Level A Investigations

## A-I Speed on the Roller Coaster

How do hills and valleys affect motion?
The objective of this Investigation is for students to learn to setup the roller coaster and measure the speed of a marble as it rolls along the track. They practice their skills by making and testing a prediction about where the speed of the marble is fastest on the roller coaster. In performing the Investigation, students begin to develop their ideas about how hills and valleys affect motion.

## A-2 Height on the Roller Coaster

How is height related to the speed of the marble on a roller coaster?
In this Investigation, students discover that the speed and height of the marble on a roller coaster are inversely related. The higher the marble, the slower it moves. The lower the marble, the faster it moves. Neither the angle of the hill nor the direction the marble is moving (up or down) appreciably affect the speed of the marble. Students arrive at their understanding by creating and interpreting a graph using the data they collect.

## Level B Investigations

## B-I Energy and the Roller Coaster

How is motion on a roller coaster related to energy?
In this B-level Investigation, students discover that the speed and height of the marble on the roller coaster are inversely related. The higher the marble, the slower it moves. The lower the marble, the faster it moves. The speed of the marble is not strongly affected by the angle of the hill or the direction the marble is moving (up or down). Students use a graph to compare height and speed data. The concepts potential energy, kinetic energy, and the conservation of energy are introduced.

## B-2 Conservation of Energy

Is energy transformation from potential to kinetic $100 \%$ ?
Energy is one of the most fundamental quantities in the universe, yet it cannot be directly touched or seen. The purpose of this Investigation is to learn how potential energy and kinetic energy relate to each other according to the law of conservation of energy. A roller coaster is a good example of a system that operates on both potential and kinetic energy. Using a roller coaster, students explore how energy is transformed from one form to another and how efficient this transformation is.

## B-3 Mass and Motion

How does mass influence the motion of a marble on a roller coaster?
In this Investigation, students compare the motion of a steel marble on the roller coaster to the motion of a plastic marble. Students discover that their expectations for how the two marbles compare may not match their observations. The speed values are close with the exception of the values for the top of the hill (position 5). From their observations, students develop theories to explain how mass influences the energy and motion of a marble on a roller coaster.

## Level C Investigations

## C-I Motion on the Roller Coaster <br> How do you predict the speed of a marble on a roller coaster?

The motion on the roller coaster track is interesting because the speed and the acceleration of the marble are constantly changing. In this Investigation, students learn to predict the speed of the marble using the law of conservation of energy and the equations for potential and kinetic energy. Students explore how energy is transformed from one form to another on the roller coaster as they compare their predicted values to measured values for speed.

## C-2 Rotational Kinetic Energy

How can you calculate the rolling energy of a marble?
A marble on the roller coaster uses two kinds of motion to get from the beginning to the end of the track. Translational motion is the forward or linear motion of the marble. Thus far in working with the roller coaster, calculations and descriptions of the marble's motion have only considered translational motion. The rolling of the marble along the track is a second kind of motion used by the marble. In this Investigation, students learn to measure and calculate the speed and kinetic energy of an object that has rotational motion.

## C-3 Mass, Motion, and Energy

How does mass influence the motion of a marble on a roller coaster?
In this C-level Investigation, students compare the motion of a steel marble on the roller coaster to the motion of a plastic marble. Students discover that their expectations for how the two marbles compare may not match their observations. The speed values are close with the exception of the values for the top of the hill (position 5). Using the law of conservation of energy, students develop a theory to explain how mass influences the marbles' motion.

In this Investigation, you will:

1. Derive an equation for the rotational kinetic energy of a marble.
2. Calculate the translational and rotational kinetic energies of a rolling marble.
3. Compare the potential energy of the marble to the sum of its kinetic energies.
4. Make and interpret a graph of the energy of the marble.

A marble on the roller coaster uses two kinds of motion to get from the beginning to the end of the track. Translational motion is the forward or linear motion of the marble. Thus far in working with the roller coaster, your calculations and descriptions of the marble's motion have only considered translational motion. The rolling of the marble along the track is the second kind of motion that the marble uses. In this Investigation, you will learn to measure and calculate the speed and kinetic energy of an object that has rotational motion.


## Getting started

Describing the motion of an object often includes providing the speed of the object. When describing and measuring translational motion, speed is represented by the variable, $v$. The speed of an object that has rotational motion is called angular speed and is represented with the variable, $\omega$ (omega).

The following equation relates the angular and linear (translational) speeds of a sphere of radius $r$ rolling on one edge.

Translational speed $=$ Angular speed $\times$ the radius of the rolling object

$$
v=\omega r
$$

The kinetic energy $\left(E_{K r}\right)$ of a rigid, rolling object can be calculated using the angular speed $(\omega)$ and the moment of inertia ( $I$ ) of the object.

$$
E_{K r}=\frac{1}{2} I \omega^{2}
$$

The moment of inertia of an object is the term that describes the mass of a rotating object. Moment of inertia $(I)$ is equal to two-fifths times the mass $(m)$ of the object times the square of the distance $(r)$. This mass is from the axis of rotation. See the diagram on the next page.
a. Use the equations on the previous page and the moment of inertia for a sphere to derive an equation for the rotational kinetic energy $\left(E_{K r}\right)$ in terms of mass $(m)$ and the linear speed $(v)$. This derived equation allows you to calculate rotational kinetic energy when you only know mass and linear speed.
b. Write down the equations for potential energy and translational (linear) kinetic energy in Table 1.
c. Using the energy equations ( $E_{K r}, E_{P}$ and $E_{K t}$ ) and the law of conservation of energy, derive a new equation that predicts the speed of the marble from the
$\mathrm{I}=\frac{\mathbf{2}}{5} \mathrm{mr}^{2}$

sphere of radius $r$ rotated about a major diameter acceleration of gravity, the initial height of the marble on the track, and the height at any later time.

Table I: Potential and Kinetic Energy Equations

## Potential Energy ( $E_{p}$ )

Translational Kinetic Energy ( $E_{K t}$ )

2 Comparing predicted and calculated speeds
a. Fill in the first four columns in Table 2 below with the data from C-1 Motion on the Roller Coaster (Table 2 in Part 6). Do not fill in the predicted speed values from $C-1$.
b. Using the equation from Part 1, question c, calculate predicted speed. Record these values in Table 2.
c. Compare the predicted speed values you just calculated and the predicted speeds from Investigation C-1 Motion on the Roller Coaster. Which predicted values might be more accurate? Why?

Table 2: Comparing Predicted and Calculated Speed

| Position on <br> roller coaster <br> $(\mathrm{m})$ | Height <br> $(\mathrm{m})$ | Time from <br> photogate $\mathbf{A}$ <br> $(\mathrm{sec})$ | Measured <br> speed <br> $(\mathrm{m} / \mathrm{sec})$ | Predicted <br> speed <br> $(\mathrm{m} / \mathrm{sec})$ |
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The predicted speed values in Table 2 take into consideration that the marble has rotational motion. Now, make a graph that shows the relationship between the measured speeds and these predicted speed values. Place predicted speed on the $x$-axis and measured speed on the $y$-axis.

## Analyzing the data

a. What can you tell from your graph? Write down one to two sentences that describe how the measured speed compares to the theoretical prediction. Your answer should contain a numerical estimate of the differences between the two values expressed as a percentage. Were the values more similar in this Investigation than they were in C-1 Motion on the Roller Coaster?
b. Is there something that is still not included in the theory that might explain why the predicted speed values do not exactly match the measured values? Comparing energies

Using your new equation from Part 1, question c, calculate total kinetic energy (translation plus rotation). Then, calculate the potential energy and total energy (potential plus total kinetic). Record your calculations in Table 3.

Table 3: Potential, Total Kinetic, and Total Energy

| Position on <br> roller coaster <br> $(\mathrm{m})$ | Height <br> $(\mathrm{m})$ | Speed <br> $(\mathrm{m} / \mathrm{sec})$ | Potential <br> energy <br> (joules) | Total kinetic <br> energy <br> (joules) | Total energy <br> (joules) |
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Use your energy data to plot a graph showing three parameters-potential energy, kinetic energy, and total energy-plotted against distance along the roller coaster. Use a solid line for potential energy, a dotted line for kinetic energy, and a dashed line for total energy. Also, make a legend indicating what each line represents.

The example shows what this graph might look like for the first few points:

Energy vs. Distance Along the Rollercoaster


## 1 Analyzing the data

a. From your graph, what can you say about the energy of the marble? Write down one to two sentences that describe the different types of energy the marble has at different places along the roller coaster.
b. What percentage of the marble's energy is left at the end of the roller coaster just before it stops (or at the last place you measured)?
c. What happens to the lost energy of the marble?

## 8 Comparing results

In this Investigation, you determined the total kinetic energy of the marble by adding the translational and rotational kinetic energies. Your handout includes comparable data, but rotational kinetic energy was not included in the calculations for this data set.

Write a paragraph that compares your data and graph made using total kinetic energy with the data and graph your teacher gave you. In your paragraph, state the advantages of including total kinetic energy (translational plus rotational) and the advantages of only including translational in your calculations. In other words, is it always necessary to include rotational kinetic energy in your calculations? Explain your responses to these questions.

## Rotational Kinetic Energy

## Question: How can you calculate the rolling energy of a marble?

## 1 Getting started

a. Use the equations and the moment of inertia for a sphere to derive an equation for the rotational kinetic energy $\left(E_{K r}\right)$ in terms of mass $(m)$ and the linear speed $(v)$. This derived equation allows you to calculate rotational kinetic energy when you only know mass and linear speed.
b. Write down the equations for potential energy and translational (linear) kinetic energy in Table 1.
c. Using the energy equations ( $E_{K r}, E_{B}$ and $E_{K t}$ ) and the law of conservation of energy, derive a new equation that predicts the speed of the marble from the acceleration of gravity, the initial height of the marble on the track, and the height at any later time.

Table I: Potential and Kinetic Energy Equations

\section*{| Potential Energy (EP) | Translatio |
| :--- | :---: |
|  |  |
| paring predicted and calculated speeds |  |}

a. Fill in the first four columns in Table 2 below with the data from C-1 Motion on the Roller Coaster (Table 2 in Part 6). Do not fill in the predicted speed values from $C-1$.
b. Using the equation from Part 1, question c, calculate predicted speed. Record these values in Table 2.
c. Compare the predicted speed values you just calculated and the predicted speeds from Investigation C-1 Motion on the Roller Coaster. Which predicted values might be more accurate? Why?

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| Position on <br> roller coaster <br> $(\mathrm{m})$ | Height <br> $(\mathrm{m})$ | Time from <br> photogate $\mathbf{A}$ <br> $(\mathrm{sec})$ | Measured <br> speed <br> $(\mathrm{m} / \mathrm{sec})$ | Predicted <br> speed <br> $(\mathrm{m} / \mathrm{sec})$ |
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The predicted speed values in Table 2 take into consideration that the marble has rotational motion. Now, make a graph that shows the relationship between the measured speeds and these predicted speed values. Place predicted speed on the $x$-axis and measured speed on the $y$-axis.

Title: $\qquad$

$x$ label:

## Analyzing the data

a. What can you tell from your graph? Write down one to two sentences that describe how the measured speed compares to the theoretical prediction. Your answer should contain a numerical estimate of the differences between the two values expressed as a percentage. Were the values more similar in this Investigation than they were in C-1 Motion on the Roller Coaster?
b. Is there something that is still not included in the theory that might explain why the predicted speed values do not exactly match the measured values?

Using your new equation from Part 1, question c, calculate total kinetic energy (translation plus rotation). Then, calculate the potential energy and total energy (potential plus total kinetic). Record your calculations in Table 3.

Table 3: Potential, Total Kinetic, and Total Energy

| Position on <br> roller coaster <br> $(\mathrm{m})$ | Height <br> $(\mathrm{m})$ | Speed <br> $(\mathrm{m} / \mathrm{sec})$ | Potential <br> energy <br> (joules) | Total kinetic <br> energy <br> (joules) | Total energy <br> (joules) |
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Title: $\qquad$


$x$ label: $\qquad$

## 1 Analyzing the data

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c. What happens to the lost energy of the marble?

Write a paragraph that compares your data and graph made using total kinetic energy with the data and graph below. In your paragraph, state the advantages of including total kinetic energy (translational plus rotational) and the advantages of only including translational in your calculations. In other words, is it always necessary to include rotational kinetic energy in your calculations? Explain your responses to these questions.

## Sample data and graph:

| Position on <br> roller coaster <br> $(\mathrm{m})$ | Height <br> $(\mathrm{m})$ | Speed <br> $(\mathrm{m} / \mathrm{sec})$ | Potential <br> energy <br> (joules) | Kinetic <br> energy <br> (joules) | Total <br> energy <br> (joules) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.05 | 0.347 | 0.420 | 0.096 | 0.00249 | 0.098 |
| 0.15 | 0.293 | 1.011 | 0.080 | 0.0144 | 0.094 |
| 0.25 | 0.226 | 1.429 | 0.062 | 0.029 | 0.091 |
| 0.35 | 0.166 | 1.681 | 0.046 | 0.040 | 0.086 |
| 0.45 | 0.140 | 1.759 | 0.039 | 0.044 | 0.083 |
| 0.55 | 0.172 | 1.638 | 0.048 | 0.038 | 0.086 |
| 0.65 | 0.241 | 1.159 | 0.067 | 0.019 | 0.086 |
| 0.75 | 0.297 | 0.748 | 0.082 | 0.0079 | 0.090 |
| 0.85 | 0.308 | 0.629 | 0.085 | 0.0056 | 0.091 |
| 0.95 | 0.270 | 0.990 | 0.075 | 0.013 | 0.088 |
| 1.05 | 0.206 | 1.429 | 0.057 | 0.029 | 0.086 |
| 1.15 | 0.137 | 1.696 | 0.038 | 0.041 | 0.079 |

## Energy versus Distance Along the Roller Coaster Graph:



## Questions

I. The resistance of an object to changes in its rotational motion is called $\qquad$ .
2. By the time the marble reaches the end of the track some of the energy has been dissipated from the system as a result of $\qquad$ .
3. According to the law of conservation of energy, how does the total energy of the marble at a point that is 18 centimeters above the work surface compare to the total energy of the marble at a position that is 28 centimeters above the work surface?
4. Use your answer to the previous question to solving the following problem. The time on the timer below was made with one photogate at the top of the hill 28 centimeters above a level table. What was the speed of the marble at position 1 which is 18 centimeters above the table?

5. Bicycle wheels are commonly made using a metal rim held in place by spokes attached to a hub in the middle. Racers who compete at the Olympic and professional levels often use solid wheels that are about the same mass as spoked wheels. What is the advantage to using solid wheels?
6. A pole vaulter runs down the approach in an attempt to clear a bar 6.5 meters in height. Using the law of conservation of energy, estimate the speed required by the vaulter to clear the bar.


## Curriculum Resource Guide: Rollercoaster

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