Revised 05-23-13

ATLANTA PUBLIC SCHOOLS
Making A Difference

CCGPS Math Grade 6 Unit 1
Number System Fluency
Traditional Schools: 08-12-14 to 09-13-14
(5 WEEKS)

Contents

Unit Overview

Students come to sixth grade with an understanding of factors and multiples. It is in sixth grade that they will expand their understanding to common factors and common multiples between two or more numbers. They will apply this understanding and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2) and 4(9) + 4(2).

At each grade level in the standards, one or two fluencies are expected. For sixth graders it is multi-digit whole number division and multi-digit decimal operations. Fluent in the standards means "fast and accurate". It might also help to think of fluency as meaning more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow; fluent is not halting, stumbling, or reversing oneself. Fluency may not happen within the scope of one unit.

There are three parts to this unit: Dividing fractions by fractions; Computing fluency with multi-digit numbers and decimals; and Factors and multiples (including using the distributive property to express a sum of two whole numbers 1-100 with a common factor a multiple of a sum of two whole numbers with no common factor). Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense.

Connections to Previous Learning

In Grade 5, students focused on: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.

Connections to Future Learning

In Grade 7, students will develop a unified understanding of numbers, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

Content Standards

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

MCC.6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a short story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find the common factors and multiples.

MCC.6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MCC.6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MCC.6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*

Learning Targets

I can …

- Use a visual model to represent the division of a fraction by a fraction. (6NS1)
- Divide fractions by fractions using an algorithm or mathematical reasoning. (6NS1)
- Justify the quotient of a division problem by relating it to a multiplication problem. (6NS1)
- Use mathematical reasoning to justify the standard algorithm for fraction division. (6NS1)
- Solve real world problems involving the division of fractions and interpret the quotient in the context of the problem. (6NS1)
- Create story contexts for problems involving the division of a fraction by a fraction. (6NS1)
- Use the standard algorithm to fluently divide multi-digit numbers. (6NS2)
- Fluently add multi-digit decimals using the standard algorithm. (6NS3)
- Fluently subtract multi-digit decimals using the standard algorithm. (6NS3)
- Fluently multiply multi-digit decimals using the standard algorithm. (6NS3)
- Fluently divide multi-digit decimals using the standard algorithm. (6NS3) Find all factors of any given number, less than or equal to 100. (6NS4)
- Find the greatest common factor of any two numbers, less than or equal to 100. (6NS4)
- Create a list of multiples for any number less than or equal to 12. (6NS4)
- Find the least common multiple of any two numbers, less than or equal to 12. (6NS4)
- Use the distributive property to rewrite a simple addition problem when the addends have a common factor. (6NS4)

Focus Standards for Mathematical Practice

SMP 1: Make Sense of Problems and Persevere in Solving Them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They see relationships between various representations. They relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. They continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.

Essential Questions

How can *what is known* help determine *where to begin* or *what to do next* when solving a problem?

SMP 4: Model with Mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In grade 6, students continue this understanding by using visual models and equations to divide whole numbers by fractions and fractions by fractions by fractions to solve word problems. Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of 1/2. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in answer with halves.

Essential Questions

How can a mathematical model help the solution process?

SMP 6: Attend to Precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign

consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

Essential Questions

What makes work clear and precise so that results are reliable and communicated effectively?

SMP 7: Look For and Make Use Of Structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see that 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They can also step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Essential Questions

How can identifying a pattern or structure help the solution process?

Resources

http://www.insidemathematics.org/index.php/standard-1 SMP 1 Inside Mathematics Website

http://www.insidemathematics.org/index.php/standard-4 SMP 4 Inside Mathematics Website

http://www.insidemathematics.org/index.php/standard-6 SMP 6 Inside Mathematics Website

http://www.insidemathematics.org/index.php/standard-7 SMP 7 Inside Mathematics Website

Grade 6 Unit 1 Prerequisite Skills

Prior to learning the content in this unit, students should know how to add and subtract decimals to the hundredths, fractions and mixed numbers with unlike denominators, solve word problems, and interpret a fraction as division of the numerator by the denominator. In the 5th grade, students developed fluency with multi-digit addition, subtraction, multiplication, and division. They applied models for decimals and decimal notation. Students used the relationship between decimals and whole numbers to understand and explain why the procedures for multiplying and dividing finite decimals make sense. Students used their understanding of patterns when multiplying and dividing by powers of ten to develop fluency with operation with multi-digit decimals. Teachers can use the Computer Adaptive Data to determine which students have not mastered these standards so they can provide remediation before the unit begins or in intervention support during or outside of class.

- MCC.5NBT.7 Add, subtract, multiply, and divide decimals to hundredths using concrete models, drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- MCC.5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
- MCC.5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.
- MCC.5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Grade 6 U1
PreTest.docx

A. Computing Fluently with Multi-Digit Numbers and Decimals

Compute fluently with multi-digit numbers and find the common factors and multiples.

MCC.6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MCC.6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Essential Questions

In what ways can quotients be represented? (6.NS.2)

How are decimals lined up to add or subtract? (6.NS.3)

Learning Targets

I can …

- Use the standard algorithm to fluently divide multi-digit numbers. (6.NS.2)
- Fluently add multi-digit decimals using the standard algorithm. (6.NS.3)
- Fluently subtract multi-digit decimals using the standard algorithm. (6.NS.3)
- Fluently multiply multi-digit decimals using the standard algorithm. (6.NS.3)
- Fluently divide multi-digit decimals using the standard algorithm. (6.NS.3)

Concept Overview

As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Procedural fluency is defined by Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations (limited to 4-digit numbers divided by 2-digit numbers), area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed. This understanding is foundational for work with fractions and decimals in 7^{th} grade.

Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations.

As a starter activity, use division problems that can reasonably be solved by using mental math (e.g., 105/25), estimation (e.g., 150 ÷ 12; 227 ÷ 30), and reasoning (e.g., when I think of 105 divided by 25, I think of 4 sets of 25 with 5 left over, the 5 left over is 5/25 which is 1/5, so the answer is $4\frac{1}{5}$). Model for students your thinking as you work through the problem.

(Note: This strategy should not apply to complex division problems for which the algorithm is most appropriate [e.g., 4567 ÷ 192]).

In 4^{th} or 5^{th} grades, students added and subtracted decimals. Multiplication and division of decimals was introduced in 5^{th} grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Standard 6.NS.3 calls for students to fluently compute with decimals. A companion of fluency is the extension of the students' existing number sense to decimals. It is insufficient to merely teach procedures about "where to move the decimal". Rather, the focus of instruction and student work should be on operations and number sense.

Vocabulary

Algorithm: a step-by-step solution to a problem.

Dividend: A number that is divided by another number.

Divisor: A number by which another number is to be divided.

Quotient: A number that is the result of division.

Sum: The number you get by adding two or more number together.

Sample Problems

Sample Problem 1

Evaluate the following: (6.NS.3)

1. 256.567 + 384.07
2. 42.8 – 13.9367
3. 14.56 × 7.43
4. 341.8 ÷ 1.2

Solutions:

1. 640.637
2. 28.8633
3. 108.1808
4. 284.8333…….

Sample Problem 2

The school had a bake sale and raised $75.55. If each cookie cost $0.05, how many cookies were sold? Explain how you got your answer (6.NS.3)

Solution: $75.55 ÷ $0.05 = 1511 cookies

Sample Problem 3

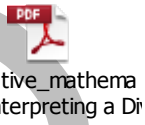
Parsley costs $.60 per ounce. If Chef Jorge needs 7.25 ounces for his recipe, how much will he spend on parsley? (6.NS.3)

A. $43.50
B. $12.08
C. $7.85
D. $4.35

Solution: D. $4.35.

Resources, Teacher Notes, and Instructional Strategies

Standard & Topic	Resources	Teacher Notes
MCC.6.NS.2 Dividing with multi-digit numbers.	http://www.amblesideprimary.com/ambleweb/mentalmaths/dividermachine.html Division Interactive Practice (S/U) - SM http://www.ixl.com/math/grade-6/divisibility-rules.	**Student Misconceptions:** Students may think… • Division is commutative, for example 3 ÷ 5 = 5 ÷

	Interactive Practice on the Divisibility Rules. (U) - RP http://www.ixl.com/math/grade-6/divide-whole-numbers-2-digit-divisors Interactive Practice. (U) - RP http://www.ixl.com/math/grade-6/divide-whole-numbers-3-digit-divisors Interactive Practice. (U) - RP http://illuminations.nctm.org.ActivityDetail.aspx?ID=173 Rectangle Division – this applet uses an array model to represent any two-digit number as a product of two numbers. Remainders are included. - SM Dividing Whole Number – Intervention (S) www.learner.org/courses/learningmath/number/session4/index.html Module examines various models for multiplication and division of whole numbers and features video, sample problems and solutions, and classroom activities. **Illustrative Mathematics Task(s)** Interpreting a Division Computation  illustrative_mathematics_Interpreting a Di	3. • Dividing always gives a smaller number. • Multiplication and division are often taught as two separate entities; consequently, students often fail to see the relationship between multiplication and division. This lack of relational thinking inhibits students from developing flexible connections related to division and multiplication. In turn, this affects students' division retrieval strategies (i.ie. 20 divided by 5 can be solved more readily if a student knows to relate 5 × 4 = 20 to such division problem) (Van de Walle, 2007) • Another conceptual hurdle for students is their ability to conceive of a group contains a given number of objects. For example, when asked, "How many cookies on 3 plates of 5 cookies each?" Students have difficulty conceptualizing the problem as three sets of five (Van de Walle, 2007) **Probing questions:** • How is moving the decimals in the divisor and dividend the same number of places like creating an equivalent fraction? **Differentiation Strategies:** • Have students turn their loose-leaf paper sideways to use the lines vertically to keep place values organized vertically. Students can also use graph paper. • Stations on dividing multi-digit numbers. • Manipulatives (e.g., base 10 squares) to model division. Readiness Differentiation: Pre-test this skill so that students that can show mastery can work on an enrichment activity (e.g., word problems, a mini-project/task, computer based learning) in a group while you work with a smaller group that still needs to master this skill. This will also give you a feel for the overall ability of the class with this skill. **Cooperative Learning Strategies:** • Think-Pair-Share (e.g., long division problem, word problem). • Partners **Literacy Strategies:** • Anticipation Guide Provide students with 5-10 statements about the topic of the lesson. (There should be two response

		columns, one for before and after the lesson.) As the activating strategy, have students respond in the first column, basing their answers on their prior knowledge or predictions about the statement. Return to the Anticipation Guide at the end of the lesson and have students respond again, this time in the "After" column, as the summarizing strategy for the lesson.
MCC.6.NS.3 Multi-digit decimal addition, subtraction, multiplication, and division.	For every topic, provide websites, activities, and worksheets that exhibit the depth and rigor of the Common Core State Standards. Make sure activities are balanced in developing knowledge, skills, and understanding. https://www.teachingchannel.org/videos/repeating-decimals-lesson-plan . Teaching Channel: Decimals Forever- Lesson Video (K/S/U) http://nlvm.usu.edu/en/nav/frames_asid_187_g_3_t_1.html?open=instructions& - Adding Decimals Interactive Game (K/S/U) – SM/RP http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=7 – LearnAlberta – "Addition and Subtraction of Decimals" Video Lesson (S/U) – BP http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=10 – LearnAlberta - "Multiplication and Division of Decimals" Video Tutorial (S/U) – BP/SM http://www.math-play.com/Decimals-Jeopardy/decimals-jeopardy.html - Jeopardy Game - (K/U/S) – BP/SM/RP http://www.learnalberta.ca/content/mesg/html/math6web/index.html?page LearnAlberta – "Solving Problems with Decimals" (K/S/U) – BP/SM http://www.decimalsquares.com/dsGames/ Decimal Games (U) – SM/RP http://insidemathematics.org/common-core-math-tasks/6th-grade/6-2009%20Sewing.pdf Mars Task Learning Task (S) – SM Dividing a Number by a Decimal Worksheet (S) Dividing Number by Decimals.pdf Dividing by Decimals Worksheet (S) dividing decimals worksheet.pdf	**Student Misconceptions:** Students may think... • That 5 × 0.4 should give 2 since is smaller than 5. In the same way, they find it hard to accept that 10 ÷ 0.1 gives 100 since 100 is much bigger than 10. For many students, to make a number bigger, you have to multiply it and to make it smaller you have to divide it. **Probing questions:** • When in real life do we do use decimals? • Explain why 12 ÷ 0.5 gives you a larger number than 12 when you are dividing 12 into parts. • How can estimation strategies help you? **Differentiation Strategies:** • Give each student a sheet of graph paper and have them to write each number in a box. This will help students to line up their decimals correctly in terms of place value. • Stations on dividing multi-digit numbers and decimals (e.g., long division, dividing decimals by decimals, dividing a decimal by a whole number) ***Readiness Differentiation:*** Use a **pre-test** to determine which students have mastered each of the decimal operations. Make sure to only put students into enrichment group if they show complete mastery. Each skill: add, subtract, multiply should be separate enrichments groups on separate days. Make sure to have an enrichment activity ready for students that have shown mastery. This will help you target students that need the skills, yet allow students that have mastered the skill to be enriched. **Cooperative Learning Strategies:** Frayer Model.doc • Frayer Model • Numbered Heads Together (mastery, thinking) 1. Students count off numbers in their groups. 2. Teacher poses a problem and gives wait time (Example: "Everyone think about how rainbows are formed. [Pause] Now make sure everyone in your team knows how rainbows are formed.") 3. Students lift up from their chairs to put their heads together, discuss and teach.

	Decimals Intervention (S)  Ready_to_Go Decimals Intervention **Illustrative Mathematics Learning Task(s)** Jayden's Snack Task - Adding and Subtracting Decimals  illustrative_mathematics_Jayden's Snack.p Gifts from Grandma Task – Multiplying and Dividing Decimals  illustrative_mathematics_Gift from Grandm Buying Gas Task – Dividing Decimals  illustrative_mathematics_Buying Gas.pdf	4. Students sit down when everyone knows the answer or has something to share or when time is up. 5. Teacher calls a number. The student with that number from each team answers question individually, using: a. Response cards b. chalkboard response c. manipulatives d. slate share

B. Factors and Multiples

Compute fluently with multi-digit numbers and find the common factors and multiples.

MCC.6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*

Essential Questions

What relationships exist between numbers and their common factors and multiples?

Learning Targets

I can …

- Find all factors of any given number, less than or equal to 100. (6NS4)
- Find the greatest common factor of any two numbers, less than or equal to 100. (6NS4)
- Create a list of multiples for any number less than or equal to 12. (6NS4)
- Find the least common multiple of any two numbers, less than or equal to 12. (6NS4)
- Use the distributive property to rewrite a simple addition problem when the addends have a common factor. (6NS4)

Concept Overview

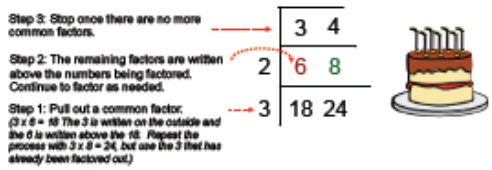
Students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by:

- Listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8) is also

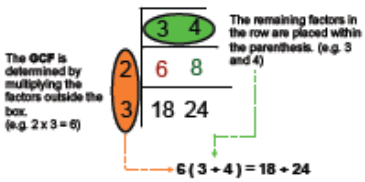
the largest number such that the other factors are relatively prime (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, the show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4are not relatively prime (have 2 in common), the 4 cannot ne the greatest common factor.

Sample Problem 4 (6.NS.4)

This strategy provides a tidy structure for helping students determine the GCF and apply the Distributive Property to the factored expression 6 (3 + 4).



DISTRIBUTIVE PROPERTY & BIRTHDAY CAKE (Expressing the sum of two whole numbers with a common factor as a multiple of two whole numbers with no common factor)



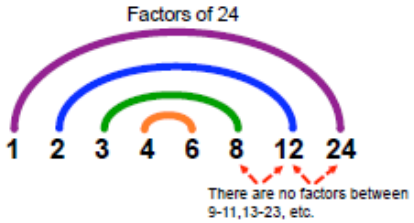
The birthday cake strategy is effective, but it is not the common middle school approach. Therefore, the factor trees strategy is recommended for most students.

Sample Problem 5 (6.NS.4)

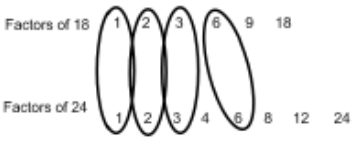
FACTOR RAINBOW: The factor rainbow assists students in seeing the range of numbers possible rather than randomly generating lists of numbers. Another benefit is that factors are arranged in order from least to greatest.

Example for finding the factors of 24: Start with 1 and 24, which is the range for finding possible factors and the greatest and smallest factors possible. Next, determine if the next prime number has a factor that creates 24. For example, 2 and 12 are factors of 24. This allows students to see that there are no possible factors between 13 and 23. Continue in this manner until all factors are determined.

Creating a factor rainbow to find the factors of 24:



Finding the GCF of two numbers using a Factor Rainbow: Determine factors using the factor rainbow and then identify common factors. The GCF is the largest factor both numbers have in common. In this case, the Greatest Common Factor is 6.



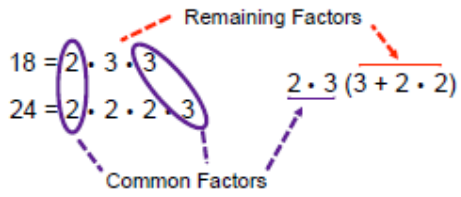
Remember 1 is a factor for every number, but it is not a PRIME factor. The Factor Rainbow is effective for finding factors, but could create some confusion about prime factors. Once students are able to find factors, they should progress to the Factor Trees strategy, which does not add confusion between factors in general and prime factors.

- Listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8). Students also understand that the greatest common factor and the distributive property of two prime numbers will be 1. Students use the greatest common factor and the distributive property to find the sum of two whole numbers. For example, 36 + 8 can be expressed as 4(9 + 2) = 4(11). Students may also use factor towers, Venn Diagram and factor trees to find LCM and GCF.

Sample Problem 6 (6.NS.4)

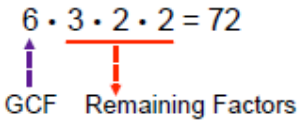
18 + 24

Greatest Common Factor



GCF is 2 • 3 = 6

Least Common Multiple



LCM is 72

The GCF and LCM are important tools for helping students develop greater capacity for seeing and using number relationships to solve problems.

Greatest common factor and least common multiple are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as

prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: "If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?" Being able to see and write the relationship between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, (36 + 24) = 12(3 + 2), where 12 is the GCF of 36 and 24. A model could be used to show that 4(9 + 2) is four groups of 9 and four groups of 2. This concept will be extended in Expressions and Equations as work progresses from understanding the number system and solving equations to simplifying and solving algebraic equations in Grade 7.

Vocabulary

Greatest Common Factor (GCF): The largest factor that two or more numbers have in common.

Least Common Factor (LCM): The smallest multiple (other than zero) that two or more numbers have in common.

Distributive Property: The sum of two addends multiplied by a number will be the sum of the product of each addend and the number. Example: a(b + c) = ab + ac

Sample Problems

Sample Problem 7

Use the greatest common factor and the distributive property to find the sum of 36 and 8. (6.NS.4)

Solution:

35 + 8 = 4 (9) + 4(2)

44 = 4 (9 + 2)

44 = 4(11)

44 = 44 check

Sample Problem 8

Hotdogs come in packs of 8. Buns come in packs of 12. How many packets of hotdogs and bags of buns would you have to buy to have an equal number of hotdogs and bun? (6.NS.4)

Solution: 3 packs of hotdogs and 2 packs of buns. The LCM is 24, because 8 × 3 = 24 and 12 × 2 = 24.

Resources, Teacher Notes, and Instructional Strategies

Standard & Topic	Resources	Teacher Notes
MCC.6.NS.4 Greatest common factor, factors, multiples and prime factorization	http://math.wonderhowto.com/how-to/find-lcm-gcf-number-set-with-venn-diagram-345722/ Instructional Video – Using the Venn Diagram to find the GCF and LM. (U) http://www.shodor.org/interactivate/lessons/SetsTheVennDiagram/ Interactive Venn Diagram. (U/K). http://www.shodor.org/interactivate/lessons/Factors/ Interactive Lesson on Factors. (U) http://www.shodor.org/interactivate/lessons/FindingFactors/ Finding the Factors. (U) http://www.learnalberta.ca/content/mesg/html/mat	**Student Misconceptions:** Students may think... • That 1 is a prime number. • Believe that even numbers are composite and odd numbers are prime. • Believe that factor of a given number must be smaller than the given; example: They do not realized that 36 is a factor of 36? **Probing questions:** • When in real life would you need to find the

	h6web/index.html?page=lessons&lesson=m6lessonshell07.swf Factors, Multiples, and Prime Factorizations. (U) http://www.coolmath.com/prealgebra/01-gcfs-lcms/index.html Math Game on the GCF and LCM. (U) http://www.mathplayground.com/howto_primefactorization.html - Prime Factorization Video (U) http://illuminations.nctm.org/LessonDetail.aspx?ID=L744. Illustrative Task – Using Distributing and Factoring using Area. (U) http://www.oswego.org/ocsd-web/match/dragflip.asp?filename=slanegcf Interactive Lesson on GCF (U/S) **State Framework Learning Task(s) –** Back to School	greatest common factor? **Differentiation Strategies:** • Vary your delivery style during each class period to appeal to several styles of learners. For example, have handouts outlining the topic for visual learners, use discussions for auditory learners, and have a hands-on activity for kinesthetic learners. Limit the time spent on lecture. **Cooperative Learning Strategies:** **Pairs Check** (mastery, communication) In pairs, student stake turns solving problems. After every two problems, they check answers and celebrate with another pair. • In teams, shoulder partners are formed. Partner A in each pair does the first problem, talking out loud. Partner B watches and coaches. Partner B praises. • Trade roles: partner B does the next problem. Partner A watches, coaches, and praises. • Pairs check with their eyeball partners after every two problems. Teammates coach and correct if needed. • The team celebrates after reaching agreement on the two problems. **Literacy Strategies:** Vocabulary Tree Graphic Organizer - The vocabulary tree is made up of a trunk, roots, branches, and leaves. The trunk holds the main concept or key term, and the branches include related terms, ideas, or examples. The leaves of the tree are used for the definitions of the terms or ideas listed in the branches. Finally, the roots of the tree are reserved for the definition or Latin root of the key term. Students may complete a Vocabulary Tree individually, in pairs, in small groups, or even as a whole class. The Vocabulary Tree strategy is extremely helpful for students to *see* vocabulary terms and how they relate to other terms or concepts in the text. Students are able to expand on the single idea or term and create visual connections that will help them see how the many concepts or vocabulary words in a text are related.
MCC.6.NS.4 Least common multiple.	http://math.wonderhowto.com/how-to/find-lcm-gcf-number-set-with-venn- diagram-345722/ Instructional Video – Using the Venn Diagram to find the GCF and LCM. (U)	**Student Misconceptions:** Students may think... • Multiples of a given number must be larger than the given number; example: They do not

	http://edweb.sdsu.edu/courses/edtec670/cardboard/Card/L/LCM/lcm.html Card Game on LCM – (K/U) http://e-learningforkids.org/Courses/EN/M1105/index.html LCM Lesson (U) **State Framework Learning Task(s) –** Secret Number	recognize 36 as multiple of 36. • Students will sometimes begin listing the multiples of a number with 1 and then continue accurately with multiples because they are confusing factoring with listing multiples. Example: Multiples of 3 are: 1, 3, 6, 9, 12…. • Students may not understand that each whole number has unique prime factorization. • Students may incorrectly write the prime factorization for numbers. **Probing questions:** • List all of the things you already know about least common multiple. • How can you decide if a problem is asking for GCF or LCM? • How are GCF and LCM alike and different? • Can you make a list of key words that might tell you whether you are solving a GCF or LCM problem? **Differentiation Strategies:** • Distribute learning packets for skills-based mathematics topics where mastery is essential. Packets can include practice sheets and assessments for students to use at their own pace. **Cooperative Learning Strategies:** **RallyTable** (Mastery, thinking, communication) *In pairs, students alternate generating written responses or solving problems.* 1. Cooperative teams are given one piece of paper and one pen or pencil. 2. Teacher poses a problem or provides a task to which there are multiple possible answers, steps, or procedures. 3. The teacher provides an example and checks for understanding. A time limit is set. 4. The teacher selects a student to begin in each team. 5. Students quickly write their word or phrase and pass their paper to the team member on the left. 6. The paper continues to go around and around the table as each student adds to the team's list. 7. The teacher calls time. All pencils/pens are placed on the team table. 8. The teams take turns sharing their responses with

		the rest of the class. 9. Students celebrate their success. **Literacy Strategies:** • Web Organizer Strategy Plan • Web Organizer Worksheet
MCC.6.NS.4 Distributive property.	http://illuminations.nctm.org/LessonDetail.aspx?ID=L744. Illustrative Task – Using Distributing and Factoring using Area. (U) http://www.youtube.com/watch?v=EWcllbr8Hqs Instructional Video on Distributive Property. (U) PowerPoint lesson on the Distributive Property  Distributive Prop.ppt More Instructional Examples on Distributive Property  Distri Prop Lesson.ppt http://www.onlinemathlearning.com/the-distributive-property.html Song, Video, Worksheets, and Stories on the Distributive Property. (K/S/U) **State Learning Task(s) –** Let's Distribute	**Model Lesson:**  Distributive Property Model Lesson.docx **Student Misconceptions:** Students may not see how expressions are equivalent when applying the distributive property. Some students may need to make connections with finding common factors and setting up expressions that relate to the distributive property. Students will distributive to the first term only. For example, 2(3 + 7) = 6 + 7 (incorrect). Using a diagram to clear this misconception. **Probing questions:** • When in real life do we use distributive property? (Ex. Buying the same multiples of items 3(1.25 + 4.80) gives you the total cost. **Differentiation Strategies:** • Edmodo Enrichment/Remediation Worksheets • GOAS *(Ga. Online Assessment System)* • Study Island • Peer-tutoring • Teacher conferencing **Cooperative Learning Strategies:** • Jeopardy Game • Place Mat  Place_Mat.docx **Literacy Strategies:** • Math Foldables

C. Dividing Fractions by Fractions

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

MCC.6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a short story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Essential Questions

In what ways can rational numbers be useful?

Learning Targets

I can …

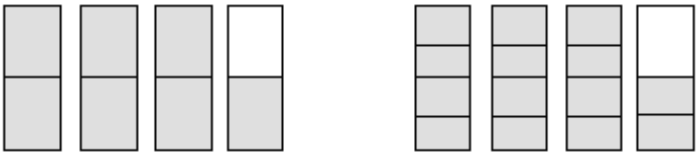
- Use a visual model to represent the division of a fraction by a fraction. (6NS1)
- Divide fractions by fractions using an algorithm or mathematical reasoning. (6NS1)
- Justify the quotient of a division problem by relating it to a multiplication problem. (6NS1)
- Use mathematical reasoning to justify the standard algorithm for fraction division. (6NS1)
- Solve real world problems involving the division of fractions and interpret the quotient in the context of the problem. (6NS1)
- Create story contexts for problems involving the division of a fraction by a fraction. (6NS1)

Concept Overview

In 5th grade, students divided whole numbers by unit fractions. Students continue this understanding in 6th grade by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought.

For example: 12 ÷3 means; How many groups of three would make 12? Or how many in each of 3 groups would make 12? Thus 7/2 ÷ 1/4 can be solved the same way. How many groups of 1/4 make 7/2 ?

Creating the picture that represents this problem makes seeing and proving the solutions easier.



Set the problem in context and represent the problem with a concrete or pictorial model.

1/4 ÷ 1/2 becomes 5/4 cups of nuts fills 1/2 of a container. How many cups of nuts will fill the entire container? Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed. Note that stages 2 and 3 are skills that should have been taught in earlier grades, but may have to be briefly revisited before teaching stage 3.

Vocabulary:

Dividend: A number that is divided by another number.

Divisor: A number by which another number is to be divided.

Quotient: A number that is the result of division.

Reciprocal: Two numbers whose product is 1. A factor by which you multiply a given number so that their product is 1.

Inverse operations: Pairs of operations that undo each other. Addition and subtraction are inverse operations For example, 1 + 4 = 5 reversely 5 – 4 = 1. Multiplication and division are inverse operations. For example, 2 × 3 = 6, reversely 6 ÷ 3 = 2.

Problems

Sample Problem 9

You have $\frac{5}{8}$ pound of Skittles. You have want give your friends $\frac{1}{4}$ pound each. How many friends you give Skittles to? Explain your answer. (6.NS.1)

Solution:

$\frac{5}{8} \div \frac{1}{4} = \frac{5}{2} = 2.5$ or $2\frac{1}{2}$ pounds each

Sample Problem 10 (6.NS.1)

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

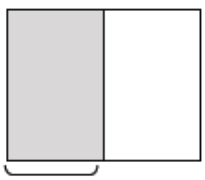
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

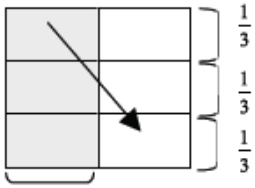
The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

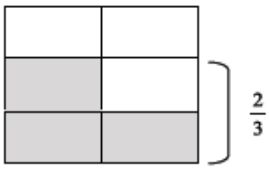
3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.



$\frac{1}{2}$



$\frac{1}{2}$



Resources, Teacher Notes, and Instructional Strategies

Standard & Topic	Resources	Teacher Notes
MCC.6.NS.1 Dividing fractions	http://www.learningplanet.com/sam/ff/index.asp Fraction Freezy: Matching Equivalent Fractions (S/U) http://www.visualfractions.com/ Visual Fractions: A tutorials that model fractions with number lines and circles. (S/U). http://www.kidsolr.com/math/fractions.html Interactive Tutorials with Fractions. (U) http://nlvm.usu.edu/en/nav/frames_asid_265_g_3_t_1.html?open=activities&from=category_g_3_t_1.html NLVM - Fraction Number Line Bars- Interactive Applet (U) http://www.visualfractions.com/divide.htm Visual Fractions - "Divide Fractions" - Interactive Applets and Game (S/U) http://www.uen.org/Lessonplan/preview.cgi?LPid=23394 UEN - "Modeling Multiplication and Division of Fractions" Lesson (U) http://learnzillion.com/lessons/204-divide-fractions-by-fractions-using-models Video on Dividing Fractions Using Models. (U) **State Learning Task(s) -** Dividing Fractions and Fractional Divisors http://www.uen.org/Lessonplan/preview?LPid=5301 – Act 01: Dividing Fractions Task http://www.uen.org/Lessonplan/preview?LPid=15443 – Sticky Note Task	**Model Lesson:** This is a model lesson on dividing fractions using visual models. DIVIDING FRACTIONS.docx **Student Misconceptions:** Students may think... • Dividing by $\frac{1}{2}$ by is the same as dividing in half. Dividing in half. Dividing by half means to find how many $\frac{1}{2}$ s there are in a quantity, whereas, dividing in half means take a quantity and split it into two equal parts. Thus 7 divided by $\frac{1}{2}$ = 14 and 7 divided in half equals equals $3\frac{1}{2}$. **Probing questions:** How might you represent dividing fractions? When you divide a whole number does the quotient increase or decrease? (Ex. 10 ÷ 5) Why? When you divide a whole number or fraction, does the quotient increase or decrease? ($10\frac{1}{2} \div \frac{1}{2}$) Why? What is the word problem asking? How might you represent dividing fractions? **Differentiation Strategies:** • Distribute learning packets for skills-based mathematics topics where mastery is essential. Packets can include practice sheets and assessments for students to use at their own pace. • Small groups • Stations • Have students to use manipulatives/cut outs/drawings to represent dividing fractions using visual models. **Cooperative Learning Strategies:** **Pairs Compare** (teambuilding, mastery, thinking) Pairs generate multiple responses to a question, then compare their answers with another pair, and

		then team up to create additional solutions. 1. Teacher provides topic or question. 2. With their shoulder partners, students RallyTable ideas or answers. 3. Teacher calls time. 4. Pairs pair up with another pair. 5. Partner A in Pair One shares; Partner A in Pair Two adds the item to the list, or if it is already listed, checks it off. 6. Partner B in Pair One shares; Partner B in Pair Two adds or checks off the item. 7. Partner A in Pair Two shares; Partner A in Pair One adds or checks off item. 8. Partner B in Pair Two shares; Partner B in Pair One adds or checks off item. 9. Steps 5 through 8 are repeated until all items are shared. **Using the *Round the Table Cooperative Learning Strategy*** have students work on problems jointly by passing the problems around the table for each member's response. (Groups of 2 or 3 will work best for dialogue and conversation) Pass Fraction Division Word Problems for each member to answer a portion of the problem. Make sure students are justifying their solutions with group as they work through the problems. **Literacy Strategies:** • The Doorbell Rang by Pat Hutchins ISBN-10: 0688092349 • Teaching Word Problems Graphic Organizer
Grade 6 Unit 1 Summative Assessment  UNIT 1 SUMMATIVE ASSESSMEN1.docx		