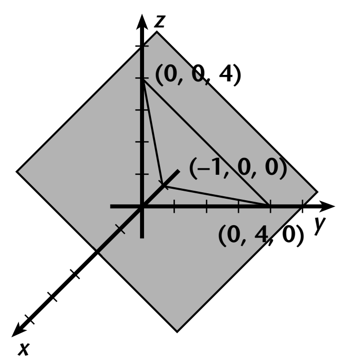
Lesson 6.5

1. (5, 0, 0); (0, −7.5, 0); (0, 0, 3)

2. (8, 0, 0); (0, 2, 0); (0, 0, −4)

3. (3, 0, 0); (0, −6, 0); (0, 0, −9)

4.



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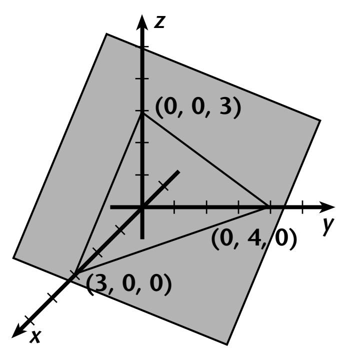


NAME ______________________ CLASS __________ DATE __________

Reteaching

6.5 Lines and Planes in Space

◆ Skill A Sketching planes in space

Recall An equation of a plane can be written in the form $Ax + By + Cz = D$, where A, B, C, and D are real numbers that are not all zero, and A is nonnegative. Just as in a plane, you can graph an equation by locating its intercepts. To determine the x-intercept, set y and z equal to 0 and solve the equation for x. Then plot the point $(x, 0, 0)$. Use a similar method to plot the points $(0, y, 0)$ and $(0, 0, z)$. Draw the triangle determined by the points and shade the plane.

◆ Example

Sketch the plane with equation $6x + 4y - 3z = -12$.

◆ Solution

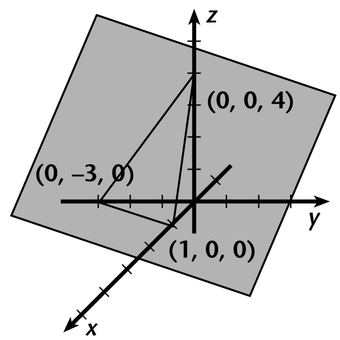
Determine the intercepts.

$6x + 4(0) - 3(0) = -12;\ x = -2$

$6(0) + 4y - 3(0) = -12;\ y = -3$

$6(0) + 4(0) - 3z = -12;\ z = 4$

Plot the points $(-2, 0, 0)$, $(0, -3, 0)$, and $(0, 0, 4)$. Sketch the triangle determined by the points and shade the plane.



Determine the *x*-, *y*-, and *z*-intercepts of the graph of the equation.

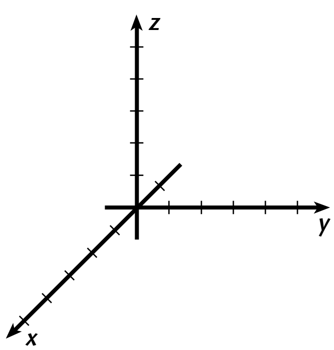
1. $3x - 2y + 5z = 15$ ____________________

2. $x + 4y - 2z = 8$ ____________________

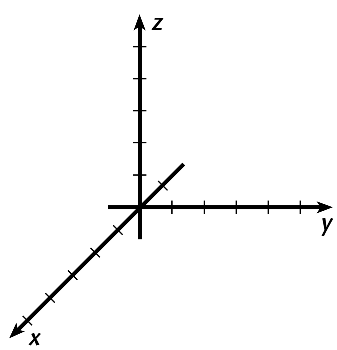
3. $6x - 3y - 2z = 18$ ____________________

Sketch the plane with the given equation.

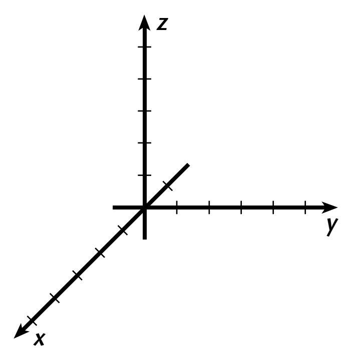
4. $4x - y - z = -4$



5. $4x + 3y + 4z = 12$



6. $x + y + z = 3$



◆ Skill A Sketching lines in space

Recall It may seem that since $Ax + By = C$ is an equation of a line in a plane, that $Ax + By + Cz = D$ should be an equation of a line in space. However, you have seen that the graph of such an equation is a plane, not a line. Then what does the equation of a line in space look like? In your study of algebra, you learned about *parametric equations*, equations that express the variables in terms of another variable, called the *parameter*. Parametric equations provide one way to describe an equation of a line in space.

◆ **Example**

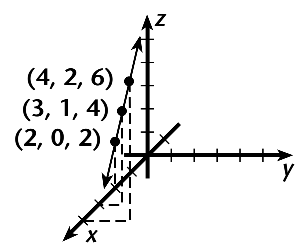
Graph the line with the given parametric equations.

$x = t + 1;\ y = t - 1;\ z = 2t$

◆ **Solution**

Find the values of x, y, and z for given values of t. In space, as in a plane, two points determine a line, so we will find coordinates of three points, using the third as a check. We will make a table, letting $t = 1$, 2, and 3, and find corresponding values of x, y, and z.

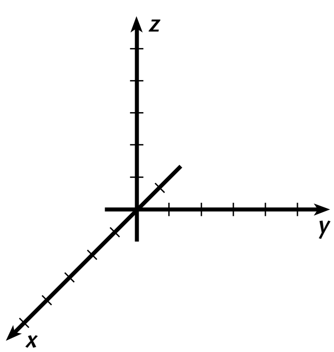
t	x	y	z
1	2	0	2
2	3	1	4
3	4	2	6



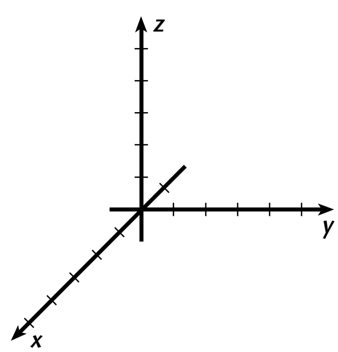
The resulting ordered triples are (2, 0, 2), (3, 1, 4), and (4, 2, 6).

Graph the line with the given parametric equations. Make a table if needed.

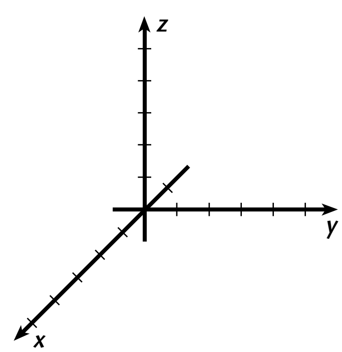
7. $x = t - 2$
$y = t + 1$
$z = 3$



8. $x = t$
$y = 2t - 3$
$z = -t + 3$



9. $x = 2t - 1$
$y = t + 2$
$z = 5 - t$



ANSWERS

10. *JKLM*; *JMRN*; *NPQR*; *KLQP*

11. *LRQ*; *KNP* **12.** 45°

Lesson 6.3

1. No; the figure does not have two parallel faces and only one face is a parallelogram.

2. Yes; right rectangular prism.

3. Yes; oblique pentagonal prism.

4. right trapezoidal prism

5. *ABCD* and *EFGH*

6. $\overline{CD}$, $\overline{EF}$, $\overline{GH}$

7. $\overline{AE}$, $\overline{BF}$, $\overline{CG}$

8. *ABFE*, *DCGH*

9. angles *BAE*, *AEF*, *ABF*, *BFE*, *CBF*, *BFG*, *BCG*, *CGF*, *DCG*, *CGH*, *GHD*, *CDH*, *DAE*, *AEH*, *EHD*, *ADH*

10. $\sqrt{122}$; 11.05 **11.** $2\sqrt{6}$; 4.90

12. $\sqrt{62}$; 7.87 **13.** $\sqrt{22}$; 4.69

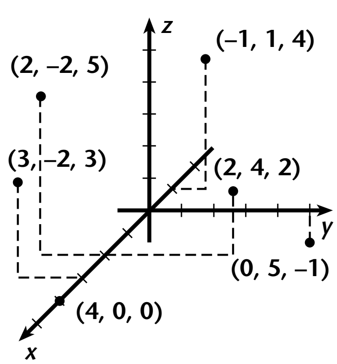
14. $3\sqrt{38}$; 18.49 **15.** $2\sqrt{70}$; 16.73

16. $2\sqrt{57}$; 15.10 **17.** $6\sqrt{5}$; 13.42

18. 8 **19.** $\sqrt{55}$; 7.42

Lesson 6.4

1–6.



7. *y*-axis

8. back-right-bottom

9. front-right-bottom

10. (2, 0, 2) **11.** (0, 0, 2) **12.** (2, 4, 2)

13. (0, 4, 0) **14** (2, 4, 0) **15.** $5\sqrt{2}$; 7.07

16. $\sqrt{2}$; 1.41 **17.** $\sqrt{66}$; 8.12

18. $\sqrt{185}$; 13.60 **19.** 15

20. 2 **21.** $2\sqrt{5}$; 4.47 **22.** $2\sqrt{2}$; 2.83

23. $2\sqrt{6}$; 4.90

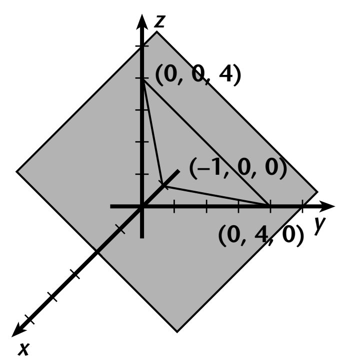
Lesson 6.5

1. (5, 0, 0); (0, −7.5, 0); (0, 0, 3)

2. (8, 0, 0); (0, 2, 0); (0, 0, −4)

3. (3, 0, 0); (0, −6, 0); (0, 0, −9)

4.



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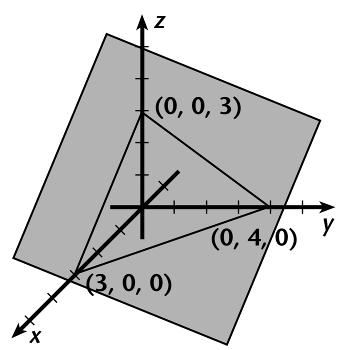

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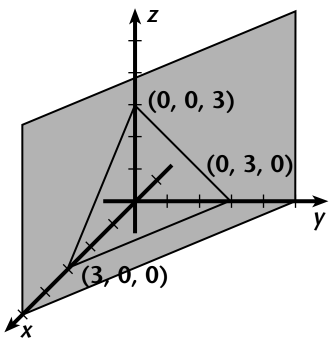

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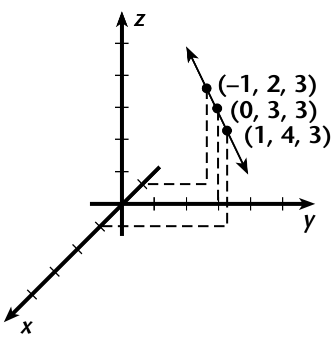

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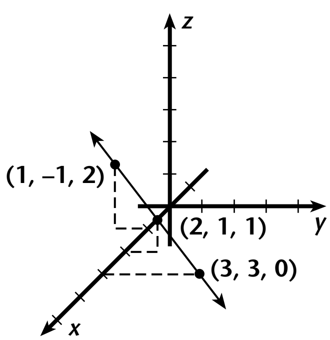

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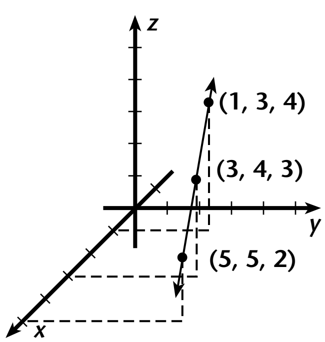


Lesson 6.6

1–6. Check drawings.

Reteaching — Chapter 7

Lesson 7.1

1. 216 square units

2. 296 square meters

3. 480 square inches

4. 118 square meters

5. 210 square units

6. 55 square centimeters

7. 254.2 cubic units

8. 90 cubic units

9. 490 cubic centimeters

10. 302.4 cubic meters

11. 1120 cubic units

12. 37.5 cubic feet

13. A; the area of the front face is 96 square inches, which is 1.6 times larger than the area of the front face of B.

14. a cube that is $\sqrt[3]{240}$, or approximately 7.83 inches on a side

15. The first package has greater surface area and would be more expensive to produce. The second, however, would be very difficult to hold in one hand for pouring.

Lesson 7.2

1. 244 square millimeters

2. 680 square feet **3.** 72 square meters

4. 380 square inches **5.** 192 cubic feet

6. 90 cubic meters **7.** 700 cubic inches

8. 800 cubic feet **9.** 252 cubic meters

10. 420 cubic meters

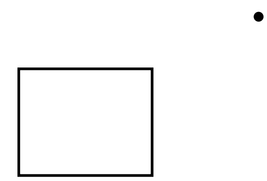
NAME ______________________ CLASS ____________ DATE ____________

Reteaching
6.6 *Perspective Drawing*

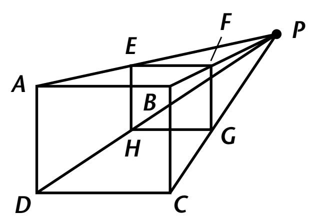
◆ Skill A Using one-point perspective

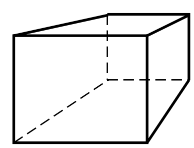
Recall In a drawing with one-point perspective, parallel lines appear to intersect at a single point called the *vanishing point*.

◆ **Example**
Draw a rectangular solid in one-point perspective. Let the rectangle be the front face of the solid and the given point be the vanishing point.



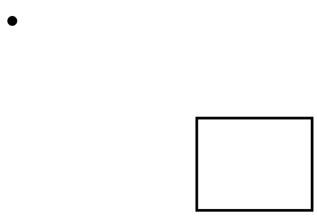
◆ **Solution**
In the drawing at the left below, labels have been added for clarity. Lightly draw $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$. Choose a point, E, on $\overline{PA}$ and draw $\overline{EF}$ parallel to $\overline{AB}$, $\overline{FG}$ parallel to $\overline{BC}$, $\overline{HG}$ parallel to $\overline{DC}$, and $\overline{EH}$ parallel to $\overline{AD}$. Erase all segments that are not edges of the rectangular solid and make hidden lines dashed.



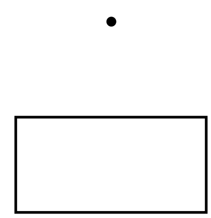


Draw a rectangular solid in one-point perspective that has the given rectangle as its front face and the given point as the vanishing point.

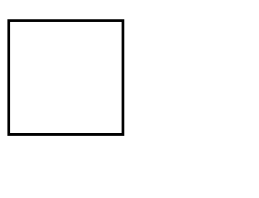
1.



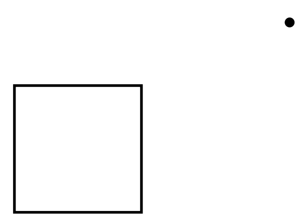
2.



3.



4.



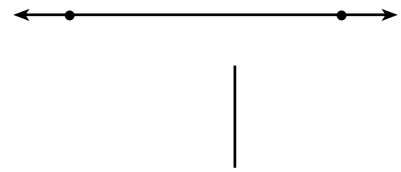
CLASS ______________ DATE ______________

◆ Skill B Using two-point perspective

Recall A drawing done in two-point perspective has two vanishing points.

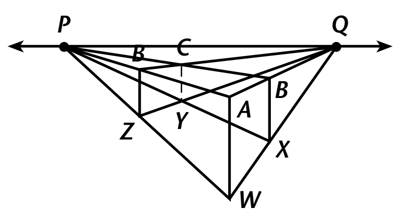
◆ **Example**

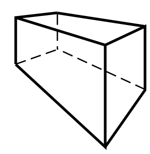
Draw a rectangular solid in two-point perspective. Let the given vertical segment be the front edge of the solid, the horizontal line be the horizon, and the given points be the vanishing points.



◆ **Solution**

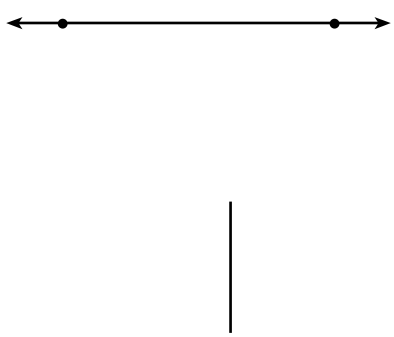
In the drawing at the left below, labels have been added for clarity. Lightly draw segments from P to A and W and from Q to A and W. Choose point D on $\overline{PA}$ and point B on $\overline{QA}$ and draw vertical segments $\overline{DZ}$ and $\overline{BX}$ as shown. Next, draw segments from P to B and X and from Q to D and Z. Let C be the intersection of $\overline{PB}$ and $\overline{QD}$ and Y be the intersection of $\overline{PX}$ and $\overline{QZ}$. Draw a dashed segment from C to Y. Erase all segments that are not edges of the rectangular solid and make hidden lines dashed.





Draw a rectangular solid in two-point perspective Let the vertical segment be the front edge of the solid. Let the horizontal line be the horizon and the two given points the vanishing points.

5.



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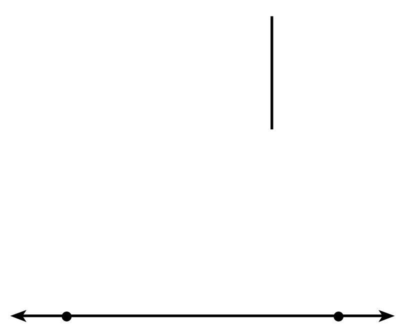

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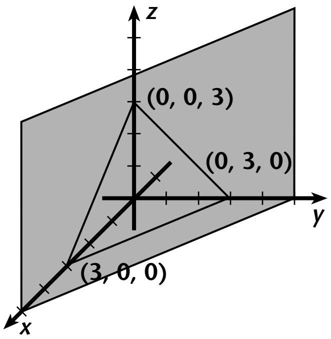

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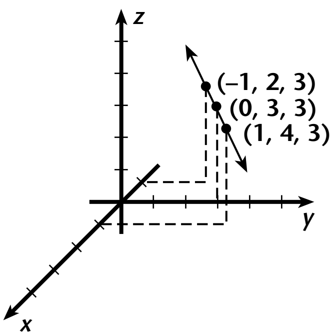

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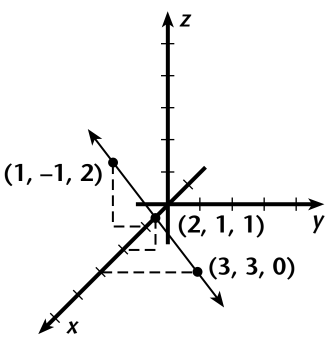

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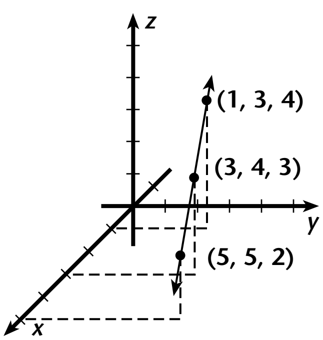


Lesson 6.6

1–6. Check drawings.

Reteaching — Chapter 7

Lesson 7.1

1. 216 square units

2. 296 square meters

3. 480 square inches

4. 118 square meters

5. 210 square units

6. 55 square centimeters

7. 254.2 cubic units

8. 90 cubic units

9. 490 cubic centimeters

10. 302.4 cubic meters

11. 1120 cubic units

12. 37.5 cubic feet

13. A; the area of the front face is 96 square inches, which is 1.6 times larger than the area of the front face of B.

14. a cube that is $\sqrt[3]{240}$, or approximately 7.83 inches on a side

15. The first package has greater surface area and would be more expensive to produce. The second, however, would be very difficult to hold in one hand for pouring.

Lesson 7.2

1. 244 square millimeters

2. 680 square feet **3.** 72 square meters

4. 380 square inches **5.** 192 cubic feet

6. 90 cubic meters **7.** 700 cubic inches

8. 800 cubic feet **9.** 252 cubic meters

10. 420 cubic meters

Menu

Print

NAME ______________________ CLASS __________ DATE __________

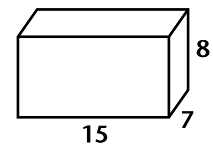
Reteaching

7.1 Surface Area and Volume

◆ **Skill A** Finding the surface area of a right rectangular prism

Recall The surface area, S, of a right rectangular prism with length ℓ, width w, and height h is $S = 2\ell w + 2\ell h + 2wh$.

◆ **Example**
Find the surface area of the right rectangular prism.



◆ **Solution**
$\ell = 15$, $w = 7$, and $h = 8$, so
$S = 2(15)(7) + 2(15)(8) + 2(7)(8) = 772$.
The surface area is 772 square units.

Find the surface area of the right rectangular prism with the given dimensions.

1. $\ell = 6$, $w = 6$, $h = 6$ ______

2. $\ell = 10$ m, $w = 5.5$ m, $h = 6$ m ______

3. $\ell = 12$ in., $w = 12$ in., $h = 4$ in. ______

4. $\ell = 8.5$ m, $w = 2$ m, $h = 4$ m ______

5. $\ell = 9$, $w = 6.5$, $h = 3$ ______

6. $\ell = 7$ cm, $w = 2$ cm, $h = 1.5$ cm ______

◆ **Skill B** Finding the volume of a right rectangular prism

Recall The volume, V, of a right rectangular prism with length ℓ, width w, and height h is $V = \ell \times w \times h$.

◆ **Example**
Refer to the right rectangular prism in the figure above. Find the volume of the prism.

◆ **Solution**
$V = \ell \times w \times h = 15 \times 7 \times 8 = 840$ The volume is 840 cubic units.

Find the volume of the right rectangular prism with the given dimensions.

7. $\ell = 10$, $w = 6.2$, $h = 4.1$ ______

8. $\ell = 7.5$, $w = 6$, $h = 2$ ______

9. $\ell = 7$ cm, $w = 7$ cm, $h = 10$ cm ______

10. $\ell = 9$ m, $w = 4.2$ m, $h = 8$ m ______

11. $\ell = 8$, $w = 7$, $h = 20$ ______

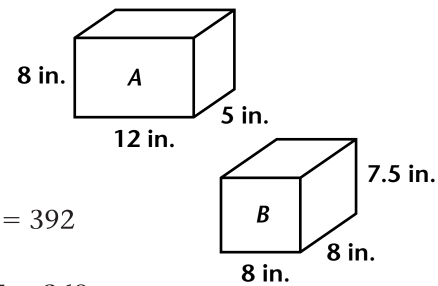
12. $\ell = 3$ ft, $w = 2.5$ ft, $h = 5$ ft ______

◆ Skill C Comparing the surface areas and volumes of right rectangular prisms

Recall For a right rectangular prism with volume, V, the prism with minimum surface area is a cube with sides of length $\sqrt[3]{V}$.

◆ **Example**

A manufacturer is considering two packages for a new laundry detergent. Compare the surface areas and volumes of the boxes. Based on cost of materials alone, which would be a better package?



◆ **Solution**

Package A: $S = 2 \cdot 12 \cdot 5 + 2 \cdot 12 \cdot 8 + 2 \cdot 5 \cdot 8 = 392$
$V = 12 \cdot 5 \cdot 8 = 480$

Package B: $S = 2 \cdot 8 \cdot 8 + 2 \cdot 8 \cdot 7.5 + 2 \cdot 8 \cdot 7.5 = 368$
$V = 10 \cdot 8 \cdot 6 = 480$

The packages will contain the same amount of detergent. However, Package B has a smaller surface area. Thus, it can be produced using a lesser amount of material. Based on cost alone, Package B would be a better package than Package A.

Refer to packages A and B in the example above.

13. Suppose the packages were places side-by-side on a supermarket shelf. Which would be more noticeable? Explain.

____

____

14. Suppose you wanted to produce another size package with half the volume of those above. Describe the dimensions of a rectangular package with the given volume and the smallest possible surface area.

____

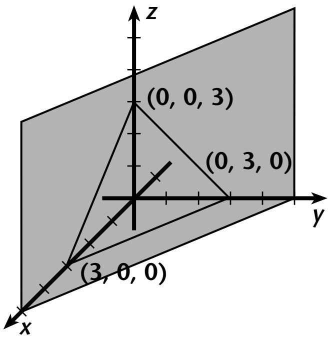
____

15. A cereal box will have an opening for pouring at the top. Compare the advantages and disadvantages of two packages, one that is 8.5 inches long, 3 inches deep, and 12 inches high and one that is a cube 6.7 inches on a side.

____

____

6.



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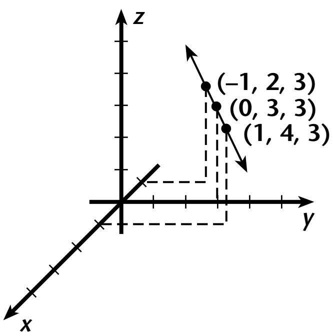

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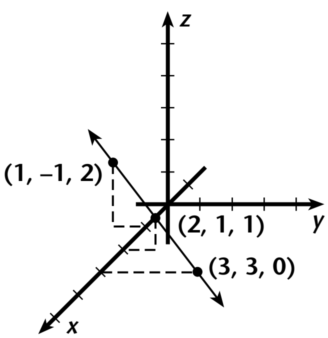

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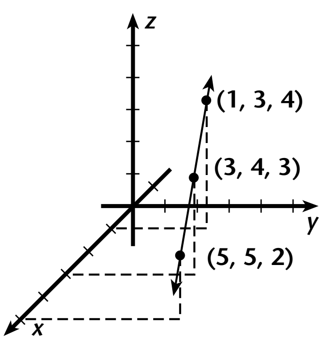

Lesson 6.6

1–6. Check drawings.

Reteaching — Chapter 7

Lesson 7.1

1. 216 square units

2. 296 square meters

3. 480 square inches

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5. 210 square units

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13. A; the area of the front face is 96 square inches, which is 1.6 times larger than the area of the front face of B.

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15. The first package has greater surface area and would be more expensive to produce. The second, however, would be very difficult to hold in one hand for pouring.

Lesson 7.2

1. 244 square millimeters

2. 680 square feet **3.** 72 square meters

4. 380 square inches **5.** 192 cubic feet

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8. 800 cubic feet **9.** 252 cubic meters

10. 420 cubic meters

NAME ______________________ CLASS ____________ DATE ____________

Reteaching

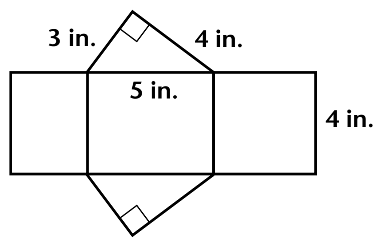
7.2 Surface Area and Volume of Prisms

◆ Skill A Finding the surface area of a right prism

Recall The surface area of any space figure is the sum of the areas of its surfaces. The surface area of any prism is equal to the sum of its lateral area and the areas of both bases, that is, $S = L + 2B$. For a right prism, if the perimeter is p, the height is h, and the area of a base is B, the surface area is $S = hp + 2B$. Note that if the base of a prism is a regular polygon with apothem a, you can use the area formula $\left(B = \frac{1}{2}ap\right)$ for such a polygon to determine that $2B = ap$ and $S = hp + ap$.

◆ Example

The figure shows a net for a right triangular prism.
Find the surface area of the prism.



◆ Solution 1

$h = 4;\ p = 3 + 4 + 5 = 12$

$B = \frac{1}{2}(3)(4) = 6$

$S = hp + 2B = 4(12) + 2(6) = 60$

Solution 2

$h = 4;\ L = 3(4) + 5(4) + 4(4) = 48$

$B = \frac{1}{2}(3)(4) = 6$

$S = L + 2B = 48 + 12 = 60$

The surface area of the prism is 60 square inches.

Find the surface area of the prism with the given dimensions.

1. $L = 124\ \text{mm}^2$, $B = 60\ \text{mm}^2$

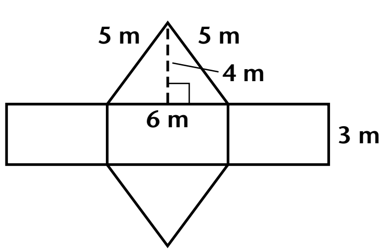
____

2. $h = 15$ ft, $p = 32$ ft, $B = 100\ \text{ft}^2$

____

Each figure shows a net for a right prism. Find the surface area of the prism.

3.



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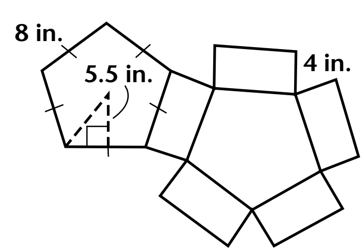

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NAME ______________________ CLASS ______________ DATE ______________

◆ Skill B Finding the volume of a prism

Recall The volume of a prism with height h and area of a base B is $V = Bh$. This formula is true for all prisms, whether they are right or oblique.

◆ **Example 1**

The bases of a triangular prism are right triangles with sides of length 5 inches, 12 inches, and 13 inches. The prism is 9 inches high. Find the volume of the prism.

◆ **Solution**

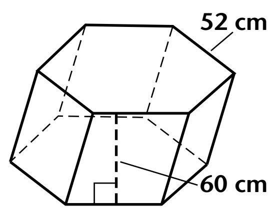
$B = \frac{1}{2}(5)(12) = 30$

$V = Bh = 30(9) = 270$

The volume of the prism is 270 cubic inches.

◆ **Example 2**

The base of the prism is a regular hexagon with apothem 45 centimeters. Find the volume of the prism.



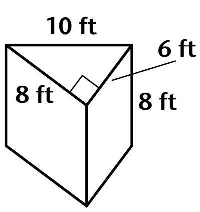
◆ **Solution**

$B = \frac{1}{2}ap = \frac{1}{2}(45)(6)(52) = 7020$

$V = Bh = (7020)(60) = 421{,}200$

The volume of the prism is 421,200 cubic centimeters.

Find the volume of each prism.

5.


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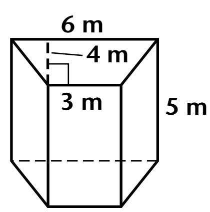
6.


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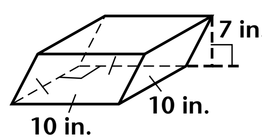
7.


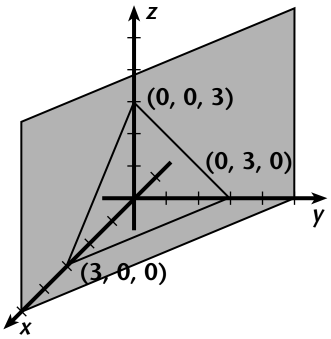
____

8. The area of a base is 80 square feet and the height is 10 feet. ______________

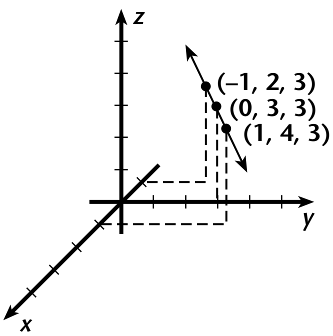
9. Each base of the right prism is a rectangle that is 7 meters long and 4 meters wide and the height of the prism is 9 meters. ______________

10. Each base of the oblique prism is a trapezoid with height 5 inches and bases that are 4 inches and 10 inches. The height of the prism is 12 inches. ______________

6.



7.



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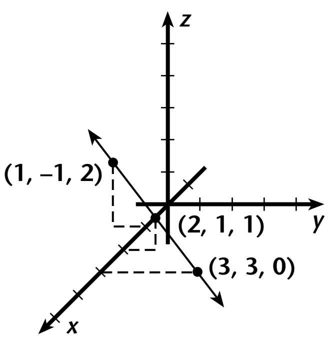

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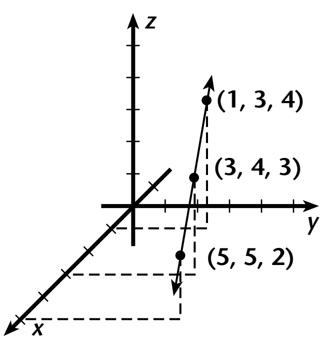

Lesson 6.6

1–6. Check drawings.

Reteaching — Chapter 7

Lesson 7.1

1. 216 square units

2. 296 square meters

3. 480 square inches

4. 118 square meters

5. 210 square units

6. 55 square centimeters

7. 254.2 cubic units

8. 90 cubic units

9. 490 cubic centimeters

10. 302.4 cubic meters

11. 1120 cubic units

12. 37.5 cubic feet

13. A; the area of the front face is 96 square inches, which is 1.6 times larger than the area of the front face of B.

14. a cube that is $\sqrt[3]{240}$, or approximately 7.83 inches on a side

15. The first package has greater surface area and would be more expensive to produce. The second, however, would be very difficult to hold in one hand for pouring.

Lesson 7.2

1. 244 square millimeters

2. 680 square feet **3.** 72 square meters

4. 380 square inches **5.** 192 cubic feet

6. 90 cubic meters **7.** 700 cubic inches

8. 800 cubic feet **9.** 252 cubic meters

10. 420 cubic meters

Menu

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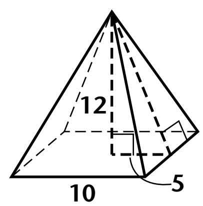
Reteaching

7.3 Surface Area and Volume of Pyramids

◆ **Skill A** Finding the surface area of a regular pyramid

Recall The surface area , S, of a regular pyramid with lateral area L and base area B is $S = L + B$. If the slant height of the pyramid is ℓ and the perimeter is p, the surface area is $S = \frac{1}{2}\ell p + B$. Also, $B = \frac{1}{2}ap$, where a is the apothem.

◆ **Example**
Find the surface area of the regular square pyramid.



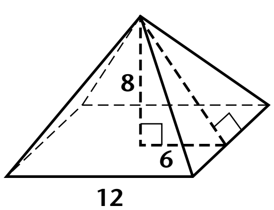
◆ **Solution**
The base is a square with sides that are 10 units long, so $B = 100$.
The perimeter of the base is 40 units.
Next use the Pythagorean Theorem to find the slant height, ℓ.

$$\begin{aligned} \ell^2 &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \\ \ell &= 13 \end{aligned}$$

Then $S = \frac{1}{2}\ell p + B = \frac{1}{2}(13)(40) + 100 = 360$.
The surface area of the pyramid is 360 square units.

Find the surface area of each regular pyramid.

1.



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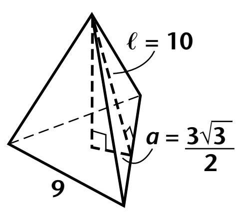
2.



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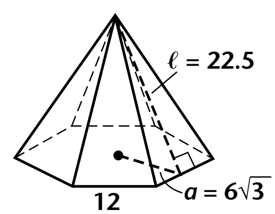
3.



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Find the surface area of each pyramid.

4. a regular pyramid whose base is a pentagon with sides 4 inches long and area 43 square inches, and slant height 10 inches ______________________

5. a regular pyramid whose base is a hexagon with sides 9 meters long and apothem 7.8 meters, and slant height 12 meters ______________________

Menu

Print

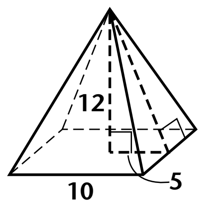
6. a regular pyramid whose base is an equilateral triangle with sides 7 feet long and apothem 2 feet, and slant height 4 feet ____________________

◆ Skill B Finding the volume of a pyramid

Recall The volume, V, of a pyramid with base area B and height h is $V = \frac{1}{3}Bh$.

◆ **Example**

Find the volume of a regular square pyramid.



◆ **Solution**

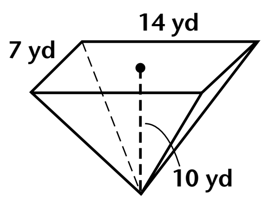
The area of the base is 100 square units and the height is 12 units.

$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(100)(12)$$
$$= 400$$

The volume is 400 cubic units.

Find the volume of each pyramid.

7.



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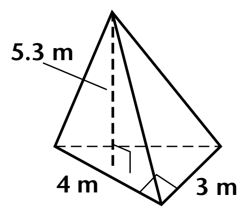
8.



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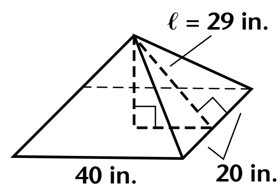
9.



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10. an octagonal pyramid with base area 412 square meters and height 15 meters ____________

11. a pentagonal pyramid with base area 108 square inches and height 7.5 inches ____________

12. a rectangular pyramid whose base is 20 centimeters long and 16 centimeters wide, and whose height is 10 centimeters ____________________

13. a pyramid whose base is a right triangle with sides of length 9 feet, 12 feet, and 15 feet and whose height is 10 feet ____________________

Menu

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ANSWERS

Lesson 7.3

1. 384 square units
2. 170.07 square units
3. 1184.12 square units
4. 143 square inches
5. 534.6 square meters
6. 63 square feet
7. 326.67 cubic yards
8. 10.6 cubic meters
9. 11,200 cubic inches
10. 2060 cubic meters
11. 270 cubic inches
12. 1066.67 cubic centimeters
13. 180 cubic feet

Lesson 7.4

1. 576π square inches; 1809.56 square inches
2. 12π square feet; 37.70 square feet
3. 2280π square centimeters; 7162.83 square centimeters
4. $\frac{3\pi}{2}$ square millimeters; 4.71 square millimeters
5. 4.2 meters **6.** 22.02 feet **7.** 348.9 yds
8. 816 cubic centimeters
9. 4π cubic inches; 12.6 cubic inches
10. 800π cubic meters; 2513.3 cubic meters
11. 57.6π cubic centimeters; 181.0 cubic centimeters
12. 100π cubic feet; 314.2 cubic feet
13. 864π cubic units; 2714.3 cubic units
14. 8 meters
15. 14π inches; 44.0 inches
16. 75π cubic meters; 235.6 cubic meters
17. 500π cubic meters; 1570.8 cubic meters

Lesson 7.5

1. 14π square inches
2. 33π square feet
3. $49\pi + 7\pi\sqrt{149}$ square units
4. 85π square inches; 267.0 square inches
5. 16π square millimeters; 50.3 square millimeters
6. $7\pi\sqrt{113} + 49\pi$ square feet; 387.7 square feet
7. $11\pi\sqrt{377} + 121\pi$ square millimeters; 1051.1 square millimeters
8. $\frac{80\pi}{3}$ cubic meters
9. 21π cubic feet
10. 18π cubic inches
11. 648π cubic inches; 2035.8 cubic inches
12. $\frac{484\pi}{3}$ cubic feet; 506.8 cubic feet
13. $\frac{25\pi}{3}$ cubic meters; 26.2 cubic meters
14. 9 feet **15.** 12 feet
16. 320π cubic inches
17. 3240π cubic millimeters

Lesson 7.6

1. 36π square inches; 113.10 square inches
2. 576π square millimeters; 1809.56 square millimeters
3. 153.76π square centimeters; 483.05 square centimeters

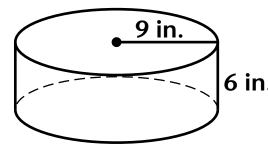
NAME ____________________ CLASS __________ DATE __________

Reteaching
7.4 *Surface Area and Volume of Cylinders*

◆ **Skill A** Finding the surface area of a right cylinder

Recall The surface area, S, of a cylinder is the sum of its lateral area and the area of both bases, that is, $S = L + 2B$. For a right cylinder, if the radius of a base is r and the height of the cylinder is h, the surface area, S, is $S = 2\pi rh + 2\pi r^2$.

◆ **Example 1**
The radius of a base of a right cylinder is 9 inches. The cylinder is 6 inches high. Find the exact surface area of the cylinder and the surface area rounded to the nearest tenth.



◆ **Solution**
$r = 9$ and $h = 6$, so $S = 2\pi(9)(6) + 2\pi(9^2) = 270\pi$.
The surface area is 270π square inches, or approximately 848.23 square inches.

◆ **Example 2**
A right cylinder that is 15 meters high has surface area 1400π square meters. Find the exact circumference of a base of the cylinder.

◆ **Solution**
To find the circumference, first find the radius. $S = 2\pi rh + 2\pi r^2$

$$1400\pi = 2\pi r(15) + 2\pi r^2$$
$$700 = 15r + r^2$$

Solve the quadratic equation: $r^2 + 15r - 700 = 0$; $r = -35$ or $r = 20$
Since r must be positive, $r = 20$. Then $C = 2\pi r = 40\pi$.
The circumference of a base is 40π meters.

The dimensions of a right cylinder are given. Find the exact surface area of the cylinder and the surface area rounded to the nearest hundredth.

1. radius of a base = 12 in., height = 12 in.

____

2. radius of a base = 2 ft, height = 1 ft

____

3. radius of a base = 30 cm, height = 8 cm

____

4. diameter of a base = 1 mm, height = 1 mm

____

Find the unknown value for each cylinder. Give the exact value as well as the value rounded to the nearest tenth.

5. surface area = 92π m^2, radius of a base = 5 m, height = ______________________

6. surface area = 507π ft^2, height = 12 ft, diameter of a base = ______________________

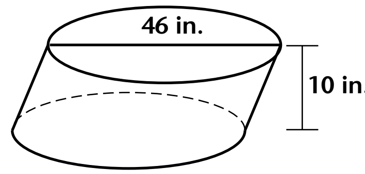
7. surface area = 7500π yd^2, height = 12 yd, circumference of a base = ______________________

◆ Skill B Finding the volume of a cylinder

Recall If the area of a base of a cylinder is B and the height of the cylinder is h, then the volume, V, of the cylinder is $V = Bh$. If the radius of the cylinder is r, then the volume is $V = \pi r^2 h$.

◆ **Example**

The diameter of a base of an oblique cylinder is 46 inches. The height of the cylinder is 10 inches. Find the exact volume of the cylinder and the volume rounded to the nearest tenth.



◆ **Solution**

$V = \pi r^2 h$

Since $d = 46$, $r = 23$.

Then, since $h = 10$,

$V = \pi(23)^2(10)$

$\quad = 5290\pi$.

The volume of the cylinder is 5290π cubic units, or approximately 16,619.0 cubic inches.

Find the exact volume of each cylinder. Then give the volume rounded to the nearest tenth.

8. area of a base = 68 cm^2, height = 12 cm

____

9. radius of a base = 1 in., height = 4 in.

____

10. diameter of a base = 20 m, height = 8 m

____

11. radius of a base = 2.4 cm, height = 10 cm

____

12. radius of a base = 5 ft, height = 4 ft

____

13. circumference of a base = 24π, height = 6

____

Find the unknown value for each cylinder. Give the exact value as well as the value rounded to the nearest tenth. In Exercises 16–17, the cylinder is a right cylinder.

14. volume = 128π cm^3, diameter of a base = 8 m, height = ______________________________

15. volume = 245π in.^{3}, height = 5 in. , circumference of a base = ______________________________

16. surface area = 80π m^2, height = 3 m, volume = ______________________________

17. surface area = 250π m^2, height = 20 m, volume = ______________________________

Menu

Print

ANSWERS

Lesson 7.3

1. 384 square units
2. 170.07 square units
3. 1184.12 square units
4. 143 square inches
5. 534.6 square meters
6. 63 square feet
7. 326.67 cubic yards
8. 10.6 cubic meters
9. 11,200 cubic inches
10. 2060 cubic meters
11. 270 cubic inches
12. 1066.67 cubic centimeters
13. 180 cubic feet

Lesson 7.4

1. 576π square inches; 1809.56 square inches
2. 12π square feet; 37.70 square feet
3. 2280π square centimeters; 7162.83 square centimeters
4. $\frac{3\pi}{2}$ square millimeters; 4.71 square millimeters
5. 4.2 meters **6.** 22.02 feet **7.** 348.9 yds
8. 816 cubic centimeters
9. 4π cubic inches; 12.6 cubic inches
10. 800π cubic meters; 2513.3 cubic meters
11. 57.6π cubic centimeters; 181.0 cubic centimeters
12. 100π cubic feet; 314.2 cubic feet
13. 864π cubic units; 2714.3 cubic units
14. 8 meters
15. 14π inches; 44.0 inches
16. 75π cubic meters; 235.6 cubic meters
17. 500π cubic meters; 1570.8 cubic meters

Lesson 7.5

1. 14π square inches
2. 33π square feet
3. $49\pi + 7\pi\sqrt{149}$ square units
4. 85π square inches; 267.0 square inches
5. 16π square millimeters; 50.3 square millimeters
6. $7\pi\sqrt{113} + 49\pi$ square feet; 387.7 square feet
7. $11\pi\sqrt{377} + 121\pi$ square millimeters; 1051.1 square millimeters
8. $\frac{80\pi}{3}$ cubic meters
9. 21π cubic feet
10. 18π cubic inches
11. 648π cubic inches; 2035.8 cubic inches
12. $\frac{484\pi}{3}$ cubic feet; 506.8 cubic feet
13. $\frac{25\pi}{3}$ cubic meters; 26.2 cubic meters
14. 9 feet **15.** 12 feet
16. 320π cubic inches
17. 3240π cubic millimeters

Lesson 7.6

1. 36π square inches; 113.10 square inches
2. 576π square millimeters; 1809.56 square millimeters
3. 153.76π square centimeters; 483.05 square centimeters

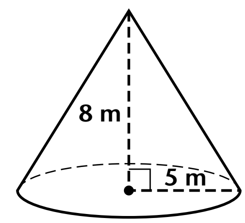
NAME ______________________ CLASS ____________ DATE ____________

Reteaching
7.5 *Surface Area and Volume of Cones*

◆ **Skill A** Finding the surface area of a right cone

Recall The surface area, S, of a cone is the sum of its lateral area and the area of its base, that is, $S = L + B$. For a right cone, if the radius of the base is r and the slant height of the cone is ℓ, the surface area, S, is $S = \pi r\ell + \pi r^2$.

◆ **Example 1**
Find the exact surface area of the right cone and the surface area rounded to the nearest tenth.



◆ **Solution**
First, use the Pythagorean Theorem to find the slant height of the cone.

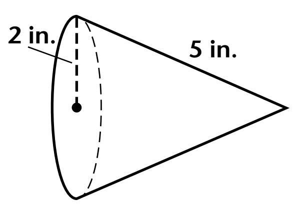
$\ell^2 = 5^2 + 8^2 = 89; \ \ell = \sqrt{89}$

$S = \pi(5)(\sqrt{89}) + \pi(5^2) = 5\pi\sqrt{89} + 25\pi$

The surface area of the cone is $5\pi\sqrt{89} + 25\pi$ square meters, or approximately 226.7 square meters.

Find the exact surface area of each right cone.

1.



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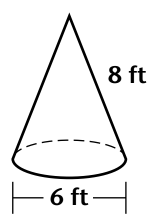
2.



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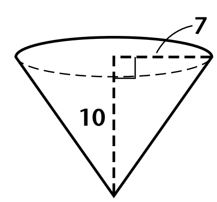
3.



____

Find the exact surface area of each right cone. Then find the surface area to the nearest tenth.

4. radius of the base = 5 in., slant height = 12 in. ______________________

5. radius of the base = 2 mm, slant height = 6 mm ______________________

6. radius of the base = 7 ft, height = 8 ft ______________________

7. diameter of the base = 22 mm, height = 16 mm ______________________

Menu

Print

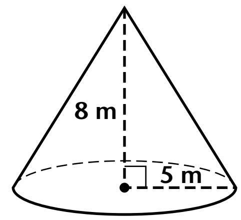
CLASS ______________ DATE ______________

◆ **Skill B** Finding the volume of a cone

Recall If the area of the base of a cone is B and the slant height of the cone is ℓ, then the volume, V, of the cone is $V = \frac{1}{3}Bh$. If the radius of the cone is r, then the volume is $V = \frac{1}{3}\pi r^2 h$.

◆ **Example**

Find the exact volume of the cone. Then find the volume to the nearest hundredth.



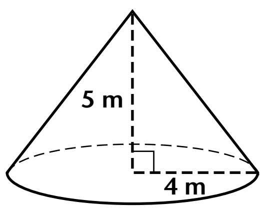
◆ **Solution**

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5^2)(8) = \frac{200\pi}{3}$

The volume of the cone is $\frac{200\pi}{3}$ cubic meters, or approximately 209.44 cubic meters.

Find the exact volume of each cone.

8.



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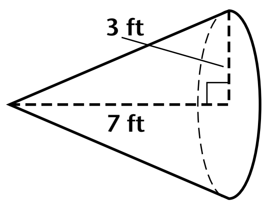
9.



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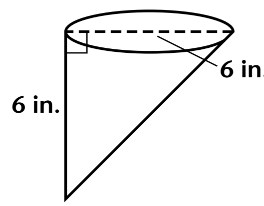
10.



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Find the exact volume of each cone. Then find the volume rounded to the nearest tenth.

11. radius of the base = 18 in., height = 6 in. ______________

12. radius of the base = 11 ft, height = 4 ft ______________

13. radius of the base = 2.5 m, height = 4 m ______________

Find the exact unknown value for each cone. In Exercises 16 and 17, the cones are right cones.

14. volume = 432π ft^2, height = 16 ft, radius of the base = ______________

15. volume = 784π m^2, diameter of the base = 28 ft, height = ______________

16. diameter of the base = 16 in., slant height = 17 in., volume = ______________

17. radius of the base 9 mm, slant height = 41 mm, volume = ______________

Menu

Print

ANSWERS

Lesson 7.3

1. 384 square units
2. 170.07 square units
3. 1184.12 square units
4. 143 square inches
5. 534.6 square meters
6. 63 square feet
7. 326.67 cubic yards
8. 10.6 cubic meters
9. 11,200 cubic inches
10. 2060 cubic meters
11. 270 cubic inches
12. 1066.67 cubic centimeters
13. 180 cubic feet

Lesson 7.4

1. 576π square inches; 1809.56 square inches
2. 12π square feet; 37.70 square feet
3. 2280π square centimeters; 7162.83 square centimeters
4. $\frac{3\pi}{2}$ square millimeters; 4.71 square millimeters
5. 4.2 meters **6.** 22.02 feet **7.** 348.9 yds
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14. 8 meters
15. 14π inches; 44.0 inches
16. 75π cubic meters; 235.6 cubic meters
17. 500π cubic meters; 1570.8 cubic meters

Lesson 7.5

1. 14π square inches
2. 33π square feet
3. $49\pi + 7\pi\sqrt{149}$ square units
4. 85π square inches; 267.0 square inches
5. 16π square millimeters; 50.3 square millimeters
6. $7\pi\sqrt{113} + 49\pi$ square feet; 387.7 square feet
7. $11\pi\sqrt{377} + 121\pi$ square millimeters; 1051.1 square millimeters
8. $\frac{80\pi}{3}$ cubic meters
9. 21π cubic feet
10. 18π cubic inches
11. 648π cubic inches; 2035.8 cubic inches
12. $\frac{484\pi}{3}$ cubic feet; 506.8 cubic feet
13. $\frac{25\pi}{3}$ cubic meters; 26.2 cubic meters
14. 9 feet **15.** 12 feet
16. 320π cubic inches
17. 3240π cubic millimeters

Lesson 7.6

1. 36π square inches; 113.10 square inches
2. 576π square millimeters; 1809.56 square millimeters
3. 153.76π square centimeters; 483.05 square centimeters

Menu

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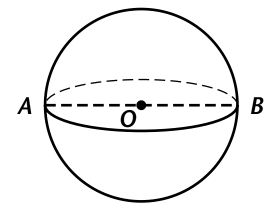
NAME ______________________ CLASS ____________ DATE ____________

Reteaching

7.6 Surface Area and Volume of Spheres

◆ Skill A Finding the surface area of a sphere

Recall In the figure at the right, O is the center of the sphere. $\overline{OA}$ and $\overline{OB}$ are radii and $\overline{AB}$ is a diameter. The radius of the sphere is OB. The diameter is AB. The surface area, S, of a sphere with radius r is $S = 4\pi r^2$.



◆ **Example**
A sphere has radius 10 meters. Find the exact surface area of the sphere and the surface area rounded to the nearest tenth.

◆ **Solution**
Since $r = 10$, $S = 4\pi(10)^2 = 400\pi$.
The surface area of the sphere is 400π square meters, or approximately 1256.6 square meters.

Find the exact surface area of each sphere and the surface area rounded to the nearest hundredth.

1. radius = 3 in. ______________

2. radius = 12 mm ______________

3. radius = 6.2 cm ______________

4. diameter = 16 yd ______________

5. diameter = 7 ft ______________

6. diameter = 4.6 in. ______________

◆ Skill B Finding the volume of a sphere

Recall The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

◆ **Example**
Find the exact volume of the sphere in the example above. Then find the volume to the nearest tenth of a cubic meter.

◆ **Solution**
$r = 10$, so $V = \frac{4}{3}\pi(10^3) = \frac{4000\pi}{3}$.

The volume is $\frac{4000\pi}{3}$ cubic meters, or approximately 4188.8 cubic meters.

Find the exact volume of each sphere and the volume rounded to the nearest tenth.

7. radius = 24 yd ______________

8. radius = 15 mm ______________

9. radius = 11 m ______________

Menu
Print

10. diameter = 18 in. ______________

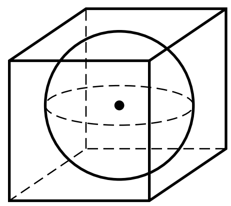
11. diameter = 17 yd ______________

12. diameter = 13 cm ______________

◆ Skill C Solving problems using the formula for the volume of a sphere

◆ **Example**

The sphere is inscribed in the cube. The volume of the cube is 4096 cubic feet. Find the volume of the sphere.



◆ **Solution**

volume of the cube $= 4096 = s^3$; $s = \sqrt[3]{4096} = 16$

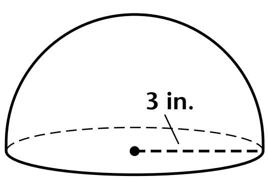
radius of the sphere $= \frac{1}{2}s = 8$

volume of the sphere $= \frac{4}{3}\pi(8)^3$

The volume of the sphere is $\frac{2048\pi}{3}$ cubic feet, or approximately 2144.66 square feet.

Find the exact volume of each space figure.

13.



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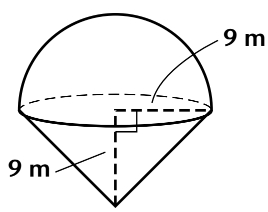

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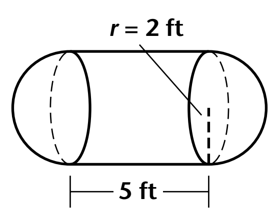

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16. A spherical scoop of ice cream has radius 1.25 inches. The scoop is placed on top of a cone with radius 1 inch and height 5 inches. Is the cone large enough to hold all the ice cream if it melts? Explain.

____

____

Menu

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ANSWERS

Lesson 7.3

1. 384 square units
2. 170.07 square units
3. 1184.12 square units
4. 143 square inches
5. 534.6 square meters
6. 63 square feet
7. 326.67 cubic yards
8. 10.6 cubic meters
9. 11,200 cubic inches
10. 2060 cubic meters
11. 270 cubic inches
12. 1066.67 cubic centimeters
13. 180 cubic feet

Lesson 7.4

1. 576π square inches; 1809.56 square inches
2. 12π square feet; 37.70 square feet
3. 2280π square centimeters; 7162.83 square centimeters
4. $\frac{3\pi}{2}$ square millimeters; 4.71 square millimeters
5. 4.2 meters **6.** 22.02 feet **7.** 348.9 yds
8. 816 cubic centimeters
9. 4π cubic inches; 12.6 cubic inches
10. 800π cubic meters; 2513.3 cubic meters
11. 57.6π cubic centimeters; 181.0 cubic centimeters
12. 100π cubic feet; 314.2 cubic feet
13. 864π cubic units; 2714.3 cubic units
14. 8 meters
15. 14π inches; 44.0 inches
16. 75π cubic meters; 235.6 cubic meters
17. 500π cubic meters; 1570.8 cubic meters

Lesson 7.5

1. 14π square inches
2. 33π square feet
3. $49\pi + 7\pi\sqrt{149}$ square units
4. 85π square inches; 267.0 square inches
5. 16π square millimeters; 50.3 square millimeters
6. $7\pi\sqrt{113} + 49\pi$ square feet; 387.7 square feet
7. $11\pi\sqrt{377} + 121\pi$ square millimeters; 1051.1 square millimeters
8. $\frac{80\pi}{3}$ cubic meters
9. 21π cubic feet
10. 18π cubic inches
11. 648π cubic inches; 2035.8 cubic inches
12. $\frac{484\pi}{3}$ cubic feet; 506.8 cubic feet
13. $\frac{25\pi}{3}$ cubic meters; 26.2 cubic meters
14. 9 feet **15.** 12 feet
16. 320π cubic inches
17. 3240π cubic millimeters

Lesson 7.6

1. 36π square inches; 113.10 square inches
2. 576π square millimeters; 1809.56 square millimeters
3. 153.76π square centimeters; 483.05 square centimeters

4. 256π square yards; 804.25 square yards

5. 49π square feet; 153.94 square feet

6. 21.16π square inches; 66.48 square inches

7. 18432π cubic yards; 57,905.8 cubic yards

8. 4500π cubic millimeters; 14,137.2 cubic millimeters

9. $\frac{5324\pi}{3}$ cubic meters; 5575.3 cubic meters

10. 972π cubic inches; 3053.6 cubic inches

11. $\frac{4913\pi}{6}$ cubic yards; 2572.4 cubic yards

12. $\frac{2197\pi}{6}$ cubic centimeters; 1150.3 cubic centimeters

13. 18π cubic inches

14. 729π cubic meters

15. $\frac{92\pi}{3}$ cubic feet

16. No; the volume of the ice cream is about 8.2 cubic inches, while the volume of the cone is only about 5.2 cubic inches.

Lesson 7.7

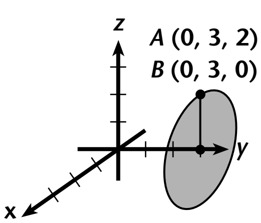
1. (2, 4, 1) 2. (2, −3, 4) 3. (5, 0, 2)

4. $A'(0, 2, 1)$; $B'(3, -3, 4)$

5. $A'(-2, 2, -2)$; $B'(4, -3, 1)$

6. $A'(0, 4, 2)$; $B'(3, 5, 3)$

7.



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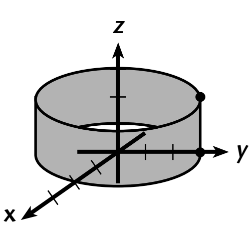


9. a cylinder with base radius 12 in. and height 10 in.

10. a cylinder with base radius 10 in. and height 12 in.

11. a cylinder with base radius 5 in. and height 12 in.

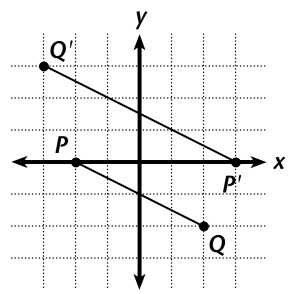
Reteaching — Chapter 8

Lesson 8.1

1. (0, 10) 2. (−4,2) 3. (8, −4)

4. $\left(-\frac{5}{2}, 6\right)$ 5. (12, −8) 6. (2, 12)

7.



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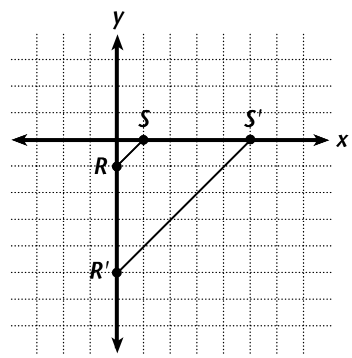

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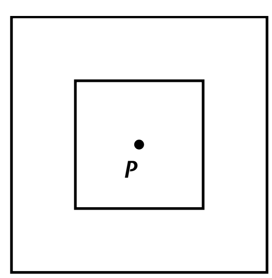

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NAME ______________________ CLASS __________ DATE __________

Reteaching

7.7 Three-Dimensional Symmetry

◆ **Skill A** Reflecting points and segments across a plane in space

Recall Reflecting the point $P(x, y, z)$ across one of the planes defined by a pair of axes in space produces a change in the coordinates according to the plane across which P is reflected.

The image of P reflected across the yz-plane is $(-x, y, z)$.
The image of P reflected across the xz-plane is $(x, -y, z)$.
The image of P reflected across the xy-plane is $(x, y, -z)$.

◆ **Example**
The segment with endpoints $A(3, -1, 2)$ and $B(1, 2, 3)$ is reflected across the xz-plane. Identify the coordinates of the endpoints of the image.

◆ **Solution**
The endpoints of the image are $A'(3, 1, 2)$ and $B'(1, -2, 3)$.

Give the coordinates of the image of the given point after reflection across the indicated plane.

1. $(2, 4, -1)$; xy-plane ______________

2. $(-2, -3, 4)$; yz-plane ______________

3. $(5, 0, 2)$; xz-plane ______________

Given A and B, find the coordinates of the endpoints of the image of $\overline{AB}$ after reflection across the indicated plane.

4. $A(0, 2, 1)$ and $B(-3, -3, 4)$
yz-plane ______________

5. $A(-2, -2, -2)$ and $B(4, 3, 1)$
xz-plane ______________

6. $A(0, 4, -2)$ and $B(3, 5, -3)$
xy-plane ______________

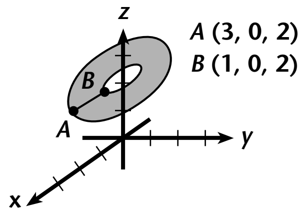
◆ **Skill B** Rotating segments about an axis in space

Recall When a segment in space is rotated about an axis to which it is perpendicular, the figure formed may be a circle or a donut-shaped region called an *annulus*. If the segment is parallel to the axis of rotation, the image is the lateral surface of a right cylinder whose axis is the axis of rotation.

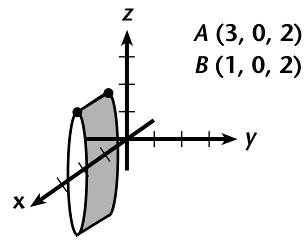
◆ **Example**
Draw the image of the segment with endpoints $A(3, 0, 2)$ and $B(1, 0, 2)$ rotated (a) about the z-axis and (b) about the x-axis.

◆ **Solution**

a.

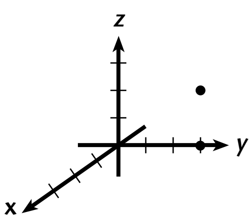


b.

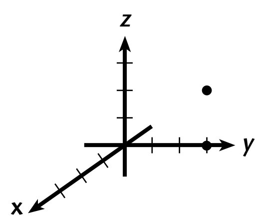


$\overline{AB}$ has endpoints $A(0, 3, 2)$ and $B(0, 3, 0)$. Sketch the figure formed by rotating $\overline{AB}$ about the indicated axis.

7. y-axis



8. z-axis

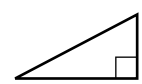


◆ **Skill C** Visualizing and describing solids of revolution

Recall A closed plane figure that is rotated about an axis in space produces a solid figure called a solid of revolution.

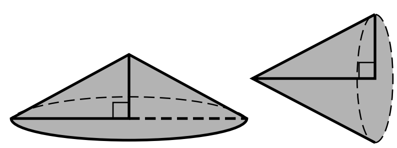
◆ **Example**

Describe the solid of revolution formed when the right triangle is rotated about (a) its vertical leg and (b) its horizontal leg.

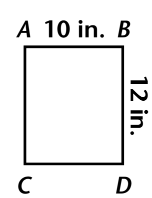


◆ **Solution**

Picture the figure rotating about the indicated axis. Both rotations produce cones. Since the cones have different heights and base radii, they have different volumes.



Describe the solid of revolution formed when the rectangle is rotated about the indicated axis. Give the dimensions of the solid of revolution.



9. $\overleftrightarrow{AB}$ ____

10. $\overleftrightarrow{BC}$ ____

11. the line through the midpoints of $\overline{AB}$ and $\overline{DC}$ ______________________________

4. 256π square yards; 804.25 square yards

5. 49π square feet; 153.94 square feet

6. 21.16π square inches; 66.48 square inches

7. 18432π cubic yards; 57,905.8 cubic yards

8. 4500π cubic millimeters; 14,137.2 cubic millimeters

9. $\frac{5324\pi}{3}$ cubic meters; 5575.3 cubic meters

10. 972π cubic inches; 3053.6 cubic inches

11. $\frac{4913\pi}{6}$ cubic yards; 2572.4 cubic yards

12. $\frac{2197\pi}{6}$ cubic centimeters; 1150.3 cubic centimeters

13. 18π cubic inches

14. 729π cubic meters

15. $\frac{92\pi}{3}$ cubic feet

16. No; the volume of the ice cream is about 8.2 cubic inches, while the volume of the cone is only about 5.2 cubic inches.

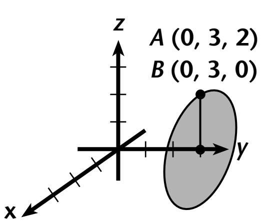
Lesson 7.7

1. (2, 4, 1) 2. (2, −3, 4) 3. (5, 0, 2)

4. $A'(0, 2, 1)$; $B'(3, -3, 4)$

5. $A'(-2, 2, -2)$; $B'(4, -3, 1)$

6. $A'(0, 4, 2)$; $B'(3, 5, 3)$

7.


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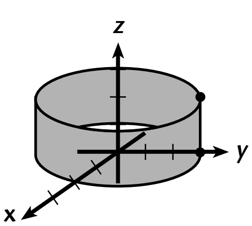


9. a cylinder with base radius 12 in. and height 10 in.

10. a cylinder with base radius 10 in. and height 12 in.

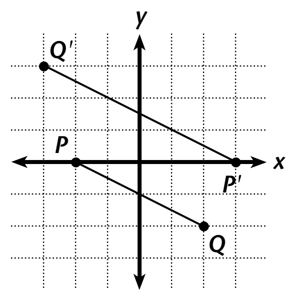
11. a cylinder with base radius 5 in. and height 12 in.

Reteaching — Chapter 8

Lesson 8.1

1. (0, 10) 2. (−4,2) 3. (8, −4)

4. $\left(-\frac{5}{2}, 6\right)$ 5. (12, −8) 6. (2, 12)

7.


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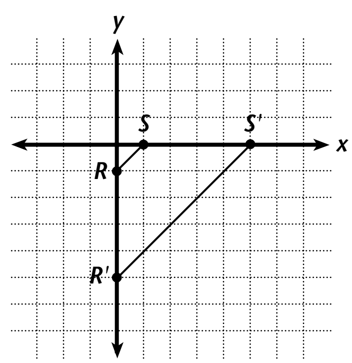

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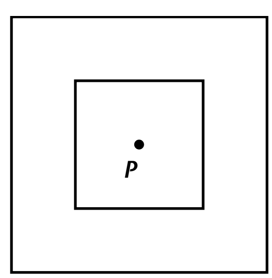
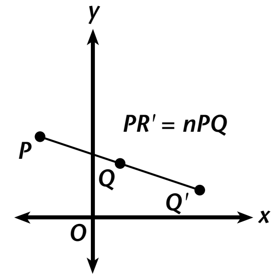


NAME ______________________ CLASS __________ DATE __________

Reteaching
8.1 Dilations and Scale Factors

◆ **Skill A** Determining the coordinates of the image of a point under a dilation

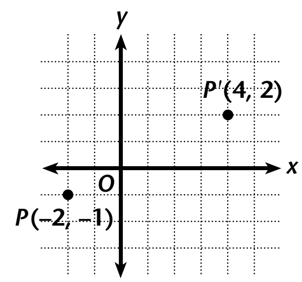
Recall Let P be a point in the coordinate plane and n a real number, n not 0, and let D be the dilation with center P and scale factor n. The image of any point Q under D is the point Q' on $\overrightarrow{OQ}$ (on the ray opposite $\overrightarrow{OQ}$ if n is negative) such that $OQ' = n \cdot OQ$. If $|n| > 1$, the dilation is called an *expansion*. If $n < 0$, the dilation is called a *contraction*.



If the center of the dilation is the origin and Q has coordinates (x, y), then Q' has coordinates (nx, ny).

◆ **Example**
Let D be a dilation with center (0, 0) and scale factor −2 and let $P = (-2, -1)$. Determine the coordinates of P'.

◆ **Solution**
Since the origin is the center of the dilation and the scale factor is −2, $P' = D(-2, -1)$ has coordinates $(-2(-2), -2(-1)) = (4, 2)$.



For each point *P*, find the coordinates of *P′*, the image of *P* under the dilation with center (0, 0) and scale factor *n*.

1. $n = 2.5$; $P(0, 4)$ ______

2. $n = -1$; $P(4, -2)$ ______

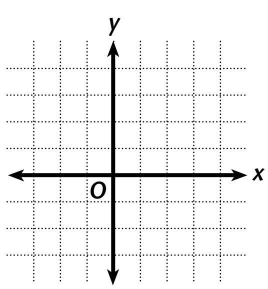
3. $n = -4$; $P(-2, 1)$ ______

4. $n = \frac{1}{2}$; $P(-5, 12)$ ______

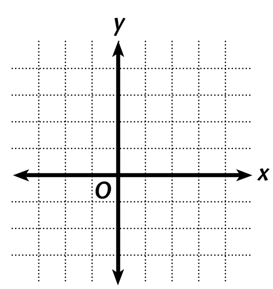
5. $n = \frac{2}{3}$; $P(18, -12)$ ______

6. $n = 3$; $P\left(\frac{2}{3}, 4\right)$ ______

7. The dilation D has center (0, 0) and scale factor $-1\frac{1}{2}$. Draw the segment with endpoints $P(-2, 0)$ and $Q(2, -2)$ and its image, $\overline{P'Q'}$, under D.



8. The dilation E has center (0, 0) and scale factor 5. Draw the segment with endpoints $R(0, 1)$ and $S(1, 0)$ and its image, $\overline{R'S'}$, under E.



◆ Skill B Drawing the image of a plane figure under a dilation

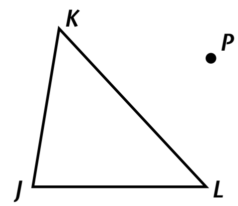
Recall To draw the image of a plane figure under a dilation with center P and scale factor n, first draw the images of the vertices of the figure.

If $n > 0$ and point A is a vertex of the figure to be dilated, draw $\overrightarrow{PA}$. Measure $\overrightarrow{PA}$ and locate A' on $\overrightarrow{PA}$ so that $PA' = PA$. A' is the image of A. Use the same method to locate the images of the other vertices and connect them.

If $n < 0$, proceed as described above, but locate A' on the ray opposite $\overrightarrow{PA}$ instead.

◆ Example

Draw the image of $\triangle JKL$ under a dilation with center L and scale factor $\frac{3}{4}$.



◆ Solution

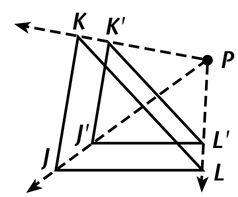
Draw $\overrightarrow{PJ}$, $\overrightarrow{PK}$, and $\overrightarrow{PL}$.
$PJ = 36$ cm, $PK = 24$ cm, and $PL = 20$ cm

So $PJ' = \frac{3}{4} \cdot 36 = 27$, $PK' = \frac{3}{4} \cdot 24 = 18$, and

$PL' = \frac{3}{4} \cdot 20 = 15$.

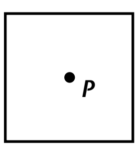
Locate J' on $\overrightarrow{PJ}$, K' on $\overrightarrow{PK}$, and L' on $\overrightarrow{PL}$.

Finally, draw $\overline{J'K'}$, $\overline{K'L'}$, and $\overline{J'L'}$. $\triangle J'K'L'$ is the image of $\triangle JKL$.

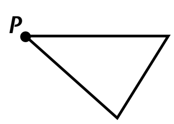


Draw the image of the figure under the dilation with center *P* and the indicated scale factor.

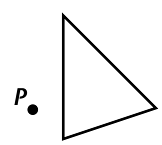
9. $n = \frac{1}{2}$



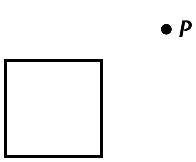
10. $n = 1.5$



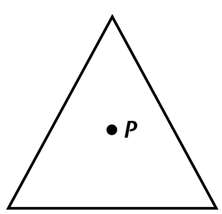
11. $n = 2$



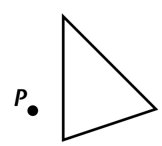
12. $n = -1$



13. $n = -\frac{1}{2}$



14. $n = -2$



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5. 49π square feet; 153.94 square feet

6. 21.16π square inches; 66.48 square inches

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12. $\frac{2197\pi}{6}$ cubic centimeters; 1150.3 cubic centimeters

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15. $\frac{92\pi}{3}$ cubic feet

16. No; the volume of the ice cream is about 8.2 cubic inches, while the volume of the cone is only about 5.2 cubic inches.

Lesson 7.7

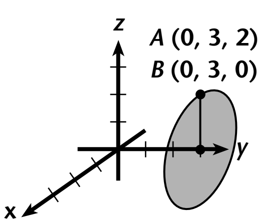
1. (2, 4, 1) 2. (2, −3, 4) 3. (5, 0, 2)

4. $A'(0, 2, 1)$; $B'(3, -3, 4)$

5. $A'(-2, 2, -2)$; $B'(4, -3, 1)$

6. $A'(0, 4, 2)$; $B'(3, 5, 3)$

7.



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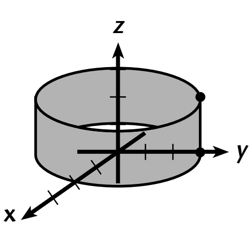


9. a cylinder with base radius 12 in. and height 10 in.

10. a cylinder with base radius 10 in. and height 12 in.

11. a cylinder with base radius 5 in. and height 12 in.

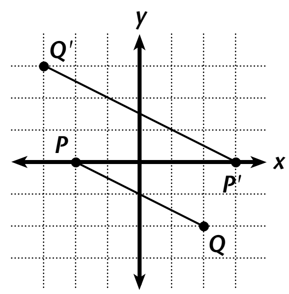
Reteaching — Chapter 8

Lesson 8.1

1. (0, 10) 2. (−4,2) 3. (8, −4)

4. $\left(-\frac{5}{2}, 6\right)$ 5. (12, −8) 6. (2, 12)

7.



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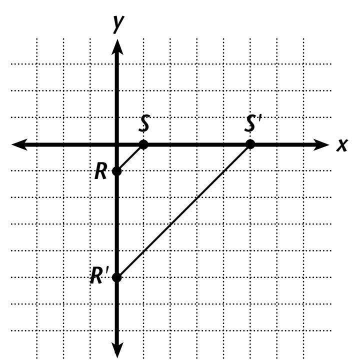

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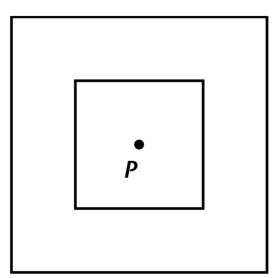

Menu
Print

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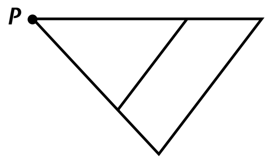

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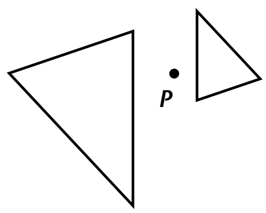
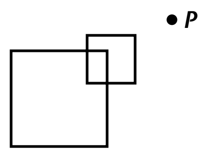


12.



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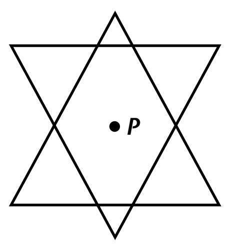

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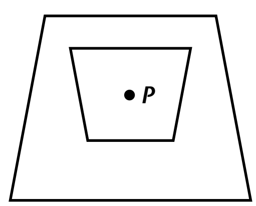


Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$

2. no

3. Yes; $DEFG \sim LMJK$

4. 7.2 **5.** 10.8 **6.** 24

7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS

2. Yes; $\triangle BCE \sim \triangle STR$; AA

3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS

4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS

5. Yes; $\triangle RST \sim \triangle MNP$; AA

6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS

7. 2 feet **8.** $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 **2.** 4.5 **3.** 4.5 **4.** 4.8

5. 9 **6.** 6.75 **7.** $3\frac{1}{3}$ **8.** 2.4

9. 2 **10.** 2 **11.** $2\frac{2}{3}$ **12.** $3\frac{1}{3}$

13. $1\frac{2}{3}$ **14.** $7\frac{2}{9}$

Lesson 8.5

1. 34 feet **2.** 45 feet **3.** 22 feet

4. 160 feet **5.** 75 feet **6.** 6 meters

7. 7.5 **8.** 9 **9.** 10 **10.** 5.6

Lesson 8.6

1. $\frac{16}{121}$; $\frac{4}{11}$ **2.** $\frac{1}{25}$; $\frac{1}{125}$ **3.** $\frac{27}{8}$; $\frac{9}{4}$

4. $\frac{4}{3}$; $\frac{4}{3}$ **5.** $\frac{729}{125}$ **6.** 25 cm

7. 144π square yards; 4500π cubic yards

8. 6 inches; 10 square inches

9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, $\overline{QD}$

2. $\overline{AC}$, $\overline{BD}$

3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$

4. $\overset{\frown}{ABC}$, $\overset{\frown}{ADC}$, $\overset{\frown}{BAD}$, $\overset{\frown}{BCD}$

Menu

Print

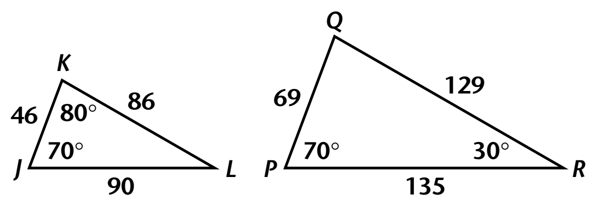
Reteaching
8.2 Similar Polygons

◆ **Skill A** Using the Polygon Similarity Postulate to determine whether two polygons are similar

Recall The Polygon Similarity Postulate:

Two polygons are similar if and only if there is a way of setting up a correspondence between their sides and angles so that:
(1) All corresponding angles are congruent.
(2) All corresponding sides are proportional.

◆ **Example**
Determine whether the triangles are similar. If so, write a similarity statement.



◆ **Solution**
By the Triangle Sum Theorem: $m\angle L = 180° - (70° + 80°) = 30°$
$m\angle Q = 180° - (70° + 30°) = 80°$

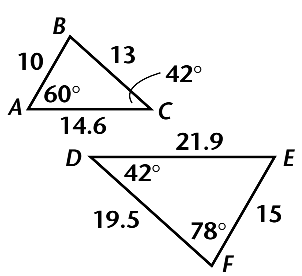
Then $\angle J \cong \angle P$, $\angle K \cong \angle Q$, and $\angle L \cong \angle R$. Then $\overline{JK}$ and $\overline{PQ}$ are corresponding sides, as are $\overline{KL}$ and $\overline{QR}$, and $\overline{JL}$ and $\overline{PR}$.

$$\frac{JK}{PQ} = \frac{46}{69} = \frac{2}{3} \qquad \frac{KL}{QR} = \frac{86}{129} = \frac{2}{3} \qquad \frac{JL}{PR} = \frac{90}{135} = \frac{2}{3}$$

The ratios of the lengths of corresponding sides are equal, so corresponding sides are in proportion. By the Polygon Similarity Postulate, $\triangle JKL \sim \triangle PQR$.

Determine whether the polygons are similar. If so, write a similarity statement.

1.



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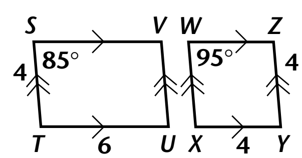
2.



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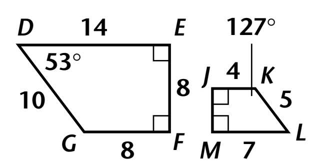

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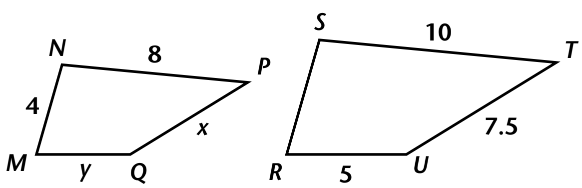
◆ Skill B

Using properties of proportions and scale factors to solve problems involving similar polygons

Recall A proportion is a statement that two ratios are equal. Given two similar polygons, then, you can write proportions comparing the ratios of any two pairs of corresponding sides. Given some of the side lengths of two smaller figures, this method allows you to determine unknown lengths. Suppose that $\triangle ABC \sim \triangle XYZ$. Then $\frac{AB}{XY} = \frac{BC}{YZ}$. By the Exchange Property of Proportions, it is also true that $\frac{AB}{BC} = \frac{XY}{YZ}$. That is, the ratio of the lengths of two sides of one of a pair of similar figures is equal to the ratio of the lengths of the corresponding sides of the other figure.

◆ **Example**

$MNPQ \sim RSTU$. Find the values of x and y.



◆ **Solution**

Since the quadrilaterals are similar, $\frac{NP}{ST} = \frac{PQ}{TU}$ or $\frac{8}{10} = \frac{x}{7.5}$.

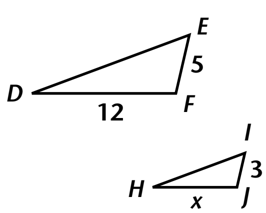
Then $10x = 60$ and $x = 6$.

Also, $\frac{MQ}{RU} = \frac{MP}{ST}$, so by the Exchange Property of Proportions, $\frac{MQ}{NP} = \frac{RU}{ST}$.

Then $\frac{y}{8} = \frac{5}{10} = \frac{1}{2}$ and $y = 4$.

Complete.

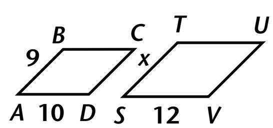
4.



$\triangle DEF \sim \triangle HIJ$, so

$x =$ ______________.

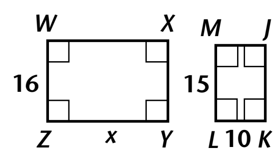
5.



$ABCD \sim STUV$, so

$x =$ ______________.

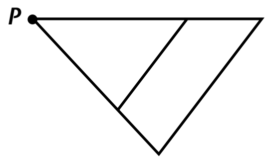
6.



$WXYZ \sim JKLM$, so

$x =$ ______________.

7. An architect is making plans for a rectangular office building that is 840 feet long and 252 feet wide. A blueprint of the floor plan for the first floor is 15 inches long. How wide is the blueprint? ______________

10.


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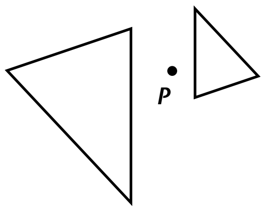

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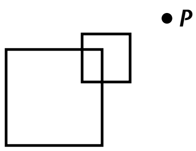

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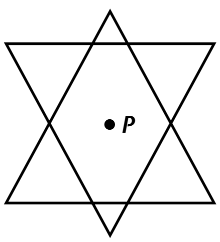

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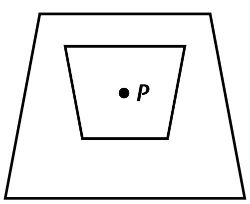


Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$

2. no

3. Yes; $DEFG \sim LMJK$

4. 7.2 **5.** 10.8 **6.** 24

7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS

2. Yes; $\triangle BCE \sim \triangle STR$; AA

3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS

4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS

5. Yes; $\triangle RST \sim \triangle MNP$; AA

6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS

7. 2 feet **8.** $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 **2.** 4.5 **3.** 4.5 **4.** 4.8

5. 9 **6.** 6.75 **7.** $3\frac{1}{3}$ **8.** 2.4

9. 2 **10.** 2 **11.** $2\frac{2}{3}$ **12.** $3\frac{1}{3}$

13. $1\frac{2}{3}$ **14.** $7\frac{2}{9}$

Lesson 8.5

1. 34 feet **2.** 45 feet **3.** 22 feet

4. 160 feet **5.** 75 feet **6.** 6 meters

7. 7.5 **8.** 9 **9.** 10 **10.** 5.6

Lesson 8.6

1. $\frac{16}{121}; \frac{4}{11}$ **2.** $\frac{1}{25}; \frac{1}{125}$ **3.** $\frac{27}{8}; \frac{9}{4}$

4. $\frac{4}{3}; \frac{4}{3}$ **5.** $\frac{729}{125}$ **6.** 25 cm

7. 144π square yards; 4500π cubic yards

8. 6 inches; 10 square inches

9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, $\overline{QD}$

2. $\overline{AC}$, $\overline{BD}$

3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$

4. $\overset{\frown}{ABC}$, $\overset{\frown}{ADC}$, $\overset{\frown}{BAD}$, $\overset{\frown}{BCD}$

NAME ______________________ CLASS __________ DATE __________

Reteaching
8.3 Triangle Similarity Postulates

◆ **Skill A** Using similarity postulates and theorems to determine whether two triangles are similar

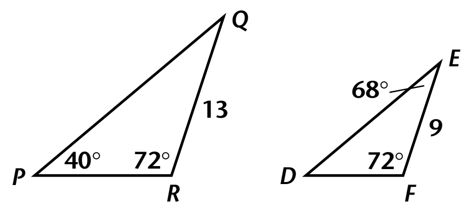
Recall **AA Similarity Postulate:** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

SSS Similarity Postulate: If the three sides of one triangle are proportional to the three sides of another triangle, the triangles are similar.

SAS Similarity Postulate: If two sides of one triangle are proportional to two sides of another triangle, and their included angles are congruent, then the triangles are similar.

◆ **Example**

Are the triangles similar?



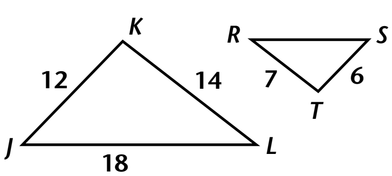
◆ **Solution**

$$m\angle Q = 180° - (m\angle P + m\angle R)$$
$$= 180° - (40° + 72°)$$
$$= 68°$$

Then $\angle Q \cong \angle E$ and $\angle R \cong \angle F$, so $\triangle PQR \sim \triangle DEF$ by the AA Similarity Postulate.

Are the triangles similar? If so, write a similarity statement and identify the postulate or theorem that justifies your answer.

1.



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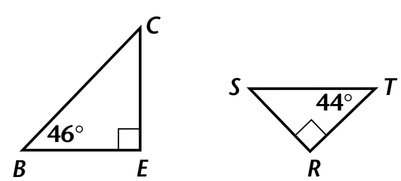

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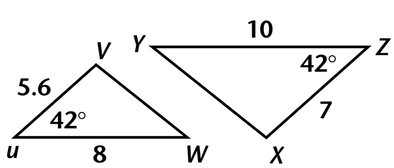

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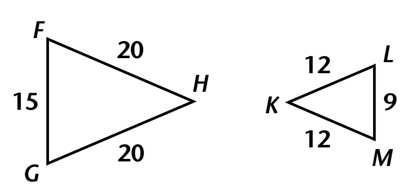

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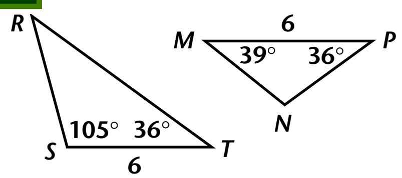

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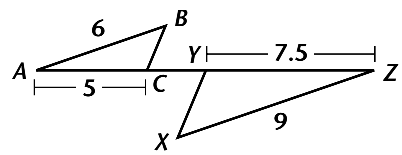
6.



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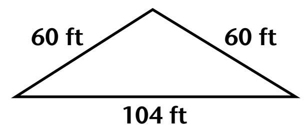
◆ Skill B Using triangle similarity to solve problems

Recall If you can show that two triangles are similar, then you can use the proportionality of the sides to solve problems.

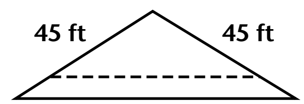
◆ **Example**

A park is in the shape of an isosceles triangle with legs 60 feet long. The base is 104 feet and is marked by a picket fence. If each of the congruent sides of the park is cut back to a length of 45 feet, how long a fence will be needed for the base?



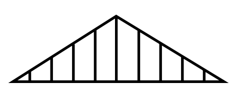
◆ **Solution**

Make a sketch. By the SAS Similarity Theorem, the new triangle is similar to the original triangle. Let x be the base of the new triangle. The ratio of the lengths of the legs is $\frac{60}{45} = \frac{4}{3}$, so the ratio of the bases is also $\frac{4}{3}$ and $\frac{104}{x} = \frac{4}{3}$. Then $x = 78$; a fence 78 feet long will be needed.



Solve each problem. Identify the postulate or theorem that justifies your solution.

7. A portion of an exterior wall of a house is marked off by the edges of the roof and a horizontal piece of trim 50 feet long. Decorative vertical strips are spaced evenly as shown. If the middle strip is 10 feet long, how long are each of the shortest strips?



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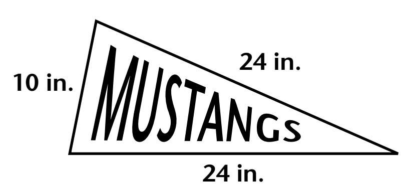
8. Students plan to make a large replica of a school banner to display at a football game. The banner is triangular as shown in the figure. If the shortest side of the replica will be $6\frac{1}{4}$ feet long, how many feet of trim will be needed to edge the replica?



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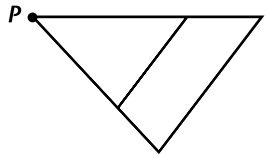
ANSWERS

10.



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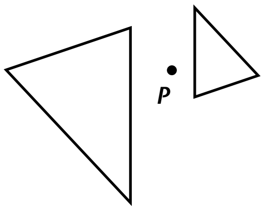

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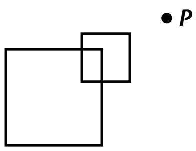

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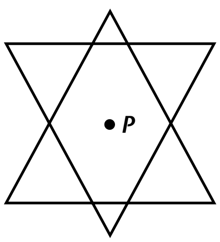

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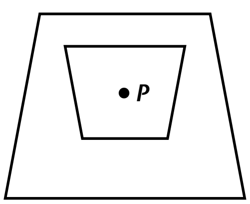


Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$

2. no

3. Yes; $DEFG \sim LMJK$

4. 7.2 **5.** 10.8 **6.** 24

7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS

2. Yes; $\triangle BCE \sim \triangle STR$; AA

3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS

4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS

5. Yes; $\triangle RST \sim \triangle MNP$; AA

6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS

7. 2 feet **8.** $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 **2.** 4.5 **3.** 4.5 **4.** 4.8

5. 9 **6.** 6.75 **7.** $3\frac{1}{3}$ **8.** 2.4

9. 2 **10.** 2 **11.** $2\frac{2}{3}$ **12.** $3\frac{1}{3}$

13. $1\frac{2}{3}$ **14.** $7\frac{2}{9}$

Lesson 8.5

1. 34 feet **2.** 45 feet **3.** 22 feet

4. 160 feet **5.** 75 feet **6.** 6 meters

7. 7.5 **8.** 9 **9.** 10 **10.** 5.6

Lesson 8.6

1. $\frac{16}{121}; \frac{4}{11}$ **2.** $\frac{1}{25}; \frac{1}{125}$ **3.** $\frac{27}{8}; \frac{9}{4}$

4. $\frac{4}{3}; \frac{4}{3}$ **5.** $\frac{729}{125}$ **6.** 25 cm

7. 144π square yards; 4500π cubic yards

8. 6 inches; 10 square inches

9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, $\overline{QD}$

2. $\overline{AC}$, $\overline{BD}$

3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$

4. $\overset{\frown}{ABC}$, $\overset{\frown}{ADC}$, $\overset{\frown}{BAD}$, $\overset{\frown}{BCD}$

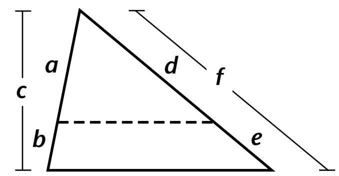
NAME ______________________ CLASS ______________ DATE ______________

Reteaching
8.4 The Side-Splitting Theorem

◆ **Skill A** Using the Side-Splitting Theorem to solve problems

Recall The **Side-Splitting Theorem** states that a line parallel to one side of a triangle divides the other two sides proportionally.

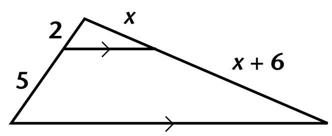
In the figure below, the dashed segment is parallel to a side of the triangle. The Side-Splitting Theorem can be used to justify any proportion equivalent to the first proportion listed. Several examples are given.



$\frac{a}{c} = \frac{d}{e}$ $\quad$ $\frac{a}{b} = \frac{c}{d}$ $\quad$ $\frac{a}{c} = \frac{b}{d}$ $\quad$ $\frac{b}{c} = \frac{d}{f}$

To use the Side-Splitting Theorem to find an unknown length x in a triangle, find a proportion in which x is isolated as the numerator or denominator of one of the ratios.

◆ **Example**
Find the value of x.

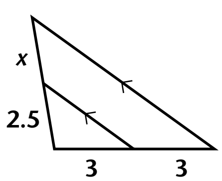


◆ **Solution**

$$\frac{2}{5} = \frac{x}{x+6}$$
$$2x + 12 = 5x$$
$$3x = 12$$
$$x = 4$$

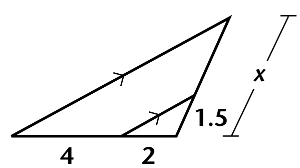
Find the value of *x*.

1.



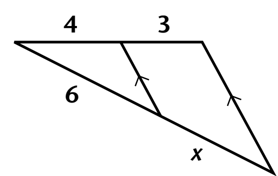
$x =$ ______________

2.



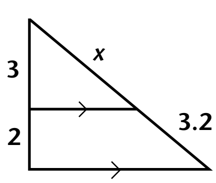
$x =$ ______________

3.



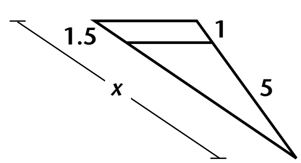
$x =$ ______________

4.



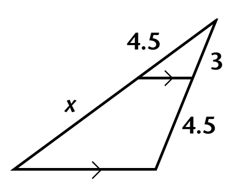
$x =$ ______________

5.



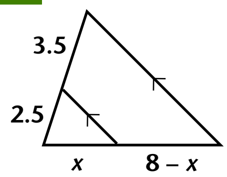
$x =$ ______________

6.



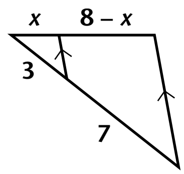
$x =$ ______________

7.



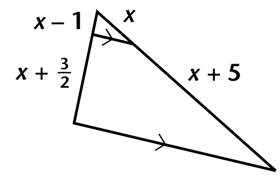
$x =$ ____________________

8.



$x =$ ____________________

9.



$x =$ ____________________

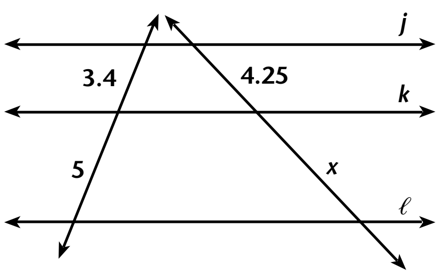
◆ **Skill B** Using the Two-Transversal Proportionality Corollary to solve problems

Recall A corollary of a theorem is a theorem that can be easily proven using the given theorem. The **Two-Transversal Proportionality Corollary** is a corollary of the Side-Splitting Theorem:

If three or more parallel lines intersect two transversals, they divide them proportionally.

◆ **Example**

In the figure, j, k, and ℓ are parallel. Find the value of x.



◆ **Solution**

Since $j \parallel k \parallel \ell$, the transversals are divided proportionally. That is, $\frac{3.4}{5} = \frac{4.25}{x}$.

Then $3.4x = 21.25$

$x = 6.25.$

In the figure, lines *m*, *n*, *p*, and *q* are parallel. Use the Two-Transversal Proportionality Corollary and similar triangles to find each value.

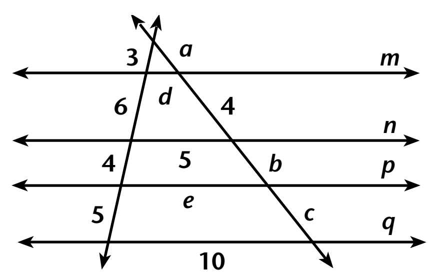
10. $a =$ ______________________________

11. $b =$ ______________________________

12. $c =$ ______________________________

13. $d =$ ______________________________

14. $e =$ ______________________________



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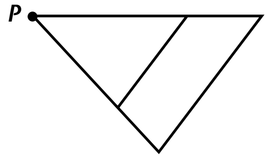
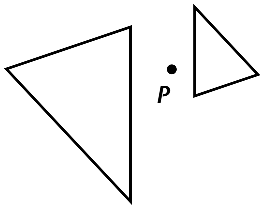
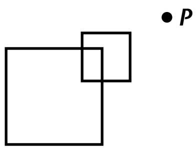
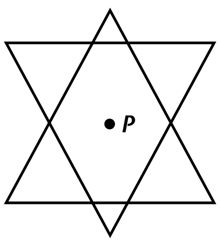
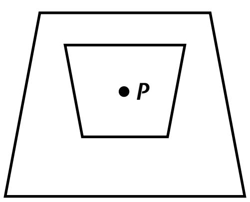
10. 

11. 

12. 

13. 

14. 

Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$

2. no

3. Yes; $DEFG \sim LMJK$

4. 7.2 **5.** 10.8 **6.** 24

7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS

2. Yes; $\triangle BCE \sim \triangle STR$; AA

3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS

4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS

5. Yes; $\triangle RST \sim \triangle MNP$; AA

6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS

7. 2 feet **8.** $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 **2.** 4.5 **3.** 4.5 **4.** 4.8

5. 9 **6.** 6.75 **7.** $3\frac{1}{3}$ **8.** 2.4

9. 2 **10.** 2 **11.** $2\frac{2}{3}$ **12.** $3\frac{1}{3}$

13. $1\frac{2}{3}$ **14.** $7\frac{2}{9}$

Lesson 8.5

1. 34 feet **2.** 45 feet **3.** 22 feet

4. 160 feet **5.** 75 feet **6.** 6 meters

7. 7.5 **8.** 9 **9.** 10 **10.** 5.6

Lesson 8.6

1. $\frac{16}{121}; \frac{4}{11}$ **2.** $\frac{1}{25}; \frac{1}{125}$ **3.** $\frac{27}{8}; \frac{9}{4}$

4. $\frac{4}{3}; \frac{4}{3}$ **5.** $\frac{729}{125}$ **6.** 25 cm

7. 144π square yards; 4500π cubic yards

8. 6 inches; 10 square inches

9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, $\overline{QD}$

2. $\overline{AC}$, $\overline{BD}$

3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$

4. $\overset{\frown}{ABC}$, $\overset{\frown}{ADC}$, $\overset{\frown}{BAD}$, $\overset{\frown}{BCD}$

NAME ______________________ CLASS __________ DATE __________

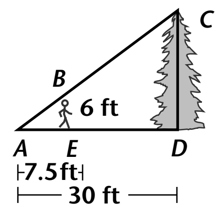
Reteaching
8.5 Indirect Measurement and Additional Similarity Theorems

◆ **Skill A** Using similar triangles to measure distances indirectly

Recall Similar triangles may be used to determine unknown lengths that may be difficult or even impossible to measure using ordinary methods.

◆ **Example**

Arvid wanted to determine the height of a tree in his yard. He stood in the shadow of the tree and positioned himself carefully so that his shadow ended at the same point as the tree's shadow. He then had his sister measure his shadow (7.5 feet) and the sun's shadow (30 feet). If Arvid is 6 feet tall, how tall is the tree?



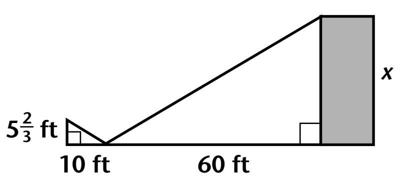
◆ **Solution**

$\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate. $\frac{CD}{BE} = \frac{AD}{AE}$ or $\frac{CD}{6} = \frac{30}{7.5}$ so $CD = 24$.

The tree is 24 feet high.

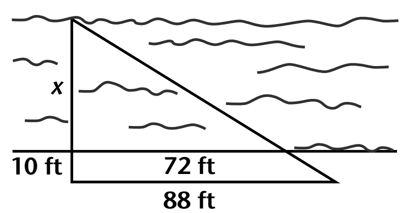
Find the value of *x*.

1.



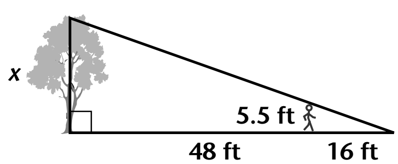
$x =$ ______________

2.



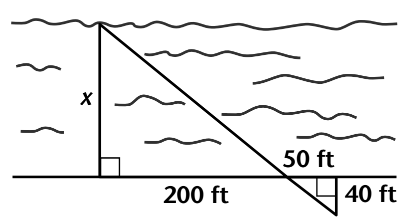
$x =$ ______________

3.



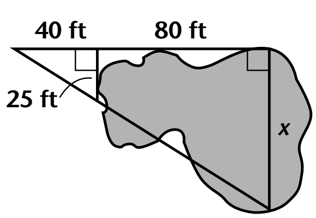
$x =$ ______________

4.



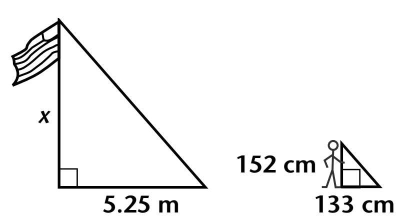
$x =$ ______________

5.



$x =$ ______________

6.



$x =$ ______________

◆ Skill B Using the Proportional Altitudes, Medians, and Angle Bisector Theorems

Recall If two triangles are similar, their sides are proportional. As the following theorems indicate, other lines related to the triangles are proportional as well.

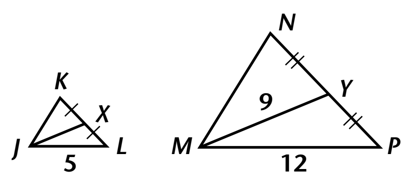
Proportional Altitudes Theorem: If two triangles are similar, then the ratio of the lengths of two corresponding altitudes is the same as the ratio of the lengths of two corresponding sides.

Proportional Medians Theorem: If two triangles are similar, then the ratio of the lengths of two corresponding medians is the same as the ratio of the lengths of two corresponding sides.

Proportional Angle Bisectors Theorem: If two triangles are similar, then the ratio of the lengths of two corresponding angle bisectors is the same as the ratio of the lengths of two corresponding sides.

◆ **Example**

If $\triangle JKL \sim \triangle MNP$, find the length of $\overline{JX}$.

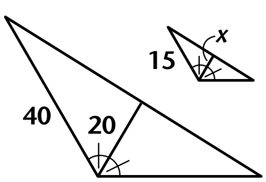


◆ **Solution**

Since X is the midpoint of $\overline{KL}$ and Y is the midpoint of $\overline{NP}$, $\overline{JX}$ and $\overline{MY}$ are medians. By the Proportional Medians Theorem, $\frac{JX}{MY} = \frac{JL}{MP}$. Then $\frac{JX}{9} = \frac{5}{12}$ and $JX = 3.75$.

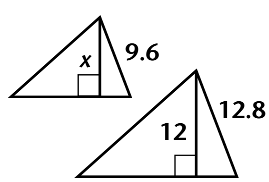
The triangles in each pair are similar. Find the value of *x*.

7.



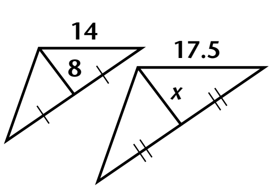
$x =$ ______________________________

8.



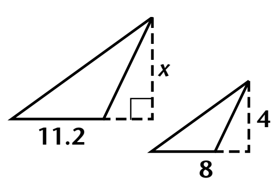
$x =$ ______________________________

9.



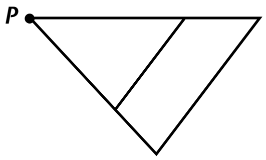
$x =$ ______________________________

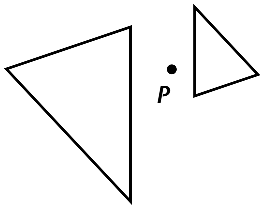
10.

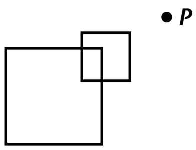


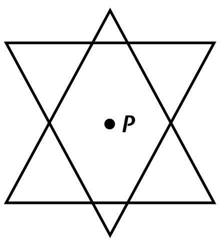
$x =$ ______________________________

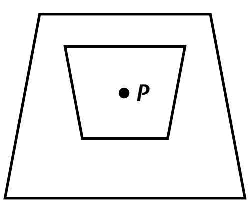
ANSWERS

10. 

11. 

12. 

13. 

14. 

Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$
2. no
3. Yes; $DEFG \sim LMJK$
4. 7.2 5. 10.8 6. 24
7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS
2. Yes; $\triangle BCE \sim \triangle STR$; AA
3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS
4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS
5. Yes; $\triangle RST \sim \triangle MNP$; AA
6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS
7. 2 feet 8. $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 2. 4.5 3. 4.5 4. 4.8

5. 9 6. 6.75 7. $3\frac{1}{3}$ 8. 2.4

9. 2 10. 2 11. $2\frac{2}{3}$ 12. $3\frac{1}{3}$

13. $1\frac{2}{3}$ 14. $7\frac{2}{9}$

Lesson 8.5

1. 34 feet 2. 45 feet 3. 22 feet

4. 160 feet 5. 75 feet 6. 6 meters

7. 7.5 8. 9 9. 10 10. 5.6

Lesson 8.6

1. $\frac{16}{121}; \frac{4}{11}$ 2. $\frac{1}{25}; \frac{1}{125}$ 3. $\frac{27}{8}; \frac{9}{4}$

4. $\frac{4}{3}; \frac{4}{3}$ 5. $\frac{729}{125}$ 6. 25 cm

7. 144π square yards; 4500π cubic yards
8. 6 inches; 10 square inches
9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, $\overline{QD}$
2. $\overline{AC}$, $\overline{BD}$
3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$
4. $\overset{\frown}{ABC}$, $\overset{\frown}{ADC}$, $\overset{\frown}{BAD}$, $\overset{\frown}{BCD}$

Menu

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NAME ______________________ CLASS __________ DATE __________

Reteaching
8.6 Area and Volume Ratios

◆ **Skill A** Determining the ratio of the perimeters and the areas of similar plane figures and the ratio of the volumes of similar solids

Recall If the scale factor of two similar plane figures is $\frac{a}{b}$, then the ratio of their perimeters is $\frac{a}{b}$ and the ratio of their areas is $\frac{a^2}{b^2}$.

If the scale factor of two similar solids is $\frac{a}{b}$, then the ratio of corresponding areas is $\frac{a^2}{b^2}$ and the ratio of their volumes is $\frac{a^3}{b^3}$.

◆ **Example 1**

The scale factor of two rectangular pyramids is $\frac{3}{7}$. Find the ratio of the perimeters of their bases, the ratio of the areas of their bases, and the ratio of their volumes.

◆ **Solution**

The scale factor is $\frac{3}{7}$, so the ratio of the perimeters of their bases is $\frac{3}{7}$.

The ratio of the areas of their bases is $\frac{3^2}{7^2} = \frac{9}{49}$. The ratio of the volumes is $\frac{3^3}{7^3} = \frac{27}{343}$.

Find the indicated ratio for the similar figures.

1. rectangles, scale factor $\frac{4}{11}$;

ratio of areas: ______________; ratio of perimeters: ______________

2. pyramids, scale factor $\frac{1}{5}$;

ratio of areas of bases: ______________; ratio of volumes: ______________

3. spheres, scale factor $\frac{3}{2}$;

ratio of volumes: ______________; ratio of surface areas: ______________

4. $\triangle PQR$ and $\triangle FGH$, ratio of areas $\frac{16}{9}$;

ratio of perimeters: ______________; $\frac{QR}{GH}$: ______________

5. square prisms, ratio of areas of bases $\frac{81}{25}$;

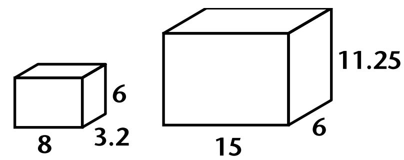
ratio of volumes: ______________; ratio of perimeters: ______________

◆ Skill B Using ratios for perimeters, areas, and volumes of similar figures

Recall The scale factor of two similar figures is related to the ratios of other linear dimensions as well as to the ratios of areas and, for space figures, volumes. Then you can use information about any one of the ratios to solve problems involving the others.

◆ **Example**

The scale factor of the similar rectangular prisms is $\frac{8}{15}$. If the volume of the smaller prism is 153.6 cubic meters, find the volume of the larger prism.



◆ **Solution**

Let V be the volume of the larger prism. Since the scale factor is $\frac{8}{15}$, the ratio of the volumes is $\frac{83}{153} = \frac{512}{3375}$. Then $\frac{153.6}{V} = \frac{512}{3375}$ and $V = 1012.5$.

The volume of the larger prism is 1012.5 cubic meters.

Complete.

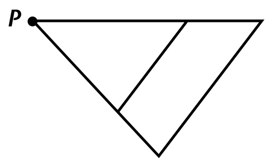
6. The ratio of the areas of two similar triangles is $\frac{16}{25}$. If the perimeter of the smaller triangle is 20 centimeters, then the perimeter of the larger is ______________.

7. The scale factor of two similar spheres is $\frac{5}{2}$. If the surface area of the larger sphere is 900π square yards, the surface area of the smaller sphere is ______________. If the volume of the smaller sphere is 288π cubic yards, then the volume of the larger sphere is ______________.

8. Square pyramids A and B are similar. The ratio of the volume of pyramid A to the volume of pyramid B is $\frac{8}{27}$. If the height of pyramid A is 4 inches, then the height of pyramid B is ______________. If the area of a lateral face of pyramid B is 22.5 square inches, then the area of a lateral face of pyramid A is ______________.

9. Cylinder X is 16 meters high. A similar cylinder, Y, is 28 meters high. If the area of the base of cylinder Y is 49π square meters, then the area of a base of cylinder X is ______________. If the volume of cylinder Y is 1372π cubic meters, then the volume of cylinder X is ______________.

ANSWERS

10.


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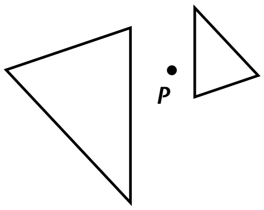

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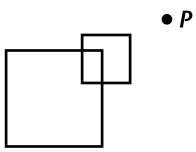

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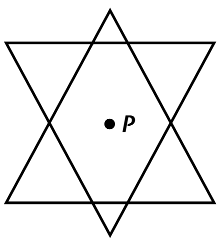

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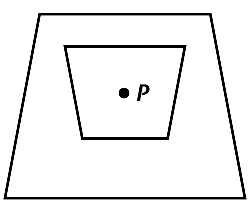


Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$

2. no

3. Yes; $DEFG \sim LMJK$

4. 7.2 **5.** 10.8 **6.** 24

7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS

2. Yes; $\triangle BCE \sim \triangle STR$; AA

3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS

4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS

5. Yes; $\triangle RST \sim \triangle MNP$; AA

6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS

7. 2 feet **8.** $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 **2.** 4.5 **3.** 4.5 **4.** 4.8

5. 9 **6.** 6.75 **7.** $3\frac{1}{3}$ **8.** 2.4

9. 2 **10.** 2 **11.** $2\frac{2}{3}$ **12.** $3\frac{1}{3}$

13. $1\frac{2}{3}$ **14.** $7\frac{2}{9}$

Lesson 8.5

1. 34 feet **2.** 45 feet **3.** 22 feet

4. 160 feet **5.** 75 feet **6.** 6 meters

7. 7.5 **8.** 9 **9.** 10 **10.** 5.6

Lesson 8.6

1. $\frac{16}{121}; \frac{4}{11}$ **2.** $\frac{1}{25}; \frac{1}{125}$ **3.** $\frac{27}{8}; \frac{9}{4}$

4. $\frac{4}{3}; \frac{4}{3}$ **5.** $\frac{729}{125}$ **6.** 25 cm

7. 144π square yards; 4500π cubic yards

8. 6 inches; 10 square inches

9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}, \overline{QB}, \overline{QC}, \overline{QD}$

2. $\overline{AC}, \overline{BD}$

3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$

4. $\overset{\frown}{ABC}, \overset{\frown}{ADC}, \overset{\frown}{BAD}, \overset{\frown}{BCD}$

NAME ______________________ CLASS __________ DATE __________

Reteaching
9.1 Chords and Arcs

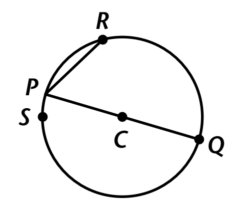
◆ **Skill A** Identifying parts of circles

Recall A circle, ⊙*P*, is the set of all points in a plane that are equidistant from a point *P*, called the center of the circle. A **radius** of ⊙*P* is a segment with one endpoint *P* and the other endpoint on the circle. A **chord** is a segment with both endpoints on the circle. A chord that contains *P* is a **diameter** of ⊙*P*. Two points on a circle determine a chord and two **arcs.** If the points are the endpoints of a diameter, both arcs are **semicircles.** If the two points are not endpoints of a diameter, they determine a **minor arc** (shorter than a semicircle) and a **major arc** (longer than a semicircle). Three points on an arc must be used to denote a semicircle or a major arc.

◆ **Example**

Given circle *C*, name each of the following.

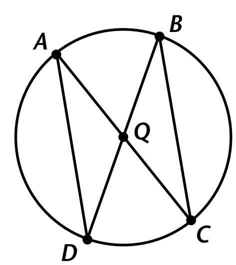
a. the center **b.** a radius or radii
c. a diameter **d.** a major arc
e. a semicircle **f.** a minor arc
g. a chord that is not a diameter and the arc of the chord



◆ **Solution**

a. *C* **b.** $\overline{CR}$, $\overline{CP}$, and $\overline{CQ}$
c. $\overline{PQ}$ **d.** $\overset{\frown}{RQS}$ or $\overset{\frown}{QRS}$
e. $\overset{\frown}{PRQ}$ or $\overset{\frown}{PSQ}$ **f.** $\overset{\frown}{PR}$, $\overset{\frown}{PS}$, $\overset{\frown}{SQ}$, $\overset{\frown}{RQ}$, or $\overset{\frown}{RS}$
g. $\overline{PR}$, $\overset{\frown}{PR}$

Given ⊙*Q*, name all of the following:



1. radii

____

2. diameters

____

3. chords that are not diameters and their arcs

____

4. semicircles

____

5. minor arcs

____

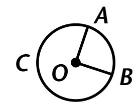
6. major arcs

____

NAME ______________________________ CLASS ______________ DATE ______________

◆ Skill B Finding the degree measure of an arc of a circle

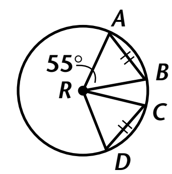
Recall A **central angle** of $\odot O$ is an angle with vertex O. In the figure, $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$ are the arcs intercepted by $\angle AOB$, and $\angle AOB$ is the central angle of arcs $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$.



The measure of a minor arc is the measure of its central angle. The measure of a major arc is 360° minus the measure of its central angle. The measure of a semicircle is 180°.
In a circle, or in congruent circles, arcs of congruent chords are congruent.

◆ **Example**
Find the measure of each arc.

a. $\overset{\frown}{AB}$ **b.** $\overset{\frown}{ADB}$ **c.** $\overset{\frown}{DC}$



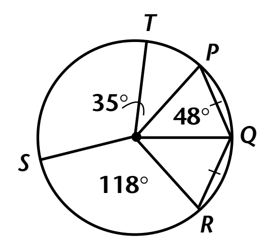
◆ **Solution**

a. $m\overset{\frown}{AB} = m\angle ARB = 55°$

b. $m\overset{\frown}{ADB} = 360° - m\angle ARB = 305°$

c. $\overline{DC} \cong \overline{AB}$, so $m\overset{\frown}{DC} = m\overset{\frown}{AB} = 55°$.

Find the measure of each arc.



7. $\overset{\frown}{PQ}$ ________________ **8.** $\overset{\frown}{QR}$ ________________

9. $\overset{\frown}{PSQ}$ ________________ **10.** $\overset{\frown}{SPR}$ ________________

11. $\overset{\frown}{QT}$ ________________ **12.** $\overset{\frown}{RS}$ ________________

◆ Skill C Finding the length of an arc of a circle

Recall If $\overset{\frown}{AB}$ is an arc of circle P with radius r and $m\overset{\frown}{AB} = M$, then the length, L, of $\overset{\frown}{AB}$ is given by $L = \frac{M}{360}(2\pi r)$.

◆ **Example**
Find the length of arc with measure 40° in a circle with radius 18 cm.

◆ **Solution**

$$L = \frac{M}{360}(2\pi r) = \frac{40}{360}(2)(\pi)(18) = 4\pi \approx 12.57$$

Find the length of an arc with the given measure in a circle with the given radius. Give your answer to the nearest hundredth.

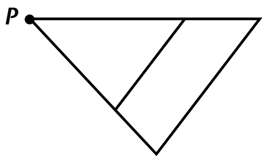
13. 45°; $r = 25$ m ________________

14. 60°; $r = 100$ in. ________________

15. 120°; $r = 15$ m ________________

16. 50°; $r = 9.5$ cm ________________

ANSWERS

10.


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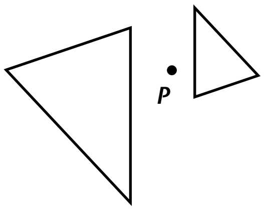

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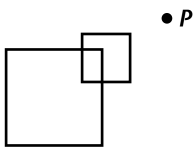
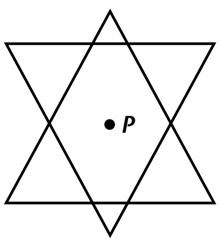
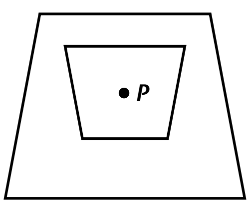


13.


14.


Lesson 8.2

1. Yes; $\triangle ABC \sim \triangle EFD$

2. no

3. Yes; $DEFG \sim LMJK$

4. 7.2 **5.** 10.8 **6.** 24

7. 4.5 inches

Lesson 8.3

1. Yes; $\triangle JKL \sim \triangle STR$; SSS

2. Yes; $\triangle BCE \sim \triangle STR$; AA

3. Yes; $\triangle UVW \sim \triangle ZXY$; SAS

4. Yes; $\triangle FGH \sim \triangle LMK$ or $\triangle FGH \sim \triangle MLK$; SSS

5. Yes; $\triangle RST \sim \triangle MNP$; AA

6. Yes; $\triangle ABC \sim \triangle ZXY$; SAS

7. 2 feet **8.** $36\frac{1}{4}$ feet

Lesson 8.4

1. 2.5 **2.** 4.5 **3.** 4.5 **4.** 4.8

5. 9 **6.** 6.75 **7.** $3\frac{1}{3}$ **8.** 2.4

9. 2 **10.** 2 **11.** $2\frac{2}{3}$ **12.** $3\frac{1}{3}$

13. $1\frac{2}{3}$ **14.** $7\frac{2}{9}$

Lesson 8.5

1. 34 feet **2.** 45 feet **3.** 22 feet

4. 160 feet **5.** 75 feet **6.** 6 meters

7. 7.5 **8.** 9 **9.** 10 **10.** 5.6

Lesson 8.6

1. $\frac{16}{121}; \frac{4}{11}$ **2.** $\frac{1}{25}; \frac{1}{125}$ **3.** $\frac{27}{8}; \frac{9}{4}$

4. $\frac{4}{3}; \frac{4}{3}$ **5.** $\frac{729}{125}$ **6.** 25 cm

7. 144π square yards; 4500π cubic yards

8. 6 inches; 10 square inches

9. 16π meters; 256π cubic meters

Reteaching — Chapter 9

Lesson 9.1

1. $\overline{QA}, \overline{QB}, \overline{QC}, \overline{QD}$

2. $\overline{AC}, \overline{BD}$

3. $\overline{AD}$ and $\overset{\frown}{AD}$, $\overline{BC}$ and $\overset{\frown}{BC}$

4. $\overset{\frown}{ABC}, \overset{\frown}{ADC}, \overset{\frown}{BAD}, \overset{\frown}{BCD}$

5. $\overarc{AB}$, $\overarc{BC}$, $\overarc{CD}$, $\overarc{AD}$

6. $\overarc{ADB}$, $\overarc{BAC}$, $\overarc{DAC}$, $\overarc{ABD}$

7. 48° 8. 48° 9. 312°

10. 242° 11. 83° 12. 118°

13. 19.63 meters

14. 104.72 inches

15. 31.42 meters

16. 8.29 centimeters

Lesson 9.2

1. 9 2. 9.4 3. 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, $\triangle APQ$ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 7. 15.6 8. 8.2 9. 28.4 10. 11.6

Lesson 9.3

1. 26° 2. 15° 3. 41°

4. 15° 5. 23° 6. 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

9. 204° 10. 90° 11. 90° 12. 38°

13. 76° 14. 104° 15. 52° 16. 27°

17. 90° 18. 126° 19. 63° 20. 158°

21. 79°

Lesson 9.4

1. 108° 2. 48° 3. 75° 4. 58.5°

5. 106° 6. 66° 7. 57° 8. 48° 9. 37°

Lesson 9.5

1. 11.25 2. 9.25 3. 9 4. 12.8 5. 8.94

6. 6.24 7. 3.33 8. 6.22 9. 7.2

Lesson 9.6

1. ±1.5; ±1.5; (0, 0); 1.5

2. ±8.49; 12, −6; (0, 3); 9

3. −7.85, 17.85; −10, 14; (5, 2); 13

4. −0.26, −7.74; none; (−4, 1); 3.87

5. $x^2 + y^2 = 400$

6. $(x - 5)^2 + y^2 = 324$

7. $(x + 2)^2 + (y + 2)^2 = 1.44$

8. $(x + 3)^2 + (y + 6)^2 = 48$

9. $(x - 5)^2 + (y - 5)^2 = 100$

10. $(x + 6)^2 + (y - 4)^2 = 18$

11. $(x - 1)^2 + y^2 = 4$

12. $(x - 3)^2 + (y - 3)^2 = 4$

13. $(x + 2)^2 + (y + 2)^2 = 16$

Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 2. 2 3. $\frac{5}{9}$; 0.56 4. $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 6. $\frac{19}{12}$; 1.58 7. 0.12 8. 0.81

9. 0.27 10. 14.301 11. 1.28 12. 0.55

13. 1.8 14. 3.08 15. 0.97 16. 5.14

NAME ______________________ CLASS __________ DATE __________

Reteaching
9.2 Tangents to Circles

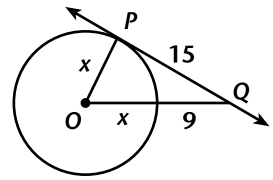
◆ **Skill A** Using the Tangent Theorem and its converse

Recall **The Tangent Theorem:** If a line is tangent to a circle, then the line is perpendicular to a radius of the circle drawn to the point of tangency.

The Converse of the Tangent Theorem: If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

◆ **Example 1**

$\overleftrightarrow{PQ}$ is tangent to $\odot O$. Find the value of x.



◆ **Solution**

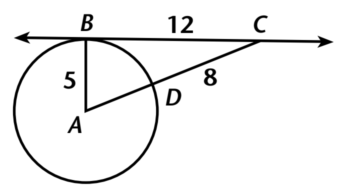
$\overleftrightarrow{PQ}$ is tangent to $\odot O$. By the Tangent Theorem, $\overleftrightarrow{PQ}$ is perpendicular to radius $\overline{OP}$.

By the Pythagorean Theorem,

$$(OQ)^2 = (PQ)^2 + (OP)^2$$
$$(x + 9)^2 = x^2 + 225$$
$$x^2 + 18x + 81 = x^2 + 225$$
$$18x = 144$$
$$x = 8$$

◆ **Example 2**

Show that $\overleftrightarrow{BC}$ is tangent to $\odot A$.



◆ **Solution**

$\overline{AB}$ is a radius of $\odot A$, so $AD = AB = 5$.

Then $AC = AD + DC = 5 + 8 = 13$.

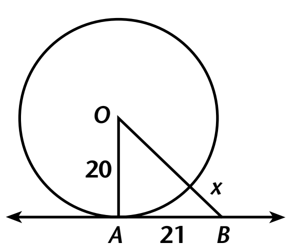
$(AB)^2 + (BC)^2 = (AC)^2$

$25 + 144 = 169.$

By the converse of the Pythagorean Theorem, $\triangle ABC$ is a right triangle and $\overleftrightarrow{BC}$ is perpendicular to radius $\overline{AB}$. By the converse of the Tangent Theorem, $\overleftrightarrow{BC}$ is tangent to $\odot A$.

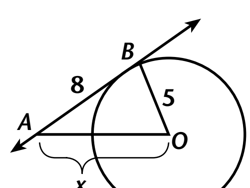
In each figure, $\overleftrightarrow{AB}$ is tangent to $\odot O$. Find the value of x to the nearest tenth.

1.



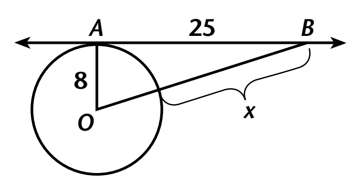
$x =$ ______________

2.



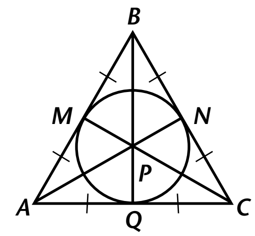
$x =$ ______________

3.



$x =$ ______________

4. In the figure, $\triangle ABC$ is an equilateral triangle with $AB = 6$. M, N, and Q are the midpoints of the sides of $\triangle ABC$ The radius of $\odot P$ is $\sqrt{3}$ and $PA = PB = PC = 2\sqrt{3}$. Show that $\odot P$ is inscribed in $\triangle ABC$, that is, show that each side of $\triangle ABC$ is tangent to $\odot P$.



______

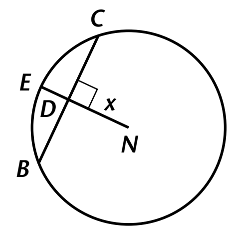
______

______

◆ Skill B Using the Radius and Chord Theorem

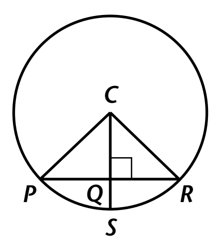
Recall **The Radius and Chord Theorem:** A radius that is perpendicular to a chord of a circle bisects the chord.

◆ **Example**
The radius of $\odot N$ is 16 and $BC = 10$. Find the value of x to the nearest tenth.



◆ **Solution**
Radius $\overline{NE}$ is perpendicular to chord $\overline{BC}$, so $\overline{NE}$ bisects $\overline{BC}$ by the Radius and Chord Theorem. Then $BD = 5$.
By the Pythagorean Theorem, $x^2 + 5^2 = (NB)^2 = 16^2$, and $x = \sqrt{231} \approx 15.2$.

Exercises 5–8 refer to $\odot C$. Give your answers to the nearest tenth, if necessary.



5. Name two pairs of congruent segments. ______

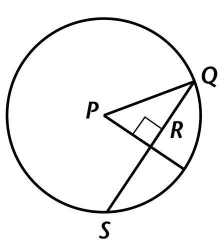
6. If $PR = 8$, find PQ. ______

7. If $CQ = 10$ and $PR = 24$, find the radius of $\odot C$. ______

8. If the radius of $\odot C$ is 9 and $QS = 1$, find PR. ______

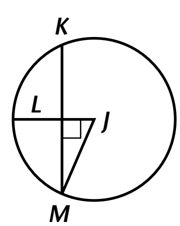
Find the indicated measure. Round your answers to the nearest tenth if necessary.

9.



$PR = 18$, $QS = 44$, $PQ =$ ______

10.



$JM = 24$, $KM = 42$, $JL =$ ______

5. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, $\overset{\frown}{AD}$

6. $\overset{\frown}{ADB}$, $\overset{\frown}{BAC}$, $\overset{\frown}{DAC}$, $\overset{\frown}{ABD}$

7. 48° 8. 48° 9. 312°

10. 242° 11. 83° 12. 118°

13. 19.63 meters

14. 104.72 inches

15. 31.42 meters

16. 8.29 centimeters

Lesson 9.2

1. 9 2. 9.4 3. 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, $\triangle APQ$ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 7. 15.6 8. 8.2 9. 28.4 10. 11.6

Lesson 9.3

1. 26° 2. 15° 3. 41°

4. 15° 5. 23° 6. 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

9. 204° 10. 90° 11. 90° 12. 38°

13. 76° 14. 104° 15. 52° 16. 27°

17. 90° 18. 126° 19. 63° 20. 158°

21. 79°

Lesson 9.4

1. 108° 2. 48° 3. 75° 4. 58.5°

5. 106° 6. 66° 7. 57° 8. 48° 9. 37°

Lesson 9.5

1. 11.25 2. 9.25 3. 9 4. 12.8 5. 8.94

6. 6.24 7. 3.33 8. 6.22 9. 7.2

Lesson 9.6

1. ± 1.5; ± 1.5; (0, 0); 1.5

2. ± 8.49; 12, -6; (0, 3); 9

3. -7.85, 17.85; -10, 14; (5, 2); 13

4. -0.26, -7.74; none; $(-4, 1)$; 3.87

5. $x^2 + y^2 = 400$

6. $(x - 5)^2 + y^2 = 324$

7. $(x + 2)^2 + (y + 2)^2 = 1.44$

8. $(x + 3)^2 + (y + 6)^2 = 48$

9. $(x - 5)^2 + (y - 5)^2 = 100$

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Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 2. 2 3. $\frac{5}{9}$; 0.56 4. $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 6. $\frac{19}{12}$; 1.58 7. 0.12 8. 0.81

9. 0.27 10. 14.301 11. 1.28 12. 0.55

13. 1.8 14. 3.08 15. 0.97 16. 5.14

NAME ______________________ CLASS __________ DATE __________

Reteaching
9.3 Inscribed Angles and Arcs

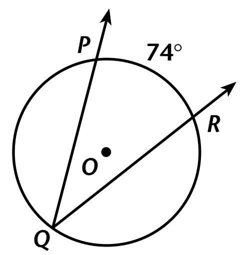
◆ **Skill A** Using the Inscribed Angle Theorem

Recall An **inscribed angle** of a circle is an angle whose vertex lies on the circle and whose sides are chords of the circle. You can use the Inscribed Angle Theorem to determine the measure of an inscribed angle of a circle.

The Inscribed Angle Theorem: The measure of an angle inscribed in a circle is equal to one-half the measure of the intercepted arc.

◆ **Example**

Find the measure of $\angle PQR$.

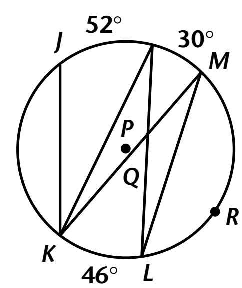


◆ **Solution**

By the Inscribed Angle Theorem,

$m\angle PQR = \frac{1}{2} \cdot m\overparen{PR} = \frac{1}{2}(74°) = 37°.$

Find the measure of each inscribed angle of $\odot P$.



1. $\angle JKN$ ______________
2. $\angle NKM$ ______________
3. $\angle JKM$ ______________
4. $\angle NLM$ ______________
5. $\angle KNL$ ______________
6. $\angle KML$ ______________

Refer to the figure for Exercises 1–6. Can you use the Inscribed Angle Theorem to determine the measure of the given angle? Explain why or why not.

7. $\angle NQM$ ______________________________
8. $\angle MLR$ ______________________________
9. Complete: $\angle A$ is inscribed in $\odot P$ and intersects $\overparen{AB}$. If $m\angle A = 102°$, then $m\overparen{AB} =$ ______________.

◆ Skill B Using the corollaries of the Inscribed Angle Theorem

Recall The following corollaries follow directly from the Inscribed Angle Theorem.

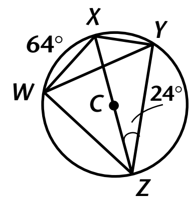
The Right-Angle Corollary: If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

The Arc-Intercept Corollary: If two inscribed angles intercept the same arc, then the angles have the same measure.

◆ **Example**

$\overline{XZ}$ is a diameter of ⊙'C.
Find the measure of each angle.

a. $\angle XYZ$
b. $\angle XWY$
c. $\angle YWZ$
d. $\angle WYZ$



◆ **Solution**

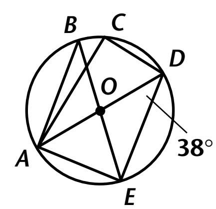
a. $\angle XYZ$ intercepts $\overset{\frown}{XWZ}$, which is a semicircle, so $\angle XYZ$ is a right angle and $m\angle XYZ = 90°$.

b. $\angle XWY$ intercepts the same arc as $\angle XZY$, so $m\angle XWY = m\angle XZY = 24°$.

c. $\angle YWZ$ intercepts $\overset{\frown}{YZ}$. $\overset{\frown}{XY}$ and $\overset{\frown}{YZ}$ are adjacent arcs, that is, they share only one point. So $m\overset{\frown}{XY} + m\overset{\frown}{YZ} = m\overset{\frown}{XYZ}$. Since $\angle XZY$ intercepts $\overset{\frown}{XY}$ and $m\angle XZY = 24°$, $m\overset{\frown}{XY} = 48°$ by the Inscribed Angle Theorem. Substituting, $48° + m\overset{\frown}{YZ} = 180°$ and $m\overset{\frown}{YZ} = 132°$. Again by the Inscribed Angle Theorem, $m\angle YWZ = 66°$.

d. $m\overset{\frown}{XW} + m\overset{\frown}{WZ} = m\overset{\frown}{XWZ} = 180°$, so $m\overset{\frown}{WZ} = 180° - 64° = 116°$. Then by the Inscribed Angle Theorem, $m\angle WYZ = 58°$.

Find the measure of each angle or arc in ⊙*O* with diameters $\overline{AD}$ and $\overline{BE}$.

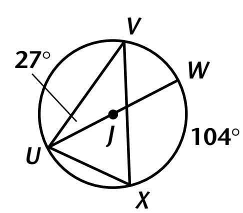


10. $\angle BAE$ ______________ **11.** $\angle ACD$ ______________

12. $\angle ABE$ ______________ **13.** $\overset{\frown}{AE}$ ______________

14. $\overset{\frown}{AB}$ ______________ **15.** $\angle AEB$ ______________

Find the measure of each angle or arc in ⊙*J* with diameter $\overline{UW}$.



16. $\angle VXW$ ______________ **17.** $\angle UXW$ ______________

18. $\overset{\frown}{UV}$ ______________ **19.** $\angle UXV$ ______________

20. $\overset{\frown}{VWX}$ ______________ **21.** $\angle VUX$ ______________

5. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, $\overset{\frown}{AD}$

6. $\overset{\frown}{ADB}$, $\overset{\frown}{BAC}$, $\overset{\frown}{DAC}$, $\overset{\frown}{ABD}$

7. 48° 8. 48° 9. 312°

10. 242° 11. 83° 12. 118°

13. 19.63 meters

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Lesson 9.2

1. 9 2. 9.4 3. 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, $\triangle APQ$ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 7. 15.6 8. 8.2 9. 28.4 10. 11.6

Lesson 9.3

1. 26° 2. 15° 3. 41°

4. 15° 5. 23° 6. 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

9. 204° 10. 90° 11. 90° 12. 38°

13. 76° 14. 104° 15. 52° 16. 27°

17. 90° 18. 126° 19. 63° 20. 158°

21. 79°

Lesson 9.4

1. 108° 2. 48° 3. 75° 4. 58.5°

5. 106° 6. 66° 7. 57° 8. 48° 9. 37°

Lesson 9.5

1. 11.25 2. 9.25 3. 9 4. 12.8 5. 8.94

6. 6.24 7. 3.33 8. 6.22 9. 7.2

Lesson 9.6

1. ± 1.5; ± 1.5; (0, 0); 1.5

2. ± 8.49; 12, -6; (0, 3); 9

3. -7.85, 17.85; -10, 14; (5, 2); 13

4. -0.26, -7.74; none; $(-4, 1)$; 3.87

5. $x^2 + y^2 = 400$

6. $(x - 5)^2 + y^2 = 324$

7. $(x + 2)^2 + (y + 2)^2 = 1.44$

8. $(x + 3)^2 + (y + 6)^2 = 48$

9. $(x - 5)^2 + (y - 5)^2 = 100$

10. $(x + 6)^2 + (y - 4)^2 = 18$

11. $(x - 1)^2 + y^2 = 4$

12. $(x - 3)^2 + (y - 3)^2 = 4$

13. $(x + 2)^2 + (y + 2)^2 = 16$

Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 2. 2 3. $\frac{5}{9}$; 0.56 4. $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 6. $\frac{19}{12}$; 1.58 7. 0.12 8. 0.81

9. 0.27 10. 14.301 11. 1.28 12. 0.55

13. 1.8 14. 3.08 15. 0.97 16. 5.14

NAME ______________________ CLASS __________ DATE __________

Reteaching
9.4 Angles Formed by Secants and Tangents

◆ **Skill A** Finding the measure of an angle formed by a secant and a tangent of a circle that intersect on the circle

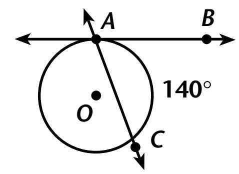
Recall **Theorem:** If a tangent and a secant or chord of a circle intersect on the circle at the point of tangency, the measure of the angle formed is half the measure of the intercepted arc.

◆ **Example**

$\overleftrightarrow{AB}$ is tangent to $\odot O$ at A. Find $m\angle BAC$.

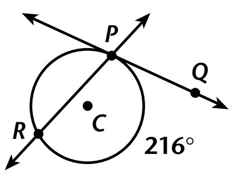
◆ **Solution**

$m\angle BAC = \frac{1}{2}m\overset{\frown}{AC} = \frac{1}{2} \cdot 140° = 70°$



$\overleftrightarrow{PQ}$ is tangent to $\odot C$ at P. Find the measure of $\angle QPR$.

1.



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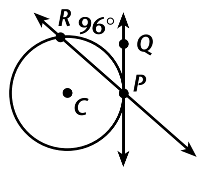
2.



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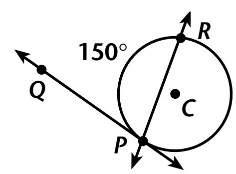
3.



____

◆ **Skill B** Finding the measure of an angle formed by two secants or chords of a circle that intersect inside the circle

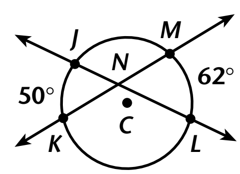
Recall The measure of an angle formed by two secants or chords that intersect in the interior of a circle is equal to half the sum of the measures of the intercepted arcs.

◆ **Example**

Find $m\angle JNK$.

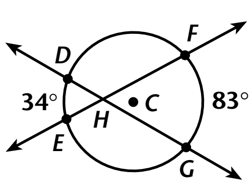
◆ **Solution**

$m\angle JNK = \frac{1}{2}(m\overset{\frown}{JK} + \overset{\frown}{ML}) = \frac{1}{2}(50° + 62°) = 56°$



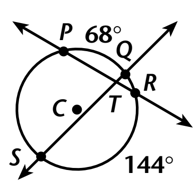
Find the measure of each angle.

4.



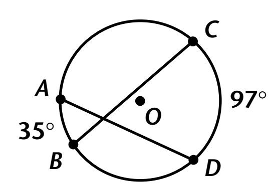
$m\angle FHG =$ ______________

5.



$m\angle STR =$ ______________

6.



$m\angle DEC =$ ______________

Menu

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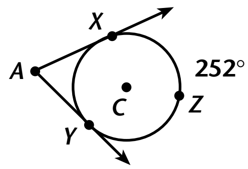
◆ **Skill C** Finding the measure of an angle formed by two secants, two tangents, or a secant and a tangent of a circle that intersect outside the circle

Recall The measure of an angle formed by two secants, two tangents, or a secant and a tangent of a circle that intersect outside the circle is one-half the difference of the measures of the intercepted arcs.

◆ **Example**

$\overleftrightarrow{AX}$ is tangent to $\odot C$ at X. $\overleftrightarrow{AY}$ is tangent to $\odot C$ at Y. Find $m\angle XAY$.



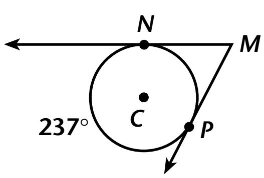
◆ **Solution**

The measure of $\angle XAY$ is one-half the difference of the measures of the intercepted arcs, $\overset{\frown}{XZY}$ and $\overset{\frown}{XY}$. Since $m\overset{\frown}{XZY} + m\overset{\frown}{XY} = 360°$,

$m\overset{\frown}{XY} = 108°$. Then $m\angle XAY = \frac{1}{2}(\overset{\frown}{XZY} - \overset{\frown}{XY}) = \frac{1}{2}(252° - 108°) = 72°$

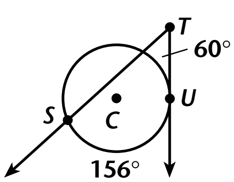
Find the measure of the each angle.

7.



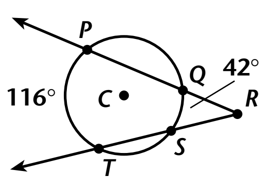
$m\angle NMP =$ ______________

8.



$m\angle STU =$ ______________

9.



$m\angle QRS =$ ______________

5. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, $\overset{\frown}{AD}$

6. $\overset{\frown}{ADB}$, $\overset{\frown}{BAC}$, $\overset{\frown}{DAC}$, $\overset{\frown}{ABD}$

7. 48° 8. 48° 9. 312°

10. 242° 11. 83° 12. 118°

13. 19.63 meters

14. 104.72 inches

15. 31.42 meters

16. 8.29 centimeters

Lesson 9.2

1. 9 2. 9.4 3. 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, $\triangle APQ$ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 7. 15.6 8. 8.2 9. 28.4 10. 11.6

Lesson 9.3

1. 26° 2. 15° 3. 41°

4. 15° 5. 23° 6. 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

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17. 90° 18. 126° 19. 63° 20. 158°

21. 79°

Lesson 9.4

1. 108° 2. 48° 3. 75° 4. 58.5°

5. 106° 6. 66° 7. 57° 8. 48° 9. 37°

Lesson 9.5

1. 11.25 2. 9.25 3. 9 4. 12.8 5. 8.94

6. 6.24 7. 3.33 8. 6.22 9. 7.2

Lesson 9.6

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10. $(x + 6)^2 + (y - 4)^2 = 18$

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12. $(x - 3)^2 + (y - 3)^2 = 4$

13. $(x + 2)^2 + (y + 2)^2 = 16$

Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 2. 2 3. $\frac{5}{9}$; 0.56 4. $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 6. $\frac{19}{12}$; 1.58 7. 0.12 8. 0.81

9. 0.27 10. 14.301 11. 1.28 12. 0.55

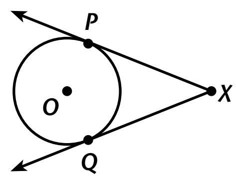
13. 1.8 14. 3.08 15. 0.97 16. 5.14

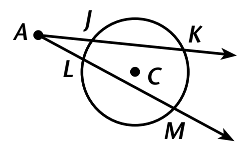
NAME ______________________ CLASS ____________ DATE ____________

Reteaching
9.5 Segments of Tangents, Secants, and Chords

◆ **Skill A** Finding lengths of segments determined by two tangents or two secants of a circle that intersect outside the circle

Recall The figures below illustrate the terms *tangent segment*, *secant segment*, and *external secant segment*. $\overleftrightarrow{XP}$ and $\overleftrightarrow{XQ}$ are tangents of $\odot O$. $\overleftrightarrow{AK}$ and $\overleftrightarrow{AM}$ are secants of $\odot C$.





$\overline{XP}$ and $\overline{XQ}$ are tangent segments.

$\overline{AK}$ and $\overline{AM}$ are secant segments.
$\overline{AJ}$ and $\overline{AL}$ are external secant segments.

In the figures, $XP = XQ$ and $AK \cdot AJ = AM \cdot AL$. These relationships are formalized in the following theorems.

If two tangents are drawn to a circle from the same external point, the tangent segments are congruent.

If two secants are drawn to a circle from the same external point, the product of the lengths of one secant segment and its external segment is equal to the product of the lengths of the other segment and its external segment.

◆ **Example**

Refer to $\odot O$ and $\odot C$ above. Complete each statement.

a. In $\odot O$, if $XQ = 7.2$, then $XP =$ _?_.

b. In $\odot C$, if $AJ = 5$, $AK = 15$, and $AM = 13.6$, then $AL =$ _?_.

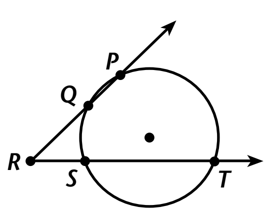
◆ **Solution**

a. $\overleftrightarrow{XP}$ and $\overleftrightarrow{XQ}$ are tangents to $\odot O$ from the same external point, so $XP = XQ = 7.2$.

b. Secants $\overleftrightarrow{AK}$ and $\overleftrightarrow{AM}$ intersect at point A outside $\odot Q$, so $AK \cdot AJ = AM \cdot AL$.
Then $AL = \frac{AK \cdot AJ}{AM} = \frac{5 \cdot 15}{13.6} = 5.5$.

Find the measure of the indicated length.

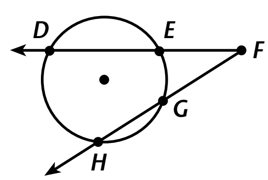
1.



$RS = 5$, $RT = 18$, $RQ = 8$

$RP =$ ______________

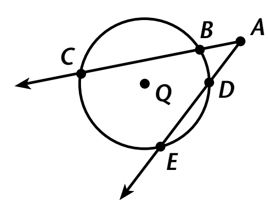
2.



$EF = 8$, $DF = 18$, $FH = 16$

$FG =$ ______________

3.



$AB = 3$, $AC = 12$, $AD = 4$

$AE =$ ______________

◆ **Skill B** Finding lengths of segments determined by a tangent and a secant of a circle that intersect outside the circle or by two chords that intersect

Recall If a secant and a tangent to a circle intersect at a point outside the circle, then the product of the lengths of the secant segment and its external segment is equal to the square of the length of the tangent segment.

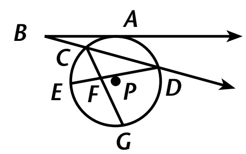
If two chords of a circle intersect, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other.

◆ **Example**

$\overleftrightarrow{AB}$ is tangent to $\odot P$.

a. If $BC = 6$ and $CD = 10$, find AB to the nearest hundredth.

b. If $EF = 4$, $FD = 8$, and $CF = 4.5$, find FG to the nearest hundredth.



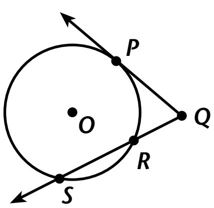
◆ **Solution**

a. Tangent $\overleftrightarrow{AB}$ and secant $\overleftrightarrow{BD}$ intersect at point B outside of $\odot P$.
Then $BC \cdot CD = (AB)^2$ and $AB = \sqrt{BC \cdot CD} = \sqrt{60} \approx 7.75$.

b. $\overline{ED}$ and $\overline{CG}$ are chords of $\odot P$. Then $EF \cdot FD = CF \cdot FG$
and $FG = \dfrac{EF \cdot CD}{CF} = \dfrac{4 \cdot 8}{4.5} = 7\frac{1}{9} \approx 7.11$.

In Exercises 4–9, $\overleftrightarrow{PQ}$ is tangent to $\odot O$. Find each length to the nearest hundredth.

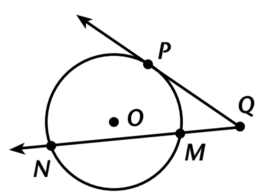
4.



$PQ = 8$, $QR = 5$

$QS =$ ______________

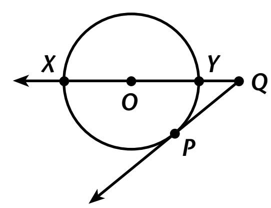
5.



$QM = 5$, $QN = 16$

$QP =$ ______________

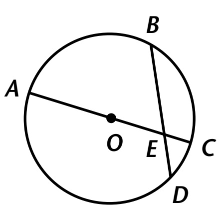
6.



$QY = 3$, $QX = 13$

$QP =$ ______________

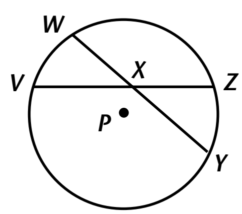
7.



$AE = 5$, $EC = 1$, $ED = 1.5$

$BE =$ ______________

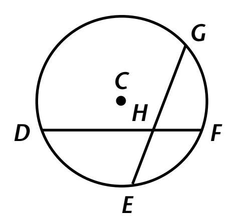
8.



$XZ = 7$, $VX = 8$, $XY = 9$

$WX =$ ______________

9.



$DH = 9$, $HF = 4$, $HE = 5$

$GH =$ ______________

5. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, $\overset{\frown}{AD}$

6. $\overset{\frown}{ADB}$, $\overset{\frown}{BAC}$, $\overset{\frown}{DAC}$, $\overset{\frown}{ABD}$

7. 48° **8.** 48° **9.** 312°

10. 242° **11.** 83° **12.** 118°

13. 19.63 meters

14. 104.72 inches

15. 31.42 meters

16. 8.29 centimeters

Lesson 9.2

1. 9 **2.** 9.4 **3.** 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, $\triangle APQ$ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 **7.** 15.6 **8.** 8.2 **9.** 28.4 **10.** 11.6

Lesson 9.3

1. 26° **2.** 15° **3.** 41°

4. 15° **5.** 23° **6.** 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

9. 204° **10.** 90° **11.** 90° **12.** 38°

13. 76° **14.** 104° **15.** 52° **16.** 27°

17. 90° **18.** 126° **19.** 63° **20.** 158°

21. 79°

Lesson 9.4

1. 108° **2.** 48° **3.** 75° **4.** 58.5°

5. 106° **6.** 66° **7.** 57° **8.** 48° **9.** 37°

Lesson 9.5

1. 11.25 **2.** 9.25 **3.** 9 **4.** 12.8 **5.** 8.94

6. 6.24 **7.** 3.33 **8.** 6.22 **9.** 7.2

Lesson 9.6

1. ±1.5; ±1.5; (0, 0); 1.5

2. ±8.49; 12, -6; (0, 3); 9

3. -7.85, 17.85; -10, 14; (5, 2); 13

4. -0.26, -7.74; none; $(-4, 1)$; 3.87

5. $x^2 + y^2 = 400$

6. $(x - 5)^2 + y^2 = 324$

7. $(x + 2)^2 + (y + 2)^2 = 1.44$

8. $(x + 3)^2 + (y + 6)^2 = 48$

9. $(x - 5)^2 + (y - 5)^2 = 100$

10. $(x + 6)^2 + (y - 4)^2 = 18$

11. $(x - 1)^2 + y^2 = 4$

12. $(x - 3)^2 + (y - 3)^2 = 4$

13. $(x + 2)^2 + (y + 2)^2 = 16$

Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 **2.** 2 **3.** $\frac{5}{9}$; 0.56 **4.** $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 **6.** $\frac{19}{12}$; 1.58 **7.** 0.12 **8.** 0.81

9. 0.27 **10.** 14.301 **11.** 1.28 **12.** 0.55

13. 1.8 **14.** 3.08 **15.** 0.97 **16.** 5.14

NAME ______________________ CLASS ______________ DATE ______________

Reteaching
9.6 Circles in the Coordinate Plane

◆ **Skill A** Using an equation of a circle to determine the intercept(s), the center, and the radius of the circle

Recall The equation $(x - k)^2 + (y - k)^2 = r^2$ is the *standard form of the equation of a circle* with center (h, k) and radius r. If the center is the origin, then $h = 0$ and $k = 0$ and the equation can be written $x^2 + y^2 = r^2$.

◆ **Example**

A circle has equation $(x + 1)^2 + y^2 = 53$. Find:

a. the x-intercept(s) and the y-intercept(s).

b. the center and radius of the circle.

◆ **Solution**

a. (1) Let $y = 0$ and solve for x.

$(x + 1)^2 = 53$

$x + 1 = \pm\sqrt{53}$

$x = -1 \pm \sqrt{53}$

(2) Let $x = 0$ and solve for y:

$1^2 + y^2 = 53$

$y^2 = 52$

$y = \pm\sqrt{52} = \pm 2\sqrt{13}$

The x-intercepts are approximately -8.28 and 6.28. The y-intercepts are approximately -7.21 and 7.21.

b. First write the equation in standard form.

$(x - k)^2 + (y - k)^2 = r^2$ $\quad$ $(x - (-1))^2 + (y - 0)^2 = \left(\sqrt{53}\right)^2$

$h = -1$, $k = 0$, and $r = \sqrt{53}$

The center is $(-1, 0)$ and the radius is $\sqrt{53} \approx 7.28$.

Find the x-intercept(s), the y-intercept(s), the center, and the radius of the circle with the given equation. If necessary, round to the nearest hundredth.

1. equation: $x^2 + y^2 = 2.25$ $\quad$ x-intercept(s): ______________

y-intercepts: ______________ center: ______________ radius: ______________

2. equation: $x^2 + (y - 3)^2 = 81$ $\quad$ x-intercept(s): ______________

y-intercepts: ______________ center: ______________ radius: ______________

3. equation: $(x - 5)^2 + (y - 2)^2 = 169$ $\quad$ x-intercept(s): ______________

y-intercepts: ______________ center: ______________ radius: ______________

4. equation: $(x + 4)^2 + (y - 1)^2 = 15$ $\quad$ x-intercept(s): ______________

y-intercepts: ______________ center: ______________ radius: ______________

NAME ______________________ CLASS ____________ DATE ____________

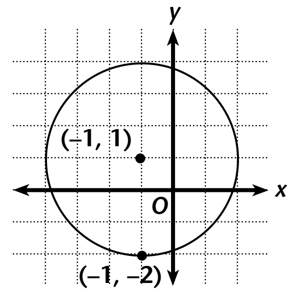
◆ Skill B Writing an equation of a circle

Recall Given the coordinates of the center of a circle and the radius of a circle, you can write an equation of the circle.

◆ Example
Write an equation in standard form for each circle.

a. the circle with center $(-7, 3)$ and radius 11

b. $\odot C$, shown at the right



◆ Solution

a. Substitute -7 for h, 3 for k, and 11 for r in the standard equation.
$(x - (-7))^2 + (y - 3)^2 = 11^2$
$(x + 7)^2 + (y - 3)^2 = 121$

b. From the graph, you can determine that the center of the circle is $(-1,1)$ and the radius is 3. Substitute -1 for h, 1 for k, and 3 for r in the standard equation.
$(x - (-1))^2 + (y - 1)^2 = 3^2$
$(x + 1)^2 + (y - 1)^2 = 9$

Write an equation in standard form for each circle.

5. the circle with center $(0, 0)$ and radius 20 ____________

6. the circle with center $(5, 0)$ and radius 18 ____________

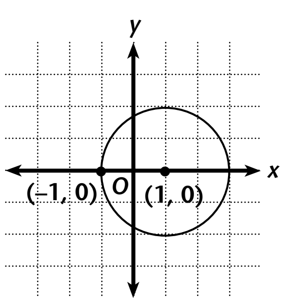
7. the circle with center $(-2, -2)$ and radius 1.2 ____________

8. the circle with center $(-3, -6)$ and radius $4\sqrt{3}$ ____________

9. the circle with center $(5, 5)$ and radius 10 ____________

10. the circle with center $(-6, 4)$ and radius $3\sqrt{2}$ ____________

11.



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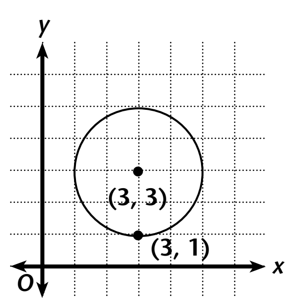
12.



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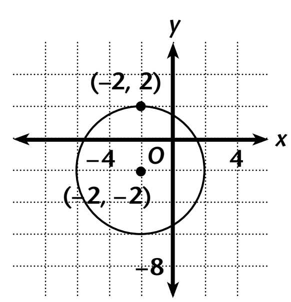
13.



____

5. $\widehat{AB}$, $\widehat{BC}$, $\widehat{CD}$, $\widehat{AD}$

6. $\widehat{ADB}$, $\widehat{BAC}$, $\widehat{DAC}$, $\widehat{ABD}$

7. 48° 8. 48° 9. 312°

10. 242° 11. 83° 12. 118°

13. 19.63 meters

14. 104.72 inches

15. 31.42 meters

16. 8.29 centimeters

Lesson 9.2

1. 9 2. 9.4 3. 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, ΔAPQ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 7. 15.6 8. 8.2 9. 28.4 10. 11.6

Lesson 9.3

1. 26° 2. 15° 3. 41°

4. 15° 5. 23° 6. 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

9. 204° 10. 90° 11. 90° 12. 38°

13. 76° 14. 104° 15. 52° 16. 27°

17. 90° 18. 126° 19. 63° 20. 158°

21. 79°

Lesson 9.4

1. 108° 2. 48° 3. 75° 4. 58.5°

5. 106° 6. 66° 7. 57° 8. 48° 9. 37°

Lesson 9.5

1. 11.25 2. 9.25 3. 9 4. 12.8 5. 8.94

6. 6.24 7. 3.33 8. 6.22 9. 7.2

Lesson 9.6

1. ± 1.5; ± 1.5; (0, 0); 1.5

2. ± 8.49; 12, -6; (0, 3); 9

3. -7.85, 17.85; -10, 14; (5, 2); 13

4. -0.26, -7.74; none; $(-4, 1)$; 3.87

5. $x^2 + y^2 = 400$

6. $(x - 5)^2 + y^2 = 324$

7. $(x + 2)^2 + (y + 2)^2 = 1.44$

8. $(x + 3)^2 + (y + 6)^2 = 48$

9. $(x - 5)^2 + (y - 5)^2 = 100$

10. $(x + 6)^2 + (y - 4)^2 = 18$

11. $(x - 1)^2 + y^2 = 4$

12. $(x - 3)^2 + (y - 3)^2 = 4$

13. $(x + 2)^2 + (y + 2)^2 = 16$

Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 2. 2 3. $\frac{5}{9}$; 0.56 4. $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 6. $\frac{19}{12}$; 1.58 7. 0.12 8. 0.81

9. 0.27 10. 14.301 11. 1.28 12. 0.55

13. 1.8 14. 3.08 15. 0.97 16. 5.14

NAME ______________________ CLASS ____________ DATE ____________

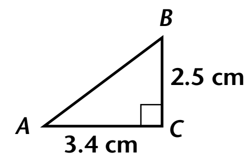
Reteaching
10.1 Tangent Ratios

◆ Skill A Finding the tangent of an acute angle

Recall If A is an acute angle of a right triangle, the **tangent** of A, written tan $\angle A$ or tan A, is the ratio of the length of the leg opposite $\angle A$ to the length of the leg adjacent to $\angle A$. This can be abbreviated $\tan A = \frac{\text{opposite}}{\text{adjacent}}$. If the measure in degrees of $\angle A$ is θ, the ratio may also be referred to as tan θ.

The tangent of an angle can be determined using a scientific or graphics calculator, or a table of trigonometric ratios, such as the one given on page 870 of your textbook.

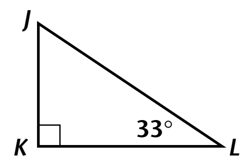
◆ **Example 1**
Find tan A. Give your answer to the nearest hundredth.



◆ **Solution**
$BC = 2.5$ cm and $AC = 3.4$ cm, so

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} = \frac{2.5}{3.4} \approx 0.74.$

◆ **Example 2**
In $\triangle JKL$, $m\angle L = 33°$. Use a calculator to find tan L. Round your answer to the nearest hundredth.



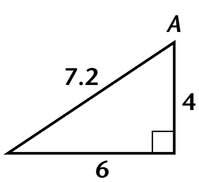
◆ **Solution**
Be sure the calculator is set in degree mode.
A sample key sequence is shown.

TAN 3 3 ENTER

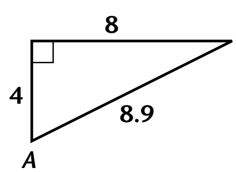
To the nearest tenth, $\tan 33° \approx 0.65$.

Find tan *A* as a fraction and as a decimal rounded to the nearest hundredth.

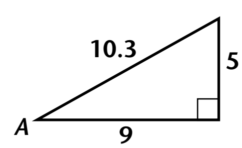
1.



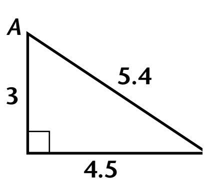
2.



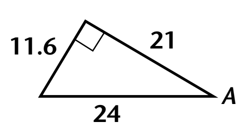
3.



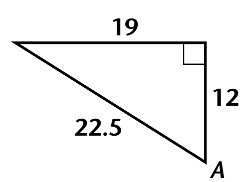
4.



5.



6.



NAME ______ CLASS ______ DATE ______

Use a scientific or graphics calculator to find the tangent of the angle with the given measure. Round your answer to the nearest hundredth.

7. 7° ______ **8.** 39° ______

9. 15° ______ **10.** 86° ______

11. 52° ______ **12.** 29° ______

13. 61° ______ **14.** 72° ______

15. 44° ______ **16.** 79° ______

◆ **Skill B** Finding the measure of an acute angle given its tangent

Recall To find the measure of an acute angle with a given tangent $r > 0$, you could write r as a fraction $\frac{a}{b}$, then construct a right triangle with legs of length a and b. You could then measure the angle opposite the side with length a. Since measurements may be inaccurate, it is better to use the **inverse tangent function,** $\tan^{-1}$, of a scientific or graphics calculator.

For $r > 0$, $\tan^{-1}(r) = \theta$, where $0° < \theta < 90°$ and $\tan \theta = r$.

◆ **Example**

A is an acute angle with $\tan A = \frac{5}{8}$. Find $m\angle A$ to the nearest degree.

◆ **Solution**

Be sure the calculator is set in degree mode. A sample key sequence is shown.

2nd TAN 5 ÷ 8) ENTER

To the nearest degree, $\tan^{-1}\frac{5}{8} \approx 32°$.

The tangent of an acute angle is given. Use a calculator to find the measure of the angle to the nearest degree.

17. $\frac{9}{10}$ ______ **18.** $\frac{4}{5}$ ______

19. $\frac{1}{3}$ ______ **20.** $\frac{5}{12}$ ______

21. 1.32 ______ **22.** 3 ______

23. 4.25 ______ **24.** 6.05 ______

25. 0.88 ______ **26.** 21.5 ______

5. $\overparen{AB}$, $\overparen{BC}$, $\overparen{CD}$, $\overparen{AD}$

6. $\overparen{ADB}$, $\overparen{BAC}$, $\overparen{DAC}$, $\overparen{ABD}$

7. 48° 8. 48° 9. 312°

10. 242° 11. 83° 12. 118°

13. 19.63 meters

14. 104.72 inches

15. 31.42 meters

16. 8.29 centimeters

Lesson 9.2

1. 9 2. 9.4 3. 18.2

4. Consider $\triangle APQ$. $PA = 2\sqrt{3}$, $PQ =$ radius of $\odot P = \sqrt{3}$, and $AQ = \frac{1}{2} \cdot 12$, so $(AQ)^2 + (PQ)^2 = (AP)^2$. By the converse of the Pythagorean Theorem, $\triangle APQ$ is a right triangle and $\overline{PQ} \perp \overline{AC}$. Then by the converse of the Tangent Theorem, $\overline{AC}$ is tangent to $\odot P$. Similarly, $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot P$.

5. $\overline{CP}$, $\overline{CR}$; $\overline{QP}$, $\overline{QR}$

6. 4 7. 15.6 8. 8.2 9. 28.4 10. 11.6

Lesson 9.3

1. 26° 2. 15° 3. 41°

4. 15° 5. 23° 6. 23°

7. No; $\angle NQM$ is not an inscribed angle.

8. No; the measure of the intercepted arc cannot be determined from the figure.

9. 204° 10. 90° 11. 90° 12. 38°

13. 76° 14. 104° 15. 52° 16. 27°

17. 90° 18. 126° 19. 63° 20. 158°

21. 79°

Lesson 9.4

1. 108° 2. 48° 3. 75° 4. 58.5°

5. 106° 6. 66° 7. 57° 8. 48° 9. 37°

Lesson 9.5

1. 11.25 2. 9.25 3. 9 4. 12.8 5. 8.94

6. 6.24 7. 3.33 8. 6.22 9. 7.2

Lesson 9.6

1. ± 1.5; ± 1.5; (0, 0); 1.5

2. ± 8.49; 12, -6; (0, 3); 9

3. -7.85, 17.85; -10, 14; (5, 2); 13

4. -0.26, -7.74; none; (-4, 1); 3.87

5. $x^2 + y^2 = 400$

6. $(x - 5)^2 + y^2 = 324$

7. $(x + 2)^2 + (y + 2)^2 = 1.44$

8. $(x + 3)^2 + (y + 6)^2 = 48$

9. $(x - 5)^2 + (y - 5)^2 = 100$

10. $(x + 6)^2 + (y - 4)^2 = 18$

11. $(x - 1)^2 + y^2 = 4$

12. $(x - 3)^2 + (y - 3)^2 = 4$

13. $(x + 2)^2 + (y + 2)^2 = 16$

Reteaching — Chapter 10

Lesson 10.1

1. $\frac{3}{2}$; 1.5 2. 2 3. $\frac{5}{9}$; 0.56 4. $\frac{3}{2}$; 1.5

5. $\frac{11.6}{21}$; 0.55 6. $\frac{19}{12}$; 1.58 7. 0.12 8. 0.81

9. 0.27 10. 14.301 11. 1.28 12. 0.55

13. 1.8 14. 3.08 15. 0.97 16. 5.14

17. 42° **18.** 39° **19.** 18° **20.** 23°

21. 53° **22.** 72° **23.** 77° **24.** 81°

25. 41° **26.** 88°

Lesson 10.2

1. $\frac{21}{29}$; $\frac{20}{29}$ **2.** $\frac{4}{5}$; $\frac{3}{5}$ **3.** $\frac{40}{41}$; $\frac{9}{41}$ **4.** 0.91; 0.41

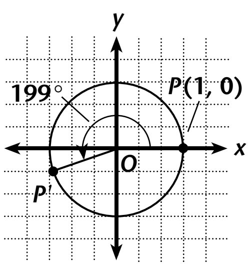
5. 0.24; 0.97 **6.** 0.45; 0.89 **7.** 9° **8.** 46°

9. 45° **10.** 57° **11.** 3° **12.** 18°

13. 107 ft **14.** 143 ft

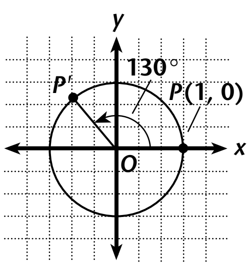
Lesson 10.3

1.



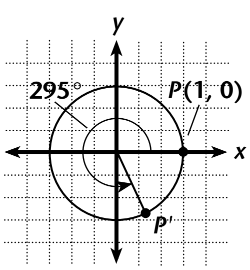
(−0.9455, −0.3256); −0.9455; −0.3256

2.



(−0.6428, 0.7660); −0.6428; 0.7660

3.



(0.4226, −0.9063); 0.4226; −0.9063

4. −0.2588; 0.9659; (−0.2588, 0.9659)

5. −0.3429; −0.9397; (−0.9397, −0.3429)

6. −0.3429; 0.9397; (0.9397, −0.3429)

7. 12°, 168° **8.** 115° **9.** 33° **10.** 129°

11. 33°, 147° **12.** 16°, 164° **13.** 53°, 127°

14. 10°, 170° **15.** 139°

Lesson 10.4

1. 17.8 **2.** 13.1 **3.** 7.1 **4.** 12.4

5–7. Answers may vary slightly due to rounding.

5. m∠C = 97°, b = 11.4, c = 20.7

6. m∠L = 66°, k = 14.8, ℓ = 19.4

7. m∠A = 65°, b = 11.0, c = 3.8

8. 42° or 138° **9.** 90°

10. no solution **11.** 49° or 131°

12–15. Answers may vary slightly due to rounding.

12. m∠K = 40°, m∠L = 48°, ℓ = 5.2

13. no solution

14. (1) m∠C = 63°, m∠A = 76°, a = 16.3
(2) m∠C = 117°, m∠A = 22°, a = 6.3

15. m∠Q = 43°, m∠R = 27°, r = 2.7

Lesson 10.5

1. 40° **2.** 7.1 **3.** 28.1

4. 5.3 **5.** 133° **6.** 69°

7–12. Answers may vary slightly due to rounding or to difference in solution methods.

7. m∠B = 80°, m∠A = 57°, m∠C = 43°

8. y = 9.3, m∠X = 29°, m∠Z = 26°

9. m∠S = 73°, m∠R = 57°, m∠Q = 50°

10. c = 34.4, m∠A = 33°, m∠B = 37°

NAME ______________________ CLASS __________ DATE __________

Reteaching
10.2 Sines and Cosines

◆ Skill A Finding the sine and the cosine of an acute angle

Recall Two other important trigonometric ratios are the **sine** of the angle and the **cosine** of the angle. The terms sine and cosine are abbreviated sin and cos.

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}} \text{ or } \frac{\text{opposite}}{\text{hypotenuse}}$$

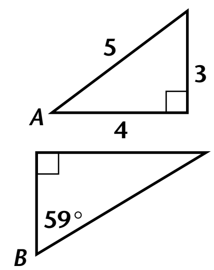
$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}} \text{ or } \frac{\text{adjacent}}{\text{hypotenuse}}$$

The sine of an angle A with measure θ can be referred to as $\sin \angle A$, $\sin A$, or $\sin \theta$. Similar expressions can be used to identify the cosine of an angle. The sine and cosine can be determined using a scientific or graphics calculator, or a table of trigonometric ratios, such as the one given on page 870 of your textbook.

◆ Example

a. Express $\sin A$ as a fraction.

b. Use a scientific or graphics calculator to find $\cos B$ to the nearest hundredth.



◆ Solution

a. $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$

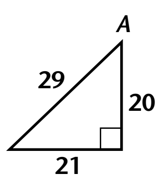
b. Be sure the calculator is set in degree mode. A sample key sequence is shown.

COS 5 9 ENTER

To the nearest tenth, $\cos B \approx 0.52$.

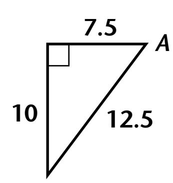
Write sin*A* and cos*A* as fractions in simplest form.

1.



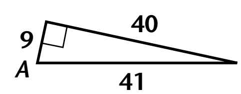
sin A: ______ cos A: ______

2.



sin A: ______ cos A: ______

3.



sin A: ______ cos A: ______

Use a scientific or graphics calculator to find sin*A* and cos*A* to the nearest hundredth.

4. $m\angle A = 66°$

sin A: ______ cos A: ______

5. $m\angle A = 14°$

sin A: ______ cos A: ______

6. $m\angle A = 27°$

sin A: ______ cos A: ______

NAME ______________________ CLASS ______________ DATE ______________

◆ Skill B Finding the measure of an acute angle given its sine or cosine

Recall You can use a scientific or graphics calculator to find the measure of an acute angle given its sine or cosine.

◆ **Example**
The cosine of an acute angle A is 0.93.
Find $m\angle A$ to the nearest degree.

◆ **Solution**
Use the $\cos^{-1}$ function: $\cos^{-1} 0.93 = 21.56518502$
$m\angle A \approx 22°$

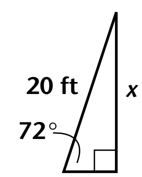
Find the measure of $\angle A$ to the nearest degree.

7. $\sin A = 0.15$, $m\angle A =$ ______________
8. $\cos A = 0.69$, $m\angle A =$ ______________
9. $\sin A = 0.71$, $m\angle A =$ ______________
10. $\cos A = 0.55$, $m\angle A =$ ______________
11. $\sin A = 0.05$, $m\angle A =$ ______________
12. $\sin A = 0.31$, $m\angle A =$ ______________

◆ Skill C Using the sine and cosine ratios to solve problems

Recall You can use the sine and cosine ratios to solve problems.

◆ **Example**
A 20 foot ladder placed against a building makes an angle of 72° with the ground. Find how high up the building the ladder reaches.

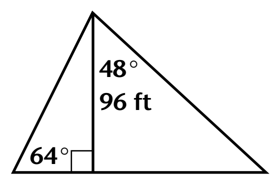


◆ **Solution**

$$\sin 72° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{20},$$

so $x = 20(\sin 72°) \approx 20(0.95) = 19$.
The ladder reaches 19 feet up the building.

The figure shows a tower that is 96 feet tall and has two attached guy wires.



13. The wire on the left meets the ground at an angle of 64°. Find the length of the wire to the nearest foot.

____

14. The wire on the right meets the tower at an angle of 48°. Find the length of the wire to the nearest foot.

____

ANSWERS

17. 42° **18.** 39° **19.** 18° **20.** 23°

21. 53° **22.** 72° **23.** 77° **24.** 81°

25. 41° **26.** 88°

Lesson 10.2

1. $\frac{21}{29}; \frac{20}{29}$ **2.** $\frac{4}{5}; \frac{3}{5}$ **3.** $\frac{40}{41}; \frac{9}{41}$ **4.** 0.91; 0.41

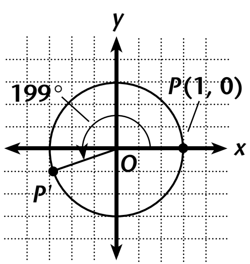
5. 0.24; 0.97 **6.** 0.45; 0.89 **7.** 9° **8.** 46°

9. 45° **10.** 57° **11.** 3° **12.** 18°

13. 107 ft **14.** 143 ft

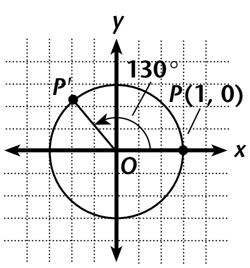
Lesson 10.3

1.



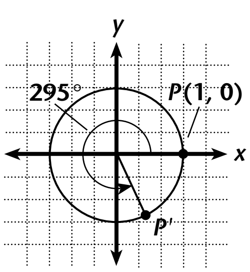
(−0.9455, −0.3256); −0.9455; −0.3256

2.



(−0.6428, 0.7660); −0.6428; 0.7660

3.



(0.4226, −0.9063); 0.4226; −0.9063

4. −0.2588; 0.9659; (−0.2588, 0.9659)

5. −0.3429; −0.9397; (−0.9397, −0.3429)

6. −0.3429; 0.9397; (0.9397, −0.3429)

7. 12°, 168° **8.** 115° **9.** 33° **10.** 129°

11. 33°, 147° **12.** 16°, 164° **13.** 53°, 127°

14. 10°, 170° **15.** 139°

Lesson 10.4

1. 17.8 **2.** 13.1 **3.** 7.1 **4.** 12.4

5–7. Answers may vary slightly due to rounding.

5. m∠C = 97°, b = 11.4, c = 20.7

6. m∠L = 66°, k = 14.8, ℓ = 19.4

7. m∠A = 65°, b = 11.0, c = 3.8

8. 42° or 138° **9.** 90°

10. no solution **11.** 49° or 131°

12–15. Answers may vary slightly due to rounding.

12. m∠K = 40°, m∠L = 48°, ℓ = 5.2

13. no solution

14. (1) m∠C = 63°, m∠A = 76°, a = 16.3
(2) m∠C = 117°, m∠A = 22°, a = 6.3

15. m∠Q = 43°, m∠R = 27°, r = 2.7

Lesson 10.5

1. 40° **2.** 7.1 **3.** 28.1

4. 5.3 **5.** 133° **6.** 69°

7–12. Answers may vary slightly due to rounding or to difference in solution methods.

7. m∠B = 80°, m∠A = 57°, m∠C = 43°

8. y = 9.3, m∠X = 29°, m∠Z = 26°

9. m∠S = 73°, m∠R = 57°, m∠Q = 50°

10. c = 34.4, m∠A = 33°, m∠B = 37°

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
10.3 Extending the Trigonometric Ratios

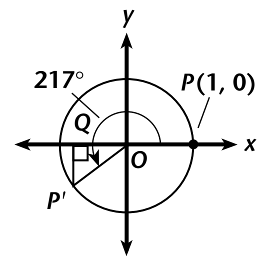
◆ **Skill A** Using the unit circle to find the sine and cosine of any angle

Recall A ray with endpoint O lies along the positive x-axis and intersects the unit circle at $P(0, 1)$. If $\overrightarrow{OP}$ is rotated about the origin, angles of rotation are produced.

Suppose $\overrightarrow{OP}$ is rotated through an angle θ. Let P' be the point where the terminal side of $\overrightarrow{OP}$ intersects the unit circle. Then the x-coordinate of P' is $\cos\theta$ and the y-coordinate of P' is $\sin\theta$ or $(\cos\theta, \sin\theta)$.

◆ **Example**

For $\theta = 217°$, find the coordinates of P' to four decimal places and use them to give $\sin 217°$ and $\cos 217°$.



◆ **Solution**

First note that since P' is in the third quadrant, both coordinates of P' are negative. Draw the unit circle and an angle of rotation of 217°.

Draw $\overline{P'Q}$ to form a right triangle with one side on the x-axis.

$m\angle QOP' = 217° - 180° = 37°$

$\cos 37° = \frac{OQ}{OP'}$ $\qquad$ $\sin 37° = \frac{QP'}{OP'}$

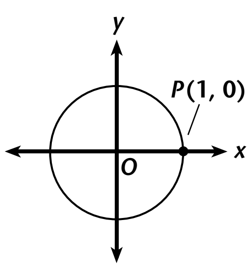
$0.7986 = \frac{OQ}{1}$ $\qquad$ $0.6018 = \frac{QP'}{1}$

$OQ = 0.7986$ $\qquad$ $QP' = 0.6018$

$P' = (-0.7986, -0.6018)$, so $\cos 217° = -0.7986$ and $\sin 217° = -0.6018$.

Use the unit circle to sketch $\overrightarrow{OP'}$, the result of rotating $\overrightarrow{OP}$ through an angle with measure θ. Find the coordinates of P to four decimal places. Then find $\cos\theta$ and $\sin\theta$.

1. $\theta = 199°$

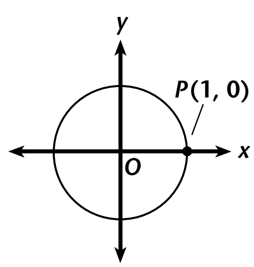


$P' =$ ______________

$\cos 199° =$ ____________

$\sin 199° =$ ____________

2. $\theta = 130°$

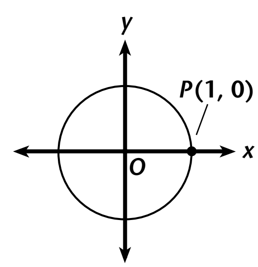


$P' =$ ______________

$\cos 130° =$ ____________

$\sin 130° =$ ____________

3. $\theta = 295°$



$P' =$ ______________

$\cos 295° =$ ____________

$\sin 295° =$ ____________

Use a scientific calculator to find the sine and cosine of each angle to four decimal places. Then give the *x*- and *y*-coordinates of the point *P′* where the terminal side of the given angle intersects the unit circle.

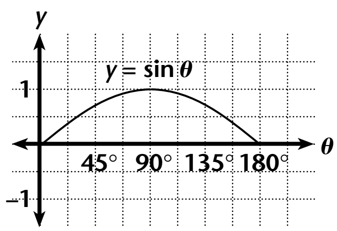
4. $\cos 105° =$ ______ $\sin 105° =$ ______ $P' =$ ______

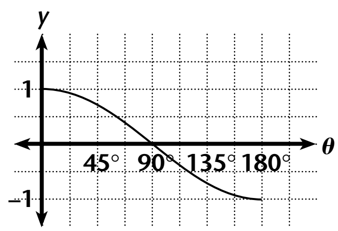
5. $\sin 200° =$ ______ $\cos 200° =$ ______ $P' =$ ______

6. $\sin 340° =$ ______ $\cos 340° =$ ______ $P' =$ ______

◆ **Skill B** Finding the measures of angles given their sines or cosines

Recall Since we will be using sines and cosines to work with triangles, we will be mainly interested in angles whose measures are between 0° and 180°. The figures below show the graphs of $y = \sin\theta$ and $y = \cos\theta$ for $0° \leq \theta \leq 180°$.





Note the following from the graphs. For $0° \leq \theta \leq 180°$, $\sin\theta$ is between 0 and 1. Also, if you draw a horizontal line through the graph of $y = \sin\theta$, you will note that for any value z between 0 and 1, there are two angles with measures between 0° and 180° whose sines are equal to z. If you use a calculator to find $\sin^{-1} z$, the calculator will give you an angle θ with measure between 0° and 90°. The other angle has measure $180° - \theta$.

For $0° \leq \theta \leq 180°$, $\cos\theta$ is between -1 and 1. For any value z between -1 and 1, there is only one angle θ, $0° \leq \theta \leq 180°$ for which $\cos\theta = z$.

◆ **Example**

Use a scientific calculator to find all values of θ, $0° \leq \theta \leq 180°$, for which $\sin\theta = 0.44$. Round your answer(s) to the nearest degree.

◆ **Solution**

$\theta = 26°$ and $\theta = 180° - 26° = 154°$

Find all values of θ between 0° and 180° for which the given statement is true. Round your answer(s) to the nearest degree.

7. $\sin\theta = 0.21$ ______ **8.** $\cos\theta = -0.43$ ______ **9.** $\cos\theta = 0.84$ ______

10. $\cos\theta = -0.63$ ______ **11.** $\sin\theta = 0.55$ ______ **12.** $\sin\theta = 0.27$ ______

13. $\sin\theta = 0.80$ ______ **14.** $\sin\theta = 0.18$ ______ **15.** $\cos\theta = -0.75$ ______

ANSWERS

17. 42° **18.** 39° **19.** 18° **20.** 23°

21. 53° **22.** 72° **23.** 77° **24.** 81°

25. 41° **26.** 88°

Lesson 10.2

1. $\frac{21}{29}; \frac{20}{29}$ **2.** $\frac{4}{5}; \frac{3}{5}$ **3.** $\frac{40}{41}; \frac{9}{41}$ **4.** 0.91; 0.41

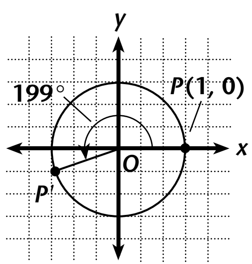
5. 0.24; 0.97 **6.** 0.45; 0.89 **7.** 9° **8.** 46°

9. 45° **10.** 57° **11.** 3° **12.** 18°

13. 107 ft **14.** 143 ft

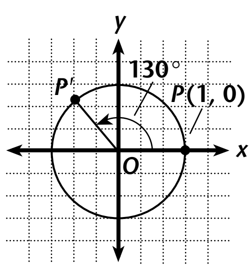
Lesson 10.3

1.



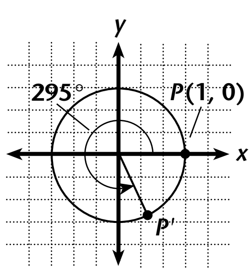
(−0.9455, −0.3256); −0.9455; −0.3256

2.



(−0.6428, 0.7660); −0.6428; 0.7660

3.



(0.4226, −0.9063); 0.4226; −0.9063

4. −0.2588; 0.9659; (−0.2588, 0.9659)

5. −0.3429; −0.9397; (−0.9397, −0.3429)

6. −0.3429; 0.9397; (0.9397, −0.3429)

7. 12°, 168° **8.** 115° **9.** 33° **10.** 129°

11. 33°, 147° **12.** 16°, 164° **13.** 53°, 127°

14. 10°, 170° **15.** 139°

Lesson 10.4

1. 17.8 **2.** 13.1 **3.** 7.1 **4.** 12.4

5–7. Answers may vary slightly due to rounding.

5. $m\angle C = 97°$, $b = 11.4$, $c = 20.7$

6. $m\angle L = 66°$, $k = 14.8$, $\ell = 19.4$

7. $m\angle A = 65°$, $b = 11.0$, $c = 3.8$

8. 42° or 138° **9.** 90°

10. no solution **11.** 49° or 131°

12–15. Answers may vary slightly due to rounding.

12. $m\angle K = 40°$, $m\angle L = 48°$, $\ell = 5.2$

13. no solution

14. (1) $m\angle C = 63°$, $m\angle A = 76°$, $a = 16.3$
(2) $m\angle C = 117°$, $m\angle A = 22°$, $a = 6.3$

15. $m\angle Q = 43°$, $m\angle R = 27°$, $r = 2.7$

Lesson 10.5

1. 40° **2.** 7.1 **3.** 28.1

4. 5.3 **5.** 133° **6.** 69°

7–12. Answers may vary slightly due to rounding or to difference in solution methods.

7. $m\angle B = 80°$, $m\angle A = 57°$, $m\angle C = 43°$

8. $y = 9.3$, $m\angle X = 29°$, $m\angle Z = 26°$

9. $m\angle S = 73°$, $m\angle R = 57°$, $m\angle Q = 50°$

10. $c = 34.4$, $m\angle A = 33°$, $m\angle B = 37°$

Menu

Print

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
10.4 The Law of Sines

◆ **Skill A** Using the Law of Sines to solve a triangle given the measures of two angles and a side

Recall The Law of Sines states that for any triangle *ABC* with sides *a*, *b*, and *c*, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. In some cases, the Law of Sines can be used to solve triangles. *Solving a triangle* means using given lengths and angle measures to determine all possible values of all other measurements. Given the measures of two angles and a side of a triangle, there is exactly one solution of the triangle.

◆ **Example**

Solve $\triangle ABC$, in which $m\angle B = 61°$, $m\angle C = 38°$, and $b = 15$.

◆ **Solution**

First note that $m\angle A = 180° - (61° + 38°) = 81°$. Next use the Law of Sines to find *a* and *c*.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad\qquad \frac{\sin C}{c} = \frac{\sin B}{b}$$

$$a = \frac{b \sin A}{\sin B} \qquad\qquad c = \frac{b \sin C}{\sin B}$$

$$= \frac{15 \sin 81°}{\sin 61°} \approx 16.9 \qquad\qquad = \frac{15 \sin 38°}{\sin 61°} \approx 10.6$$

Find the indicated measure in $\triangle ABC$. Give lengths to the nearest tenth.

1. $m\angle B = 42°$, $m\angle C = 70°$, $c = 25$, $b =$ ____________

2. $m\angle A = 54°$, $m\angle C = 45°$, $a = 15$, $c =$ ____________

3. $m\angle B = 105°$, $m\angle C = 40°$, $b = 12$, $a =$ ____________

4. $m\angle A = 22°$, $m\angle B = 62°$, $c = 14$, $b =$ ____________

Solve each triangle. Give lengths to the nearest tenth and angle measures to the nearest degree.

5. $\triangle ABC$: $m\angle A = 50°$, $m\angle B = 33°$, $a = 16$ ____________

6. $\triangle JKL$: $m\angle J = 70°$, $m\angle K = 44°$, $j = 20$ ____________

7. $\triangle PQR$: $m\angle P = 95°$, $m\angle Q = 20°$, $r = 10$ ____________

◆ **Skill B** Using the Law of Sines to solve a triangle given the measures of two sides and an angle that is opposite one of the sides

Recall If you are given the lengths of two sides of a triangle and the measure of an angle that is opposite one of the sides, it is possible that there is no solution , one solution, or two solutions. It is sometimes helpful to make a sketch that is roughly to scale.

◆ **Example**

Solve each triangle.

a. $m\angle A = 65°$, $a = 6$, $b = 8$

b. $m\angle A = 47°$, $a = 8$, $b = 9$

◆ **Solution**

a. By the Law of Sines, $\frac{\sin A}{a} = \frac{\sin B}{b}$. Then $\frac{\sin 65°}{6} = \frac{\sin B}{8}$, and

$\sin B = \frac{8 \sin 65}{6} \approx 1.21$. Since the sine of every angle is between -1 and 1, there is no such angle, so there is no such triangle.

b. By the Law of Sines, $\frac{\sin A}{a} = \frac{\sin B}{b}$. Then $\frac{\sin 47°}{8} = \frac{\sin B}{9}$, and

$\sin B = \frac{9 \sin 47°}{8} \approx 0.8228$. Two possible angles are 55° and $180° - 55° = 125°$. If $m\angle B = 55°$, then $m\angle C = 78°$ and $c \approx 10.7$. If $m\angle B = 125°$, then $m\angle C = 8°$ and $c \approx 1.5$.

Find the measure of the indicated angle of $\triangle ABC$ to the nearest degree. If there is no solution, write *no solution*. If there is more than one solution, give both.

8. $m\angle C = 34°$, $c = 25$, $b = 30$, $m\angle B =$ ____________________

9. $a = 6$, $c = 12$, $m\angle A = 30°$, $m\angle C =$ ____________________

10. $m\angle C = 29°$, $a = 16$, $c = 7$, $m\angle A =$ ____________________

11. $m\angle B = 33°$, $b = 13$, $c = 18$, $m\angle C =$ ____________________

Solve each triangle. Give lengths to the nearest tenth and angle measures to the nearest degree. If there is no solution, write *no solution*. If there is more than one solution, give both.

12. $\triangle JKL$: $m\angle J = 92°$, $j = 7$, $k = 4.5$ ____________________

13. $\triangle DEF$: $m\angle D = 96°$, $d = 5$, $f = 8$ ____________________

14. $\triangle ABC$: $m\angle B = 41°$, $b = 11$, $c = 15$ ____________________

15. $\triangle PQR$: $p = 5.5$, $q = 4$, $m\angle P = 110°$ ____________________

ANSWERS

17. 42° **18.** 39° **19.** 18° **20.** 23°

21. 53° **22.** 72° **23.** 77° **24.** 81°

25. 41° **26.** 88°

Lesson 10.2

1. $\frac{21}{29}; \frac{20}{29}$ **2.** $\frac{4}{5}; \frac{3}{5}$ **3.** $\frac{40}{41}; \frac{9}{41}$ **4.** 0.91; 0.41

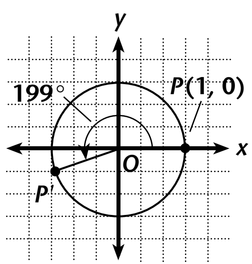
5. 0.24; 0.97 **6.** 0.45; 0.89 **7.** 9° **8.** 46°

9. 45° **10.** 57° **11.** 3° **12.** 18°

13. 107 ft **14.** 143 ft

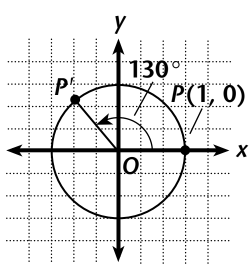
Lesson 10.3

1.



(−0.9455, −0.3256); −0.9455; −0.3256

2.



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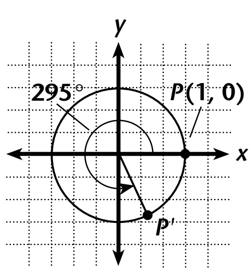
3.



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7. 12°, 168° **8.** 115° **9.** 33° **10.** 129°

11. 33°, 147° **12.** 16°, 164° **13.** 53°, 127°

14. 10°, 170° **15.** 139°

Lesson 10.4

1. 17.8 **2.** 13.1 **3.** 7.1 **4.** 12.4

5–7. Answers may vary slightly due to rounding.

5. m∠C = 97°, b = 11.4, c = 20.7

6. m∠L = 66°, k = 14.8, ℓ = 19.4

7. m∠A = 65°, b = 11.0, c = 3.8

8. 42° or 138° **9.** 90°

10. no solution **11.** 49° or 131°

12–15. Answers may vary slightly due to rounding.

12. m∠K = 40°, m∠L = 48°, ℓ = 5.2

13. no solution

14. (1) m∠C = 63°, m∠A = 76°, a = 16.3
(2) m∠C = 117°, m∠A = 22°, a = 6.3

15. m∠Q = 43°, m∠R = 27°, r = 2.7

Lesson 10.5

1. 40° **2.** 7.1 **3.** 28.1

4. 5.3 **5.** 133° **6.** 69°

7–12. Answers may vary slightly due to rounding or to difference in solution methods.

7. m∠B = 80°, m∠A = 57°, m∠C = 43°

8. y = 9.3, m∠X = 29°, m∠Z = 26°

9. m∠S = 73°, m∠R = 57°, m∠Q = 50°

10. c = 34.4, m∠A = 33°, m∠B = 37°

NAME ______________________ CLASS ____________ DATE ____________

Reteaching
10.5 The Law of Cosines

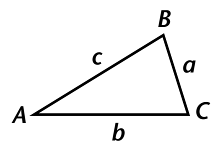
◆ **Skill A** Using the Law of Cosines to find the measure of an angle or side of a triangle

Recall The Law of Cosines states that for any triangle ABC with sides a, b, and c, each of the following are true:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



The Law of Cosines can be used in solving triangles where the measures of two sides and the included angle are known or where the lengths of all three sides are known.

The following theorem may simplify your work when solving such triangles.

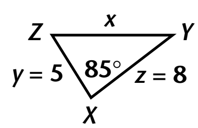
Theorem: If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second.

It follows from the theorem that the largest angle of a triangle is opposite the longest side.

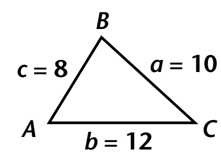
It is also helpful to recall that, for any numbers x, $-1 \leq x \leq 1$, there is only value of θ between 0° and 180° for which $\cos \theta = x$.

◆ **Example**

a. Find x.



b. Find m∠C.



◆ **Solution**

a. By the Law of Cosines, $x^2 = y^2 + z^2 - 2yz \cos X = 25 + 64 - 2(40)(\cos 85°)$ $= 82$. Then $x = \sqrt{82} \approx 9.1$.

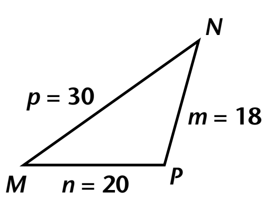
b. By the Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, so

$\cos C = \dfrac{c^2 - a^2 - b^2}{-2ab} = \dfrac{64 - 225 - 144}{-2(12)(15)} \approx 0.8472.$

Then $m\angle C = \cos^{-1} 0.8472 \approx 32°$.

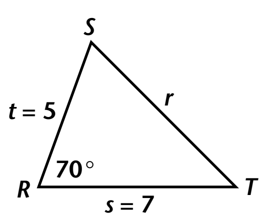
Find each indicated measure. Give lengths to the nearest tenth and angles to the nearest degree.

1.



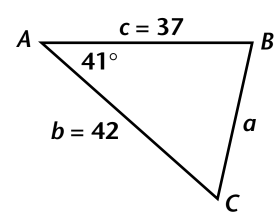
m∠N = ____________

2.



r = ____________

3.



a = ____________

4. $\triangle DEF$: $m\angle D = 25°$, $e = 12.5$, $f = 11$, $d =$ ______________

5. $\triangle MNP$: $m = 18$, $n = 12$, $p = 7.5$, $m\angle M =$ ______________

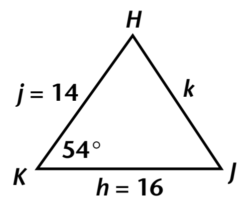
6. $\triangle STU$: $s = 30$, $t = 35$, $u = 32$, $m\angle U =$ ______________

◆ Skill B Using the Law of Cosines along with the Law of Sines to solve triangles

Recall The Law of Cosines may be used together with the Law of Sines to solve triangles. For example, consider $\triangle ABC$ in the example on the preceding page. It is possible to use the Law of Cosines to determine the measure of each angle of the triangle. However, once you have found the measure of one angle, it is much simpler to use the Law of Sines to find the measure of a second and the Triangle Sum Theorem to find the measure of the third.

◆ **Example**

Solve $\triangle HJK$.



◆ **Solution**

By the Law of Cosines, $k^2 = h^2 + j^2 - 2hj\cos K$

$= 256 + 196 - 2(16)(14)\cos 54°$

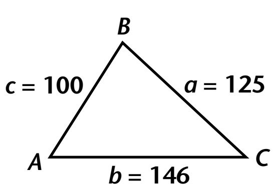
$\approx 189.$

Then $k \approx 13.7$.

By the Law of Sines, $\frac{\sin J}{14} = \frac{\sin 54°}{13.7}$, so $\sin J = \frac{14 \sin 54°}{13.7}$ ≈ 0.8267. Since $\angle J$ is not the largest angle, $\angle J$ must be acute, so $m\angle J = \sin^{-1} 0.8267 \approx 56°$.

Solve each triangle. Give lengths to the nearest tenth and angle measures to the nearest degree.

7.



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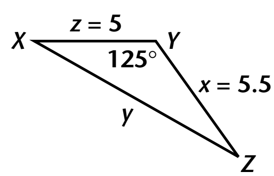
8.



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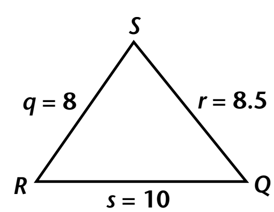
9.



____

10. $\triangle ABC$: $m\angle C = 110°$, $a = 20$, $b = 22$ ______________

11. $\triangle FGH$: $m\angle G = 85°$, $f = 12$, $h = 15$ ______________

12. $\triangle VWX$: $v = 73$, $w = 40$, $x = 37$ ______________

17. 42° **18.** 39° **19.** 18° **20.** 23°

21. 53° **22.** 72° **23.** 77° **24.** 81°

25. 41° **26.** 88°

Lesson 10.2

1. $\frac{21}{29}; \frac{20}{29}$ **2.** $\frac{4}{5}; \frac{3}{5}$ **3.** $\frac{40}{41}; \frac{9}{41}$ **4.** 0.91; 0.41

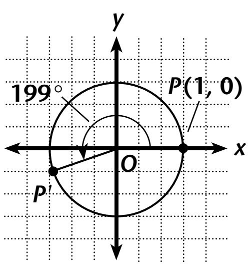
5. 0.24; 0.97 **6.** 0.45; 0.89 **7.** 9° **8.** 46°

9. 45° **10.** 57° **11.** 3° **12.** 18°

13. 107 ft **14.** 143 ft

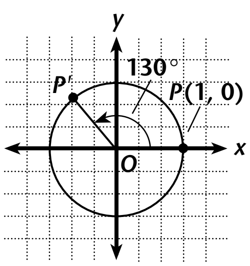
Lesson 10.3

1.



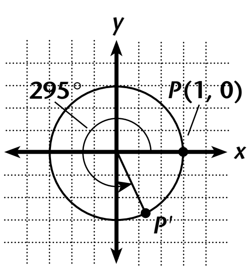
(−0.9455, −0.3256); −0.9455; −0.3256

2.



(−0.6428, 0.7660); −0.6428; 0.7660

3.



(0.4226, −0.9063); 0.4226; −0.9063

4. −0.2588; 0.9659; (−0.2588, 0.9659)

5. −0.3429; −0.9397; (−0.9397, −0.3429)

6. −0.3429; 0.9397; (0.9397, −0.3429)

7. 12°, 168° **8.** 115° **9.** 33° **10.** 129°

11. 33°, 147° **12.** 16°, 164° **13.** 53°, 127°

14. 10°, 170° **15.** 139°

Lesson 10.4

1. 17.8 **2.** 13.1 **3.** 7.1 **4.** 12.4

5–7. Answers may vary slightly due to rounding.

5. $m\angle C = 97°, b = 11.4, c = 20.7$

6. $m\angle L = 66°, k = 14.8, \ell = 19.4$

7. $m\angle A = 65°, b = 11.0, c = 3.8$

8. 42° or 138° **9.** 90°

10. no solution **11.** 49° or 131°

12–15. Answers may vary slightly due to rounding.

12. $m\angle K = 40°, m\angle L = 48°, \ell = 5.2$

13. no solution

14. (1) $m\angle C = 63°, m\angle A = 76°, a = 16.3$
(2) $m\angle C = 117°, m\angle A = 22°, a = 6.3$

15. $m\angle Q = 43°, m\angle R = 27°, r = 2.7$

Lesson 10.5

1. 40° **2.** 7.1 **3.** 28.1

4. 5.3 **5.** 133° **6.** 69°

7–12. Answers may vary slightly due to rounding or to difference in solution methods.

7. $m\angle B = 80°, m\angle A = 57°, m\angle C = 43°$

8. $y = 9.3, m\angle X = 29°, m\angle Z = 26°$

9. $m\angle S = 73°, m\angle R = 57°, m\angle Q = 50°$

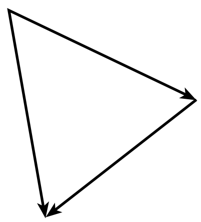
10. $c = 34.4, m\angle A = 33°, m\angle B = 37°$

11. $g = 18.4$, $m\angle F = 41°$, $m\angle H = 54°$

12. $m\angle V = 143°$, $m\angle W = 19°$, $m\angle X = 18°$

Lesson 10.6

1.



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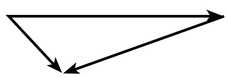

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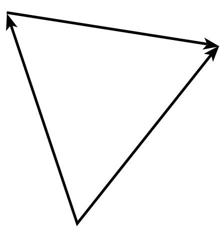

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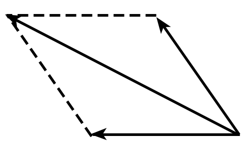

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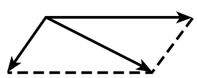
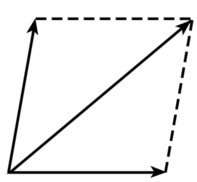


6.



7. 54.2 mi/h; 12° **8.** 48.8 mi/h; 20°

Lesson 10.7

1. (0, −2) **2.** (−2.60, 1.5)

3. (−0.64, 1.26) **4.** (3.57, 0.52)

5. (−2.66, 3.59) **6.** (−2.94, −1.16)

7. (1, −2) **8.** (2, 1) **9.** (−2, 2)

10. (0.71, 2.12), (4.95, 0.71), (−2.12, 4.95)

11. (4.19, 2.73), (1.71, 4.70), (4.95, −3.93)

12. (7.31, 6.82), (−7.80, −0.44), (−7.90, −3.26)

13. (−4.69, 9.33), (−1.22, −10.37), (−7.88, −1.39)

14. (0.62, −7.04), (−3.31, 5.39), (−8.95, 4.90)

15. (1.22, −0.71), (2.26, −4.57), (6.57, −1.34), (7.61, −5.21)

Reteaching — Chapter 11

Lesson 11.1

1. 6.18 ft **2.** 6.47 yd **3.** 12.14 m

4. 35.60 mm **5.** 123.61 ft

6. 5.87 in. **7.** $2x$ **8.** $x\sqrt{5} - x$

9. $\frac{TU}{QT} = \frac{2x}{x\sqrt{5} - x} = \frac{2}{\sqrt{5} - 1} = \frac{2\sqrt{5} + 2}{4}$
$= \frac{1 + \sqrt{5}}{2}$

10–11. Check drawings.

Lesson 11.2

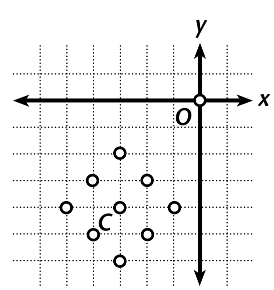
1–3. Check drawings.

1. 5 **2.** 4 **3.** 7 **4.** 20 **5.** 23 **6.** 24

7. 80; 160 **8.** 24; 48 **9.** 40; 80

10. 48; 96 **11.** 28; 56 **12.** 64; 128

13.



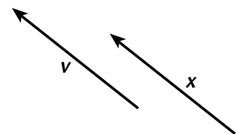
NAME ______________________ CLASS __________ DATE __________

Reteaching
10.6 Vectors in Geometry

◆ **Skill A** Drawing the sum of vectors using the head-to-head method and the parallelogram method

Recall A vector is a quantity that has both magnitude, or size, and direction. It may be represented by an arrow. The length of the arrow represents the magnitude of the vector. To describe the direction of the vector, picture superimposing a coordinate grid on the vector with the origin at the tail of the vector. The direction is the angle the vector makes with the positive *x*-axis.

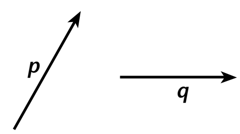
The figure at the right shows $\vec{v}$. The head of the vector is indicated by the arrow. The tail is the other end. The magnitude of $\vec{v}$ is denoted $|\vec{v}|$. Two vectors are equivalent if they have the same direction and magnitude. For any point P in the plane, a vector equivalent to $\vec{v}$ can be drawn with its tail at P. This allows you to draw a vector equivalent to $\vec{v}$ anywhere in the plane. In the figure, $\vec{v}$ and $\vec{x}$ are equivalent vectors.



There are several methods for finding the resultant, or sum, of two vectors.

◆ **Example**

Draw the sum of $\vec{p}$ and $\vec{q}$.



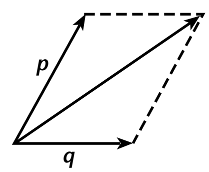
◆ **Solution**

a. Head-to-tail method

Since you can draw a vector with its tail at any point in the plane, draw $\vec{q}$ with its head at the tail of $\vec{p}$. The resultant, or sum, $\vec{p} + \vec{q}$ is the vector with tail at the tail of $\vec{p}$ and head at the head of $\vec{q}$.

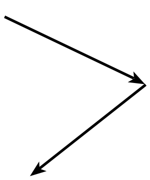
b. Parallelogram method

Draw $\vec{p}$ and $\vec{q}$ with their tails at the same point. Draw a parallelogram with sides $\vec{p}$ and $\vec{q}$. The resultant vector is represented by a diagonal of the parallelogram. Its tail is at the common point of $\vec{p}$ and $\vec{q}$ and its head at the opposite vertex of the parallelogram.



Draw the resultant of the vectors using the head-to-tail method.

1.



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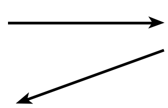

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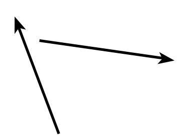
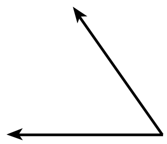
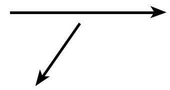
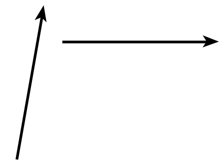


Draw the resultant of the vectors using the parallelogram method.

4. 

5. 

6. 

◆ **Skill B** Using trigonometric ratios to find the resultant of two vectors

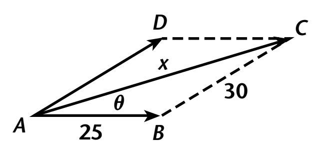
Recall Trigonometric ratios can be used to find the resultant of two vectors.

◆ **Example**

A boat is traveling at a speed of 30 mi/h at a direction of 28°. The wind is blowing from the west at a speed of 25 mi/h. Draw $\vec{b}$, the velocity of the boat, and $\vec{w}$, the wind velocity, and find $\vec{b} + \vec{w}$.

◆ **Solution**

Draw $\vec{b} + \vec{w}$ using the parallelogram method.
$m\angle DAB = 28°$, so $m\angle B = 180° - 28° = 152°$.



Let x be the magnitude and θ the direction of $\vec{b} + \vec{w}$.

By the Law of Cosines, $x^2 = 25^2 + 30^2 - 2(25)(30) \cos 152° \approx 2849$, so $x \approx 53.4$ mi/h.

By the Law of Sines, $\frac{\sin \theta}{30} = \frac{\sin 152°}{53.4}$, so $\sin^{-1} \theta \approx 0.2637$ and $\theta \approx 15°$.

Suppose $\vec{b}$ and $\vec{w}$ in the example are changed as indicated. Find the magnitude to the nearest tenth and the direction to the nearest degree of the resultant vector.

7. The direction of $\vec{b}$ is 20°; $\vec{w}$ and the magnitude of $\vec{b}$ are unchanged.

____

8. The directions of both vectors are unchanged, but $|\vec{b}| = 35$ mi/h and $|\vec{w}| = 15$ mi/h.

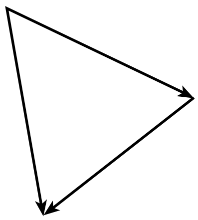
____

11. $g = 18.4$, $m\angle F = 41°$, $m\angle H = 54°$

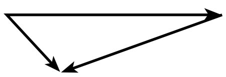
12. $m\angle V = 143°$, $m\angle W = 19°$, $m\angle X = 18°$

Lesson 10.6

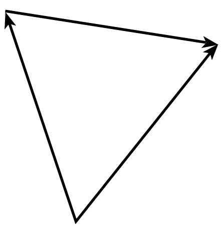
1.



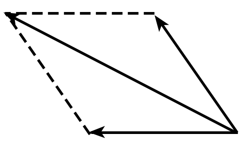
2.



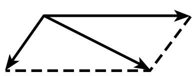
3.



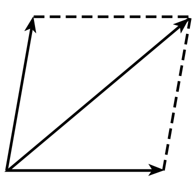
4.



5.



6.



7. 54.2 mi/h; 12° **8.** 48.8 mi/h; 20°

Lesson 10.7

1. (0, −2) **2.** (−2.60, 1.5)

3. (−0.64, 1.26) **4.** (3.57, 0.52)

5. (−2.66, 3.59) **6.** (−2.94, −1.16)

7. (1, −2) **8.** (2, 1) **9.** (−2, 2)

10. (0.71, 2.12), (4.95, 0.71), (−2.12, 4.95)

11. (4.19, 2.73), (1.71, 4.70), (4.95, −3.93)

12. (7.31, 6.82), (−7.80, −0.44), (−7.90, −3.26)

13. (−4.69, 9.33), (−1.22, −10.37), (−7.88, −1.39)

14. (0.62, −7.04), (−3.31, 5.39), (−8.95, 4.90)

15. (1.22, −0.71), (2.26, −4.57), (6.57, −1.34), (7.61, −5.21)

Reteaching — Chapter 11

Lesson 11.1

1. 6.18 ft **2.** 6.47 yd **3.** 12.14 m

4. 35.60 mm **5.** 123.61 ft

6. 5.87 in. **7.** $2x$ **8.** $x\sqrt{5} - x$

9. $\frac{TU}{QT} = \frac{2x}{x\sqrt{5} - x} = \frac{2}{\sqrt{5} - 1} = \frac{2\sqrt{5} + 2}{4}$
$= \frac{1 + \sqrt{5}}{2}$

10–11. Check drawings.

Lesson 11.2

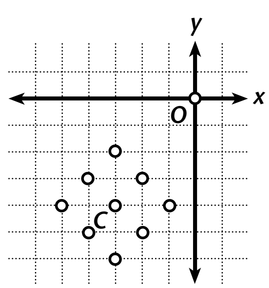
1–3. Check drawings.

1. 5 **2.** 4 **3.** 7 **4.** 20 **5.** 23 **6.** 24

7. 80; 160 **8.** 24; 48 **9.** 40; 80

10. 48; 96 **11.** 28; 56 **12.** 64; 128

13.



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NAME ______________________ CLASS __________ DATE __________

Reteaching

10.7 Rotations in the Coordinate Plane

◆ **Skill A** Using transformations equations to identify the image of a point under a rotation about the origin

Recall In Lesson 10.3, you saw that the x-coordinate of the image of the point $P(1, 0)$ under a rotation of θ about the origin is $\cos\theta$. The y-coordinate of the image is $\sin\theta$. In general, if R is the point (x, y), the x-coordinate of the image under the same rotation is $x\cos\theta - y\sin\theta$, and the y-coordinate of the image is $x\sin\theta + y\cos\theta$.

Two transformation equations describe the image $R'(x', y')$ of $R(x, y)$ under a rotation of θ degrees about the origin.

$$x' = x\cos\theta - y\sin\theta \text{ and } y' = x\sin\theta + y\cos\theta$$

◆ **Example**

The point $P(4, -2)$ is rotated 72° about the origin. Find the coordinates of the image P' rounded to the nearest hundredth.

◆ **Solution**

The angle of rotation is 72°; $\cos 72° \approx 0.3090$ and $\sin 72° \approx 0.9511$.

$x' = x\cos\theta - y\sin\theta = 4\cos 72° - (-2)\sin 72° \approx 3.1382$

$y' = x\sin\theta + y\cos\theta = 4\sin 72° - 2\cos 72° \approx 3.1862$

The image of P is $P'(3.14, 3.19)$.

A point P and an angle θ of rotation are given. Find the coordinates of the image of P to the nearest hundredth.

1. $P(2, 0)$; $\theta = -90°$ ______________________

2. $P(0, 3)$; $\theta = 60°$ ______________________

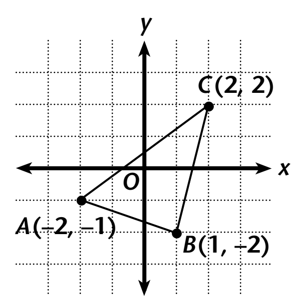
3. $P(1, 1)$; $\theta = 72°$ ______________________

4. $P(3, -2)$; $\theta = 42°$ ______________________

5. $P(2, -4)$; $\theta = 190°$ ______________________

6. $P(3, -1)$; $\theta = 220°$ ______________________

The triangle shown in the figure is rotated 90° about the origin. Find the coordinates of the images of the vertices.



7. $A' =$ ______________________

8. $B' =$ ______________________

9. $C' =$ ______________________

CLASS ______________ DATE ______________

◆**Skill B** Using rotation matrices to identify the image of a point under a rotation about the origin

Recall A rotation of θ about the origin can be represented by the rotation matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. The coordinates of the image $P'(x',y')$ of a point $P(x,y)$ can be found by multiplying the rotation matrix by the matrix representing the point, $\begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = [x\cos\theta - y\sin\theta \quad \sin\theta + y\cos\theta]$$

Rotation matrices can be used to rotate more than one point at a time. Simply add a column to the matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ for each point. For example, use the matrix $\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$ to rotate the points (x_1, y_1), (x_2, y_2), and (x_3, y_3). If you use a graphics calculator to multiply the matrices, you do not need to find the values of $\sin\theta$ and $\cos\theta$ first. You can simply enter the indicated values in the matrices. For example, the rotation matrix for a 77° rotation would be entered $\begin{bmatrix} \cos 77° & -\sin 77° \\ \sin 77° & \cos 77° \end{bmatrix}$.

◆ **Example**

A triangle with vertices $A(4, 6)$, $B(2, -3)$, and $C(7, -5)$ is rotated 82° about the origin. Find the coordinates of the images of the vertices.

◆ **Solution**

The rotation matrix is $\begin{bmatrix} \cos 82° & -\sin 82° \\ \sin 82° & \cos 82° \end{bmatrix}$.

$$\begin{bmatrix} \cos 82° & -\sin 82° \\ \sin 82° & \cos 82° \end{bmatrix} \times \begin{bmatrix} 4 & 2 & 7 \\ 6 & -3 & -5 \end{bmatrix} = \begin{bmatrix} -5.38 & 3.25 & 5.93 \\ 4.80 & 1.56 & 6.24 \end{bmatrix}$$

To the nearest hundredth, the images of the vertices are $A'(-5.38, 4.80)$, $B'(3.25, 1.56)$, and $C'(5.93, 6.24)$.

The polygon with the given vertices is rotated about the origin through the given angle. Find the coordinates of the vertices of the image. Round each coordinate to the nearest hundredth.

10. (2, 1), (4, −3), (2, 5); $\theta = 45°$ ______________

11. (3, 4), (0, 5), (6, −2); $\theta = -20°$ ______________

12. (0, −10), (5, 6), (3, 8); $\theta = 133°$ ______________

13. (10, 3), (−10, 3), (0, 8); $\theta = 100°$ ______________

14. (5, 5), (2, 6), (−2, 10); $\theta = 50°$ ______________

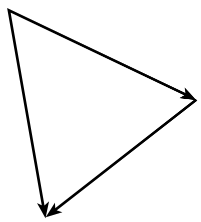
15. (1, 1), (5, 1), (3, 6), (7, 6); $\theta = -75°$ ______________

11. $g = 18.4$, $m\angle F = 41°$, $m\angle H = 54°$

12. $m\angle V = 143°$, $m\angle W = 19°$, $m\angle X = 18°$

Lesson 10.6

1.



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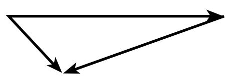

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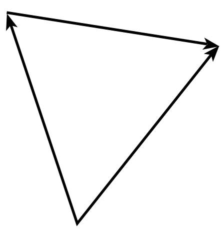

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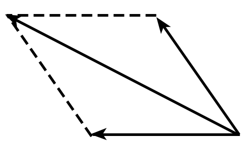

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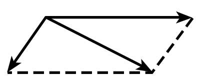
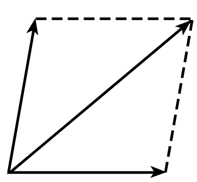


6.



7. 54.2 mi/h; 12° **8.** 48.8 mi/h; 20°

Lesson 10.7

1. (0, −2) **2.** (−2.60, 1.5)

3. (−0.64, 1.26) **4.** (3.57, 0.52)

5. (−2.66, 3.59) **6.** (−2.94, −1.16)

7. (1, −2) **8.** (2, 1) **9.** (−2, 2)

10. (0.71, 2.12), (4.95, 0.71), (−2.12, 4.95)

11. (4.19, 2.73), (1.71, 4.70), (4.95, −3.93)

12. (7.31, 6.82), (−7.80, −0.44), (−7.90, −3.26)

13. (−4.69, 9.33), (−1.22, −10.37), (−7.88, −1.39)

14. (0.62, −7.04), (−3.31, 5.39), (−8.95, 4.90)

15. (1.22, −0.71), (2.26, −4.57), (6.57, −1.34), (7.61, −5.21)

Reteaching — Chapter 11

Lesson 11.1

1. 6.18 ft **2.** 6.47 yd **3.** 12.14 m

4. 35.60 mm **5.** 123.61 ft

6. 5.87 in. **7.** $2x$ **8.** $x\sqrt{5} - x$

9. $\frac{TU}{QT} = \frac{2x}{x\sqrt{5} - x} = \frac{2}{\sqrt{5} - 1} = \frac{2\sqrt{5} + 2}{4} = \frac{1 + \sqrt{5}}{2}$

10–11. Check drawings.

Lesson 11.2

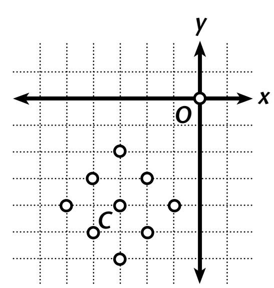
1–3. Check drawings.

1. 5 **2.** 4 **3.** 7 **4.** 20 **5.** 23 **6.** 24

7. 80; 160 **8.** 24; 48 **9.** 40; 80

10. 48; 96 **11.** 28; 56 **12.** 64; 128

13.



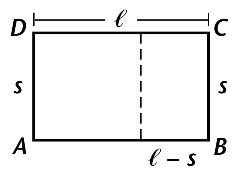
NAME ______________________ CLASS ____________ DATE ____________

Reteaching
11.1 Golden Connections

◆ **Skill A** Using the golden ratio to find side lengths of golden rectangles

Recall A rectangle is a **golden rectangle** if, when a square is cut off one end of the rectangle, the remaining rectangle and the original rectangle are similar. Let *ABCD* be a rectangle with longer side ℓ and shorter side s. *ABCD* is a golden rectangle if $\frac{\ell}{s} = \frac{s}{\ell - s}$.

In the figure, *ABCD* is a golden rectangle.



The ratio of the length of the longer side to the length of the shorter side of a golden rectangle, called the **golden ratio,** is the same for all golden rectangles. Let *PQRS* be a golden rectangle with longer side x and shorter side 1. The ratio for this golden rectangle is $\frac{x}{1} = \frac{1}{x - 1}$. The numerical value of the golden ratio can be determined by cross-multiplying and using the quadratic formula to solve the resulting quadratic equation.

$x^2 - x = 1 \quad \rightarrow \quad x^2 - x - 1 = 0$

$$a = 1,\ b = -1, \text{ and } c = -1 \ \rightarrow \ x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 + \sqrt{5}}{2} \text{ (Since } x \text{ is a length, } x \text{ must be positive.)}$$

$$\approx 1.618$$

◆ **Example**

The shorter side of a golden rectangle is 18 meters long. Find the length of the longer side to the nearest hundredth of a meter.

◆ **Solution**

$\frac{\ell}{s} = \frac{\ell}{18} \approx 1.618$, so $\ell \approx 29.124$.

The longer side is approximately 29.12 m long.

Given the length of one side of a golden rectangle, find the length of the other side. Round your answers to the nearest hundredth.

1. longer side = 10 ft

shorter side = ____________

2. shorter side = 4 yd

longer side = ____________

CLASS ______________ DATE ______________

3. shorter side = 7.5 m ______________

longer side = ______________

4. shorter side = 22 mm ______________

longer side = ______________

5. longer side = 200 ft ______________

shorter side = ______________

6. longer side = 9.5 in. ______________

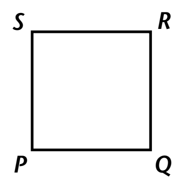
shorter side = ______________

◆ Skill B Constructing a golden rectangle

Recall You can use a square to construct a golden rectangle.

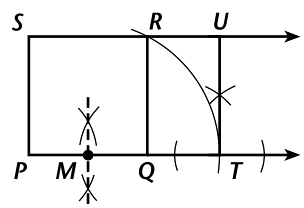
◆ **Example**

PQRS is a square. Construct a golden rectangle with shorter side *PQ*.



◆ **Solution**

Draw $\overleftrightarrow{PQ}$ and $\overleftrightarrow{SR}$. Construct the midpoint, M, of $\overline{PQ}$. Draw an arc with center M and radius MR, intersecting $\overleftrightarrow{PQ}$ at T. Construct a perpendicular to $\overleftrightarrow{PQ}$ at T, intersecting $\overleftrightarrow{SR}$ at U. Draw $\overline{TU}$.To show that $PTUS$ is a golden rectangle, let $PQ = 2x$. Then $MQ = x$ and $MT = MR = \sqrt{(2x)^2 + x^2} = x\sqrt{5}$. So, $\frac{PT}{TU} = \frac{x + x\sqrt{5}}{2x} = \frac{1 + \sqrt{5}}{2}$, the golden ratio.



Refer to *QTUR* in the figure above. Complete.

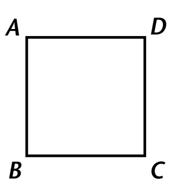
7. TU = ______________

8. QT = ______________

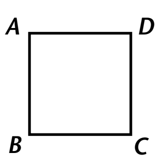
9. Show that *QTUR* is a golden rectangle. ______________

***ABCD* is a square. Construct a golden rectangle with shorter side *AB*.**

10.



11.

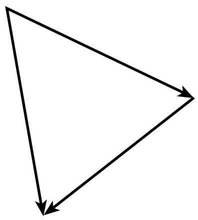


11. $g = 18.4$, $m\angle F = 41°$, $m\angle H = 54°$

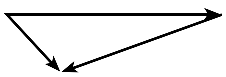
12. $m\angle V = 143°$, $m\angle W = 19°$, $m\angle X = 18°$

Lesson 10.6

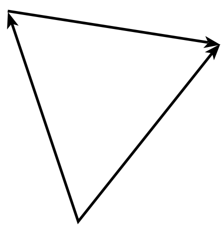
1.



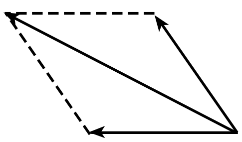
2.



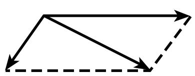
3.



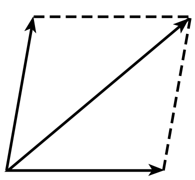
4.



5.



6.



7. 54.2 mi/h; 12° **8.** 48.8 mi/h; 20°

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1. (0, −2) **2.** (−2.60, 1.5)

3. (−0.64, 1.26) **4.** (3.57, 0.52)

5. (−2.66, 3.59) **6.** (−2.94, −1.16)

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15. (1.22, −0.71), (2.26, −4.57), (6.57, −1.34), (7.61, −5.21)

Reteaching — Chapter 11

Lesson 11.1

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10–11. Check drawings.

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1–3. Check drawings.

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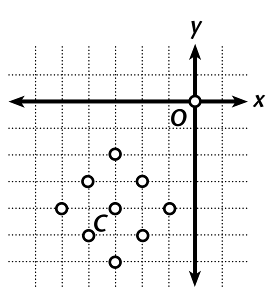


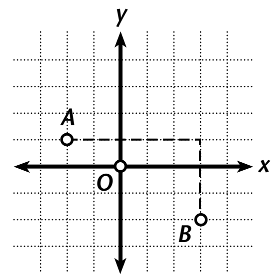
NAME ______________________ CLASS ______________ DATE ______________

Reteaching

11.2 Taxicab Geometry

◆ **Skill A** Finding the taxidistance between two points

Recall In the taxicab grid at the right, A has coordinates $(-2, 1)$ and B has coordinates $(3, -2)$. The dashed line shows one path from A to B.



The **taxidistance** between two points is the smallest number of blocks a taxi must drive to get from one point to the other. (Notice that the taxi could not drive on a diagonal, but only along the horizontal and vertical streets.)

The taxidistance between two points (x_1, y_1) and (x_2, y_2) is $|x_2 - x_1| + |y_2 - y_1|$. If A and B lie on the same line, there is only one minimum distance path between them. If not, there is more than one such path.

◆ **Example**

Find the taxidistance between points A and B in the figure above.

◆ **Solution**

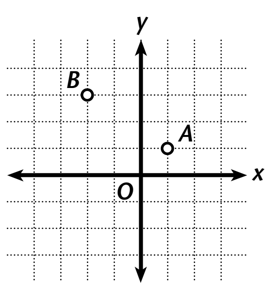
A has coordinates $(-2, 1)$ and B has coordinates $(3, -2)$.

$$\begin{aligned} |x_2 - x_1| + |y_2 - y_1| &= |3 - (-2)| + |-2 - 1| \\ &= |5| + |-3| \\ &= 5 + 3 \\ &= 8 \end{aligned}$$

The taxidistance from point A to point B is 8 blocks.

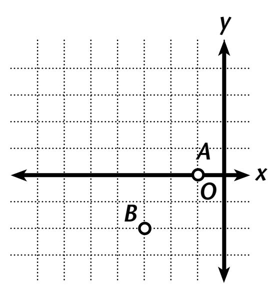
Find the taxidistance, *d*, between points *A* and *B* and draw at least two paths from *A* to *B* that are *d* blocks long.

1.



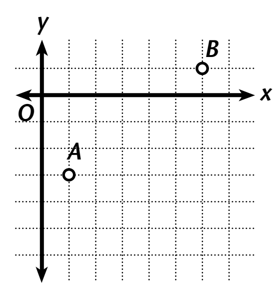
d = ______________

2.



d = ______________

3.



d = ______________

Find the taxidistance, *d*, between the points.

4. $(-7, 5)$ and $(12, 4)$

d = ______________

5. $(14, 8)$ and $(-6, 5)$

d = ______________

6. $(-7, -7)$ and $(4, 6)$

d = ______________

◆ Skill B Plotting taxicab circles

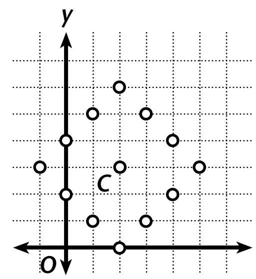
Recall In taxicab geometry, a circle is the set of all points in the taxicab plane that are a given distance r from a point C, called the center. Unlike a Euclidean circle, a taxicab circle is square and contains a finite number of points. In fact, if the radius of the circle is n, then the circle contains $4n$ points. The circumference of a taxicab circle with radius r is $8r$.

◆ **Example 1**
A taxicab circle has radius 18. Determine the number of points on the circle and find its circumference.

◆ **Solution**
Since $r = 18$, the circle has $4(18) = 72$ points and its circumference is $8(18) = 144$.

◆ **Example 2**
Plot the taxicab circle with center $C(2, 3)$ and radius 4.

◆ **Solution**
Plot the point C. Then plot each point in the plane that is 4 blocks from C.



Find the number, n, of points on the taxicab circle with the given radius. Then find the circumference, C, of the circle.

7. $r = 20$
$n =$ ________ $C =$ ________

8. $r = 6$
$n =$ ________ $C =$ ________

9. $r = 10$
$n =$ ________ $C =$ ________

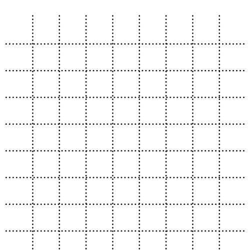
10. $r = 12$
$n =$ ________ $C =$ ________

11. $r = 7$
$n =$ ________ $C =$ ________

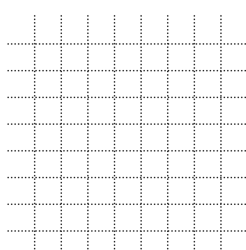
12. $r = 16$
$n =$ ________ $C =$ ________

Plot the taxicab circle with center C and radius r.

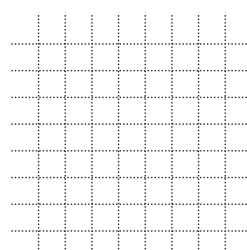
13. $C(-3, -4)$; $r = 2$



14. $C = (-3, 0)$; $r = 4$



15. $C(-2, 2)$; $r = 3$



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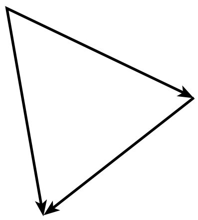
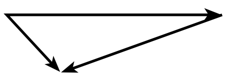
Lesson 10.6

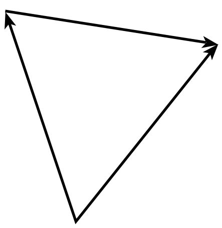
1.



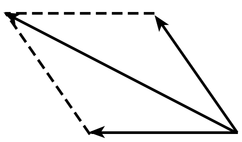
2.



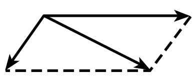
3.



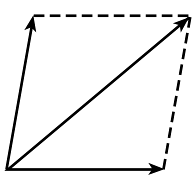
4.



5.



6.



7. 54.2 mi/h; 12° **8.** 48.8 mi/h; 20°

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Reteaching — Chapter 11

Lesson 11.1

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4. 35.60 mm **5.** 123.61 ft

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9. $\frac{TU}{QT} = \frac{2x}{x\sqrt{5} - x} = \frac{2}{\sqrt{5} - 1} = \frac{2\sqrt{5} + 2}{4}$
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10–11. Check drawings.

Lesson 11.2

1–3. Check drawings.

1. 5 **2.** 4 **3.** 7 **4.** 20 **5.** 23 **6.** 24

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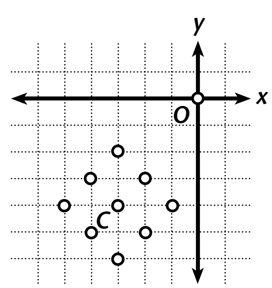
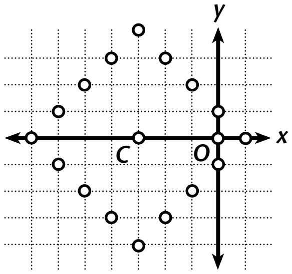
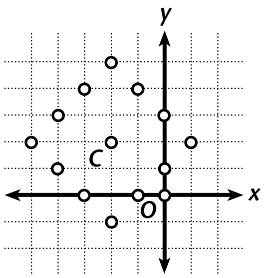


ANSWERS

14.



15.



Lesson 11.3

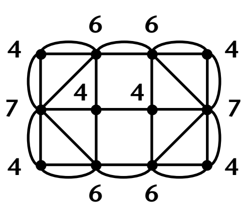
1. Yes; the graph would have exactly two odd vertices, A and C.

2. Yes; the graph would have exactly two odd vertices, A and E.

3. Yes. **4.** Yes. **5.** No. **6.** Yes.

7. Yes. **8.** Yes. **9.** No; yes. **10.** No; yes.

11.



12. No.

Lesson 11.4

1. Yes. **2.** No. **3.** Yes. **4.** No.

5. The torus and sphere are not topologically equivalent.

6. The sphere is topologically equivalent to a cube, which has Euler characteristic 2.

7. -2 **8.** -2

Lesson 11.5

1. A segment consists of two points on a line and all the points of the line between them (an arc of a great circle); sample: $\overline{EF}$

2. $\overleftrightarrow{CD}$ and the small circle through A and B; no; the small circle is not a line.

3. $\overleftrightarrow{CF}$ and $\overleftrightarrow{HD}$ **4.** $\overleftrightarrow{AG}$

5. not a line **6.** not a line **7.** not a line

Lesson 11.6

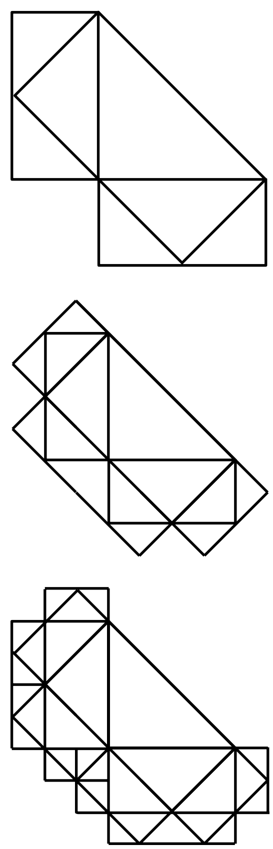
1. $1; \frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \frac{1}{16}; \frac{1}{32}; \frac{1}{2^n}$

2. $1; 3; 7; 15; 31; 63; 2^{n+1} - 1$

3. $1; 2; 3; 4; 5; 6; n + 1$

4. The total length of the branches increases without limit.

5.



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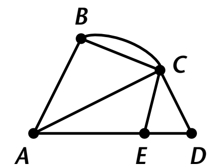
NAME ______________________ CLASS ____________ DATE ____________

Reteaching
11.3 Graph Theory

◆ **Skill A** Determining whether a graph contains an Euler path

Recall The diagram at the right is called a **graph** (or a **network**). Each point is a **vertex**. Each segment or curve that links two vertices is called an **edge**. (An edge may also connect a single vertex to itself.) The **degree** of a vertex is the number of edges at the vertex. (An edge that connects a vertex to itself is counted twice.) A vertex is **even** if its degree is even, and **odd** if its degree is odd.



An **Euler path** is a path that travels continuously from one vertex of a graph to each other vertex, traveling along each edge exactly once. A graph contains an Euler path if and only if it has at most two odd vertices.

◆ **Example**

Determine whether the graph above contains an Euler path.

◆ **Solution**

There are three edges at vertex A, so A has degree 3 and A is odd.
There are three edges at vertex B, so B has degree 3 and B is odd.
There are five edges at vertex C, so C has degree 5 and C is odd.
There are two edges at vertex D, so D has degree 2 and D is even.
There are three edges at vertex E, so E has degree 3 and E is odd.
Since there are more than two odd vertices in the graph, the graph does not contain an Euler path.

Suppose the indicated change were made to the graph in the figure at the top of the page. Would the resulting graph contain an Euler path? Explain.

1. An edge is added from vertex B to vertex E. ____________________

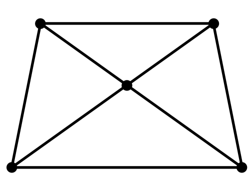
____

2. One edge from vertex B to C is removed. ____________________

____

Does the graph contain an Euler path? Write *Yes* or *No*.

3.



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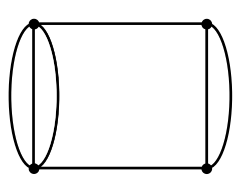
4.



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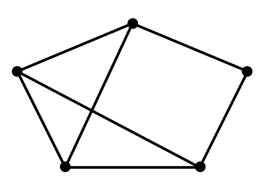

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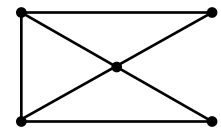
Each vertex in the graph at the right represents one room of an apartment. An edge connecting two vertices indicates a doorway between the rooms.



6. Does the graph contain an Euler path? ____________________

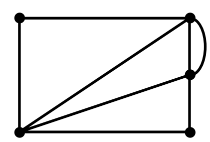
7. Can you walk through the apartment on a continuous path, passing through each doorway exactly once? ____________________

◆ **Skill B** Determining whether a graph contains an Euler circuit

Recall An Euler path has a starting point and an ending point. An **Euler circuit** is an Euler path for which any vertex is both a starting point and an ending point.

For every route into an even vertex, there is another route out, so that an even vertex can be anywhere on an Euler path. However, the edges at an odd vertex do not occur in pairs, so if one route is taken in and another out, there is a vertex left over. Then an odd vertex can only be at the beginning or end of an Euler path. It is clear that if a graph contains any odd vertices, it cannot contain an Euler circuit. Any graph that contains only even vertices contains an Euler circuit.

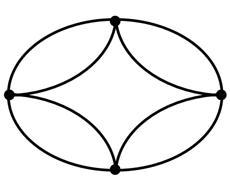
◆ **Example**
Does the graph contain an Euler circuit? If not, does it contain an Euler path?



◆ **Solution**
Yes. The graph has five vertices, each of which is even. Therefore, the graph contains an Euler circuit.

Does the graph contain an Euler circuit? If not, does it contain an Euler path?

8.



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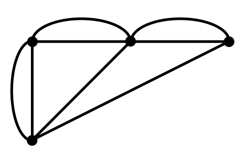

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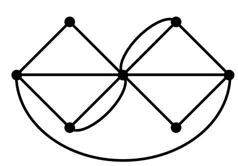

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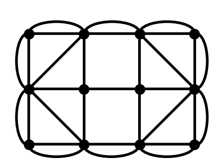
Refer to the figure at the right.

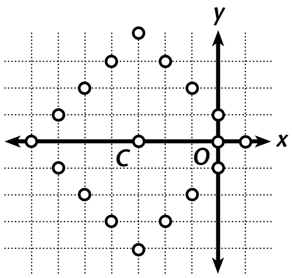


11. Label each vertex with its degree.

12. Is it possible to trace the figure in one stroke without lifting the pencil or retracing any part of the figure? ____________________

ANSWERS

14.



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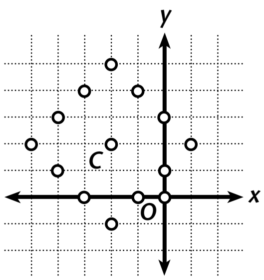


Lesson 11.3

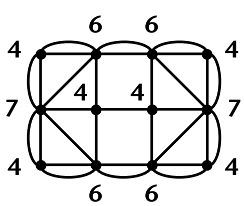
1. Yes; the graph would have exactly two odd vertices, *A* and *C*.

2. Yes; the graph would have exactly two odd vertices, *A* and *E*.

3. Yes. **4.** Yes. **5.** No. **6.** Yes.

7. Yes. **8.** Yes. **9.** No; yes. **10.** No; yes.

11.



12. No.

Lesson 11.4

1. Yes. **2.** No. **3.** Yes. **4.** No.

5. The torus and sphere are not topologically equivalent.

6. The sphere is topologically equivalent to a cube, which has Euler characteristic 2.

7. -2 **8.** -2

Lesson 11.5

1. A segment consists of two points on a line and all the points of the line between them (an arc of a great circle); sample: $\overline{EF}$

2. $\overleftrightarrow{CD}$ and the small circle through *A* and *B*; no; the small circle is not a line.

3. $\overleftrightarrow{CF}$ and $\overleftrightarrow{HD}$ **4.** $\overleftrightarrow{AG}$

5. not a line **6.** not a line **7.** not a line

Lesson 11.6

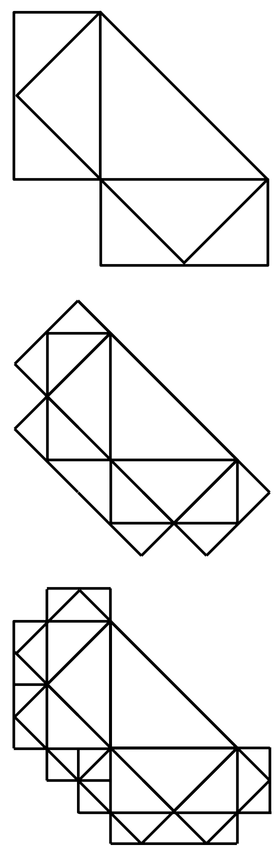
1. $1; \frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \frac{1}{16}; \frac{1}{32}; \frac{1}{2^n}$

2. $1; 3; 7; 15; 31; 63; 2^{n+1} - 1$

3. $1; 2; 3; 4; 5; 6; n + 1$

4. The total length of the branches increases without limit.

5.



NAME ______________________ CLASS ____________ DATE ____________

Reteaching
11.4 Topology: Twisted Geometry

◆ **Skill A** Exploring topologically equivalent plane figures

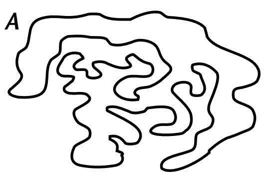
Recall Two figures are topologically equivalent if one can be stretched, shrunk, or otherwise distorted into the other without cutting tearing, or intersecting itself, or compressing a segment or a curve into a point. You can imagine that the figures are made of extremely elastic material and picture whether you can manipulate one (without tearing it or puncturing any holes in it) until its appearance matches the other.

A simple closed curve is a curve that is topologically equivalent to a circle. According to the **Jordan Curve Theorem,** every simple closed curve in a plane divides the plane into two regions, the inside and the outside. Every curve that connects a point inside the curve to a point outside the curve must intersect the curve.

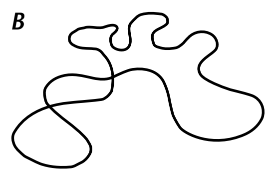
◆ **Example**

Determine whether Figure *A* and Figure *B* are topologically equivalent.

A



B

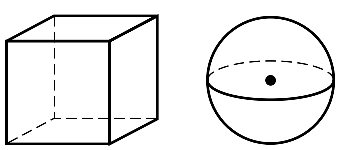


◆ **Solution**

Figure *A* and Figure *B* are not topologically equivalent. Figure *A* is actually a simple closed curve. Figure *B* intersects itself.

Are the figures topologically equivalent?

1.



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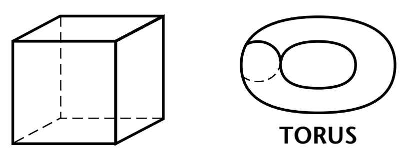
2.



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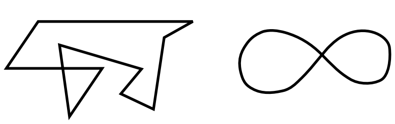
3.



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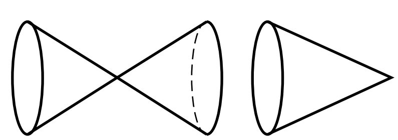
4.



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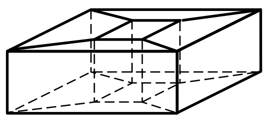
◆ **Skill B** Determining the Euler characteristic of a topological figure

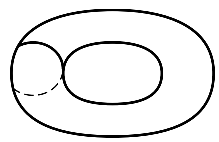
Recall In Chapter 6, you learned Euler's formula. For any polyhedron with V vertices, E edges, and F faces, $V - E + F = 2$. For any space figure, the number $V - E + F$ is called the **Euler characteristic** of the figure. (Notice, then, that the Euler characteristic of any polyhedron is 2.)

When a figure is deformed into a topologically equivalent figure, the Euler characteristic remains unchanged. That is, the Euler characteristic is *invariant*. Properties that are changed in the process of deforming are not invariant. For example, distance, area, and volume are not invariant.

◆ **Example**

The figures are topologically equivalent. The figure on the left has 16 vertices, 32 edges, and 16 faces. The donut-shaped figure on the right is called a *torus*. Find the Euler characteristic of a torus.





◆ **Solution**

The Euler characteristic of the figure on the left is $16 - 32 + 16 = 0$. Since the torus is topologically equivalent to the figure, the Euler characteristic of the torus is 0.

Refer to the text and the example above.

5. Explain why the Euler characteristic of a sphere is *not* 0.

____

____

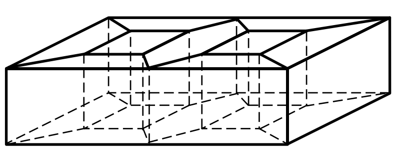
6. Explain why the Euler characteristic of a sphere is 2.

____

____

The figure in Exercise 7 has 28 vertices, 56 faces, and 26 edges and is topologically equivalent to the figure in Exercise 8. Find the Euler characteristic of each figure.

7.



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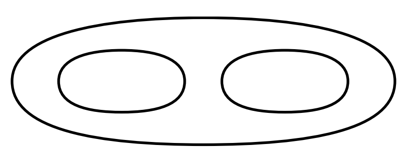

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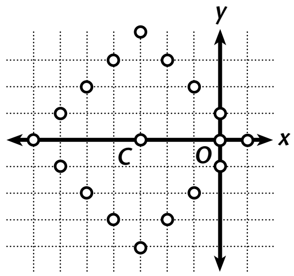
ANSWERS

14.



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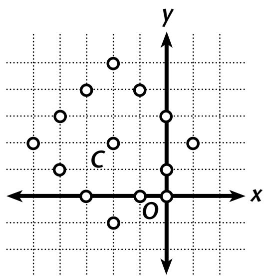


Lesson 11.3

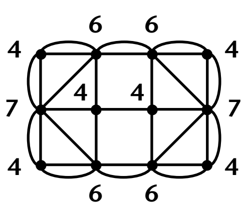
1. Yes; the graph would have exactly two odd vertices, A and C.

2. Yes; the graph would have exactly two odd vertices, A and E.

3. Yes. **4.** Yes. **5.** No. **6.** Yes.

7. Yes. **8.** Yes. **9.** No; yes. **10.** No; yes.

11.



12. No.

Lesson 11.4

1. Yes. **2.** No. **3.** Yes. **4.** No.

5. The torus and sphere are not topologically equivalent.

6. The sphere is topologically equivalent to a cube, which has Euler characteristic 2.

7. -2 **8.** -2

Lesson 11.5

1. A segment consists of two points on a line and all the points of the line between them (an arc of a great circle); sample: $\overline{EF}$

2. $\overleftrightarrow{CD}$ and the small circle through A and B; no; the small circle is not a line.

3. $\overleftrightarrow{CF}$ and $\overleftrightarrow{HD}$ **4.** $\overleftrightarrow{AG}$

5. not a line **6.** not a line **7.** not a line

Lesson 11.6

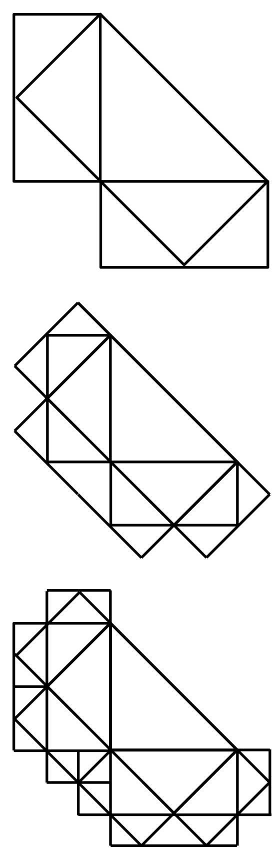
1. $1; \frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \frac{1}{16}; \frac{1}{32}; \frac{1}{2^n}$

2. $1; 3; 7; 15; 31; 63; 2^{n+1} - 1$

3. $1; 2; 3; 4; 5; 6; n + 1$

4. The total length of the branches increases without limit.

5.



NAME ______________________ CLASS ____________ DATE ____________

Reteaching
11.5 Euclid Unparalleled

◆ **Skill A** Identifying lines and relationships among lines in spherical geometry

Recall In Euclidean geometry, the Parallel Postulate (which you studied in Chapter 5) guarantees that through a point not on a line there is exactly one line through the given point parallel to the given line. If the Parallel Postulate is not true, what are the alternatives? There are two.

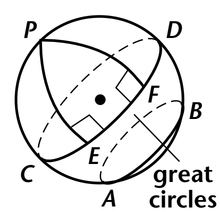
1. There is no line through the given point parallel to the given line.
2. There is more than one line through the given point parallel to the given line.
The first statement is true for spherical geometry. The second is true for hyperbolic geometry.

Recall that the terms *point*, *line*, and *plane* are not defined in Euclidean geometry. Suppose you thought of a plane not as a flat surface extending infinitely in all directions, but as a sphere. The Euclidean interpretation of a straight line would make no sense; there would be no such lines in the plane. However, you can use your understanding of Euclidean geometry to understand the concepts of lines on a sphere. In the text that follows, the terms *straight line* and *line* will be used to distinguish between a line in a plane and a line on a sphere.

If a plane intersects a sphere, the intersection is a circle. If the plane intersects the center of the sphere, the circle is a **great circle** that divides the sphere into two halves. If you consider that a great circle extends infinitely in either direction, it makes sense that in spherical geometry, a great circle is a line. On a sphere, any two great circles intersect in two points. The great circle through points A and B may be referred to as $\overleftrightarrow{AB}$.

Since the Parallel Postulate of Euclidean Geometry is not true for spherical geometry, it is clear that many of the theorems of Euclidean geometry that rely on that postulate for proof are not true either. For example, the sum of the measures of a triangle on a sphere is always between 180° and 540°.

Example
Tell whether each figure is a line.
a. the circle that passes through A and B
b. the circle that passes through C and D
c. the arc from E to F



◆ **Solution**
a. No; the circle through A and B is not a great circle.
b. Yes; the great circle is $\overleftrightarrow{CD}$.
c. No; the arc from E to F is not a great circle.

Refer to the figure in the example.

1. Consider the definition of *segment* in Euclidean geometry and write a definition of *segment* for spherical geometry. Then identify a segment in the figure.

____

____

2. Name two figures on the sphere in the example on the preceding page that appear to be parallel. Is this a contradiction? Explain.

____

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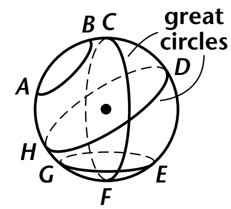
3. Refer to the figure at right. Identify all lines.



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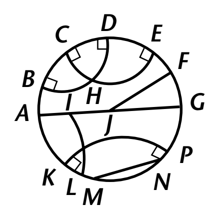
◆ **Skill B** Identifying lines and their relationships in hyperbolic geometry

Recall There are several models of hyperbolic geometry in which through any given line and a point not on the line, there are infinitely many lines parallel to the given line. One such model is the Poincaré model. In this model, the surface is neither a plane nor a sphere, but a circle .

A **line** in Poincaré's model is an arc that has endpoints on the circle, is inside the circle, and is orthogonal to it. (*Orthogonal* means perpendicular.)

◆ **Example**
In the figure, H is not on $\overleftrightarrow{KP}$. Name two different lines through H that are parallel to $\overleftrightarrow{KP}$.



◆ **Solution**
$\overleftrightarrow{BD}$ and $\overleftrightarrow{CE}$

Refer to the figure in the example above. If the curve through the given points is a line, write its name. If not, write *not a line*.

4. A and G ______________________

5. J and F ______________________

6. I and L ______________________

7. M and N ______________________

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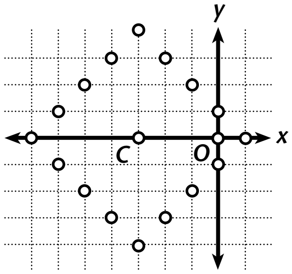
14.



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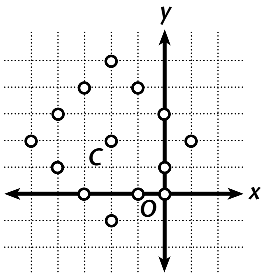


Lesson 11.3

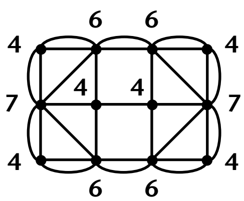
1. Yes; the graph would have exactly two odd vertices, *A* and *C*.

2. Yes; the graph would have exactly two odd vertices, *A* and *E*.

3. Yes. **4.** Yes. **5.** No. **6.** Yes.

7. Yes. **8.** Yes. **9.** No; yes. **10.** No; yes.

11.



12. No.

Lesson 11.4

1. Yes. **2.** No. **3.** Yes. **4.** No.

5. The torus and sphere are not topologically equivalent.

6. The sphere is topologically equivalent to a cube, which has Euler characteristic 2.

7. -2 **8.** -2

Lesson 11.5

1. A segment consists of two points on a line and all the points of the line between them (an arc of a great circle); sample: $\overline{EF}$

2. $\overleftrightarrow{CD}$ and the small circle through *A* and *B*; no; the small circle is not a line.

3. $\overleftrightarrow{CF}$ and $\overleftrightarrow{HD}$ **4.** $\overleftrightarrow{AG}$

5. not a line **6.** not a line **7.** not a line

Lesson 11.6

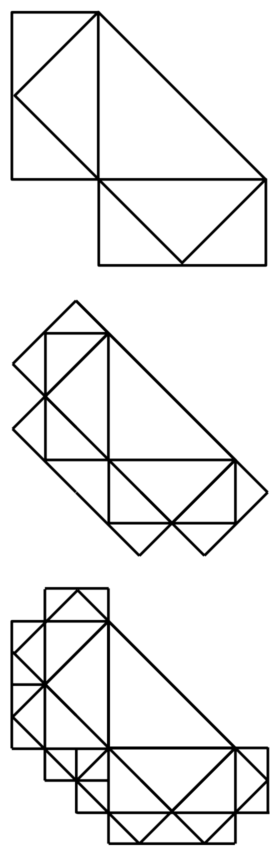
1. $1; \frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \frac{1}{16}; \frac{1}{32}; \frac{1}{2^n}$

2. $1; 3; 7; 15; 31; 63; 2^{n+1} - 1$

3. $1; 2; 3; 4; 5; 6; n + 1$

4. The total length of the branches increases without limit.

5.



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NAME ______________________ CLASS ____________ DATE ____________

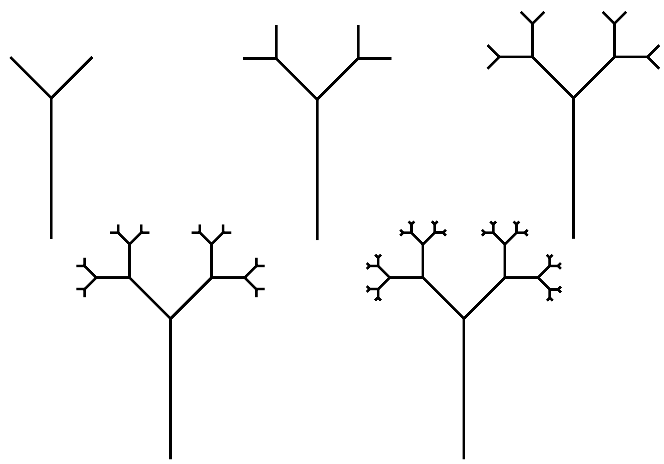
Reteaching
11.6 Fractal Geometry

◆ **Skill A** Examining the properties of fractals

Recall A **fractal** is a geometric figure that is self-similar. The figure appears the same, whether at a distance or up close. Each part of the figure is similar to the entire figure. A fractal can be produced by starting with a simple figure, changing it in some prescribed way, and then repeating the process over and over. This repetition is called *iteration*. Many objects in nature can be modeled by fractals.

◆ **Example**

The growth of a tree can be modeled by a fractal. The fractal begins with a single branch (which will become the trunk) with a length of 1. In the first iteration, two new branches grow. Each is half the length of the original branch. For each subsequent iteration, two branches grow out of each new branch from the previous iteration. Again, each is half the length of the branches from the previous iteration. Make a table showing the number of new branches after each iteration and write an expression for the number of new branches after n iterations.



◆ **Solution**

iterations	0	1	2	3	4	5	n
number of new branches	1	2	4	8	16	32	2^n

Refer to the example. Complete each table.

1.

iterations	0	1	2	3	4	5	n
length of new branches							

2.

iterations	0	1	2	3	4	5	n
total number of branches							

3.

iterations	0	1	2	3	4	5	*n*
total length of branches							

4. Describe what happens to the total length of the branches as the number of iterations increases.

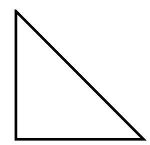
____

____

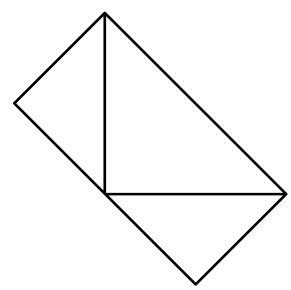
◆ Skill B Using iteration to draw fractals

Recall To draw a fractal, begin with a simple figure, change it in some prescribed way, and then repeat the process over and over.

◆ **Example**
An isosceles right triangle is shown. Sketch the first iteration of a fractal by drawing a right triangle on each leg of the given triangle.



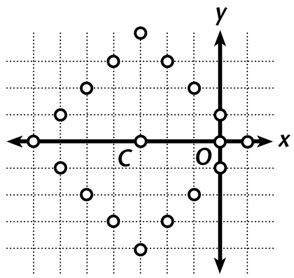
◆ **Solution**
When the two triangles are drawn, the resulting figure will be a rectangle. To sketch the triangles, sketch a line parallel to the hypotenuse of the original triangle and then two perpendicular segments to form the rectangle.



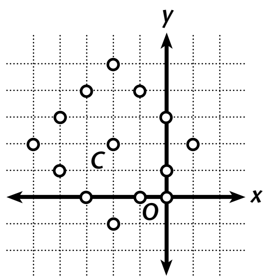
5. Draw the second, third, and fourth iterations of the fractal in the example.

ANSWERS

14.



15.



Lesson 11.3

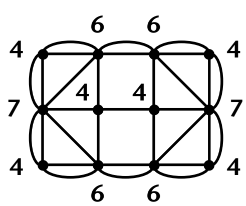
1. Yes; the graph would have exactly two odd vertices, *A* and *C*.

2. Yes; the graph would have exactly two odd vertices, *A* and *E*.

3. Yes. **4.** Yes. **5.** No. **6.** Yes.

7. Yes. **8.** Yes. **9.** No; yes. **10.** No; yes.

11.



12. No.

Lesson 11.4

1. Yes. **2.** No. **3.** Yes. **4.** No.

5. The torus and sphere are not topologically equivalent.

6. The sphere is topologically equivalent to a cube, which has Euler characteristic 2.

7. −2 **8.** −2

Lesson 11.5

1. A segment consists of two points on a line and all the points of the line between them (an arc of a great circle); sample: $\overline{EF}$

2. $\overleftrightarrow{CD}$ and the small circle through *A* and *B*; no; the small circle is not a line.

3. $\overleftrightarrow{CF}$ and $\overleftrightarrow{HD}$ **4.** $\overleftrightarrow{AG}$

5. not a line **6.** not a line **7.** not a line

Lesson 11.6

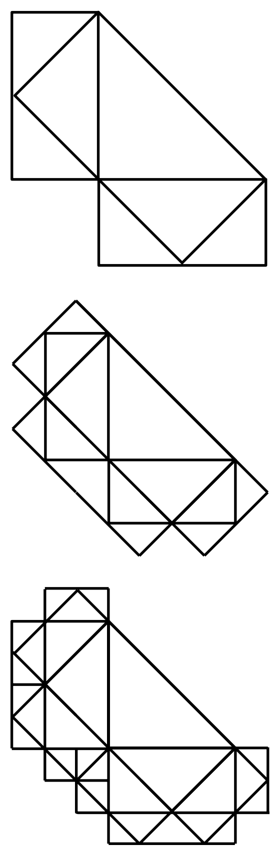
1. $1; \frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \frac{1}{16}; \frac{1}{32}; \frac{1}{2^n}$

2. $1; 3; 7; 15; 31; 63; 2^{n+1} - 1$

3. $1; 2; 3; 4; 5; 6; n + 1$

4. The total length of the branches increases without limit.

5.



NAME ______________________ CLASS ____________ DATE ____________

Reteaching
11.7 Other Transformations: Projective Geometry

◆ **Skill A** Drawing images of figures under affine transformations

Recall The transformations we have studied so far (reflections, rotations, translations, and dilations) are all special types of affine transformations.

An **affine transformation** transforms a plane figure in such a way that the images of collinear points are collinear points, the images of straight lines are straight lines, the images of intersecting lines are intersecting lines, and the images of parallel lines are parallel lines. Affine transformations can be described using coordinates.

◆ **Example**

Let T be the affine transformation $T(x, y) = \left(\frac{3}{2}x, -\frac{1}{2}y\right)$ and let $ABCD$ be the quadrilateral with vertices $A(0, 0)$, $B(6, 0)$, $C(6, 4)$, and $D(2, 4)$. Draw $ABCD$ and its image, $A'B'C'D'$, under T.

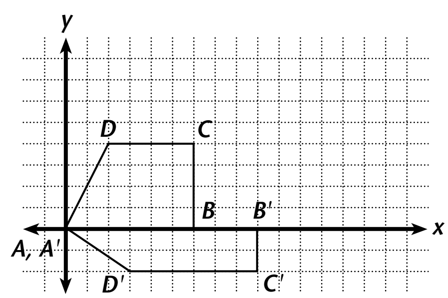
◆ **Solution**

$A' = \left(\frac{3}{2}(0), -\frac{1}{2}(0)\right) = (0, 0)$

$B' = \left(\frac{3}{2}(6), -\frac{1}{2}(0)\right) = (9, 0)$

$C' = \left(\frac{3}{2}(6), -\frac{1}{2}(4)\right) = (9, -2)$

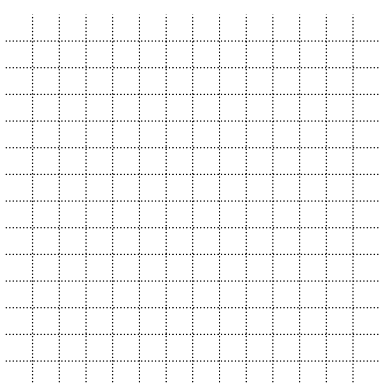
$D' = \left(\frac{3}{2}(2), -\frac{1}{2}(4)\right) = (3, -2)$



Given an affine transformation and the vertices of a figure, draw the figure and its image under the transformation.

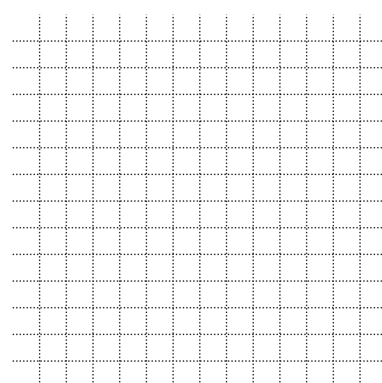
1. $T(x, y) = (-2x, 2y)$

$A(0, 0)$, $B(-3, 2)$, $C(2, 5)$



2. $T(x, y) = \left(-\frac{1}{2}x, \frac{3}{2}y\right)$

$P(-3, -3)$, $Q(-3, 3)$, $R(4, 3)$, $S(6, -3)$



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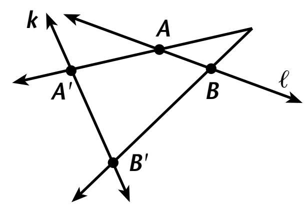
______________________________ CLASS ______________ DATE ______________

◆Skill B Drawing projection images of figures

Recall When a movie is shown in a theater, the image of a piece of film is projected onto a screen. The process involves a **central projection**. Given a point O, called the center of projection, the image of any point A is a point A' on $\overrightarrow{OA}$. The rays from O are called **projective rays**.

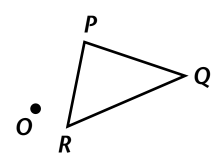
In the figure, the points A and B on line ℓ are projected onto line k. The center of projection is point P.



Projective geometry is the study of the properties of figures that do not change under a projection. Projective geometry includes no concepts of size, measurement, or congruence. Its theorems deal with the concepts of position of points and intersections of lines. In drawing projection images, only an unmarked straightedge may be used.

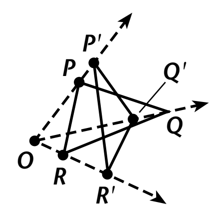
◆ **Example**

Let point O be a center of projection and draw a projection image of $\triangle PQR$.



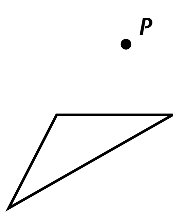
◆ **Solution**

First note that there are infinitely many possible images. Begin by drawing $\overrightarrow{OP}$, $\overrightarrow{OQ}$, and $\overrightarrow{OR}$. Choose a random point P' on $\overrightarrow{OP}$, a random point Q' on $\overrightarrow{OQ}$, and a random point R' on $\overrightarrow{OR}$. Draw $\Delta P'Q'R'$, a projection image of $\triangle PQR$.

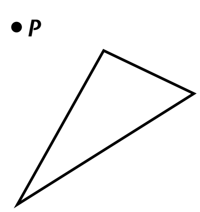


Given *P*, the center of projection, draw a projection image of the given triangle.

3.

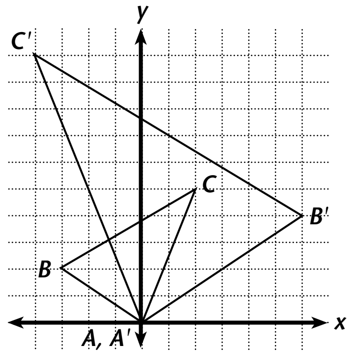


4.

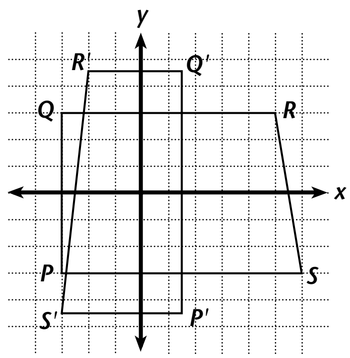


Lesson 11.7

1.



2.



3. Check drawings.

4. Check drawings.

Reteaching — Chapter 12

Lesson 12.1

1. *modus tollens;* valid

2. denying the antecedent; invalid

3. affirming the consequent; invalid

4. *modus ponens;* valid

5. no conclusion

6. You will buy a pizza.

7. My age is not divisible by 9.

Lesson 12.2

1. All squares are rectangles and a yard is three feet long; true.

2. A triangle is a polygon and a hexagon has three sides; true.

3. Pine trees are evergreens or gorillas are pink; false.

4. The moon is a planet or Neptune is a star; false.

5. Ice hockey is a sport or Abraham Lincoln's portrait is on the nickel; true.

6. not truth functionally equivalent

7. truth functionally equivalent

Lesson 12.3

1. If not y, then not x; if not y is true and not x is false (that is, y is false and x is true)

2. If I finish my report, then the library is open; if I finish my report and the library is not open.

3. If there is not an orange in my lunch, then it is not Tuesday; if there is not an orange in my lunch and it is Tuesday.

4. If $n + 1$ is odd, then n is even; if $n + 1$ is odd and n is odd.

5. If a number is even, then it is a multiple of 2.

6. If a triangle is isosceles, then it has at least two congruent sides.

7. If a number is negative, then it does not have a real square root.

Lesson 12.4

1. Not a contradiction

2. Not a contradiction

3. contradiction

4. contradiction

5. Let $x^2 \neq y^2$ and assume temporarily that $x = y$.

Menu

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NAME ______________________ CLASS ____________ DATE ____________

Reteaching

12.1 Truth and Validity in Logical Arguments

◆ **Skill A** Identifying valid and invalid arguments

Recall In logic, a **statement** is a sentence that is either true or false. An **argument** is a sequence of statements. The initial statements, called **premises,** lead to the final statement, called the **conclusion.** A **valid** argument is one in which whenever the premises are true, the conclusion is true. In this case, the conclusion is said to be a logical consequence of the premises. The validity of an argument is determined by its form, not by the truth of its premises. Two valid argument forms are the argument form, called *modus ponens* (proposing mode), and the law of indirect reasoning, called *modus tollens* (removing mode).

	modus ponens	*modus tollens*
Valid argument →	*modus ponens*	*modus tollens*
true premises →	If p then q p	If p then q Not q
true conclusion →	Therefore, q	Therefore, not p

An argument is **invalid** if the conclusion does not follow logically from the premises. Two invalid forms are called *affirming the consequent,* and *denying the antecedent.* In affirming the consequent, you assume that because the conclusion of a conditional statement is true, the hypothesis is true. In denying the antecedent, you assume that if the hypothesis is not true, then the conclusion must also not be true. Both are logical errors or *fallacies.*

Invalid argument →	*affirming the consequent*	*denyinng the antecedent*
true premises →	If p then q q	If p then q Not p
conclusion → (not necessarily true)	Therefore, p	Therefore, not q

◆ **Example**

Determine whether the argument is valid.

a. If Aurelia lives in New York City, then she lives in Argentina. Aurelia lives in New York City. Therefore, she lives in Argentina.

b. If Louis wears warm mittens, then it is winter. Louis does not wear warm mittens. Therefore, it is not winter.

◆ **Solution**

a. Let p = Aurelia lives in New York City and q = she lives in Argentina. The argument can be written: If p then q; p. Therefore, q.
The argument has the *modus ponens* form, so it is a valid argument even though the premise is not true.

b. Let p = Louis wears warm mittens and q = it is winter. The argument can be written: If p then q; not p. Therefore, not q.
The argument has the *denying the antecedent* form, so it is invalid.

Menu

Print

Identify the form of the argument and tell whether it is valid or invalid.

1. If a bicycle has wheels, then pigeons can skate. Pigeons cannot skate. Therefore, a bicycle does not have wheels. ____________________

2. If it raining, the parade will be canceled. It is not raining. Therefore, the parade will not be canceled. ____________________

3. If a quadrilateral is a parallelogram, then it has a pair of opposite sides that are parallel. *JKLM* has a pair of opposite sides that are parallel. Therefore, *JKLM* is a parallelogram. ____________________

4. If I pass the test, then I will pass the course. I pass the test. Therefore, I pass the course. ____________________

◆ **Skill B** Reaching conclusions

Recall If a valid argument can be written using a sequence of true statements, then a true conclusion can be reached.

◆ **Example**
Determine what, if any, conclusion can be reached if both premises are true. Explain.
If a rectangle is a square, then it is a rhombus.
Rectangle *ABCD* is not a rhombus.

◆ **Solution**
Let p = a rectangle is a square and q = it is a rhombus. The premises can be written: If p, then q; not q.
This is the form of the premises of the *modus tollens* argument. The conclusion that follows is "not p." Rectangle *ABCD* is not a square.

Tell what, if any, conclusion can be reached if both premises are true.

5. If money grows on trees, then the streets are paved with gold. Money does not grow on trees. ____________________

6. If I rent a video, then you will buy a pizza. I rent a video.

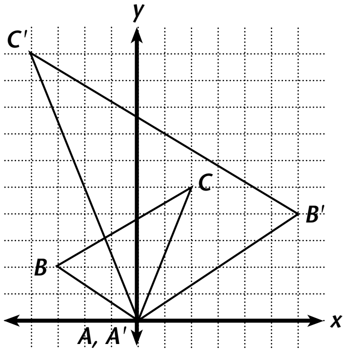
____

7. If a number is divisible by 9, then it is divisible by 3. My age is not divisible by 3.

____

Lesson 11.7

1.



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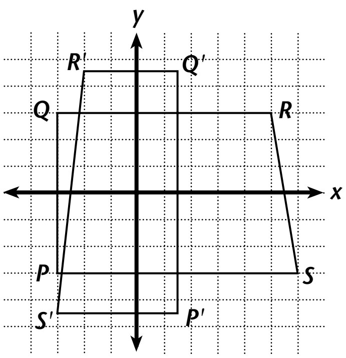

3. Check drawings.

4. Check drawings.

Reteaching — Chapter 12

Lesson 12.1

1. *modus tollens;* valid

2. denying the antecedent; invalid

3. affirming the consequent; invalid

4. *modus ponens;* valid

5. no conclusion

6. You will buy a pizza.

7. My age is not divisible by 9.

Lesson 12.2

1. All squares are rectangles and a yard is three feet long; true.

2. A triangle is a polygon and a hexagon has three sides; true.

3. Pine trees are evergreens or gorillas are pink; false.

4. The moon is a planet or Neptune is a star; false.

5. Ice hockey is a sport or Abraham Lincoln's portrait is on the nickel; true.

6. not truth functionally equivalent

7. truth functionally equivalent

Lesson 12.3

1. If not y, then not x; if not y is true and not x is false (that is, y is false and x is true)

2. If I finish my report, then the library is open; if I finish my report and the library is not open.

3. If there is not an orange in my lunch, then it is not Tuesday; if there is not an orange in my lunch and it is Tuesday.

4. If $n + 1$ is odd, then n is even; if $n + 1$ is odd and n is odd.

5. If a number is even, then it is a multiple of 2.

6. If a triangle is isosceles, then it has at least two congruent sides.

7. If a number is negative, then it does not have a real square root.

Lesson 12.4

1. Not a contradiction

2. Not a contradiction

3. contradiction

4. contradiction

5. Let $x^2 \neq y^2$ and assume temporarily that $x = y$.

Menu

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NAME ______________________ CLASS __________ DATE __________

Reteaching
12.2 And, Or, *and* Not *in Logical Arguments*

◆ **Skill A** Writing conjunctions and determining their truth values

Recall A **conjunction** is a compound statement in which two statements are joined by the word *and*. The truth table below shows when the conjunction "*p* AND *q*" is true and when it is false.

p	*q*	*p* AND *q*
T	T	T
T	F	F
F	T	F
F	F	F

The conjunction *p* AND *q* is true if and only if both *p* and *q* are true.

◆ **Example**
Write a conjunction using the two statements and determine whether the conjunction is true or false: A cube has 8 vertices. The American flag is green.

◆ **Solution**
A cube has 8 vertices *and* the American flag is not green.
The conjunction is false because only one of the statements is true.

Write a conjunction using the given statements and determine whether the conjunction is true or false.

1. All squares are rectangles. A yard is three feet long. ______________________

2. A triangle is a polygon. A hexagon has three sides. ______________________

◆ **Skill B** Writing disjunctions and determining their truth values

Recall A **disjunction** is a compound statement in which two statements are joined by the word *or*. The truth table for the disjunction "*p* OR *q*" is shown.

p	*q*	*p* OR *q*
T	T	T
T	F	T
F	T	T
F	F	F

The disjunction *p* OR *q* is true if either *p* is true, *q* or true, or both *p* and *q* are true. The disjunction is false if only if both *p* and *q* are false.

◆ **Example**
Write a conjunction using the two statements and determine whether the conjunction is true or false: Cats have wings. Baseballs are not cubes.

◆ **Solution**
Cats have wings *or* baseballs are not cubes. The disjunction is true because one statements is true.

CLASS ______ DATE ______

Write a disjunction using the given statements and determine whether the disjunction is true or false.

3. Pine trees are evergreens. Gorillas are pink. ______

4. The moon is a planet. Neptune is a star. ______

5. Ice hockey is a sport. Abraham Lincoln's portrait is on the nickel. ______

______

◆ Skill C Determining whether statements are truth functionally equivalent

Recall Given any statement p, you can write the statement NOT p (or $\sim p$), called the negation of p. For example, if p = the car is green, $\sim p$ = the car is not green. For any statement p, p and $\sim p$ have opposite truth values.

Two statements are **truth functionally equivalent** if they have the same truth value.

◆ **Example**

The truth table for $p \rightarrow q$ is given. Complete the table to determine whether $p \rightarrow q$ and $\sim p$ OR q are truth functionally equivalent.

p	q	$p \rightarrow q$	$\sim p$ OR q
T	T	T	
T	F	F	
F	T	T	
F	F	T	

◆ **Solution**

The disjunction $\sim p$ OR q is true if p is true, if q is true, or if both p and q are true. Since $p \rightarrow q$ and $\sim p$ OR q have the same truth value, they are truth functionally equivalent.

p	q	$p \rightarrow q$	$\sim p$ OR q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Determine whether the statements are truth functionally equivalent.

6.

p	q	$\sim p$	$\sim q$	$\sim p$ AND q	p OR $\sim q$
T	T				
T	F				
F	T				
F	F				

______

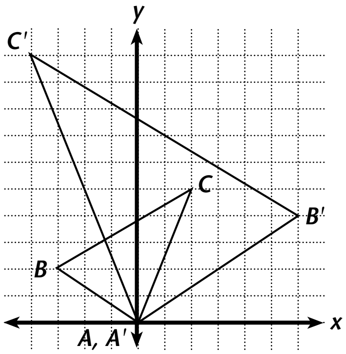
7.

p	q	$\sim p$	$\sim q$	$\sim(p$ AND $q)$	p OR $\sim q$
T	T				
T	F				
F	T				
F	F				

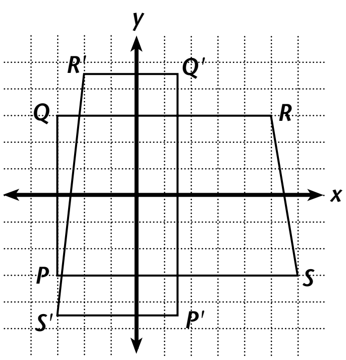
______

Lesson 11.7

1.



2.



3. Check drawings.

4. Check drawings.

Reteaching — Chapter 12

Lesson 12.1

1. *modus tollens;* valid

2. denying the antecedent; invalid

3. affirming the consequent; invalid

4. *modus ponens;* valid

5. no conclusion

6. You will buy a pizza.

7. My age is not divisible by 9.

Lesson 12.2

1. All squares are rectangles and a yard is three feet long; true.

2. A triangle is a polygon and a hexagon has three sides; true.

3. Pine trees are evergreens or gorillas are pink; false.

4. The moon is a planet or Neptune is a star; false.

5. Ice hockey is a sport or Abraham Lincoln's portrait is on the nickel; true.

6. not truth functionally equivalent

7. truth functionally equivalent

Lesson 12.3

1. If not y, then not x; if not y is true and not x is false (that is, y is false and x is true)

2. If I finish my report, then the library is open; if I finish my report and the library is not open.

3. If there is not an orange in my lunch, then it is not Tuesday; if there is not an orange in my lunch and it is Tuesday.

4. If $n + 1$ is odd, then n is even; if $n + 1$ is odd and n is odd.

5. If a number is even, then it is a multiple of 2.

6. If a triangle is isosceles, then it has at least two congruent sides.

7. If a number is negative, then it does not have a real square root.

Lesson 12.4

1. Not a contradiction

2. Not a contradiction

3. contradiction

4. contradiction

5. Let $x^2 \neq y^2$ and assume temporarily that $x = y$.

NAME ______________________ CLASS __________ DATE __________

Reteaching
12.3 A Closer Look at If-Then Statements

◆ **Skill A** Writing the converse, the inverse, and the contrapositive of a given conditional and determining their truth values

Recall The conditional statement "If p then q" or "p implies q" can be written in symbols as $p \rightarrow q$. The hypothesis of the conditional is p, and the conclusion is q. Given the conditional $p \rightarrow q$, you can write three related statements.

Statement	Symbols	Description
converse	$q \rightarrow p$	The hypothesis and conclusion are interchanged. (If q then p.)
inverse	$\sim p \rightarrow \sim q$	The hypothesis and the conclusion are negated. (If not p then not q.)
contrapositive	$\sim q \rightarrow \sim p$	The hypothesis and the conclusion are negated and interchanged. (If not q then not p.)

As you will see in the truth table below, a conditional is false only if the hypothesis is true and the conclusion is false. Since the converse, inverse, and contrapositive of a conditional are all conditionals as well, each is also only false when the hypothesis is true and the conclusion is false.

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

A conditional and its contrapositive are truth functionally equivalent. The converse and the inverse are also truth functionally equivalent.

◆ **Example**

Given the statement, "If Ricky has brown eyes, then Lucy has red hair" write the converse, inverse, and contrapositive. Determine the conditions under which each of the four statements is false.

◆ **Solution**

converse: If Lucy has red hair, then Ricky has brown eyes.
The converse is false if Lucy has red hair but Ricky does not have brown eyes.

inverse: If Ricky does not have brown eyes, then Lucy does not have red hair.
The inverse is false if Ricky does not have brown eyes but Lucy has red hair.

contrapositive: If Lucy does not have red hair, then Ricky does not have brown eyes.
The contrapositive is false if Lucy does not have red hair Ricky does have brown eyes.

NAME ______ CLASS ______ DATE ______

A conditional statement is given. Write the indicated related conditional, and describe the circumstances under which the related conditional is false.

1. If x, then y. (contrapositive) ______

2. If the library is open, then I will finish my report. (converse) ______

3. If there is an orange in my lunch, then it is Tuesday. (inverse) ______

4. If n is even, then $n + 1$ is odd. (converse) ______

◆ **Skill B** Writing conditional statements in if-then form

Recall Not every conditional is written in if-then form. It may be helpful to rewrite such sentences when using them in logic.

◆ **Example**
Write each statement in if-then form.
a. All triangles are polygons.
b. No rectangles are trapezoids.

◆ **Solution**
a. If a figure is a triangle, then it is a polygon.
b. If a figure is a rectangle, then it is not a trapezoid.

Rewrite each statement in if-then form.

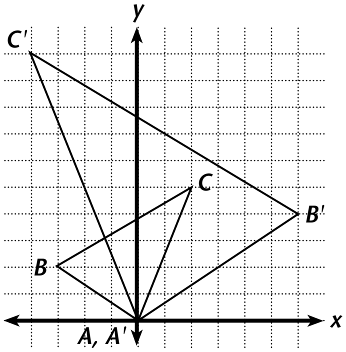
5. Every even number is a multiple of 2. ______

6. All isosceles triangles have at least two congruent sides. ______

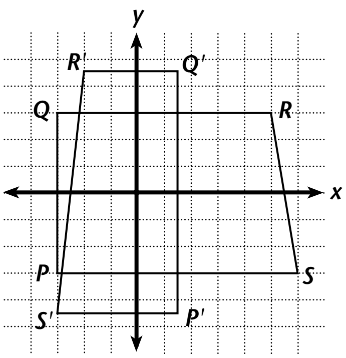
7. No negative number has a real square root. ______

Lesson 11.7

1.



2.



3. Check drawings.

4. Check drawings.

Reteaching — Chapter 12

Lesson 12.1

1. *modus tollens;* valid

2. denying the antecedent; invalid

3. affirming the consequent; invalid

4. *modus ponens;* valid

5. no conclusion

6. You will buy a pizza.

7. My age is not divisible by 9.

Lesson 12.2

1. All squares are rectangles and a yard is three feet long; true.

2. A triangle is a polygon and a hexagon has three sides; true.

3. Pine trees are evergreens or gorillas are pink; false.

4. The moon is a planet or Neptune is a star; false.

5. Ice hockey is a sport or Abraham Lincoln's portrait is on the nickel; true.

6. not truth functionally equivalent

7. truth functionally equivalent

Lesson 12.3

1. If not y, then not x; if not y is true and not x is false (that is, y is false and x is true)

2. If I finish my report, then the library is open; if I finish my report and the library is not open.

3. If there is not an orange in my lunch, then it is not Tuesday; if there is not an orange in my lunch and it is Tuesday.

4. If $n + 1$ is odd, then n is even; if $n + 1$ is odd and n is odd.

5. If a number is even, then it is a multiple of 2.

6. If a triangle is isosceles, then it has at least two congruent sides.

7. If a number is negative, then it does not have a real square root.

Lesson 12.4

1. Not a contradiction

2. Not a contradiction

3. contradiction

4. contradiction

5. Let $x^2 \neq y^2$ and assume temporarily that $x = y$.

Menu

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NAME ______________________ CLASS ____________ DATE ____________

Reteaching
12.4 Indirect Proof

◆ **Skill A** Identifying contradictions

Recall You know that if a logical statement p is true, then its negation, $\sim p$, is false. Consider the conjunction p AND $\sim p$. Since a conjunction is true if and only if both statements are true, the conjunction p AND $\sim p$ is *never* true. In fact, this statement is called a **contradiction.**

If a logical argument results in a contradiction, the premise of the argument must be rejected. This type of argument is called *reductio ad absurdum,* which is Latin for "reduction to absurdity."

◆ **Example**

Determine whether the given conjunction is a contradiction.

a. A triangle is a quadrilateral and a triangle is not a quadrilateral.

b. A square is a rhombus and a square is a parallelogram.

◆ **Solution**

a. Let p = triangle is a quadrilateral. Then $\sim p$ = a triangle is not a quadrilateral, so the given statement has the form p AND $\sim p$ and is a contradiction.

b. Let p = a square is a rectangle. The second part of the conjunction (a square is a parallelogram) is not equivalent to $\sim p$. Then the statement is not a contradiction.

Determine whether the given conjunction is a contradiction. If it is not, write a contradiction using one of the two statements.

1. A cow is a mammal and a cow is not a biped. ______________________________

____

2. The number is negative and the number is positive. ______________________________

____

3. The Allards live in Houston and the Allards do not live in Texas. ______________________________

____

4. A regular hexagon is equilateral and a regular hexagon does not have congruent sides.

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◆ **Skill B** Using contradictions in indirect proof

Recall To prove a statement is true by **contradiction** (or **indirect proof**), assume that it is false and show that this assumption leads to a contradiction. That is, to prove $p \rightarrow q$, assume that both p and $\sim q$ are true and show that this leads to a contradiction.

DeMorgan's Laws (one of which you learned in Lesson 12.2) are often useful in indirect proof.

$\sim (p \text{ AND } q)$ is truth functionally equivalent to $\sim p \text{ OR} \sim q$.
$\sim (p \text{ OR } q)$ is truth functionally equivalent to $\sim p \text{ AND} \sim q$.

◆ **Example**

Write an indirect proof that if $ab = 0$, either $a = 0$ or $b = 0$.

◆ **Solution**

Begin by assuming the hypothesis and the negation of the conclusion. The conclusion is in the form p OR q, the negation of which is truth functionally equivalent to $\sim p$ AND $\sim q$. Then let $ab = 0$, and assume temporarily that $a \neq 0$ and $b \neq 0$. Since $a \neq 0$, a has a multiplicative inverse, $\frac{1}{a}$.

$$ab = 0$$
$$\frac{1}{a} \cdot (ab) = \frac{1}{a} \cdot 0$$
$$\left(\frac{1}{a} \cdot a\right) \cdot b = 0$$
$$1 \cdot b = 0$$

This contradicts the assumption that $b \neq 0$. Therefore the assumption that $a \neq 0$ and $b \neq 0$ is false. That is, if $ab = 0$, then $a = 0$ or $b = 0$.

Write the first line of an indirect proof of the given statement.

5. If $x^2 \neq y^2$, then $x \neq y$. ______________________________

6. If the product of integers a and b is even, then a is even or b is even. ______________________________

____

Write an indirect proof.

7. If a is rational and b is irrational, then $a + b$ is irrational. (Hint: If a and $a + b$ are rational, there are integers m, n, x, and y such that $a = \frac{m}{n}$ and $a + b = \frac{x}{y}$.)

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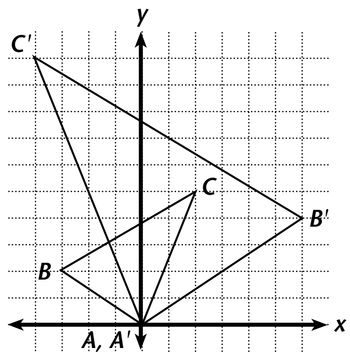
Lesson 11.7

1.



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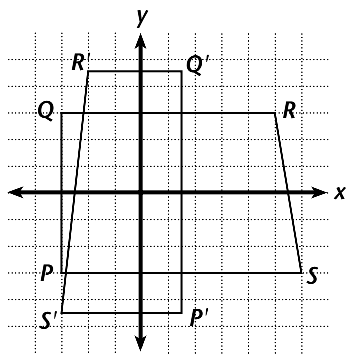


3. Check drawings.

4. Check drawings.

Reteaching — Chapter 12

Lesson 12.1

1. *modus tollens;* valid

2. denying the antecedent; invalid

3. affirming the consequent; invalid

4. *modus ponens;* valid

5. no conclusion

6. You will buy a pizza.

7. My age is not divisible by 9.

Lesson 12.2

1. All squares are rectangles and a yard is three feet long; true.

2. A triangle is a polygon and a hexagon has three sides; true.

3. Pine trees are evergreens or gorillas are pink; false.

4. The moon is a planet or Neptune is a star; false.

5. Ice hockey is a sport or Abraham Lincoln's portrait is on the nickel; true.

6. not truth functionally equivalent

7. truth functionally equivalent

Lesson 12.3

1. If not y, then not x; if not y is true and not x is false (that is, y is false and x is true)

2. If I finish my report, then the library is open; if I finish my report and the library is not open.

3. If there is not an orange in my lunch, then it is not Tuesday; if there is not an orange in my lunch and it is Tuesday.

4. If $n + 1$ is odd, then n is even; if $n + 1$ is odd and n is odd.

5. If a number is even, then it is a multiple of 2.

6. If a triangle is isosceles, then it has at least two congruent sides.

7. If a number is negative, then it does not have a real square root.

Lesson 12.4

1. Not a contradiction

2. Not a contradiction

3. contradiction

4. contradiction

5. Let $x^2 \neq y^2$ and assume temporarily that $x = y$.

ANSWERS

6. Let a and b be integers with ab even and assume temporarily that a is not even and b is not even.

7. Let a be rational and b irrational and assume temporarily that $a + b$ is rational. Then there are integers m, n, x, and y such that $a = \frac{m}{n}$ and $a + b = \frac{x}{y}$. Then $b = (a + b) - a = \frac{x}{y} - \frac{m}{n} = \frac{x - m}{y - n}$. Since b is a ratio of two integers, b is rational.This contradicts the fact that b is irrational, which was given. Then the assumption that $a + b$ is rational is false and $a + b$ is irrational.

Lesson 12.5

1.

p	q	$\sim q$	p AND $\sim q$	NOT (p AND $\sim q$)
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

2.

p	q	$\sim p$	$\sim q$	$\sim p$ OR $\sim q$	($\sim p$ OR$\sim q$) AND p
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	T	F

3. (p OR q) AND (r OR s)

4. ($\sim$ (p OR q)) AND ($\sim$ r)

5. NOT ($\sim$ p AND q) OR ($\sim$ r)

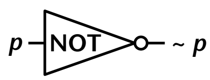
6. ($\sim$ p AND q) OR (p OR r)

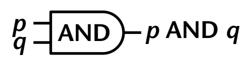
NAME ______________________ CLASS __________ DATE __________

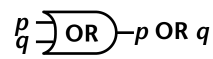
Reteaching
12.5 Computer Logic

◆ Skill A Constructing input-output tables for networks of logic gates

Recall In performing any operation, a computer goes through a series of operations which can be thought of as on (or 1) and off (or 0). A logic gate is an electronic circuit that responds to an electrical impulse. Three such logic gates are the NOT, AND, and OR gates, which act like their counterparts, the logical operators NOT, AND, and OR. An input-output table, similar to a truth table, may be constructed for each logical gate with 1 corresponding to *true* and 0 corresponding to *false*.







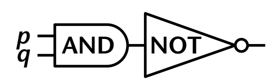
NOT	
Input	Output
p	NOT *p*
1	0
0	1

AND		
Input		Output
p	*q*	*p* AND *q*
1	1	1
1	0	0
0	1	0
0	0	0

OR		
Input		Output
p	*q*	*p* OR *q*
1	1	1
1	0	1
0	1	1
0	0	0

◆ Example

Construct an input-output table for the network.



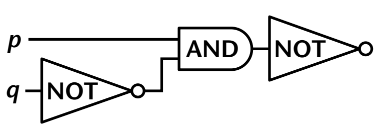
◆ Solution

Read from left to right. The first gate is AND, so the output is *p* AND *q*. The second is NOT, and the output is NOT (*p* and *q*). Consider all combinations of possible input values.

Input		Output	
p	*q*	*p* AND *q*	NOT (*p* and *q*)
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

Complete the input-output table for the network.

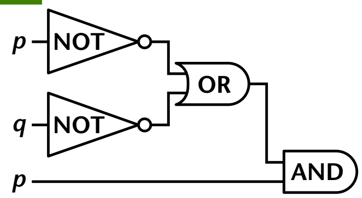
1.



p	*q*	$\sim q$	*p* AND $\sim q$	NOT (*p* AND $\sim q$)

CLASS ______________ DATE ______________

2.

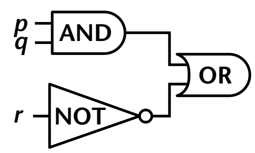


p	q	$\sim p$	$\sim q$	$\sim p$ OR $\sim q$	($\sim p$ OR $\sim q$) AND p

◆ **Skill B** Writing a logical expression corresponding to a network of logical gates

Recall A network of logical gates is read from left to right, one branch at a time. The output from each gate is then the input for the next gate.

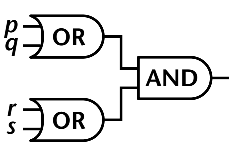
◆ **Example**
Write a logical expressions that corresponds to the given network.



◆ **Solution**
Begin at the upper left. The inputs for the AND gate are p and q, so the output is p AND q. The input for the NOT gate at the bottom left is r, so the output is NOT r. Finally, the inputs for the OR gate at the right are (p AND q) and ($\sim r$), so the output is (p AND q) OR ($\sim r$). The logical expression that corresponds to the network of logical gates is (p AND q) OR ($\sim r$).

Write a logical expression that corresponds to the given network.

3.



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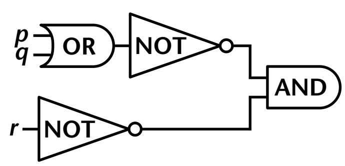
4.



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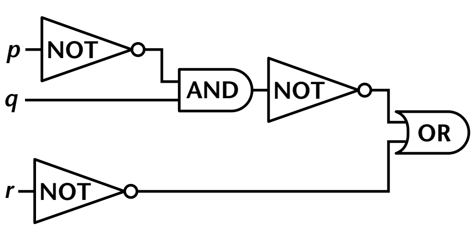
5.



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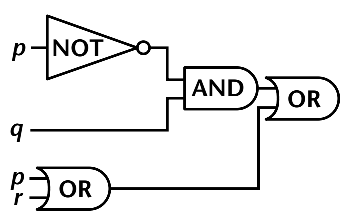
6.



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ANSWERS

6. Let a and b be integers with ab even and assume temporarily that a is not even and b is not even.

7. Let a be rational and b irrational and assume temporarily that $a + b$ is rational. Then there are integers m, n, x, and y such that $a = \frac{m}{n}$ and $a + b = \frac{x}{y}$. Then $b = (a + b) - a = \frac{x}{y} - \frac{m}{n} = \frac{x - m}{y - n}$. Since b is a ratio of two integers, b is rational.This contradicts the fact that b is irrational, which was given. Then the assumption that $a + b$ is rational is false and $a + b$ is irrational.

Lesson 12.5

1.

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T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

2.

p	q	$\sim p$	$\sim q$	$\sim p$ OR $\sim q$	($\sim p$ OR$\sim q$) AND p
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	T	F

3. (p OR q) AND (r OR s)

4. ($\sim$ (p OR q)) AND ($\sim$ r)

5. NOT ($\sim$ p AND q) OR ($\sim$ r)

6. ($\sim$ p AND q) OR (p OR r)