

Review for Final Exam



General Information

- The final exam is cumulative. It includes everything covered in the lectures and homework assignments from Chapters 1, 3, 4, 5, 6 and 7.
- The lecture notes and homework assignments can be found at:
<http://www.sci.brooklyn.cuny.edu/~dzhu/busn3430/>
- The book used for this course is:
*Quantitative Analysis for Management
Tenth Edition
Render, Stair, Hanna*

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General Information

- The format of the final exam will be multiple choice, true/false, and problems (similar to the quizzes and the homeworks, no need for computers)
- Grading and Evaluation Criteria

	Percentage
Homework	15%
Quizzes	50%
Final	30%
Attendance	5%

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General Information

- You are allowed to bring ONE page of notes (one sided, letter-sized 8.5" x 11") to the exam. It can be any size font as long as you can read it.
- NO electronic devices are allowed during the exam (except calculators). Phones must be turned off.

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General Information

- **Date and Time:**
 - Monday, May 21, 2012
(8:00am – 10:00am)
- **Place:**
 - Room 234NE

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General Information

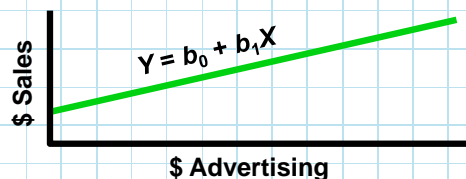
- This review does not include everything that will be covered on the final exam
- It is focused on the QA models and their applications
- You should understand each model and know what kind of problem each model is used for
- You should be able to use the models to solve problems similar to those in this review

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1. What is Quantitative Analysis?

Quantitative analysis (QA) is a scientific approach to managerial decision making – no whim, emotions and guesswork. The heart of QA is the processing and manipulating of raw data into meaningful information

Quantitative analysis models are realistic, solvable, and understandable mathematical representations of a situation



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1. Models Categorized by Risk

- Mathematical models that do not involve risk are called **deterministic** models
 - We know all the values used in the model with complete certainty
 - E.g. – profit and break-even models
- Mathematical models that involve risk, chance, or uncertainty are called **probabilistic** models
 - Values used in the model are estimates based on probabilities
 - E.g. – market for a new product:
good (60%), not good (40%)

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1. Breakeven Analysis

- The **break-even point** (BEP) is the number of units sold that will result in \$0 profit

$$\begin{aligned} \text{Profit} &= \text{Revenue} - \text{Expenses} \\ &= sX - [f + vX] \\ &= sX - f - vX \end{aligned}$$

$$BEP = \frac{f}{s - v}$$

where

s = selling price per unit

v = variable cost per unit

f = fixed cost

X = number of units sold

3. Decision Analysis

Three Types of Environments:

Type 1: Decision making under certainty

- Decision maker **knows with certainty** the consequences of every alternative or decision choice – so choose the option that will result in the best outcome (e.g. opening savings account vs buying Treasury bond)

Type 2: Decision making under uncertainty

- The decision maker **does not know** the probabilities of the various outcomes

Type 3: Decision making under risk

- The decision maker **knows the probabilities** of the various outcomes

3. Decision Making Under Uncertainty

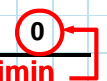
Most managers are not fortunate enough to make decisions under certainty. There are several criteria for making decisions under uncertainty:

1. Maximax (optimistic)
2. Maximin (pessimistic)
3. Criterion of realism (Hurwicz)
4. Equally likely (Laplace)
5. Minimax regret

3. Maximin

- Locate the minimum payoff for each alternative
- Select the alternative with the maximum number

ALTERNATIVE	STATE OF NATURE		MINIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	-180,000
Construct a small plant	100,000	-20,000	-20,000
Do nothing	0	0	0

Maximin 

3. Equally Likely (Laplace)

- Find the average payoff for each alternative
- Select the alternative with the highest average

ALTERNATIVE	STATE OF NATURE		ROW AVERAGE (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	10,000
Construct a small plant	100,000	-20,000	40,000
Do nothing	0	0	0

Equally likely

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3. Decision Making Under Risk

- Decision making when there are several possible states of nature and we know the probabilities associated with each possible state
- Most popular method is to choose the alternative with the highest **expected monetary value (EMV)**
- EMV is the weighted sum of all possible payoffs for an alternative and is the long run average value for that decision

$$\begin{aligned}
 \text{EMV (alternative } i) = & (\text{payoff of first state of nature}) \\
 & \times (\text{probability of first state of nature}) \\
 & + (\text{payoff of second state of nature}) \\
 & \times (\text{probability of second state of nature}) \\
 & + \dots + (\text{payoff of last state of nature}) \\
 & \times (\text{probability of last state of nature})
 \end{aligned}$$

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3. Expected Value of Perfect Information (EVPI)

- Marketing research company can find out what the exact outcome will be – perfect information – decision under risk → decision under certainty – with fee
- EVPI places an upper bound on what you should pay for additional information

$$\text{EVPI} = \text{EV}_{w\text{PI}} - \text{Maximum EMV}$$

- EV_wPI is the long run average return if we have perfect information before a decision is made

$$\begin{aligned}
 \text{EV}_{w\text{PI}} = & (\text{best payoff for first state of nature}) \\
 & \times (\text{probability of first state of nature}) \\
 & + (\text{best payoff for second state of nature}) \\
 & \times (\text{probability of second state of nature}) \\
 & + \dots + (\text{best payoff for last state of nature}) \\
 & \times (\text{probability of last state of nature})
 \end{aligned}$$

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3. Computing EMV, EV_wPI & EVPI

Alternative	State of Nature			EMV
	Good Market (\$)	Average Market (\$)	Poor Market (\$)	
Construct a large plant	75,000	25,000	-40,000	?
Construct a small plant	100,000	35,000	-60,000	?
Do nothing	0	0	0	?
Probability	0.25	0.50	0.25	

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3. Computing EMV, EVwPI & EVPI

$$\text{EMV}(\text{large}) = \$75,000 \cdot 0.25 + \$25,000 \cdot 0.5 + (-\$40,000 \cdot 0.25) = \mathbf{\$21,250}$$

$$\text{EMV}(\text{small}) = \$100,000 \cdot 0.25 + \$35,000 \cdot 0.5 + (-\$60,000 \cdot 0.25) = \mathbf{\$27,500}$$

$$\text{EMV}(\text{nothing}) = 0 \cdot 0.25 + 0 \cdot 0.5 + 0 \cdot 0.25 = \mathbf{\$0}$$

$$\text{Max. EMV} = \mathbf{\$27,500}$$

$$\text{EVwPI} = \$100,000 \cdot 0.25 + \$35,000 \cdot 0.50 + 0 \cdot 0.25 = \mathbf{\$42,500}$$

$$\text{EVPI} = \text{EVwPI} - \max(\text{EMV}) \\ = \$42,500 - \$27,500 = \mathbf{\$15,000}$$

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4. Regression Analysis

- There are generally two purposes for regression analysis
 1. To understand the relationship between variables
 - E.g. the relationship between the sales volume and the advertising spending amount, and the relationship between the price of a house and the square footage
 2. To predict the value of one variable based on the value of another variable

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4. Regression Analysis

Three types of regression models have been studied

- Simple linear regression models have only two variables
- Multiple regression models have more than two variables
- Nonlinear regression models are used when the relationships between the variables are not linear

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4. Simple Linear Regression

- True values for the slope and intercept are not known so they are estimated using sample data (computer program can be used)

$$\hat{Y} = b_0 + b_1 X$$

where

\hat{Y} = dependent variable (response)

X = independent variable (predictor or explanatory)

b_0 = intercept (value of Y when $X = 0$)

b_1 = slope of the regression line

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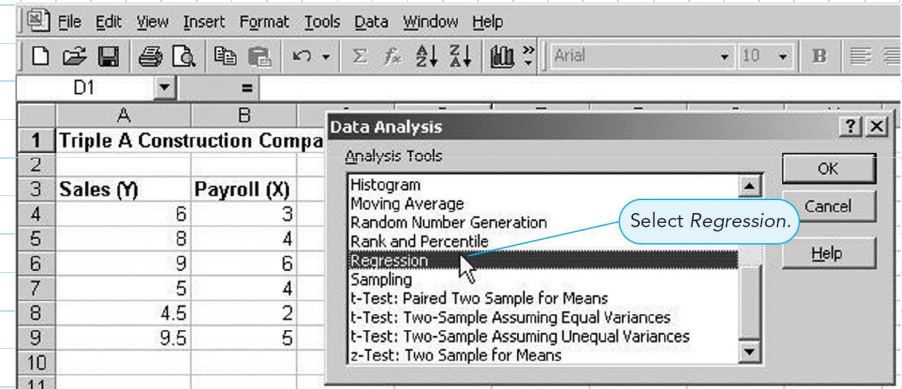
4. Triple A Construction Example

- Triple A's revenues and the total wage earnings for the past six years are listed below. Develop a linear regression model for Triple A Construction

TRIPLE A'S SALES (\$100,000's)	LOCAL PAYROLL (\$100,000,000's)
6	3
8	4
9	6
5	4
4.5	2
9.5	5

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4. Using Computer Software for Regression



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4. Using Computer Software for Regression

Correlation coefficient (r) is Multiple R in Excel

	D	E	F	G	H	I	J	K	L	
1	SUMMARY OUTPUT									
2										
3	Regression Statistics									
4	Multiple R	0.8333								
5	R Square	0.6944								
6	Adjusted R Square	0.6181								
7	Standard Error	1.3110								
8	Observations	6								
9										
10	ANOVA									
11		df	SS	MS	F	Significance F				
12	Regression	1	15.6250	15.6250	9.0909	0.0394				
13	Residual	4	6.8750	1.7188						
14	Total	5	22.5							
15										
16		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%	
17	Intercept	2	1.7425	1.1477	0.3150	-2.8381	6.8381	-2.8381	6.8381	
18	Payroll (X)	1.25	0.4146	3.0151	0.0394	0.0989	2.4011	0.0989	2.4011	

A high r^2 (close to 1) is desirable.

A low significance level means the model is useful in predicting Y.

The regression coefficients are found here.

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4. Triple A Construction Example

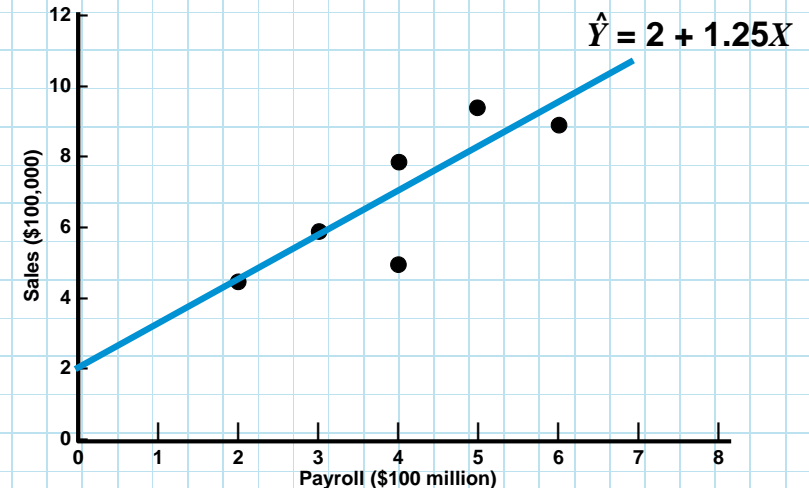


Figure 4.1: Scatter Diagram for Triple A Construction Company Data in Table 4.1

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4. Multiple Regression Analysis

- **Multiple regression models** are extensions to the simple linear model and allow the creation of models with several independent variables

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + \dots + b_kX_k$$

where

\hat{Y} = predicted value of Y

b_0 = sample intercept

b_i = sample coefficient of the i th variable

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4. Jenny Wilson Realty Example

SELLING PRICE (\$)	SQUARE FOOTAGE	AGE	CONDITION
95,000	1,926	30	Good
119,000	2,069	40	Excellent
124,800	1,720	30	Excellent
135,000	1,396	15	Good
142,000	1,706	32	Mint
145,000	1,847	38	Mint
159,000	1,950	27	Mint
165,000	2,323	30	Excellent
182,000	2,285	26	Mint
183,000	3,752	35	Good
200,000	2,300	18	Good
211,000	2,525	17	Good
215,000	3,800	40	Excellent
219,000	1,740	12	Mint

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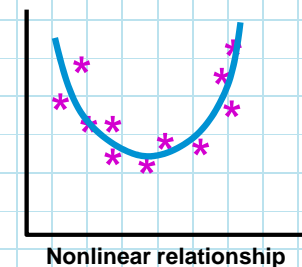
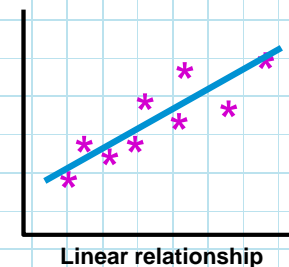
4. Jenny Wilson Realty Example

	A	B	C	D	E	F
1	Sell Price (Y)	SF (X1)	Age (X2)	Excellent (X3)	Mint (X4)	Condition
2	95000	1926	30	0	0	Good
3	119000	2069	40	1	0	Excellent
4	124800	1720	30	1	0	Excellent
5	135000	1396	15	0	0	Good
6	142800	1706	32	0	1	Mint
7	145000	1847	38	0	1	Mint
8	159000	1950	27	0	1	Mint
9	165000	2323	30	1	0	Excellent
10	182000	2285	26	0	1	Mint
11	183000	3752	35	0	0	Good
12	200000	2300	18	0	0	Good
13	211000	2525	17	0	0	Good
14	215000	3800	40	1	0	Excellent
15	219000	1740	12	0	1	Mint

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4. Nonlinear Regression

- In some situations relationships between variables are not linear
- Transformations may be used to turn a nonlinear model into a linear model to use linear regression analysis programs – e.g. Excel



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5. Forecasting Models

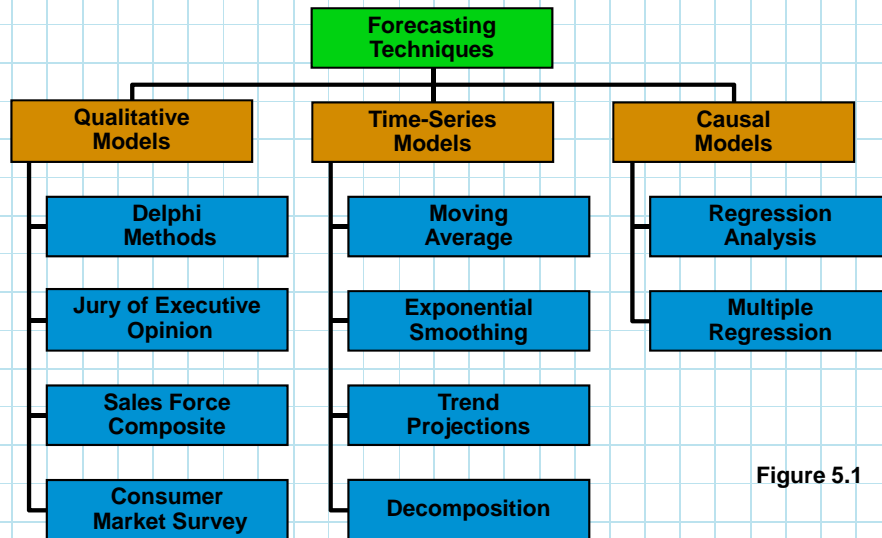


Figure 5.1

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5. Forecasting Models

- **Time-series models** attempt to predict the future by using historical (past) data
- **Causal models** use variables or factors that might influence the quantity being forecasted (may also include past data)
- **Qualitative models** incorporate judgmental or subjective factors into the model – *time-series* and *causal* models relying on quantitative data

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5. Moving Averages

Moving average forecast = $\frac{\text{Sum of demands in previous } n \text{ periods}}{n}$

- Used when demand is stable
- Mathematically

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \dots + Y_{t-n+1}}{n}$$

where

F_{t+1} = forecast for time period $t + 1$
 Y_t = actual value in time period t
 n = number of periods to average

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5. Weighted Moving Averages

- **Weighted moving averages** use weights to put more emphasis on recent periods – more responsive to recent changes
- Often used when a trend or other pattern is emerging

$$F_{t+1} = \frac{\sum (\text{Weight in period } i)(\text{Actual value in period } i)}{\sum (\text{Weights})}$$

- Mathematically

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \dots + w_n Y_{t-n+1}}{w_1 + w_2 + \dots + w_n}$$

where

w_i = weight for the i th observation

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5. Exponential Smoothing

- Mathematically

$$F_{t+1} = F_t + \alpha(Y_t - F_t) = \alpha Y_t + (1 - \alpha)F_t$$

where

F_{t+1} = new forecast (for time period $t + 1$)

F_t = pervious forecast (for time period t)

α = smoothing constant ($0 \leq \alpha \leq 1$)

Y_t = pervious period's actual demand

- Easy to use and involves little record keeping
- Simple – the new estimate is the old estimate plus a fraction of the error in the last period
- α can be increased to give more weight to recent data or decreased to give more weight to past data

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5. PM Computer Example

Actual Demand	2 Month MA	Abs. Dev.	3 Month WMA (4,2,1)	Abs. Dev.	Exp. Sm. ($\alpha=0.7$)	Abs. Dev.
37					40.00	3.00
40					37.9	2.1
41	38.50	2.50			39.37	1.63
37	40.50	3.50	40.14	3.14	40.51	3.51
45	39.00	6.00	38.57	6.43	38.05	6.59
50	41.00	9.00	42.14	7.86	42.92	7.08
43	47.50	4.50	46.71	3.71	47.87	4.87
47	46.50	0.50	45.29	1.71	44.46	2.54
56	45.00	11.00	46.29	9.71	46.24	9.76
TOTAL		37		32.57		41.45
MAD		5.29		5.43		4.61

Exponential smoothing resulted in the lowest MAD.

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5. Exponential Smoothing with Trend Adjustment

- It can cope with the trend component
- The basic approach is to develop a simple exponential smoothing forecast (unadjusted) then adjust for positive or negative lag in the trend

$$T_{t+1} = (1 - \beta)T_t + \beta(F_{t+1} - F_t)$$

where

T_{t+1} = smoothed trend for period $t+1$

T_t = smoothed trend for preceding period

β = trend smooth constant that we select

F_{t+1} = simple exponential smoothed forecast for period $t+1$

F_t = simple exponential smoothed forecast for pervious period

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5. Trend Projection

- Another method for forecasting time series with trend
- The mathematical form is

$$\hat{Y} = b_0 + b_1 X$$

where

\hat{Y} = predicted value

b_0 = intercept

b_1 = slope of the line

X = time period (i.e., $X = 1, 2, 3, \dots, n$)

- The regression line that minimizes the sum of squared errors (SSE) of the time series is used (can be computed using software)

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5. The Decomposition Method of Forecasting

- **Decomposition** is the process of isolating linear trend and seasonal factors to develop more accurate forecasts
- There are five steps to decomposition
 1. Compute seasonal indices using CMAs
 2. Deseasonalize the data by dividing each number by its seasonal index
 3. Find the equation of a trend line using the deseasonalized data
 4. Forecast for future periods using the trend line
 5. Multiply the trend line forecast by the appropriate seasonal index

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5. San Diego Hospital Example

- A San Diego hospital used 66 months of adult inpatient hospital days to develop the following seasonal indices

MONTH	SEASONALITY INDEX	MONTH	SEASONALITY INDEX
January	1.0436	July	1.0302
February	0.9669	August	1.0405
March	1.0203	September	0.9653
April	1.0087	October	1.0048
May	0.9935	November	0.9598
June	0.9906	December	0.9805

Table 5.12

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5. San Diego Hospital Example

- Using this data they developed the following equation for the trend

$$\hat{Y} = 8,091 + 21.5X$$

where

$$\begin{aligned} \hat{Y} &= \text{forecast of patient days} \\ X &= \text{time period in months} \end{aligned}$$

- Based on this model, the forecast for patient days for the next period (67, January) is

$$\text{Patient days} = 8,091 + (21.5)(67) = 9,532 \text{ (trend only)}$$

$$\begin{aligned} \text{Patient days} &= (9,532)(1.0436) \\ &= 9,948 \text{ (trend and seasonal)} \end{aligned}$$

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5. Multiple Regression

- **Multiple regression** can be used to forecast both trend and seasonal components in a time series
 - One independent variable is time
 - Dummy independent variables are used to represent the seasons – for 4 seasons 3 dummy variables are used
- The model is an additive decomposition model

$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

where

$$\begin{aligned} X_1 &= \text{time period} \\ X_2 &= 1 \text{ if quarter 2, } 0 \text{ otherwise} \\ X_3 &= 1 \text{ if quarter 3, } 0 \text{ otherwise} \\ X_4 &= 1 \text{ if quarter 4, } 0 \text{ otherwise} \end{aligned}$$

If $X_2=X_3=X_4=0$, then the quarter would be quarter 1
Which X_i is used for which season is chosen arbitrarily

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5. Multiple Regression

- Suppose the resulting regression equation is:

$$\hat{Y} = 104.1 + 2.3X_1 + 15.7X_2 + 38.7X_3 + 30.1X_4$$

- Using the model to forecast sales for the first two quarters of next year (period 13 and 14)

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

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6. Inventory Usage Over Time for EOG Model

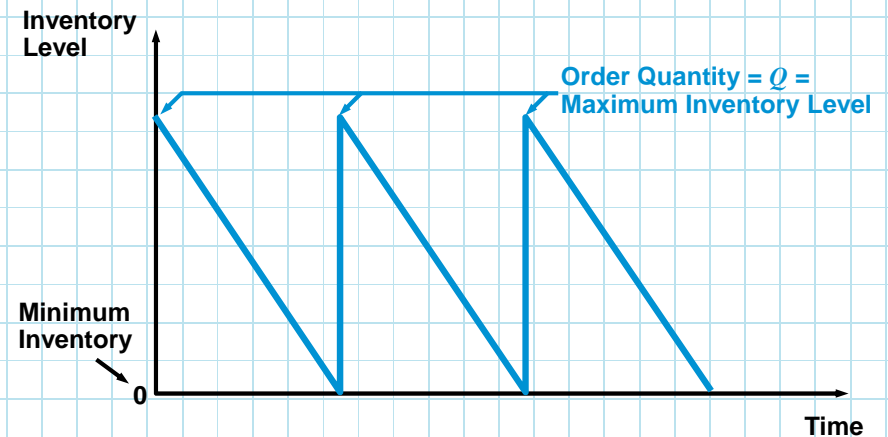


Figure 6.2

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6. Economic Order Quantity (EOQ) Model

- When the EOQ assumptions are met, total cost is minimized when

Annual ordering cost = Annual holding cost

$$\text{Annual ordering cost} = \frac{D}{Q} C_o$$

- Solving for Q

$$2DC_o = Q^2 C_h$$

$$\text{Annual holding cost} = \frac{Q}{2} C_h$$

$$\frac{2DC_o}{C_h} = Q^2$$

$$\frac{D}{Q} C_o = \frac{Q}{2} C_h$$

$$\sqrt{\frac{2DC_o}{C_h}} = Q = \text{EOQ} = Q^*$$

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6. Reorder Point: Determining When To Order

- Once the order quantity is determined, the next decision is **when to order**
- The time between placing an order and its receipt is called the **lead time (L)**
- When to order is generally expressed as a **reorder point (ROP)** – the inventory level at which an order should be placed
 - If an order is placed when the inventory level reaches the ROP, the new inventory arrives at the same instant the inventory is reaching 0

$$\text{ROP} = \left(\text{Demand per day} \right) \times \left(\text{Lead time for a new order in days} \right) = d \times L$$

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6. EOQ Without The Instantaneous Receipt Assumption

- The **production run model** eliminates the instantaneous receipt assumption
- The maximum inventory will be less than the production (or order) quantity Q (why ?)

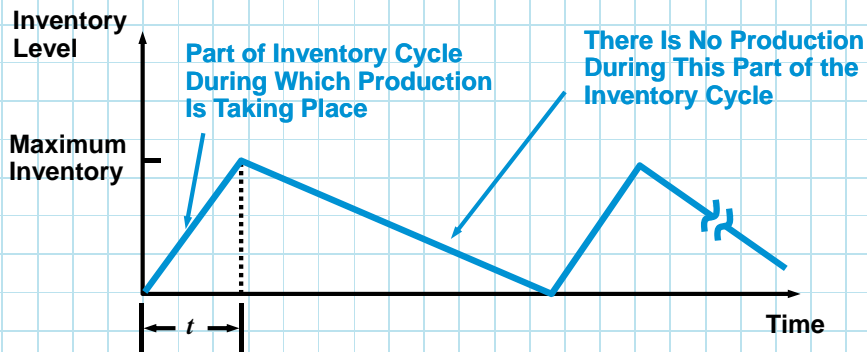


Figure 6.5

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6. Production Run Model

- Summary of equations

$$\text{Annual holding cost} = \frac{Q}{2} \left(1 - \frac{d}{p}\right) C_h$$

$$\text{Annual setup cost} = \frac{D}{Q} C_s$$

$$\text{Optimal production quantity } Q^* = \sqrt{\frac{2DC_s}{C_h \left(1 - \frac{d}{p}\right)}}$$

- If the situation does not involve production but receipt of inventory over a period of time, use the same model but replace C_s with C_o

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6. Quantity Discount Models

- If quantity discounts are available, the basic EOQ model needs to be adjusted by adding the purchase or material cost because it becomes a relevant cost

Total cost = Material cost + Ordering cost + Holding cost

$$\text{Total cost} = DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

where

D = annual demand in units

C_o = ordering cost of each order

C = cost per unit

C_h = holding or carrying cost per unit per year

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6. Steps in Discount Model

- The first step is to compute EOQ values for each discount
- The second step is adjust quantities below the allowable discount range
- The third step is to compute the total cost for each quantity

$$\text{Total cost} = DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

- The fourth step is to choose the alternative with the lowest total cost

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6. Use of Safety Stock

- If EOQ assumptions are met, the **stockouts** can be completely avoided
- However, if demand or the lead time are uncertain, the exact demand during lead time (ROP) will not be known with certainty
- To prevent stockouts, it is necessary to carry extra inventory called **safety stock (SS)**
- Safety stock can be implemented by adjusting the ROP

$$\text{Adjusted ROP} = d \times L + SS$$

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6. Use of Safety Stock

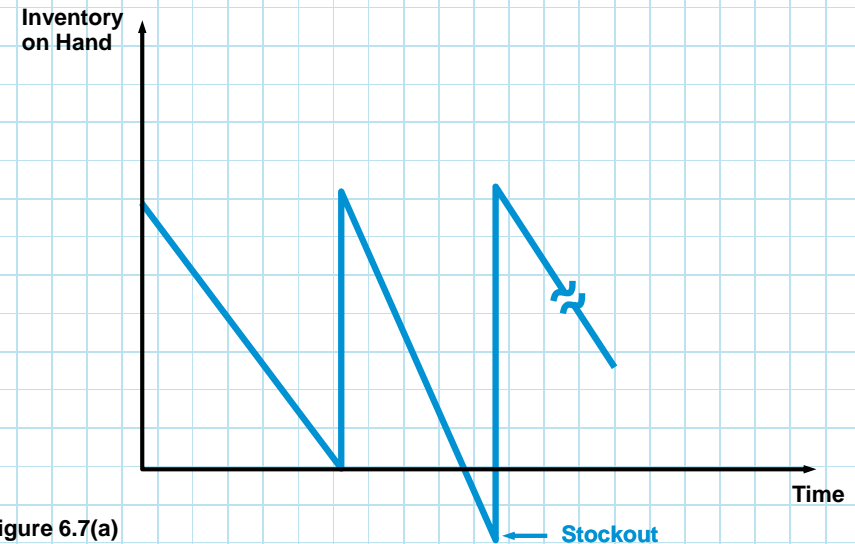


Figure 6.7(a)

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6. Use of Safety Stock

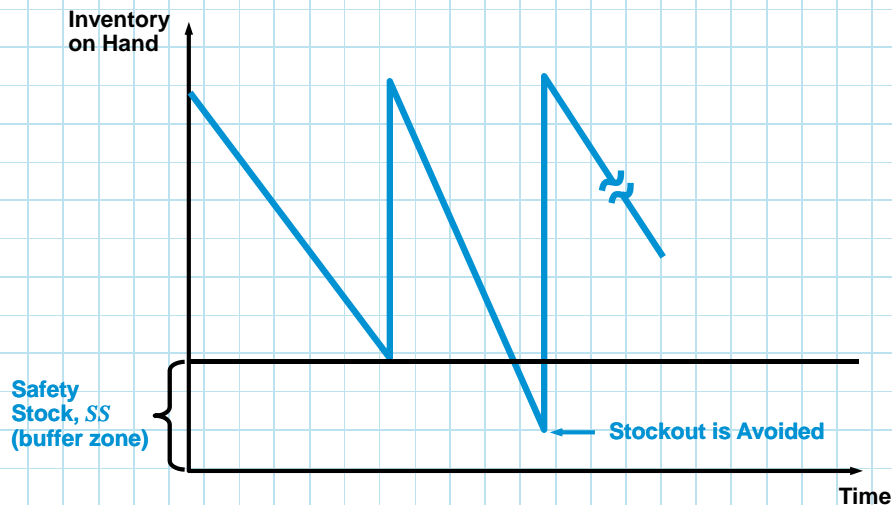


Figure 6.7(b)

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6. Safety Stock Models

- The objective is to find the safety stock quantity that will minimize the total of stockout cost and additional holding cost for the safety stock
- If the stockout cost per unit and the probability distribution of demand during lead time is known, SS can be computed using discrete distribution model
- If the stockout cost is not known, a normal distribution of demand is assumed and the service level is used for the calculation using normal distribution model

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6. Single-Period Inventory Models

- So far we have been assuming that ordered products will be sold in the future
- However some products have no future value beyond the current period (weekly magazines, daily newspapers, certain food with a short life, some seasonal clothes ...)
- These situations are called *news vendor problems* or *single-period inventory models*
- *Marginal analysis* is a simpler approach to find the optimal inventory level for news vendor problems

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6. Single-Period Inventory Models

- With a manageable number of states of nature and alternatives where the probability of each state of nature is known, *marginal analysis with discrete distributions* can be used
- When there are a large number of alternatives or states of nature and the probability distribution can be described with a normal distribution, *marginal analysis with the normal distribution* may be used
- Marginal analysis is based on manipulating marginal profit (MP) and marginal loss (ML)
 - MP is the additional profit achieved if one additional unit is stocked and sold
 - ML is the loss that occurs when an additional unit is stocked but cannot be sold

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6. Marginal Analysis with Discrete Distributions

- Decision rule – we stock an additional unit only if the expected marginal profit for that unit exceeds the expected marginal loss
- Assume

$P =$ probability that demand will be greater than or equal to a given supply (or the probability of selling at least one additional unit)

$1 - P =$ probability that demand will be less than supply (or the probability that one additional unit will not sell)

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6. Marginal Analysis with Discrete Distributions

- The expected marginal profit is $P(MP)$
- The expected marginal loss is $(1 - P)(ML)$
- The optimal decision rule is to stock the additional unit if

$$P(MP) \geq (1 - P)ML$$

- With some basic manipulation

$$P(MP) \geq ML - P(ML)$$

$$P(MP) + P(ML) \geq ML$$

$$P(MP + ML) \geq ML$$

or

$$P \geq \frac{ML}{ML + MP}$$

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6. Steps of Marginal Analysis with Discrete Distributions

1. Determine the value of $\frac{ML}{ML+MP}$ for the problem
2. Construct a probability table and add a cumulative probability column
3. Keep ordering inventory as long as the probability (P) of selling at least one additional unit is greater than $\frac{ML}{ML+MP}$

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6. Chicago Cheese Example

- Chicago Cheese sells E-Z Spread Cheese for \$100 per case and the cost is \$75
- Any cheese not sold at the end of the week is sold to a local food processor for \$50 per case, for Chicago Cheese never sells cheese that is more than a week old
- How many cases of E-Z Spread Cheese should be produced each week to maximize the profit?
- The marginal profit per carton is

$$MP = \text{Marginal profit} = \$100 - \$75 = \$25$$

- The marginal loss is

$$ML = \text{Marginal loss} = \$75 - \$50 = \$25$$

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6. Chicago Cheese Example

- Step 1.** Determine the value of $\frac{ML}{ML+MP}$ for the decision rule

$$P \geq \frac{ML}{ML+MP} = \frac{\$25}{\$25 + \$25} = \frac{\$25}{\$50} = 0.5$$

$$P \geq 0.5$$

- Step 2.** The probability distribution of the demand for the cheese is given. Now add a new cumulative probability column to the table to reflect the probability that E-Z Spread Cheese sales will be at each level or greater

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6. Chicago Cheese Example

- Marginal analysis

WEEKLY SALES (CASES OF CHEESE)	PROBABILITY (P) THAT DEMAND WILL BE AT THIS LEVEL	PROBABILITY (P) THAT DEMAND WILL BE AT THIS LEVEL OR GREATER
10	0.2	$1.0 \geq 0.5$
11	0.3	$0.8 \geq 0.5$
12	0.2	$0.5 \geq 0.5$
13	0.2	0.3
14	0.1	0.1
Total	1.00	

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6. Chicago Cheese Example

- Step 3.** Keep ordering additional cases as long as the probability of selling at least one additional case is greater than or equal to 0.5, which is the indifference probability

$$P \text{ at 12 cases} = 0.5 \geq 0.5$$

Therefore Chicago Cheese should produce 12 cases of E-Z Spread Cheese each week, since 0.5 is the smallest cumulative probability value that is

greater than or equal to $\frac{ML}{ML + MP} = 0.5$

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7. Linear Programming

- So far decisions are made assuming no limit to the access of resources
- Many management decisions involve trying to make the most effective use of limited resources
 - Machinery, labor, money, time, warehouse space, raw materials
- **Linear programming (LP)** is a widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation

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7. Requirements of a Linear Programming Problem

- All LP problems have 4 properties in common
 1. All problems seek to **maximize** or **minimize** some quantity, usually profit or cost (the **objective function**)
 2. The presence of restrictions or **constraints** that limit the degree to which we can pursue our objective
 3. There must be alternative courses of action to choose from
 4. The objective and constraints in problems must be expressed in terms of **linear** equations or **inequalities**
 - $2A + 5B = 10$ (linear equation)
 - $3A + 2B < 25$ (linear inequality)
 - $2A^2 + 5B^3 + 3AB < 10$ (non-linear inequality)

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7. Formulating LP Problems

- The steps in formulating a linear program are
 1. Completely understand the managerial problem being faced
 2. Identify the objective and constraints
 3. Define the decision variables
 4. Use the decision variables to write mathematical expressions for the objective function and the constraints

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7. Summary of Graphical Solution Methods

ISOPROFIT/ISOCOST METHOD

1. Graph all constraints and find the feasible region.
2. Select a specific profit (or cost) line and graph it to find the slope.
3. Move the objective function line in the direction of increasing profit (or decreasing cost) while maintaining the slope. The last point it touches in the feasible region is the optimal solution.
4. Find the values of the decision variables at this last point and compute the profit (or cost).

CORNER POINT METHOD

1. Graph all constraints and find the feasible region.
2. Find the corner points of the feasible region.
3. Compute the profit (or cost) at each of the feasible corner points.
4. Select the corner point with the best value of the objective function found in step 3. This is the optimal solution.

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7: Vase Maker Example

- Hal the vase maker has 24 lb. of clay and 16 lb. of special glazing
- He needs 1 lb. of clay and 1 lb. of special glazing to make a small vase
- To make a large vase, Hal needs 4 lb. of clay and 2 lb. of special glazing
- The smaller vases sell for \$3 each, while the larger vases would bring \$9 each.
- Help Hal to determine the best mix of small and large vases in order to maximize the income

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7: Vase Maker Example

- There are two decision variables
 - Let X_1 be the number of small vases
 - Let X_2 be the number of large vases
- Objective function
 - Maximize: Income = $\$3X_1 + \$9X_2$
- Subject to
 - $1X_1 + 4X_2 \leq 24$ (clay constraint)
 - $1X_1 + 2X_2 \leq 16$ (glaze constraint)

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7. Vase Maker Example

Draw the two constraint lines

- Clay constraint:
($X_1=0$; $X_2=6$)
($X_1=24$; $X_2=0$)
- Glaze constraint:
($X_1=0$; $X_2=8$)
($X_1=16$; $X_2=0$)

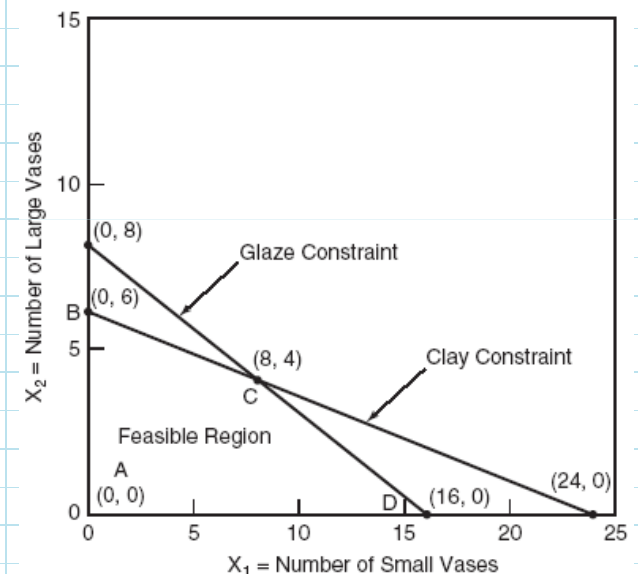
Solving equations:

$$1X_1 + 4X_2 = 24$$

$$1X_1 + 2X_2 = 16$$

We have:

$$(X_1=8; X_2=4)$$



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7. Vase Maker Example

- Use corner point method

$$(\text{Income} = \$3X_1 + \$9X_2)$$

Point	X_1	X_2	Income
A	0	0	\$ 0
B	0	6	54
C	8	4	60*
D	16	0	48

- *Optimum income of \$60 will occur by making and selling 8 small vases and 4 large vases