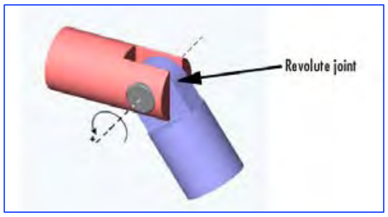
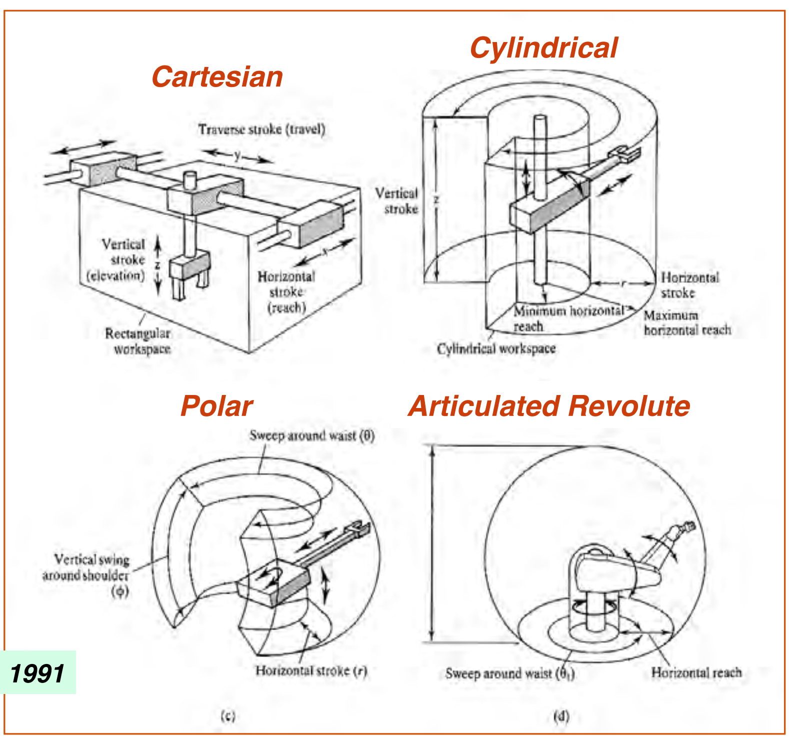
Articulated Robots

Robert Stengel
Robotics and Intelligent Systems
MAE 345, Princeton University, 2017



- **Robot configurations**
- **Joints and links**
- **Joint-link-joint transformations**
  - Denavit-Hartenberg representation

Assembly Robot Configurations

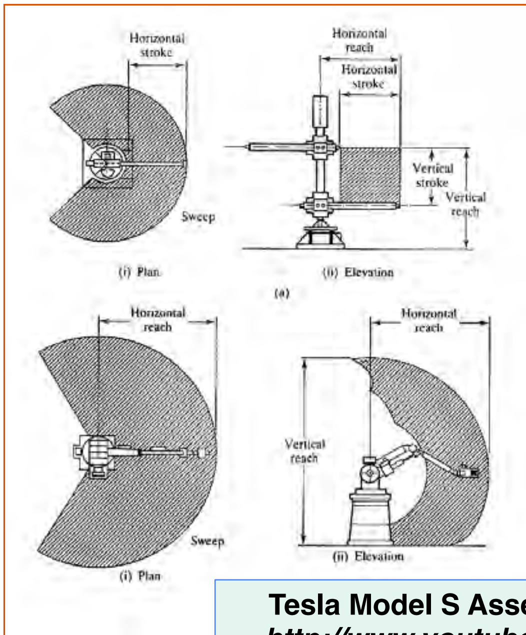


McKerrow, 1991

Assembly Robot Workspaces

Cylindrical

Articulated Revolute



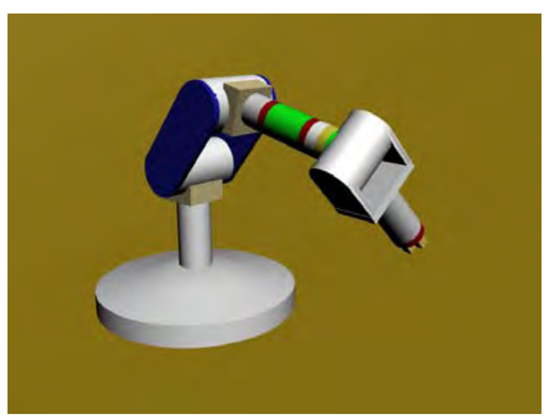
McKerrow, 1991

Tesla Model S Assembly
http://www.youtube.com/watch?v=8_lfxPI5ObM

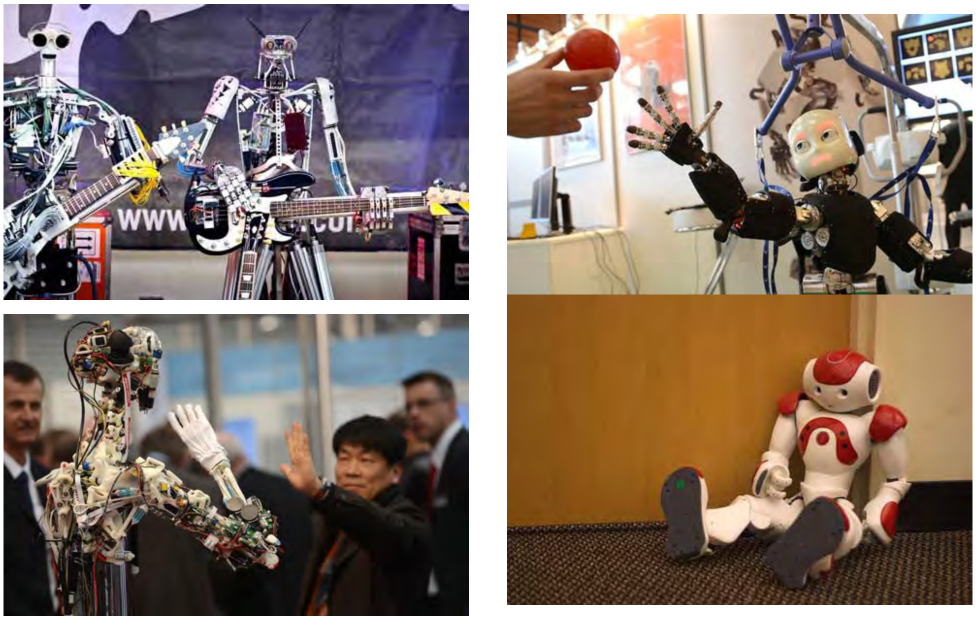
Serial Robotic Manipulators

***Proximal link**: closer to the base*
***Distal link**: farther from the base*

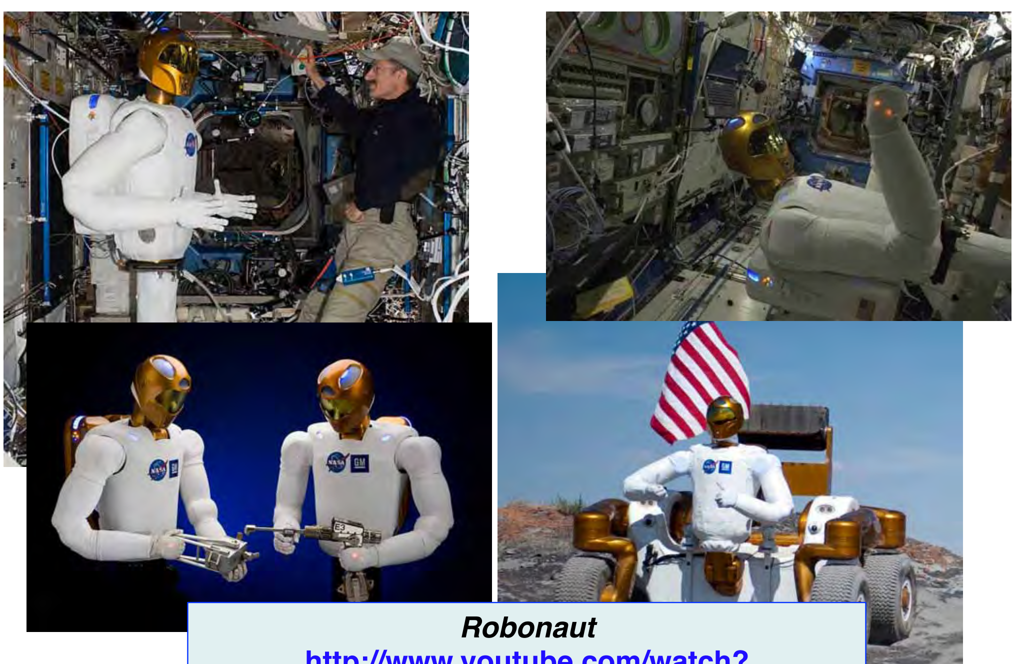
- **Serial chain of robotic links and joints**
  - **Large workspace**
  - **Low stiffness**
  - **Cumulative errors from link to link**
  - **Proximal links carry the weight and load of distal links**
  - **Actuation of proximal joints affects distal links**
  - **Limited load-carrying capability at end effecter**



Humanoid Robots



NASA/GM Robonaut



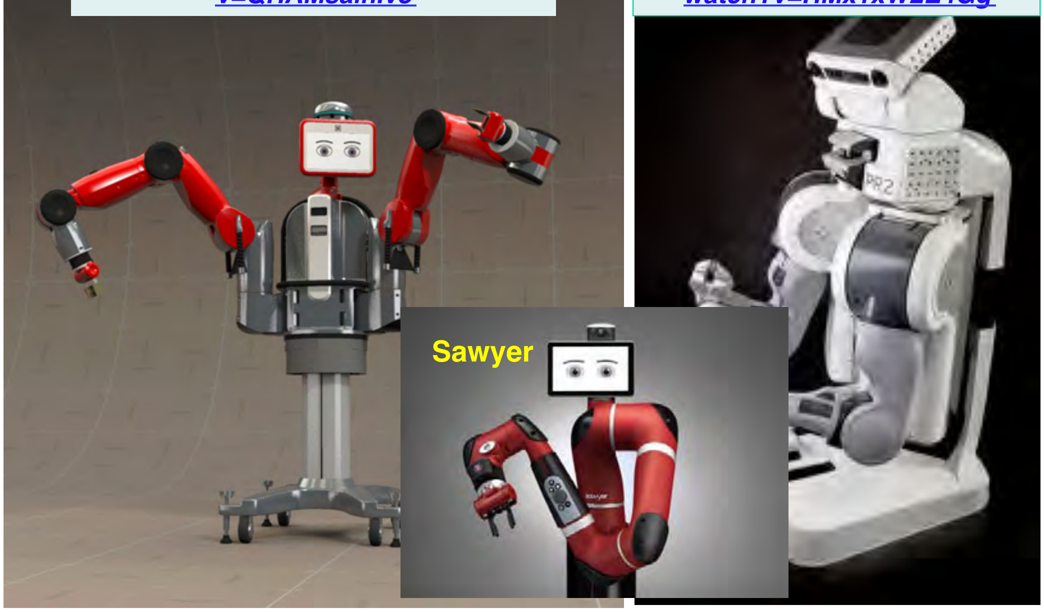
Robonaut
http://www.youtube.com/watch?v=g3u48T4Vx7k

Disney Audio-Animatronics, 1967

Baxter, Sawyer, and the PR2

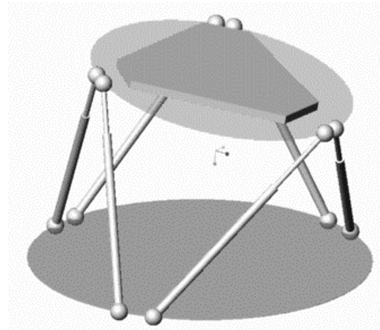
Baxter
http://www.youtube.com/watch?v=QHAMsalhlv8

PR2
http://www.youtube.com/watch?v=HMx1xW2E4Gg



Parallel Robotic Mechanisms

- **End plate is directly actuated by multiple links and joints** (*kinematic chains*)
  - Restricted workspace
  - Common link-joint configuration
  - Light construction
  - Stiffness
  - High load-carrying capacity

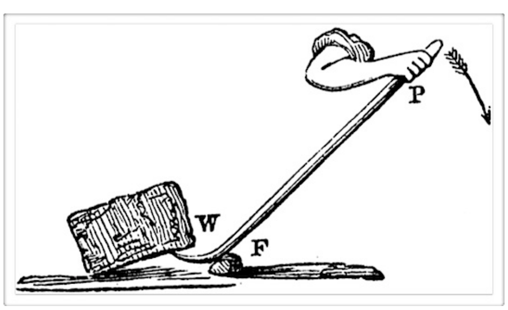


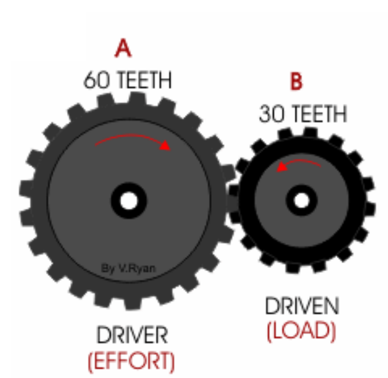
Stewart Platform
http://www.youtube.com/watch?v=QdKo9PYwGaU

Pick-and-Place Robot
http://www.youtube.com/watch?v=i4oBExI2KiQ

Gearing and Leverage

Force multiplication
Displacement ratios

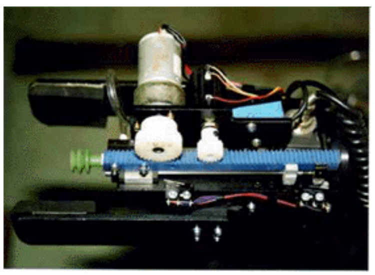


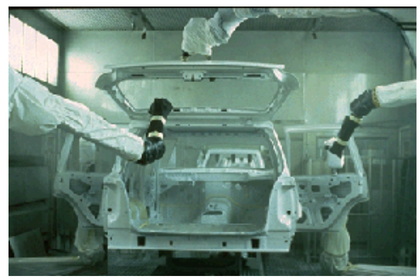


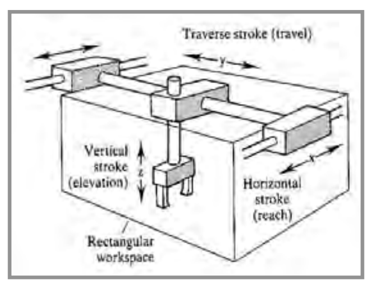
End Effecters

- **Machine tools**
  - Grinding, sanding
  - Inserting screws
  - Drilling
  - Hammering
- **Paint sprayer**
- **Gripper, clamp**
- **Multi-digit hand**

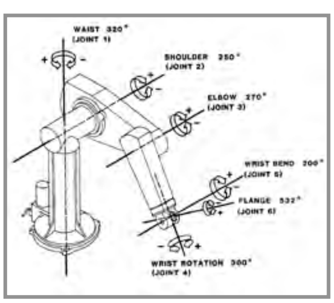
DARPA Prosthetic Hand
http://www.youtube.com/watch?v=QJg9igTnjlo&feature=related





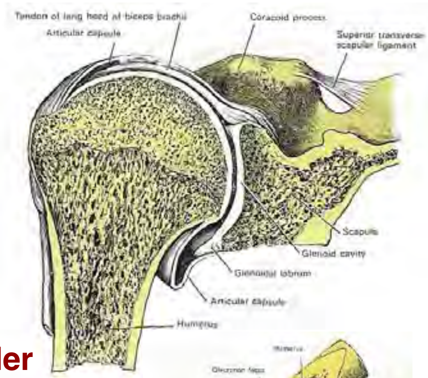


Links and Joints



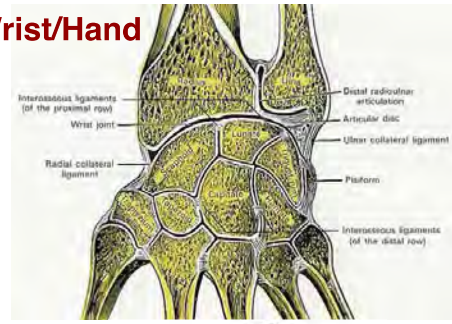
Human Joints

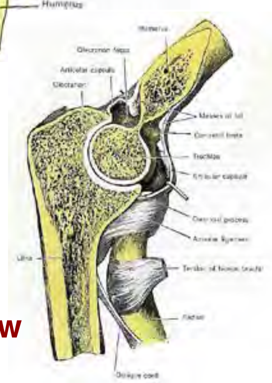
Gray's Anatomy, 1858



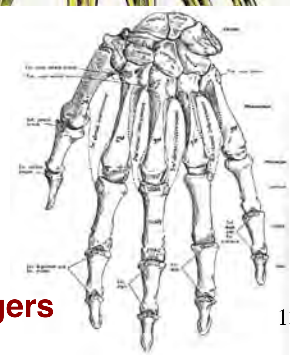
Shoulder

Wrist/Hand



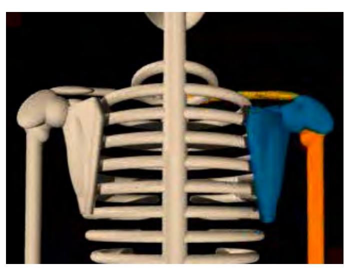


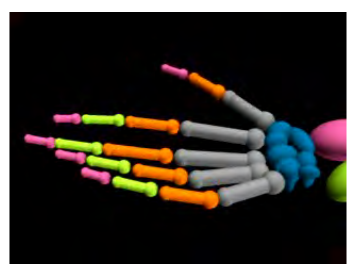
Elbow

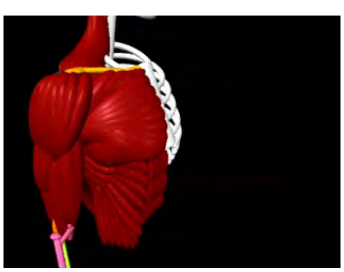


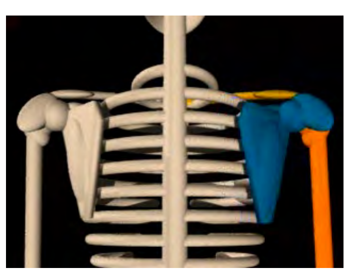
Hand/Fingers

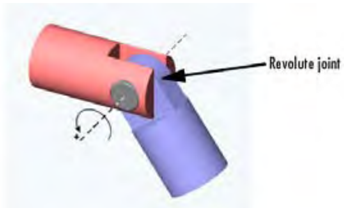
Skeleton and Muscle-Induced Motion







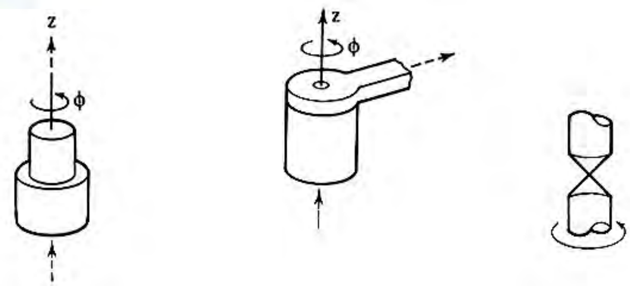




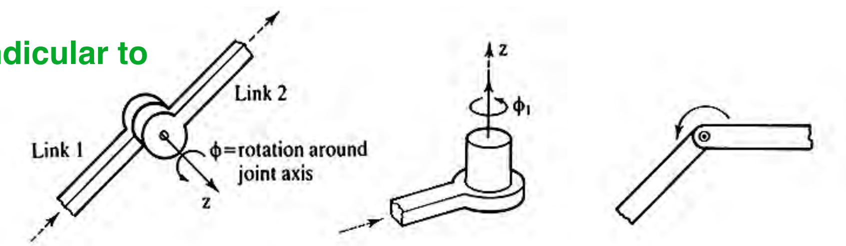
Revolute Robotic Joints

Rotation about a single axis

Parallel to Link

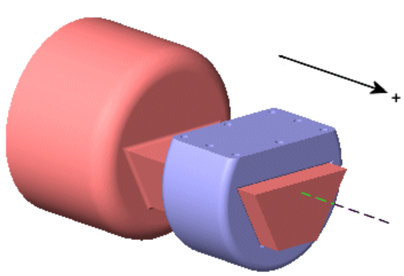


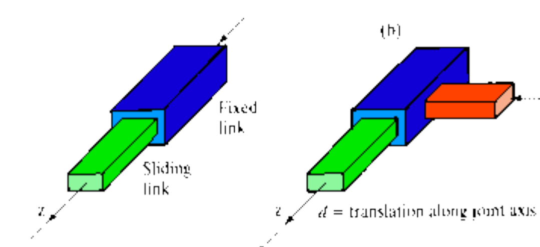
Perpendicular to Link



Prismatic Robotic Joints

Sliding along a single axis



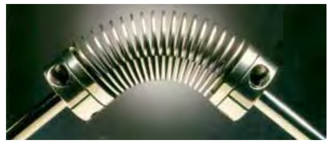


Other Robotic Joints

Universal

Constant-Velocity

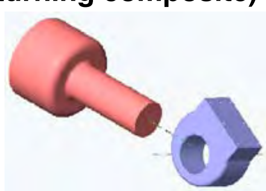
Flexible



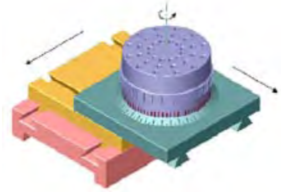
Spherical (or ball)

Roller Screw

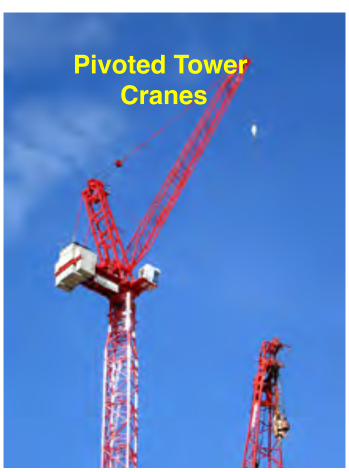
Cylindrical (sliding and turning composite)

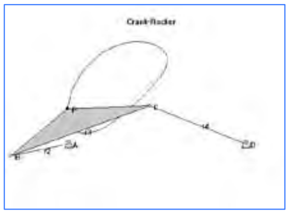


Planar (sliding and turning composite)

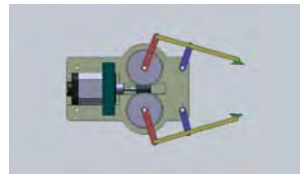


Construction Cranes

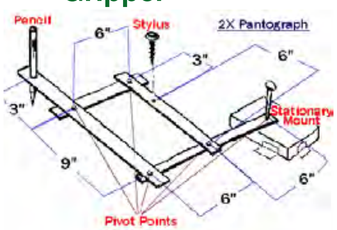




Four-Bar Linkage



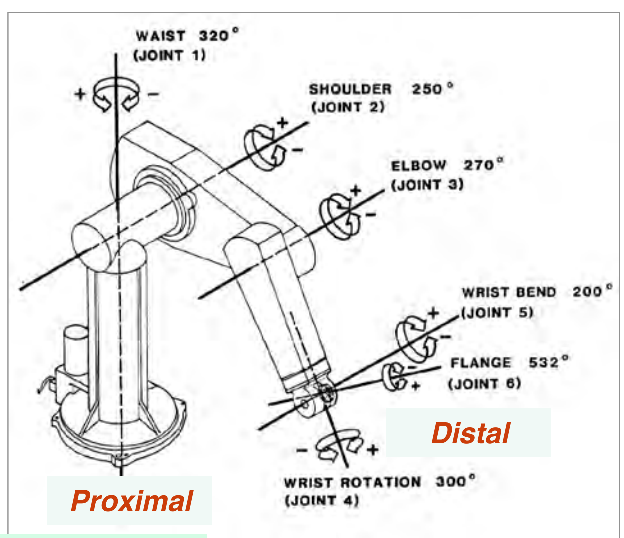
- Closed-loop structure
- Rotational joints
- Planar motion
- Proportions of link lengths determine pattern of motion
- Examples
  - Double wishbone suspension
  - Pantograph
  - Scissor lift
  - Gripper



Characteristic Transformation of a Link

Link: solid structure between two joints

- **Each link type** has a **characteristic transformation matrix** relating the proximal joint to the distal joint
- **Link *n* has**
  - Proximal end: Joint *n*, coordinate frame *n* – 1
  - *Distal end*: Joint *n* + 1, coordinate frame *n*



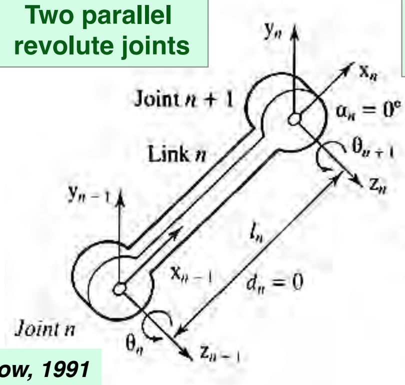
McKerrow, 1991

Links Between Revolute Joints

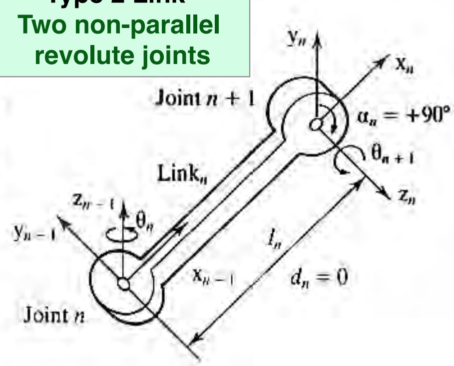
- **Link: solid structure between two joints**
  - **Proximal end: closer to the base**
  - **Distal end: farther from the base**

- **<u>4 Link Parameters</u>**
  - **Length of the link between rotational axes, *l*, along the common normal**
  - **Twist angle between axes, α**
  - **Angle between 2 links, θ (revolute)**
  - **Offset between links, *d* (prismatic)**
- **<u>Joint Variable</u>: single <u>link parameter</u> that is free to vary**

Type 1 Link
Two parallel revolute joints



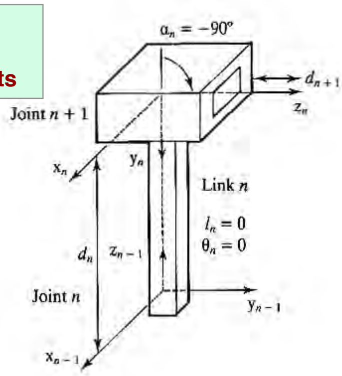
Type 2 Link
Two non-parallel revolute joints

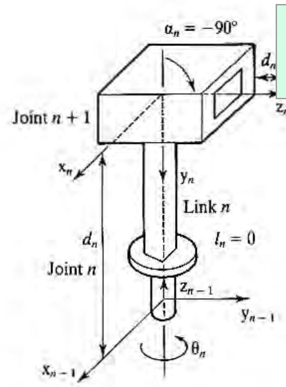


McKerrow, 1991

Links Involving Prismatic Joints

Type 5 Link
Intersecting prismatic joints





Type 6 Link
Intersecting revolute and prismatic joints

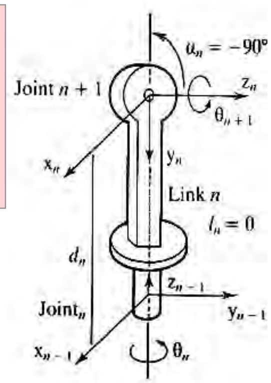
- **Link *n* extends along z_{n-1} axis**
  - **$l_n = 0$, along x_{n-1}**
  - **d_n = length, along z_{n-1} (variable)**
  - **$\theta_n = 0$, about z_{n-1}**
  - **α_n = fixed orientation of $n + 1$ prismatic axis about x_{n-1}**

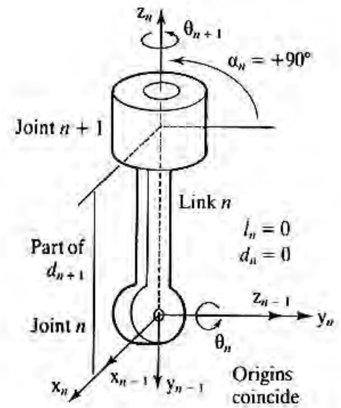
- **Link *n* extends along z_{n-1} axis**
  - **$l_n = 0$, along x_{n-1}**
  - **d_n = length, along z_{n-1} (fixed)**
  - **θ_n = variable joint angle *n* about z_{n-1}**
  - **α_n = fixed orientation of $n + 1$ prismatic axis about x_{n-1}**

McKerrow, 1991

Links Between Revolute Joints - 2

Type 3 Link
Two revolute joints with intersecting rotational axes (e.g., shoulder)





Type 4 Link
Two perpendicular revolute joints with common origin (e.g., elbow-wrist)

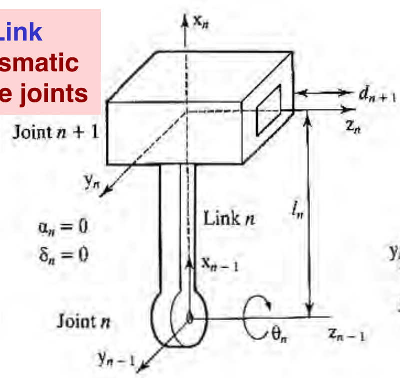
- Link n extends along z_{n-1} axis
  - $l_n = 0$, along x_{n-1}
  - d_n = length, along z_{n-1} (fixed)
  - θ_n = variable joint angle n about z_{n-1}
  - α_n = fixed orientation of $n + 1$ rotational axis about x_{n-1}

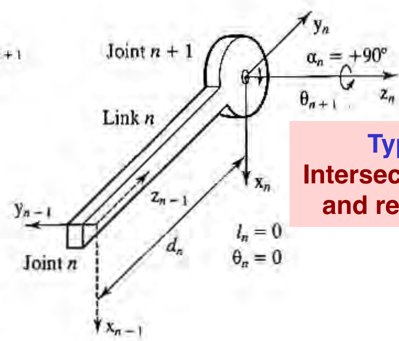
- Link n extends along $-z_n$ axis
  - $l_n = 0$, along x_{n-1}
  - $d_n = 0$, along z_{n-1}
  - θ_n = variable joint angle n about z_{n-1}
  - α_n = fixed orientation of $n + 1$ rotational axis about x_{n-1}

McKerrow, 1991

Links Involving Prismatic Joints - 2

Type 7 Link
Parallel prismatic and revolute joints



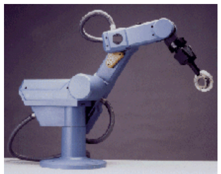


Type 8 Link
Intersecting prismatic and revolute joints

- Link n extends along x_{n-1} axis
  - l_n = length along x_{n-1}
  - $d_n = 0$, along z_{n-1}
  - θ_n = variable joint angle n about z_{n-1}
  - $\alpha_n = 0$, orientation of $n + 1$ prismatic axis about x_{n-1}

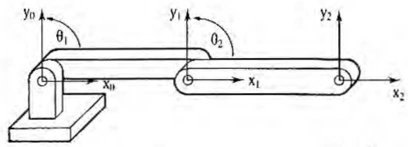
- Link n extends along z_{n-1} axis
  - $l_n = 0$, along x_{n-1}
  - d_n = length, along z_{n-1} (variable)
  - $\theta_n = 0$, about z_{n-1}
  - α_n = fixed orientation of $n + 1$ rotational axis about x_{n-1}

McKerrow, 1991

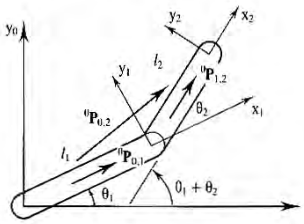


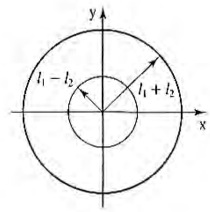
Two-Link/Three-Joint Manipulator

Parallel Rotation Axes



Manipulator in zero position





Workspace

Assignment of coordinate frames

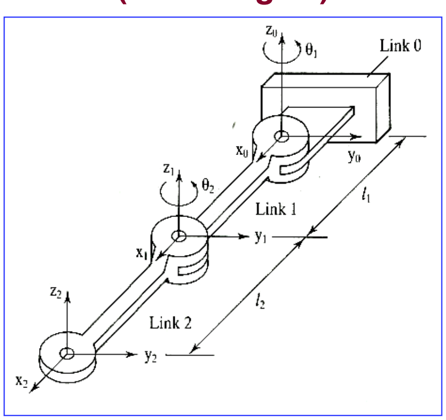
Parameters and Variables for 2-link manipulator

- Link lengths (fixed)
- Joint angles (variable)

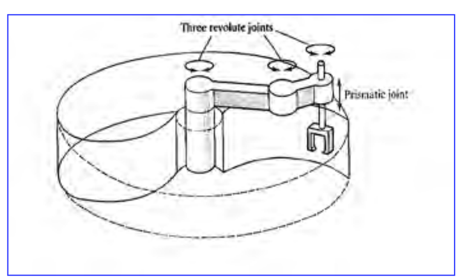
McKerrow, 1991

Four-Joint (SCARA*) Manipulator

Arm with Three Revolute Link Variables (Joint Angles)



Operation
http://www.youtube.com/watch?v=3-sbtCCyJXo



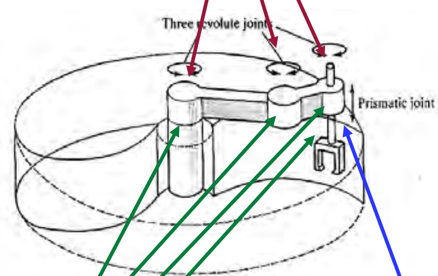
McKerrow, 1991

**Selective Compliant Articulated Robot Arm*

Joint Variables Must Be Actuated and Observed for Control

- Frames of Reference for Actuation and Control
  - World coordinates
  - Actuator coordinates
  - Joint coordinates
  - Tool coordinates

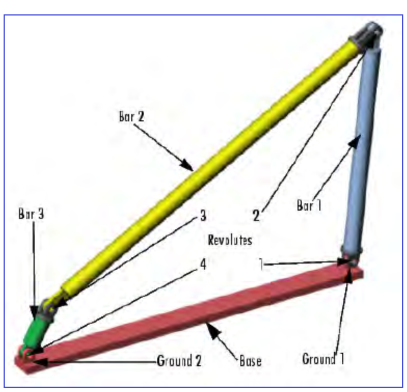
Rotary Actuators

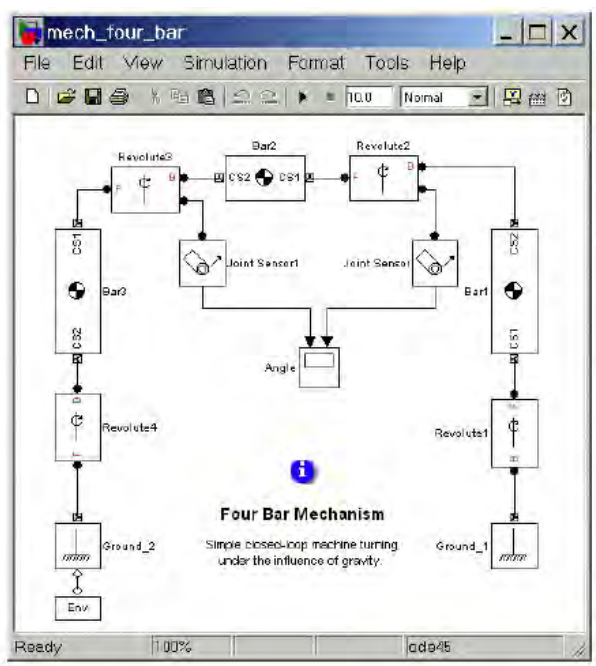


Sensors May Observe Joints Directly, Indirectly, or Not At All

Linear Actuator

Simulink/SimMechanics Representation of Four-Bar Linkage

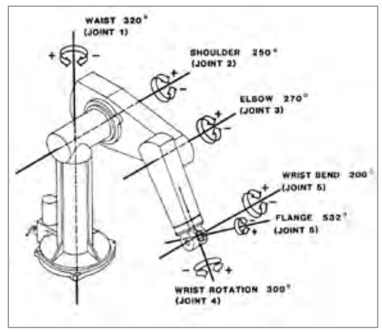




Recall: Homogeneous Transformation

$$\mathbf{s}_{new} = \left[\begin{array}{c:c} \begin{pmatrix}\text{Rotation}\\ \text{Matrix}\end{pmatrix}_{old}^{new} & \begin{pmatrix}\text{Location}\\ \text{of Old}\\ \text{Origin}\end{pmatrix}_{new} \\ \hdashline \begin{pmatrix}0 & 0 & 0\end{pmatrix} & 1 \end{array}\right] \mathbf{s}_{old} = \mathbf{A}_{old}^{new}\ \mathbf{s}_{old}$$

Transform from one joint to the next

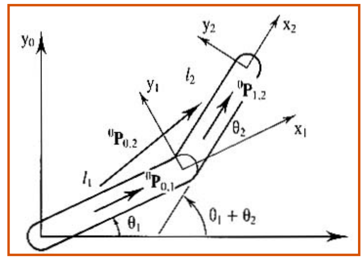


Rotation Matrix can be Derived from Euler Angles or Quaternions

$$\mathbf{A} = \left[\begin{array}{c:c} \mathbf{H}_{old}^{new} & \mathbf{r}_{old_{new}} \\ \hdashline \begin{pmatrix}0 & 0 & 0\end{pmatrix} & 1 \end{array}\right] = \left[\begin{array}{ccc:c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

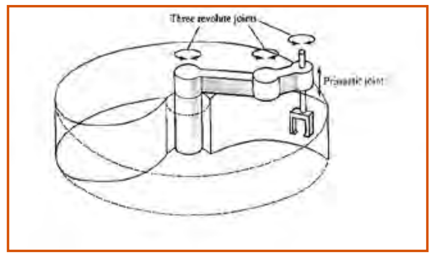
Series of Homogeneous Transformations

Two serial transformations can be combined in a single transformation



$$\mathbf{s}_2 = \mathbf{A}_1^2 \mathbf{A}_0^1 \ \mathbf{s}_0 = \mathbf{A}_0^2 \ \mathbf{s}_0$$

Four transformations for SCARA robot

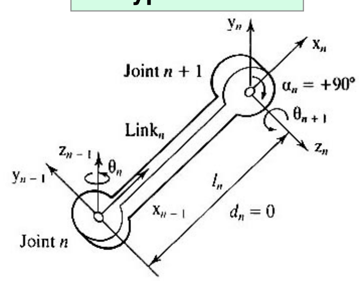


$$\mathbf{s}_4 = \mathbf{A}_3^4 \mathbf{A}_2^3 \mathbf{A}_1^2 \mathbf{A}_0^1 \ \mathbf{s}_0 = \mathbf{A}_0^4 \ \mathbf{s}_0$$

Transformation for a Single Robotic Joint-Link

- Each joint-link requires four sequential transformations:
  - Rotation about α
  - Translation along d
  - Translation along l
  - Rotation about θ

Type 2 Link



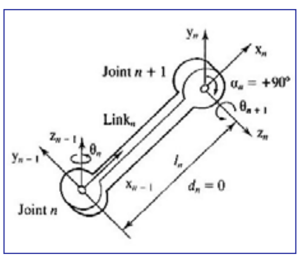
$$\mathbf{s}_{n+1} = \mathbf{A}_3^{n+1} \mathbf{A}_2^3 \mathbf{A}_1^2 \mathbf{A}_n^1 \ \mathbf{s}_n = \mathbf{A}_n^{n+1} \ \mathbf{s}_n$$
$$= \mathbf{A}_\theta \mathbf{A}_d \mathbf{A}_l \mathbf{A}_\alpha \ \mathbf{s}_n = \mathbf{A}_n^{n+1} \ \mathbf{s}_n$$

- ... axes for each transformation (along or around) must be specified

4th 3rd 2nd 1st

$$\mathbf{s}_{n+1} = \mathbf{A}\left(z_{n-1},\theta_n\right)\mathbf{A}\left(z_{n-1},d_n\right)\mathbf{A}\left(x_{n-1},l_n\right)\mathbf{A}\left(x_{n-1},\alpha_n\right)\mathbf{s}_n = \mathbf{A}_n^{n+1} \ \mathbf{s}_n$$

Denavit-Hartenberg Representation of Joint-Link-Joint Transformation



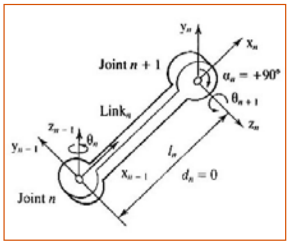
Denavit-Hartenberg Representation of Joint-Link-Joint Transformation

- Like Euler angle rotation, transformational effects of the 4 link parameters are defined in a **specific application sequence (right to left): {θ, d, l, α}**

- **4 link parameters**
  - Angle between 2 links, θ (revolute)
  - Distance (offset) between links, d (prismatic)
  - Length of the link between rotational axes, l, along the common normal (prismatic)
  - Twist angle between axes, α (revolute)

$$\mathbf{A}_n = \mathbf{A}(z_{n-1},\theta_n)\mathbf{A}(z_{n-1},d_n)\mathbf{A}(x_{n-1},l_n)\mathbf{A}(x_{n-1},\alpha_n)$$
$$= \mathrm{Rot}(z_{n-1},\theta_n)\ \mathrm{Trans}(z_{n-1},d_n)\ \mathrm{Trans}(x_{n-1},l_n)\ \mathrm{Rot}(x_{n-1},\alpha_n)$$
$$\triangleq {}^{n}\mathbf{T}_{n+1} \quad \text{in some references (e.g., McKerrow, 1991)}$$

Denavit-Hartenberg Demo
http://www.youtube.com/watch?v=10mUtjfGmzw



Four Transformations from One Joint to the Next

(Single Link)

Rotation of θ_n about the z_{n-1} axis

$$\mathrm{Rot}(z_{n-1},\theta_n)=\begin{bmatrix}\cos\theta_n & -\sin\theta_n & 0 & 0\\ \sin\theta_n & \cos\theta_n & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Translation of d_n along the z_{n-1} axis

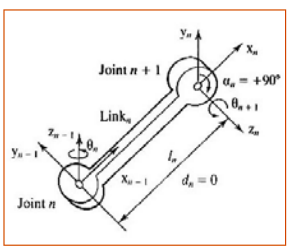
$$\mathrm{Trans}(z_{n-1},d_n)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d_n\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Translation of l_n along the x_{n-1} axis

$$\mathrm{Trans}(x_{n-1},l_n)=\begin{bmatrix}1 & 0 & 0 & l_n\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rotation of α_n about the x_{n-1} axis

$$\mathrm{Rot}(x_{n-1},\alpha_n)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\alpha_n & -\sin\alpha_n & 0\\ 0 & \sin\alpha_n & \cos\alpha_n & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

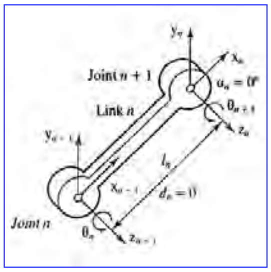


Denavit-Hartenberg Representation of Joint-Link-Joint Transformation

Then **Then** **Then** **First**

$$\mathbf{A}_n^{n-1}=\begin{bmatrix}\cos\theta_n & -\sin\theta_n & 0 & 0\\ \sin\theta_n & \cos\theta_n & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d_n\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & l_n\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\alpha_n & -\sin\alpha_n & 0\\ 0 & \sin\alpha_n & \cos\alpha_n & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\mathbf{A}_n^{n-1}=\begin{bmatrix}\cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & l_n\cos\theta_n\\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & l_n\sin\theta_n\\ 0 & \sin\alpha_n & \cos\alpha_n & d_n\\ 0 & 0 & 0 & 1\end{bmatrix}$$



Example: Joint-Link-Joint Transformation, Type 1 Link

Joint Variable = θ_n

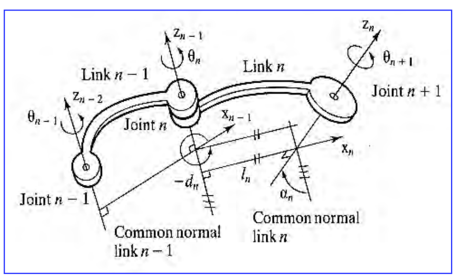
$$\mathbf{A}_n^{n-1} = \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & l_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & l_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \theta &= \text{variable} \\ d &= 0 \text{ m} \\ l &= 0.25 \text{ m} \\ \alpha &= 90 \text{ deg} \end{aligned}$$

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} \cos\theta_n & 0 & \sin\theta_n & 0.25\cos\theta_n \\ \sin\theta_n & 0 & -\cos\theta_n & 0.25\sin\theta_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \theta &\triangleq 30 \text{ deg} \\ d &= 0 \text{ m} \\ l &= 0.25 \text{ m} \\ \alpha &= 90 \text{ deg} \end{aligned}$$

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} 0.866 & 0 & 0.5 & 0.217 \\ 0.5 & 0 & -0.866 & 0.125 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Forward and Inverse Transformations

Forward transformation: proximal to distal frame (Expression of proximal frame in distal frame)

$$\mathbf{s}_1 = \mathbf{A}_0^1\mathbf{s}_0 \quad ; \quad \mathbf{s}_2 = \mathbf{A}_1^2\mathbf{s}_1 = \mathbf{A}_1^2\mathbf{A}_0^1\mathbf{s}_0 = \mathbf{A}_0^2\mathbf{s}_0$$

Reverse transformation: distal to proximal frame = inverse of forward transformation

$$\mathbf{s}_0 = \left(\mathbf{A}_0^2\right)^{-1}\mathbf{s}_2 = \mathbf{A}_2^0\mathbf{s}_2 = \mathbf{A}_1^0\mathbf{A}_2^1\mathbf{s}_2$$

Homogeneous Transformation Matrix is not Orthonormal

$$\boxed{\mathbf{A}_2^0 = \left(\mathbf{A}_0^2\right)^{-1} \neq \left(\mathbf{A}_0^2\right)^T}$$

…but a useful identity makes inversion simple

Matrix Inverse Identity

Given: a square matrix, A, and its inverse, B

$$\mathbf{A} = \left[\begin{array}{c|c} \underset{m\times m}{\mathbf{A}_1} & \underset{m\times n}{\mathbf{A}_2} \\ \hline \underset{n\times m}{\mathbf{A}_3} & \underset{n\times n}{\mathbf{A}_4} \end{array}\right] \quad ; \quad \mathbf{B} \triangleq \mathbf{A}^{-1} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix}$$

Then

$$\mathbf{A}\,\mathbf{B} = \mathbf{A}\,\mathbf{A}^{-1} = \mathbf{I}_{m+n}$$

$$= \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_n \end{bmatrix} = \begin{bmatrix} \left(\mathbf{A}_1\mathbf{B}_1 + \mathbf{A}_2\mathbf{B}_3\right) & \left(\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_4\right) \\ \left(\mathbf{A}_3\mathbf{B}_1 + \mathbf{A}_4\mathbf{B}_3\right) & \left(\mathbf{A}_3\mathbf{B}_2 + \mathbf{A}_4\mathbf{B}_4\right) \end{bmatrix}$$

Equating like parts, and solving for $\mathbf{B}_i$

$$\begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} = \left[\begin{array}{c|c} \left(\mathbf{A}_1 - \mathbf{A}_2\mathbf{A}_4^{-1}\mathbf{A}_3\right)^{-1} & -\mathbf{A}_1^{-1}\mathbf{A}_2\left(\mathbf{A}_4 - \mathbf{A}_3\mathbf{A}_1^{-1}\mathbf{A}_2\right)^{-1} \\ \hline -\mathbf{A}_4^{-1}\mathbf{A}_3\left(\mathbf{A}_1 - \mathbf{A}_2\mathbf{A}_4^{-1}\mathbf{A}_3\right)^{-1} & \left(\mathbf{A}_4 - \mathbf{A}_3\mathbf{A}_1^{-1}\mathbf{A}_2\right)^{-1} \end{array}\right]$$

Apply to Homogeneous Transformation

Forward transformation (to distal frame)

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{proximal}^{distal} & \mathbf{r}_{o_{proximal}} \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{bmatrix}$$

Inverse transformation (to proximal frame)

$$\begin{bmatrix} \mathbf{H}_{proximal}^{distal} & \mathbf{r}_o \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} = \left[\begin{array}{c:c} \mathbf{H}_{distal}^{proximal} & -\mathbf{H}_{distal}^{proximal}\mathbf{r}_{o_{distal}} \\ \hdashline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array}\right]$$

Apply to Homogeneous Transformation

Forward transformation

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} == \left[\begin{array}{ccc:c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Inverse transformation

$$\left[\begin{array}{ccc:c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]^{-1} = \left[\begin{array}{ccc:c} h_{11} & h_{21} & h_{31} & -\left(h_{11}x_o + h_{21}y_o + h_{31}z_o\right) \\ h_{12} & h_{22} & h_{32} & -\left(h_{12}x_o + h_{22}y_o + h_{32}z_o\right) \\ h_{13} & h_{23} & h_{33} & -\left(h_{13}x_o + h_{23}y_o + h_{33}z_o\right) \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Next Time:
Transformations, Trajectories, and Path Planning

Supplemental Material

Multi-Bar-Linkage Gripper

http://www.youtube.com/watch?v=YDd6VBx9oqU&feature=related