CHAPTER

4

Right Triangle Geometry

Ladders have been used since prehistoric times to climb and descend steep surfaces. Ladders are made up of vertical members called stringers and horizontal steps called rungs. You will use right triangle geometry to determine the length of a ladder needed to climb to the top of a building.

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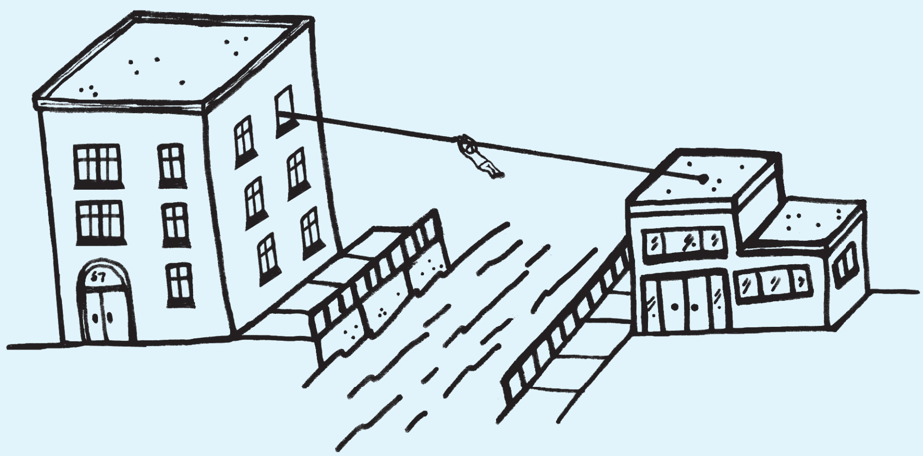
Introductory Problem for Chapter 4

The Right Triangle Escape from East Berlin

In October 1990 the German Democratic Republic (East Germany) joined the Federal Republic of Germany (West Germany). East Berlin also reunified with West Berlin and later became Germany's capital.

Before the German unification, a family lived in an apartment building in East Berlin adjacent to a river that divided them from West Berlin. They planned to escape to West Berlin by stringing a cable from their apartment window to the rooftop of a smaller apartment building directly across the river. The family made a device they could use to slide along the cable to freedom.



They knew the following:

- Their window was 34 feet above the ground.
- The rooftop of the apartment building on the other side was 26 feet above the ground.
- The river was 150 feet wide.

1. Draw a diagram of the situation.

2. What is the minimum length of cable needed to slide from the window to the rooftop of the other apartment building?

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Be prepared to share your solutions and methods.

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4.1 Interior and Exterior Angles of a Triangle

Triangle Sum, Exterior Angle, and Exterior Angle Inequality Theorems

OBJECTIVES

In this lesson you will:

- Prove the Triangle Sum Theorem.
- Explore the relationship between the interior angle measures and the side lengths of a triangle.
- Identify the remote interior angles of a triangle.
- Identify the exterior angle of a triangle.
- Explore the relationship between the exterior angle measures and two remote interior angles of a triangle.
- Prove the Exterior Angle Theorem.
- Prove the Exterior Angle Inequality Theorem.

KEY TERMS

- Triangle Sum Theorem
- remote interior angles of a triangle
- Exterior Angle Theorem
- Exterior Angle Inequality Theorem

PROBLEM 1 Triangle Interior Angle Sums

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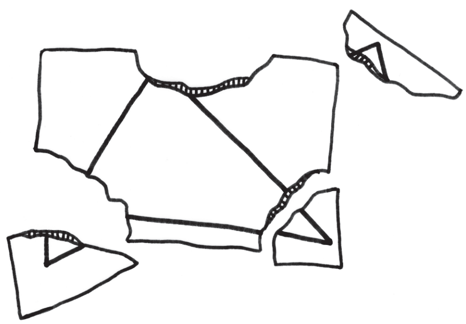
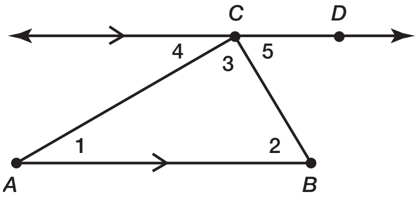
1. Draw any triangle on a piece of paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles. What do you notice about the sum of these three angles?



The **Triangle Sum Theorem** states: "The sum of the measures of the interior angles of a triangle is 180°."

2. Create a proof of the Triangle Sum Theorem using the diagram shown.



Given: Triangle ABC with $\overline{AB} \parallel \overleftrightarrow{CD}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Statements	Reasons

PROBLEM 2 Analyzing Triangles

1. Draw an acute triangle that is not an equiangular triangle. Measure each interior angle and label the angle measures in your diagram.

   a. Measure the length of each side of the triangle. Label the side lengths in your diagram.

   b. The longest side of the triangle lies opposite which interior angle?

   c. The shortest side of the triangle lies opposite which interior angle?

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2. Draw an obtuse triangle. Measure each interior angle and label the angle measures in your diagram.

a. Measure the length of each side of the triangle. Label the side lengths in your diagram.

b. The longest side of the triangle lies opposite which interior angle?

c. The shortest side of the triangle lies opposite which interior angle?

3. Draw a right triangle. Measure each interior angle and label the angle measures in your diagram.

a. Measure each side length of the triangle. Label the side lengths in your diagram.

b. The longest side of the triangle lies opposite which interior angle?

c. The shortest side of the triangle lies opposite which interior angle?

4. The measures of the three interior angles of a triangle are 57°, 62°, and 61°. Describe the location of each side with respect to the measures of the opposite interior angles without drawing or measuring any part of the triangle.

a. Longest side of the triangle

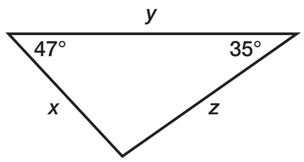
b. Shortest side of the triangle

5. One angle of a triangle decreases in measure and the sides of the angle remain the same length. Describe what happens to the side opposite the angle.

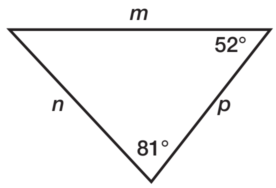
6. An angle of a triangle increases in measure and the sides of the angle remain the same length. Describe what happens to the side opposite the angle.

7. List the side lengths from shortest to longest for each diagram.

a.



b.



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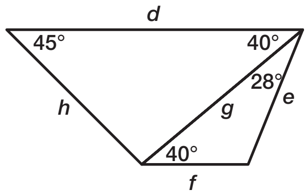
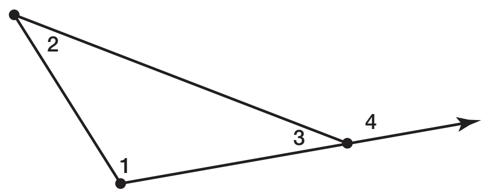


PROBLEM 3 Exterior Angles

Use the diagram shown to answer Questions 1 through 12.



1. Name the interior angles(s) of the triangle.

2. Name the exterior angle(s) of the triangle.

3. What did you need to know to answer Questions 1 and 2?

4. What does $m\angle 1 + m\angle 2 + m\angle 3$ equal? Explain your reasoning.

5. What does $m\angle 3 + m\angle 4$ equal? Explain your reasoning.

6. Why does $m\angle 1 + m\angle 2 = m\angle 4$? Explain your reasoning.

7. Consider the sentence "The location of the buried treasure is on a remote island." What does the word "remote" mean?

8. Angle 4 is an exterior angle of a triangle and $\angle 1$ and $\angle 2$ are interior angles of the same triangle. Why would $\angle 1$ and $\angle 2$ be referred to as "remote" interior angles with respect to the exterior angle?

The **remote interior angles of a triangle** are the two angles that are non-adjacent to the specified exterior angle.

9. Rewrite $m\angle 4 = m\angle 1 + m\angle 2$ as an English sentence using the words sum, remote interior angles of a triangle, and exterior angle of a triangle.

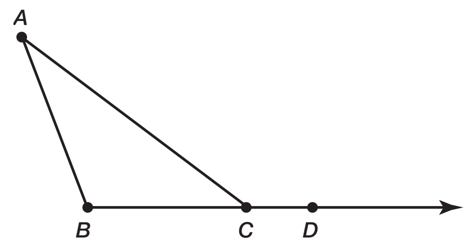
10. Is the sentence in Question 9 considered a postulate or a theorem? Explain.

11. The diagram was drawn as an obtuse triangle with one exterior angle. If the triangle had been drawn as an acute triangle, would this have changed the relationship between the measure of the exterior angle and the sum of the measures of the two remote interior angles? Explain.

12. If the triangle had been drawn as a right triangle, would this have changed the relationship between the measure of the exterior angle and the sum of the measures of the two remote interior angles? Explain.

The **Exterior Angle Theorem** states: "The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle."

13. Create a proof for the Exterior Angle Theorem using the diagram shown.



Given: Triangle ABC with exterior $\angle ACD$

Prove: $m\angle A + m\angle B = m\angle ACD$

Statements	Reasons

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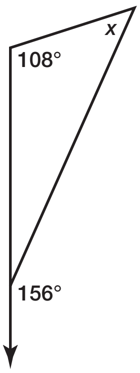
14. Solve for x in each diagram.

a.



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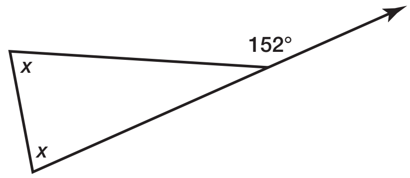

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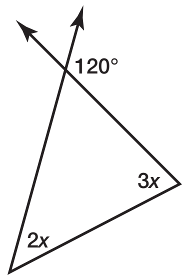
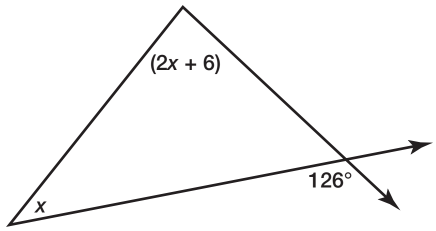


d.



The **Exterior Angle Inequality Theorem** states: “The measure of an exterior angle of a triangle is greater than the measure of either of the remote interior angles of the triangle.”

15. Why is it necessary to prove two different statements to completely prove this theorem?

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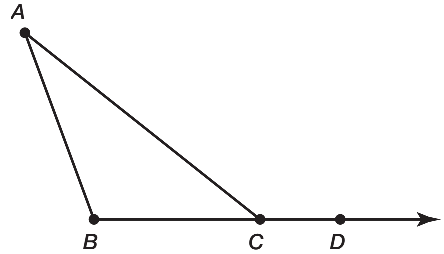
16. Prove both parts of the Exterior Angle Inequality Theorem using the diagram shown.



a. Part 1

Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle A$

Statements	Reasons
1.	**1.** Given
2.	**2.** Triangle Sum Theorem
3.	**3.** Linear Pair Postulate
4.	**4.** Definition of linear pair
5.	**5.** Substitution Property using step 2 and step 4
6.	**6.** Subtraction Property of Equality
7.	**7.** Definition of an angle measure
8.	**8.** Inequality Property (if $a = b + c$ and $c > 0$, then $a > b$)

b. Part 2

Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle B$

Statements	Reasons

4

Be prepared to share your solutions and methods.

4.2 Installing a Satellite Dish

Simplifying Radicals, Pythagorean Theorem, and Its Converse

OBJECTIVES

In this lesson you will:

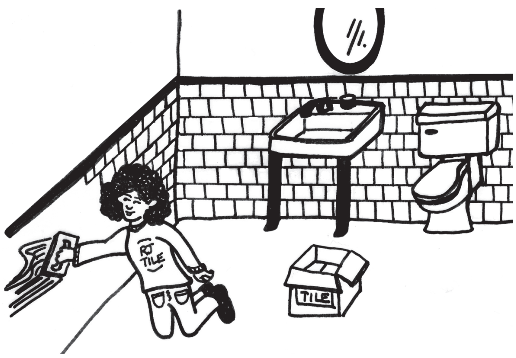
- Simplify square roots.
- Simplify fractions with radicals.
- Prove the Pythagorean Theorem.
- Use the Pythagorean Theorem.
- Write the Converse of the Pythagorean Theorem.
- Prove the Converse of the Pythagorean Theorem.
- Use the Converse of the Pythagorean Theorem.

KEY TERMS

- square root
- radical sign
- radicand
- radical expression
- simplest form
- hypotenuse
- legs

PROBLEM 1 Lots of Tiles

Part of a bathroom wall is being covered with square ceramic tiles. The rectangular area to be covered is 7 feet tall and 5 feet wide. One box of the tiles covers 10 square feet and contains 40 tiles.



1. How many boxes of tiles will be needed for the job? Keep in mind that only whole boxes of tile can be purchased.

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2. How many inches are in one foot?

3. How many square inches are in one square foot? Explain your reasoning.

4. How many square inches does one box of tiles cover?

5. How many square inches does one tile cover?

6. What are the length and width of one tile? Explain your reasoning.

You determined the answer to Question 5 by calculating a *square root* of 36. Remember that a number b is a **square root** of a number a if $b^2 = a$. An expression that involves a **radical** sign is called a **radical expression**. The **radicand** is the expression written under the radical sign.

7. Calculate the value of each radical expression.

a. $\sqrt{16}$

b. $\sqrt{4} \cdot \sqrt{4}$

c. $\sqrt{36}$

d. $\sqrt{4} \cdot \sqrt{9}$

4

A simple radical expression like $\sqrt{5}$ is in **simplest form** when the radicand contains no factors that are perfect squares. The radical expression $\sqrt{8}$ is not in simplest form because 8 contains the perfect square factor 4. The following is an example of simplifying $\sqrt{8}$.

$$\begin{aligned}\sqrt{8} &= \sqrt{4 \cdot 2}\\ &= \sqrt{4} \cdot \sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

8. Tell whether each expression is in simplest form. If not, simplify the expression.

a. $\sqrt{15}$

b. $\sqrt{12}$

c. $\sqrt{28}$

d. $\sqrt{49}$

9. Simplify the expression $(\sqrt{5})^2$.

Sometimes, a fraction like $\frac{3}{\sqrt{2}}$ contains a radical in the denominator.

This fraction is not considered to be in simplest form. An example of rationalizing the denominator is shown.

$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$	Multiply fraction by $1 = \frac{\sqrt{2}}{\sqrt{2}}$.
$= \frac{3}{\sqrt{2}} \frac{(\sqrt{2})}{(\sqrt{2})}$	Multiply numerators and multiply denominators.
$= \frac{3\sqrt{2}}{\sqrt{4}}$	Write product as single square root.
$= \frac{3\sqrt{2}}{2}$	Simplify.

10. Explain how to simplify any fraction that has a radical in the denominator.

11. Identify the expression you will use to simplify the fraction. Then, simplify the fraction.

$\frac{1}{\sqrt{3}}$ $\qquad$ $\frac{2}{\sqrt{2}}$

PROBLEM 2 Who's Correct?

1. A teacher asked her students to compute $\sqrt{1 + 1}$.
Bob got an answer of 1.414 and Bill got an answer of $\sqrt{2}$. Both students claimed their answer was correct.
   a. Whose answer is correct? Explain.

   b. What is the difference between an exact answer and an approximate answer?

   c. Would Bob's answer be considered exact or approximate? Explain.

   d. Would Bill's answer be considered exact or approximate? Explain.

2. A teacher asked her students to simplify $\sqrt{1} + \sqrt{1}$. Joe got an answer of $\sqrt{2}$, James got an answer of $2\sqrt{1}$, and Lee got an answer of 2. How did each student get their answer? Who is correct? Explain.

PROBLEM 3 Coming Up Short

Dish It Out Satellite Company has been hired to install a satellite dish on the roof of a building. Mrs. Hannon, the building's owner, has given specific instructions that the plants and shrubs around the base of the building are not to be disturbed in any way. Carrie, a worker for Dish It Out, must choose a ladder that will be long enough to reach the roof of the building while not disturbing the plants. A diagram is shown.

1. On the figure, draw the triangle that is formed by the ladder, the building, and the ground. Label the vertices of the triangle.

2. Classify the angles in your triangle.

3. Without measuring, identify the longest side of the triangle. How do you know that this side is the longest side? Explain your reasoning.

4. Suppose that you know that the building is 16 feet tall and that the foot of the ladder must be placed 12 feet from the building. What do you know about the length of the ladder? Explain your reasoning.

5. Draw a right triangle ABC with $\angle C$ as the right angle.

The **hypotenuse** of a right triangle is the side opposite the right angle.

a. Name the hypotenuse of triangle ABC.

The **legs** of a right triangle are the sides that form the right angle.

b. Name the legs of triangle ABC.

The sides of a right triangle can be labeled using a single lower case letter. The side opposite $\angle A$ is labeled "a," the side opposite $\angle B$ is labeled "b," and the side opposite $\angle C$ is labeled "c."

c. Label the sides of triangle ABC.

d. How does the length of the hypotenuse of any right triangle compare to the lengths of the legs? Explain.

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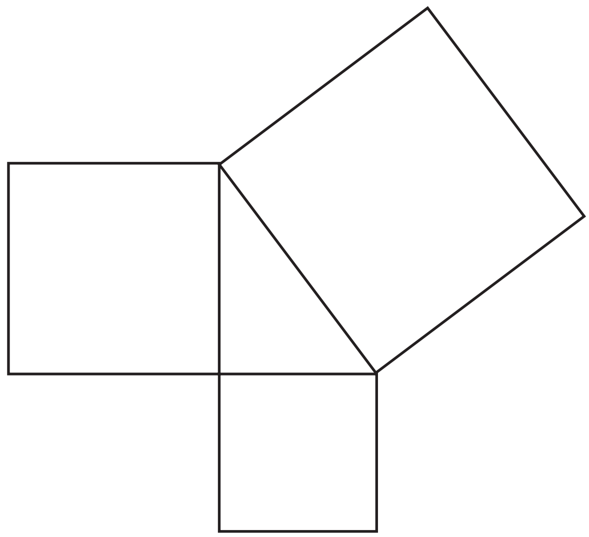
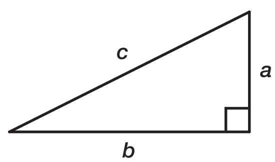
6. The figure shown is composed of one right triangle and three squares. Determine the relationship between the areas of the three squares. Use a ruler and measure in inches. Describe the relationship.



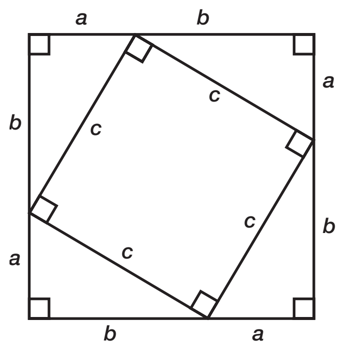
7. Write an equation that represents the relationship between the areas of the squares, if the length of the shortest leg of the right triangle is "*a*," the length of the longest leg of the right triangle is "*b*," and the length of the hypotenuse is "*c*."

The **Pythagorean Theorem** states: "If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$."



The Pythagorean Theorem can be proven in many ways. One proof involves manipulating the areas of triangles and the areas of squares.

8. Use the diagram shown and the following questions to prove the Pythagorean Theorem.



a. What is the area of the larger square?

b. What is the total area of the four right triangles?

c. What is the area of the smaller square?

d. What is the relationship between the area of the four right triangles, the area of the smaller square, and the area of the larger square?

9. How long must the ladder be so that Carrie can reach the top of the building while avoiding the plants and bushes? Explain your reasoning.

10. If Carrie uses a ladder that is 25 feet long and places it so that the top of the ladder meets the top of the building described in Problem 3, how far away from the building is the base of the ladder? Round your answer to the nearest tenth, if necessary.

11. Carrie is installing a satellite dish at a different building, but the parameters are the same: do not disturb the plants or shrubs. The base of the ladder must be placed 10 feet from the building to avoid the bushes planted at the base of the building. Carrie has a ladder that is 18 feet long that can be extended to 25 feet long. Describe the possible building heights that this ladder will reach. Round your answers to the nearest foot, if necessary.

12. According to ladder safety rules, a ladder should be placed so that it makes an angle of about 75 degrees with the ground. When the ladder is placed the required distance from the building to avoid disturbing the shrubbery in Question 4, it makes an angle of about 53 degrees with the ground. If you were one of the workers in this situation, what could you do?

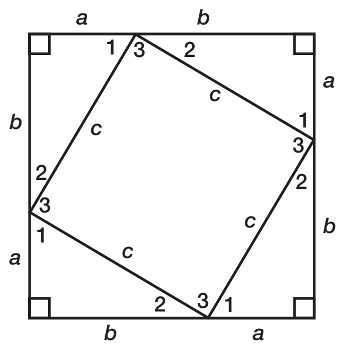
The converse of a conditional statement is formed by reversing the hypothesis and the conclusion.

13. Identify the hypothesis of the Pythagorean Theorem.

14. Identify the conclusion of the Pythagorean Theorem.

15. Write the Converse of the Pythagorean Theorem.

16. Use the diagram shown and the following questions to prove the Converse of the Pythagorean Theorem.
A large square is composed with four identical right triangles in its corners.



a. What can you conclude about $m\angle 1 + m\angle 2 + m\angle 3$?

b. Use the Triangle Sum Theorem to determine $m\angle 1 + m\angle 2$.

c. Knowing $m\angle 1 + m\angle 2$, what can you conclude about $m\angle 3$?

d. What does $m\angle 3$ tell you about the quadrilateral inside of the large square?

e. What is the area of one of the right triangles?

f. What is the area of the quadrilateral inside the large square?

g. Write an expression that represents the combined areas of the four right triangles and the quadrilateral inside the large square. Use your answers to Questions 16, parts (e) and (f).

h. Write an expression to represent the area of the large square, given one side is expressed as $(a + b)$. Simplify your answer.

4

i. Write an equation using the two different expressions representing the area of the large square from Questions 16(g) and (h). Then, solve the equation to prove the Converse of the Pythagorean Theorem.

17. Use the Converse of the Pythagorean Theorem to determine if each set of numbers are or are not possible lengths for the sides of a right triangle.

a. 1, 2, 3

b. 2, 3, 4

c. 3, 4, 5

d. 6, 8, 10

e. $3x$, $4x$, $5x$

18. Carrie successfully installed the satellite dish on the top of Mrs. Hannon's building in Question 4. The satellite dish is 15 feet away from an air conditioning unit and 25 feet from an antenna that is also located on the roof. The antenna is 20 feet from the air conditioning unit. Draw a diagram of this situation. Then determine whether the satellite dish, the air conditioning unit, and the antenna form the vertices of a right triangle.

Be prepared to share your solutions and methods.

4.3 Special Right Triangles

Properties of a 45°–45°–90° Triangle

OBJECTIVES

In this lesson you will:

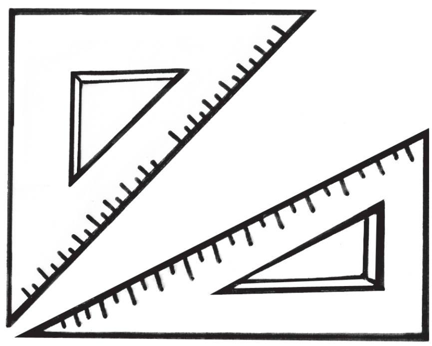
- Use the Pythagorean Theorem to prove that the length of the hypotenuse of an isosceles right triangle is the product of the leg length and $\sqrt{2}$.
- Calculate side lengths of 45°–45°–90° triangles.
- Calculate areas of 45°–45°–90° triangles.

KEY TERMS

- 45°–45°–90° triangle
- 45°–45°–90° Triangle Theorem

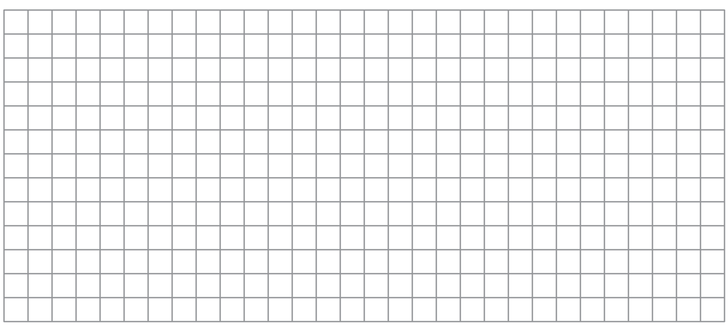
PROBLEM 1 Drawing a Triangle

Architects and engineers must be skilled at making technical drawings as part of their job. Technical drawings are precise models of objects, buildings, or parts that indicate exact shapes and give precise measurements. Most of today's technical drawings are done on the computer by using a CAD (computer aided design) program. However, some drawings can be done by hand using right triangles.



One of the right triangles used to create a technical drawing is an isosceles right triangle.

1. Use the grid to draw an isosceles right triangle.



2. Identify the lengths of your triangle and label these lengths. Label the length of the hypotenuse as c.

3. Apply the Pythagorean Theorem using the lengths of the legs of the triangle. Let c represent the length of the hypotenuse. Solve for c.

4. Draw a different isosceles right triangle on the grid in Question 1. Label the legs and hypotenuse of the triangle like you did in Question 2.

5. Write a radical expression in simplest form for the length of the hypotenuse of the right triangle in Question 4.

6. What can you conclude about the lengths of the legs when compared to the length of the hypotenuse in an isosceles right triangle?

4

PROBLEM 2 45°–45°–90° Triangles

1. Measure the angles in your triangles from Question 1. What do you notice?

2. What can you conclude about the angles that are not equal to 90° in an isosceles right triangle?

An isosceles right triangle is often called a **45°–45°–90° triangle**.

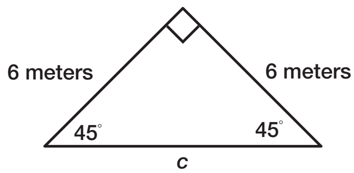
The **45°–45°–90° Triangle Theorem** states: "The length of the hypotenuse in a 45°–45°–90° triangle is $\sqrt{2}$ times the length of a leg."

3. Prove the 45°–45°–90° Theorem. Justify algebraically the relationship between the lengths of the legs and the length of the hypotenuse in an isosceles right triangle.

4. Apply the Pythagorean Theorem. Let ℓ represent the length of each leg and h represent the length of the hypotenuse. Solve for h.

5. Calculate the unknown side length in each triangle. Do not evaluate the radicals.

a.



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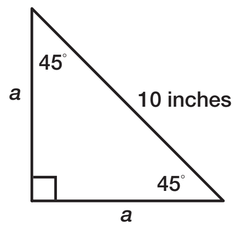
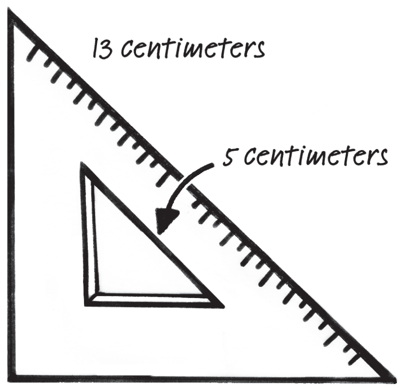


6. Drafting triangles that are 45°–45°–90° triangles are usually described by the length of their hypotenuse. An architect has a 21-centimeter 45°–45°–90° triangle and a 31-centimeter 45°–45°–90° triangle. What are the lengths of the legs of these triangles? Round your answers to the nearest tenth of a centimeter.

4

7. Drafting triangles usually have an open triangular area in the center of the triangle as shown. The interior angles of the open triangle have the same measures as the interior angles of edges of the drafting triangle. Calculate the area of the shaded region of the 45°–45°–90° drafting triangle. Do not evaluate any radicals.



PROBLEM 3 Construction

Isosceles Right Triangle

1. Construct an isosceles right triangle with $\overline{CB}$ as a leg and $\angle C$ as the right angle.

After completing the construction, use a protractor and a ruler to confirm each:

- $m\angle A = 45°$
- $m\angle B = 45°$
- $AC = BC$
- $AB = AC\sqrt{2}$
- $AB = BC\sqrt{2}$

2. Explain why an alternate method for constructing a 45°–45°–90° triangle could begin by constructing a square.

Be prepared to share your solutions and methods.

4.4 Other Special Right Triangles

Properties of a 30°–60°–90° Triangle

OBJECTIVES

In this lesson you will:

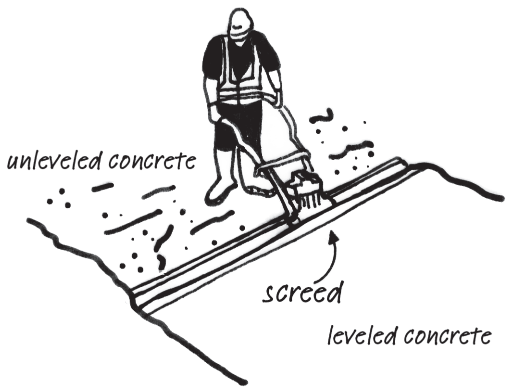
- Use the Pythagorean Theorem to prove that the length of the side opposite the 60° angle of a 30°–60°–90° triangle is half the length of the hypotenuse times $\sqrt{3}$.
- Calculate side lengths of 30°–60°–90° triangles.
- Calculate areas of 30°–60°–90° triangles.

KEY TERMS

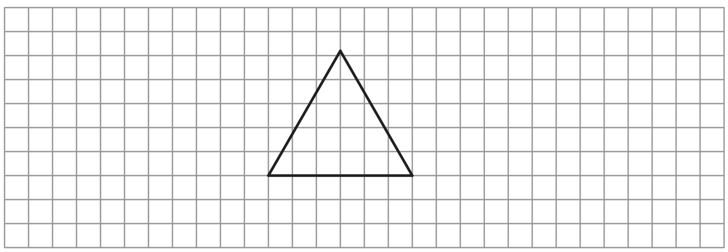
- 30°–60°–90° triangles
- 30°–60°–90° Triangle Theorem

PROBLEM 1 Leveling Off

When pouring concrete, the concrete must be leveled off before it hardens. For large construction jobs, like sections of highway, a power screed is used to level off the concrete. Power screeds are gas-powered machines that have bars called screed bars that level off the concrete by vibrating along the concrete's surface. The shape of the screed bars can vary.



The base of a screed bar can be the shape of an equilateral triangle as shown.
One grid square represents a square that is one inch long and one inch wide.



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1. An equilateral triangle is a regular polygon. What are the interior angle measures of this triangle? Explain your reasoning.

Take Note

All equilateral triangles are also equiangular.

2. Draw the altitude of the equilateral triangle on the diagram. Do you know the exact length of the altitude? Explain your reasoning.

3. What kinds of triangles are formed within the equilateral triangle by drawing the altitude?

4. How do these triangles compare to each other?

5. What are the interior angle measures of these triangles?

4

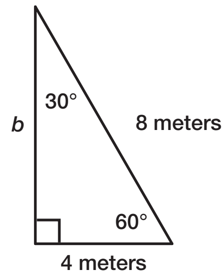
6. Which sides of these triangles are the longest? Which sides are the shortest? Explain your reasoning.

7. Determine the altitude of the equilateral triangle. Do not evaluate any radicals.

8. How does the length of the hypotenuse appear to be related to the length of the shorter leg?

9. Do you think that this relationship is true of any right triangle? Why or why not?

10. Consider the right triangle shown. Use the Pythagorean Theorem to solve for the unknown length. Write your answer as a radical expression in simplest form.



11. How does the length of the longer leg relate to the length of the shorter leg?

12. Is this relationship between the lengths of the legs the same in the right triangles you drew in Question 2? Justify your answer.

The right triangles in this lesson are called **30º–60º–90º triangles**.

The **30º–60º–90º Triangle Theorem** states: "The length of the hypotenuse in a 30º–60º–90º triangle is two times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg."

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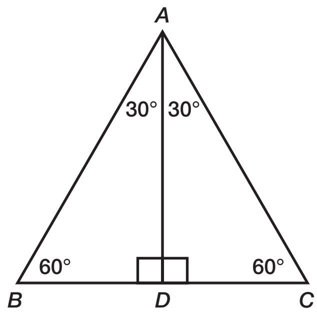
13. Prove the 30°–60°–90° Theorem. Justify algebraically the relationship between the lengths of the hypotenuse and the shorter leg, and relationship between the lengths of the longer leg and the shorter leg in a 30°–60°–90° triangle.

Triangles *ABD* and *ACD* are identical 30°–60°–90° triangles with shared side *AD*.



a. What can you conclude about triangle *ABC*? Explain.

b. What can you conclude about the relationship between $\overline{BD}$ and $\overline{AB}$? Explain.

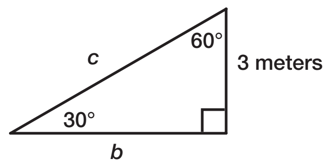
c. What can you conclude about the relationship between $\overline{DC}$ and $\overline{AC}$? Explain.

d. If x represents $\overline{BD}$, what expression represents $\overline{AB}$?

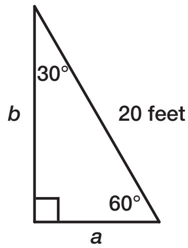
e. Apply the Pythagorean Theorem to solve for $\overline{AD}$ in triangle *ABD*.
Let x represent $\overline{BD}$.

14. Calculate the unknown side lengths in each triangle. Do not evaluate the radicals.

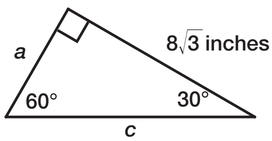
a.



b.



c.



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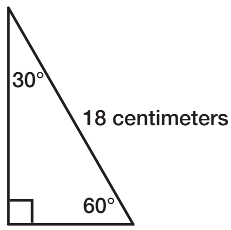
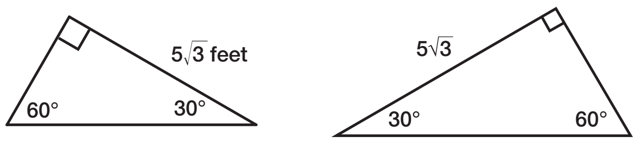
15. Calculate the area of each triangle. Round your answer to the nearest tenth.

a.



b.



PROBLEM 2 Construction

30°–60°–90° Triangle

Construct a 30°–60°–90° triangle by constructing an equilateral triangle and one altitude.

After completing the construction, use a protractor and a ruler to confirm that:

- One angle measure is 30°.
- One angle measure is 60°.
- One angle measure is 90°.
- The side opposite the 30° angle is one half the length of the hypotenuse.
- The side opposite the 60° angle is one half the hypotenuse times $\sqrt{3}$.

Be prepared to share your solutions and methods.

4.5 Pasta Anyone?

Triangle Inequality Theorem

OBJECTIVES

In this lesson you will:

- Explore the relationships between the side lengths of a triangle and the measures of its interior angles.
- Prove the Triangle Inequality Theorem.

KEY TERMS

- Triangle Inequality Theorem
- auxiliary line

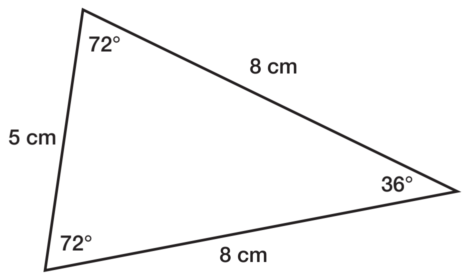
PROBLEM 1 Side and Angle Relationships

1. Draw a scalene triangle. Measure each side and angle.

a. The largest interior angle of the triangle lies opposite which side?

b. The smallest interior angle of the triangle lies opposite which side?

2. Draw an isosceles triangle that is not an equiangular triangle. Measure each side and angle.



a. The largest interior angle(s) of the triangle lies opposite which side(s)?

b. The smallest interior angle(s) of the triangle lies opposite which side(s)?

3. Draw an equilateral triangle. Measure each side and angle

a. The largest interior angle of the triangle lies opposite which sides?

b. The smallest interior angle of the triangle lies opposite which side?

4

4. The lengths of the sides of a triangle are 4 cm, 5 cm, and 6 cm. Describe the location of each angle with respect to the lengths of the opposite sides without drawing or measuring any part of the triangle. Explain your reasoning.

   a. Largest interior angle

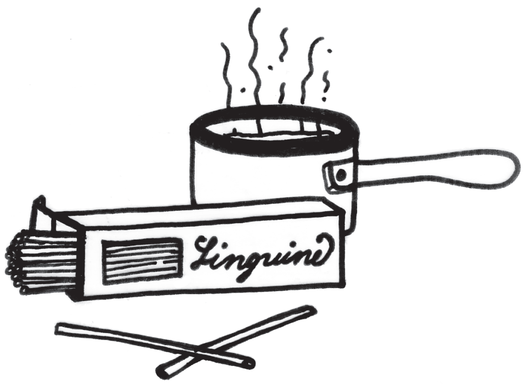
   b. Smallest interior angle

5. A side of a triangle decreases in measure and the other sides of the angle remain the same length. Describe what happens to the angle opposite the side.

6. A side of a triangle increases in measure and the other sides of the angle remain the same length. Describe what happens to the angle opposite the side.

PROBLEM 2 Who is Correct?

Sarah claims that any three lengths will determine three sides of a triangle. Sam does not agree. He thinks some combinations will not work. Who is correct? Remember, you need one counterexample to disprove a statement.



Sam then claims that he can just look at the three lengths and know immediately if they will work. Sarah is unsure. She decides to explore this for herself.

4

Help Sarah by working through the following activity.

To begin, you will need a piece of strand pasta (like linguine). Break the pasta at two random points so the strand is divided into three pieces.

1. Try to form a triangle from your three pieces of pasta.
2. Measure each of your three pieces of pasta in centimeters.

3. Collect and record your classmate's measurements.

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a Triangle? (yes or no)

4. How many students are in your class?

5. How many students' pasta pieces formed a triangle when the three pieces were connected end to end?

6. How many students' pasta pieces did not form a triangle when the three pieces were connected end to end?

7. Examine the lengths of the pasta pieces that did form a triangle. Compare them with the lengths of the pasta pieces that did not form a triangle. What observations can you make?

8. Under what conditions is it possible to form a triangle?

9. Under what conditions is it impossible to form a triangle?

10. Based upon your observations, determine if it is possible to form a triangle using segments with the following measurements. Explain.
    a. 2 cm, 5.1 cm, 2.4 cm

    b. 9.2 cm, 7 cm, 1.9 cm

The rule that Sam was using is known as the Triangle Inequality Theorem.

The **Triangle Inequality Theorem** states: "The sum of the lengths of any two sides of a triangle is greater than the length of the third side."

To prove the Triangle Inequality Theorem, consider the statements and figure shown.

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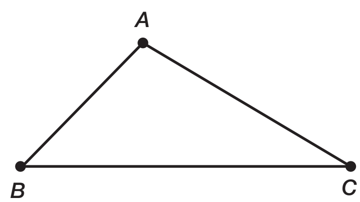
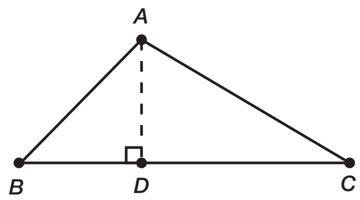


Given: Triangle ABC
Prove: $AB + AC > BC$

It is impossible to prove $AB + AC > BC$ using the figure shown because there is not enough detail. An **auxiliary line** is a line that is not shown in a diagram, but can be drawn using the points in the diagram. When proving statements, it is often necessary to modify given figures using auxiliary lines, segments, or rays. This can be done by constructing one or more lines, segments, or rays that can show parallel or perpendicular relationships to part of the given figure. Adding auxiliary lines should *never* change any parts of the original figure.

To complete the proof of the Triangle Inequality Theorem, a perpendicular segment AD is constructed through point A to side BC. Any auxiliary lines need to be noted as part of the proof. When writing a statement such as "Draw $\overline{AD} \perp \overline{BC}$," the justification or reason for drawing the auxiliary line segment is "Construction," as shown in the proof that follows.

11. Complete the proof of the Triangle Inequality Theorem by writing the reason for each statement.



Given: Triangle ABC
Prove: $AB + AC > BC$

Statements	Reasons
1. Triangle ABC	**1.**
2. Draw $\overline{AD} \perp \overline{BC}$	**2.** Construction
3. $BD^2 + AD^2 = AB^2$	**3.**
4. $CD^2 + AD^2 = AC^2$	**4.**
5. $AB^2 > BD^2$	**5.**
6. $AC^2 > DC^2$	**6.**
7. $AB > BD$	**7.**
8. $AC > DC$	**8.**
9. $AB + AC > BD + DC$	**9.**
10. $BD + DC = BC$	**10.**
11. $AB + AC > BC$	**11.**

Be prepared to share your solutions and methods.

Chapter 4 Checklist

KEY TERMS

- remote interior angles of a triangle (4.1)
- square root (4.2)
- radical sign (4.2)
- radicand (4.2)
- radical expression (4.2)
- simplest form (4.2)
- hypotenuse (4.2)
- legs (4.2)
- 45°–45°–90° triangle (4.3)
- 30°–60°–90° triangle (4.4)
- auxiliary line (4.5)

THEOREMS

- Triangle Sum Theorem (4.1)
- Exterior Angle Theorem (4.1)
- Exterior Angle Inequality Theorem (4.1)
- Pythagorean Theorem (4.2)
- Converse of the Pythagorean Theorem (4.2)
- 45°–45°–90° Triangle Theorem (4.3)
- 30°–60°–90° Triangle Theorem (4.4)
- Triangle Inequality Theorem (4.5)

CONSTRUCTIONS

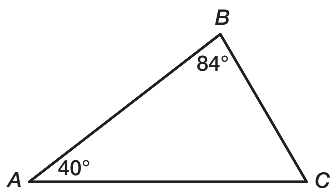
- isosceles right triangle (4.3)
- 30°–60°–90° triangle (4.4)

4.1

Using the Triangle Sum Theorem

The Triangle Sum Theorem states: "The sum of the measures of the interior angles of a triangle is 180°."

Example:



$$m\angle A + m\angle B + m\angle C = 180°$$

$$40° + 84° + m\angle C = 180°$$

$$m\angle C = 180° - (40° + 84°)$$

$$m\angle C = 180° - 124°$$

$$m\angle C = 56°$$

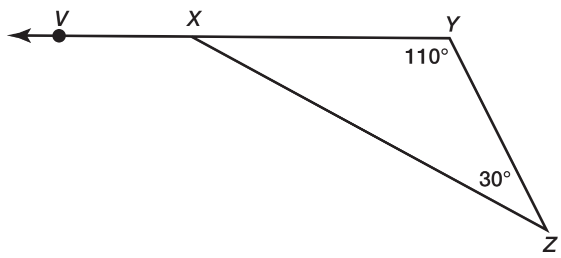
4

4.1

Using the Exterior Angle Theorem

The Exterior Angle Theorem states: "The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle."

Example:



$m\angle VXZ = m\angle Y + m\angle Z$

$m\angle VXZ = 110° + 30°$

$m\angle VXZ = 140°$

4.2

Simplifying Square Roots

An expression that involves a radical sign is called a radical expression. The radicand is the expression written under the radical sign. A simple radical expression involving a square root is in simplest form when the radicand contains no factors that are perfect squares.

Example:

$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$

The expression $\sqrt{10}$ is in simplest form.

4

4.2

Simplifying Fractions with Radical Denominators

A fraction that contains a radical in the denominator is not considered to be in simplest form. To simplify a fraction with a radical in the denominator, multiply the fraction by a form of 1 by using the radical that is in the denominator. Then, simplify the fraction.

Example:

$\frac{9}{\sqrt{5}} = \frac{9}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply fraction by 1 $= \frac{\sqrt{5}}{\sqrt{5}}$.

$= \frac{9\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$ Multiply numerators and multiply denominators.

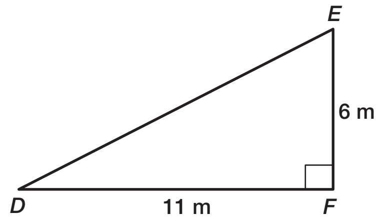
$= \frac{9\sqrt{5}}{\sqrt{25}}$ Write the product as a single square root.

$= \frac{9\sqrt{5}}{5}$ Simplify.

4.2 Using the Pythagorean Theorem

The Pythagorean Theorem states: "If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$."

Example:

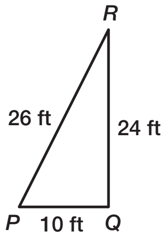


$$EF^2 + DF^2 = DE^2$$
$$6^2 + 11^2 = DE^2$$
$$36 + 121 = DE^2$$
$$157 = DE^2$$
$$\sqrt{157} = DE$$
$$DE \approx 12.53\text{ m}$$

4.2 Using the Converse of the Pythagorean Theorem

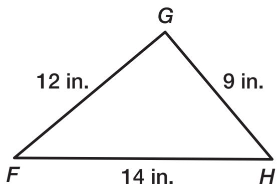
The Converse of the Pythagorean Theorem states: "If $a^2 + b^2 = c^2$, then a and b are the lengths of the legs of a right triangle and c is the length of its hypotenuse."

Examples:



$$PQ^2 + QR^2 \stackrel{?}{=} PR^2$$
$$10^2 + 24^2 \stackrel{?}{=} 26^2$$
$$100 + 576 \stackrel{?}{=} 676$$
$$676 = 676$$

Triangle PQR is a right triangle.



$$FG^2 + GH^2 \stackrel{?}{=} FH^2$$
$$12^2 + 9^2 \stackrel{?}{=} 14^2$$
$$144 + 81 \stackrel{?}{=} 196$$
$$225 \neq 196$$

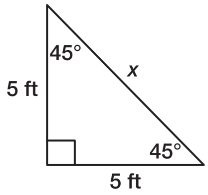
Triangle FGH is not a right triangle.

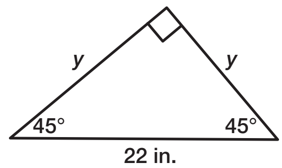
4

4.3 Using the 45°–45°–90° Triangle Theorem

The 45°–45°–90° Triangle Theorem states: "The length of the hypotenuse in a 45°–45°–90° triangle is $\sqrt{2}$ times the length of a leg."

Examples:





$x = 5\sqrt{2}$ ft

The length of the hypotenuse is $5\sqrt{2}$ feet.

$y\sqrt{2} = 22$

$$y = \frac{22}{\sqrt{2}} = \frac{22 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{22 \cdot \sqrt{2}}{2} = 11\sqrt{2} \text{ in.}$$

The length of each leg is $11\sqrt{2}$ inches.

4.4 Using the 30°–60°–90° Triangle Theorem

The 30°–60°–90° Triangle Theorem states: "The length of the hypotenuse in a 30°–60°–90° triangle is two times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg."

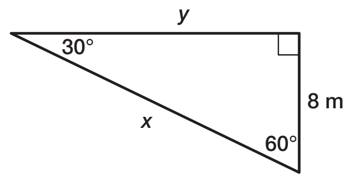
Examples:

Hypotenuse:

$x = 2(8) = 16$ m

Longer leg:

$y = 8\sqrt{3}$ m



Shorter leg:

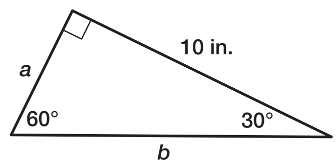
$a\sqrt{3} = 10$

$$a = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3} \text{ in.}$$

Hypotenuse:

$$b = 2a = 2\left(\frac{10\sqrt{3}}{3}\right)$$

$$= \frac{20\sqrt{3}}{3} \text{ in.}$$



Shorter leg:

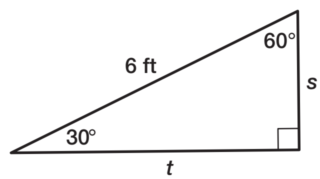
$2s = 6$

$s = 3$ ft

Longer leg:

$t = s\sqrt{3}$

$t = 3\sqrt{3}$ ft



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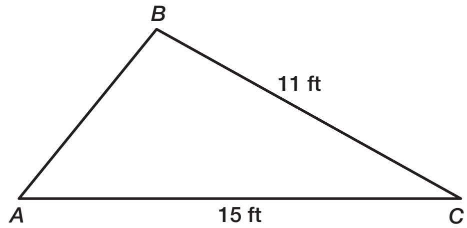
4.5 Using the Triangle Inequality Theorem

The Triangle Inequality Theorem states: "The sum of the lengths of any two sides of a triangle is greater than the length of the third side."

Example:



$AB < BC + AC$	$BC < AB + AC$	$AC < AB + BC$
$AB < 11 + 15$	$11 < AB + 15$	$15 < AB + 11$
$AB < 26$	$-4 < AB$	$4 < AB$

So, *AB* must be greater than 4 feet and less than 26 feet. (A length cannot be negative, so disregard the negative number.)

4

4